Towards the phase diagram of QCD and its critical endpoint

Based on
[arXiv:2408.08413]

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ITP Heidelberg

In Collaboration with
Friederike Ihssen, Jan M. Pawlowski, Nicolas Wink

ERG2024,

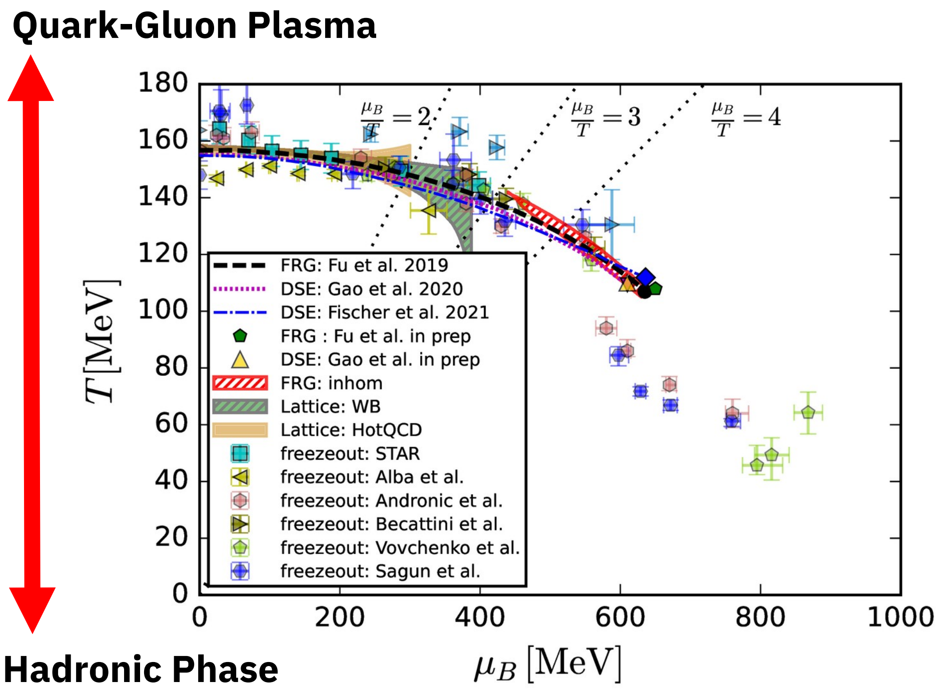
Les Diablerets

September 23, 2024

STRUCTURES CLUSTER OF EXCELLENCE

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

The QCD phase diagram



Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]
Gao, Pawlowski [Phys.Lett.B820(2021) 136584]
Gunkel, Fischer [Phys.Rev.D 104 (2021) 5, 054022]
Bellwied et al. (WB) [Phys.Lett.B 751 (2015) 559-564]
Bazavov et al. (HotQCD) [Phys.Lett.B 795 (2019) 15-21]

Phase diagram shows
Chiral symmetry breaking
i.e. condensation of $\langle \bar{q}q \rangle$

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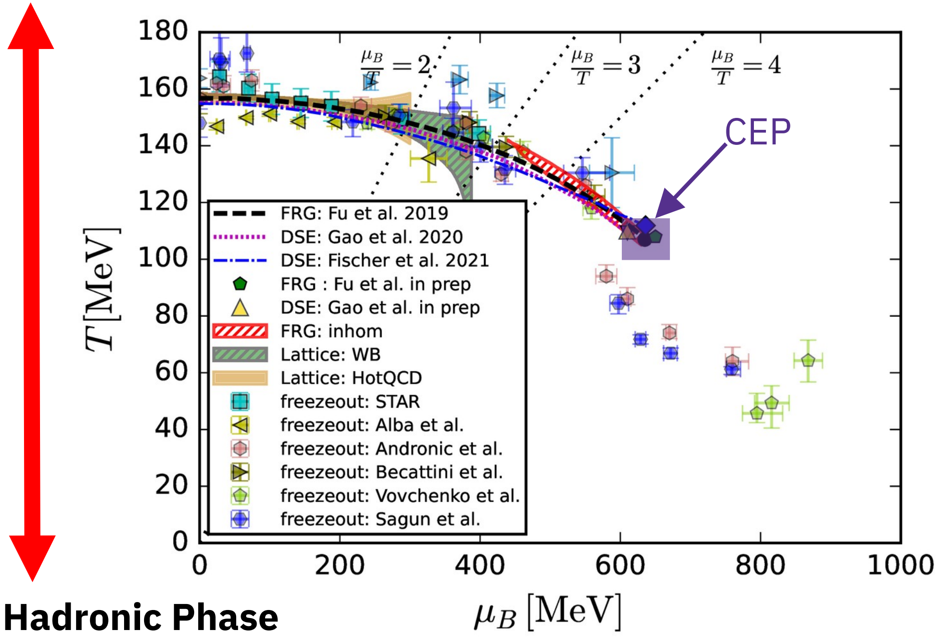

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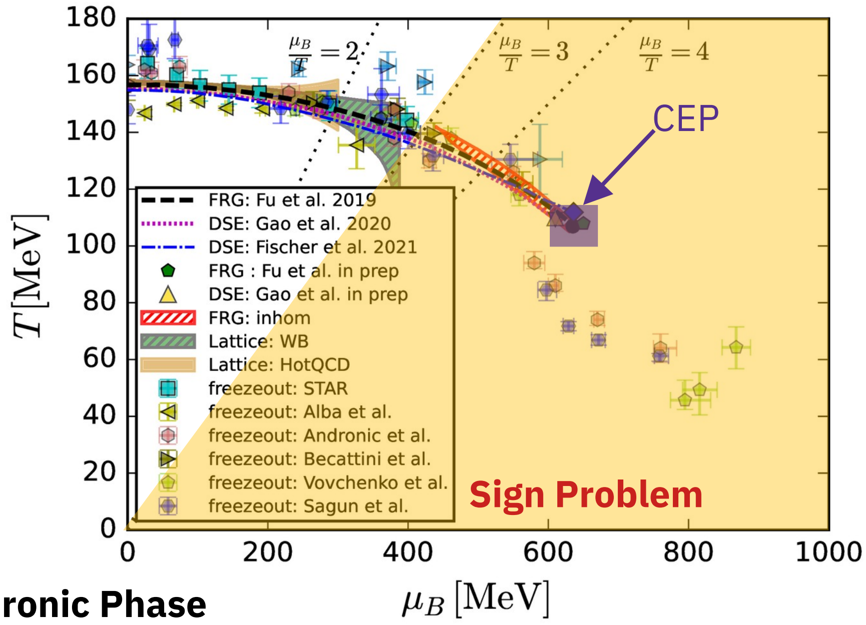
The QCD phase diagram

Quark-Gluon Plasma



Hadronic Phase

Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]
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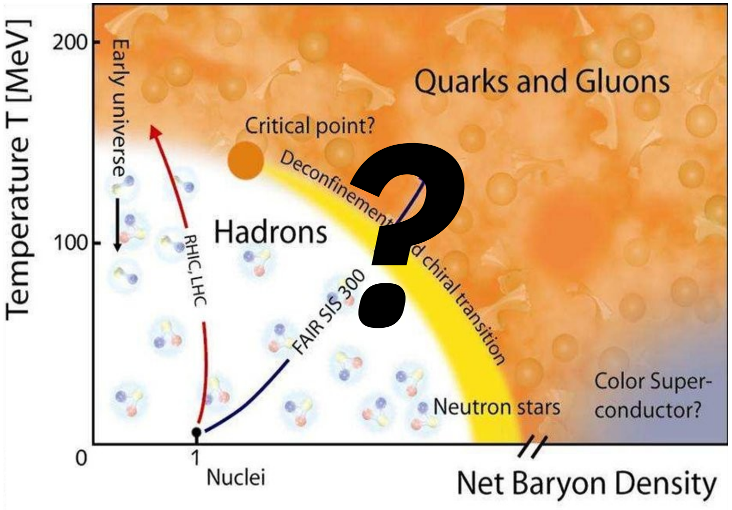
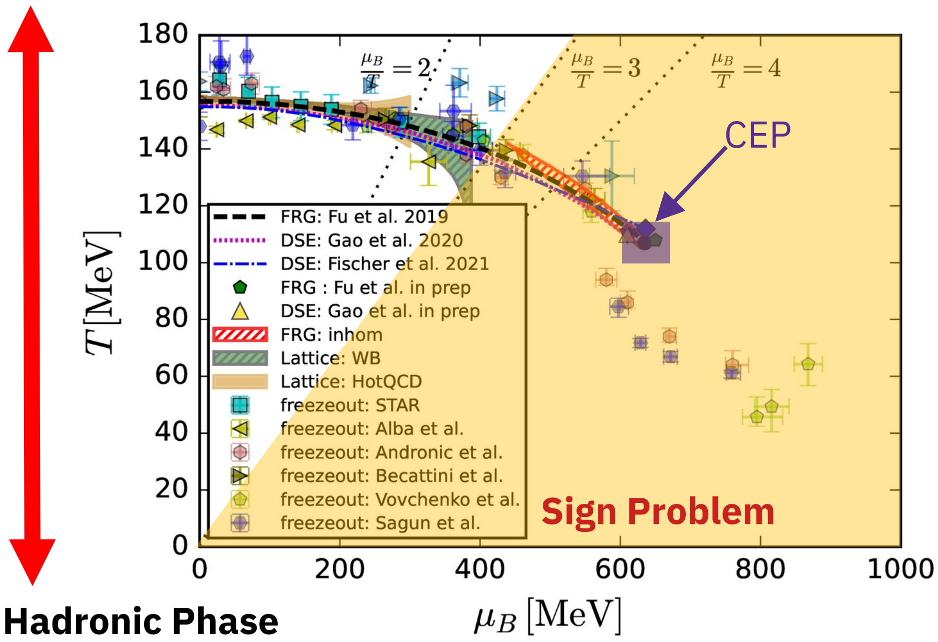
Phase Diagram for intermediate μ not know



Sketch by GSI Darmstadt

The QCD phase diagram



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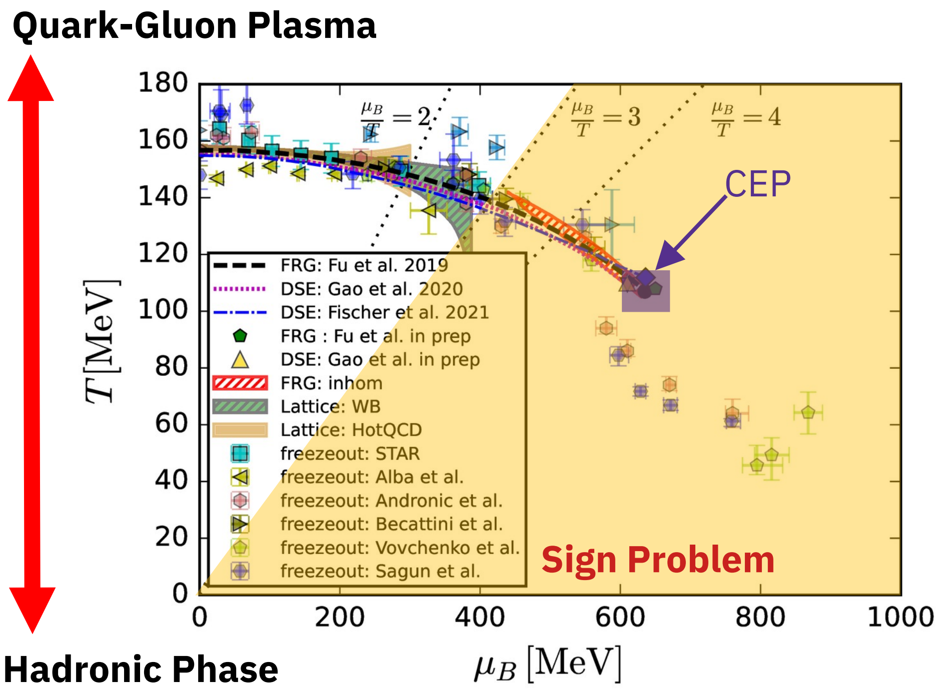

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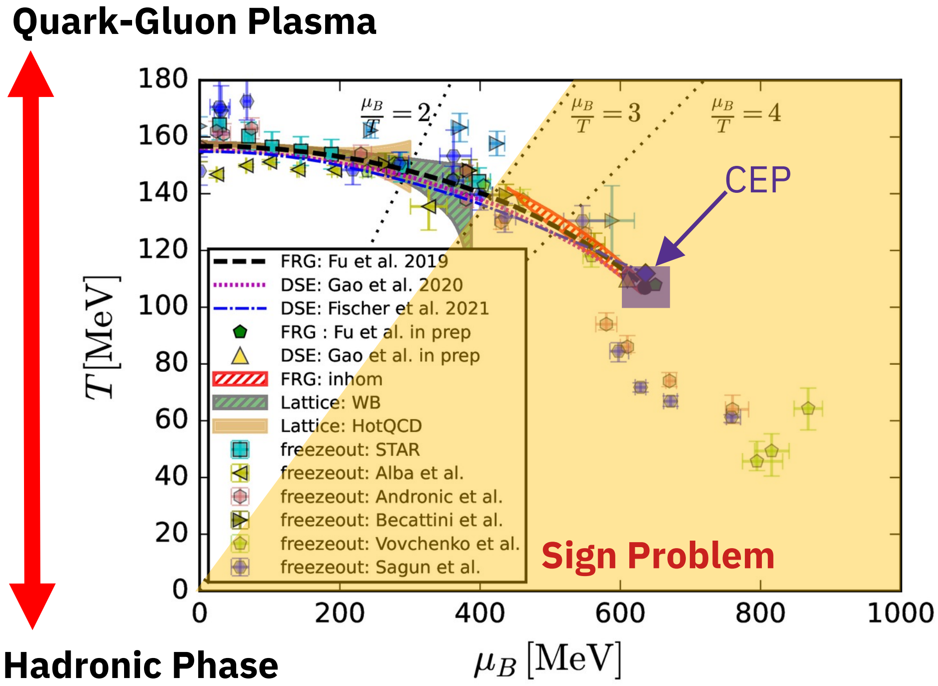
Direct access to phase structure using the

functional Renormalization Group

Nnon-perturbative, with **direct access to finite μ**.

The QCD phase diagram



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Direct access to phase structure using the

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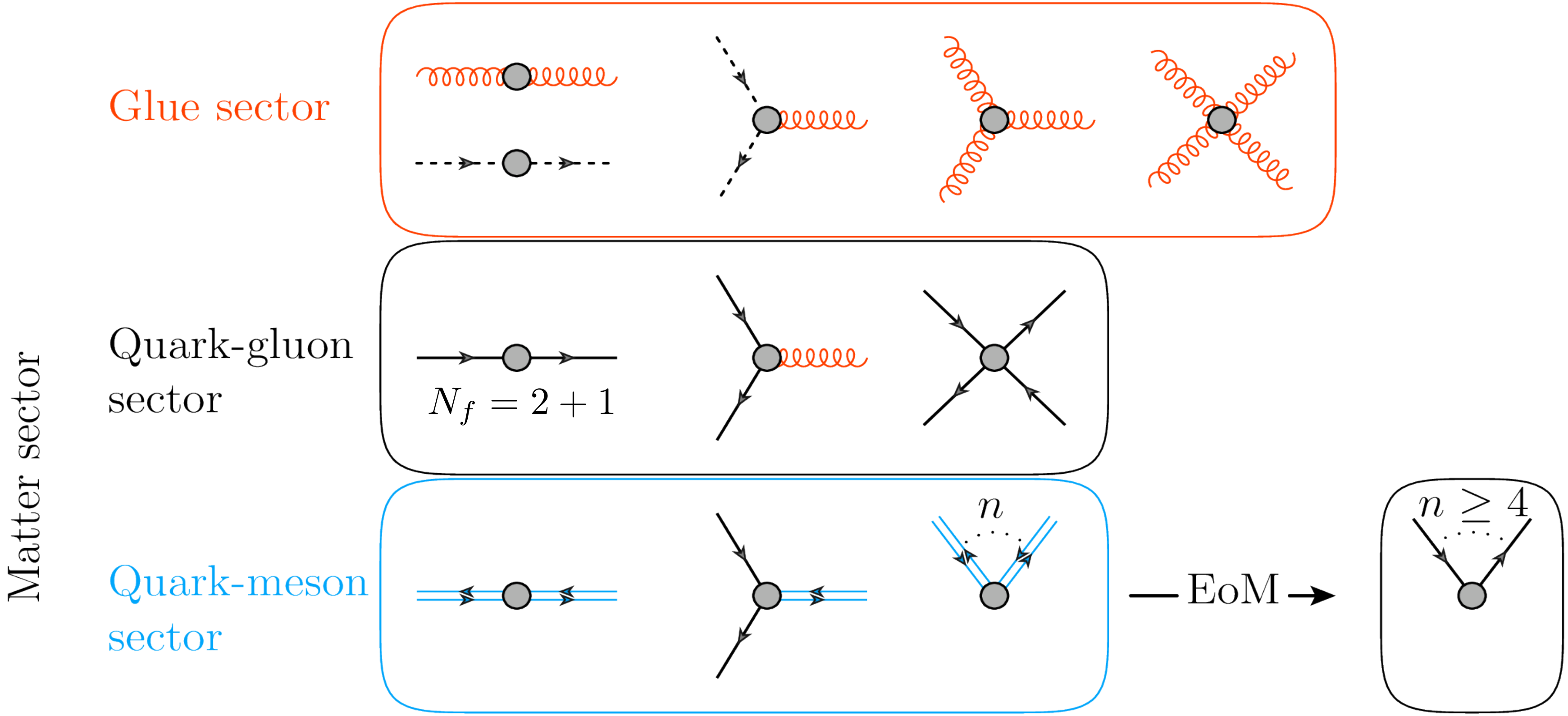
Here, first step:

- Setup
- Systematics

Vacuum

Ihssen, Pawlowski, Sattler, Wink
[arXiv:2408.08413]

Current vertex expansion



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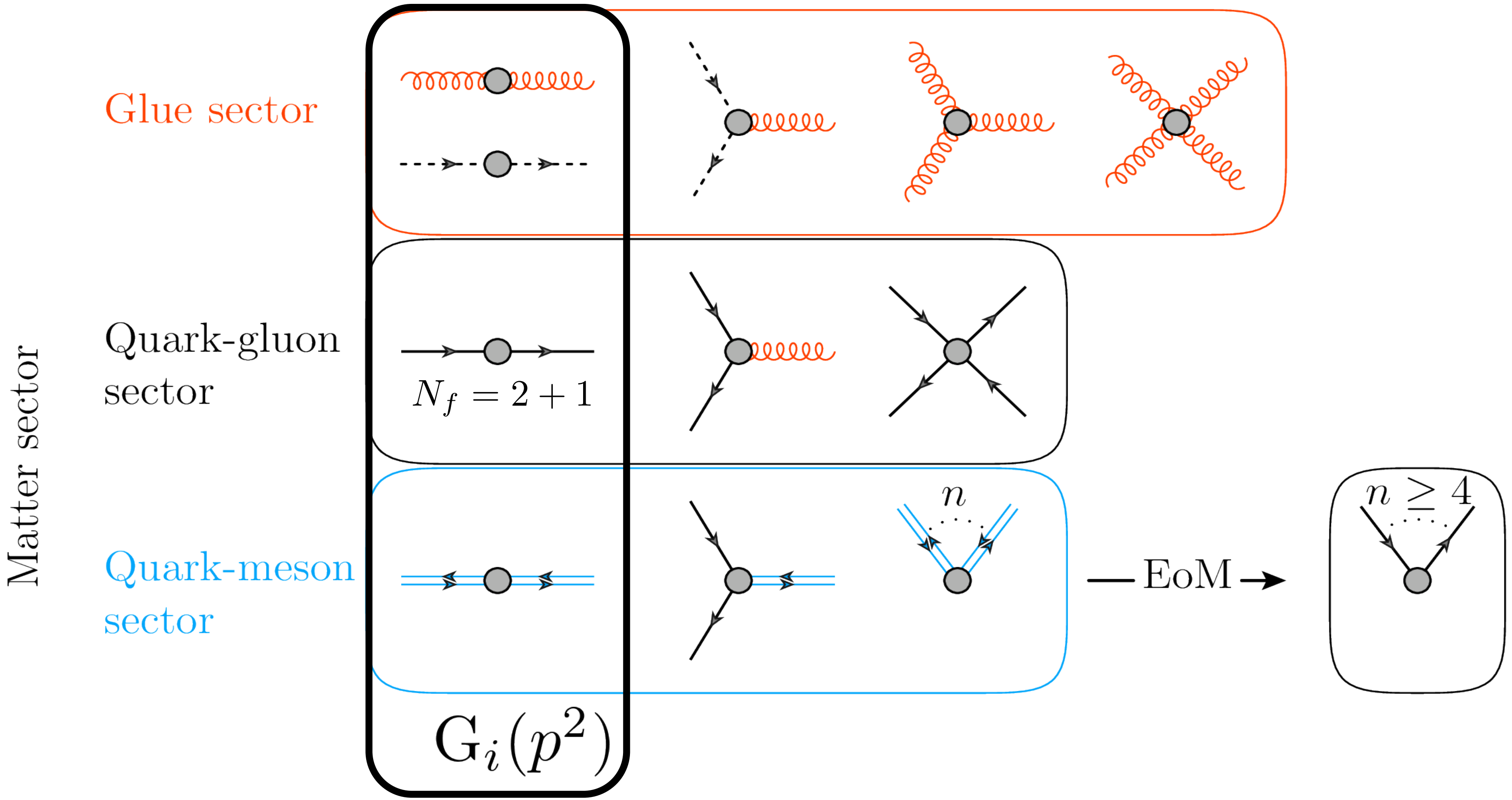

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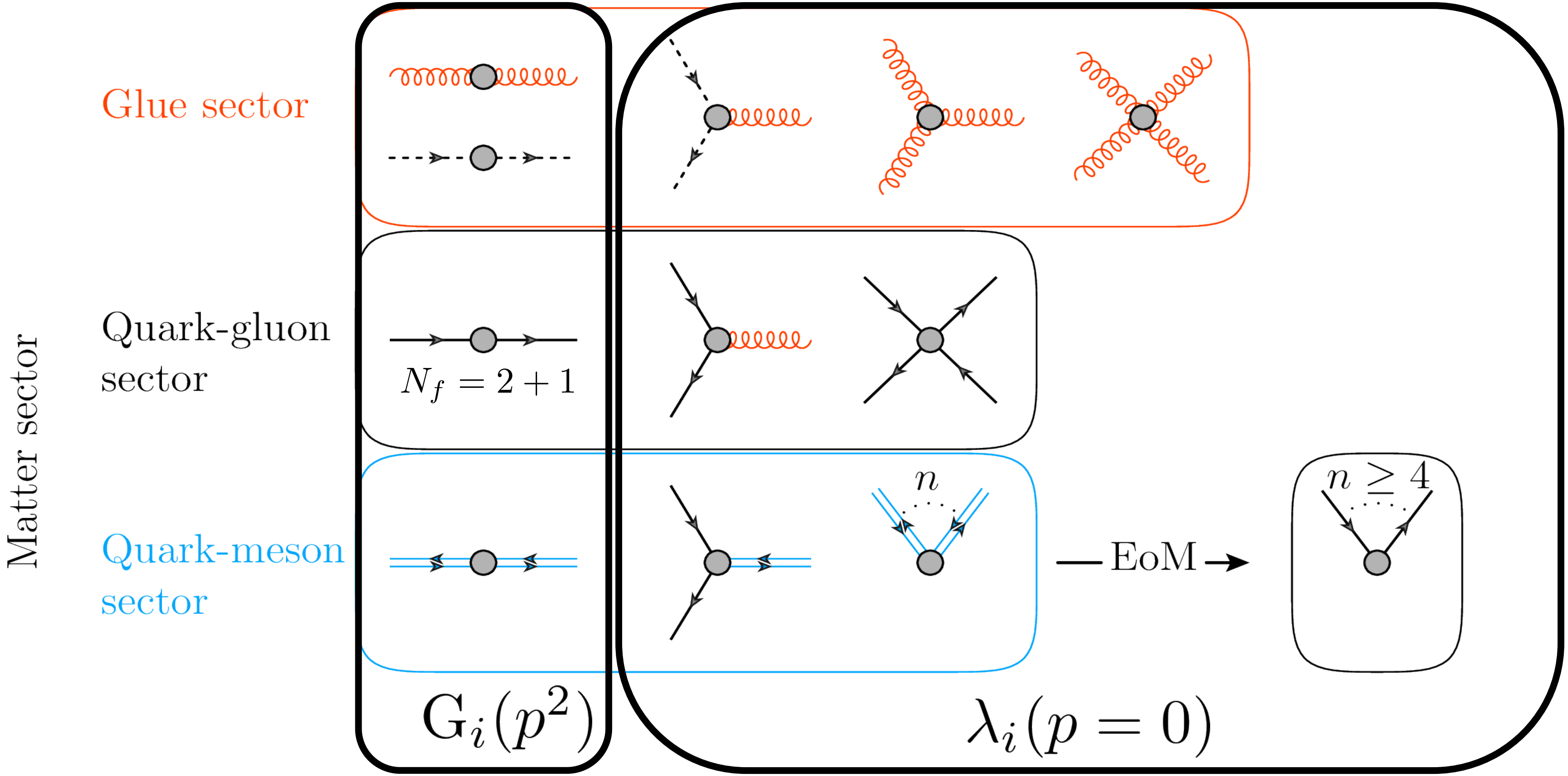

TensorBases Mathematica package

With J. Braun,
J. Pawlowski, A. Geißel,
N. Wink

- Automatically derived projectors
- Library of tensor bases, extendable by everyone

```
Needs["TensorBases`"]

In[2]:= TBGetProjector["transAqbq", 1, {p1, mu, a}, {p2, d2, A2, F2}, {p3, d3, A3, F3}]
TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 1] // FormTrace // Simplify
TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 8] // FormTrace // Simplify
```

Out[2]= $\frac{1}{6\,\mathrm{Nf} - 6\,\mathrm{Nc}^2\,\mathrm{Nf}}\, i\, \mathrm{deltaFundFlav}[F2, F3] \times \mathrm{gamma}[nu\$20834, d2, d3] \times \mathrm{TCol}[a, A2, A3] \times \mathrm{transProj}[p1, mu, nu\$20834]$

Out[3]= 1

Out[4]= 0

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DiFfRG framework

- Automatically derived projectors
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- Automatic derivation and code generation for large fRG systems
- Hydrodynamic methods for full field dependences
- GPU accelerated

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Open Source available around November

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- Hydrodynamic methods for full field dependences
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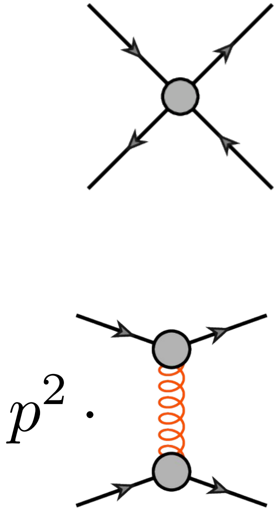
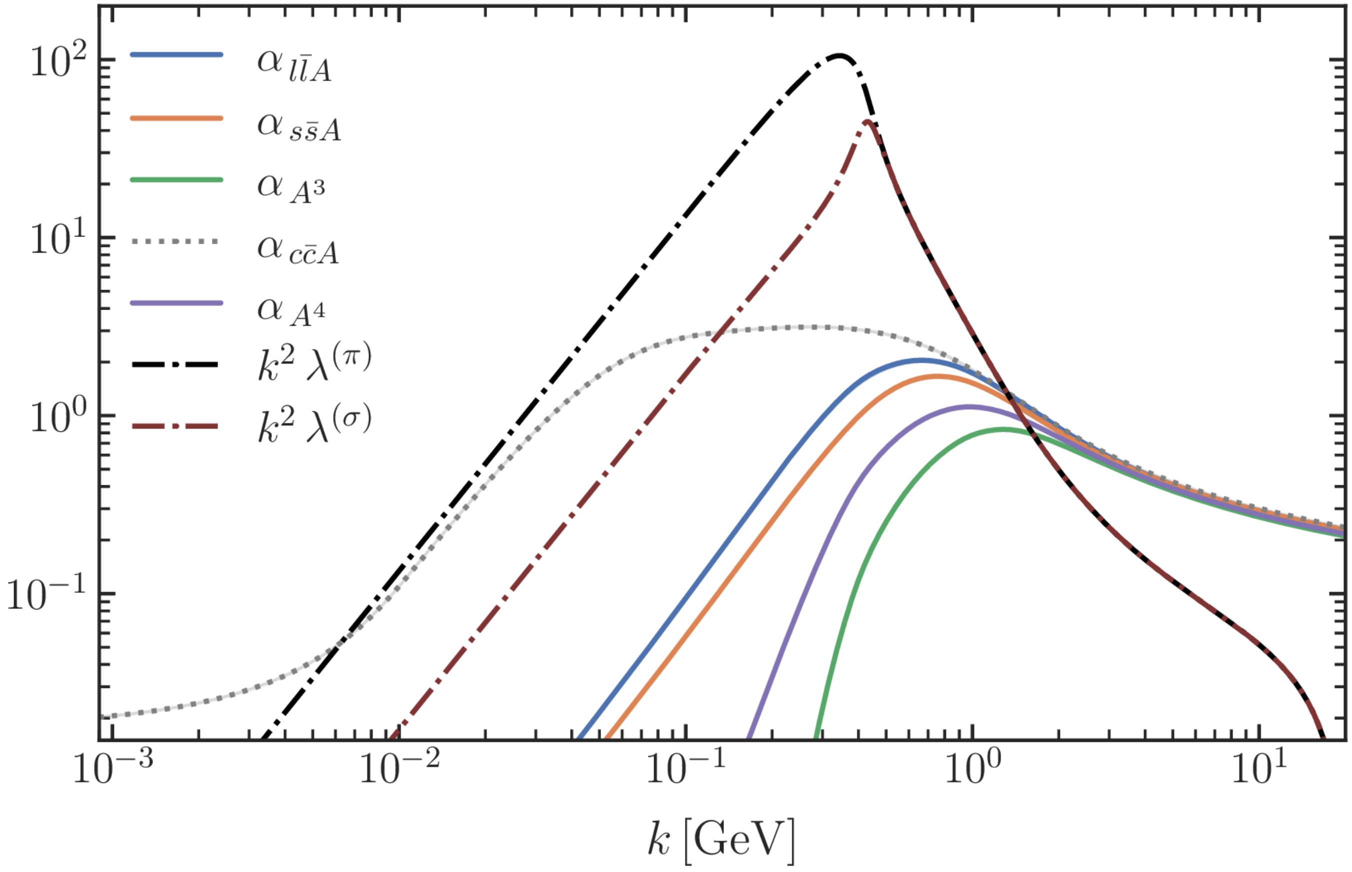
Dynamical Hadronisation in fRG





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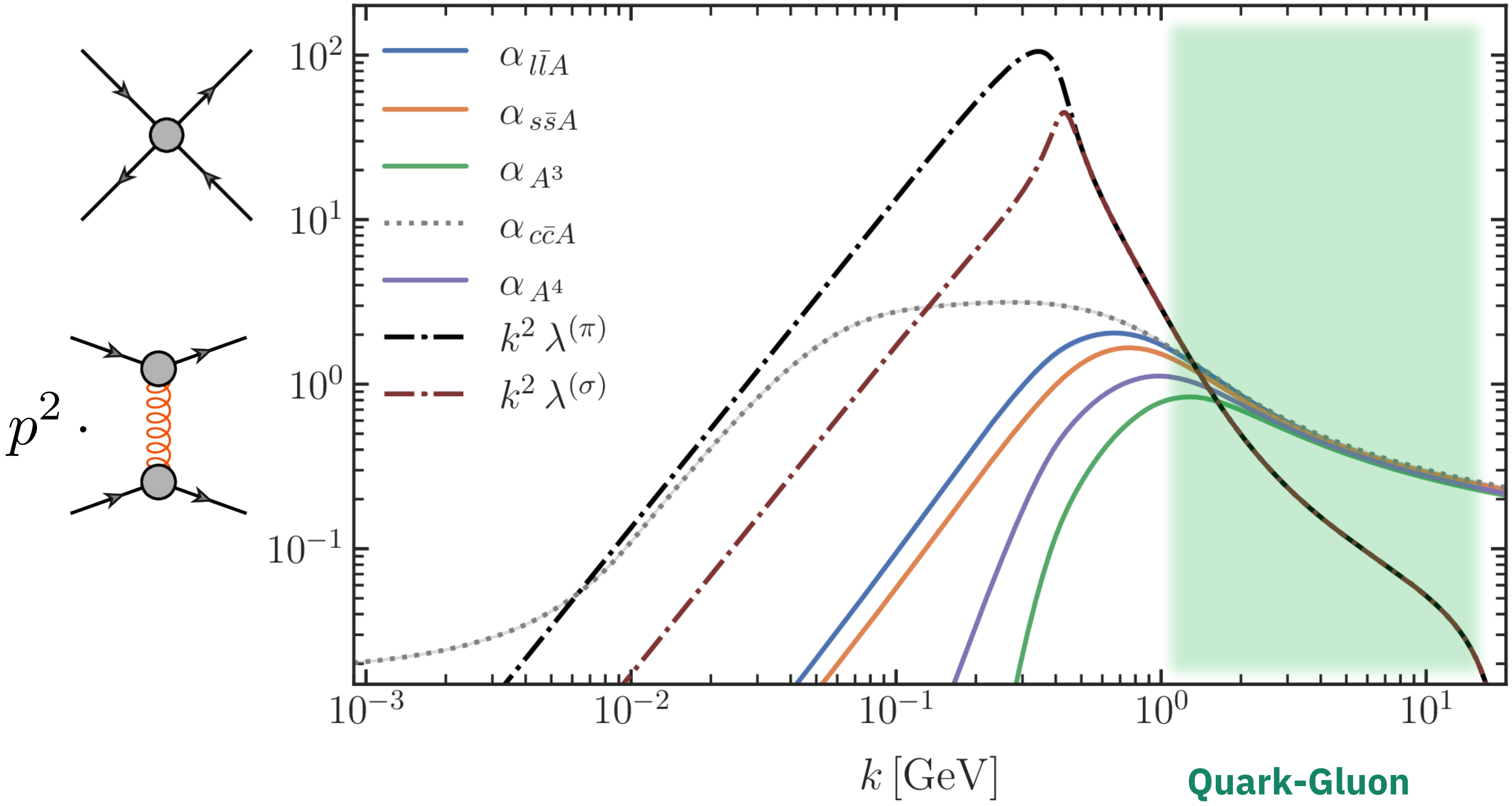
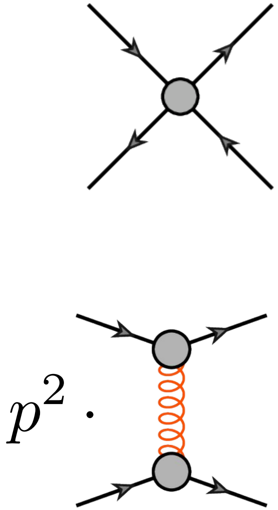
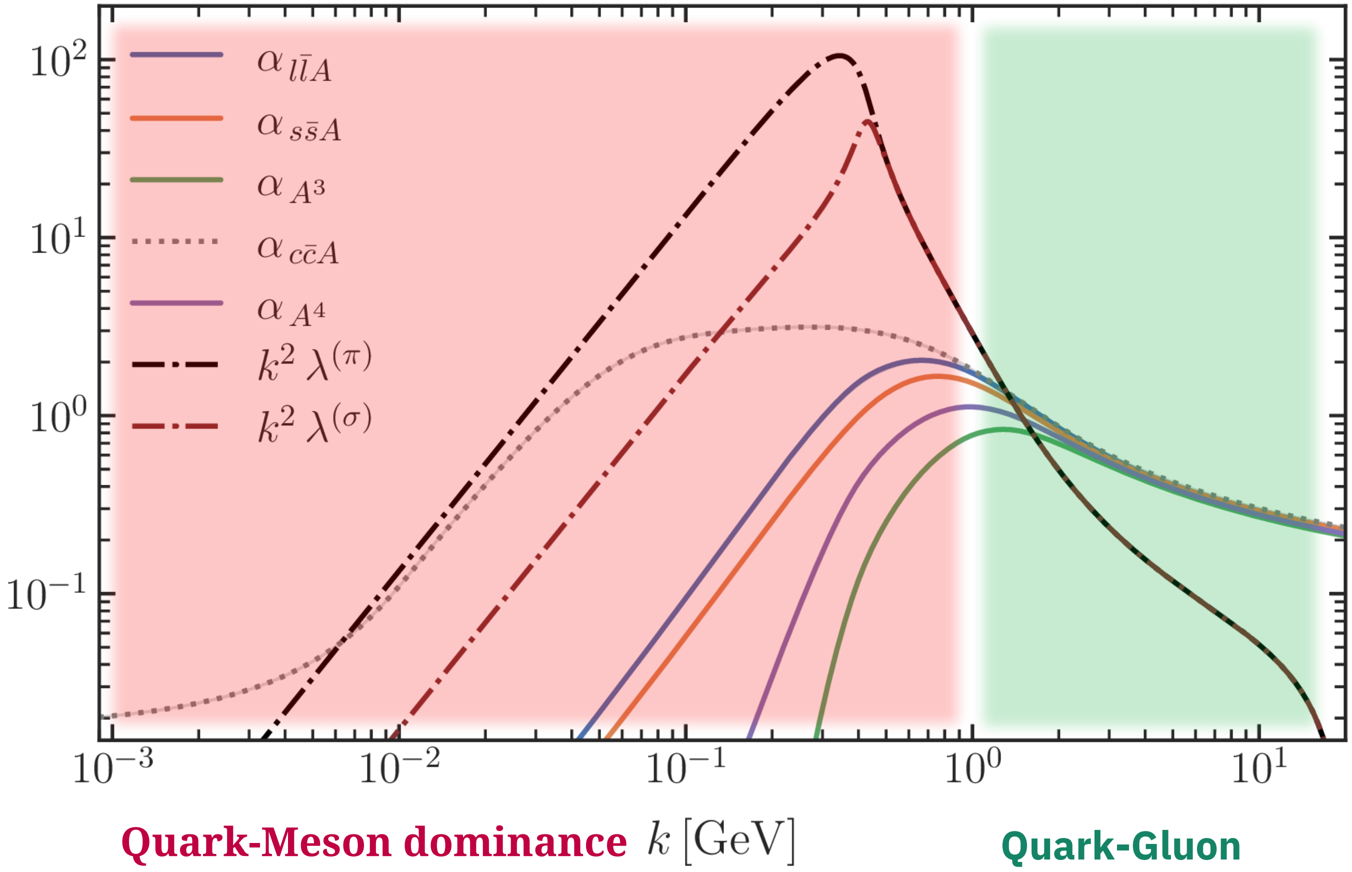


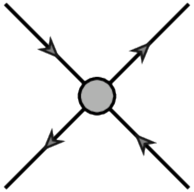
Quark-Gluon dominance

Dynamical Hadronisation in fRG





$\sigma - \pi -$ four-quark flow



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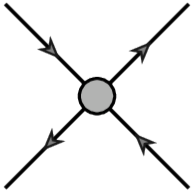

(Resonant)

Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

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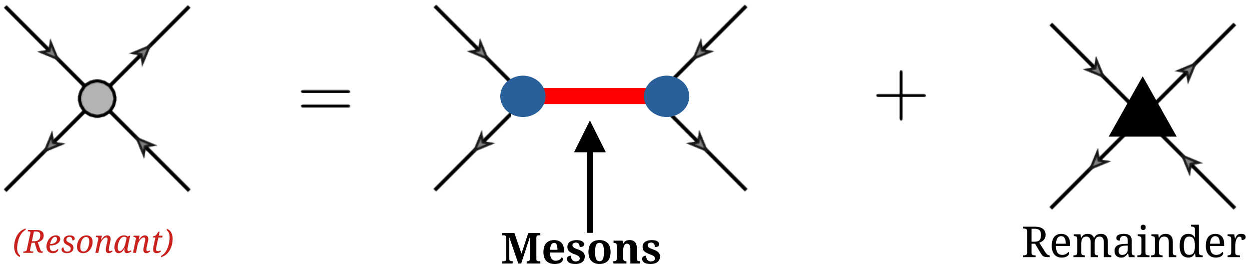

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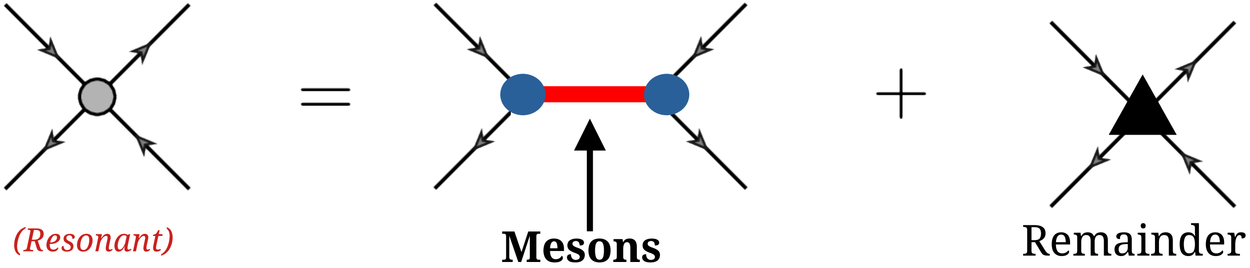

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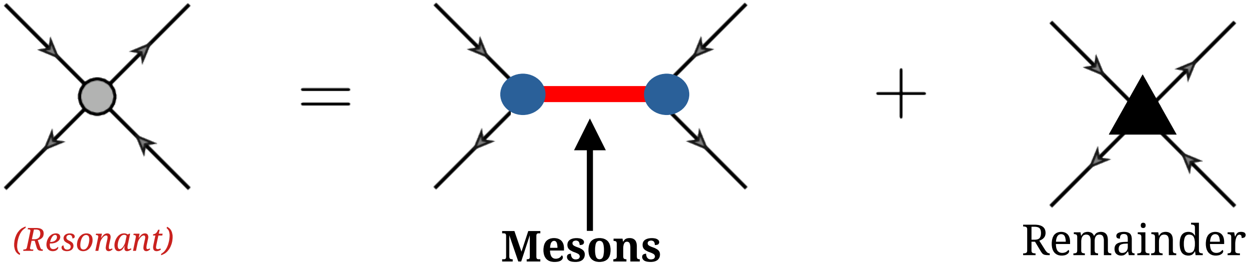
BS WF

$$h_\phi(p,q) \cdot \frac{1}{Z_\phi(t^2)(t^2+m_\phi^2)} \cdot h_\phi(p,q)$$

Meson propagator

$\sigma - \pi -$ four-quark flow



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Full scattering potential

$$V\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{2}\right)$$

All orders of n-meson scatterings

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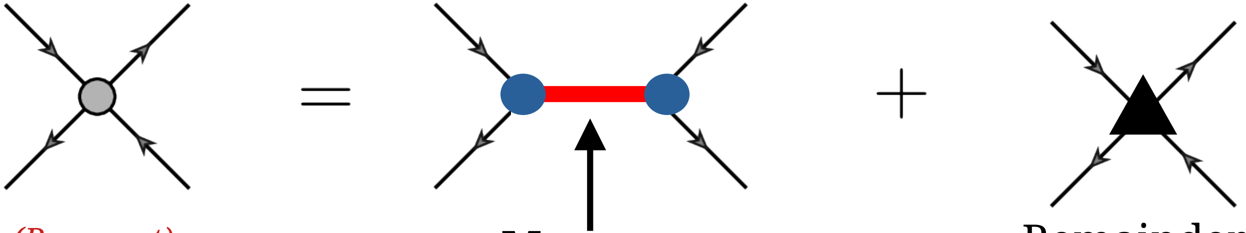

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Physical DoFs emergent from full QCD

Dynamical hadronization / Emergent Composites

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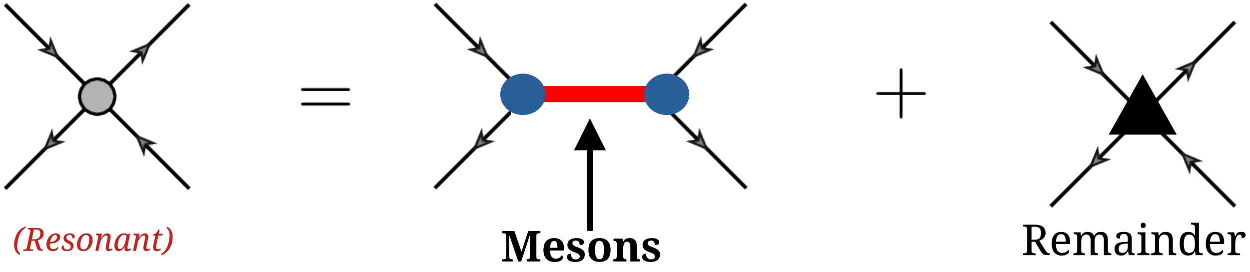

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Full mesonic potential of QCD

Field space:

Finite element method

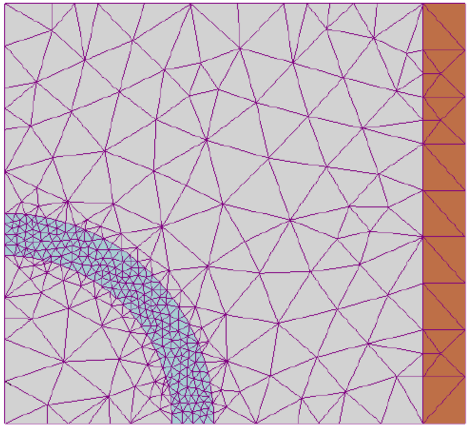


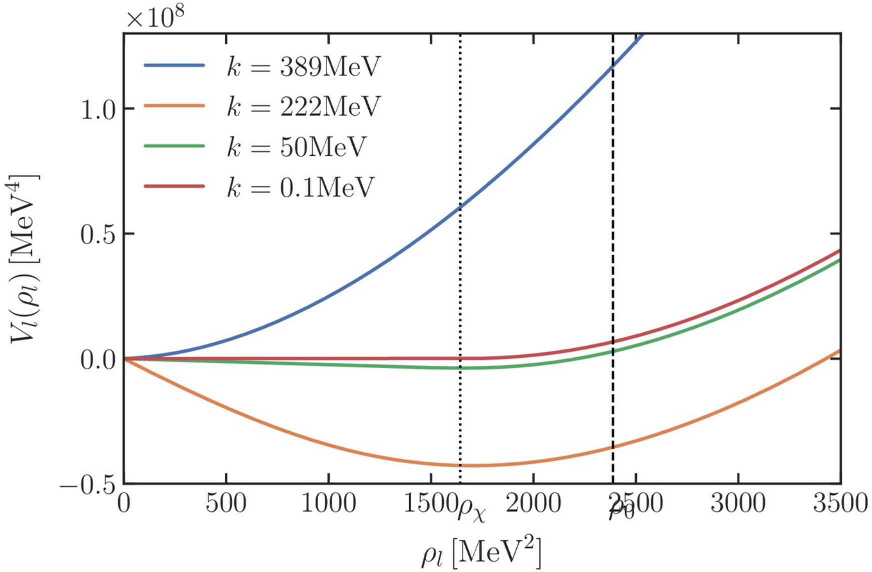
Image source: wikipedia.org

Grossi, Wink [SciPost Phys.Core 6 (2023) 071]
Grossi, Ihssen, Pawlowski, Wink [Phys.Rev.D 104 (2021) 1, 016028]
Ihssen, Pawlowski, Sattler, Wink [arXiv:2309.07335], [Comput.Phys.Commun. 300 (2024) 109182]

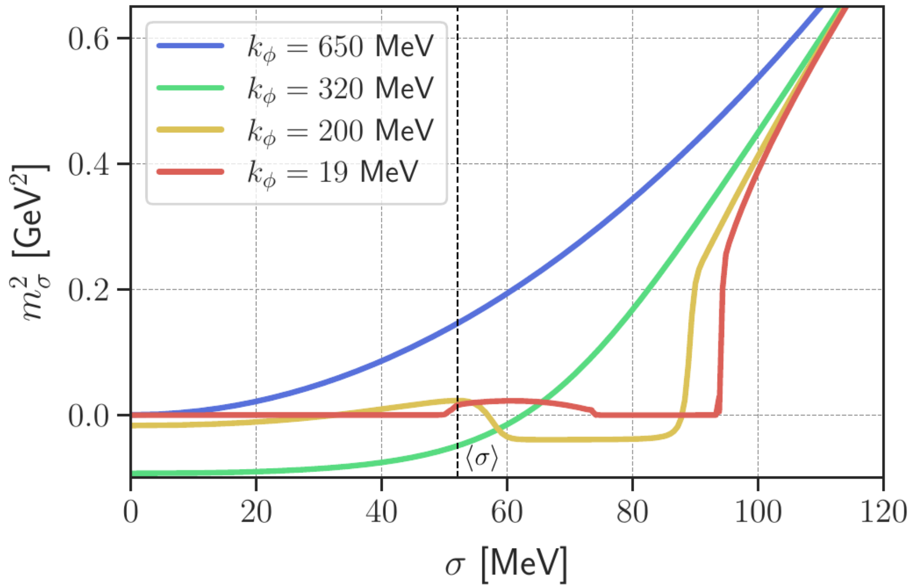
\+

sensible RG-scale integration

Ihssen, Sattler, Wink (Phys.Rev.D 107 (2023) 11, 114009)



Full mesonic potential of QCD

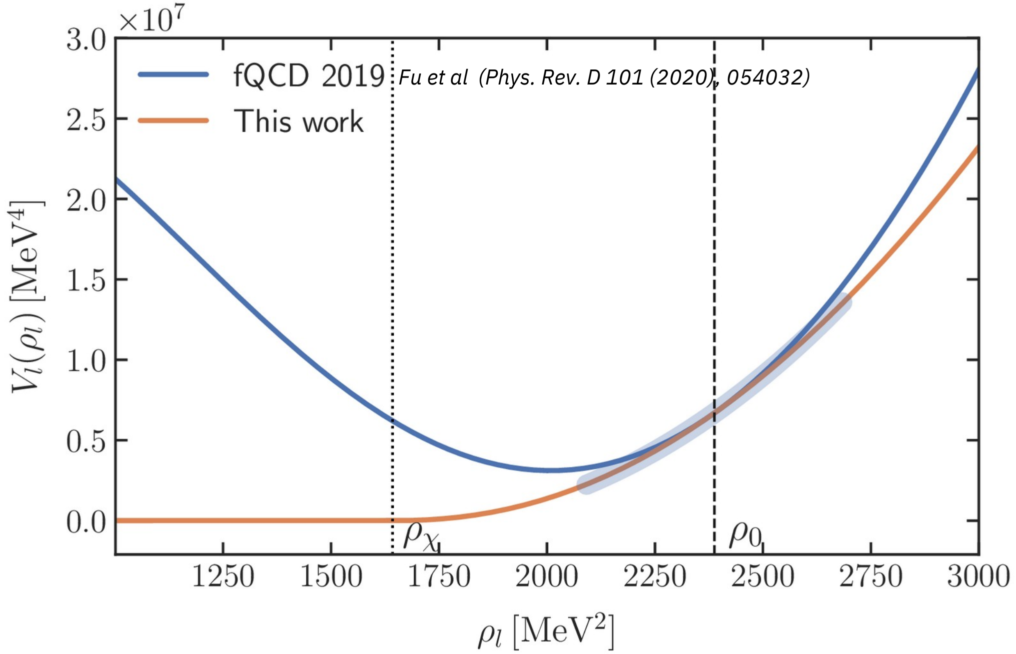


Ihssen, Pawlowski, Sattler, Wink [arXiv:2309.07335]

Hydro methods allow to access the full Potential.

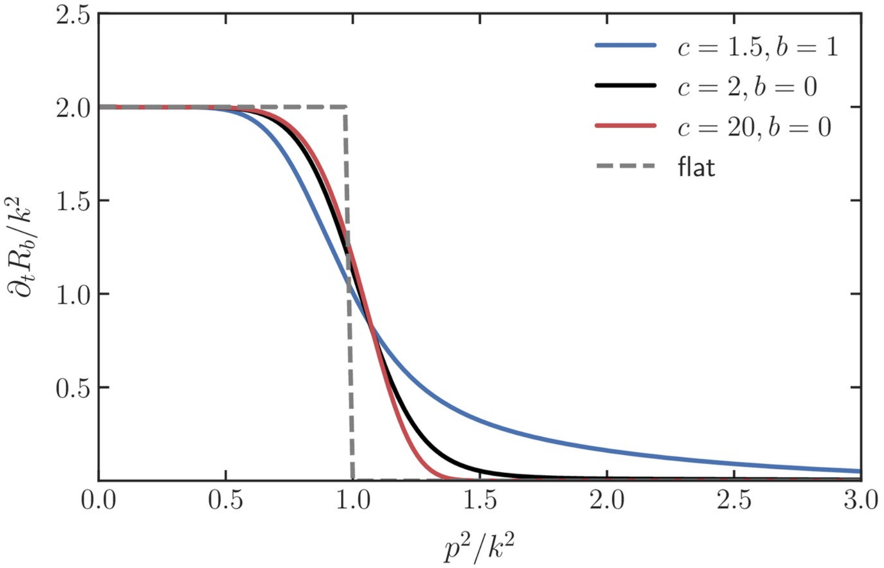
Important for phase transitions at high μ

Quantitative access to chiral limit!



Systematic errors I: Regulator dependence

Easy regulator variation thanks to numerical framework:

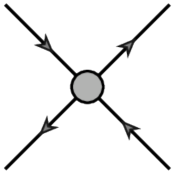


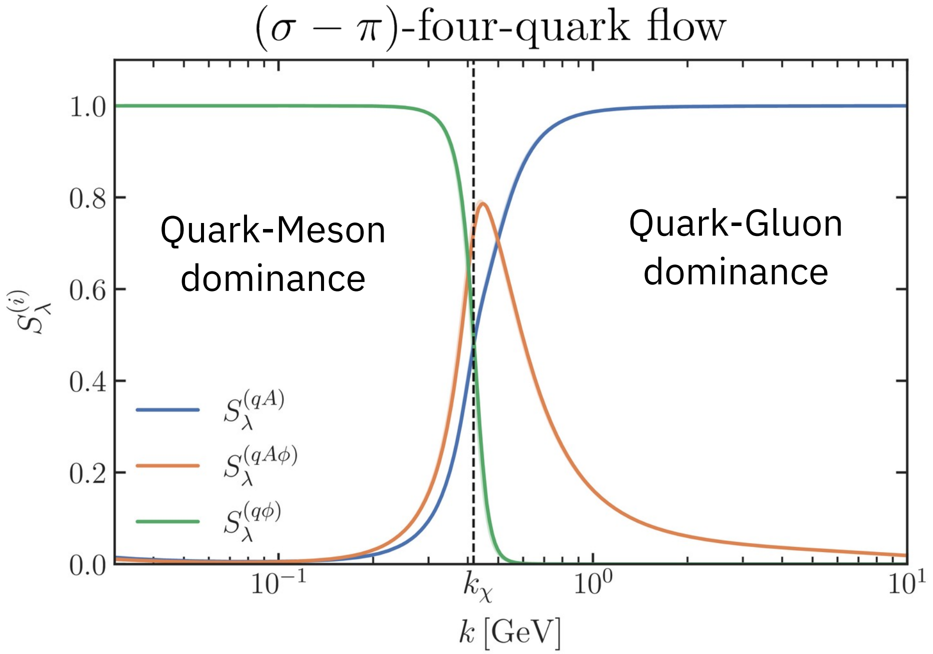
Observable	Value
$(f_K/f_\pi)_\chi$	$1.2168^{+0.0006}_{-0.0007}$
$f_{\pi,\chi}$ [MeV]	$93.2^{+3.5}_{-3.1}$
$m_{l,\chi}$ [MeV]	$311.6^{+0.3}_{-0.1}$
$m_{s,\chi}$ [MeV]	$446.7^{+0.3}_{-0.2}$
$m_{\sigma,\chi}$ [MeV]	$214.7^{+5.4}_{-9.3}$
$\sigma_{l,0,\chi}$ [MeV]	$67.1^{+1.2}_{-0.0}$

Chiral limit observables

Sattler et al. (in preperation)

Systematic errors II: The LEGO® principle





$$\mathcal{S}_\lambda^{(i)} = \frac{\left|\mathrm{Flow}_\lambda^{(i)}\right|}{\sqrt{\sum_j \left(\mathrm{Flow}_\lambda^{(j)}\right)^2}}\,, \qquad i,j = qA\,,\ q\phi\,,\ qA\phi$$

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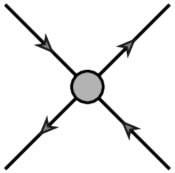
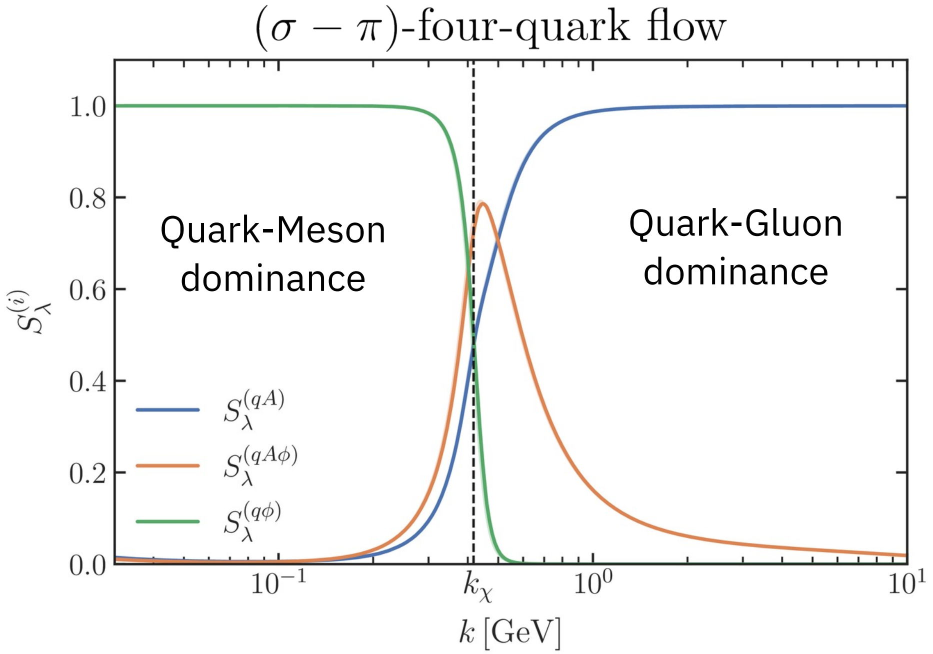
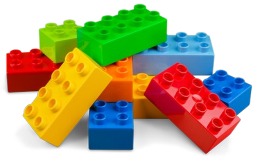


Separate LEGO® blocks:

- Glue subsystem $\{\lambda_{\mathrm{glue}}\} = \{\alpha_{A^3}\,,\, \alpha_{A^4}\,,\, \alpha_{c\bar{c}A}\}$
- Matter subsystem $\{\lambda_{\mathrm{mat}}\} = \{h_\phi(\rho_0)\,,\, \lambda_{\phi,n}(\rho_0)\}$
- Interface blocks $\{\lambda_{\mathrm{inter}}\} = \{\alpha_{l\bar{l}A}\}$





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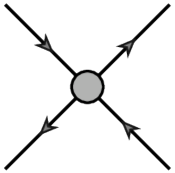

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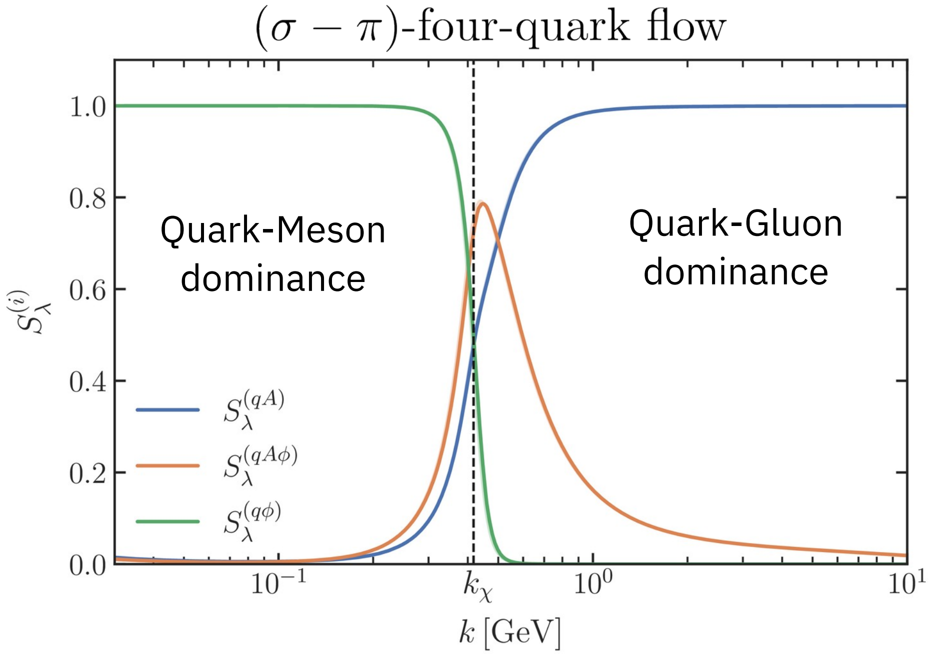
➔ Systematic error estimates from subsystems; preliminary estimate 10%.

➔ Low-energy effective theories.



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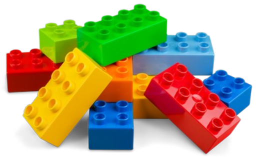
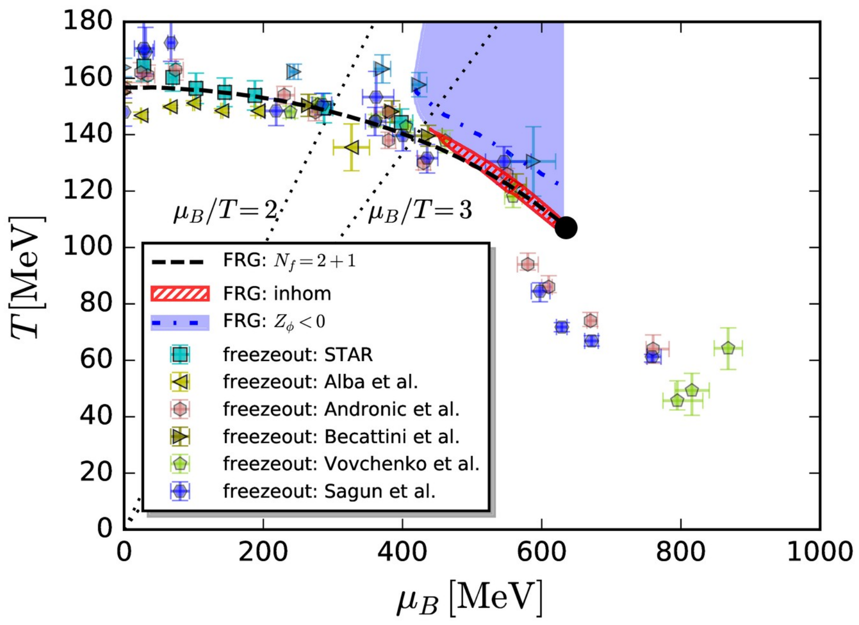
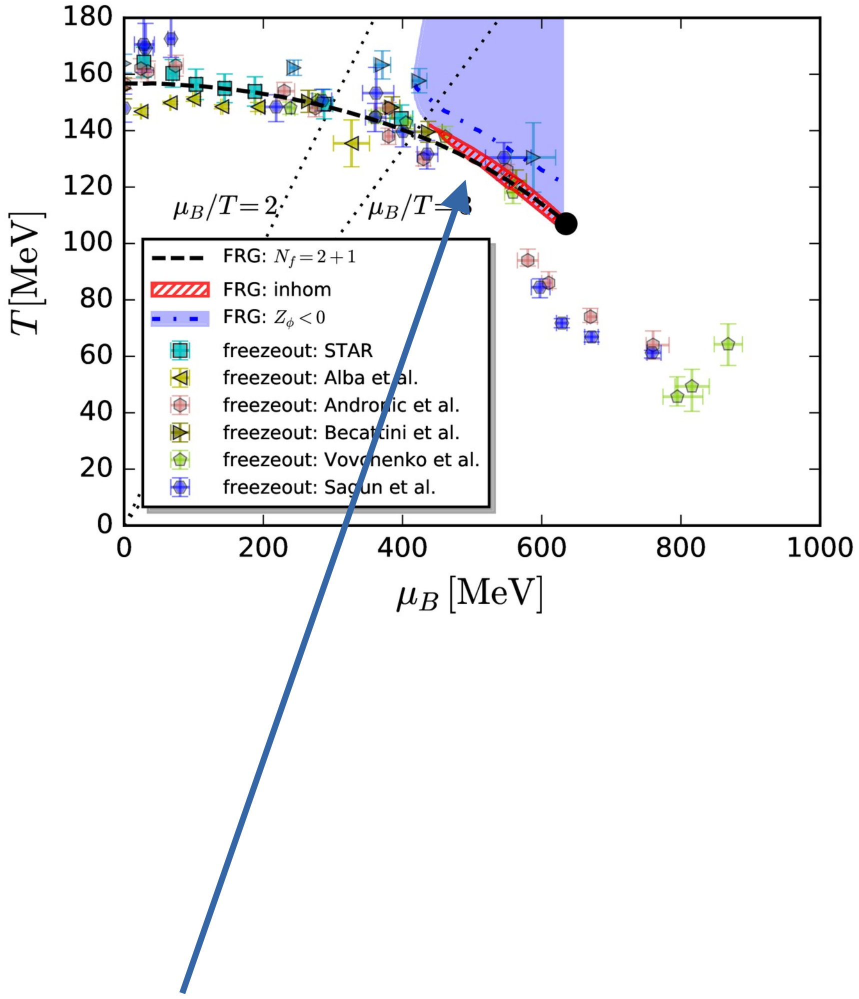
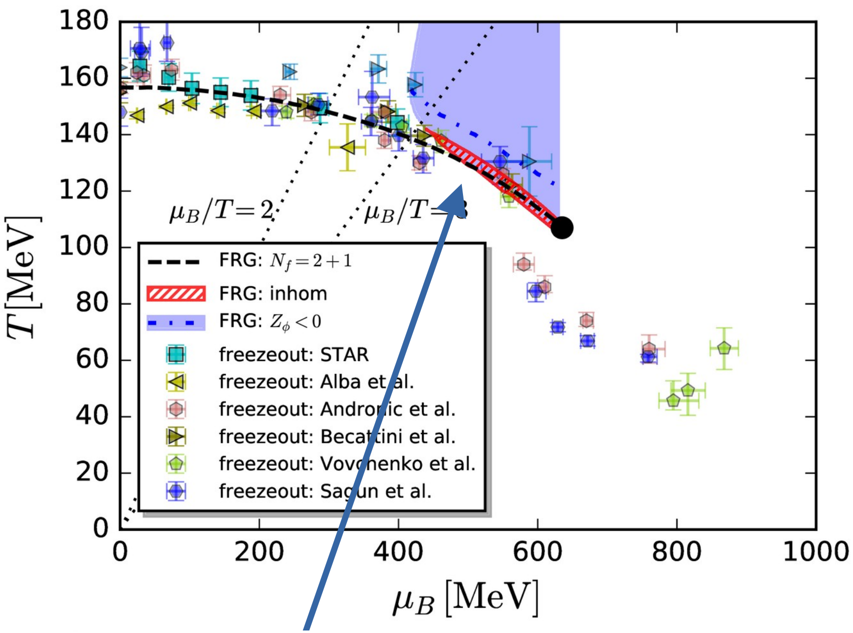




$\mu_B/T=2$
$\mu_B/T=3$
FRG: $N_f=2+1$
FRG: inhom
FRG: $Z_\phi<0$
freezeout: STAR
freezeout: Alba et al.
freezeout: Andronic et al.
freezeout: Becattini et al.
freezeout: Vovchenko et al.
freezeout: Sagun et al.
$T\,[\mathrm{MeV}]$
$\mu_B\,[\mathrm{MeV}]$



Moat regime at high μ
(possible inhomogeneous phase)



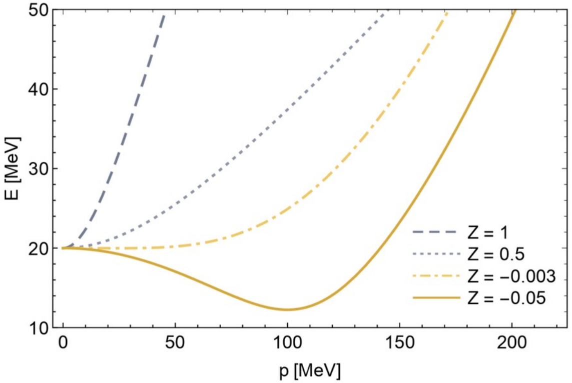
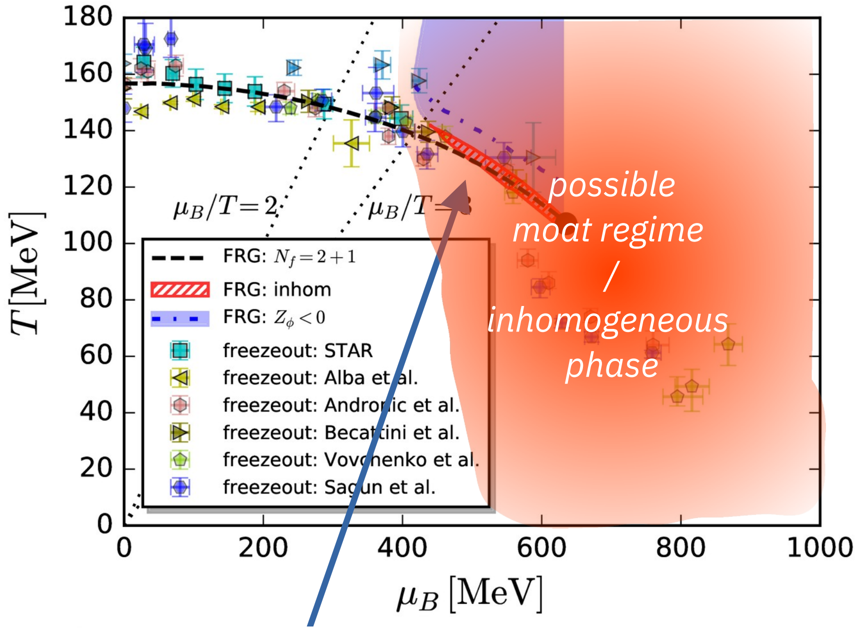


Figure by Rennecke, Pisarski [PoS CPOD2021 (2022) 016]

Moat regime at high μ

(possible inhomogeneous phase)



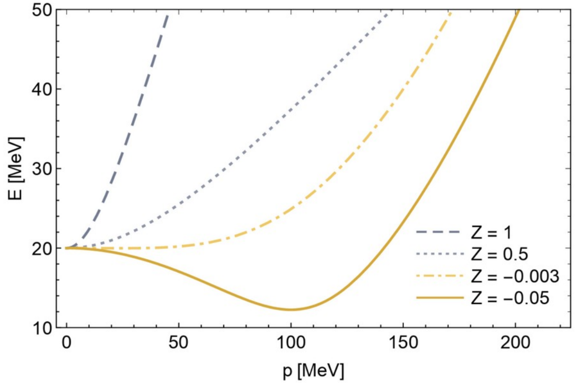


Figure by Rennecke, Pisarski [PoS CPOD2021 (2022) 016]

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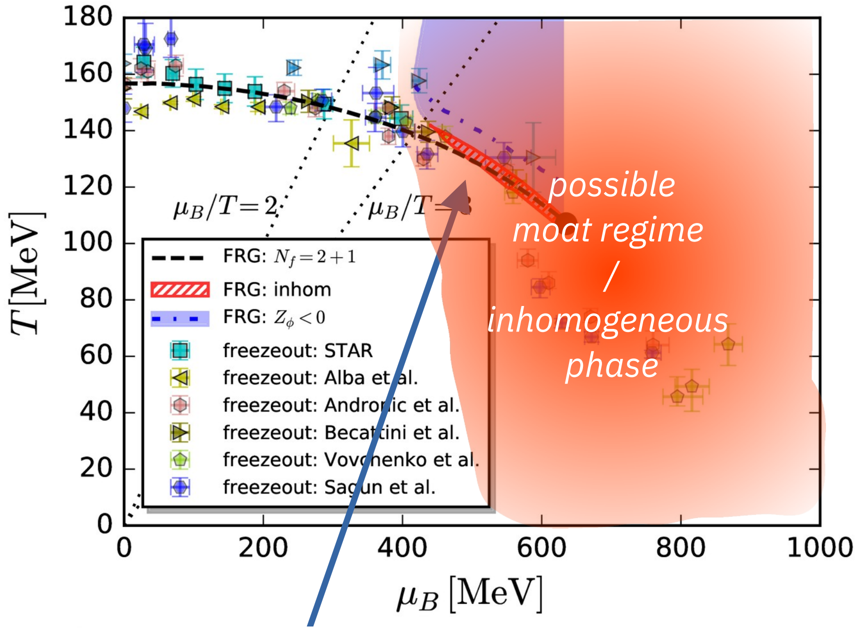
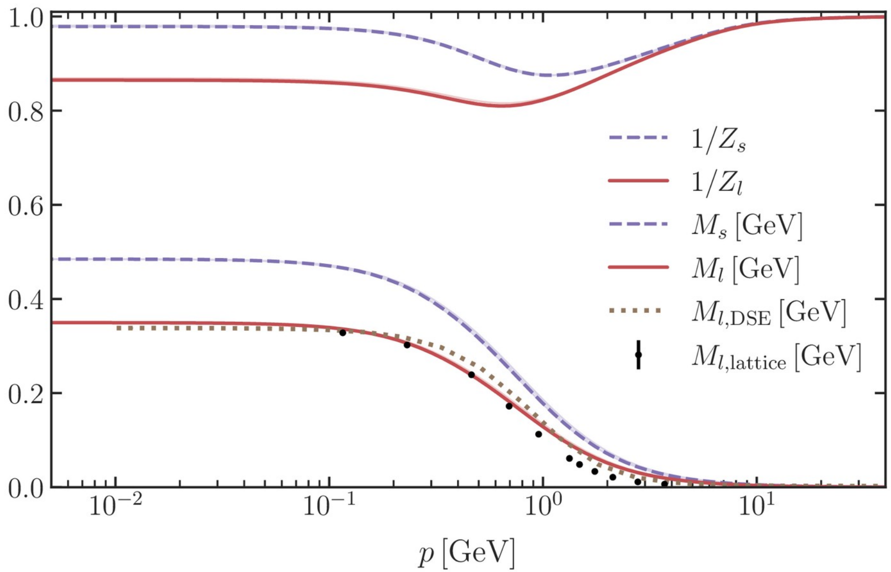


Full momentum resolution of propagators



Lattice data: Cheng et al [Phys. Rev. D 104 (2021), 094509]
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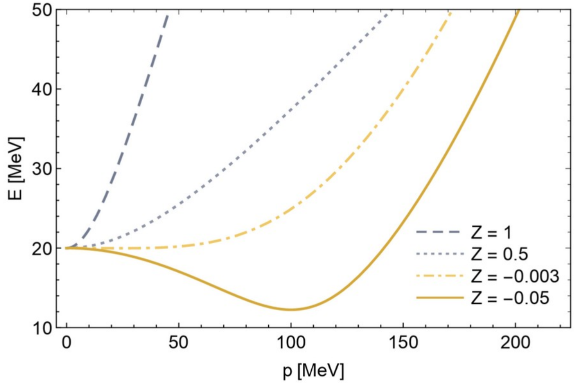

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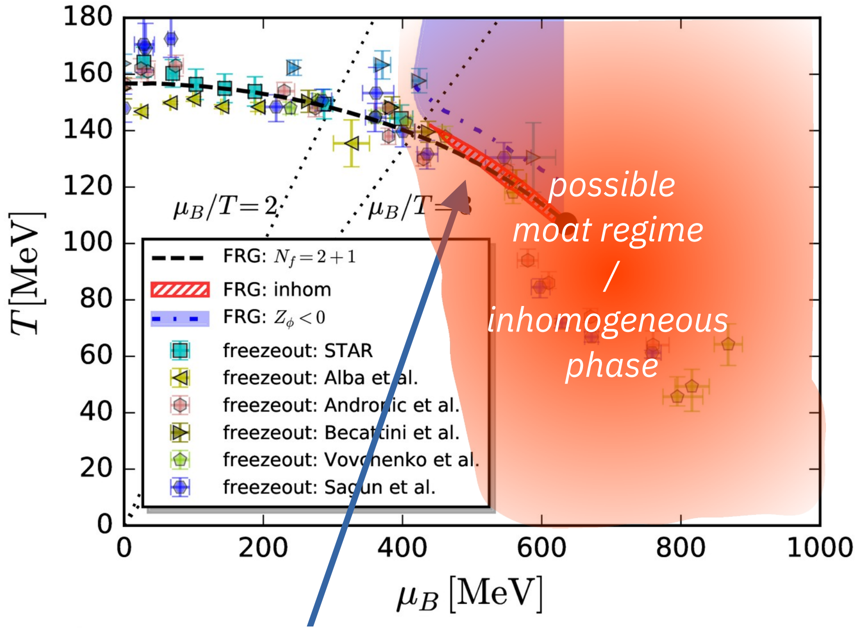

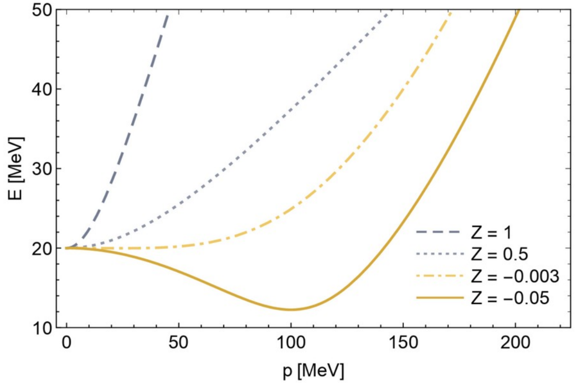

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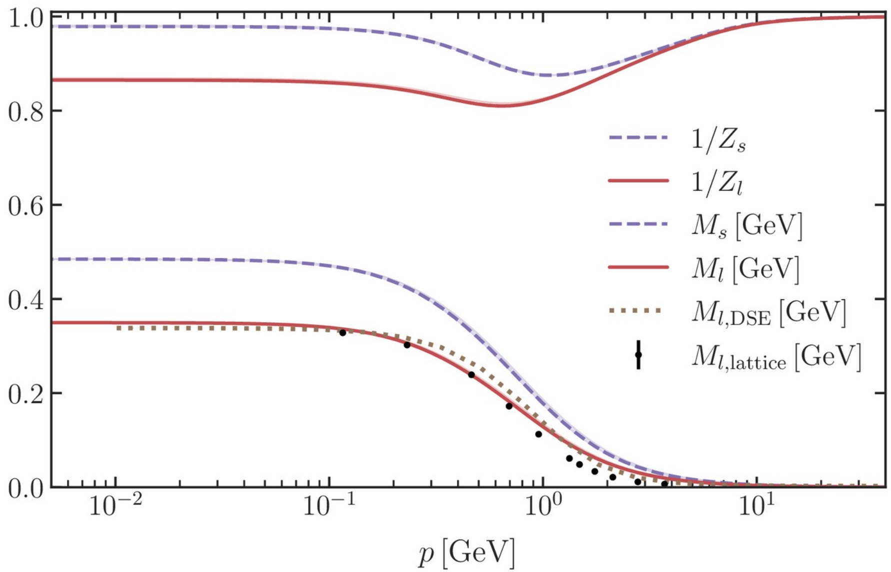

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Access to pole masses:

$$m_{\pi,\mathrm{vacuum}}^{(N_f=2)} = 139(12)\,\mathrm{MeV}$$
$$m_{\pi,\mathrm{vacuum}}^{(N_f=2+1)} = 138(9)\,\mathrm{MeV}$$

Conclusions

- **Motivation:**
  Direct access to phase structure of QCD through fRG
- **Quantitative Vacuum** results in agreement with Lattice & other functional approaches
- **Systematic error estimates**
- **Easily extendable setup**

Outlook

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- More **momentum dependences** *(done in vacuum)*
- Rebosonisation of **further channels** *(in progress)*
- Increase **number of tensor structures** *(done in vacuum)*

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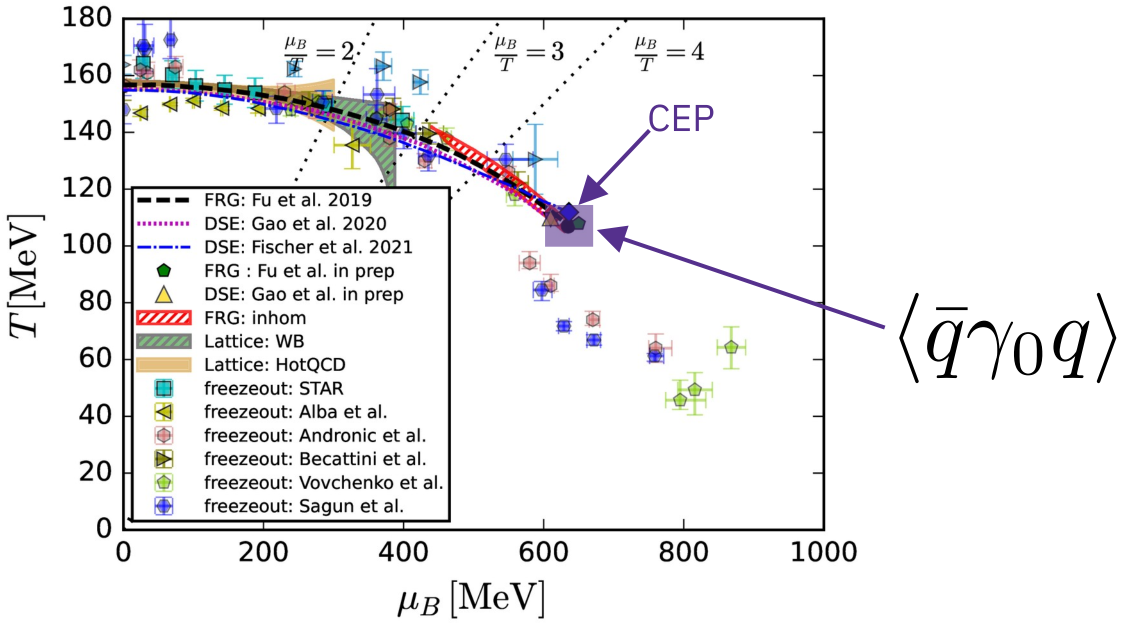
fQCD Collaboration:

Braun
Chen
Fu
Gao
Geissel
Huang
Ihssen
Lu
Pawlowski
Rennecke
Sattler
Schallmo
Stoll
Tan
Töpfel
Turnwald
Wen
Wessely
Wink
Yin
Zorbach

Thank you for your attention!

Backup slides

Mapping out the phase diagram



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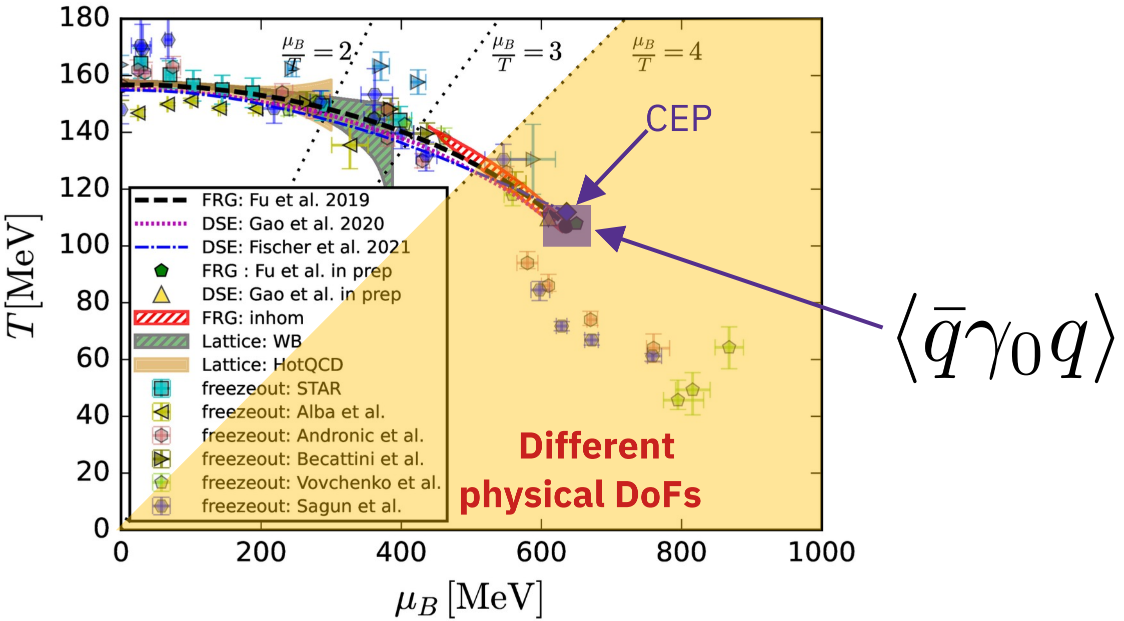

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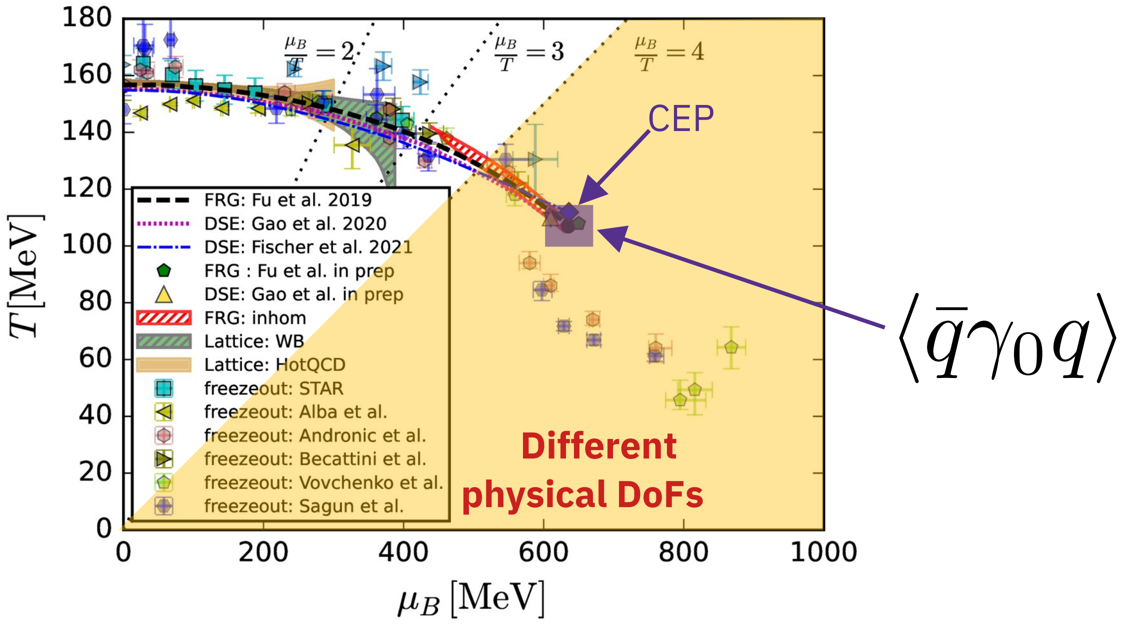

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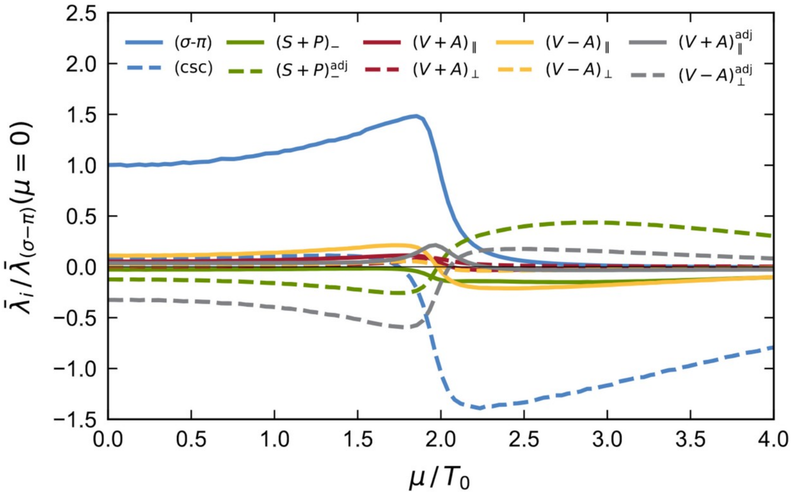
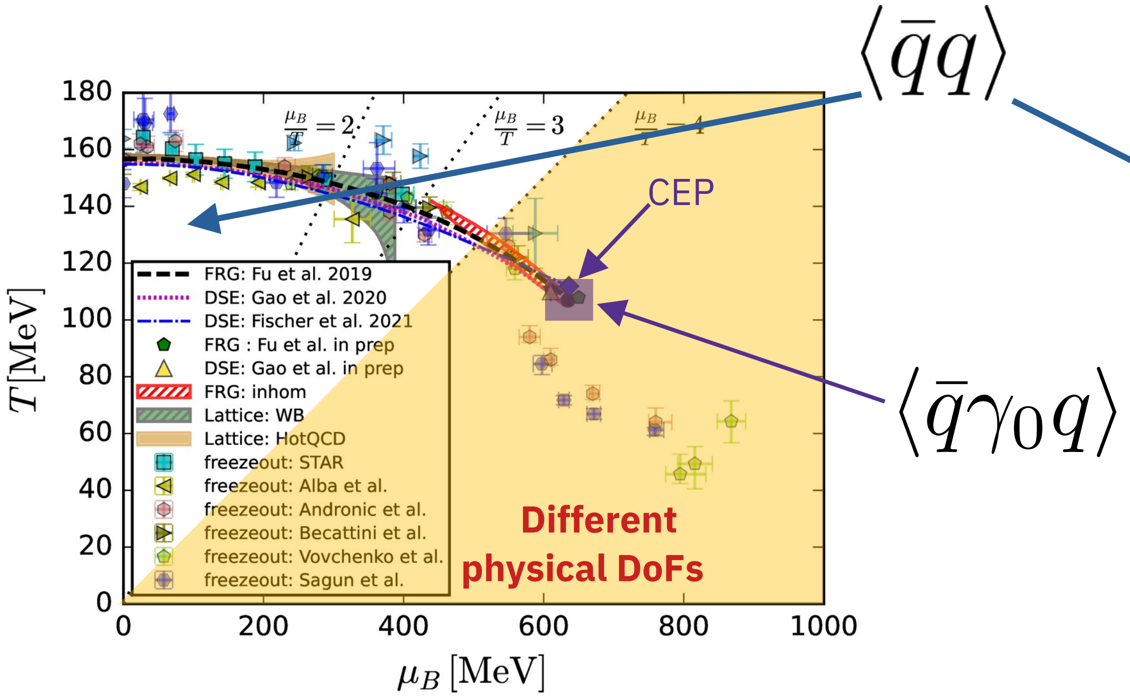
Other four-fermi channels



quark chemical potential

Braun, Leonhardt, Pospiech (Phys.Rev.D 101 (2020) 3, 036004)

Mapping out the phase diagram



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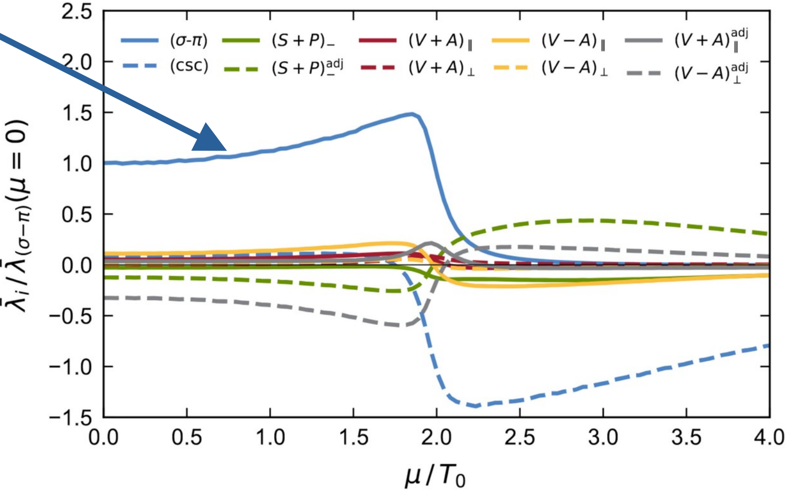

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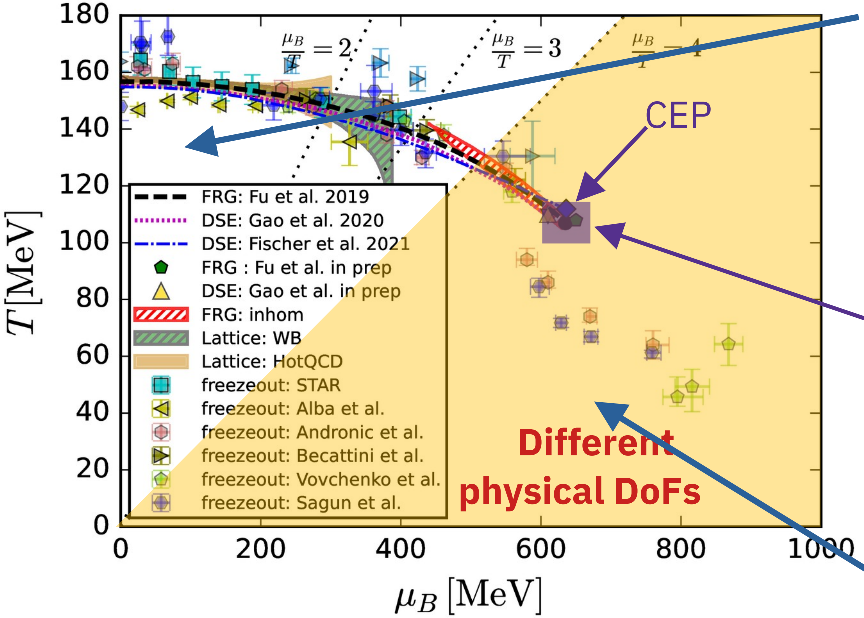
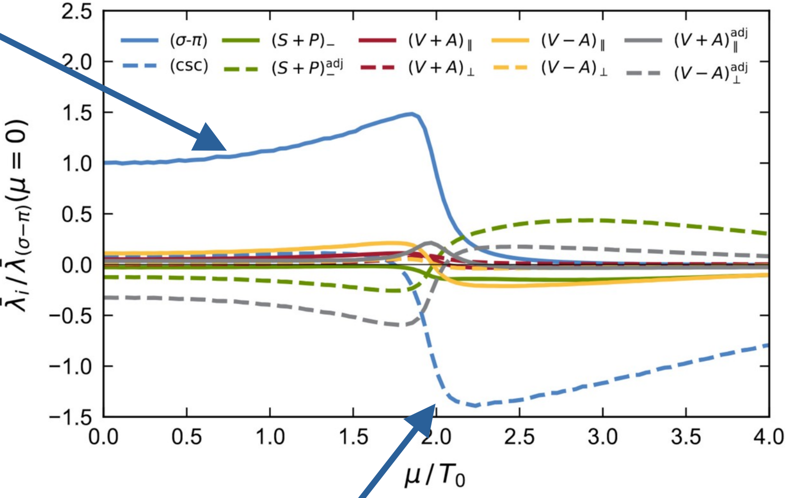
Mapping out the phase diagram

$$\langle \bar{q} q \rangle$$



Other four-fermi channels



quark chemical potential

$$\langle \bar{q} \gamma_0 q \rangle$$

$$\langle d^* d \rangle$$

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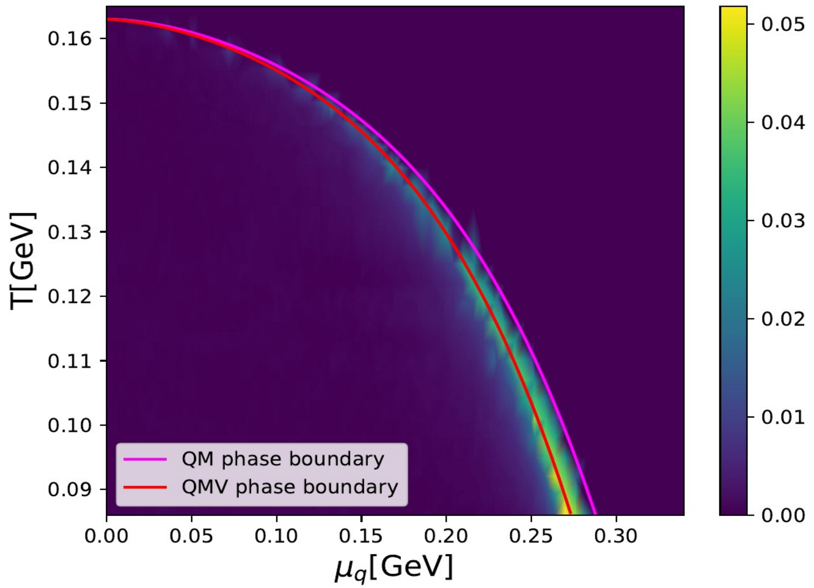
Inclusion of Density mode $\langle \bar{q}\gamma_0 q \rangle$ and Diquarks

Ihssen, Hendricks, Pawlowski, Sattler (in preparation)



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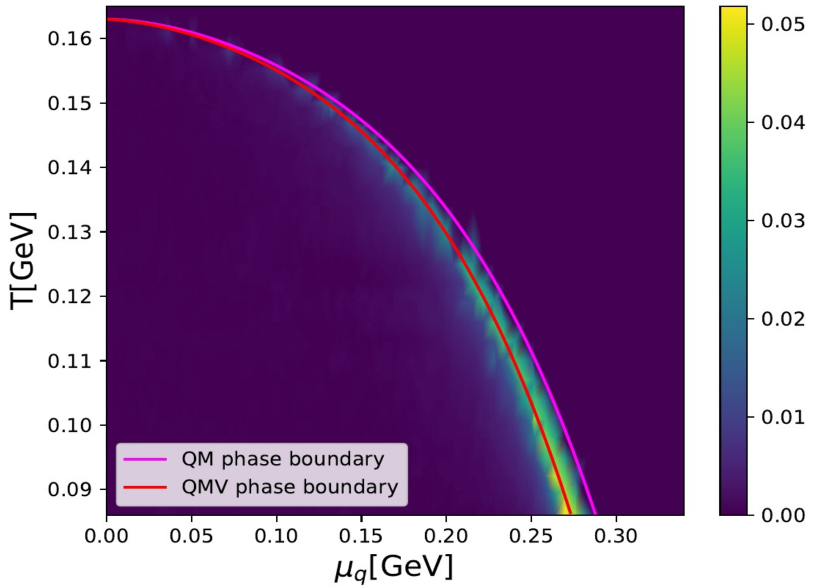


Full Nf = 2+1

Pawlowski, Sattler, Steck (in preparation)

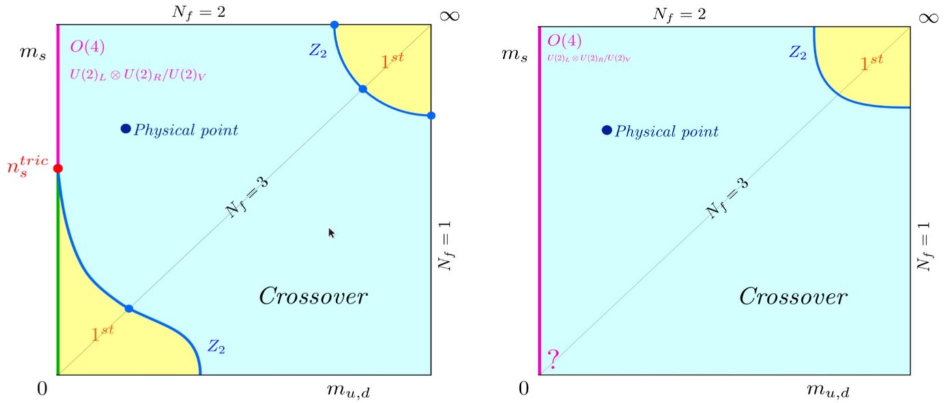
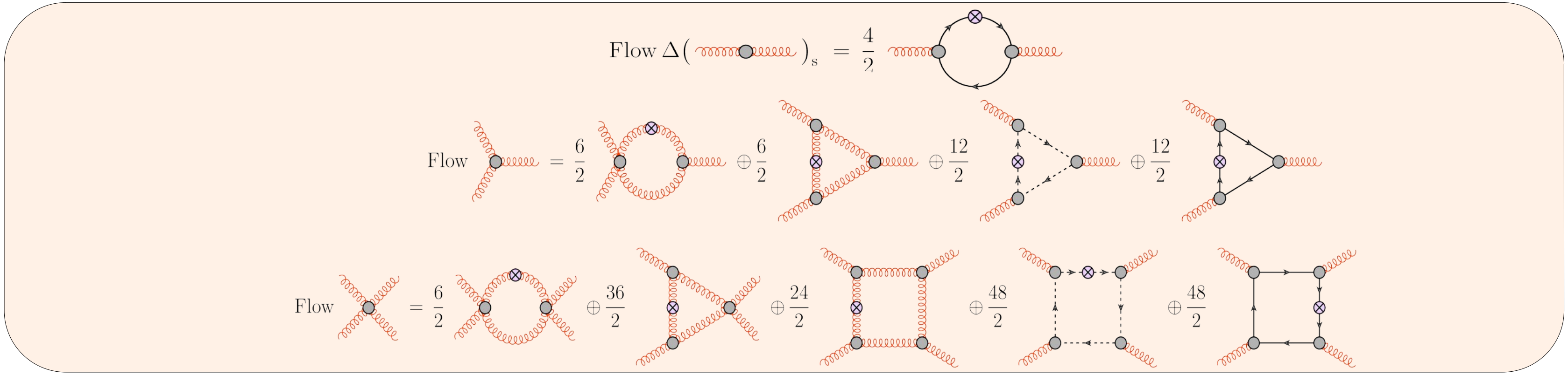
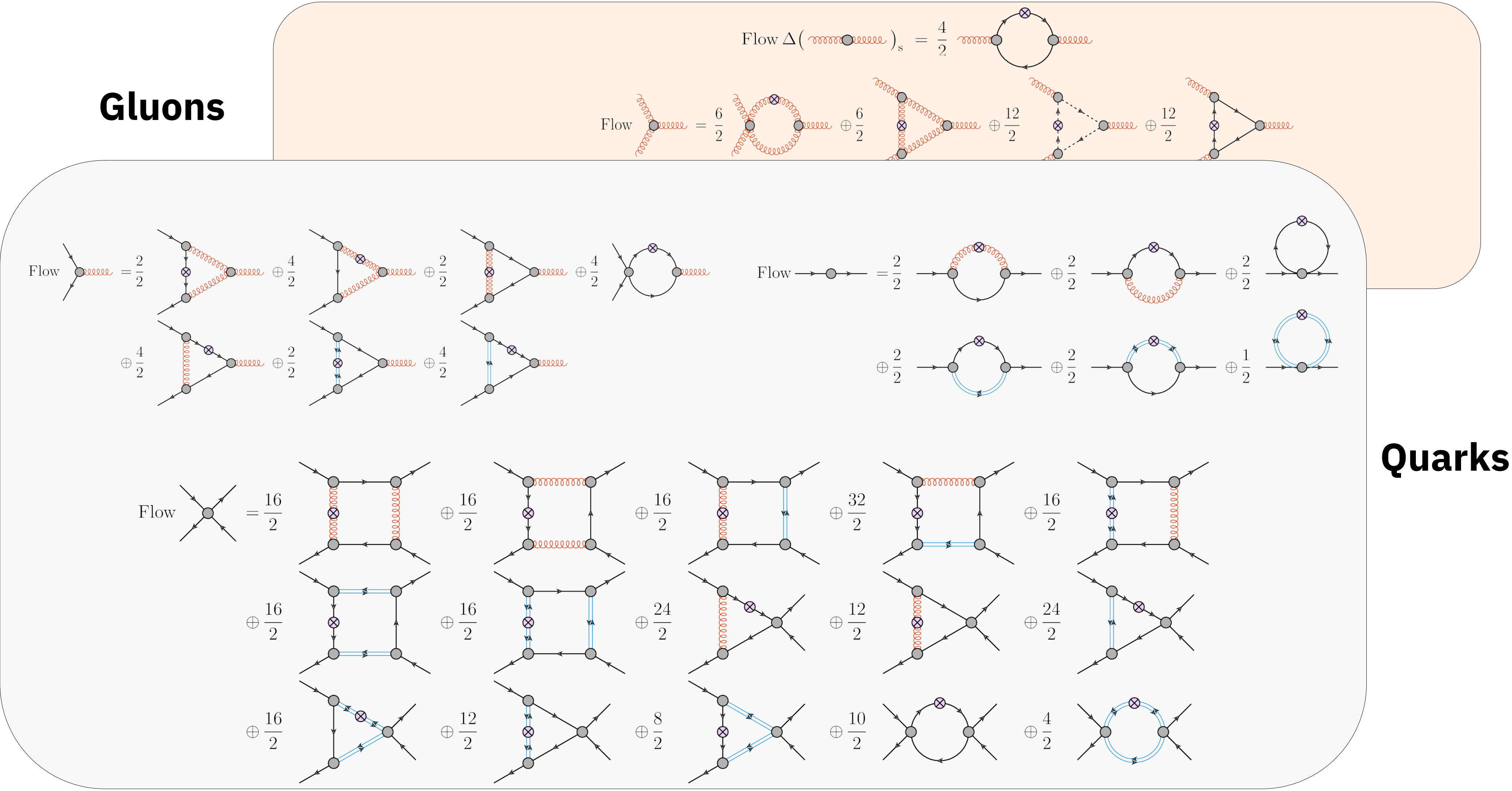
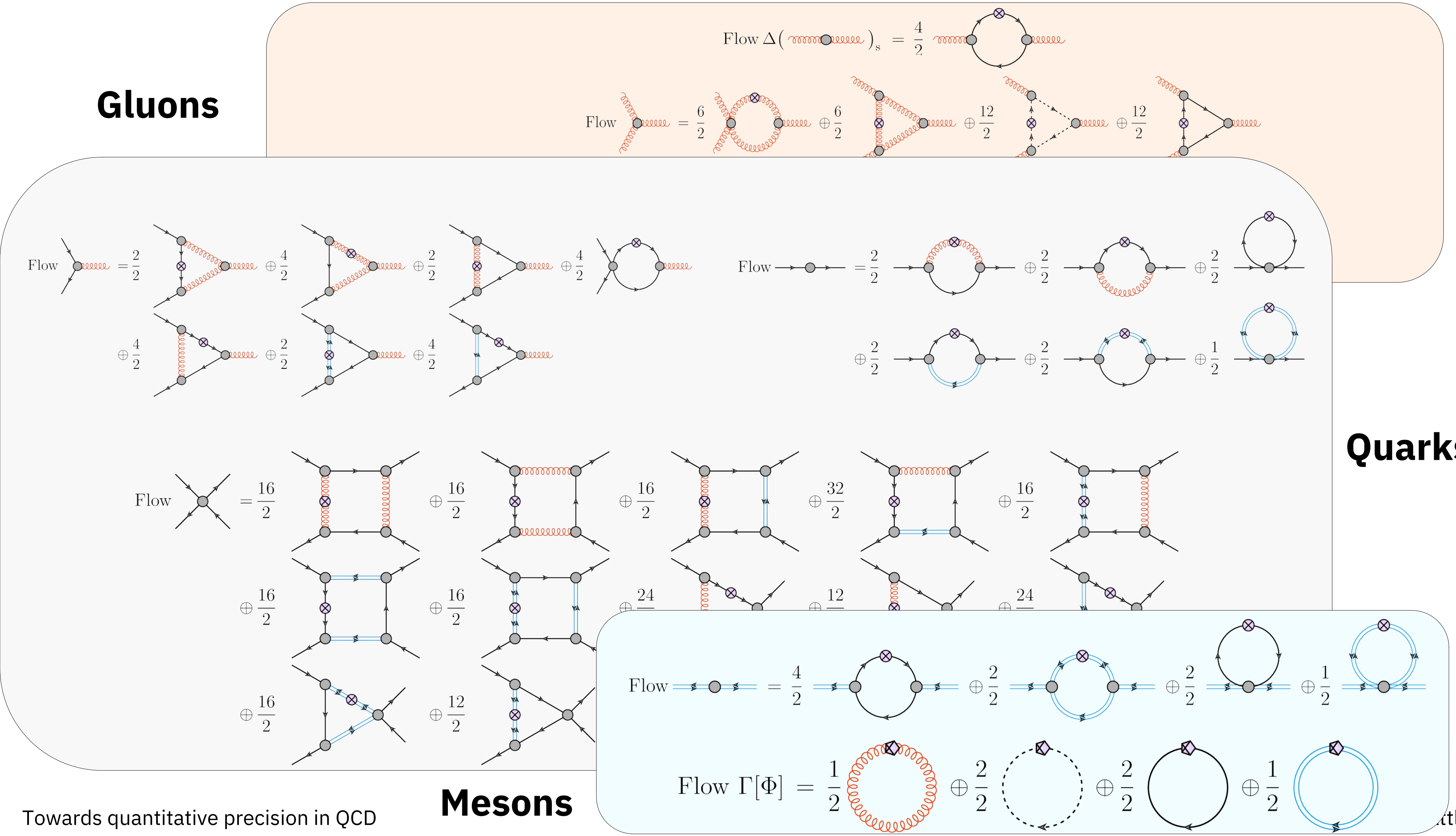


Figure from Owe Philipsen (arXiv:2111.03590)

Gluons

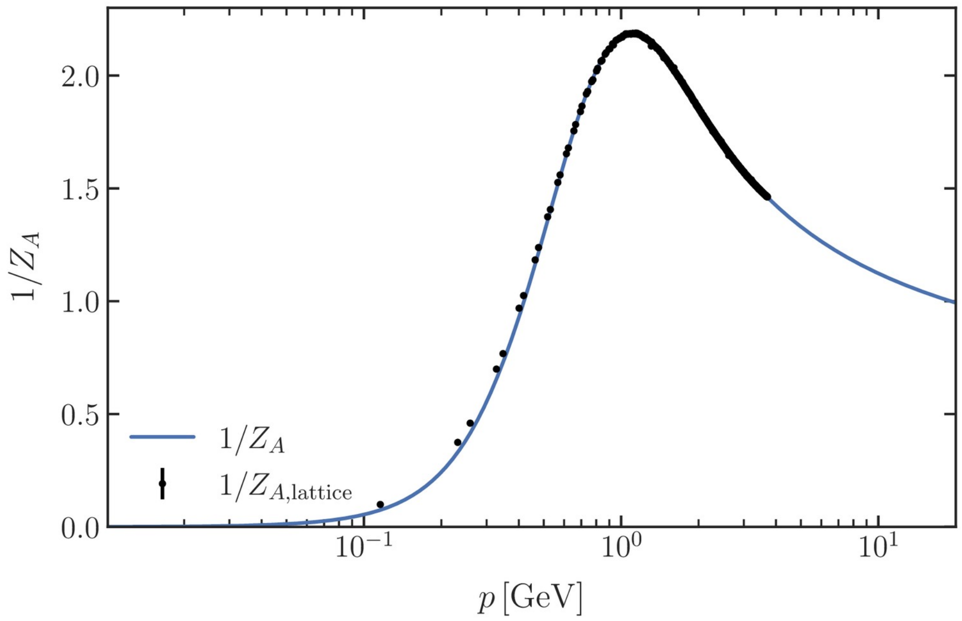



Gluons
Quarks
Flow
Flow
Flow
Flow
Flow


Gluons
Flow
Quarks
Flow
Flow
Flow
Mesons
Flow
Flow Γ[Φ]

Observables	Value	Parameter in $\Gamma_{\Lambda_{\mathrm{UV}}}$
$m_{\pi,\mathrm{pol}}$ [MeV]	138(9)	$c_{\sigma_l} = 4.67\,\mathrm{GeV}^3$
f_K/f_π	1.1914	$\Delta m_{sl} = 134.2\,\mathrm{MeV}$
$\alpha_{l\bar{l}A,\Lambda_{\mathrm{UV}}}$		$\alpha_{l\bar{l}A,\Lambda_{\mathrm{UV}}} = 0.227$
m_l [MeV]	350	$a = 0.0251 \quad b = 2\,\mathrm{GeV}$
f_π [MeV]	$97.2^{+4.0}_{-2.2}$	———
m_s [MeV]	$485.0^{+0.0}_{-0.3}$	———
$m_{\pi,\mathrm{cur}}$ [MeV]	138	———
m_σ [MeV]	$388.1^{+0.0}_{-1.1}$	———
$\sigma_{0,l}$ [MeV]	$69.^{+1.2}_{-0.2}$	———

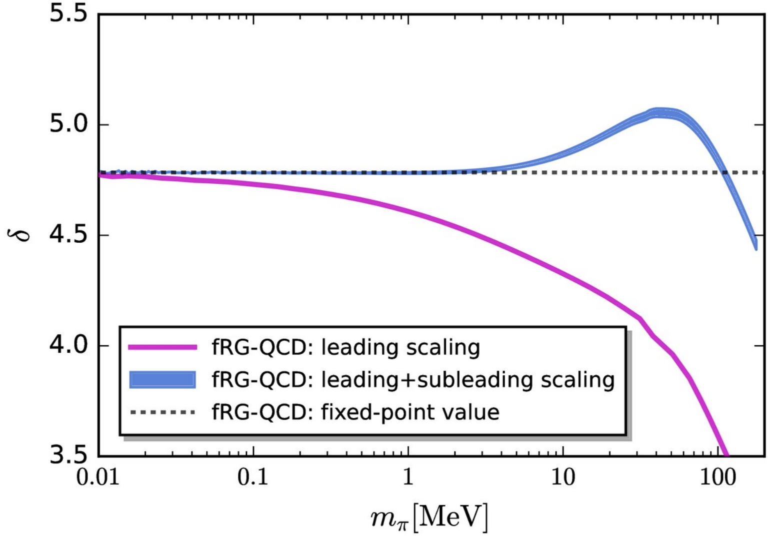
Results on physical point of QCD



Lattice data from Boucaud et al.
[Phys.Rev.D 98 (2018) 11, 114515]

Soft modes in hot QCD matter

*Braun, Chen, Fu, Gao, Huang, Ihssen, Pawlowski, Rennecke, **Sattler**, Tan, Wen, Yin (arXiv:2310.19853)*



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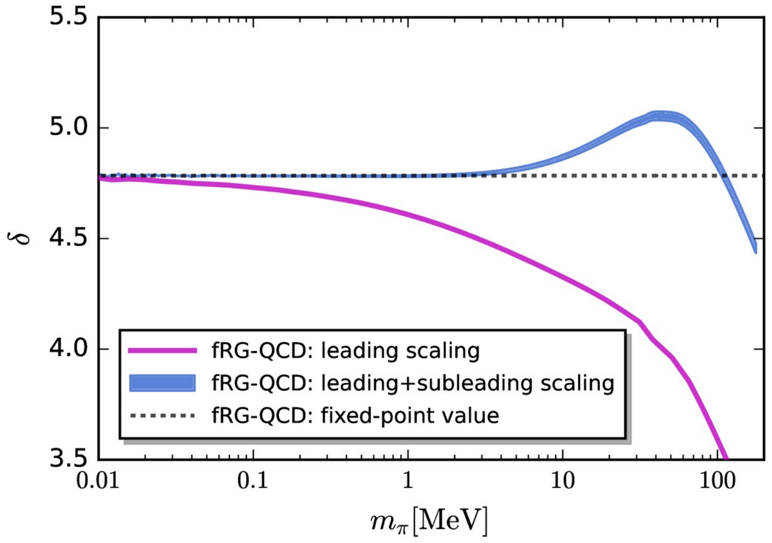
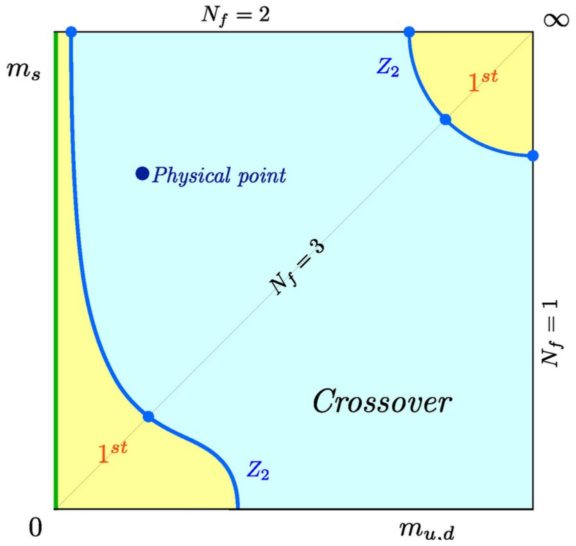


Columbia Plot

*Pawlowski, **Sattler**, Steck (in preparation)*



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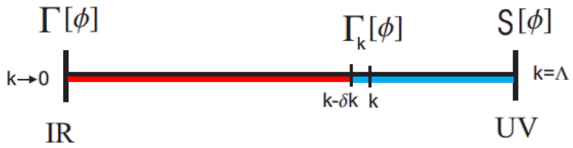
Wilsonian approach:

Integrate out momentum shells



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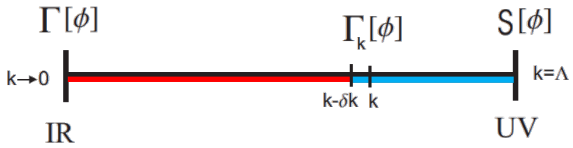

$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi] + \int d^d x J^a(x) \varphi_a(x)}$$

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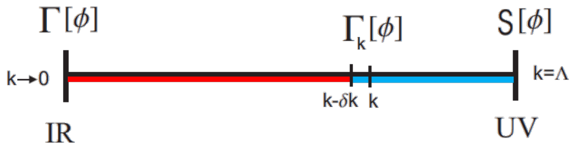

Introduce mass-like **"Regulator"**

$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi] + \int d^d x J^a(x) \varphi_a(x)}$$

$$[D\varphi]_k = [D\varphi]_{\mathrm{ren}}\, e^{-\frac{1}{2}\int d^d x \varphi_a(x) R_k^{ab}(x) \varphi_b(x)}$$

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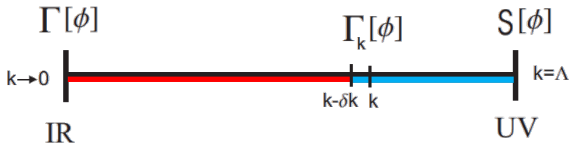

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Obtain Flow equation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\sum_{\mathrm{a,b}} \int \frac{d^d p}{(2\pi)^d} G^{(2)}_{ab}[\phi](p)\partial_t R_k^{ab}(p)$$

Infinite Tower of Functional equations

$$\partial_t \Gamma[\bar{\phi}] = \frac{1}{2} \mathrm{Tr}\, G_k\, \partial_t R_k\,,$$

$$\partial_t \Gamma^{(1)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr}\, \Gamma_k^{(3)} \left(G_k\, \partial_t R_k\, G_k \right),$$

$$\partial_t \Gamma^{(2)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(4)} - 2\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \right] \left(G_k\, \partial_t R_k\, G_k \right),$$

$$\partial_t \Gamma^{(3)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr} \left[\Gamma_k^{(5)} - 6\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} + 6\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \right] \left(G_k\, \partial_t R_k\, G_k \right),$$

$$\begin{aligned}\partial_t \Gamma^{(4)}[\bar{\phi}] = -\frac{1}{2} \mathrm{Tr} \Big[& \Gamma_k^{(6)} - 8\, \Gamma_k^{(5)}\, G_k\, \Gamma_k^{(3)} - 6\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(4)} + 18\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \\ & + 12\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} - 24\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \cdot G_k\, \Gamma_k^{(3)} \Big] \left(G_k\, \partial_t R_k\, G_k \right),\end{aligned}$$

$$\vdots \qquad\qquad \vdots$$

Infinite Tower of Functional equations

=

Infinite Tower of Diagrams

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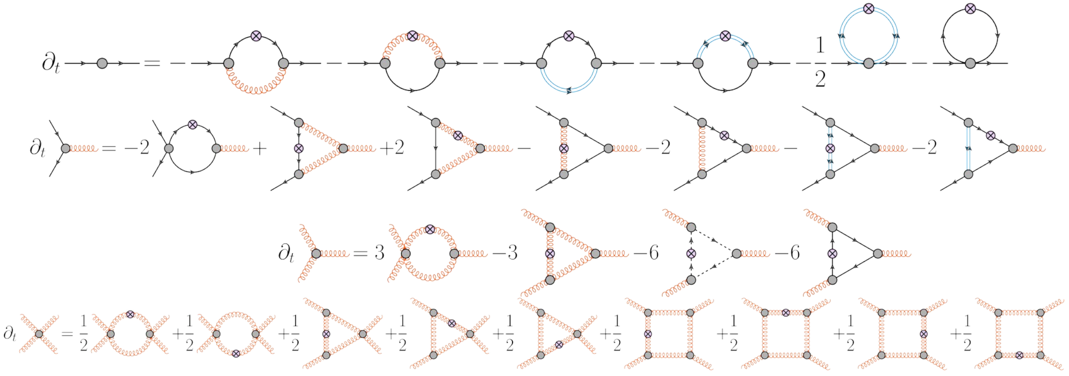
$$+ 12\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} - 24\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \cdot G_k\, \Gamma_k^{(3)} \Big] \left(G_k\, \partial_t R_k\, G_k \right),$$

⋮ ⋮

=



⋮ ⋮ ⋮ ⋮

R. Sattler