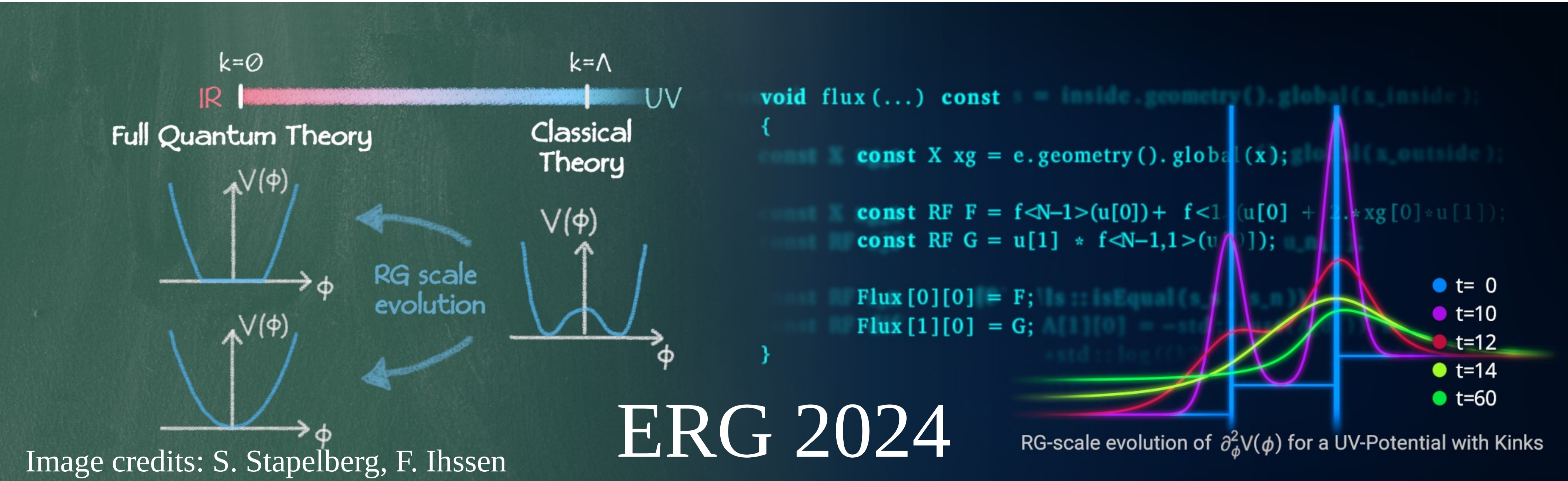
Physics-Informed Renormalisation Group flows

Friederike Ihssen

Universität Heidelberg/ETH Zürich

STRUCTURES
CLUSTER OF
EXCELLENCE



ERG 2024

Image credits: S. Stapelberg, F. Ihssen

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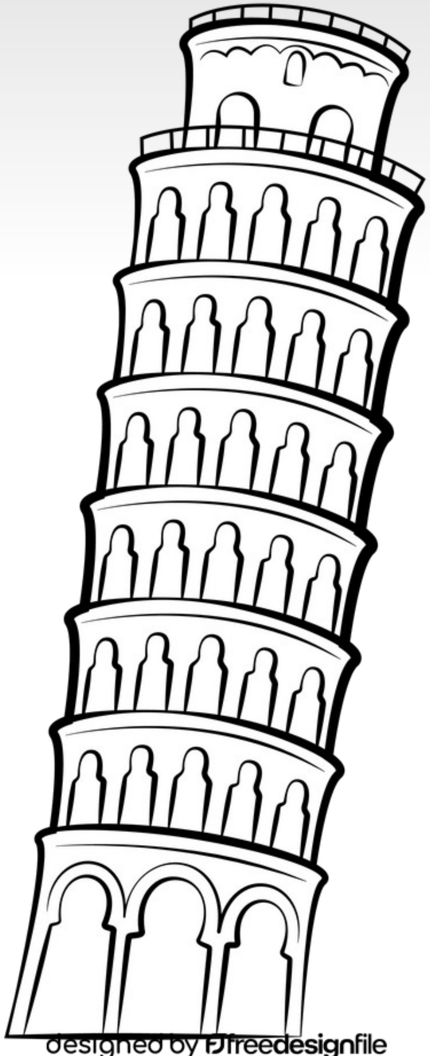

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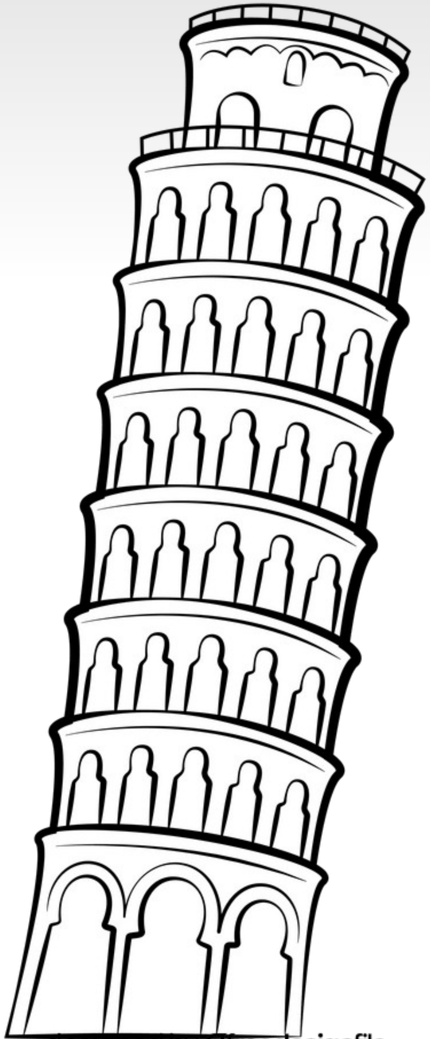
Use Physics-Informed RG to decouple the dynamics of the RG flow



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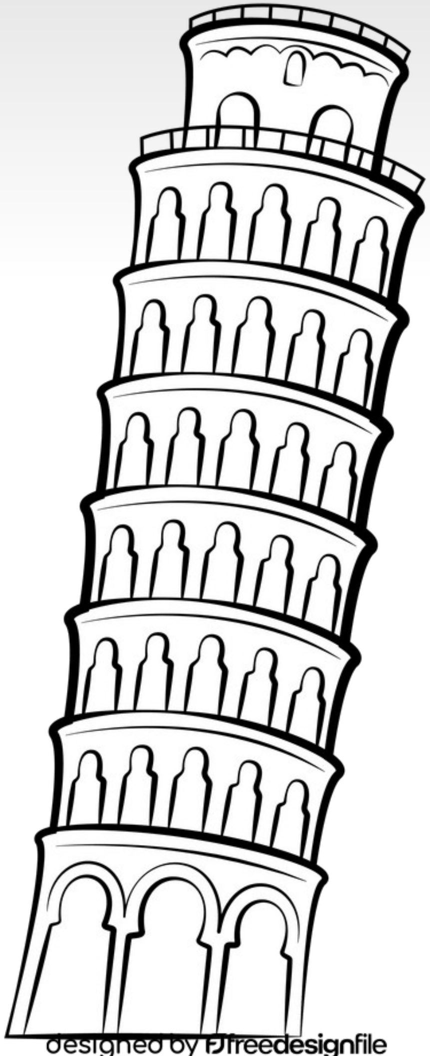
In the fRG: Choose a desired **target action** $\Gamma_T[\phi]$



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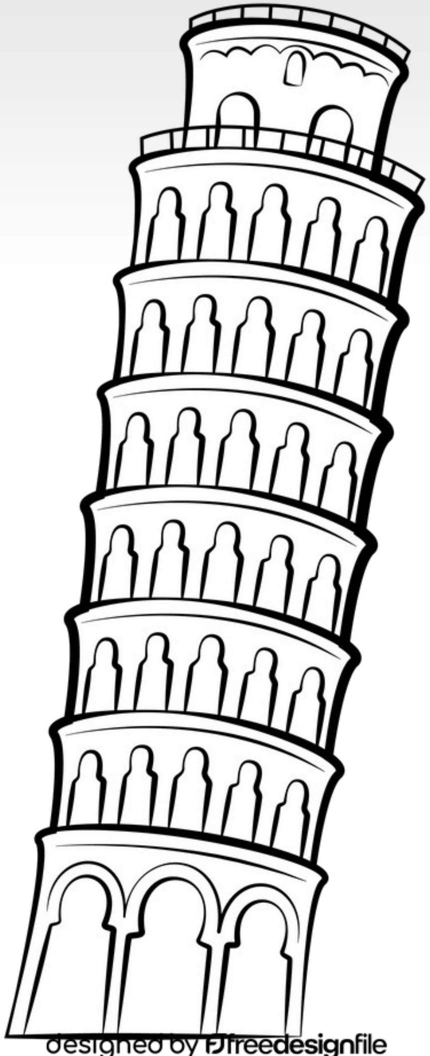
  → **Method of characteristics** for finite N



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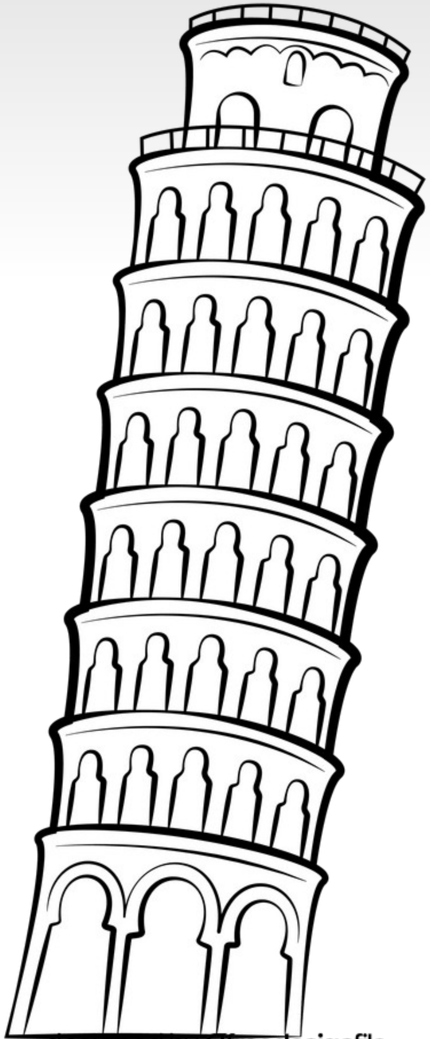

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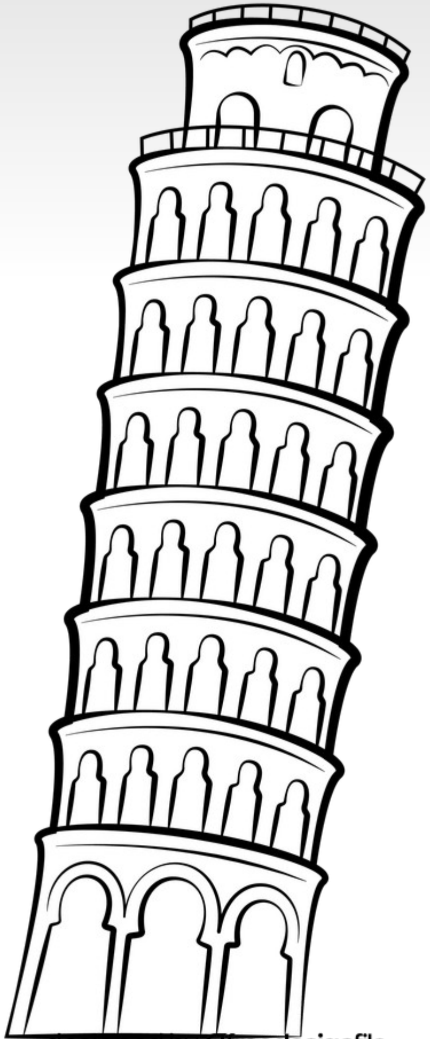

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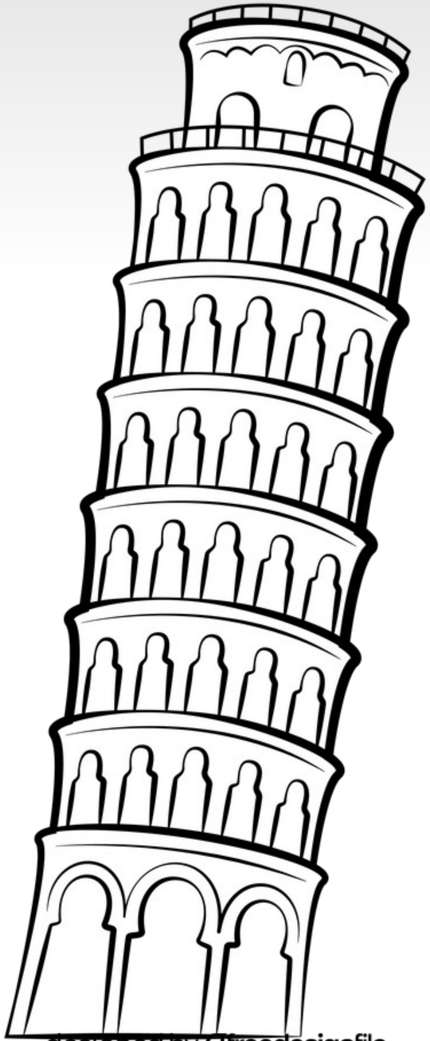

Applications of the Physics-Informed RG

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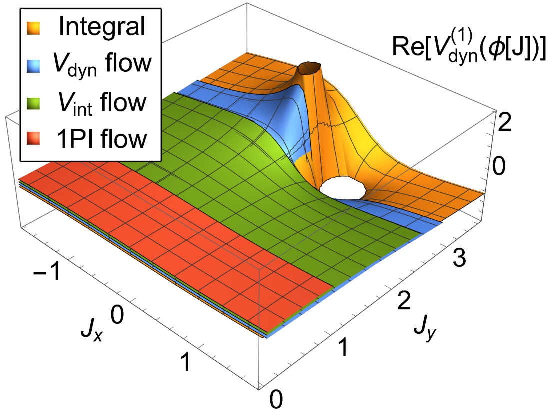
Expansion about the 2PT function
(Polchinski flow)

Salmhofer ‘07
FI, Pawlowski ‘22
Cotler, Rezchikov ‘22



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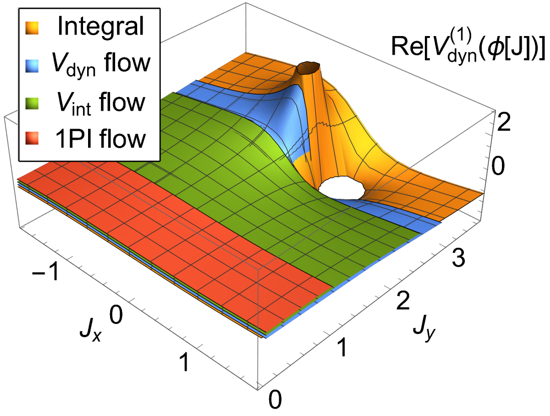

Physical fields

Lamprecht ‘07,
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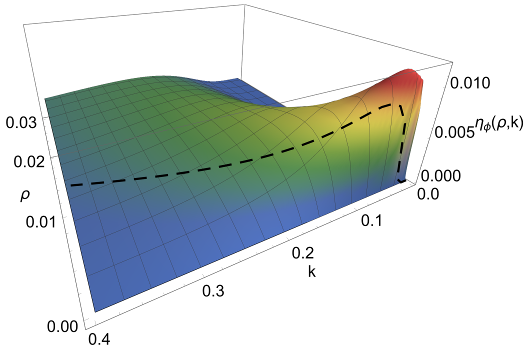

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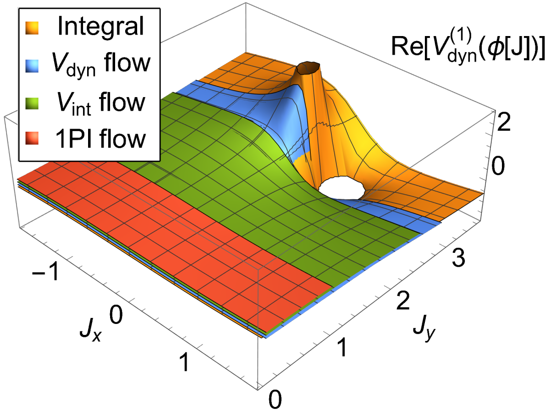

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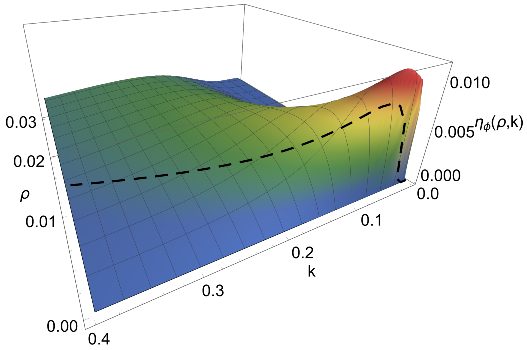
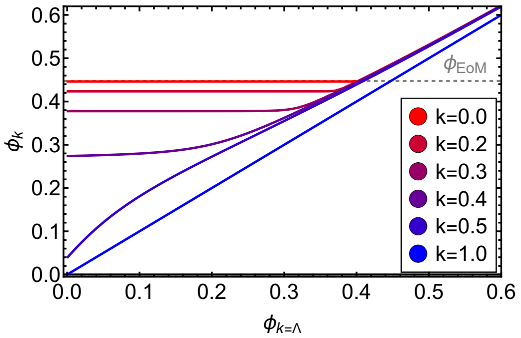


Computational simplifications

FI, Pawlowski '24



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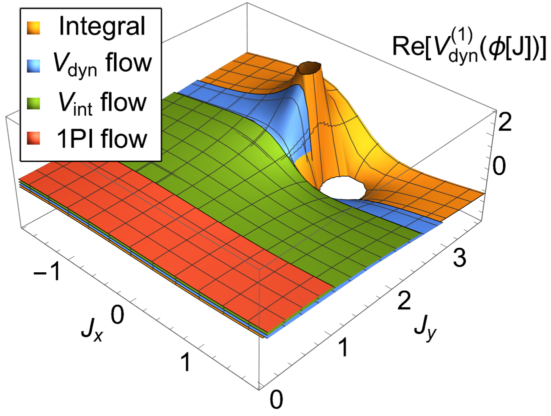

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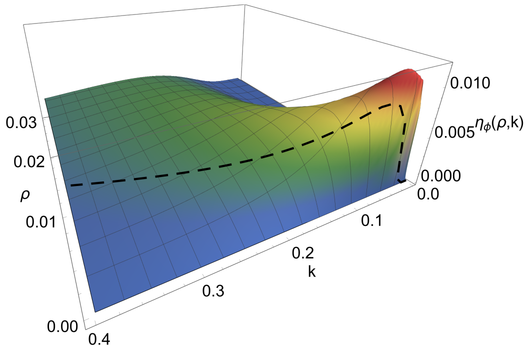

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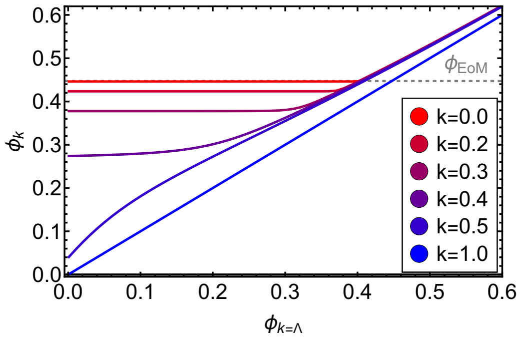

The generating functional for general composites

$$Z[J_\varphi] \simeq \int d\mu[\hat{\varphi}]\, e^{-\Delta S_k[\hat{\varphi}] + J_\varphi \hat{\varphi}}$$

$$d\mu = d\hat{\varphi}\, e^{-S[\hat{\varphi}]}$$

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Two novelties: (1) different current, (2) different Regulator

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$$\frac{\delta^n Z[J_\varphi]}{\delta J_\varphi^n} \simeq \langle \hat\varphi \cdots \hat\varphi \rangle$$

Correlation functions

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$$\varphi = \langle \hat\varphi \rangle$$

Mean field

$$\phi = \langle \hat\phi[\hat\varphi] \rangle$$

$$\boxed{\Gamma_\varphi[\varphi] \neq \Gamma_\phi[\phi]}$$

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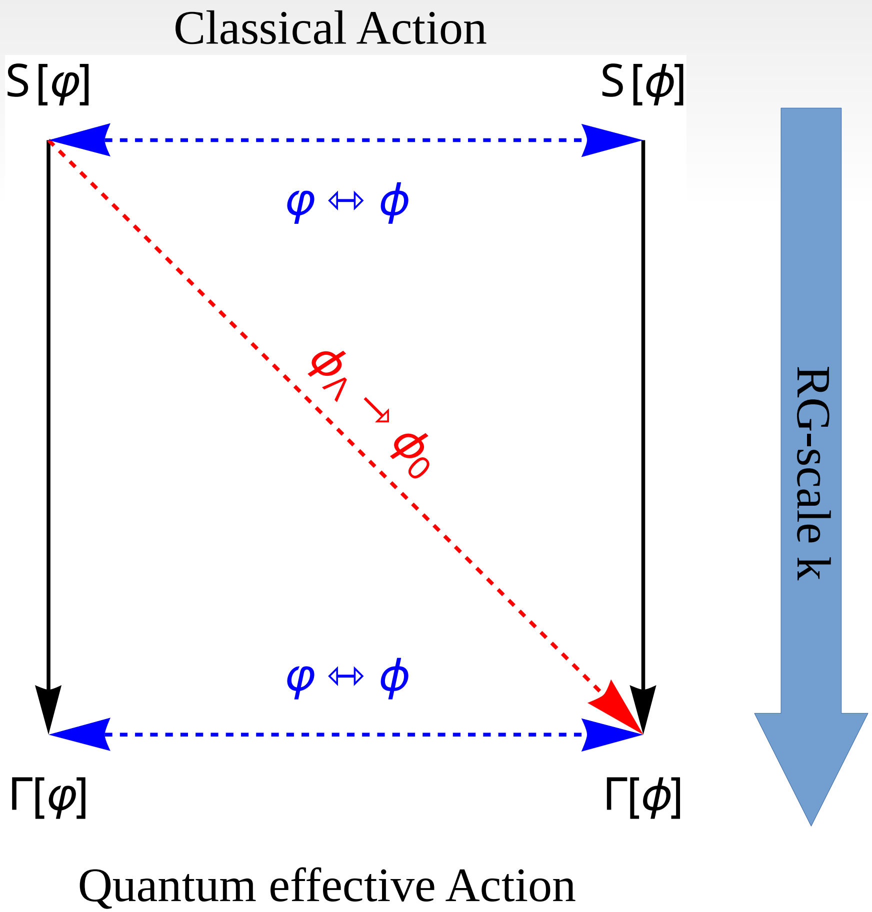
$$\Gamma_\varphi[\varphi] \neq \Gamma_\phi[\phi]$$

$\longrightarrow$

The pair is PI $(\Gamma_\phi, \phi[\varphi])$

General field transformations in the fRG



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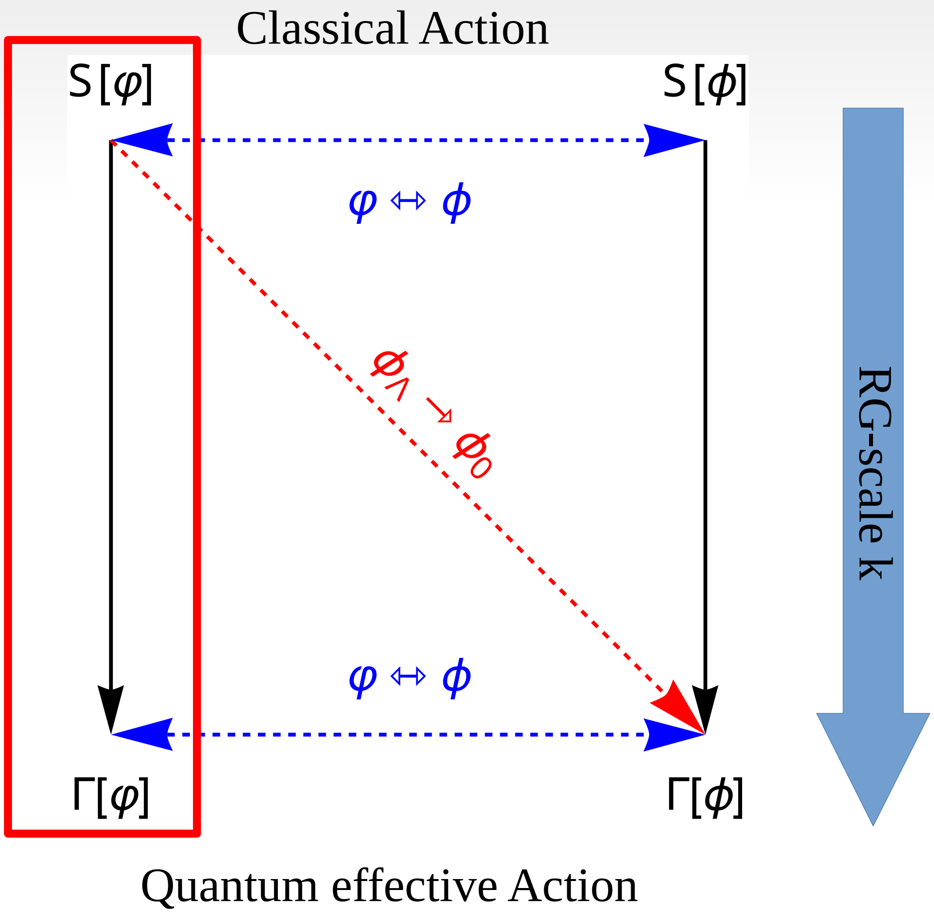

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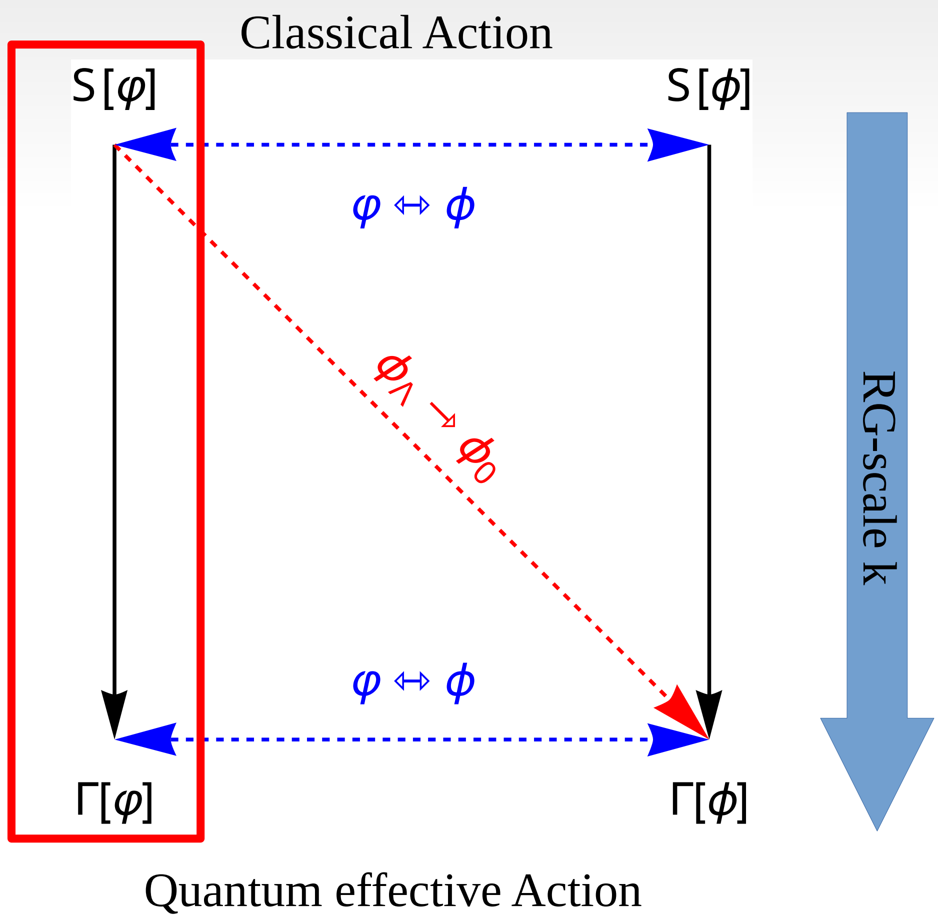

- Infrared regularised theory ↔ classical theory

$$\Gamma_\Lambda[\varphi] = S[\varphi]$$

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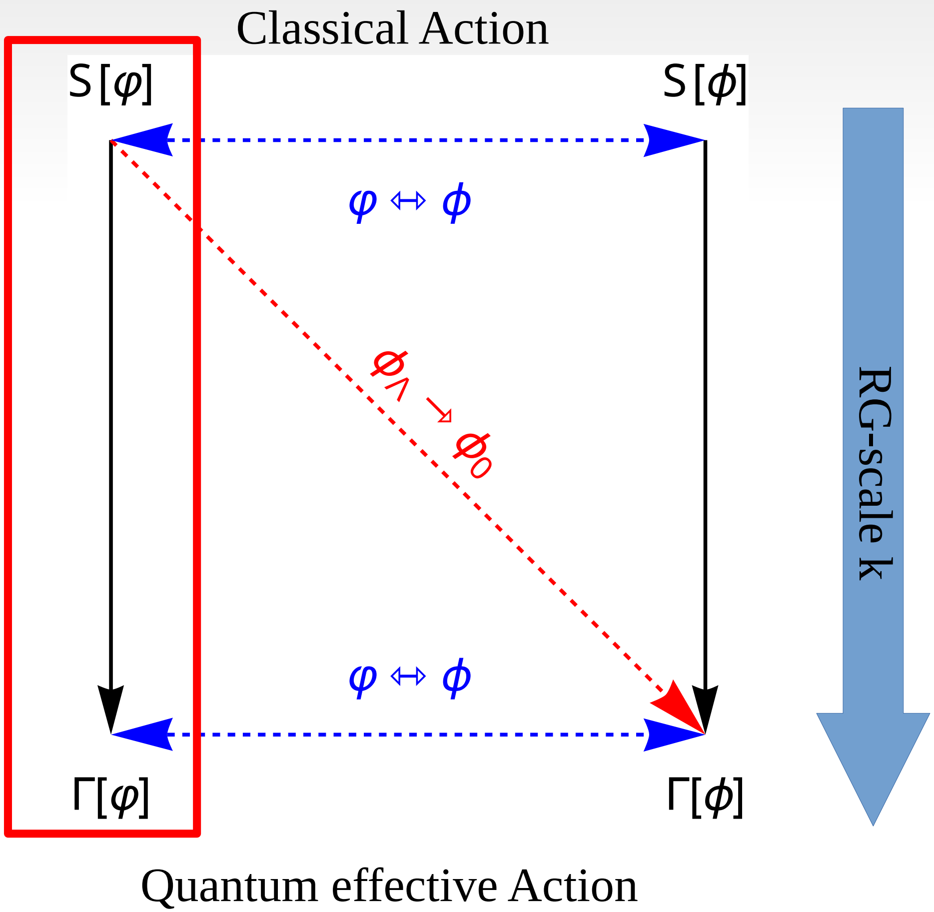

- Infrared regularised theory $\leftrightarrow$ classical theory

$$\Gamma_\Lambda[\varphi] = S[\varphi]$$

- Solve RG-flow by integrating over RG-time/RG-scale

$$\partial_t \Gamma_k[\varphi] = \frac{1}{2}\, G_k[\varphi]\, \partial_t R_k$$

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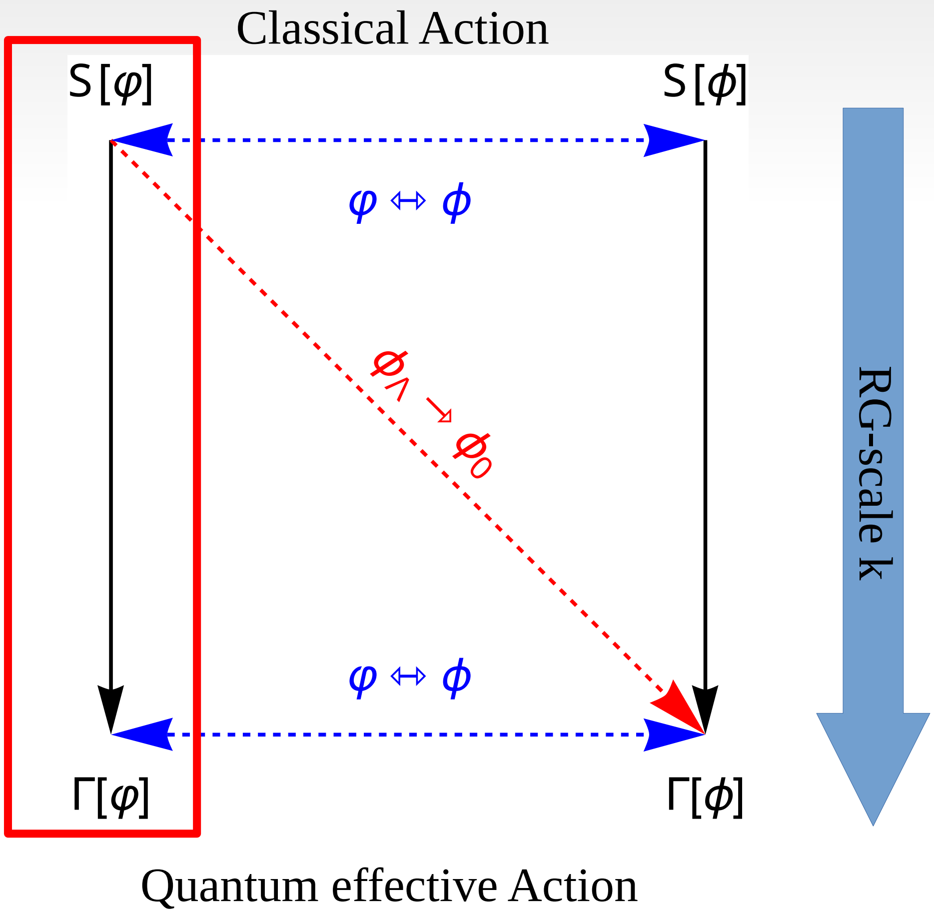

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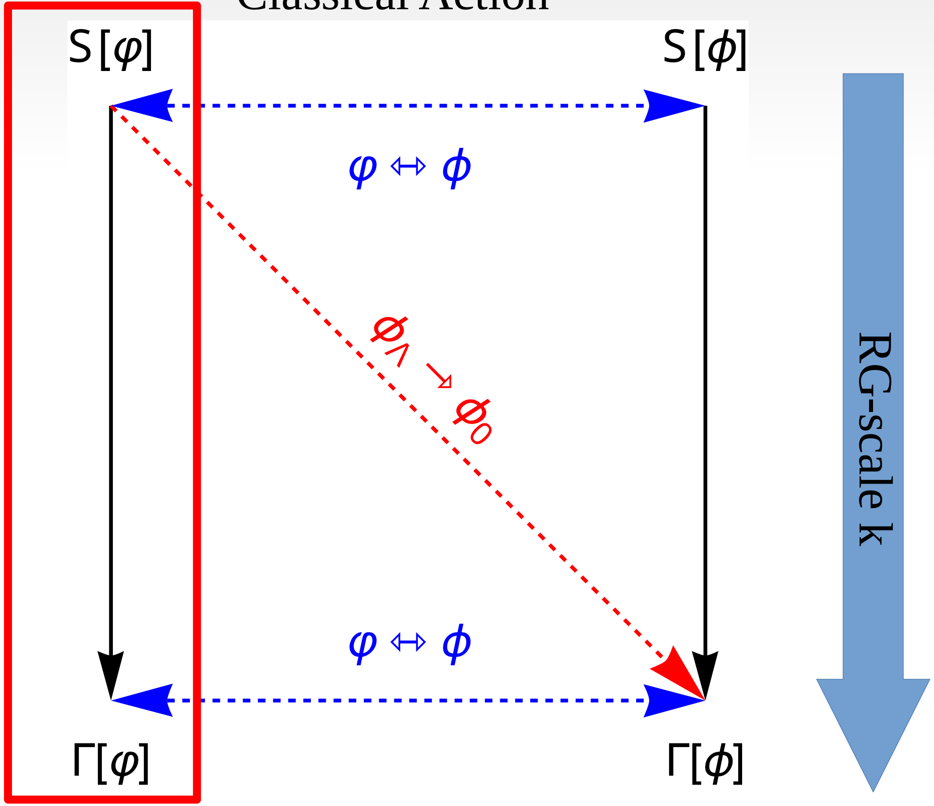
RG-time

$$t = \log\left(\frac{k}{\Lambda}\right)$$

General field transformations in the fRG

Classical Action



Quantum effective Action

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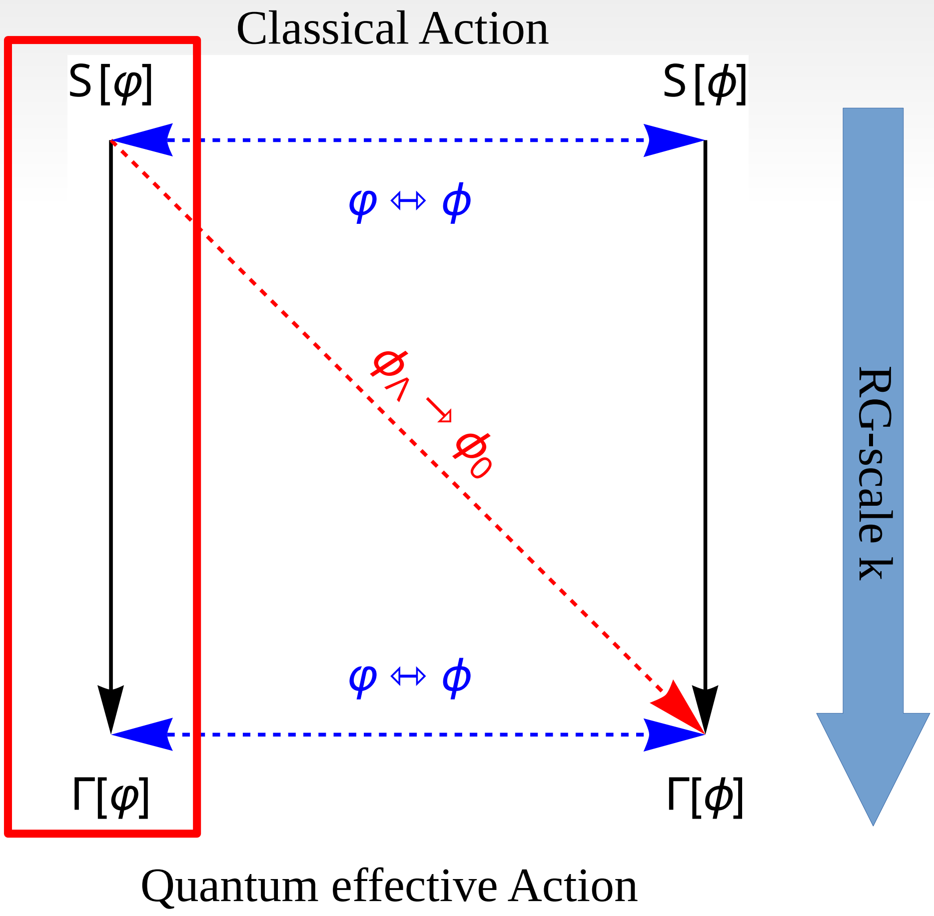

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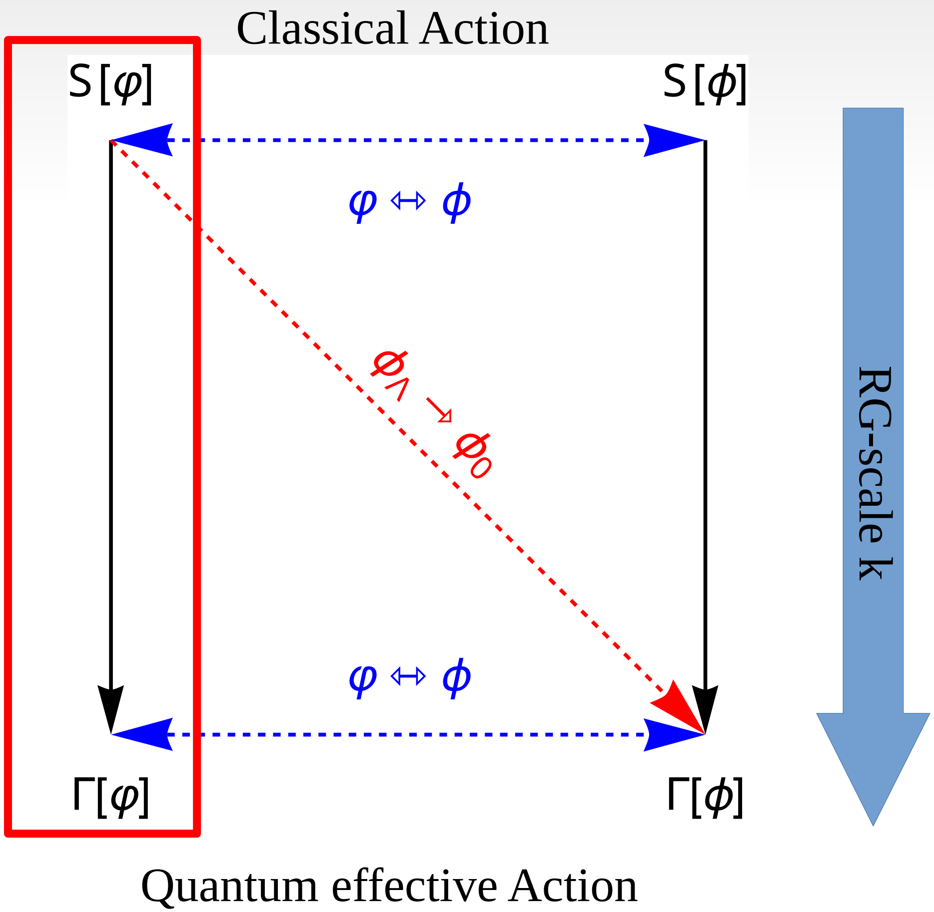

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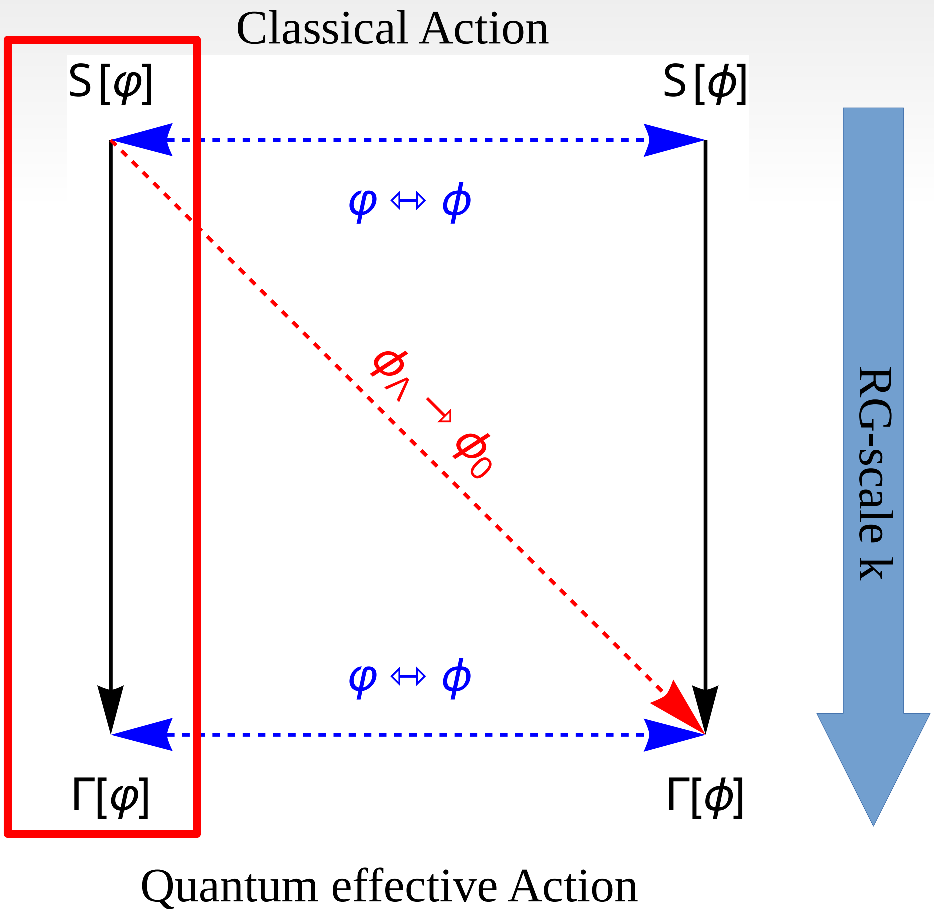

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  Wetterich'92

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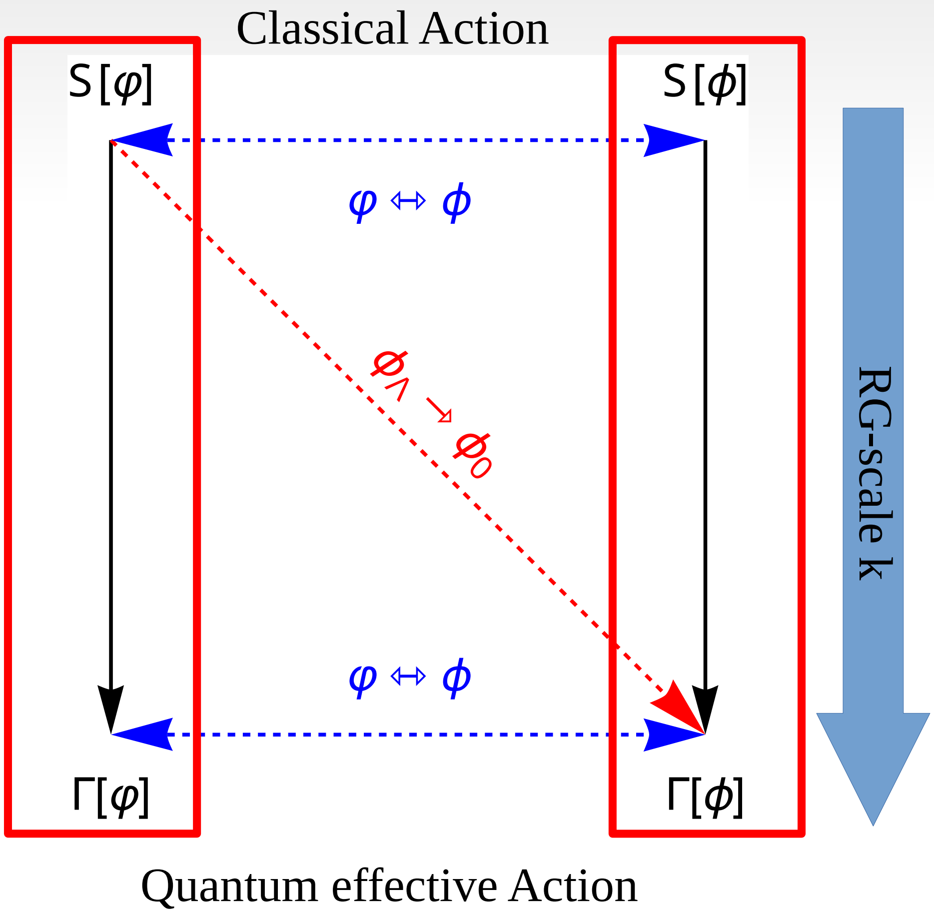

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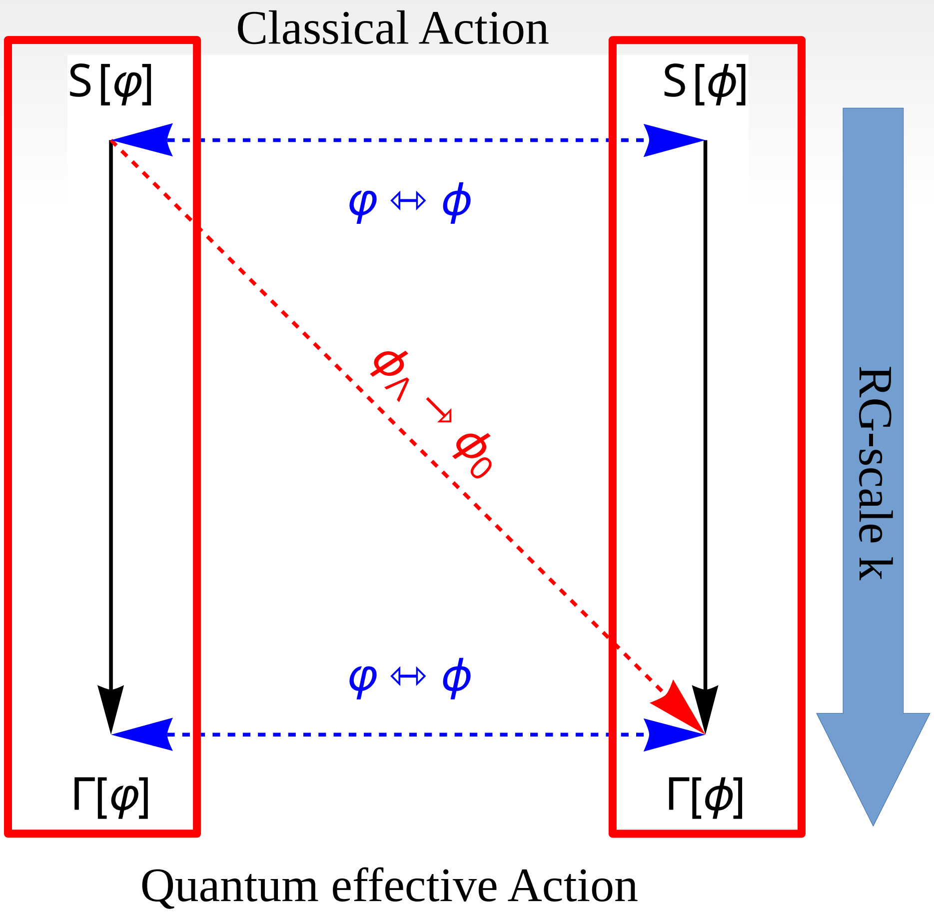

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  - Solve PDE for all generated couplings in the effective action

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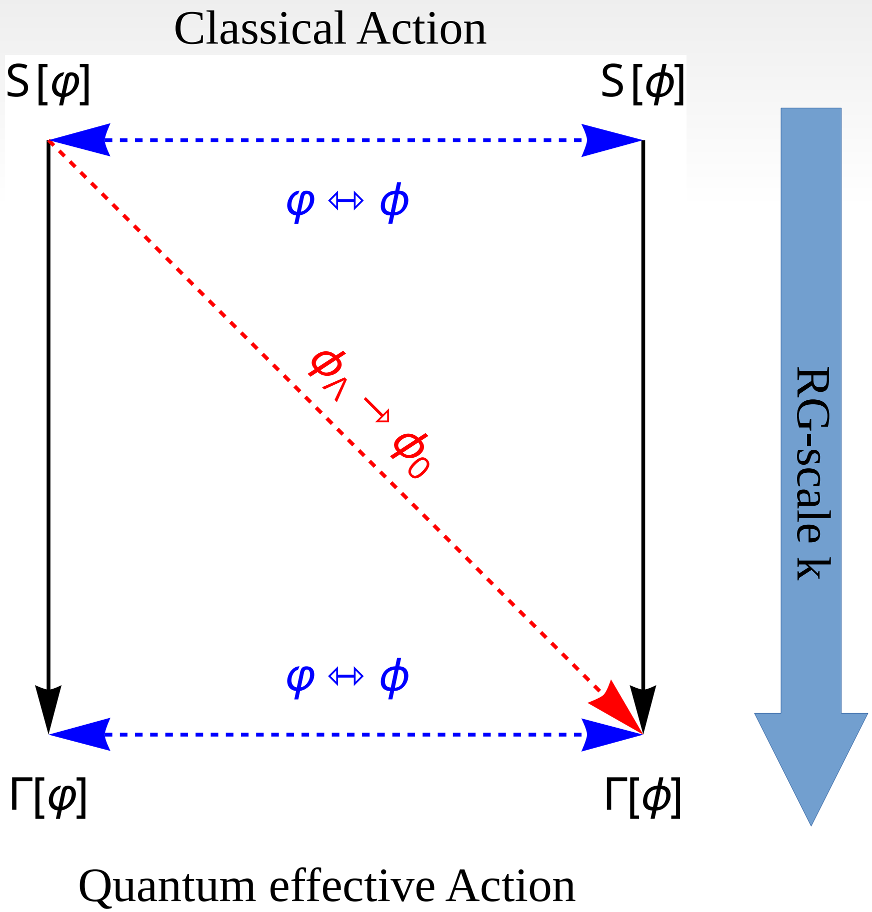

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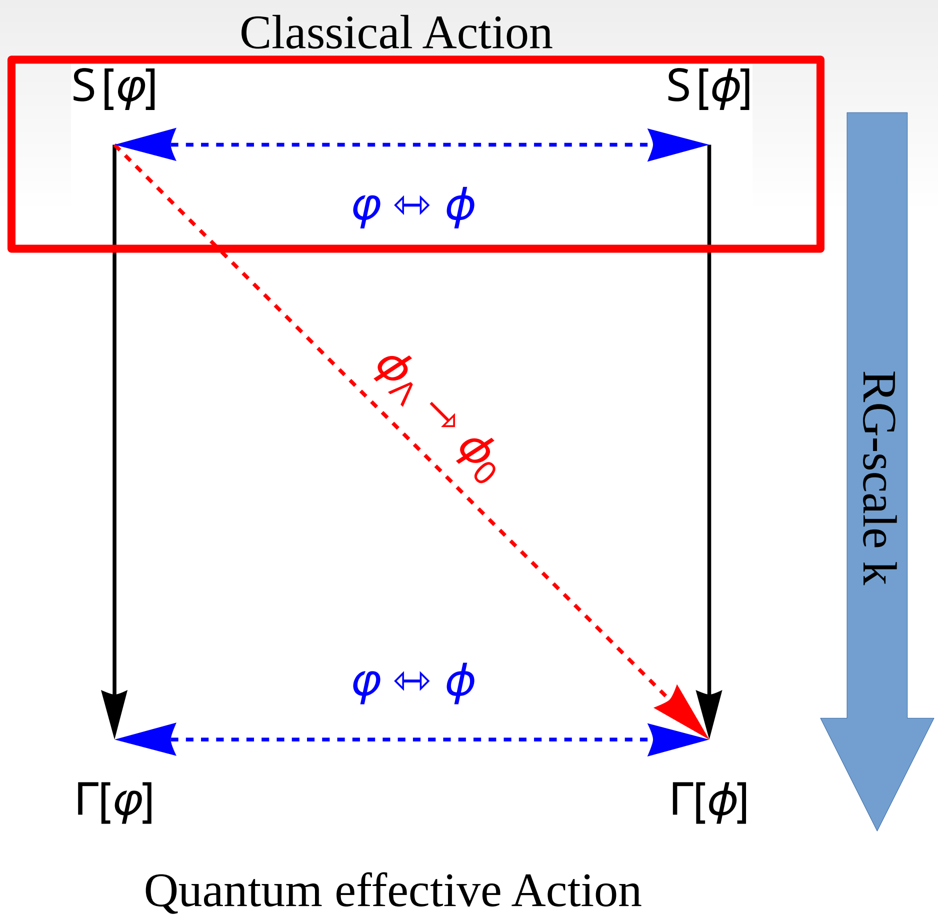

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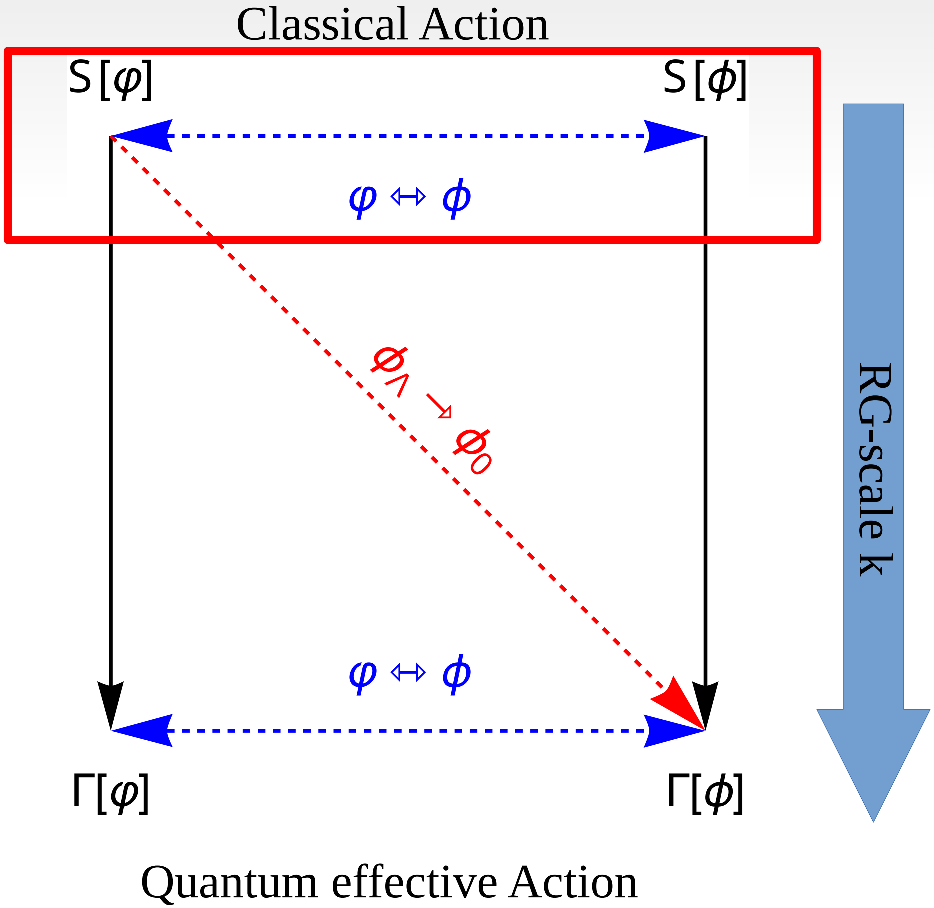

- Explicit field transformations (also possible with the RG)

$$Z[J_\varphi] \simeq \int [d\hat{\varphi}]\, e^{-S[\hat{\varphi}]+\int_{\boldsymbol{x}} J_\varphi \hat{\varphi}}$$

$$Z[J_\phi] \simeq \int [d\phi]\ \det\left|\frac{\delta\hat{\varphi}}{\delta\hat{\phi}}\right| e^{-S[\hat{\varphi}[\hat{\phi}]]+\int_{\boldsymbol{x}} J_\phi \hat{\phi}}$$

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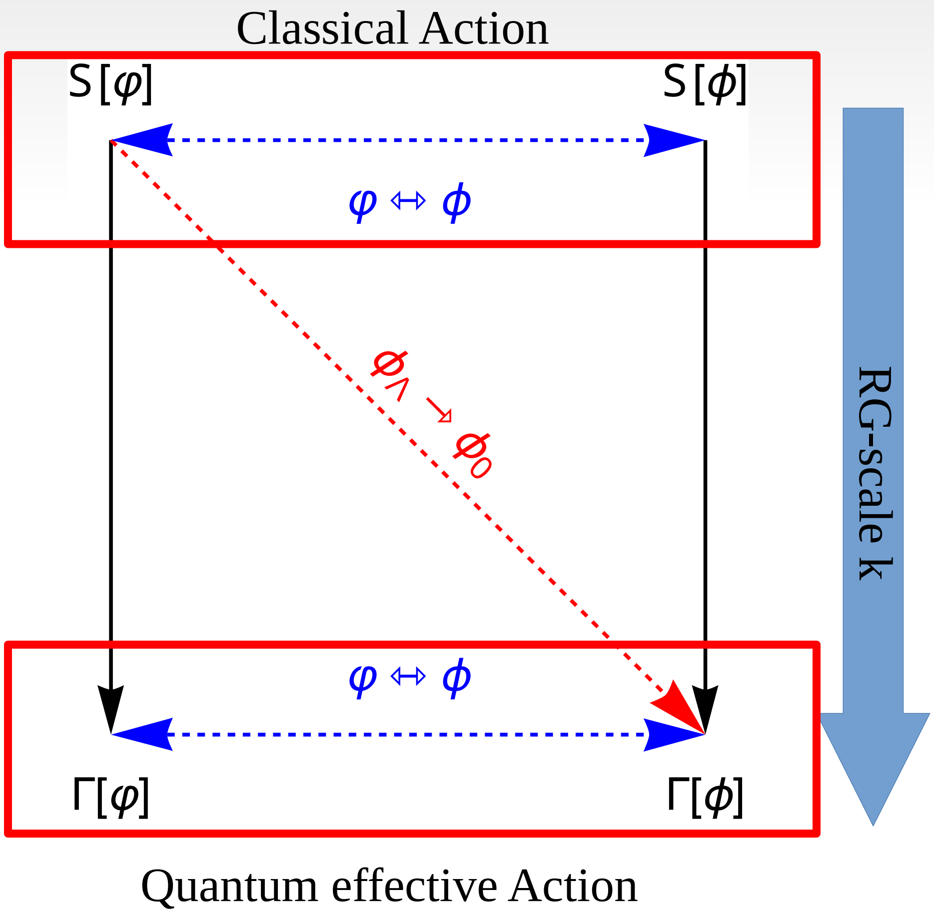

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$$\langle \hat{\phi} \cdots \hat{\phi} \rangle_c = 0$$
$$\langle \hat{\varphi} \cdots \hat{\varphi} \rangle_c \neq 0$$

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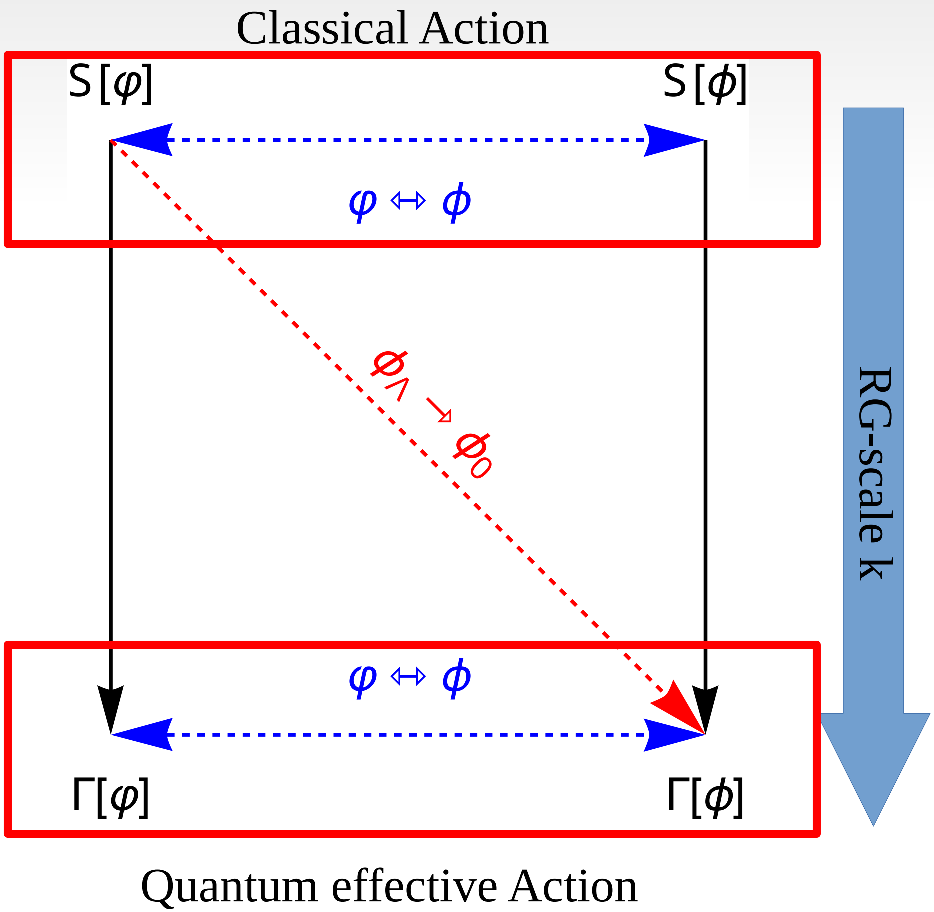

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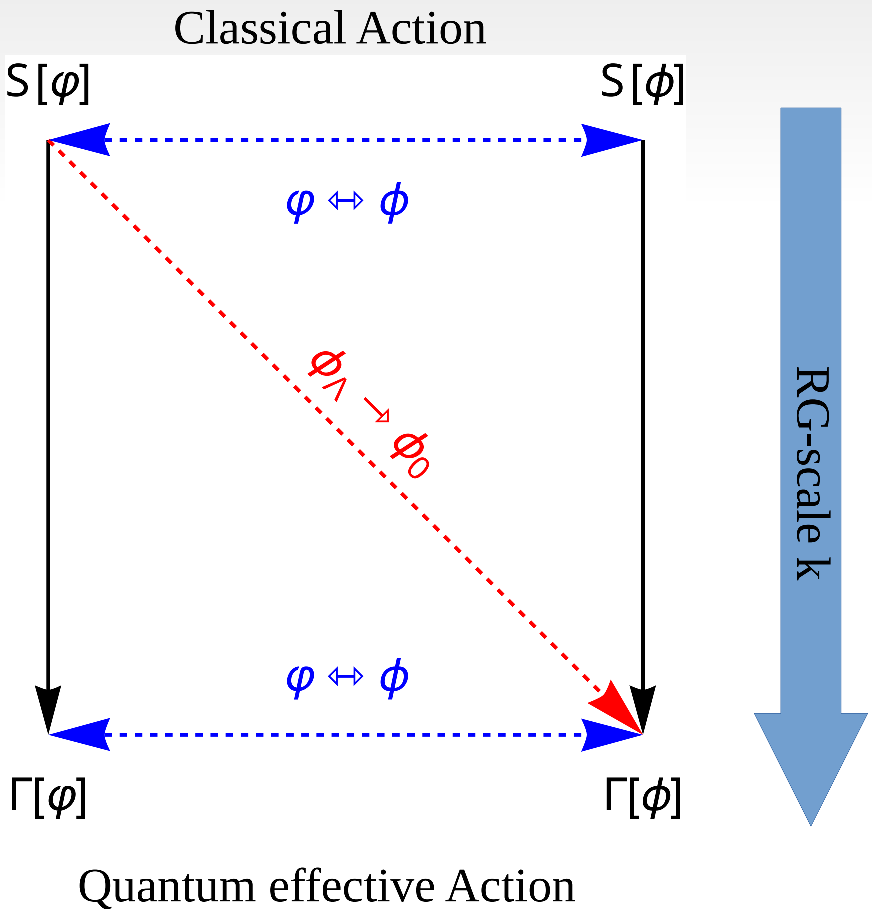
where a free theory is mapped on an interacting one Albergo et al. ‘21

General field transformations in the fRG



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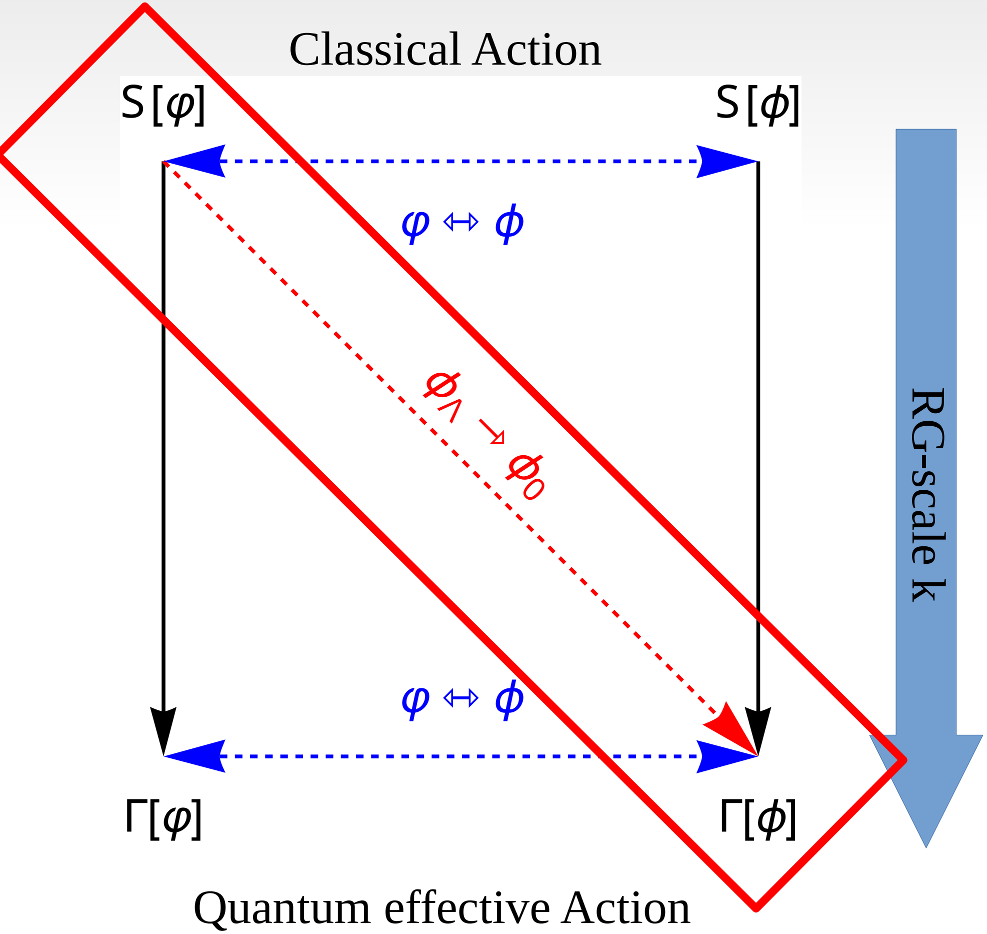
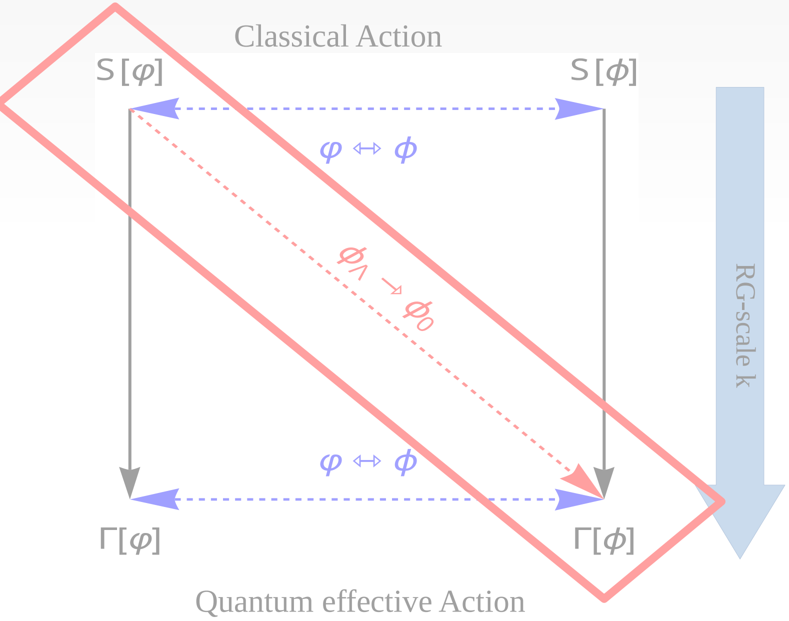


General field transformations in the fRG



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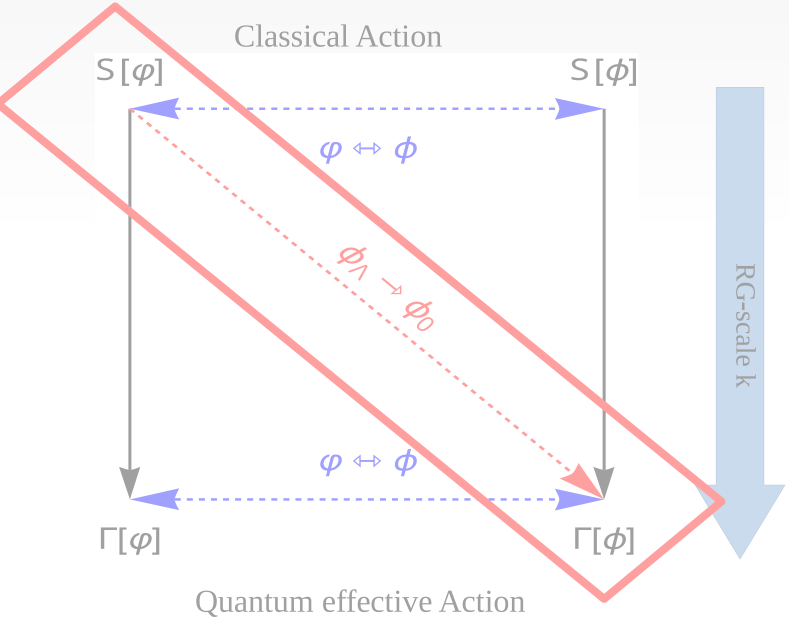

- Generalised functional Flows

RG-time $\quad$ 1PI gen. funct. $\quad$ Propagator $\quad$ Regulator

$$t = \log\left(\frac{k}{\Lambda}\right)$$

$$\left(\partial_t + \int_x \dot{\phi}\frac{\delta}{\delta\phi}\right)\Gamma_k[\phi] = \frac{1}{2}\left[G[\phi]\left(\partial_t + 2\frac{\delta\dot{\phi}}{\delta\phi}\right)R_k\right]$$

Pawlowski ‘05

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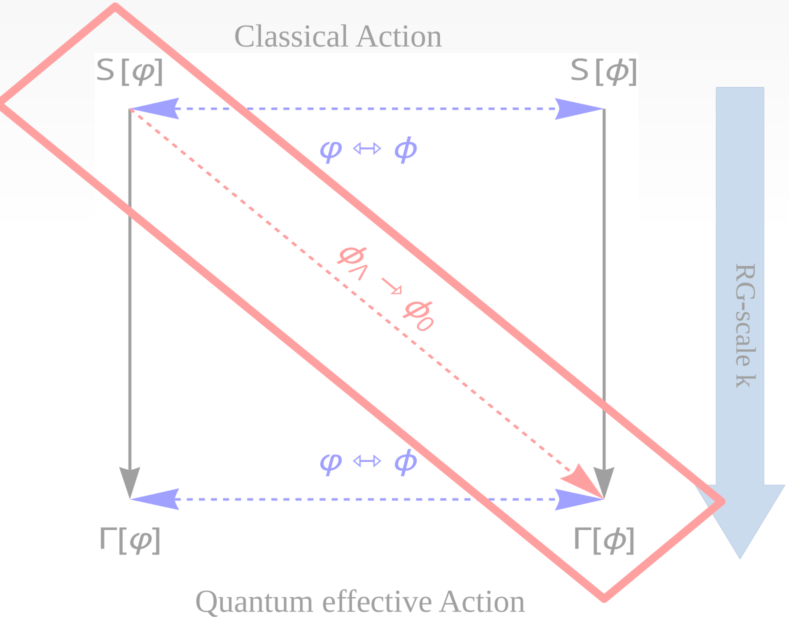

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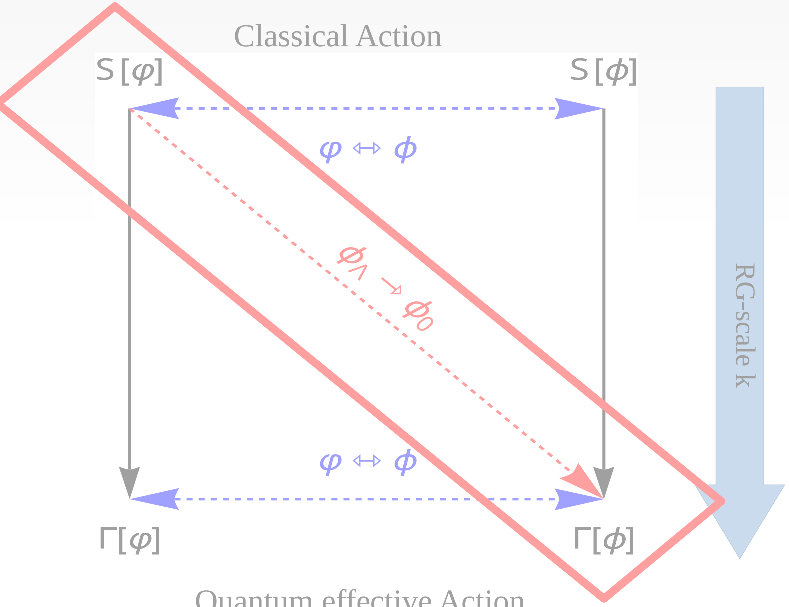

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Pawlowski '05

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$$\phi = \langle\hat{\phi}[\hat{\varphi}]\rangle \qquad \dot{\phi}[\phi] = \langle\partial_t\hat{\phi}_k\rangle[\phi]$$

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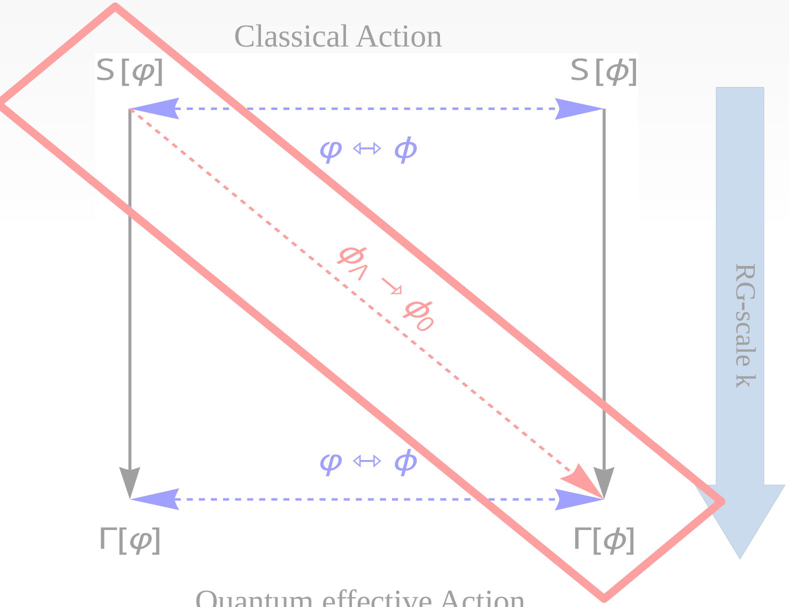

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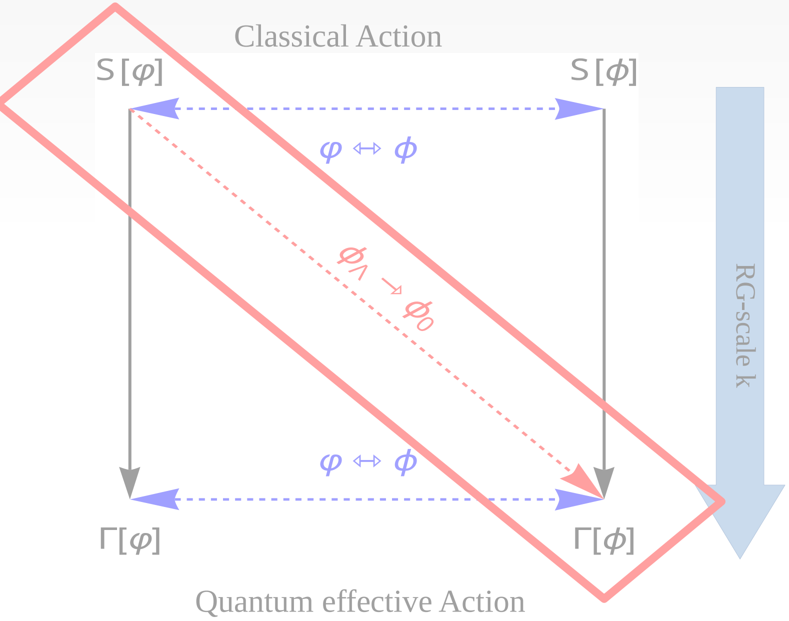

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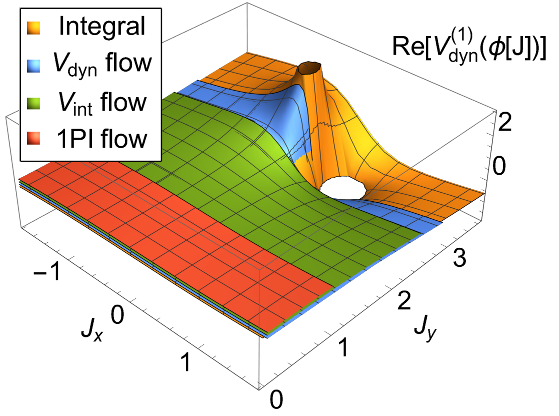
- 1PI formulation of general transformations of the path integral Wegner '74
- At the level of the effective action, physics is stored in the pair, which is physics informed

$$(\Gamma_\phi, \phi[\varphi])$$

Applications of the Physics-Informed RG

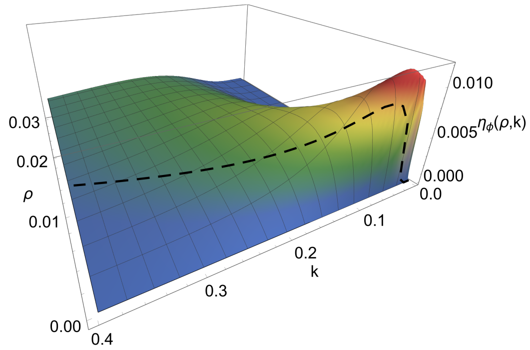
Expansion about the 2PT function (Polchinski flow)

Salmhofer ‘07
FI, Pawlowski ‘22
Cotler, Rezchikov ‘22



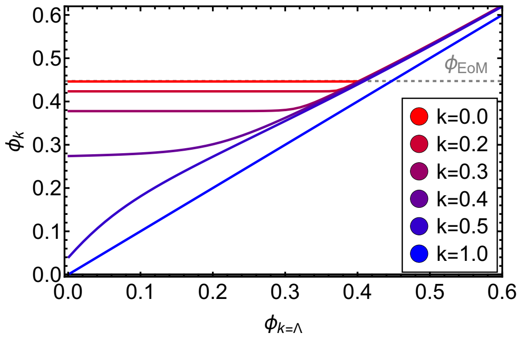
Physical field basis

Lamprecht ‘07,
Isaule, Birse, Walet ‘18 ‘20
Baldazzi, Zinati, Falls ‘21
FI, Pawlowski ‘23



Computational simplifications

FI, Pawlowski ‘24



Examples & Applications: Physics inspired

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- Implementation of "emergent composites" with Gies, Wetterich '01

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$$\dot{\phi} = \boxed{\dot{A}}\boxed{\phi} + \boxed{\dot{B}}\boxed{\bar{q}\tau q}$$

Hadronisation functions

Scalar terms, O(4)

Fermions	$\bar{q},\ q$
Scalar composites	$\phi = (\sigma, \vec{\pi})^t$

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Talk by F. R. Sattler

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"Absorption of functions"
Baldazzi, Zinati, Falls 21', Baldazzi, Falls 21',
FI, Pawlowski 23'

Absorb flows of correlation functions into the field

$$\phi_k(\varphi, k) \;\to \partial_t \Gamma^{(n)} \equiv 0$$

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"Geometric transformations"
Flow from a Cartesian to a polar basis
Lamprecht '07, Isaule, Birse, Walet '18, Isaule, Birse, Walet'19,
Daviet, Dupuis '21

$$\phi^t = (\rho, \theta)$$

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An expansion about the ground state

FI, Pawlowski ‘23 : arXiv:2305.00816

O(N) model: $\varphi^t = (\varphi_1, \ldots, \varphi_N)$ vs. $\phi^t = (\phi_1, \ldots, \phi_N)$

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and its derivatives

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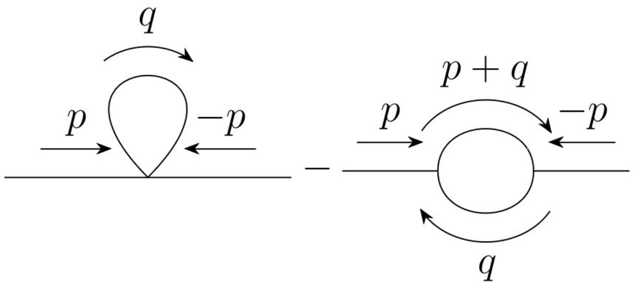

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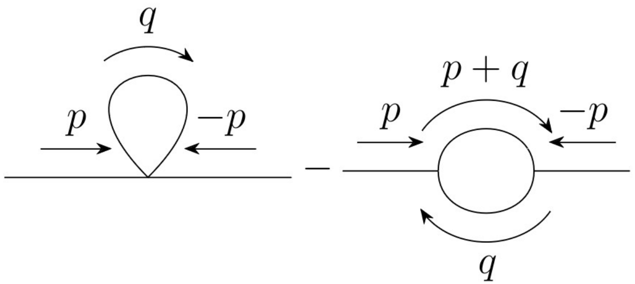

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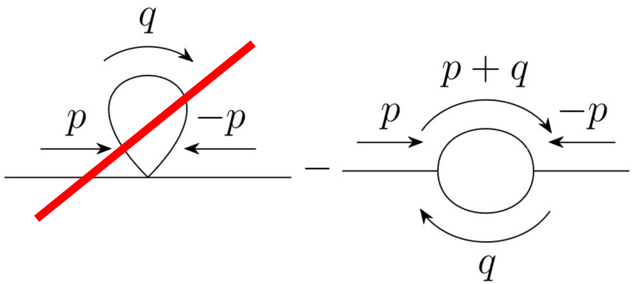

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$$\dot{\phi}_k(\phi, k) \;\rightarrow\; Z_{\phi,k}(\phi, p) \equiv 1$$

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Field dependent wave function renormalisation and its derivatives

Take away message:

- Expansion about **classical dispersion**:
  → **Optimised expansion** (quicker convergence)
- Technical simplification with improved truncation

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Parametrisation:

$$\dot{\phi} = -\frac{1}{2}\eta_\phi(\rho)\,\phi$$

And accordingly:

$$\eta_\phi(\rho) = -\frac{\partial_t Z_\varphi(\rho)}{Z_\varphi(\rho)}$$

FI, Pawlowski ‘23 : arXiv:2305.00816 **Poster by M. Alhajkhouder**

- Application: $Z_\phi(\rho, p) \approx Z_\phi(\rho)$ (1st order deriv. exp.)
- Task: Solve two equations

1) $\partial_t Z_\phi = 0$: determines $\eta_\phi(\rho)$
2) $\partial_t V_k = \ldots$: PDE, integrate $k \to k - \Delta k$

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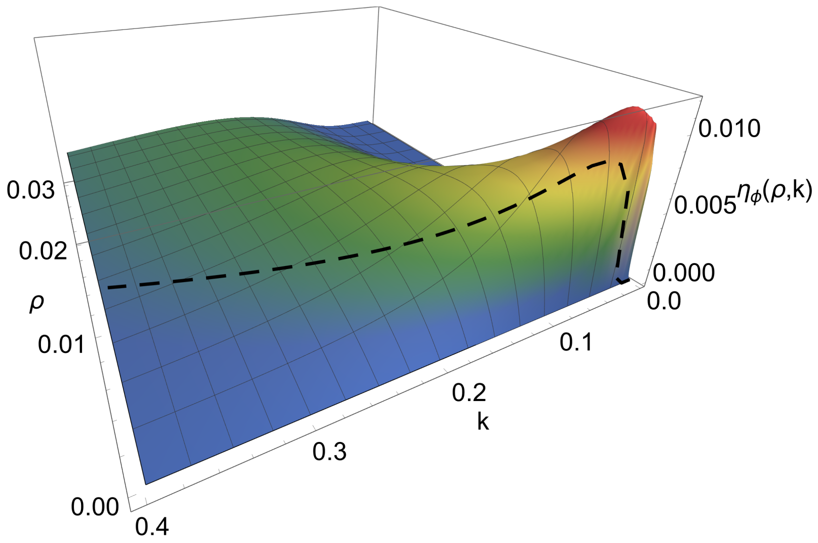

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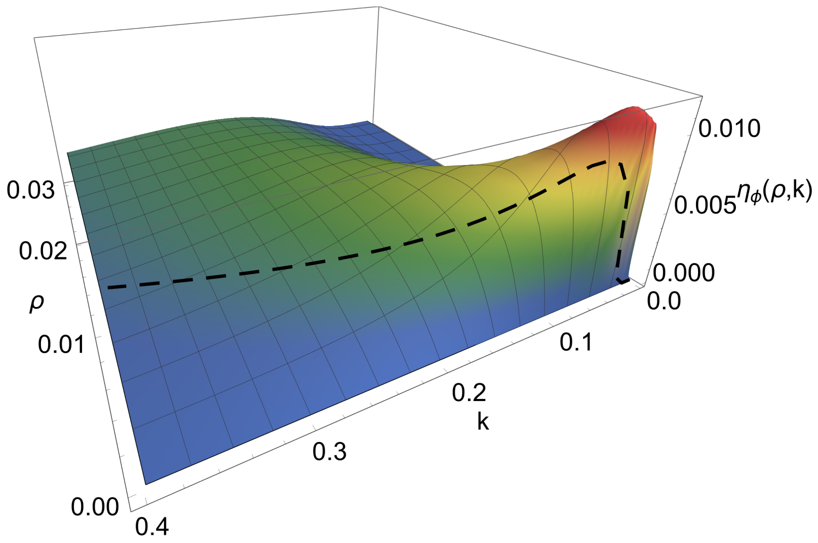

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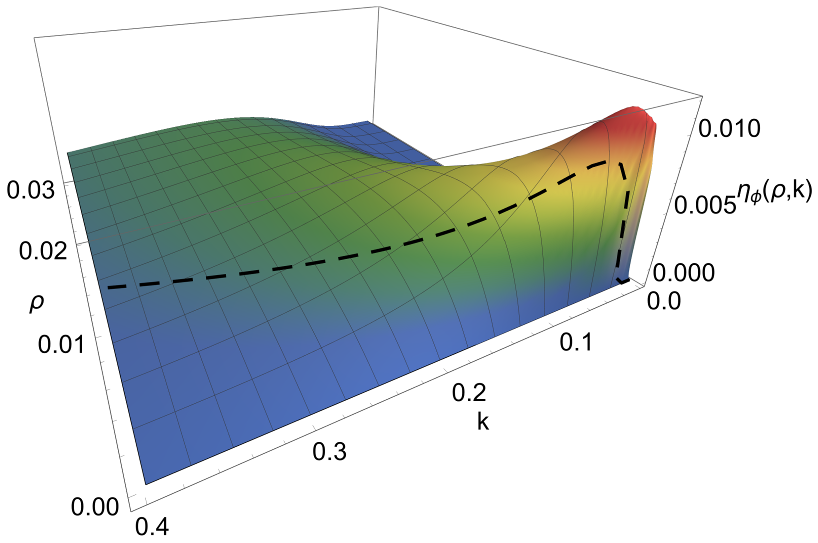
Take away message:

- Reminder: standard 1st order derivative expansion is a system of 2 coupled PDEs

  → **Technical simplification**



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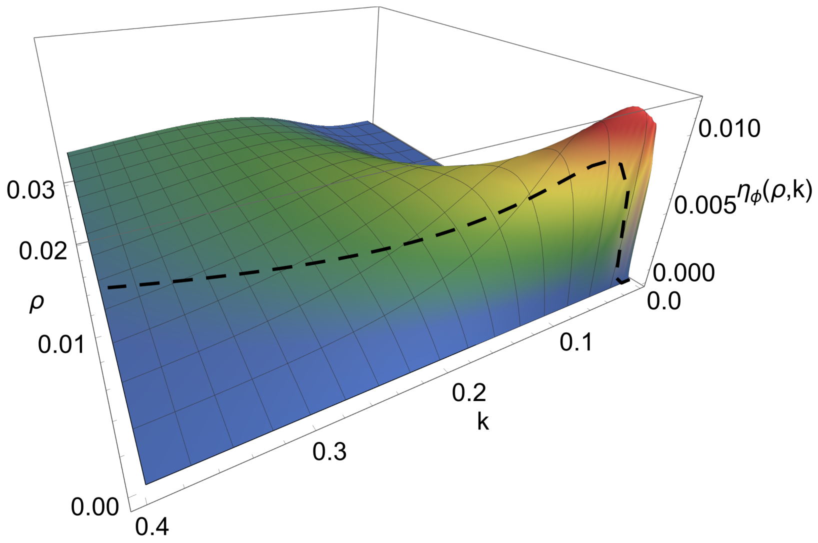
  → **Technical simplification**

- At the same time, the approximation is better

  → Includes more momentum dependences, due to **optimised expansion**



FI, Pawlowski '23 : arXiv:2305.00816 **Poster by M. Alhajkhouder**

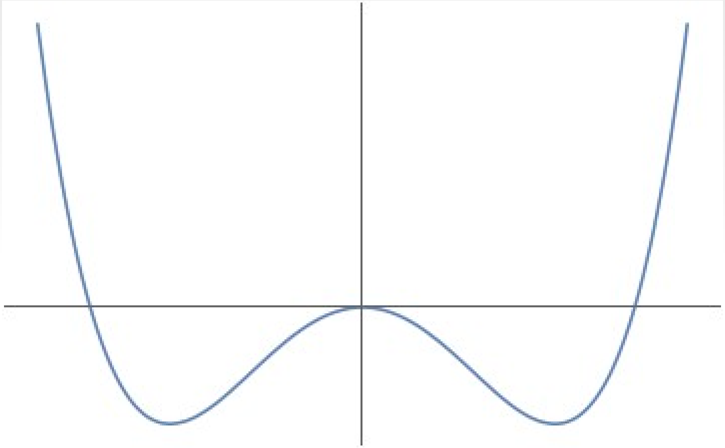
Application to the anharmonic oscillator

In Prep.
Bonanno, FI, Pawlowski

Application to the anharmonic oscillator

Setup with tunnelling

$$V_\Lambda(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4$$



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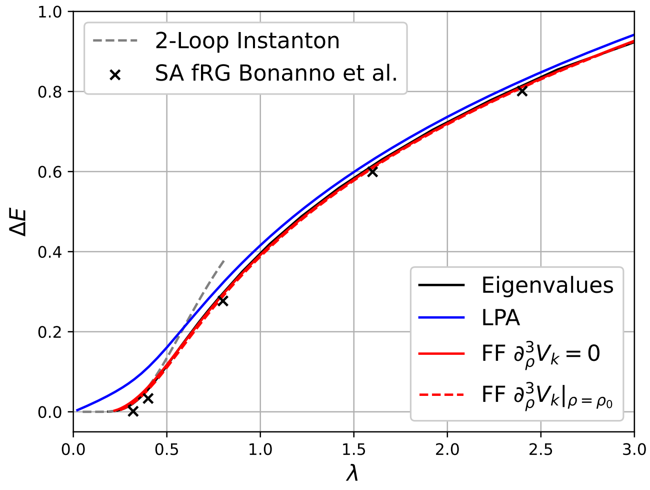

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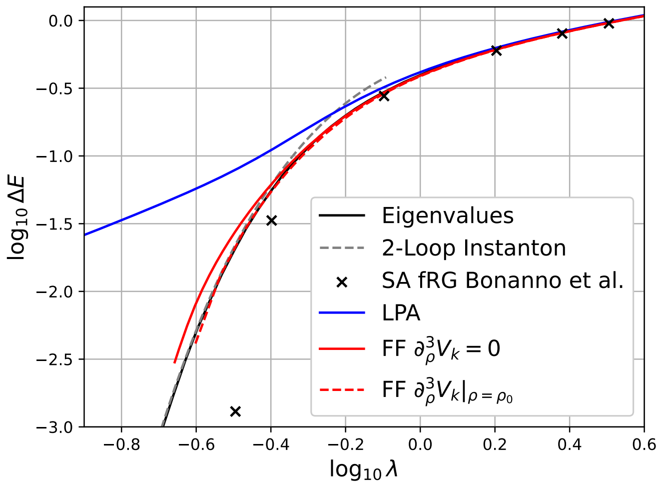

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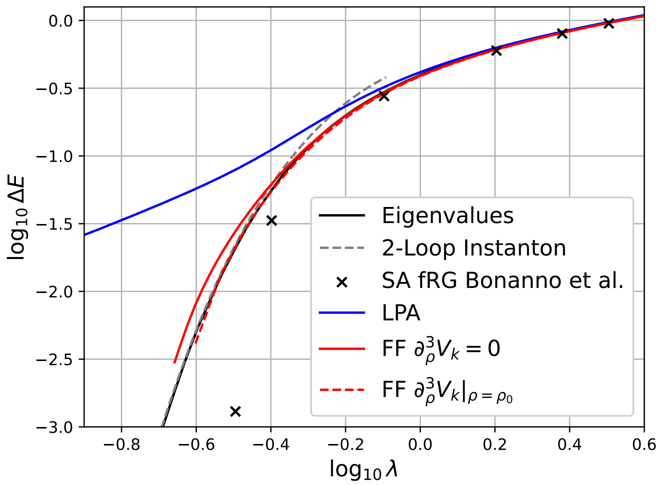

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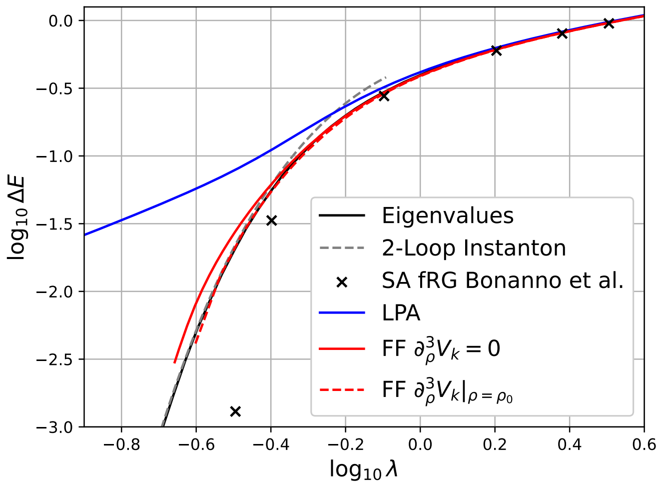

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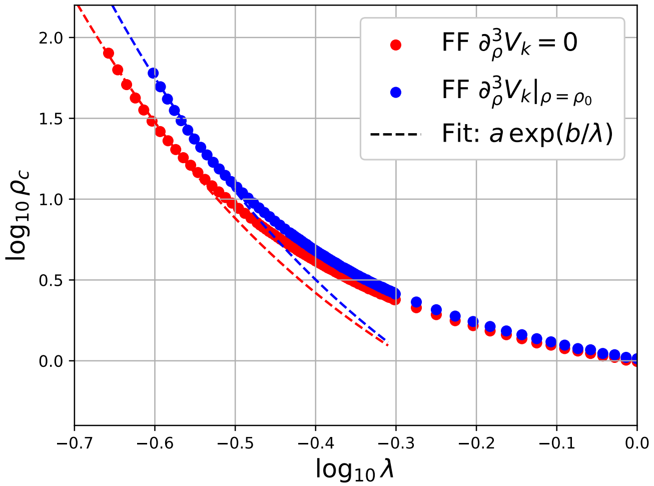

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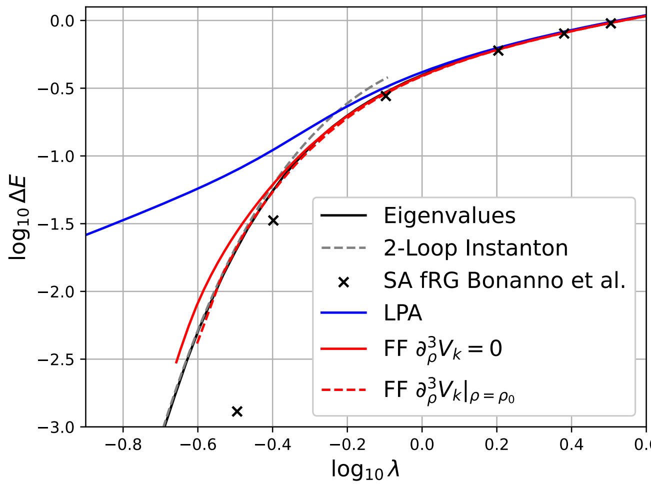

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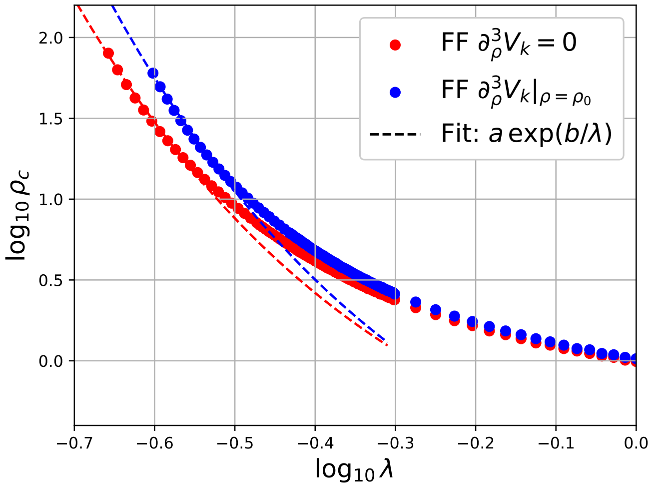

Traces of the instanton solution:

$$b_{\text{inst.}} = 4\frac{\sqrt{2}}{3} \approx 1.886$$

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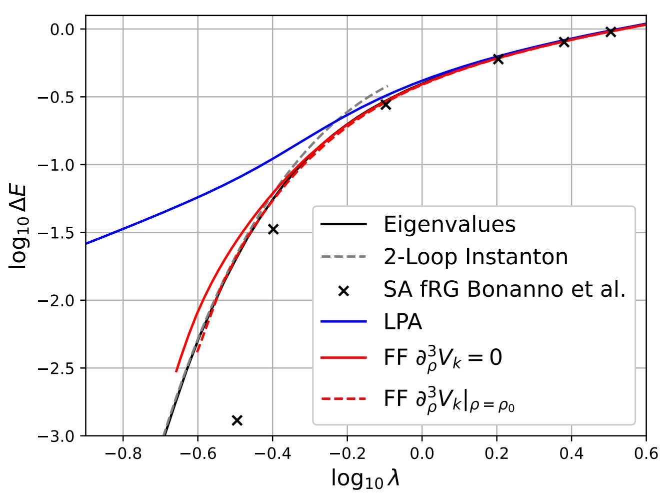

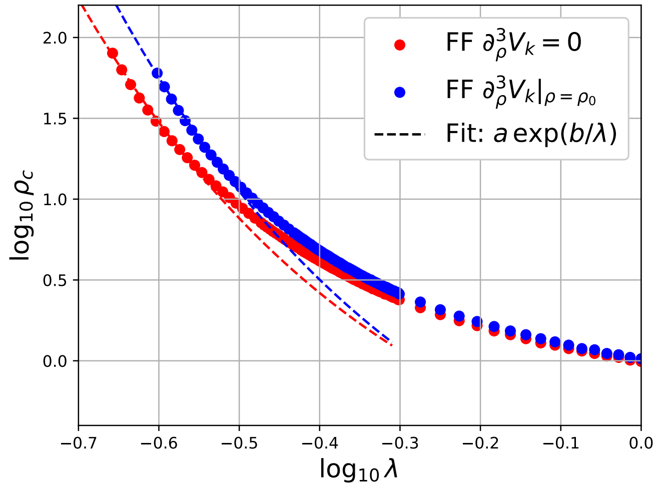

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$$b_r = 1.811 \quad b_b = 2.115$$

In Prep.
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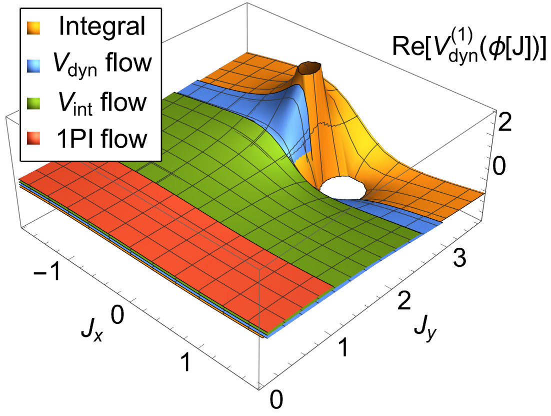
Applications of the Physics-Informed RG

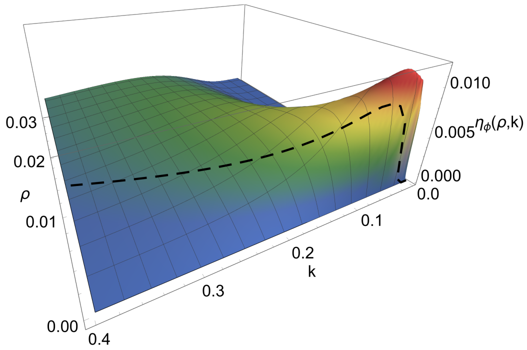
Expansion about the 2PT function (Polchinski flow)

Salmhofer ‘07
FI, Pawlowski ‘22
Cotler, Rezchikov ‘22



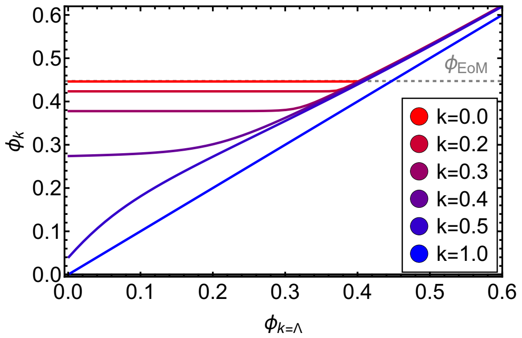
Physical field basis

Lamprecht ‘07,
Isaule, Birse, Walet ‘18 ‘20
Baldazzi, Zinati, Falls ‘21
FI, Pawlowski ‘23



Computational simplifications

FI, Pawlowski ‘24



Target Actions

FI, Pawlowski 24’ , arXiv:2409.13679

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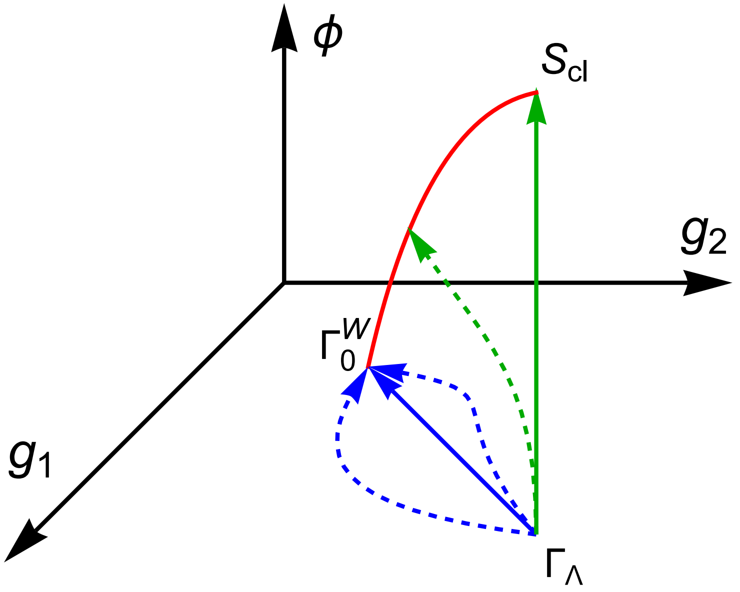

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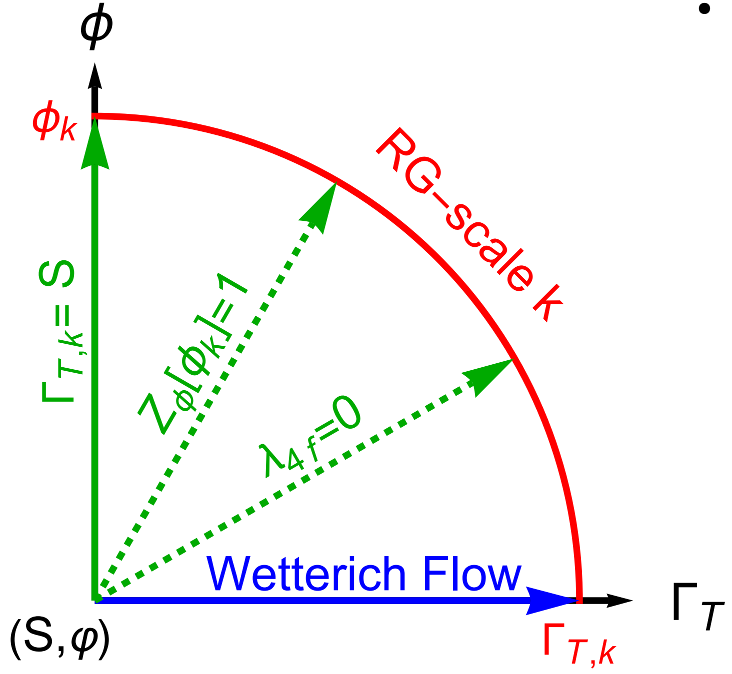

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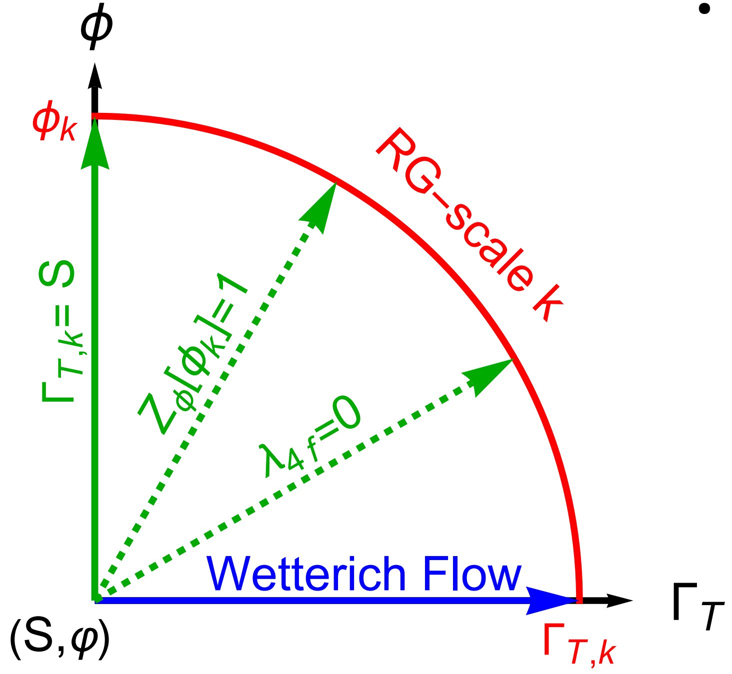

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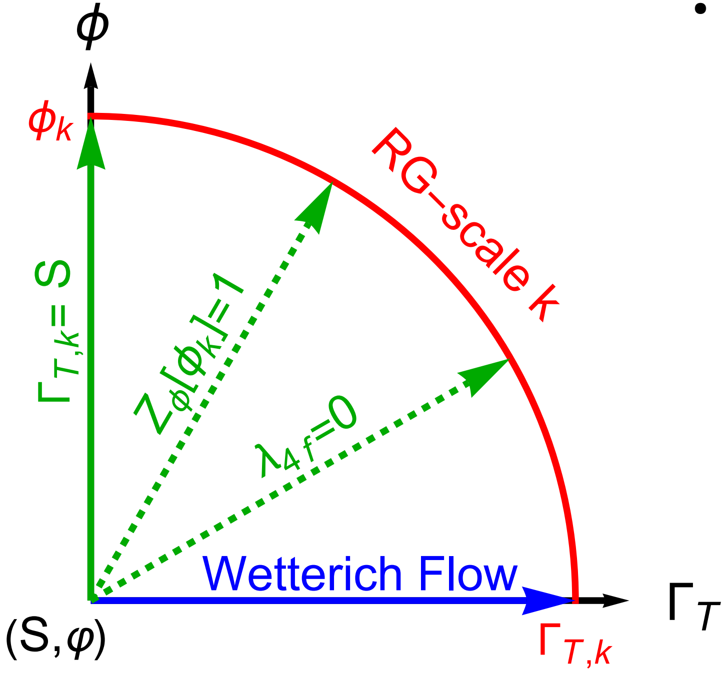

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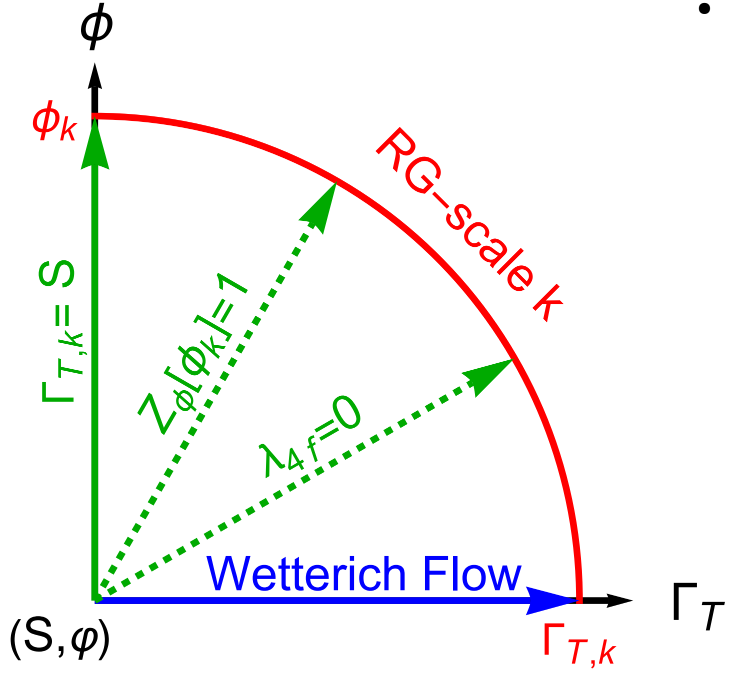

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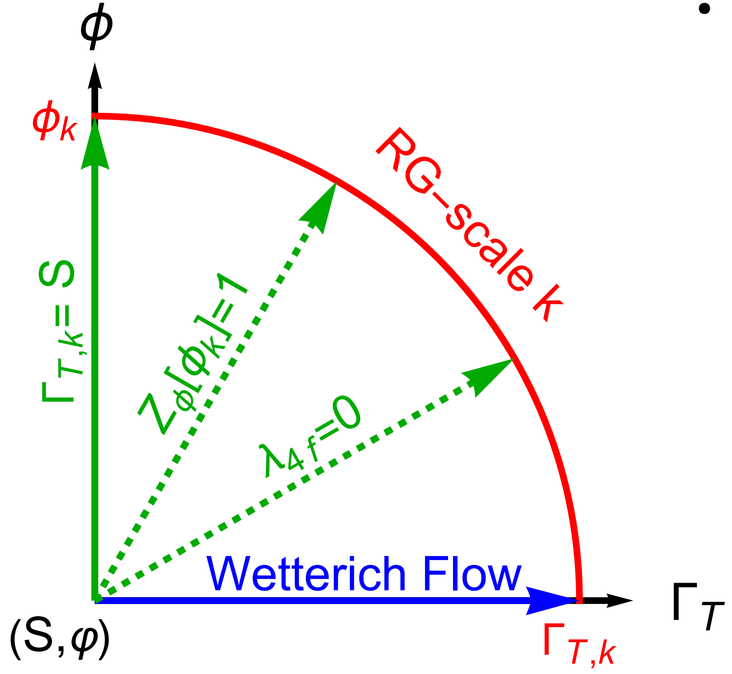

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FI, Pawlowski 24' , arXiv:2409.13679

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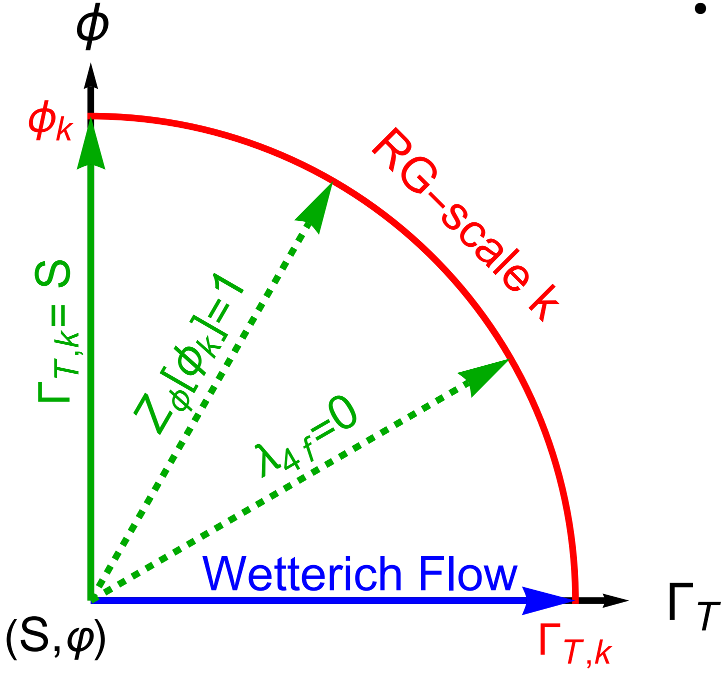

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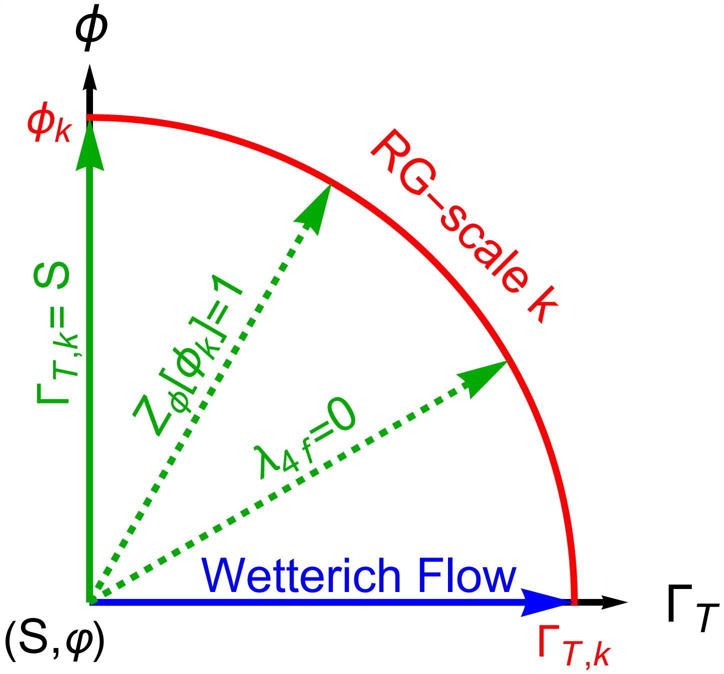
Reconstruction?



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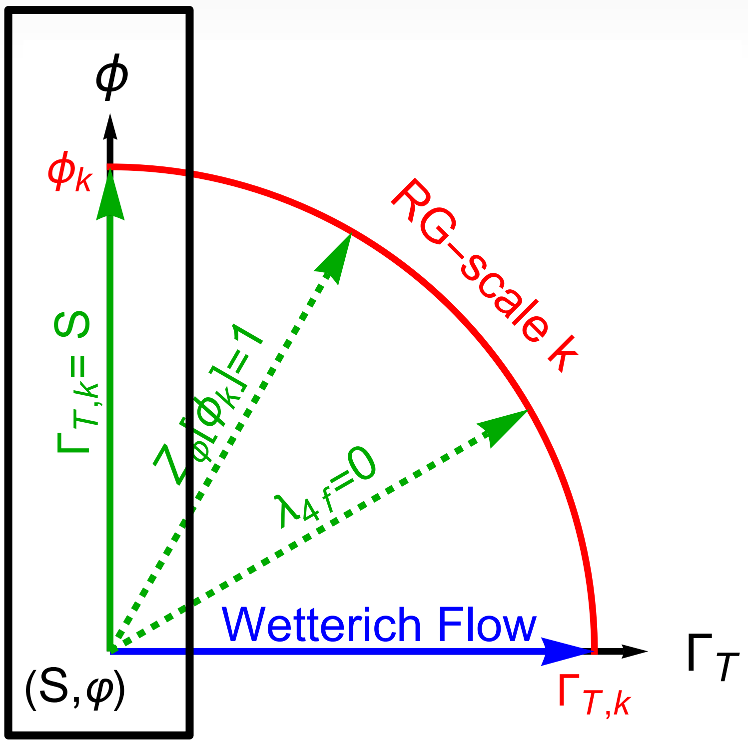
Example: Classical Target Actions

...most extreme example



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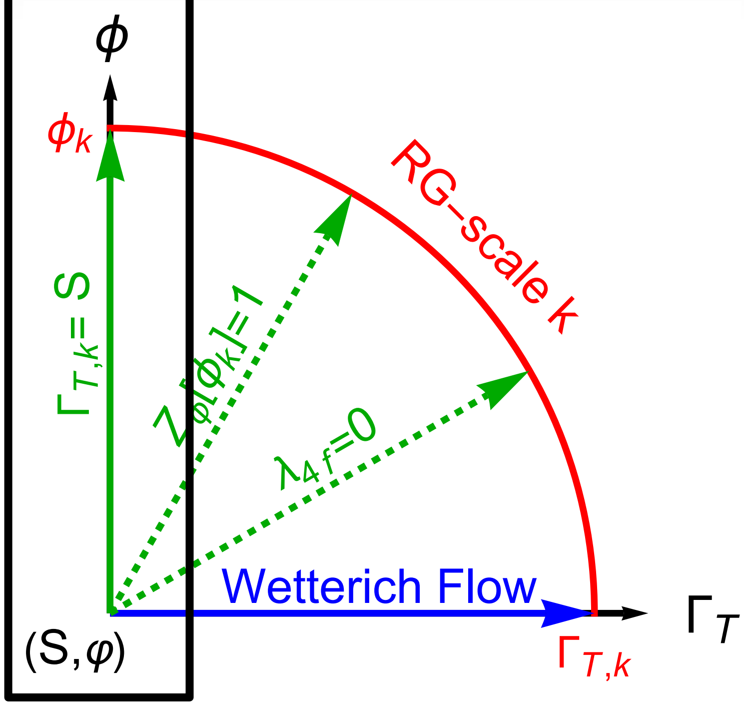

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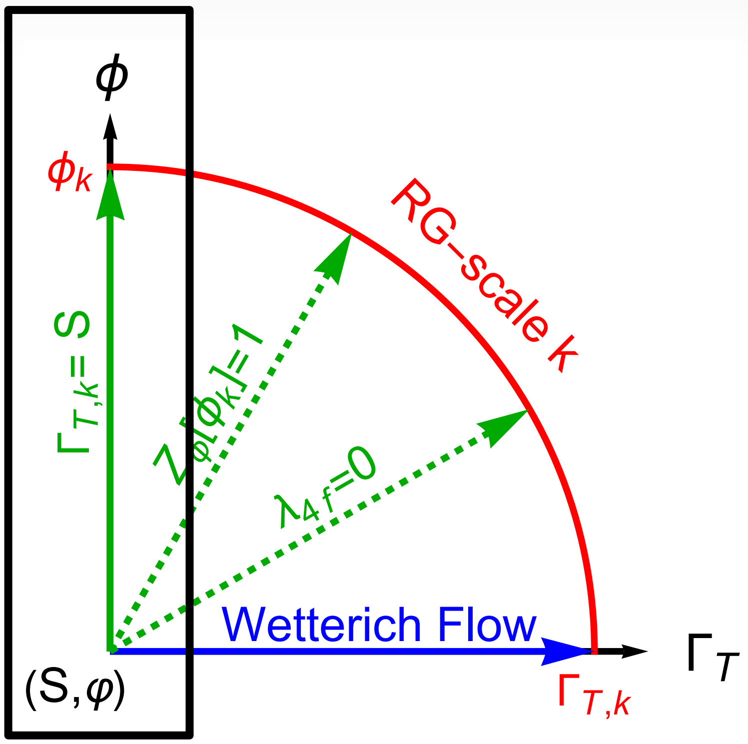

- The potential does NOT flow

$$\Gamma_T[\phi] = S[\phi] + \mathcal{C}_k$$

=> Entirety of physics in the field transformation

- Benchmark in d=0

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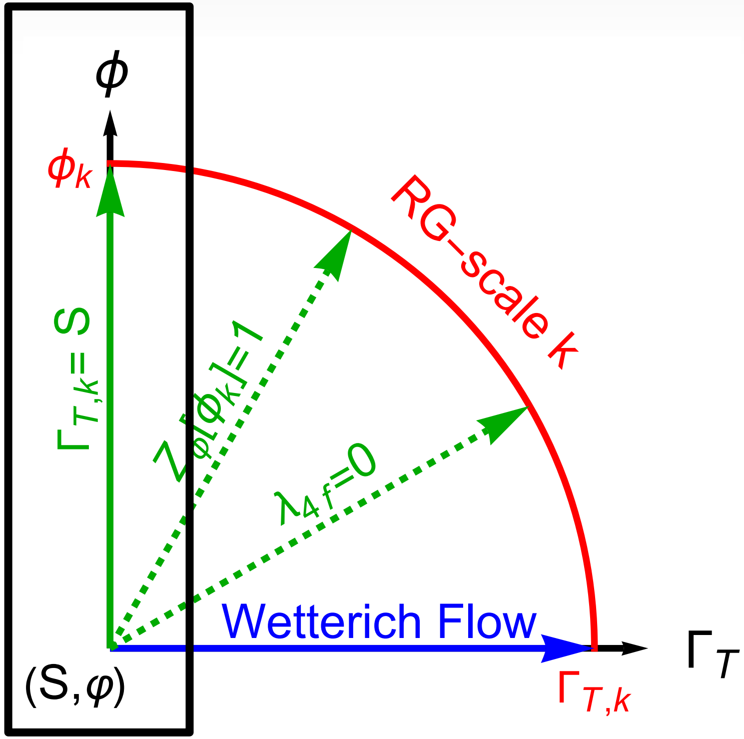

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(+) Comparison to exact solution
(-) No estimate of 'truncation artefacts'

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Existence?

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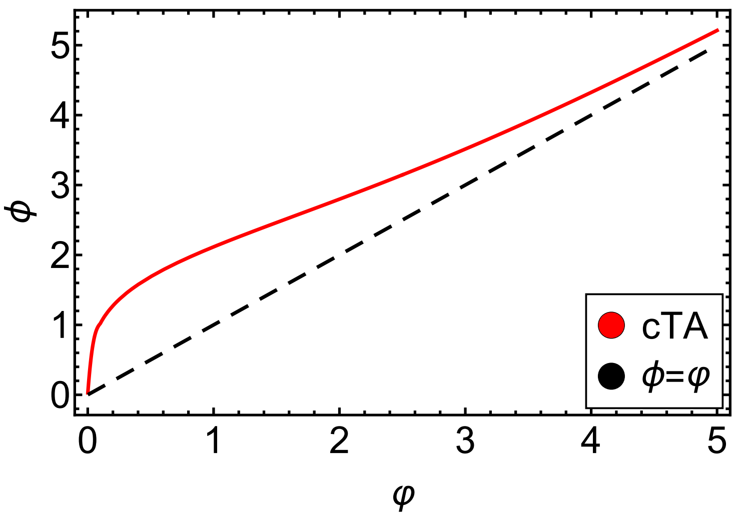

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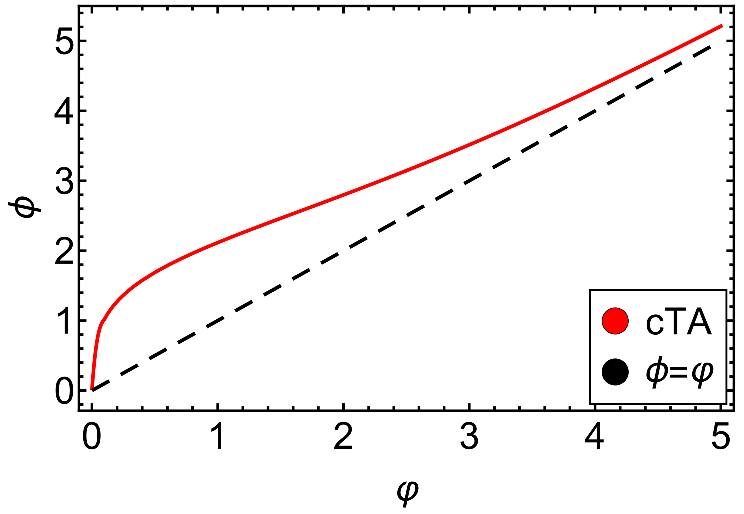

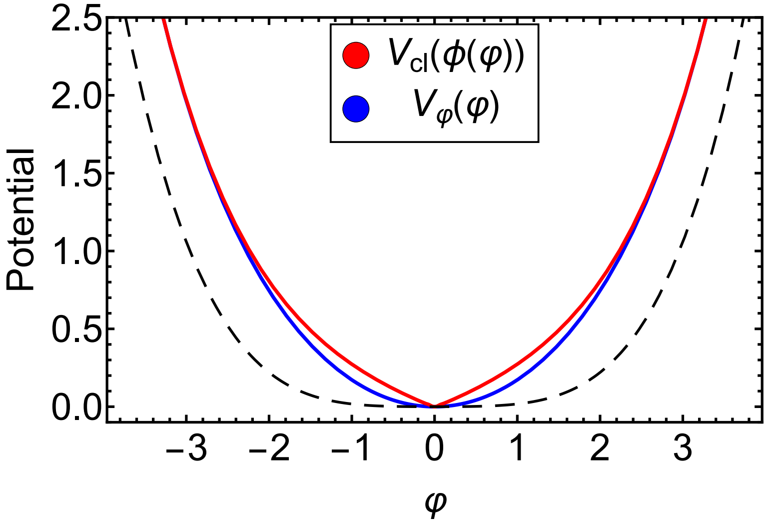

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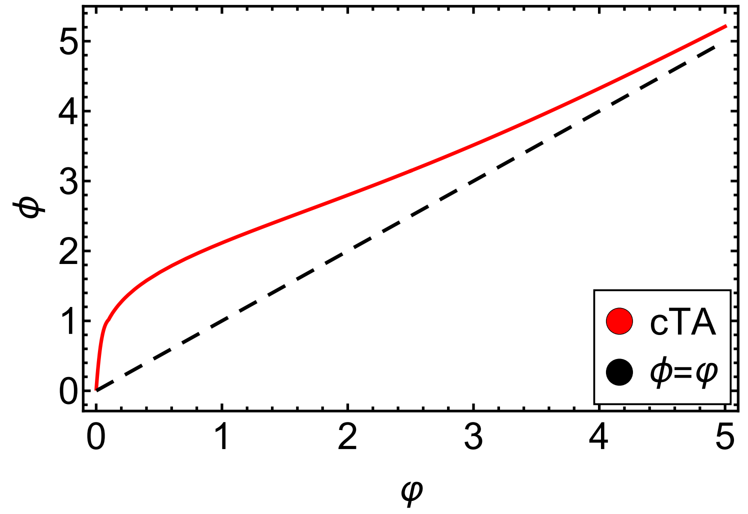
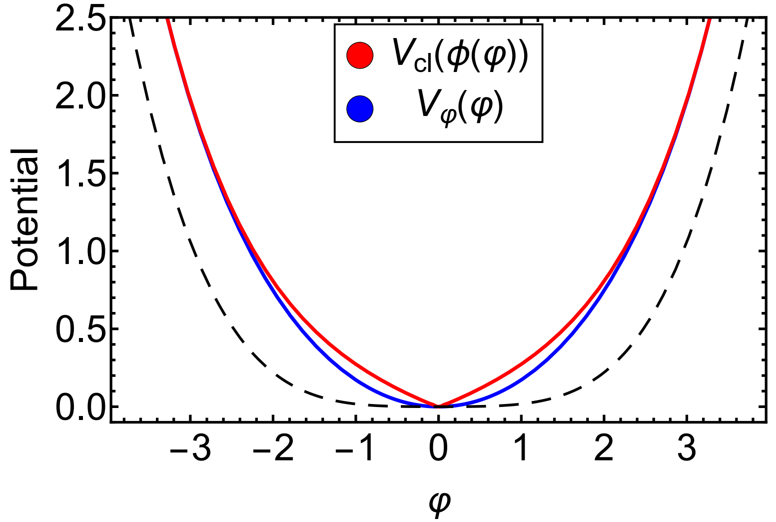
$$\boxed{\dot{\phi}\, V_T^{(1)} - \frac{2}{d+2}\dot{\phi}'\,\frac{A_d k^{d+2}}{k^2+V_T^{(2)}}} = \boxed{\frac{A_d k^{d+2}}{k^2+V_T^{(2)}} - \partial_t V_T}$$

2) Global existence of $\phi_k[\varphi]$

$$\phi[\varphi] = \varphi + \int_\Lambda^0 \frac{dk}{k}\,\dot{\phi}[\phi]$$

=> Potentials do not coincide in d=0!





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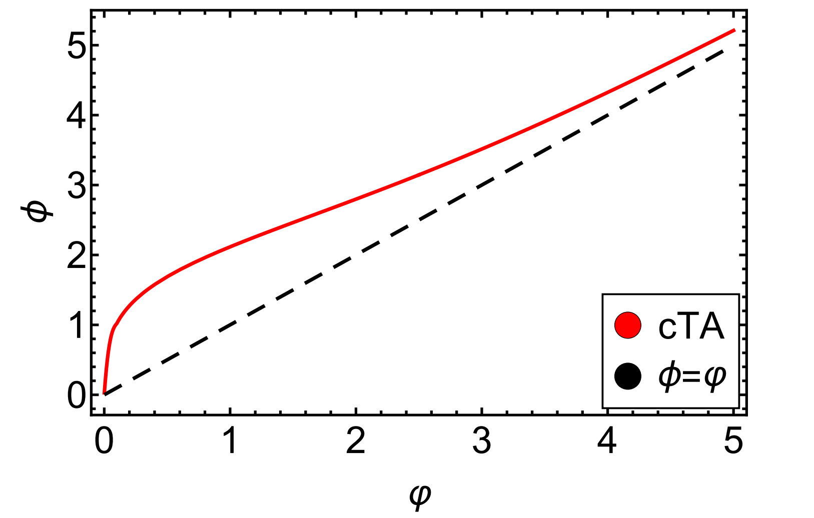

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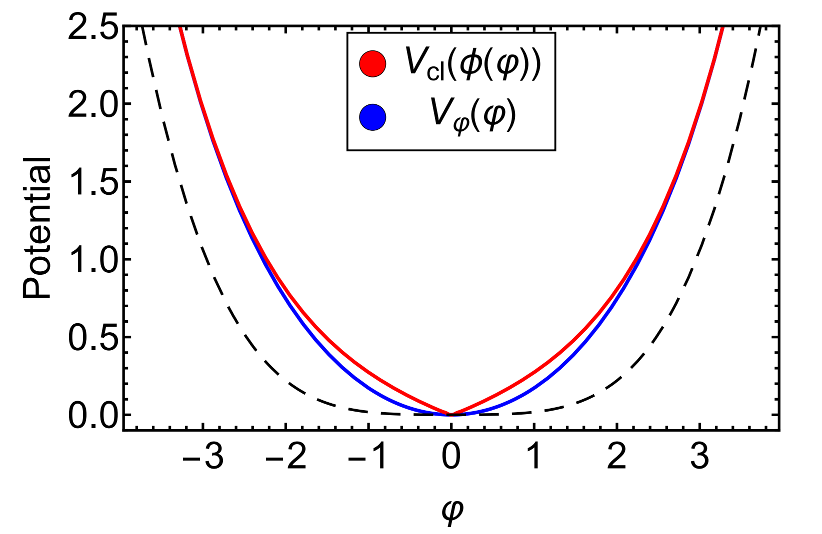

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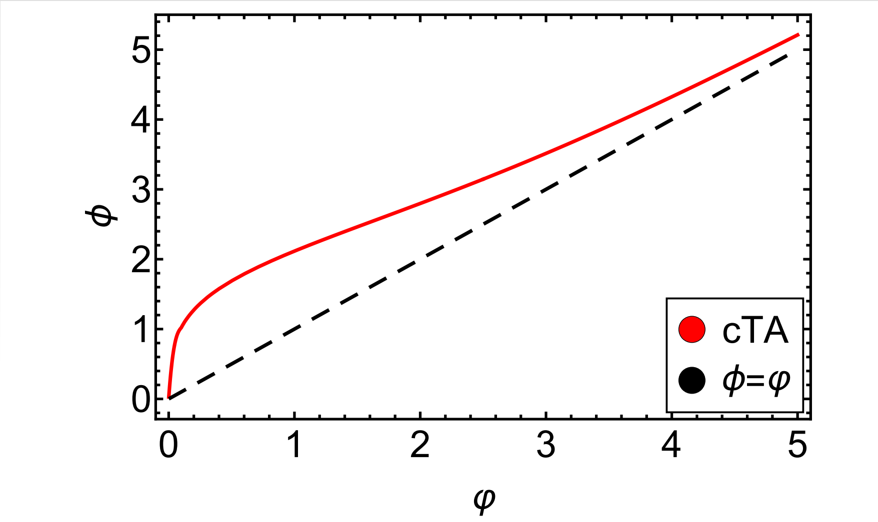

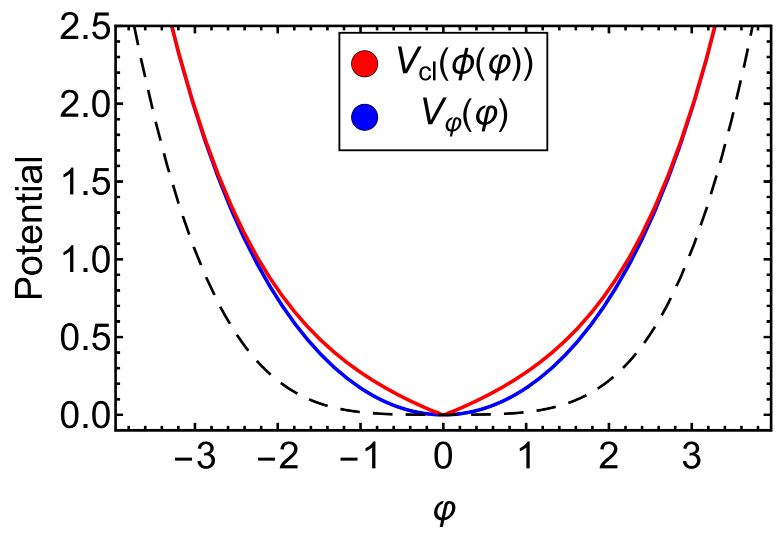

Reconstruction

Comprehensive test in d=0:

=> Physics needs to coincide

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1) Direct construction via the field transformation

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Comprehensive test in d=0:

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2) Reconstruction of correlation functions

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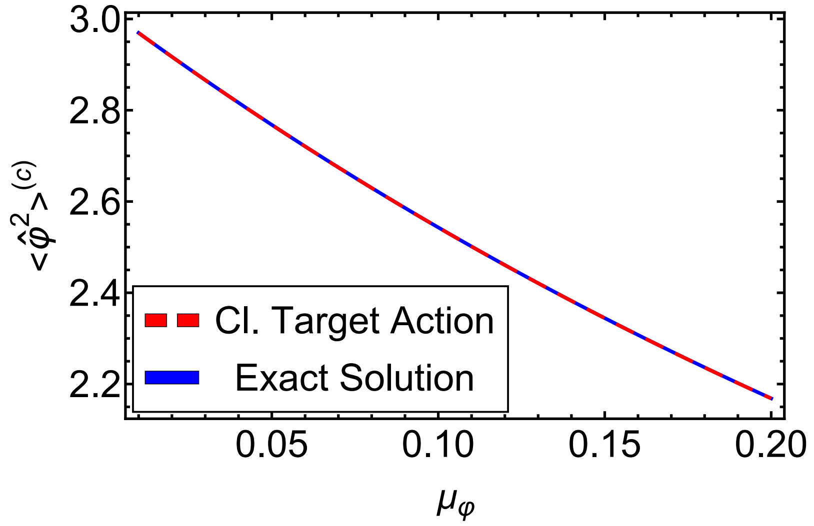

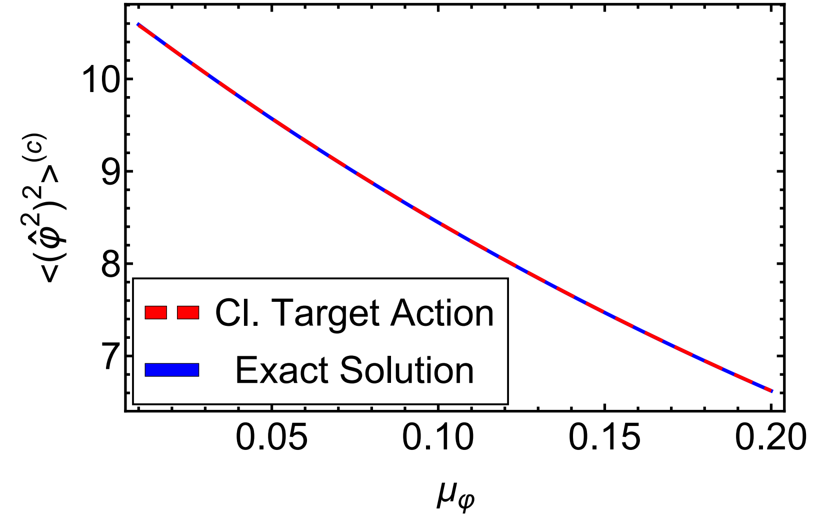

Classical target action flows

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Lets go to d=3

→ Non-convex target action

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$$\left\{ (\phi, \Gamma_\phi[\phi]) \,\middle|\, \Gamma^{(2)}_{\phi,k}[\phi] + R_{\phi,k} \geq 0 \quad \forall\, \phi \in \mathcal{S}_\phi \right\}$$

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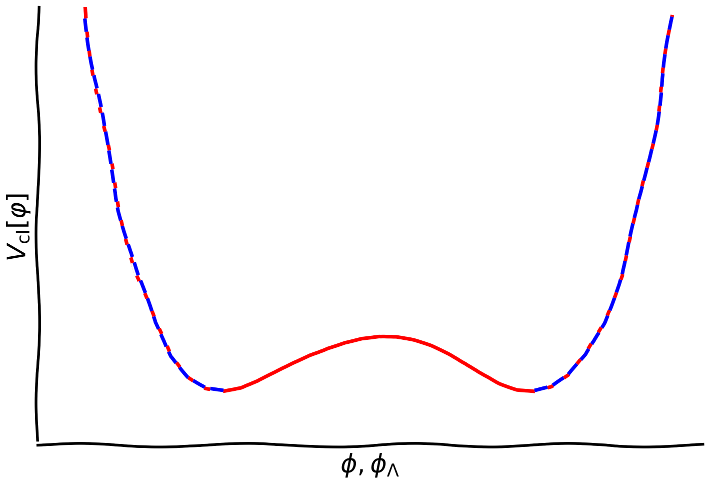

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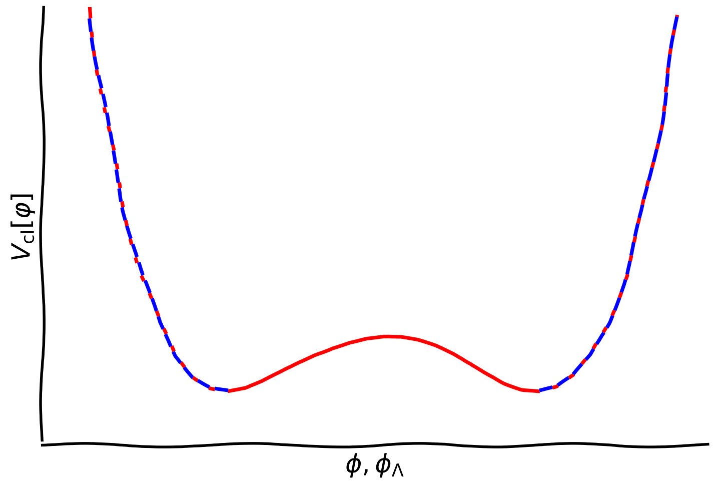

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→ Method of characteristics:

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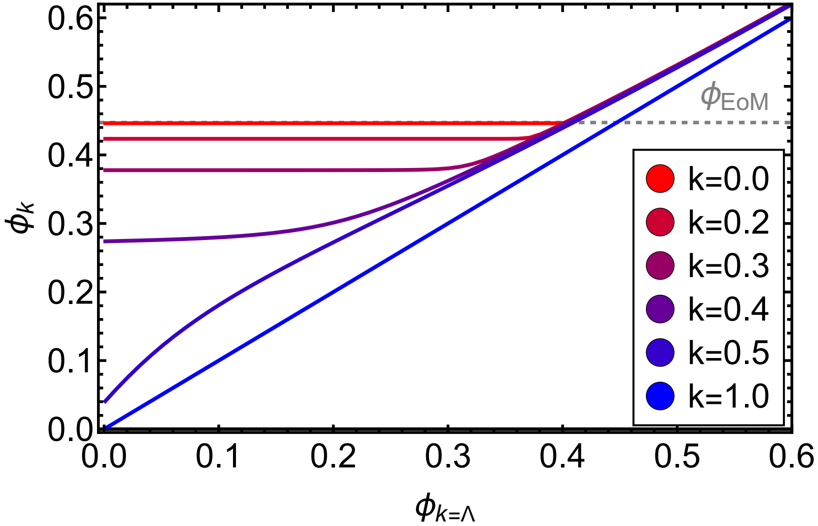

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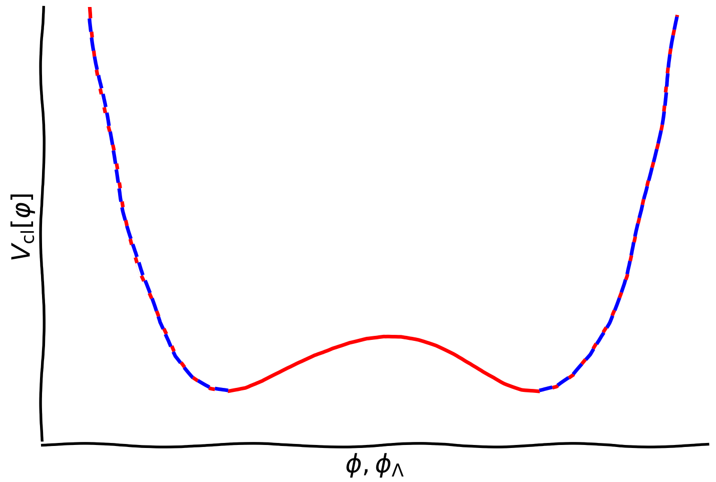

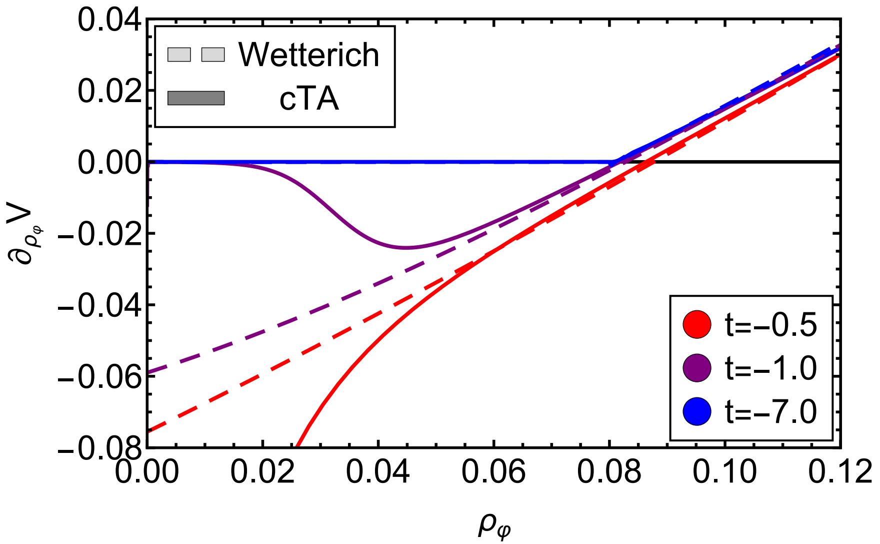

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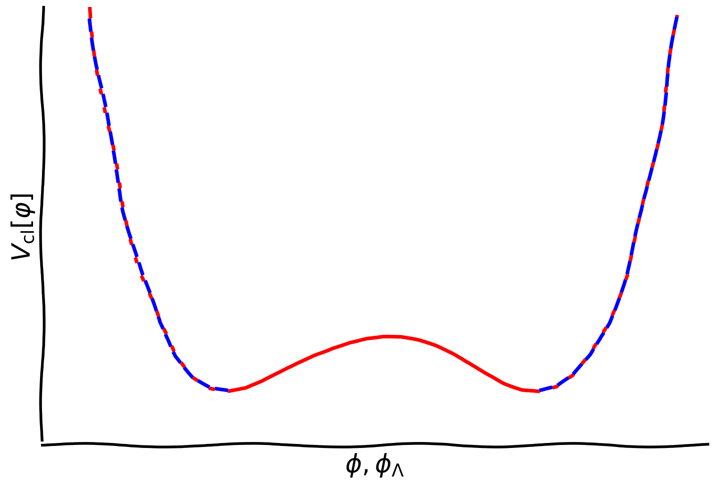
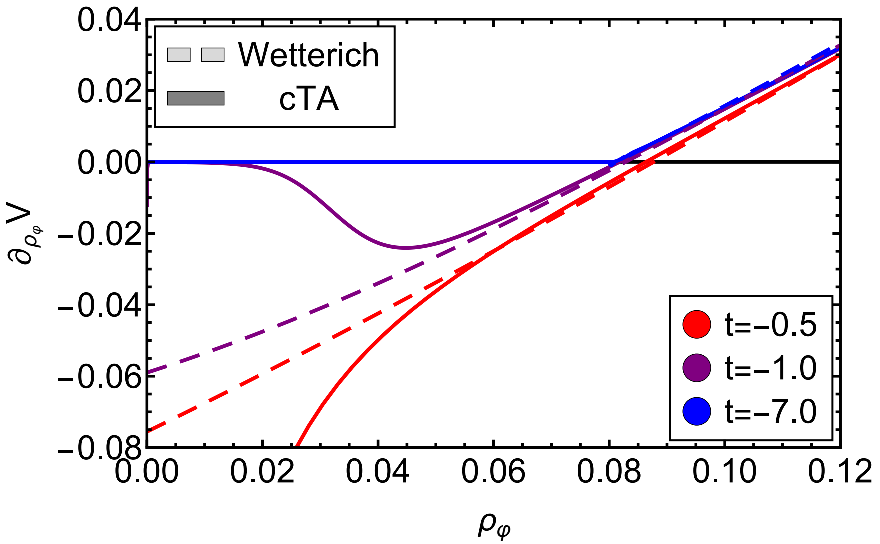
Reconstruction for Phase-Transition





Directions for Optimisation

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Convergence of expansion

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Computational complexity

Directions for Optimisation

Convergence of expansion

Example: 1st order derivative expansion

$$\partial_t Z_\phi[\phi](p) \overset{!}{=} 0 \quad \longrightarrow \quad \dot{\phi}_k[\phi]$$

Computational complexity

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Directions for Optimisation

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→ here 1st order derivative

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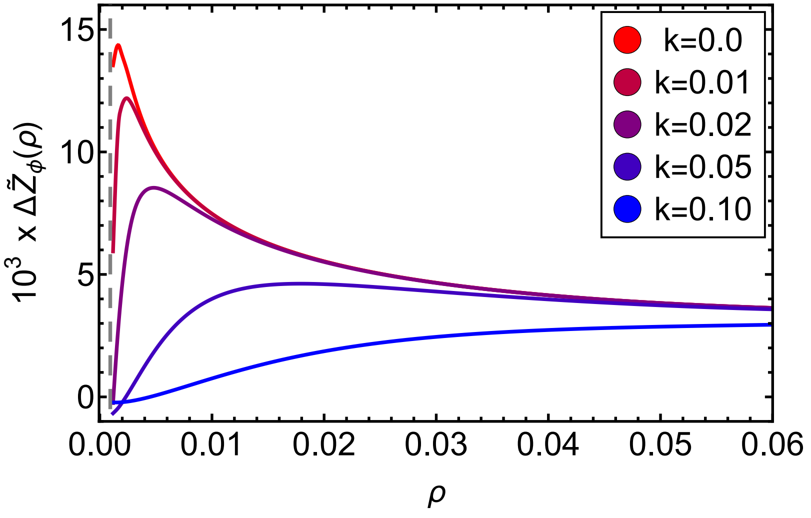

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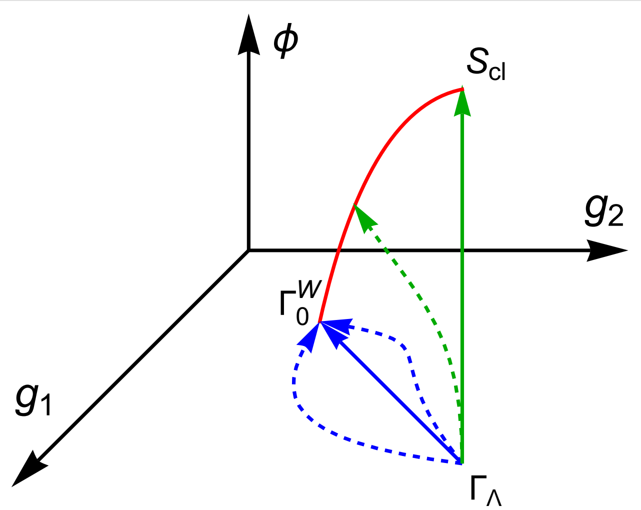
Summary & Conclusion



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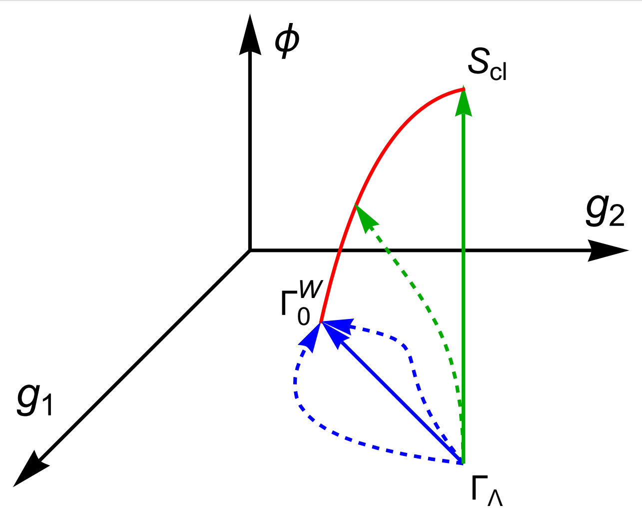
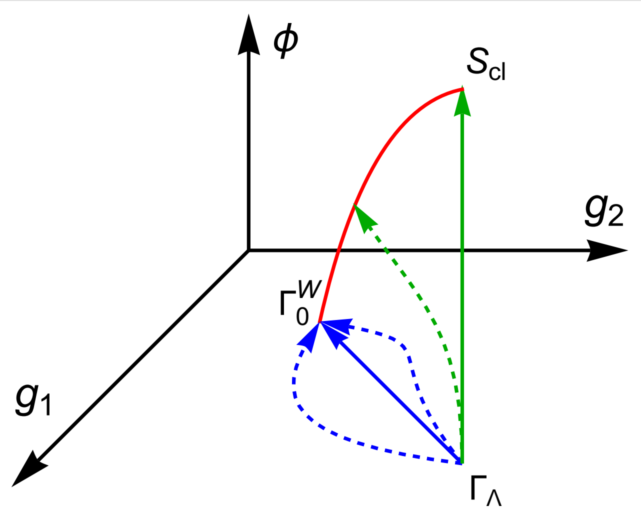


- Generalised flow equation for composite operators

Summary & Conclusion



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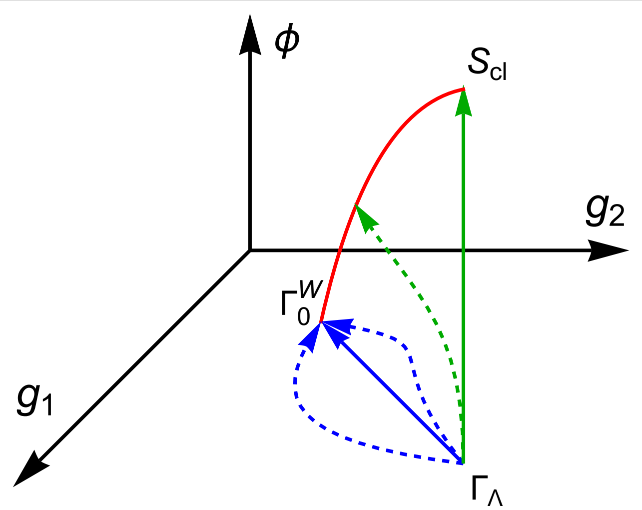

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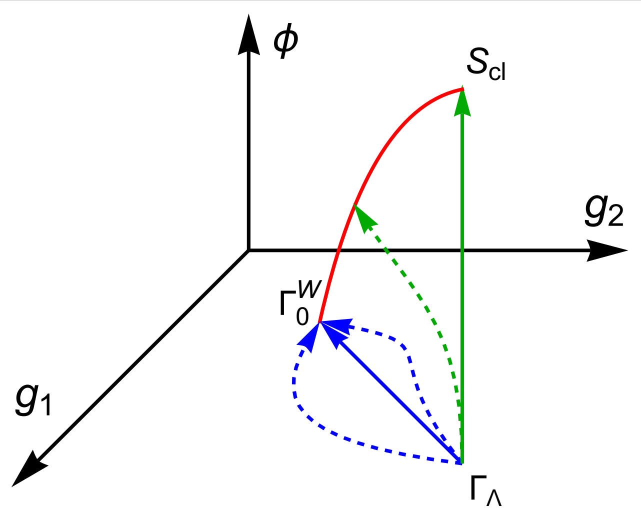

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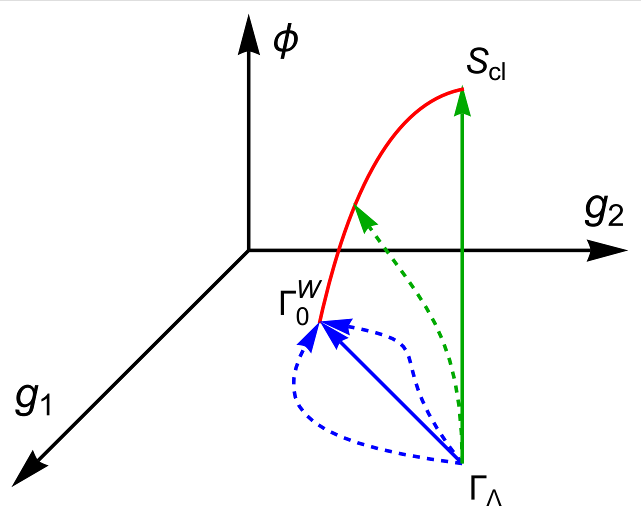

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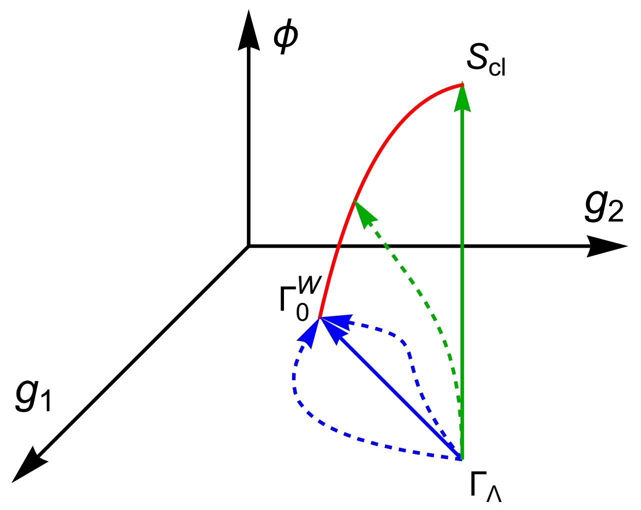
Summary & Conclusion



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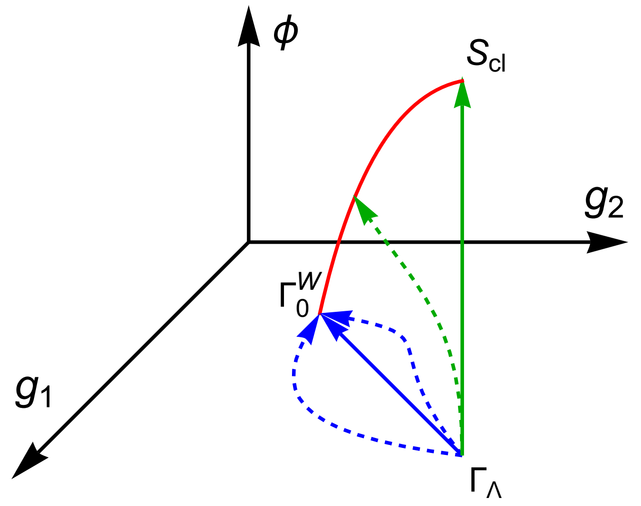
- Two types of applications

Summary & Conclusion



- Generalised flow equation for composite operators
  - → Flowing fields $\phi = \langle \hat{\phi}[\hat{\varphi}] \rangle$ $\dot{\phi}[\phi] = \langle \partial_t \hat{\phi}_k \rangle [\phi]$
  - → Existence $\phi[\varphi] = \varphi + \int_\Lambda^0 \frac{dk}{k}\, \dot{\phi}[\phi]$
  - → Reconstruction $\Big\langle \prod_{i=1}^{n} \int_{x_i} \hat{\varphi}^2(x_i) \Big\rangle^{(c)} = (-2)^n \frac{d^n \log Z_\phi^{(c)}[0]}{d(\mu_\varphi)^n}$
- PIRG setup with flowing fields and target action

$$\big(\Gamma_T \,,\, \dot{\phi}[\phi]\big)$$

- Two types of applications
  - → Physically motivated: Ground state expansion, dynamical hadronisation
  - → Computationally motivated: Classical target action, Feed-down flows

Thank you for your attention