Low-energy effective theories for metals

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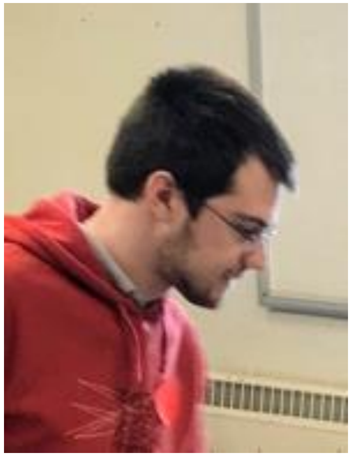
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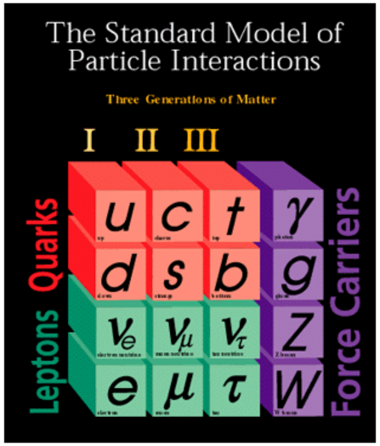
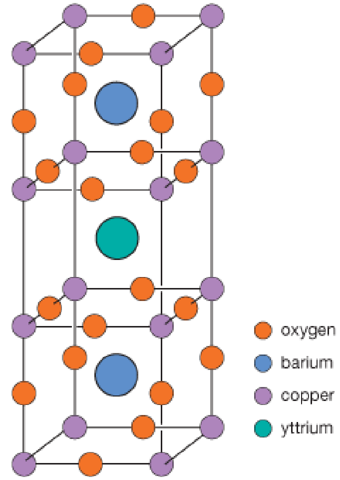
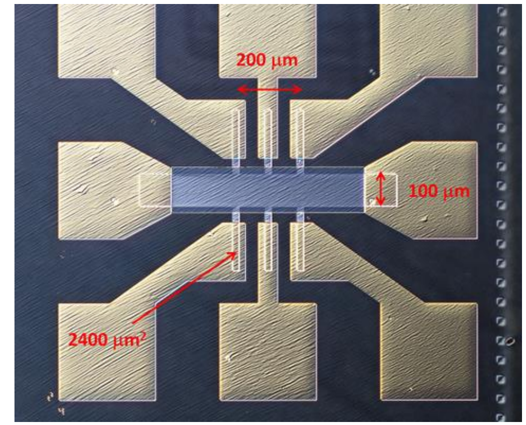
Emergent Phenomena







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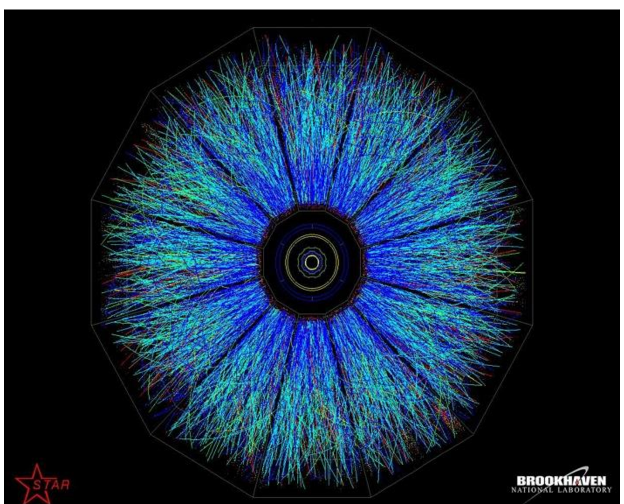
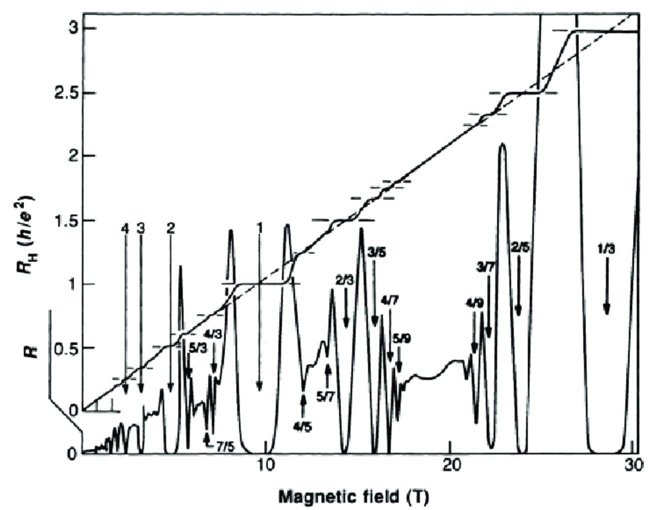


Quark-gluon plasma

High temperature superconductor



Fractional quantum Hall state

From theories to observables

$$S\left(\phi,\psi;\{g_j\};\Lambda\right) \quad \Rightarrow \quad \Gamma_n\left(\{\omega_i,k_i\};\{g_j\};\Lambda\right)$$

- The exact vertex function contains the full dynamical information
  - hard to extract

- Mapping the space of low-energy phases is easier due to simplicity that arises in the low-energy limit

Universality of low-energy physics

There exists a set of low-energy parameters defined through observables measured at energy scale $\mu \ll \Lambda$

$$\mathbf{g}^{(m)}(\mu) = \Gamma_m \left(\{\omega_i^{(m)}, k_i^{(m)}\}; \{g_j\}; \Lambda \right), \quad \omega_i^{(m)} \sim \mu$$

Those low-energy parameters determine all observables at that energy scale and below within errors that vanish in powers of μ/Λ

$$\Gamma_n \left(\{\omega_i, k_i\}; \{g_j\}; \Lambda \right) = f_n \left(\{\omega_i, k_i\}; \{\mathbf{g}^{(m)}(\mu)\} \right) + (\mu/\Lambda)^a$$

$$\text{for} \quad \omega_i \leq \mu$$

*Extracting $f_n \left(\{\omega_i, k_i\}; \{\mathbf{g}^{(m)}(\mu)\} \right)$ is still highly non-trivial

For relativistic QFTs

- The space of low-energy parameters is composed of a finite number of marginal and relevant parameters that carry non-negative scaling dimensions
- RG flow within the finite-dimensional space

$$\mathbf{g}^{(m)}(\mu)$$

- Fixed points of the renormalization group flow represent universality classes

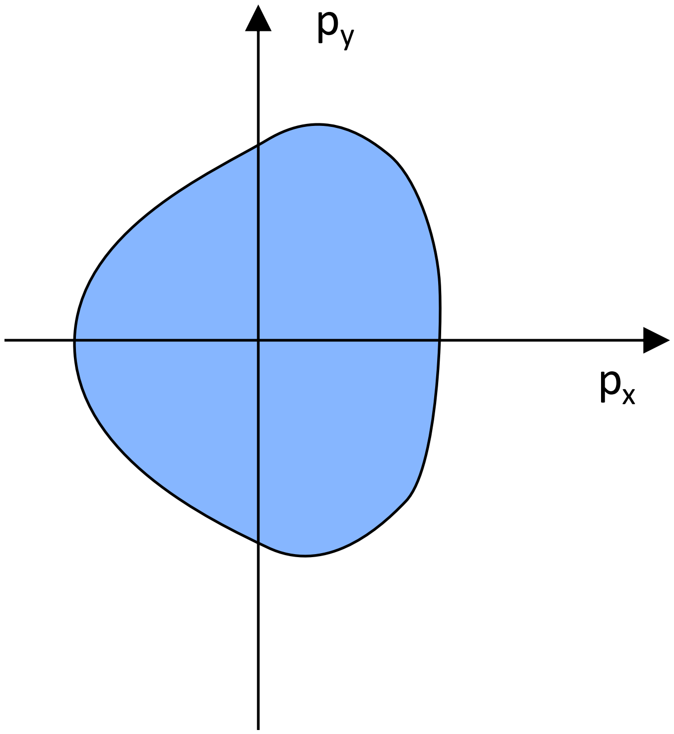
$$\lim_{\mu \to 0} \mathbf{g}^{(m)}(\mu)$$

Metals

A non-zero density of fermions form a droplet of occupied states in the momentum space

Infinitely many gapless modes that describe soft shape deformations of the droplet (particle-hole excitations near Fermi surface)

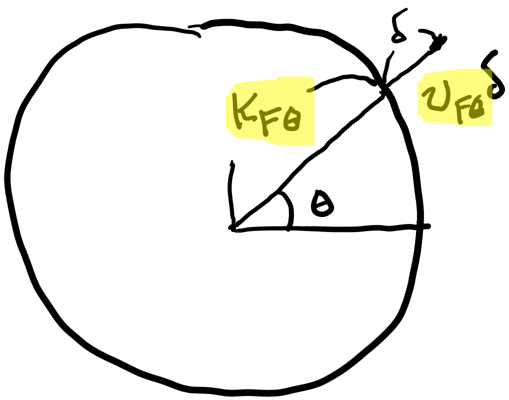
Metals subject to strong quantum fluctuations remain poorly understood

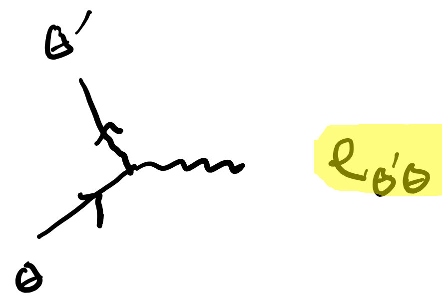


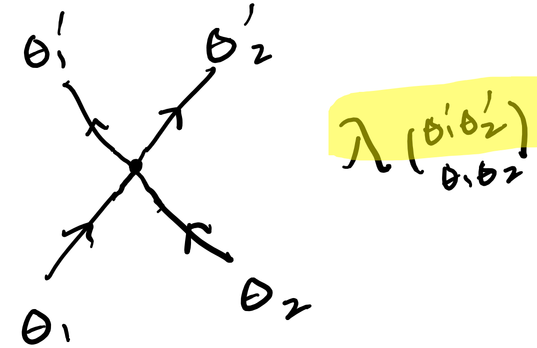
- Metals with vector flavors can behave like matrix models (non-trivial large N limit)
- Dynamical kinematic energy quenching
- **UV/IR mixing**

Theoretical challenge : infinite-dimensional space of low-energy theories

- Low-energy theories are characterized by Fermi momentum, Fermi velocity, couplings, which are functions of angles around FS





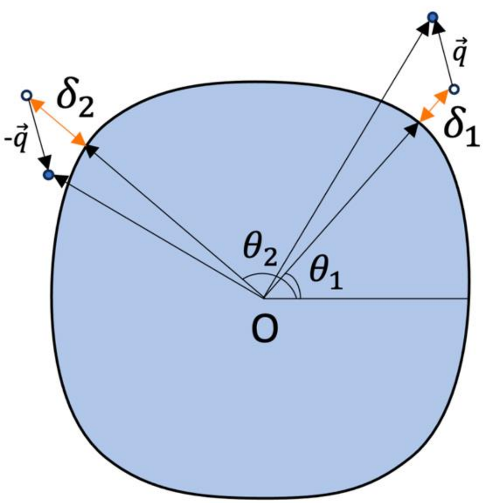


- To chart the space of metallic universality classes, one needs to track a functional renormalization group flow for a minimal set of coupling functions
  - Field-theoretic functional RG

Borges, Borissov, Singh, Schlief, SL (2023)

Theoretical challenge : absence of scale invariance

- There is no scale invariance in metals due to Fermi momentum, which is a large momentum(UV) scale but a low-energy(IR) scale
- Fermi momentum, measured in the unit of μ, grows incessantly under scale transformations

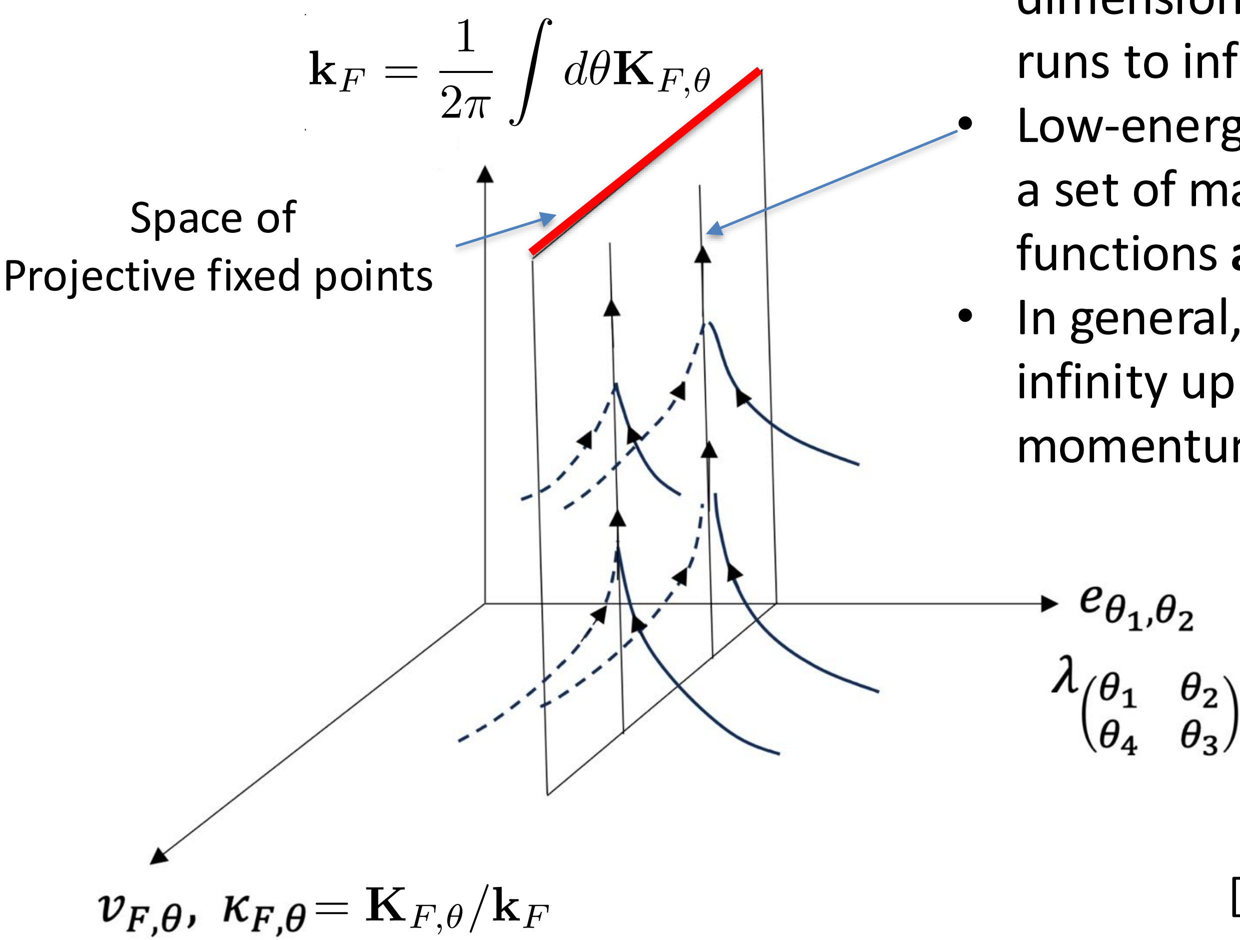


$$\Delta\theta \sim \frac{q}{K_F}$$

$$q \to q/b, \quad \theta \to \theta$$

$$K_F \to bK_F$$

Metallic Universality classes correspond to projective fixed points



- RG flow is attracted toward an one-dimensional manifold in which k_F runs to infinity
- Low-energy observables are fixed by a set of marginal/relevant coupling functions **and** Fermi momentum
- In general, one can not set k_F to infinity up front because large Fermi momentum limit can be singular

[Kukreja, Besharat, SL (2024)]

Physical consequences of projective nature of fixed points

- Mismatches between scaling dimension of couplings and their relevance
  - Coupling functions with negative dimensions can become marginal/relevant

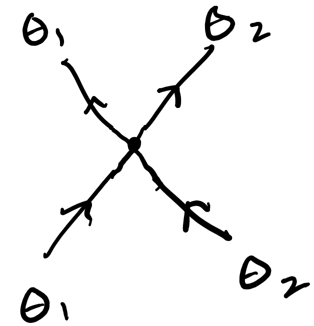
- A lack of unique dynamical critical exponent
  - The vertex function in different kinematic regimes exhibit scale invariance under q $\rightarrow sq, \omega \rightarrow s^z \omega$ with different z

Fermi liquids

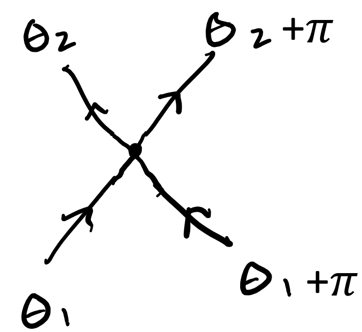
[Landau]

[Benfatto, Gallavotti] [Shankar] [Polchinski]

- Marginal functions :
  - Fermi momentum, Fermi velocity, Landau function (the forward scattering)



- Vanishing pairing interaction at the projective fixed point



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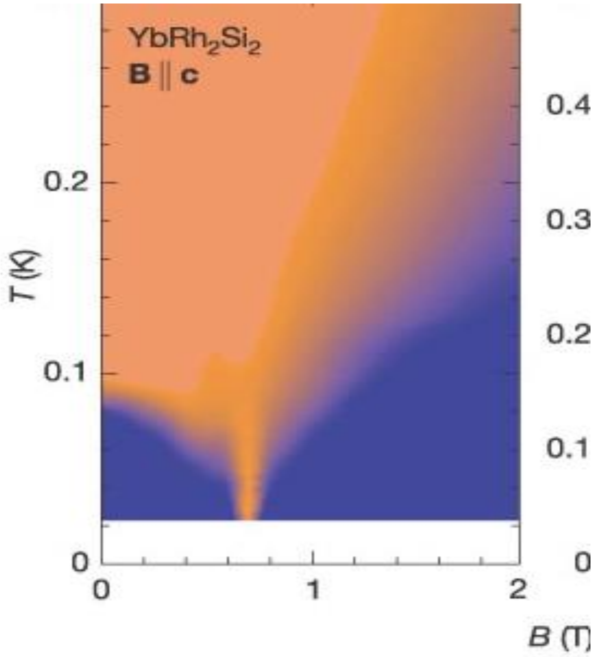
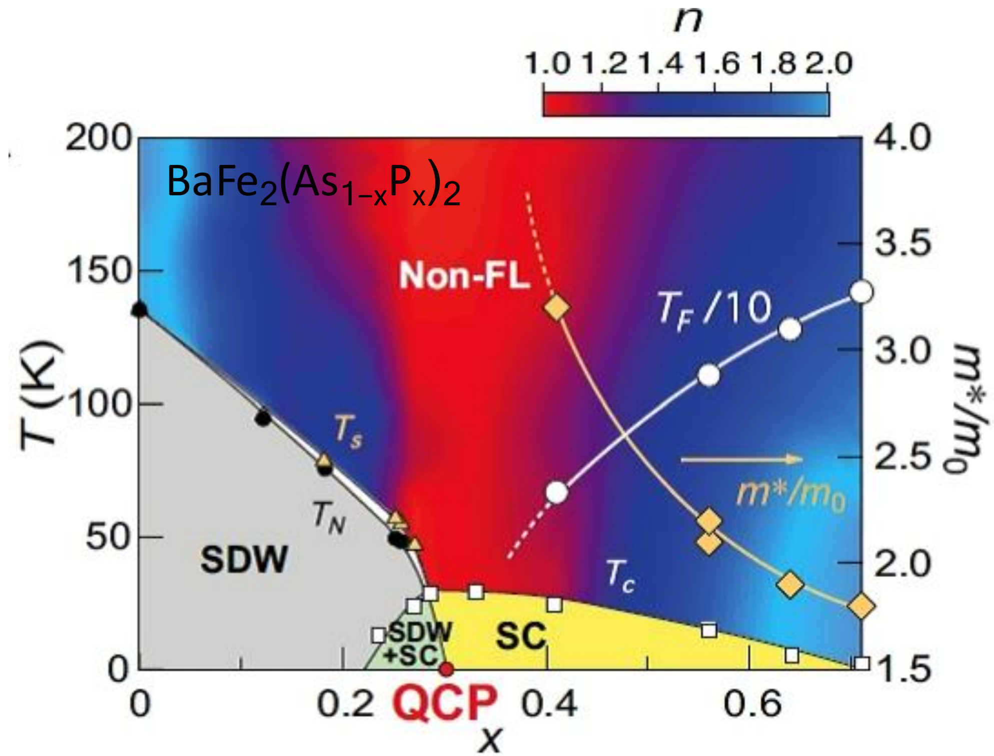
Non-Fermi liquids at Quantum Critical Points



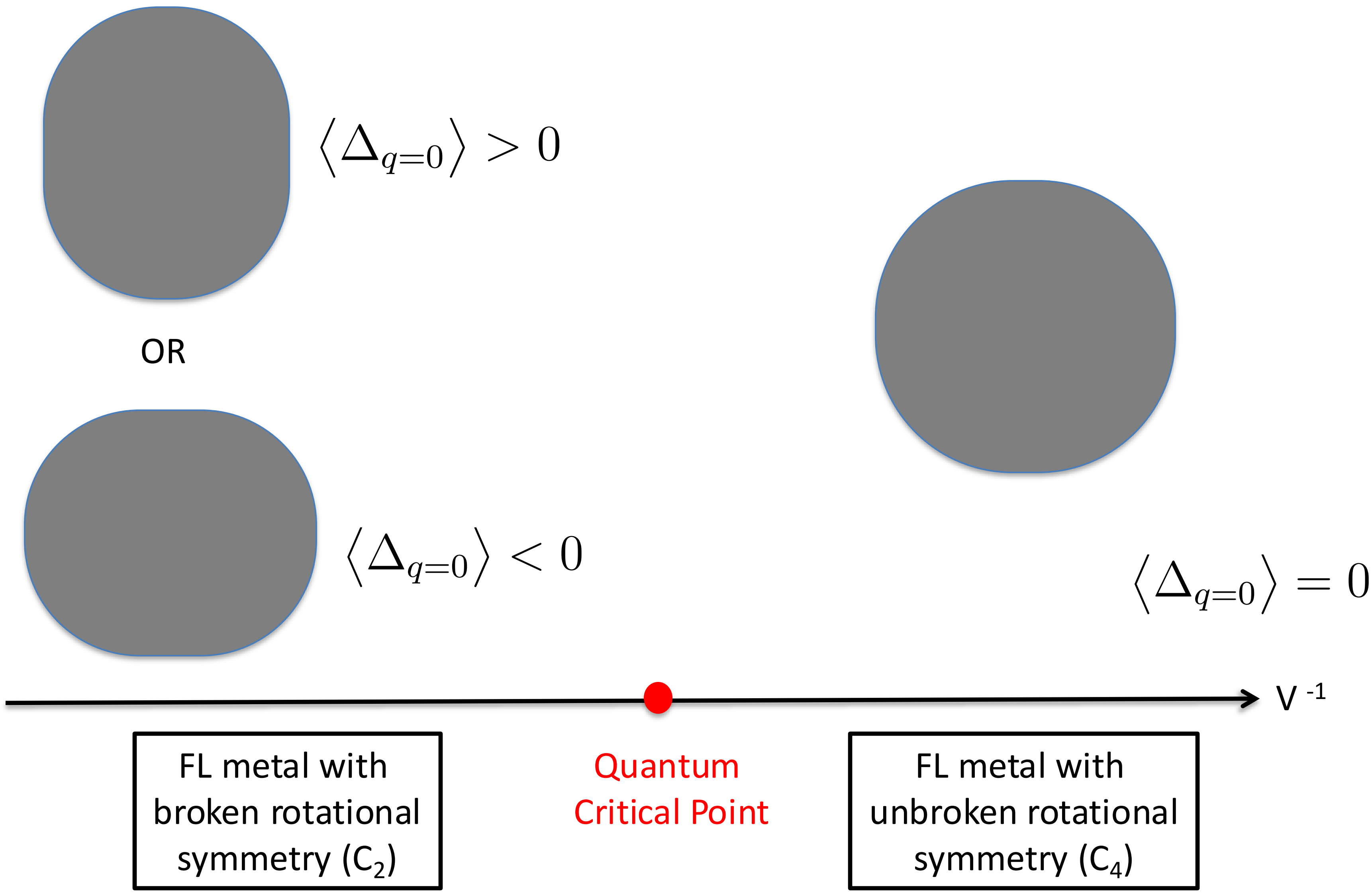
[Custers et al.(2003)]



[Hashimoto et al. Science 336, 1554 (2012)]

- Gapless collective boson coupled with particle-hole excitations creates qualitative deviations from Fermi liquids
- Strongly interacting metals realized in 2+1 dimension – the focus of this talk

Ising-Nematic QCP



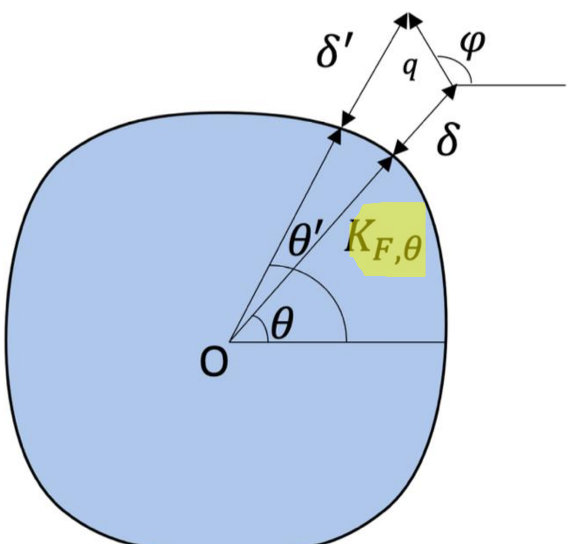
Theory of the Ising-nematic quantum critical metal

$$S = \int d_f^3\mathbf{k}\ \psi_j^\dagger(k_0,\delta,\theta)\left[ik_0 + v_{F,\theta}\delta\right]\psi_j(k_0,\delta,\theta) + \frac{1}{2}\int d_b^3\mathbf{q}\ |\mathbf{q}|^2\phi(-\mathbf{q})\phi(\mathbf{q})$$

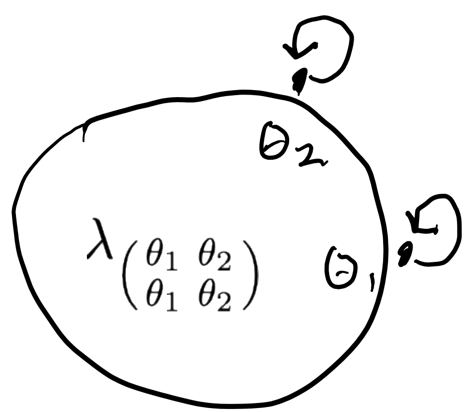
$$+\frac{1}{\sqrt{N}}\int d_f^3\mathbf{k}\ d_b^3\mathbf{q}\ \boldsymbol{e}_{\Theta(\theta,\vec{q}),\theta}\phi(q_0,\vec{q})\psi_j^\dagger\left(k_0+q_0,\Delta(\delta,\theta,\vec{q}),\Theta(\theta,\vec{q})\right)\psi_j(k_0,\delta,\theta)$$

$$+\int d_f^3\mathbf{k}\ d_f^3\mathbf{k}'\ d_b^3\mathbf{q}\ \boldsymbol{\lambda}_{\begin{pmatrix}\Theta(\theta,\vec{q}) & \theta' \\ \theta & \Theta(\theta',\vec{q})\end{pmatrix}}$$

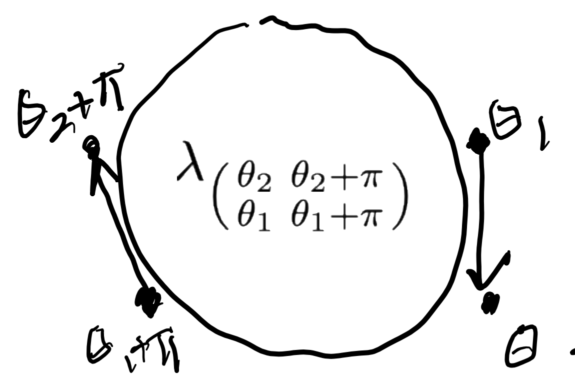
$$\psi_{j_1}^\dagger\left(k_0+q_0,\Delta(\delta,\theta,\vec{q}),\Theta(\theta,\vec{q})\right)\psi_{j_1}(k_0,\delta,\theta)\psi_{j_2}^\dagger(k_0',\delta',\theta')\psi_{j_2}\left(k_0'+q_0,\Delta(\delta',\theta',\vec{q}),\Theta(\theta',\vec{q})\right)$$



Forward scattering

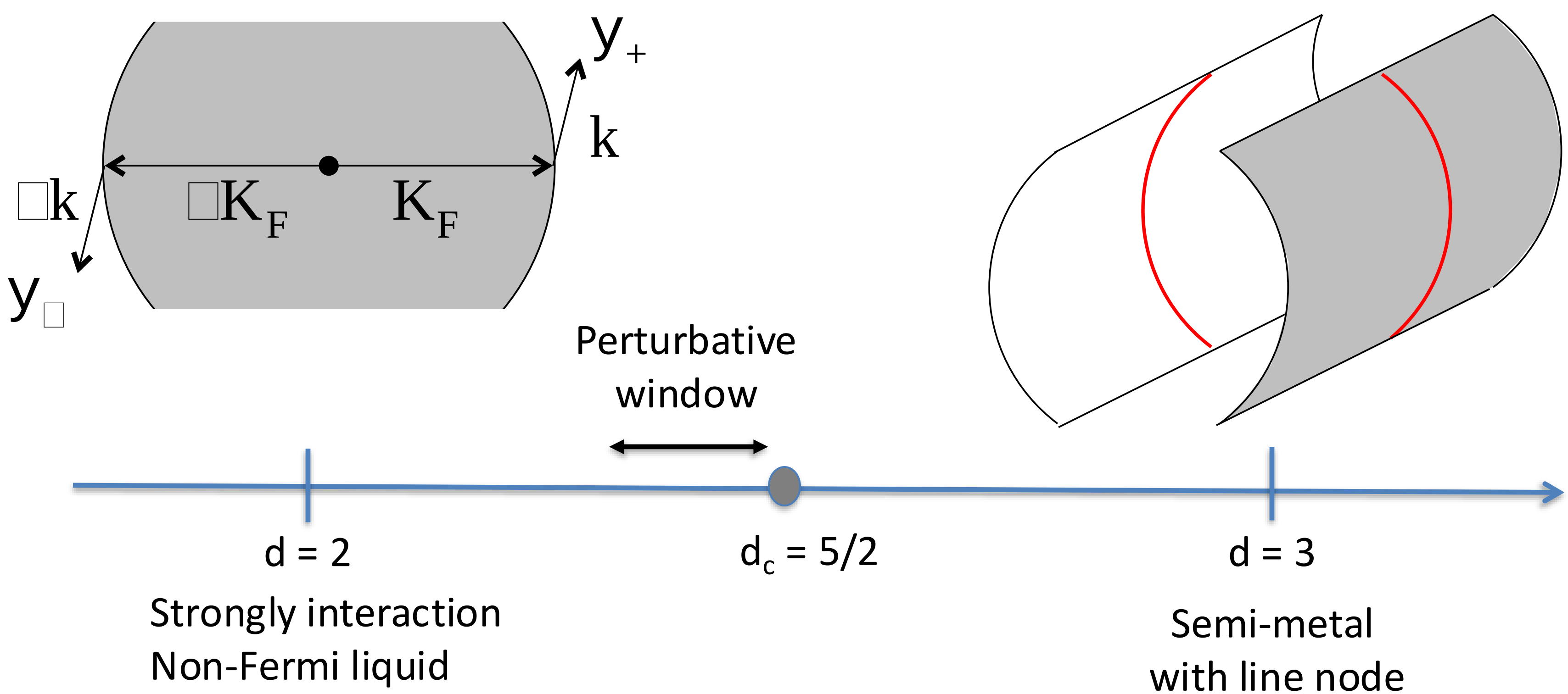


Pairing interaction



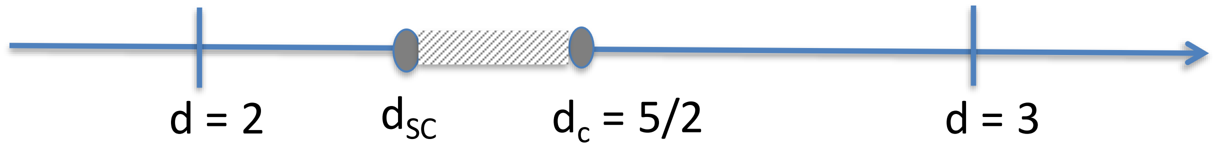
A dimensional regularization : an analytic continuation of the 2d metal to a semi-metal with line node in 3d

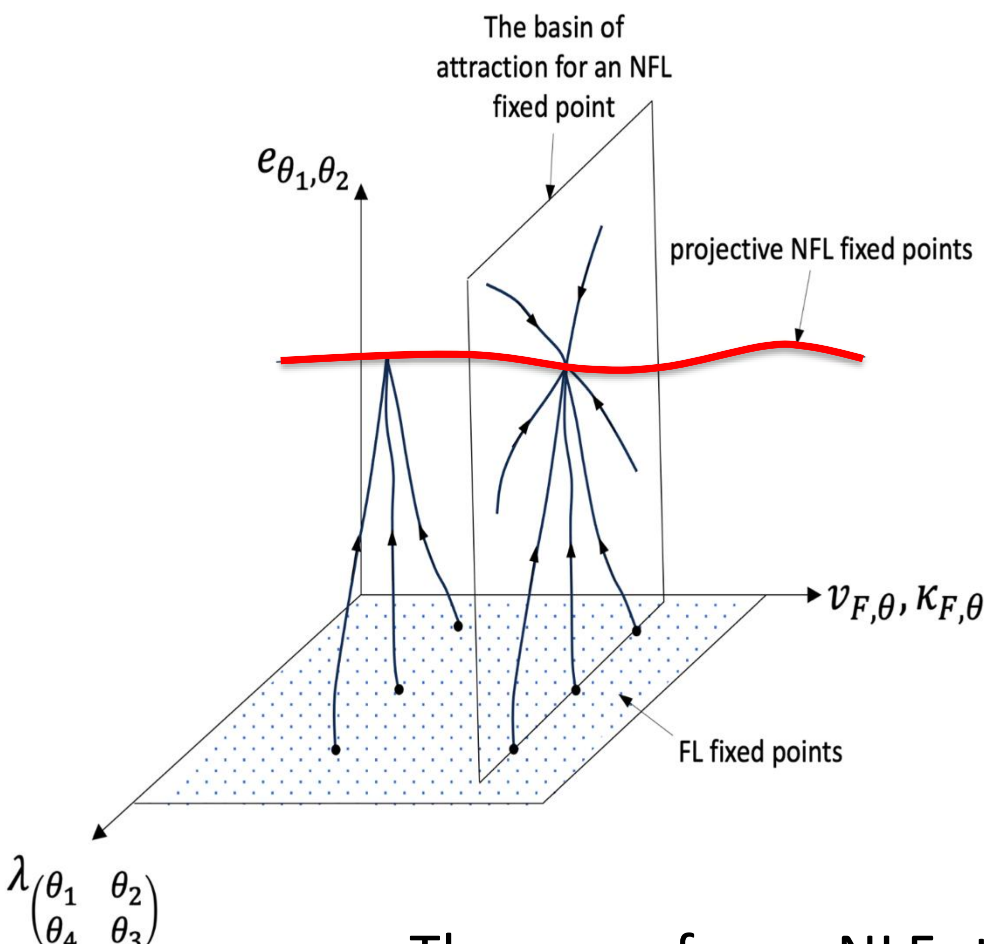
[D. Dalidovich, SL (2013)]



- Upper critical dimension : $d_c = 5/2$

$d > d_{SC}$





- Stable projective NFL fixed points
- Only two marginal parameters for the shape of FS and Fermi velocity

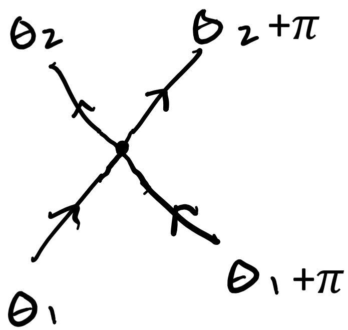
$$\kappa_{F,\theta},\, v_{F,\theta}$$

- Landau function and pairing interaction are non-zero, but fixed by the marginal parameters and Fermi momentum

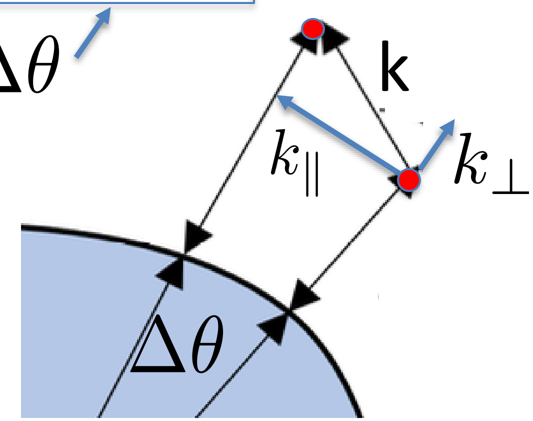
- There are fewer NLFs than FLs
- SC fluctuations are intrinsic parts of NFLs

Universal pairing interaction (d > d_{SC})

Dimensionless pairing interaction for Cooper pairs with center of mass momentum $\vec{q}$ and energy ω



$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left| \underbrace{\boxed{\left(\frac{\omega^{1/z}}{K_F}\right)^{1/2}}}_{\Delta\theta} \frac{1}{\theta_1-\theta_2} \right|^{2\Delta}$$



$$\Delta\theta \sim k_{\parallel}/K_F \sim \sqrt{\omega^{1/z}/K_F}$$

$$k_{\perp} \sim \omega^{1/z}, \quad k_{\parallel}^2 \sim k_{\perp}K_F \sim \omega^{1/z}K_F$$

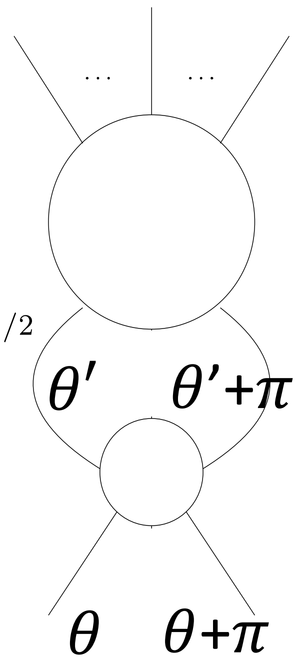
z : a dynamical critical exponent

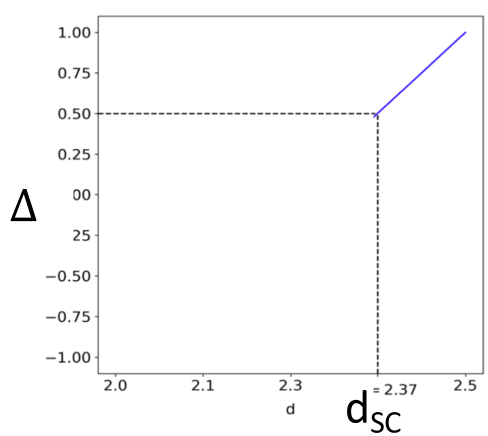
Δ: minus the scaling dimension of the four-fermion coupling function

UV/IR mixing

$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q}=0,\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F}\right)^{1/2} \frac{1}{\theta_1-\theta_2} \right|^{2\Delta}$$

$$\int d\theta' \left(\frac{K_F}{\omega^{1/z}}\right)^{1/2}$$



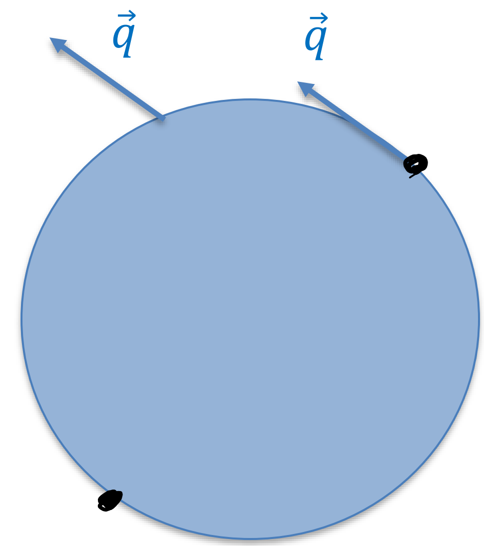


- Four-fermion coupling has a negative scaling dimension and *naively* irrelevant for $\Delta > 0$
- The pairing interaction becomes marginal for $\Delta = 1/2$
  - The growth of the number of patches compensates the decay of the interaction strength

Universal pairing interaction for $\vec{q} \neq 0$

Crossover function

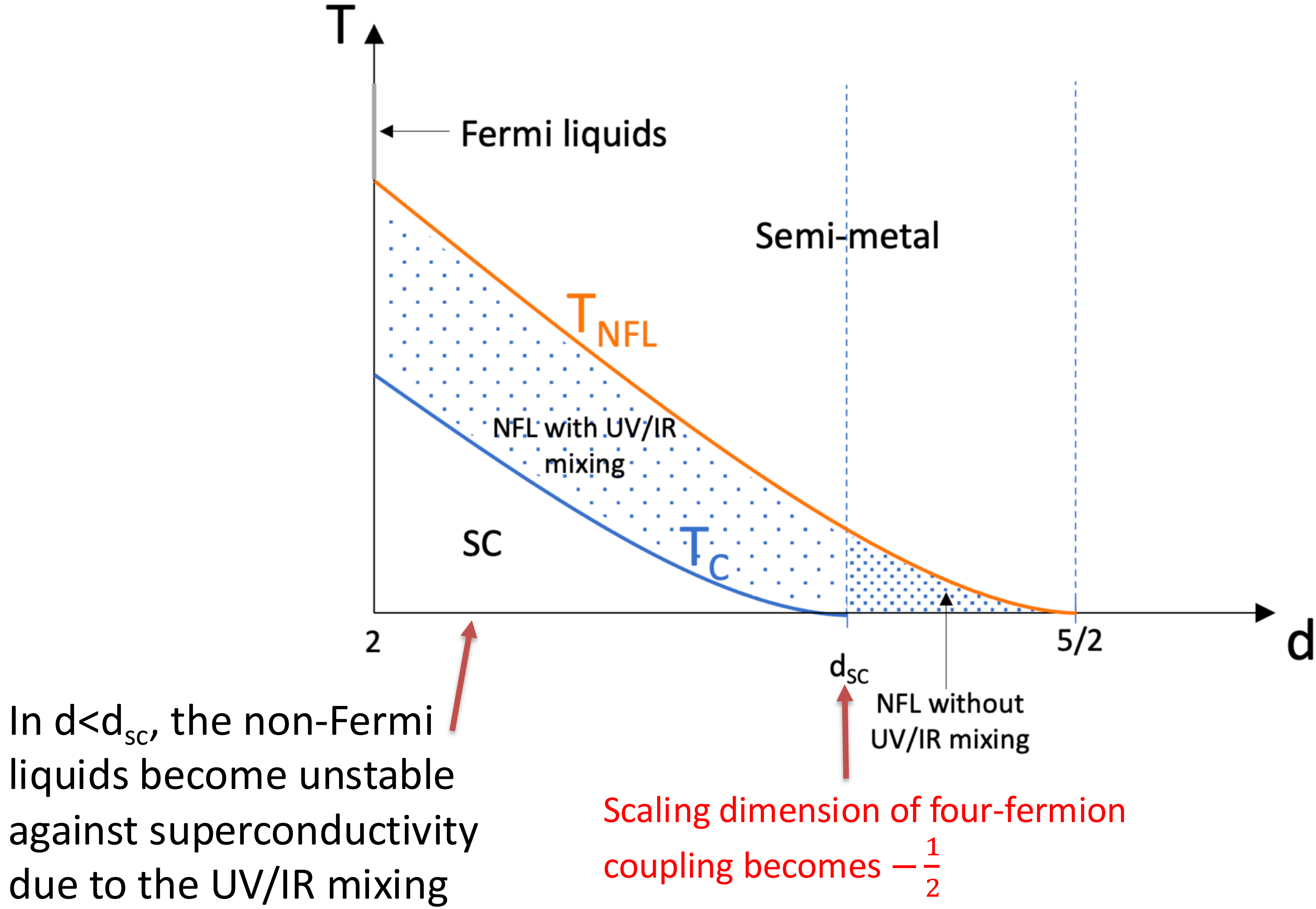
$$\Gamma^{(4)}_{\theta_1,\theta_2}(\vec{q},\omega) \sim \left| \left(\frac{\omega^{1/z}}{K_F} \right)^{1/2} \frac{1}{\theta_1 - \theta_2} \right|^{2\Delta} \mathcal{V}_{\theta_1}(\vec{q}) \mathcal{V}_{\theta_2}(\vec{q})$$



For large q,

$$\mathcal{V}_\theta(\vec{q}) \sim \begin{cases} \left(\frac{\omega^{1/z}}{q} \right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ not parallel to FS at } \theta \\ \left(\frac{\omega^{1/z} K_F}{q^2} \right)^{\frac{\eta_d}{2}}, & \vec{q} \text{ almost parallel to FS at } \theta \end{cases}$$

No unique dynamical critical exponent that sets the relative scaling between q and ω



T
Fermi liquids
Semi-metal
T_{NFL}
NFL with UV/IR mixing
SC
T_C
2
d_{SC}
5/2
d
NFL without UV/IR mixing
In $d<d_{sc}$, the non-Fermi liquids become unstable against superconductivity due to the UV/IR mixing
Scaling dimension of four-fermion coupling becomes $-\frac{1}{2}$

Summary

- The space of metallic universality classes is infinite dimensional
- Due to Fermi momentum, fixed points of metals are defined only projectively
  - Mismatch between scaling dimensions and relevancy of couplings
  - No unique dynamical critical exponent
- Universality classes of non-Fermi liquids being mapped out