# Entanglement in an Expanding Universe

Nikolaos Tetradis

University of Athens

Work with K. Boutivas, D. Katsinis, G. Pastras

# Introduction

- Consider a quantum mechanical system with many degrees of freedom, such as a spin chain or a quantum field.
- Assume it is in the ground state $|\Psi\rangle$, which is a pure state.
- The density matrix of the total system is $\rho_{\rm tot} = |\Psi\rangle\langle\Psi|$.
- The von Neumann entropy $\mathrm{S}_{\rm tot} = -\mathrm{tr}\rho_{\rm tot}\log\rho_{\rm tot}$ vanishes.
- Now divide the total system into subsystems A and B and assume that B is inaccessible to A.
- Trace out the part B of the Hilbert space in order to obtain the reduced density matrix of A: $\rho_{\rm A} = \mathrm{tr}_{\rm B}\rho_{\rm tot}$.
- The entropy $\mathrm{S}_{\rm A} = -\mathrm{tr}_{\rm A}\rho_{\rm A}\log\rho_{\rm A}$ is a measure of the entanglement between A and B.
- It is nonvanishing and $\mathrm{S}_{\rm A} = \mathrm{S}_{\rm B}$.

- For spatially separated systems in the ground state in a static background, the leading contribution is proportional to the area of the entangling surface between A and B:

$$S_A \sim \frac{\partial A}{\epsilon^{d-1}} + \text{subleading terms.}$$

- Massless scalar field in 3+1 dimensions:

$$S_A = s\,(R/\epsilon)^2 + c\ \log(R/\epsilon) + d$$

$s \simeq 0.3$ (scheme-dependent) (Srednicki 1993)
$c = -\frac{1}{90}$ (universal)
(Lohmayer, Neuberger, Schwimmer, Theisen 2009).

- Conformal field theory in 1+1 dimensions, with central charge c. System of length L, divided into pieces of lengths $\ell$ and $L-\ell$:

$$S_A = \frac{c}{6} \ln\left(\frac{2L}{\pi\epsilon} \sin\frac{\pi\ell}{L}\right) + \bar{c}'_1$$

$\bar{c}'_1$ scheme-dependent. (Korepin 2004, Calabrese, Cardy 2004)

- How does the entanglement entropy evolve in a time-dependent background?
- de Sitter space (Maldacena, Pimentel 2013).
- Relevance for the expanding Universe.
- We generalize Srednicki's approach to expanding backgrounds.

## References


- K. Boutivas, G. Pastras and N. Tetradis
  "Entanglement and expansion"
  arXiv:2302.14666 [hep-th], JHEP 05 (2023) 199
- D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement of Harmonic Systems in Squeezed States"
  arXiv:2304.04241 [hep-th], JHEP 10 (2023) 039
- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement in Cosmology"
  arXiv:2310.17208 [gr-qc], JCAP 04 (2024) 017
- D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement Entropy of a Scalar Field in a Squeezed State"
  arXiv:2403.03136 [hep-th]
- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "Entanglement Entropy as a Probe Beyond the Horizon"
  arXiv:2407.07824 [hep-th]
- K. Boutivas, D. Katsinis, G. Pastras and N. Tetradis
  "A Numerical Calculation of Entanglement Entropy in dS Space"
  arXiv:2407.07811 [hep-th]

## Expanding the field in momentum modes

- Consider a free scalar field $\phi(\tau, \mathrm{x})$ in a FRW background

$$\mathrm{ds}^2 = \mathrm{a}^2(\tau)\left(\mathrm{d}\tau^2 - \mathrm{dr}^2 - \mathrm{r}^2\mathrm{d}\Omega^2\right).$$

- With the definition $\phi(\tau, \mathrm{x}) = \mathrm{f}(\tau, \mathrm{x})/\mathrm{a}(\tau)$, the action becomes

$$\mathrm{S} = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\mathrm{x} \left(\mathrm{f}'^2 - (\nabla \mathrm{f})^2 + \left(\frac{\mathrm{a}''}{\mathrm{a}} - \mathrm{a}^2\mathrm{m}^2\right)\mathrm{f}^2\right).$$

The field $\mathrm{f}(\tau, \mathrm{x})$ has a canonically normalized kinetic term.

- For de Sitter: $\mathrm{a}(\tau) = -1/(\mathrm{H}\tau)$ with $-\infty < \tau < 0$, and

$$\mathrm{S} = \frac{1}{2}\int \mathrm{d}\tau\, \mathrm{d}^3\mathrm{x} \left(\mathrm{f}'^2 - (\nabla \mathrm{f})^2 + \frac{2\kappa}{\tau^2}\mathrm{f}^2\right),$$

where $\kappa = 1 - \mathrm{m}^2/2\mathrm{H}^2$.

- The eom (Mukhanov-Sasaki equation) in Fourier space is

$$f_k'' + k^2 f_k - \frac{2\kappa}{\tau^2} f_k = 0.$$

- Its general solution is

$$f_k(\tau) = A_1 \sqrt{-\tau}\, J_\nu(-k\tau) + A_2 \sqrt{-\tau}\, Y_\nu(-k\tau) \qquad \nu = \frac{1}{2}\sqrt{1+8\kappa}.$$

- Bunch-Davies vacuum: $A_1 = -\frac{\sqrt{\pi}}{2}$, $A_2 = -\frac{\sqrt{\pi}}{2}i$. For $\tau \to -\infty$

$$f_k(\tau) \simeq \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$

- For $\kappa = 1$ (massless scalar), the full solution reads

$$f_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}\left(1 - \frac{i}{k\tau}\right).$$

For $k\tau \to 0^-$ the mode becomes superhorizon and the oscillations stop. The mode freezes.

- The quantum field can be expressed as

$$\hat{f}(\tau, x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ f_k(\tau)\hat{a}_k + f_k^*(\tau)\hat{a}_k^\dagger \right] e^{ik\cdot x}$$

where $\hat{a}_k^\dagger$, $\hat{a}_k$ are standard creation and annihilation operators.

- For superhorizon modes with $k\tau \to 0^-$ the growing term dominates and

$$\hat{\pi}(\tau, x) \simeq -\frac{1}{\tau}\hat{f}(\tau, x).$$

- The field and its conjugate momentum commute.
- For most of its properties the field can be viewed as a classical stochastic field.
- However, the full quantum field and its conjugate always obey the canonical commutation relation. This is guaranteed by the presence of the subleading term in the mode function.

- The entanglement entropy is of purely quantum origin, for which a classical description is inadequate. It does not vanish for superhorizon modes.
- We are interested in the entanglement between degrees of freedom localized within two spatial regions separated by an entangling surface.
- For a dS background one may consider the entanglement between the interior of a horizon-size region of radius 1/H and the exterior.

## Expanding the field in coordinate space

- For spherical entangling surfaces, expand in spherical harmonics
- Discretize the radial coordinate as $r_j = j\epsilon$, $1 \leq j \leq N$.
- UV cutoff: $1/\epsilon$. IR cutoff: $1/L$ with $L = N\epsilon$.
- Trace out the oscillators with $j\epsilon > R$.
- $\epsilon$, L, R are comoving scales.
- The 'ground state' of the system is the product of the 'ground states' of the modes that diagonalize the Hamiltonian.
- In the Bunch-Davies vacuum, the 'ground state' is the solution of the Schrödinger equation that reduces to the usual simple harmonic oscillator ground state as $\tau \to -\infty$.
- The discretized Hamiltonian for the free field during inflation is

$$H = \frac{1}{2\epsilon} \sum_{l,m} \sum_{j=1}^{N} \left[ \tilde{\pi}^2_{lm,j} + \left( \omega^2_{lm,j} - \frac{2\kappa}{(\tau/\epsilon)^2} \right) \tilde{f}^2_{lm,j} \right],$$

  where $\tilde{f}_{lm,j}$ are the canonical modes.
- We need to solve for the harmonic oscillator with a time-dependent eigenfrequency of the form $\omega_0^2 - 2\kappa/\tau^2$.

## de Sitter era

- Oscillator with time-dependent frequency

$$\omega^2(\tau) = \omega_0^2 - \frac{2\kappa}{\tau^2}.$$

- Find the general solution of the Ermakov equation

$$\mathrm{b}''(\tau) + \omega^2(\tau)\mathrm{b}(\tau) = \frac{\omega_0^2}{\mathrm{b}^3(\tau)}.$$

- For the Bunch-Davies vacuum, $\mathrm{b}(\tau)$ must tend to 1 as $\tau \to -\infty$.

$$\mathrm{b}^2(\tau) = -\frac{\pi}{2}\omega_0\tau \left(\mathrm{J}_\nu^2\left(-\omega_0\tau\right) + \mathrm{Y}_\nu^2\left(-\omega_0\tau\right)\right).$$

- The solution of the Schrödinger equation can now be expressed as

$$\mathrm{F}(\tau, \mathrm{f}) = \frac{1}{\sqrt{\mathrm{b}(\tau)}} \exp\left(\frac{\mathrm{i}}{2}\frac{\mathrm{b}'(\tau)}{\mathrm{b}(\tau)}\mathrm{f}^2\right) \mathrm{F}^0\left(\int \frac{d\tau}{\mathrm{b}^2(\tau)}, \frac{\mathrm{f}}{\mathrm{b}(\tau)}\right),$$

where $\mathrm{F}^0(\tau, \mathrm{f})$ is a solution with constant frequency $\omega_0$.
- For $\kappa > 0$ and $\tau \to 0^-$, we have $\Delta\mathrm{f}/\Delta\pi \to 0$: Squeezed state.

## de Sitter (dS) era with $a(\tau) \sim -1/\tau$

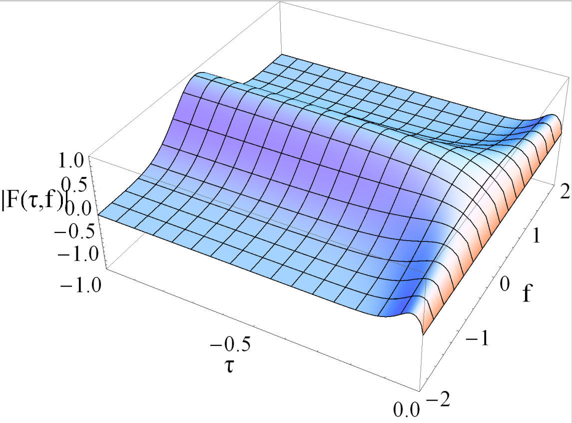


Figure: The amplitude of the 'ground-state' wave function for $\omega_0 = 5$.

# Transition to radiation domination (RD) with a($\tau$) $\sim \tau$

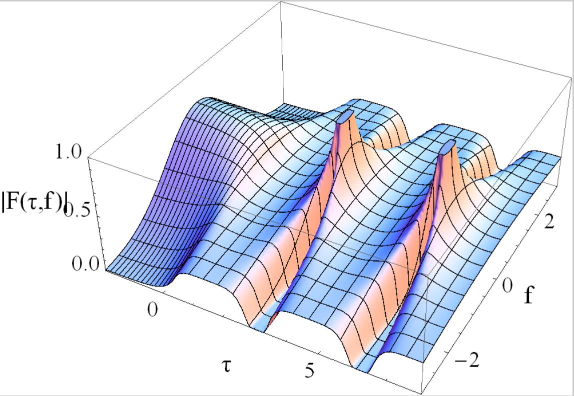


Figure: Left plot: The amplitude of the 'ground-state' wave function for the transition from a dS to a RD background at $\tau = 0.5$, for $\omega_0 = 1$, H = 2.

# Transition to matter domination (MD) with $a(\tau) \sim \tau^2$

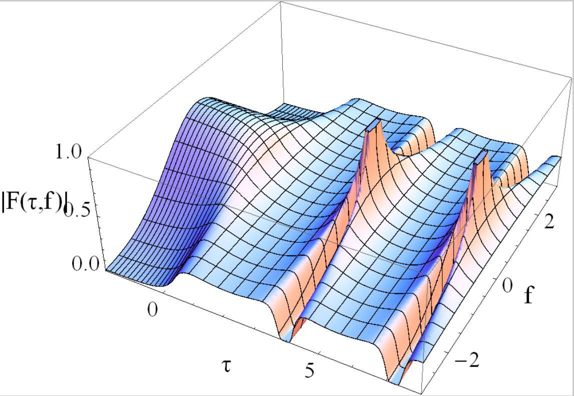


Figure: Left plot: The amplitude of the 'ground-state' wave function for the transition from a dS to a MD background at $\tau = 0.5$, for $\omega_0 = 1$, $H = 2$.

## Entanglement entropy of two quantum oscillators

- Hamiltonian (we switched from f to x)

$$H = \frac{1}{2}\left[p_1^2 + p_2^2 + k_0(x_1^2 + x_2^2) + k_1(x_1 - x_2)^2 - \lambda(\tau)(x_1^2 + x_2^2)\right].$$

- For oscillators arising from a massive field in dS, $\lambda(\tau) = 2\kappa/\tau^2$. For a massless field in a general background, $\lambda(\tau) = a''/a$.
- The Hamiltonian can be rewritten as

$$H = \frac{1}{2}\left[p_+^2 + p_-^2 + w_+^2(\tau)x_+^2 + w_-^2(\tau)x_-^2\right],$$

$x_\pm = \frac{x_1 \pm x_2}{\sqrt{2}}$, $\omega_{0+}^2 = k_0, \omega_{0-}^2 = k_0 + 2k_1$, $w_\pm^2(\tau) = \omega_{0\pm}^2 - \lambda(\tau)$.

- The 'ground state' is the tensor product of the 'ground states' of the two decoupled canonical modes:

$$\psi_0(x_+, x_-) = \left(\frac{\Omega_+\Omega_-}{\pi^2}\right)^{\frac{1}{4}} \exp\left[-\frac{1}{2}\left(\Omega_+ x_+^2 + \Omega_- x_-^2\right) + \frac{i}{2}\left(G_+ x_+^2 + G_- x_-^2\right.\right.$$

$$\Omega_\pm(\tau) \equiv \frac{\omega_{0\pm}}{b^2(\tau;\omega_{0\pm})}, \quad G_\pm(\tau) \equiv \frac{b'(\tau;\omega_{0\pm})}{b(\tau;\omega_{0\pm})}.$$

- Express the wave function in terms of $x_1$, $x_2$.
- The reduced density matrix is given by

$$\rho(x_2, x_2') = \int_{-\infty}^{+\infty} dx_1 \psi_0(x_1, x_2)\psi_0^*(x_1, x_2').$$

- The Gaussian integration gives

$$\rho(x_2, x_2') = \sqrt{\frac{\gamma - \beta}{\pi}} \exp\left(-\frac{\gamma}{2}(x_2^2 + x_2'^2) + \beta x_2 x_2'\right) \exp\left(i\frac{\delta}{2}(x_2^2 - x_2'^2)\right),$$

where $\gamma$, $\beta$, $\delta$ are functions of $\Omega_\pm$, $G_\pm$.
- The eigenfunctions of the reduced density matrix satisfy

$$\int_{-\infty}^{+\infty} dx_2' \rho(x_2, x_2') f_n(x_2') = p_n f_n(x_2).$$

- One finds

$$f_n(x) = H_n(\sqrt{\alpha}x) \exp\left(-\frac{\alpha}{2}x^2\right) \exp\left(i\frac{\delta}{2}x^2\right),$$

where $\alpha = \sqrt{\gamma^2 - \beta^2}$ and $H_n$ is a Hermite polynomial.

- The eigenvalues $\mathrm{p_n}$ are

$$\mathrm{p_n} = \sqrt{\frac{2(\gamma-\beta)}{\gamma+\alpha}}\left(\frac{\beta}{\gamma+\alpha}\right)^{\mathrm{n}} = (1-\xi)\xi^{\mathrm{n}},$$

where

$$\xi = \frac{\beta}{\gamma+\alpha}.$$

They satisfy

$$\sum_{\mathrm{n}=0}^{\infty} \mathrm{p_n} = (1-\xi)\sum_{\mathrm{n}=0}^{\infty}\xi^{\mathrm{n}} = 1.$$

- The entanglement entropy can be calculated as

$$\mathrm{S} = -\sum_{\mathrm{n}=0}^{\infty}(1-\xi)\xi^{\mathrm{n}}\ln\left[(1-\xi)\xi^{\mathrm{n}}\right] = -\ln(1-\xi) - \frac{\xi}{1-\xi}\ln\xi.$$

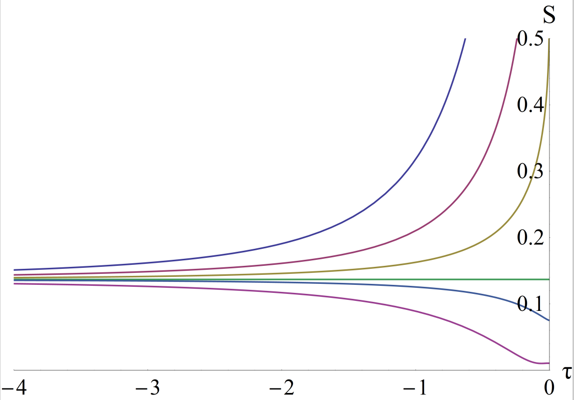


Figure: Left plot: The entanglement entropy in a dS background as a function of conformal time $\tau$ for $\omega_+ = 1$, $\omega_- = 2$ and $\kappa$ =1, 0.5, 0.2, 0, −0.1, −0.5 (from top to bottom).

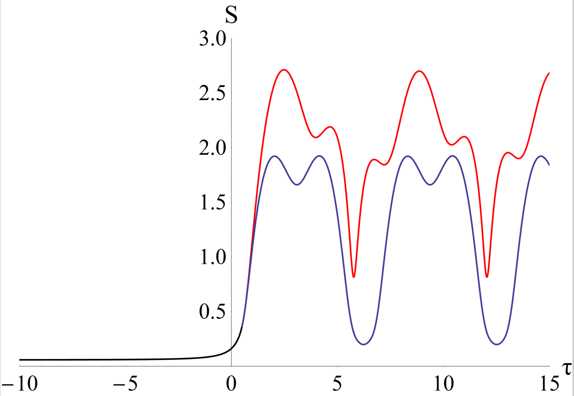


Figure: Left plot: The entanglement entropy as a function of conformal time $\tau$ for $\omega_+ = 1$, $\omega_- = 1.5$, H = 2 and $\tau_0 = 0.5$. The black line corresponds to a dS background, with a transition at $\tau_0$ to either a RD era (blue line) or to a MD era (red line).

## Entanglement entropy of many quantum oscillators

- Consider N coupled oscillators with a Hamiltonian

$$H = \frac{1}{2}\sum_{i}^{N} p_i^2 + \frac{1}{2}\sum_{i,j=1}^{N} x_i K_{ij} x_j.$$

- Diagonalize $K = O^T K_D O$ through an orthogonal matrix O. $\Omega_D = K_D^{1/2}$ contains the eigenfrequencies of the corresponding canonical modes $\tilde{X} = OX$.
- The wave function of the system is the product of the wave functions of the canonical modes.
- The system is assumed to lie in the ground state of each canonical mode in the asymptotic past (Bunch-Davies vacuum).
- At late times this becomes a squeezed state.

- When the density matrix is expressed in terms of the original coordinates $x_i$ it has the form

$$\rho(X, X') \sim \mathsf{exp}\left[-\frac{1}{2}\left(X^T W X + X'^T W^* X'\right)\right].$$

- One can introduce the block notation

$$W = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix}, \qquad X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

where A is a complex symmetric $n \times n$ matrix, C a complex symmetric $(N - n) \times (N - n)$ matrix, $X_1$ an n-dimensional vector, and so on.

- When the $N - n$ oscillators in $X_2$ are traced out, the reduced density matrix is

$$\rho_1(X_1, X_1') \sim$$

$$\exp\left(-\frac{1}{2}X_1^T\gamma\, X_1 - \frac{1}{2}X_1'^T\gamma\, X_1' + X_1'^T\beta\, X_1 + \frac{i}{2}X_1^T\delta\, X_1 - \frac{i}{2}X_1'^T\delta\, X_1'\right),$$

where

$$\gamma - i\delta = A - \frac{1}{2}B\,\mathrm{Re}\,(C)^{-1}\,B^T,$$

$$\beta = \frac{1}{2}B^*\mathrm{Re}\,(C)^{-1}\,B^T.$$

- $\gamma$ and $\delta$ are $n \times n$ real symmetric matrices, while $\beta$ is a $n \times n$ Hermitian matrix. (In a static background it would be real symmetric.)
- The eigenvalues of the density matrix do not depend on $\delta$.

- The matrices $\gamma$ and $\beta$ cannot be diagonalized through real orthogonal transformations in order to identify the eigenvalues of the reduced density matrix.
- These are real, but the determination of their exact values requires an extensive analysis.
- A method has been developed for the computation of the eigenvalues. A detailed presentation is given in the publications.
- It has been shown (analytically and numerically) that the results are identical to those obtained through the covariance matrix.

## Cosmology: choice of a UV cutoff

- In flat (3+1)-dimensional spacetime the entropy scales $\sim 1/\epsilon^2$, with $\epsilon$ a short-distance cutoff.
- There is a certain mode of comoving wavenumber $\mathrm{k_s}$ which crossed the horizon at the end of inflation and immediately re-entered. Modes with $\mathrm{k} > \mathrm{k_s}$ remained subhorizon at all times.
- The modes with $\mathrm{k} < \mathrm{k_s}$ are the ones directly accessible to experiment and constitute the observable Universe.
- The entanglement of interest is between modes with wavelengths above a UV cutoff $\epsilon \sim 1/\mathrm{k_s} \sim 1/\mathrm{H_{infl}}$.
- Modes that exited the horizon at the end of inflation have a frequency today $\mathrm{f} \sim 10^8$ Hz, which sets the cutoff in the spectrum of gravitational waves generated by inflation. The corresponding wavelength is $\lambda_\mathrm{s} \sim 1$ m.

## Entanglement entropy of a quantum field in $1+1$ dimensions

- Consider a toy model of a massless scalar field in $1+1$ dimensions.
- Assume a background given by the FRW metric, neglecting the angular part. The curvature scalar R is equal to $-2\mathrm{H}^2$.
- The field is canonically normalized.
- The state of a canonical mode in (3+1)-dimensional de Sitter space can be mimicked by including an effective mass term through a non-minimal coupling to gravity $-\mathcal{R}\phi^2/2$.
- The radiation dominated era can be mimicked by assuming a transition to a flat background with $\mathcal{R}=0$ at some time $\tau_0$.
- The toy model describes the $\mathrm{l}=0$ mode of the $3+1$-dimensional case.

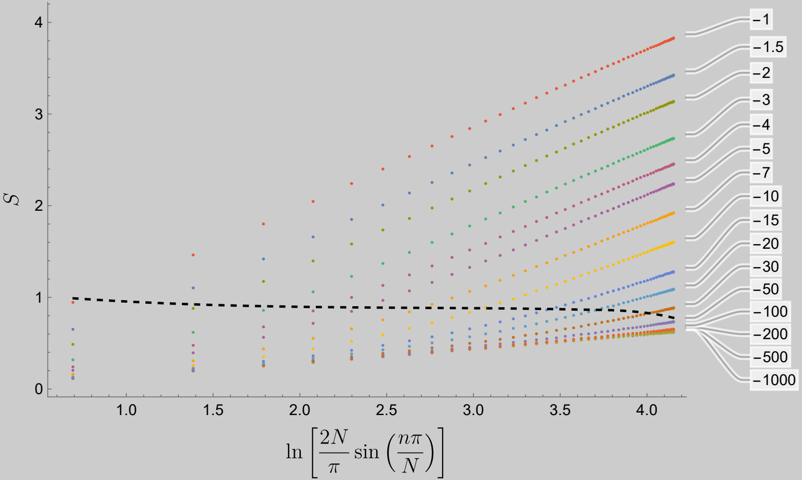


Figure: The entanglement entropy resulting from tracing out the part $n < k \leq N$ of a one-dimensional chain at various times, for a dS background.

- For $\tau \to -\infty$, the entanglement entropy can be described very well by the expression

$$S = \frac{c}{6} \ln\left(\frac{2L}{\pi\epsilon} \sin\frac{\pi\ell}{L}\right) + \bar{c}'_1,$$

with $c = 1$, in agreement with Calabrese, Cardy 2004.

- For $\tau \to 0^-$ the entanglement entropy can be described very well by the expression

$$S = \ln\left(\frac{2L\, a(\tau)}{\pi\epsilon} \sin\frac{\pi\ell}{L}\right) + d,$$

where $a(\tau) = -1/(H\tau)$.

- The entropy grows with the number of efoldings $\mathcal{N} = \ln a(\tau)$.

## Infrared effect in de Sitter space in 1+1 dimensions (preliminary)

- The limit $\tau \to 0^-$ is not physical because

$$-\frac{\tau}{\epsilon} = \frac{1/\mathrm{H}}{\mathrm{a}(\tau)\epsilon} < 1$$

corresponds to a physical lattice spacing (physical UV cutoff) $\epsilon_\mathrm{p} = \mathrm{a}(\tau)\epsilon$ larger than the Hubble radius.

- Consider an $1/\tau$ expansion around $\tau = -\infty$.
- The numerical analysis, verified by an analytical calculation, indicates the presence of a leading correction

$$\Delta\mathrm{S} = \frac{\ell^2}{\tau^2}\left[\frac{1}{3}\log\left(\frac{\mathrm{L}}{2\pi\ell}\right) + \frac{4}{9}\right] = \mathrm{H}^2\ell_\mathrm{p}^2\left[\frac{1}{3}\log\left(\frac{\mathrm{L_p}}{2\pi\ell_\mathrm{p}}\right) + \frac{4}{9}\right]$$

for $\ell_\mathrm{p} \ll 1/\mathrm{H} \ll \mathrm{L_p}$.

- Dependence on the size of the total system.
- A similar term is expected in 3+1 dimensions.

## Entanglement entropy of a quantum field in 3 + 1 dimensions

- Massless scalar field in $3+1$ dimensions.
- Hamiltonian:

$$H = \frac{1}{2\epsilon}\sum_{l,m}\sum_{j=1}^{N}\Bigg[\pi^2_{lm,j} + \left(j+\frac{1}{2}\right)^2\left(\frac{f_{lm,j+1}}{j+1}-\frac{f_{lm,j}}{j}\right)^2 + \left(\frac{l(l+1)}{j^2}-\frac{2\kappa}{(\tau/\epsilon)^2}\right)f^2_{lm,j}\Bigg],$$

with $\kappa=1$ for dS and $\kappa=0$ for RD.

- Trace out the oscillators with $j\epsilon > R$.
- Sum over $l, m$.

## A volume effect

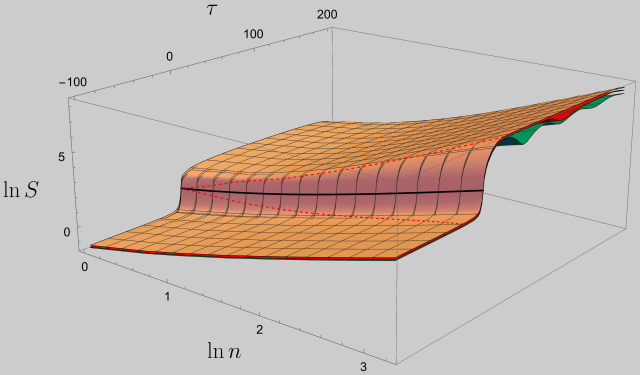


Figure: The entanglement entropy for a spherical region as a function of the entangling radius at various times for $\mathrm{H}\epsilon = 1$. The radius of the spherical lattice is $\mathrm{L} = \mathrm{N}\epsilon$. Results for $\mathrm{N} = 200$ (brown), $\mathrm{N} = 100$ (red), $\mathrm{N} = 50$ (green). We indicate the entropy at the dS to RD transition (black curve) and the location of the comoving horizon (dashed, red curve).

Fit the result with a function ($\epsilon = 1$)

$$\mathrm{S} = \mathrm{s}(\tau)\,\mathrm{R}^2 + \mathrm{c}(\tau)\,\mathrm{R}^3.$$

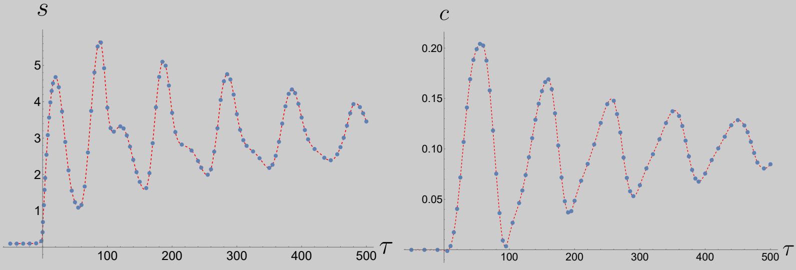


Figure: The coefficents s (left plot) and c (right plot) of the quadratic and cubic term, respectively, as a function of time.

## Entanglement entropy in de Sitter space in 3+1 dimensions

- Conjectured form of the entanglement entropy in de Sitter space (Maldacena, Pimentel 2013):

$$S = c_1 \frac{\mathcal{A}}{\epsilon_p^2} + \frac{c_2}{2} \log\left(\frac{\mathcal{A}}{\epsilon_p^2}\right) + c_4 \log(H\epsilon_p) H^2 \mathcal{A} + c_5 H^2 \mathcal{A} + \frac{c_6}{2} \log(H^2 \mathcal{A}).$$

- $\mathcal{A} = 4\pi a^2(\tau) R^2$: proper area of the entangling surface
- $\epsilon_p = a(\tau)\epsilon$: physical UV cutoff
- $c_1$ (scheme-dependent) (Srednicki 1993)
- $c_2 = -1/90$ (Lohmayer, Neuberger, Schwimmer, Theisen 2009)
- $c_6 = 1/90$ (Maldacena, Pimentel 2013)
- No volume term.
- Additional term: $\frac{1}{3} H^2 R_p^2 \log\left(\frac{R_p^{tot}}{\epsilon_p}\right)$

## Coefficient of the $H^2\mathcal{A}$ term (preliminary)

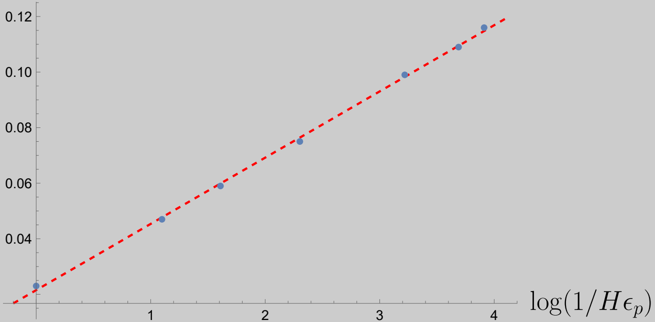


Figure: Coefficient of the $H^2\mathcal{A}$ term.

# Summary

- Modes that start as quantum fluctuations in the Bunch-Davies vacuum are expected to freeze upon horizon exit and transmute into classical stochastic fluctuations.
- This is only part of the picture. Even though its classical features are dominant, the field never loses its quantum nature.
- The various modes evolve into squeezed states.
- The squeezing triggers an enhancement of quantum entanglement. The effect is visible in the entanglement entropy.
- The entanglement entropy survives during the eras of radiation or matter domination. A volume effect appears during these eras.
- Observable consequences?
- A look behind the horizon?
- Weakly interacting, very light fields that stay coherent during the cosmological evolution (gravitational waves).

- Interpretation of the entropy as thermodynamic? A quantum-mechanical realization of reheating after inflation?
- It is consistent with the quantum to classical transition.
- It is intriguing that the appearance of a volume term, a feature of thermodynamic entropy, occurs in the RD era.
- For two oscillators, the reduced density matrix is that of a single oscillator at a thermal state. For a free field there is no unique temperature. The thermalization hypothesis suggests that in an interacting theory the reduced density matrix would be thermal.
- If we estimate the entropy through the volume term, we get $\sim (\mathrm{H}_0\lambda_\mathrm{s})^{-3} \sim 10^{78}$, to be compared with the standard thermodynamic entropy $\sim 10^{88}$ associated with the plasma in the early Universe, transferred to the photons and neutrinos today.
- Can the entropy of the Universe be attributed to the presence of the cosmological horizon?

# How to measure entanglement entropy

**Verification of the area law of mutual information in a quantum field simulator** #3

Mohammadamin Tajik (Vienna, Tech. U., Atominst.), Ivan Kukuljan (Munich, Max Planck Inst. Quantenopt. and MCQST, Munich), Spyros Sotiriadis (Freie U., Berlin), Bernhard Rauer (Vienna, Tech. U., Atominst. and Paris, Lab. Kastler Brossel), Thomas Schweigler (Vienna, Tech. U., Atominst.) et al. (Jun 21, 2022)

Published in: *Nature Phys.* 19 (2023) 7, 1022-1026 • e-Print: 2206.10563 [cond-mat.quant-gas]

## Experimental verification of the area law of mutual information in a quantum field simulator

Mohammadamin Tajik,[1,*] Ivan Kukuljan,[2,3] Spyros Sotiriadis,[4] Bernhard Rauer,[1,5]
Thomas Schweigler,[1] Federica Cataldini,[1] João Sabino,[1,6,7] Frederik Møller,[1]
Philipp Schüttelkopf,[1] Si-Cong Ji,[1] Dries Sels,[8,9] Eugene Demler,[10] and Jörg Schmiedmayer[1,†]

[1] *Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, Stadionallee 2, 1020 Vienna, Austria*
[2] *Max-Planck Institute of Quantum Optics, Hans-Kopfermann-Str. 1, 85748 Garching, Germany*
[3] *Munich Center for Quantum Science and Technology (MCQST), Schellingstr. 4, D-80799 München*
[4] *Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, Berlin, Germany*
[5] *Laboratoire Kastler Brossel, ENS-Université PSL, CNRS, Sorbonne Université, Collége de France, 24 rue Lhomond, 75005 Paris, France*
[6] *Instituto de Telecomunicações, Physics of Information and Quantum Technologies Group, Av. Rovisco Pais 1, 1049-001, Lisbon, Portugal*
[7] *Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001, Lisbon, Portugal*
[8] *Department of physics, New York University, New York City, NY 10003, USA*
[9] *Center for Computational Quantum Physics, Flatiron Institute, 162 5th Ave, New York, NY 10010, USA*
[10] *Institute for Theoretical Physics, ETH Zurich, Wolfgang-Pauli-Str. 27, 8093 Zurich, Switzerland*

(Dated: March 8, 2023)

Theoretical understanding of the scaling of entropies and the mutual information has led to significant advances in the research of correlated states of matter, quantum field theory, and gravity. Measuring von Neumann entropy in quantum many-body systems is challenging as it requires complete knowledge of the density matrix. In this work, we measure the von Neumann entropy of spatially extended subsystems in an ultra-cold atom simulator of one-dimensional quantum field theories. We experimentally verify one of the fundamental properties of equilibrium states of gapped quantum many-body systems, the area law of quantum mutual information. We also study the dependence of mutual information on temperature and the separation between the subsystems. Our work is a crucial step toward employing ultra-cold atom simulators to probe entanglement in quantum field theories.

#13

## Quantum field simulator for dynamics in curved spacetime

Celia Viermann (Kirchhoff Inst. Phys.), Marius Sparn (Kirchhoff Inst. Phys.), Nikolas Liebster (Kirchhoff Inst. Phys.), Maurus Hans (Kirchhoff Inst. Phys.), Elinor Kath (Kirchhoff Inst. Phys.) et al. (Feb 21, 2022)

Published in: *Nature* 611 (2022) 7935, 260-264 • e-Print: 2202.10399 [cond-mat.quant-gas]

### Quantum field simulator for dynamics in curved spacetime

Celia Viermann,[1] Marius Sparn,[1] Nikolas Liebster,[1] Maurus Hans,[1] Elinor Kath,[1] Álvaro Parra-López,[2,3] Mireia Tolosa-Simeón,[2] Natalia Sánchez-Kuntz,[2] Tobias Haas,[2] Helmut Strobel,[1] Stefan Floerchinger,[2,4] and Markus K. Oberthaler[1]

[1] *Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany*

[2] *Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany*

[3] *Departamento de Física Teórica and IPARCOS, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Ciudad Universitaria, 28040 Madrid, Spain*

[4] *Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, 07743 Jena, Germany*

**The observed large-scale structure in our Universe is seen as a result of quantum fluctuations amplified by spacetime evolution [1]. This, and related problems in cosmology, asks for an understanding of the quantum fields of the standard model and dark matter in curved spacetime. Even the reduced problem of a scalar quantum field in an explicitly time-dependent spacetime metric is a theoretical challenge [2–4] and thus a quantum field simulator can lead to new insights. Here, we demonstrate such a quantum field simulator in a two-dimensional Bose-Einstein condensate with a configurable trap [5, 6] and adjustable interaction strength to implement this model system. We explicitly show the realisation of spacetimes with positive and negative spatial curvature by wave packet propagation and confirm particle pair production in controlled power-law expansion of space. We find quantitative agreement with new analytical predictions for different curvatures in time and space. This benchmarks and thereby establishes a quantum field simulator of a new class. In the future, straightforward upgrades offer the possibility to enter new, so far unexplored, regimes that give further insight into relativistic quantum field dynamics.**

densate of potassium-39 with configurable de
bution and additional dynamic control of a
actions. With that we implement curved me
phononic field of the form (see methods: 'Cu
metric')

$$ds^2 = -dt^2 + a^2(t)\left(\frac{du^2}{1-\kappa u^2} + u^2 d\varphi$$

This corresponds to the standard cosmolo
of a $2+1$ dimensional homogeneous and is
verse, the Friedmann-Lemaître-Robertson-W
ric (FLRW) in reduced circumference coordi
This metric is parametrised by intrinsic and e
vature: the intrinsic curvature, $\kappa$, is the curv
spatial part of the metric, while the extrins
arises from the time dependence of the scale
In our atomic implementation both parame
ture and scale factor, can be adjusted indep
Phonons in the central region of a h
trapped Bose-Einstein condensate experien
with $\kappa < 0$. In cosmological settings, this
bolic two-dimensional spatial geometry with
ordinate of infinite range, as depicted in
Through the Poincaré transformation, the in
bolic space is mapped to a finite disc, perf
to be implemented in finite-size ultracold