Bosonization for fermionic fRG at weak and strong couplings

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In collaboration with:

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1. Introduction to SBE fRG
2. Investigations at weak couplings
3. Investigations at strong couplings

Introduction to SBE fRG

Consider a fermionic theory with a quartic interaction:

$$S\left[\overline{\psi},\psi\right] = -\sum_{\alpha_1,\alpha_2} \overline{\psi}_{\alpha_1} G^{-1}_{0,\alpha_1|\alpha_2} \psi_{\alpha_2} + \frac{1}{4} \sum_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} U_{\alpha_1\alpha_2|\alpha_3\alpha_4} \overline{\psi}_{\alpha_1} \overline{\psi}_{\alpha_2} \psi_{\alpha_3} \psi_{\alpha_4}$$

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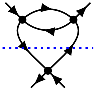
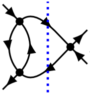
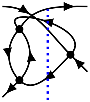
Diagrammatic classification:

- 2P-reducibility:

ph : , $\overline{ph}$: , pp : 

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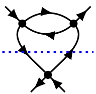
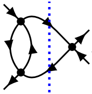
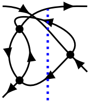
  ph : , $\overline{ph}$: , pp : 

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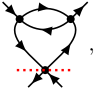
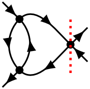
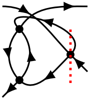
  ph : , $\overline{ph}$: , pp : 

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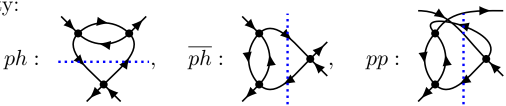

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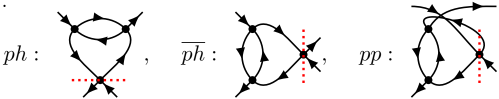
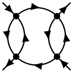


⚠ U-reducibility $\Rightarrow$ 2P-reducibility

⚠ 2P-reducibility $\not\Rightarrow$ U-reducibility



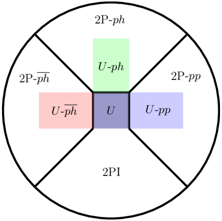
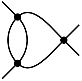


Fig. adapted from Krien, Valli, Capone, PRB 100, 155149 (2019)

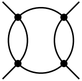
Single-boson exchange (SBE) decomposition:

[Krien, Valli, Capone, PRB 100, 155149 (2019)]

$$\Gamma^{(4)} = \left.\frac{\delta^4 \Gamma\left[\overline{\psi},\psi\right]}{\delta\overline{\psi}\delta\overline{\psi}\delta\psi\delta\psi}\right|_{\psi=\overline{\psi}=0} = \sum_{\mathrm{X}=\{pp,ph,\overline{ph}\}} \left(\nabla^{\mathrm{X}} + M^{\mathrm{X}} - U\right) + \mathcal{I}^{2\mathrm{PI}}$$

with $\nabla^{\mathrm{X}} =$  $+ \ldots$

only U-reducible diagrams

$M^{\mathrm{X}} =$  $+ \ldots$

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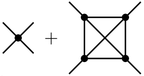
$\mathcal{I}^{2\mathrm{PI}} =$  $+ \ldots$

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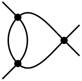
with $\nabla^{\mathrm{X}} =$  $+ \ldots$

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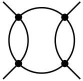
$M^{\mathrm{X}} =$  $+ \ldots$

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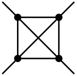
$\mathcal{I}^{2\mathrm{PI}} =$  $+$  $+ \ldots$

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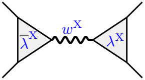
Heart of the SBE decomposition:

$\nabla^{\mathrm{X}} =$ 

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w^{X} : bosonic propagator
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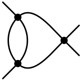
with $\nabla^{\mathrm{X}} =$  $+ \ldots$

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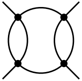
$M^{\mathrm{X}} =$  $+ \ldots$

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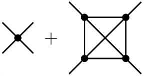
$\mathcal{I}^{\mathrm{2PI}} =$  $+ \ldots$

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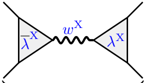
Heart of the SBE decomposition: $\nabla^{\mathrm{X}} =$ 

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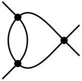

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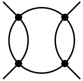

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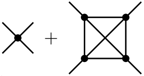

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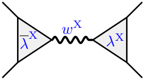

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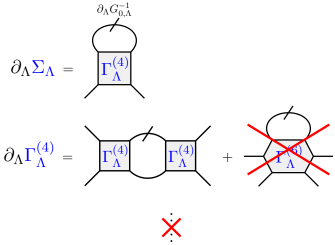
$\Rightarrow$ SBE decomposition introduces **bosonic dofs** and treats **all channels equitably**

Start from 1ℓ flow equations obtained from the vertex expansion:
[Morris, IJMP A09, 2411 (1994)]
[Metzner, Salmhofer, Honerkamp, Meden, Schönhammer, RMP 84, 299 (2012)]



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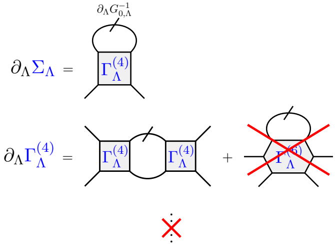
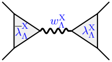
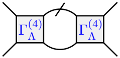


Insert SBE decomposition $\Gamma^{(4)}_\Lambda = \sum_{\mathrm{X}}$ 

$+ \ldots$ to substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$ 

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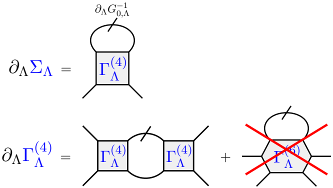
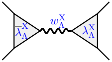
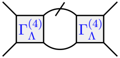


Insert SBE decomposition $\Gamma_\Lambda^{(4)} = \sum_{\mathrm{X}}$  + ... to substitute $\partial_\Lambda \Gamma_\Lambda^{(4)} =$  with:

[Bonetti, Toschi, Hille, Andergassen, Vilardi, PRR 4, 013034 (2022)]
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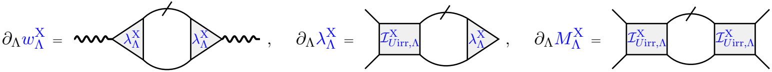

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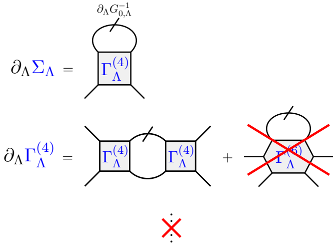
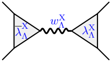
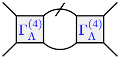


Insert SBE decomposition $\Gamma_\Lambda^{(4)} = \sum_{\mathrm{X}}$  + ... to substitute $\partial_\Lambda \Gamma_\Lambda^{(4)} =$  with:

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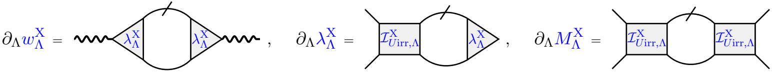
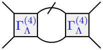


⚠ SBE formalism and Hubbard-Stratonovich transformations can yield same flow equations at 1ℓ

Multiloop fRG:

[Kugler, von Delft, PRL 120, 057403 (2018), PRB 97, 035162 (2018), NJP 20, 123029 (2018)]
(Related work for bosonic theory: [Blaizot, Pawlowski, Reinosa, PLB 696, 523 (2011), AP 431, 168549 (2021)])

Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$ 

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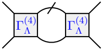
Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$ 

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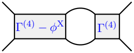
$\phi^{\mathrm{X}} =$ 

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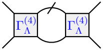
Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$ 

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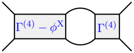
$\phi^{\mathrm{X}} =$ 

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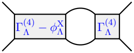
$$\Big\Downarrow \; G_0 \to G_{0,\Lambda}$$

$\partial_\Lambda \phi^{\mathrm{X}}_\Lambda = \partial_\Lambda \Bigg($ 

$\Bigg)$

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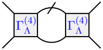
[Kugler, von Delft, PRL 120, 057403 (2018), PRB 97, 035162 (2018), NJP 20, 123029 (2018)]
(Related work for bosonic theory: [Blaizot, Pawlowski, Reinosa, PLB 696, 523 (2011), AP 431, 168549 (2021)])

Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$ 

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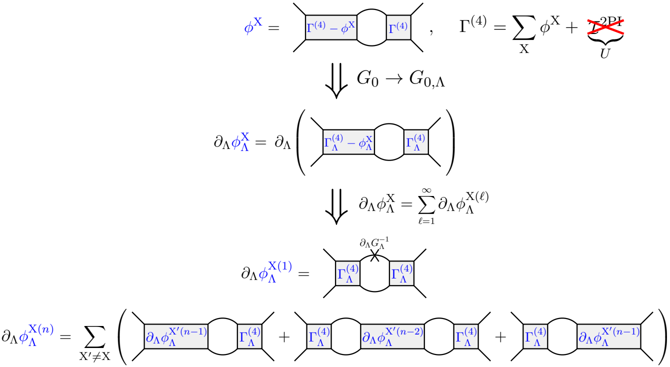

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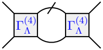
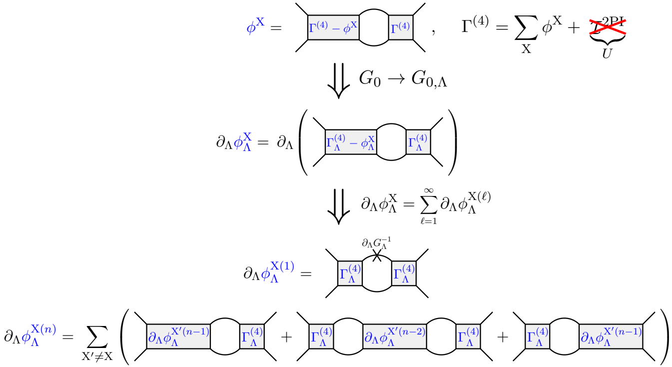
Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$  with flow equations derived from the **Bethe-Salpeter equation**:



What about the self-energy flow?

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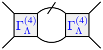
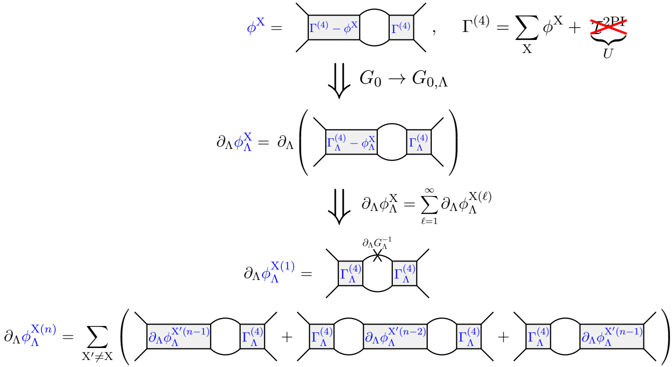
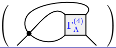
Idea: substitute $\partial_\Lambda \Gamma^{(4)}_\Lambda =$  with flow equations derived from the **Bethe-Salpeter equation**:



What about the self-energy flow?

[Hille, Rohe, Honerkamp, Andergassen, PRR 2, 033068 (2020)]
[Patricolo, Gievers, Fraboulet, Heinzelmann, Al-Eryani, Bonetti, Toschi, Vilardi, Andergassen, in prep.]
[Talk by **M. Patricolo**]

Advantageous to use **Schwinger-Dyson equation** within the truncated unity approach: $\partial_\Lambda \Sigma_\Lambda = \partial_\Lambda$ 

Insert SBE decomposition into multiloop fRG equations:

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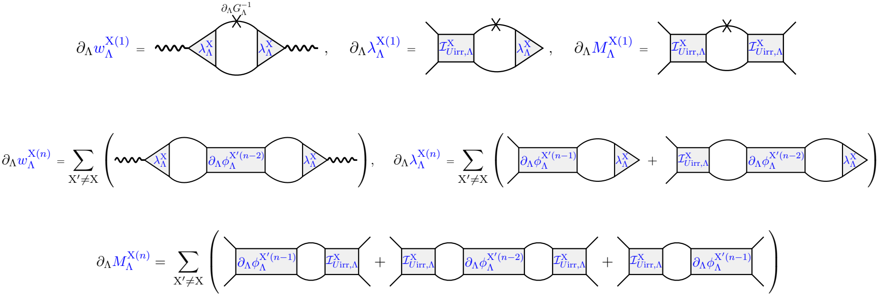
[Gievers, Walter, Ge, von Delft, Kugler, EPJB 95, 108 (2022)]



$$\partial_\Lambda w_\Lambda^{\mathrm{X}} = \sum_{\ell=1}^{\infty} \partial_\Lambda w_\Lambda^{\mathrm{X}(\ell)}, \quad \partial_\Lambda \lambda_\Lambda^{\mathrm{X}} = \sum_{\ell=1}^{\infty} \partial_\Lambda \lambda_\Lambda^{\mathrm{X}(\ell)}, \quad \partial_\Lambda M_\Lambda^{\mathrm{X}} = \sum_{\ell=1}^{\infty} \partial_\Lambda M_\Lambda^{\mathrm{X}(\ell)}$$

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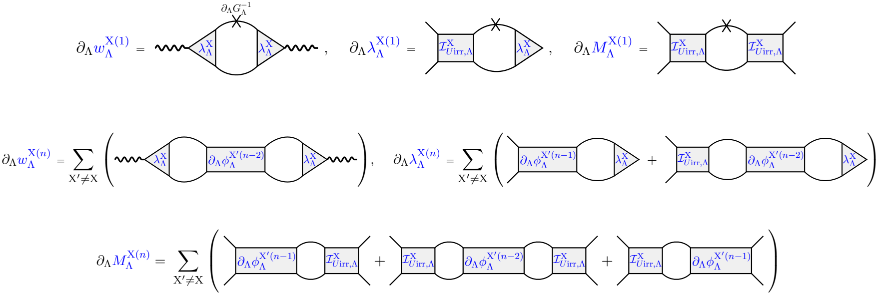
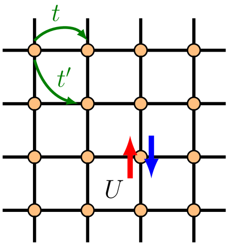


$$\partial_\Lambda w_\Lambda^{X} = \sum_{\ell=1}^{\infty} \partial_\Lambda w_\Lambda^{X(\ell)}, \quad \partial_\Lambda \lambda_\Lambda^{X} = \sum_{\ell=1}^{\infty} \partial_\Lambda \lambda_\Lambda^{X(\ell)}, \quad \partial_\Lambda M_\Lambda^{X} = \sum_{\ell=1}^{\infty} \partial_\Lambda M_\Lambda^{X(\ell)}$$

$\Rightarrow$ Multiloop SBE fRG enables us to go **beyond conventional 1ℓ fermionic fRG** by means of flowing **bosonic** vertices

Investigations at weak couplings



Hubbard model Hamiltonian:

[Hubbard, PRSL A 276, 238 (1963)]
[Gutzwiller, PRL 10, 159 (1963)]
[Kanamori, PTP 30, 275 (1963)]
[Qin, Schäfer, Andergassen, Corboz, Gull, ARCMP 13, 275 (2022)]

$$\mathcal{H} = \sum_{i \neq j, \sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$

- t_{ij} : **hopping** between nearest-neighbor sites (t) or between next-nearest-neighbor sites (t')
- U : on-site Coulomb **interaction**

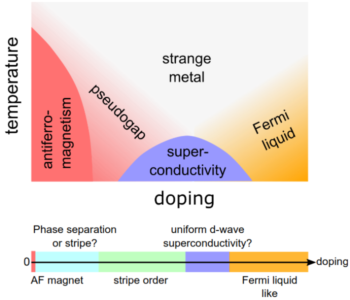


Fig. adapted from Qin, Schäfer, Andergassen, Corboz, Gull, ARCMP 13, 275 (2022)

Conclusions from 1ℓ SBE fRG study:

[Fraboulet, Heinzelmann, Bonetti, Al-Eryani, Vilardi, Toschi, Andergassen, EPJB 95, 202 (2022)]

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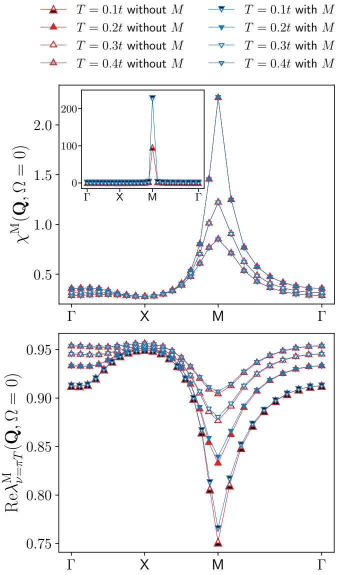
- Divergence when approaching pseudo-critical transition fully encompassed in bosonic propagator w^{M} ($w^{\mathrm{M}} = U + U^2\chi^{\mathrm{M}}$)



Parameters: $U = 2t$, half-filling

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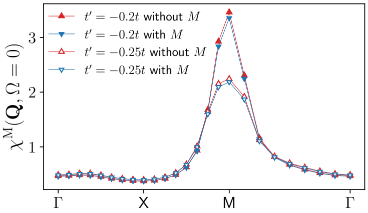
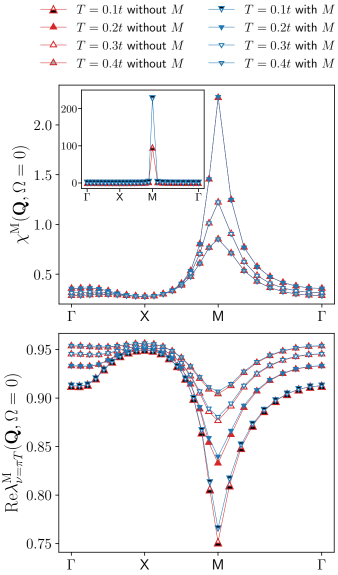


Parameters: $U = 2t$, $T = 0.2t$, $\mu = 4t'$



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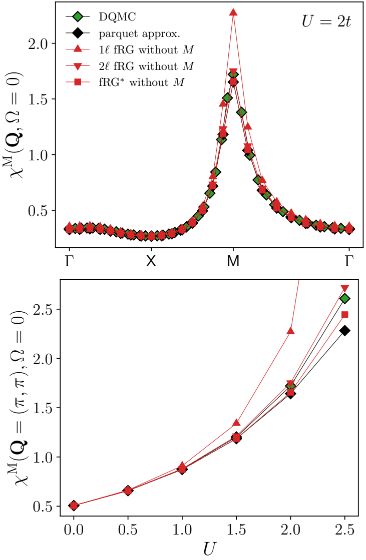

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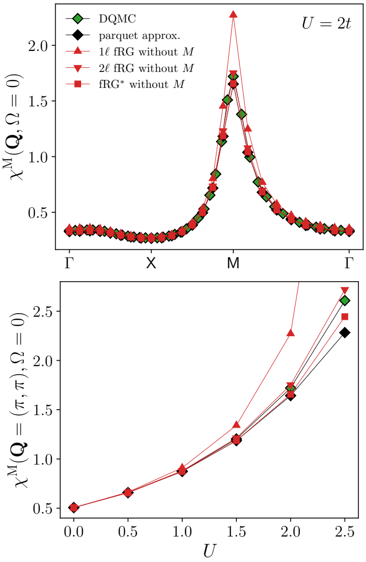
How can we extend multiloop SBE fRG to access **strong couplings**?



Parameters: $T = 0.2t$, half-filling

Investigations at strong couplings

Dynamical mean-field theory:

[Metzner, Vollhardt, PRL 62, 324 (1989)]
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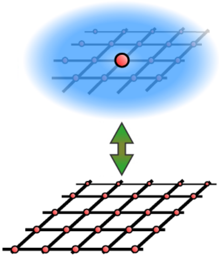
$$\mathcal{S}_{\text{imp}} = -\int_0^\beta d\tau d\tau' \sum_\sigma c_\sigma^\dagger(\tau) G_{0,\text{imp}}^{-1}(\tau-\tau') c_\sigma(\tau') + \mathcal{S}_{\text{int}}$$

$$G_{0,\text{imp}}^{-1}(\nu) = i\nu + \mu - \Delta(\nu)$$



$$\mathcal{S}_{\text{latt}} = -\int_0^\beta d\tau d\tau' \sum_{\mathbf{k},\sigma} c_\sigma^\dagger(\mathbf{k},\tau) G_{0,\text{latt}}^{-1}(\mathbf{k},\tau-\tau') c_\sigma(\mathbf{k},\tau') + \mathcal{S}_{\text{int}}$$

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Fig. adapted from Bonetti, Toschi, Hille, Andergassen, Vilardi, PRR 4, 013034 (2022)

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Determination of the dynamical mean-field $\Delta(\nu)$:

Make **local** approximation $\Sigma_{\text{imp}}(\nu) = \Sigma_{\text{latt},ii}(\nu)$ and solve Dyson equation with it

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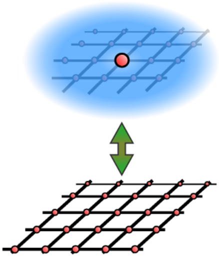

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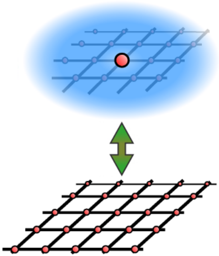

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⚠ **Local** correlations fully captured

⚠ DMFT mapping exact in infinite dimensions

Fig. adapted from Bonetti, Toschi, Hille, Andergassen, Vilardi, PRR 4, 013034 (2022)

DMF^{2}RG (DMFT+fRG):

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$$G_{0,\Lambda}(\mathbf{k},\nu) = (1-\Lambda)\,G_{0,\mathrm{imp}}(\nu) + \Lambda G_{0,\mathrm{latt}}(\mathbf{k},\nu)$$

$$\text{At } \Lambda = \Lambda_{\mathrm{ini}} = 0: \quad G_{0,\Lambda_{\mathrm{ini}}}(\mathbf{k},\nu) = G_{0,\mathrm{imp}}(\nu)$$
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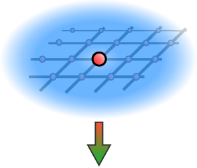

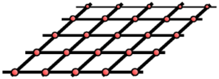

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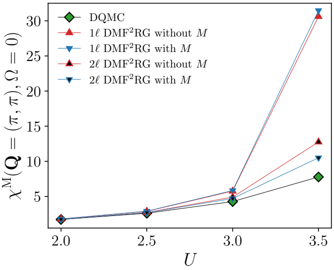

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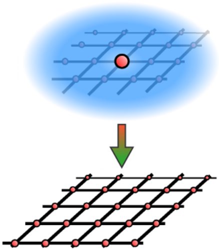

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Conclusion

Main conclusions:

- Efficient bosonization of fermionic fRG provided by the **SBE formalism**
- Possibility to combine SBE formalism with **multiloop fRG** and/or **DMF^{2}RG** to achieve better (quantitative) accuracy and/or to tackle strong couplings

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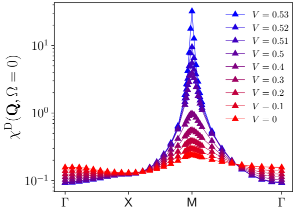
Further remarks:

- SBE fRG already used to investigate other models (extended Hubbard model, Hubbard-Holstein model, X-ray-edge singularity)
  [Heinzelmann, Al-Eryani, Fraboulet, Krien, Andergassen, in prep.]
  [Posters by **A. Al-Eryani** and **M. Gievers**]
- Ongoing work to improve starting point of DMF²RG with cellular DMFT
  [Krämer, Meixner, Fraboulet, Bonetti, Vilardi, Schäfer, Toschi, Andergassen, in prep.]
  [Poster by **M. Krämer**]



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