Global fixed point potential approach to frustrated magnets

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$O(N) \times O(2)$ models

• For Stacked Triangular Antiferromagnets (STA), the order parameter is $N \times 2$ matrix. d Irignallar Ar u ilianuulal _{mi} approaches—in dimensions *d*!2"\$ and *d*!4#\$—conflict. around *d*!4 or directly in *d*!3 succeed in reproducing sat- \mathbf{v} isfactorily the phenomenology. We show \mathbf{v}

- **For** N above (below) $N_c(d)$, the transition is of second a mechanism of $\mathbb{P}(X|X)$ and $\mathbb{P}(X|X)$ fied time and \bullet 1.8 is a long time and \bullet \mathbf{I} is the interesting in the occurrence of \mathbf{I} \cap ineinwithout \cup \cup σ (wording σ) in a slowing σ \mathbf{v} region in constants space. This allows us to the constant space. This allows us to the constant space. $F = \pm \frac{1}{2}$. The ground-state configurations $\pm \frac{1}{2}$ triangular lattice and "b# of the order parameter made of two orthono di universitute t
- (first) order. the *O*(4) fixed point obtained within a low-temperature approach in *d*!2"\$. Second, our approach provides a description of the physics in *d*!3, in terms of weakly first ϵ sec. X possible experimental and ϵ numerical tests of our scenario. We then comment "Sec. XI#

$O(N) \times O(2)$ models $R(X,Y)$ indeed, in the MZM fixed-dimensional scheme (see footnote 24), such a scheme (see football scheme (see football sch a turnaround does happen for *d slightly* below 3 (see the discussion in [14] below Fig. 3), \mathcal{L}

• $N_c(d = 3)$ is found to be very close to 3 and its precise determination is crucial to know whether the transition is first or second order for the systems realized in nature. region which is disconnected from the region at large *N. The region* still allowed scenarios in the still allowed scenarios in the scenarios of the scenarios in the scenarios of the scenarios of the scenarios of the scena

. It is also interesting for physics in $d = 3$ to study the part of the curve $N_c(d)$ below $d = 3$ (However it was difficult to study it within previous approximations of NPRG). \bullet It is also interesting for physics in a $$ scenario on the left, where the boundary curve has a turnaround point (*•*) at a *d• >* 3, is excluded bart of the curve $N_c(d)$ below $d = 3$ (Hc allowed region is not connected, are still allowed. Acknowledgments

Numerical simulations for STA

• Numerical simulations for XY and Heisenberg STA found first order transition (Loison-Schotte, Itakura, Thanh Ngo-Diep).

Numerical simulations for STA

• A recent simulation of Heisenberg STA with a very large lattice size 384^3 found second order transition corresponding to a focus FP with a complex-valued correction-to-scaling exponent (Nagano-Kawamura).

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Monte Carlo study of the critical properties of noncollinear Heisenberg magnets: $O(3) \times O(2)$ universality class

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The critical properties of the antiferromagnetic Heisenberg model on the three-dimensional stacked-triangular lattice are studied by means of a large-scale Monte Carlo simulation in order to get insight into the controversial issue of the criticality of the noncollinear magnets with the $O(3) \times O(2)$ symmetry. The maximum size studied is 3843, considerably larger than the sizes studied by the previous numerical works on the model. Availability of such large-size data enables us to examine the detailed critical properties including the effect of corrections to the leading scaling. Strong numerical evidence of the continuous nature of the transition is obtained. Our data indicate the existence of significant corrections to the leading scaling. Careful analysis by taking account of the possible corrections yields critical exponents estimates, $α = 0.44(3)$, $β = 0.26(2)$, $γ = 1.03(5)$, $ν = 0.52(1)$, $\eta = 0.02(5)$, and the chirality exponents, $\beta_{\kappa} = 0.40(3)$ and $\gamma_{\kappa} = 0.77(6)$, supporting the existence of the *O*(3) chiral [or $O(3) \times O(2)$] universality class governed by a "chiral" fixed point. We also obtain an indication that the underlying fixed point is of the focus type, characterized by the complex-valued correction-to-scaling exponent, $\omega = 0.1_{-0.05}^{+0.4} + i \cdot 0.7_{-0.4}^{+0.1}$. The focus-like nature of the chiral fixed point accompanied by the spiral-like renormalization-group (RG) flow is likely to be the origin of the apparently complicated critical behavior. The results are compared and discussed in conjunction with the results of other numerical simulations, several distinct types of RG calculations including the higher-order perturbative massive and massless RG calculations and the nonperturbative functional RG calculation, and the conformal-bootstrap program.

Renormalization group studies on $O(N) \times O(2)$ models

- With ϵ -expansion, the value of $N_c(d=3)$ is systematically found larger than 3 as it is also the case for the NPRG calculations
- On the contrary, the perturbative calculation performed directly in $d = 3$ at six loops yields a focus fixed point for $N = 2.3$, suggesting the second order phase transition.

Conformal bootstrap studies on $O(N) \times O(2)$ models

 \therefore An early study found a critical FP for $N = 2,3$ in $d = 3$ and the critical exponents in good agreement with the focus FPs found with the perturbative fixed dimensional approach.

(Ohtsuki-Nakayama PRD 2014,2015)

- In a recent refined conformal bootstrap study, a lower bound for systems satisfying reflection positivity is $N_c(d) > 3.78$.
	- (Reehorst-Rychkov-Sirois-van Rees, arXiv 2024)

Experimental results Let us also notice that a possible source of error in the estimation of the critical exponents themselves could be the existence of corrections to scaling that could bias all the results. As we now argue, we can however expect that these effects are not dramatic. Let us consider the well-documented case of the ferromagnetic Ising model in d = 3. Most of the time corrections to scaling are not [92] 0.21(2) 1.01(8) 0.54(3) \blacksquare \blacksquare [95] 0.39(9) $\overline{}$ $\overline{}$ CsNiCl³ [98] 0.37(8)

• Helimagnets are expected to belong to the same universal class. nents, i.e. at most of few percents (see for instance [87] and $\overline{}$. Hal \sim 1.101 [100] 0.243 (5) IANNATC ruynolo aro y

TABLE II: The critical exponents of the XY helimagnets.

TABLE IV: The critical exponents of the Heisenberg STA. The abbreviations A, B and C stand for $Cu(HCOO)_22CO(ND_2)_22D_2O$, $Fe[S_2CN(C_2H_5)_2]_2Cl$ and $\text{CsMn}(Br_{0.19}I_{0.81})_3$ respectively. The data in brackets are suspected to be incorrect. They are given for completeness.

i) As in the XY case, the Heisenberg materials fall into B. Delamotte, D. Mouhanna, and M. Tissier Phys. Rev. B (2004) B. Delamotte, D. Mouhanna, and M. Tissier $\Omega(\Lambda)$ in Eq. (37), one obtains, for the anomalous dimensions, for the anomalous dimension: σ

Revisiting $O(N) \times O(2)$ models with NPRG (after some refinement) might be useful. \sim \sim \sim uis in Li icantly, contrarily to what happens in group 1. retinement) For \mathfrak{p} \overline{z} + D_{Q} presents a showld between \overline{P} with the same critical exponents. For Case of Case of the transition of the transition of the transition of th sition is found to be weakly of first order, i.e. with small Ω Ω Ω Ω in Ω i the following discussion, we mainly use the exponent β \sim and \sim \sim \sim \sim \sim \sim \mathbf{t} there are two groups of \mathbf{t} characterized by a set of exponent set of exponents, \mathcal{C}

Approximations in NPRG

- Field expansion of the local potential
- Derivative expansiton (DE) of effective action

Discrepancy in $d = 3$ between NPRG and some bootstrap studies may be due to the approximations used in NPRG.

In this study, we do not make any field expansion of the local potential (but do derivative expansions).

$O(N) \times O(2)$ models

. The order parameter for STA is a $N \times 2$ matrix $\Phi = (\phi_1, \phi_2)$ that satisfies $\phi_i \cdot \phi_2 = \delta_{i,j}$ for $i, j = 1,2$. The constraint can be replaced by a soft potential $U(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2)$ whose minima are given by $\phi_i \cdot \phi_2 = const \delta_{i,j}$. The effective Hamiltonian is

$$
H = \int d^d \mathbf{x} \left(\frac{1}{2} \left[\left(\partial \boldsymbol{\phi}_1 \right)^2 + \left(\partial \boldsymbol{\phi}_2 \right)^2 \right] + U \left(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2 \right) \right)
$$

Derivative expansion (DE) of the effective action

$$
\Gamma_k = \int_x \left\{ U_k(\rho, \tau) + \frac{1}{2} Z_k \left((\partial \varphi_1)^2 + (\partial \varphi_2)^2 \right) + \frac{1}{4} \omega_k (\varphi_1 \cdot \partial \varphi_2 - \varphi_2 \cdot \partial \varphi_1)^2 \right\}.
$$

. This approximation, that we call LPA' with ω_k , reproduces one-loop ${\sf results}$ by $\epsilon = 4 - d$ and $\epsilon = d - 2$ expansions. We call the

approximation setting $\omega_k = 0$ here as LPA'. At criticality $Z_{k\to 0} \sim \Big($ *k* $\overline{\Lambda}$) −*η* .

. The approximation with $\eta = 0$ ($Z_k = const$) and $\omega_k = 0$ is called LPA.

Flow equation

$$
\partial_t \Gamma_k[\boldsymbol{\varphi}_i] = \frac{1}{2} \text{Tr} \int_{x,y} \partial_t R_k(x-y) \left(\frac{\delta^2 \Gamma_k[\boldsymbol{\varphi}_i]}{\delta \varphi_i^{\alpha}(\mathbf{x}) \delta \varphi_{i'}^{\alpha'}(\mathbf{y})} + R_k(\mathbf{x}-\mathbf{y}) \delta_{i,i'} \delta_{\alpha,\alpha'} \right)^{-1}
$$

•
$$
\alpha, \alpha' = 1, 2, \dots, N
$$
 and $i, i' = 1, 2$

• We employ a family of regulators

$$
R_k(\mathbf{q}^2) = \beta Z_k k^2 \left(1 - \frac{\mathbf{q}^2}{k^2}\right)^{\alpha} \Theta\left(k^2 - \mathbf{q}^2\right)
$$
, which is useful for analytical

treatment when $\alpha = \beta = 1$

. We evaluate η and ω_k at the minimum of the potential.

Local potential of $O(N) \times O(2)$ models

- For any φ_1 and φ_2 , we can "diagonalize" the matrix $\Psi = (\varphi_1, \varphi_2)$ as $M \equiv O_1 \Psi O_2$ so that
- **.** Because of $O(N) \times O(2)$ symmetry $U_k(\Psi) = U_k(M)$.
- . The potential is a function of the $O(N) \times O(2)$ invariants

$$
\rho = \psi_1^2 + \psi_2^2
$$
 and $\tau = \frac{1}{4} (\psi_1^2 - \psi_2^2)^2$, which satisfies that
\n $U_k(\psi_1, \psi_2) = U_k(-\psi_1, \psi_2) = U_k(\psi_1, -\psi_2) = U_k(\psi_2, \psi_1)$

Dimensionless variables and FP potential

• In order to find FP solutions, we employ the following dimensionless quantities

$$
\tilde{\psi}_i = \left(Z_k k^{2-d}\right)^{1/2} \psi_i, \quad \tilde{U}_k(\tilde{\psi}_i) = k^{-d} U_k \left(\psi_i\right).
$$

- The variations of $N_c(d = 3)$ with respect to the parameter α seem much smaller than the uncertainty associated with the truncation of the DE (LPA, LPA', LPA' with ω_{k}).
- This confirms previous NPRG results with "semi-expansion" $N_c^{\text{semi}}(d=3) \simeq 4.68(2), 5.24(2)$ at LPA, LPA' with ω_k .
- NPRG results seem to converge well within each approximation (LPA, LPA', LPA' with ω_k).

- Previous NPRG ("semi-expansion" with a different regulator) results give $\nu(N = 6) = 0.695(5)$ and $\eta(N = 6) = 0.042(2)$. Assuming that these results are converging, the dependence on the regulator is not small contrary to $N_c(d=3)$.
- The correction-to-scaling exponent (not shown) is real.

 $N_c(d = 2.5)$

- . In lower dimensions, the roles of η and ω_k are important.
- We recall that $N_c(d \leq 3)$ is interesting for physics in $d = 3$
- The solutions are numerically under control.

• The spreading of $N_c(d, \alpha, \beta)$ when α is varied between 1 and 5 and β between 1 and 2 is almost invisible on this scale.

2.5 3.0 3.5 4.0

- **.** This suggests that S-shape behavior of $N_c(d)$ around $d = 3$ is not probable.
- As for the approach to $d = 2$, we have found good agreement with the ϵ expansion results after resummation.

Multicritical problems that it also becomes vertical at large N while being this time asymptotic to the d \mathbb{N} \blacksquare \mathbf{A} and \mathbf{A} clearly a subject that must be further studied, see, however, \Box . \Box \Box \Box \Box \Box \Box \Box of the FP potential of C2 and C3 should be studied in the FP point \mathcal{L} Future as well as welld \blacksquare such an out what the basins of attraction out what the basins of attraction of attraction of attraction of

 M_2, M_3 \cdots 2,3-unstable nonperturbative FP found with field expansion

S. Yabunaka and B. Delamotte, PRL 2017

- . For multicritical FPs in $O(N)$ models, we found boundary layer behavior at Large-N, which cannot be captured by field expansions. The two nonperturbative FRS and M3, and M3, appear on the M3, appear on the M3 and M3 and M3 and M concerns only C² and T² and we could wonder whether \bullet same indeed for \bullet $\overline{}$ c_dd^a f $\overline{}$ the two curves meet and that shares σ d α at α at α at all is found. Above this region and α iavior at Large-N, which cannot be for the line on the squares the right joining the squares the squares the square the square the square t_{1} is the crosses, C₁ and M3 collapse. In each region, C_{1}
	- Therefore it may be interesting to study the multicritical FP in $O(N) \times O(2)$ models without any field expansion (in progress).

Summary

- We have obtained $N_c(d)$ in $2.3 \le d \le 4$ solving the FP equation without any field expansion (but with derivative expansion).
- \bullet In $d = 3$, our results confirm what was found with field expansions of NPRG and recent conformal bootstrap results.
- As for the approach to $d = 2$, we have found good agreement with the ϵ expansion after resummation.