The role of quenched disorder in polymerized membranes

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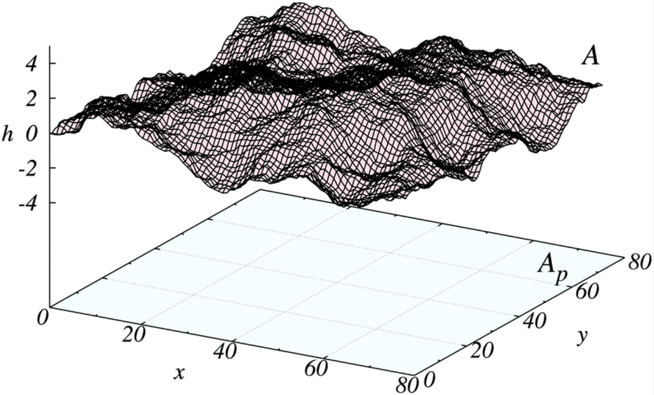
Outline

1. Fluid membranes and polymerized membranes
2. RG approaches to pure membranes
3. RG approaches to disordered membranes
4. Conclusion

Introduction

- Membranes: D-dimensional extended objects embedded in a d-dimensional space subject to quantum and/or thermal and/or disorder fluctuations



Generic questions :

- effects of – thermal – fluctuations ?

  $\Longrightarrow$ phase transition ?

  $\Longrightarrow$ ordered, flat, phase at low temperatures ?

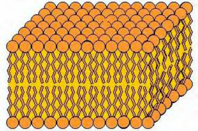
- effects of quenched disorder ?

- (effects of quantum fluctuations as $T \to 0$?)

$\Longrightarrow$ depends crucially on the nature of the membrane

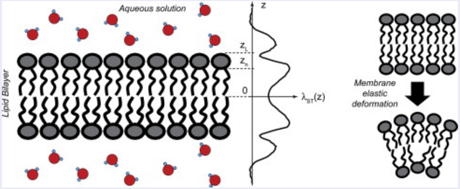
Fluid membranes *vs* polymerized membranes

Fluid membranes



- weakly interacting molecules
  - free diffusion inside the membrane plane $\Longrightarrow$ no shear modulus
  - very small compressibility and elasticity $\Longrightarrow$ no elastic energy

$\Longrightarrow$ only curvature energy



Fluid membranes

Free energy:

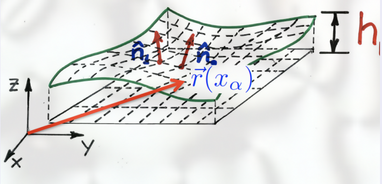
$$F = \frac{\kappa}{2} \int d^2\sigma \, \sqrt{g} \; H^2$$

- H: extrinsic curvature
- κ: rigidity constant
- $\sqrt{g} = \sqrt{\det g_{\mu\nu}}$ ensures reparametrization invariance
- $g_{\mu\nu} = \partial_\mu \mathbf{r}.\partial_\nu \mathbf{r} \equiv$ metric induced by the embedding $\mathbf{r}(\boldsymbol{\sigma})$

Fluctuations ?

Fluid membranes

- Low temperatures: Monge parametrization
  $x = \sigma_1$, $y = \sigma_2$ and $z = h(x, y)$ with h height, capillary, mode



- $\mathbf{r}(x, y) = (x, y, h(x, y))$ parametrizes points

- $\hat{\mathbf{n}}(x, y) = \dfrac{(-\partial_x h, -\partial_y h, 1)}{\sqrt{1 + (\partial_i h)^2}}$
- $\hat{\mathbf{n}}(x, y).\, \mathbf{e_z} = \cos\theta(x, y) = \dfrac{1}{\sqrt{1 + (\partial_i h)^2}}$

Fluid membranes

- Flat phase ? $\Longrightarrow$ harmonic fluctuations of $\theta(x,y)$:

$$\begin{aligned} \langle \theta(x,y)^2 \rangle &\simeq k_B T \int d^2q \; \langle \partial_i h(\mathbf{q}) \partial_i h(-\mathbf{q}) \rangle \\ &= k_B T \int d^2q \; \frac{q^2}{\kappa\, q^4} \simeq \frac{k_B T}{\kappa} \log\left(\frac{L}{a}\right) \longrightarrow \infty \end{aligned}$$

$\Longrightarrow$ no long range order between the normals (Mermin-Wagner)

$\Longrightarrow$ strong analogy with 2D-nonlinear σ model with $N-2 \longrightarrow \dfrac{d}{2}$

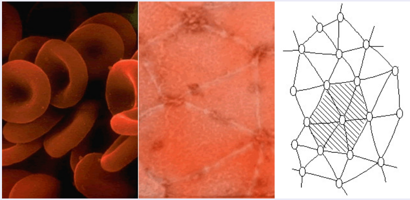
- exp. decreasing correlations: $\langle \hat{\mathbf{S}}(\mathbf{r}).\hat{\mathbf{S}}(\mathbf{0}) \rangle \sim e^{-r/\xi}$
- correlation length – mass gap: $\xi \simeq a\, e^{2\pi\kappa/(3k_BT(d/2))}$

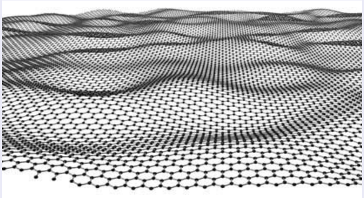
Polymerized membranes

Polymerized membranes

- chemical physics/biology: red blood cell, . . .

- condensed matter physics: graphene, phosphorene, . . .





- strongly interacting molecules by $V(|\mathbf{r_i} - \mathbf{r_j}|)$

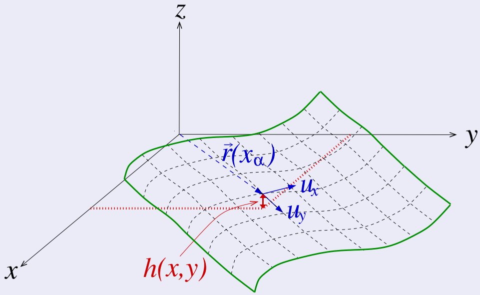
  $\Longrightarrow$ bending and elastic energy contributions

Free energy of crystalline membranes

Polymerized membranes

- Flat reference configuration: $\mathbf{r}_0(x, y) = (x, y, z = 0)$
- Fluctuations: $\mathbf{r}(x, y) = \mathbf{r}_0 + u_x(x, y)\,\mathbf{e}_1 + u_y(x, y)\,\mathbf{e}_2 + h(x, y)\,\hat{\mathbf{n}}$

  $h \equiv$ height field and $u_i \equiv$ phonon fields



Polymerized membranes

- Free energy: curvature + elasticity/shear

$$F \simeq \int d^2\mathbf{x} \; \left[\frac{\kappa}{2}(\Delta h)^2 + \lambda g_{ab}^2 + \mu g_{aa}^2\right]$$

$g_{ab} = \frac{1}{2}\left[\partial_a u_b + \partial_b u_a + \partial_a \mathbf{u}.\partial_b \mathbf{u} + \partial_a h\, \partial_b h\right]$

$g_{ab} \equiv$ stress tensor $\sim$ encodes fluctuations with respect to the flat configuration $\mathbf{r}_0$

λ, μ: Lamé coefficients

Polymerized membranes

- Free energy: curvature + elasticity/shear

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$g_{ab} \equiv$ stress tensor $\sim$ encodes fluctuations with respect to the flat configuration $\mathbf{r}_0$

λ, μ: Lamé coefficients

- coupling between height and phonon fluctuations

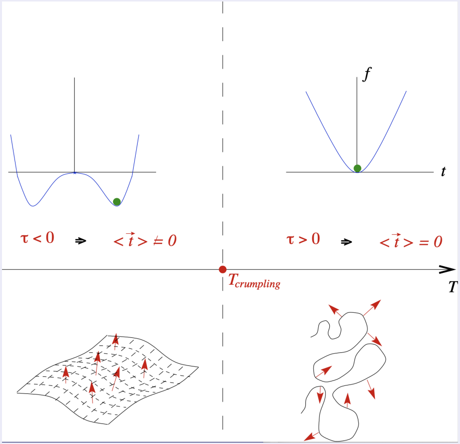
$\Longrightarrow$ *frustration* of height fluctuations:

$$\langle \theta(x,y)^2 \rangle \simeq T\left(\frac{L}{a}\right)^{-\eta} \longrightarrow 0$$

$\Longrightarrow$ long range order between normals in $D = 2$ (and less) !

Polymerized membranes

- spontaneous symmetry breaking in $D = 2$ and even in $D < 2$
  $\Longrightarrow$ crumpled-to-flat transition



Polymerized membranes

$\Longrightarrow$ low-temperature, ordered, flat, phase with non-trivial correlations in the I.R.

$$\begin{cases} G_{hh}(\mathbf{q}) \sim q^{-(4-\eta)} \\ G_{uu}(\mathbf{q}) \sim q^{-(6-D-2\eta)} \end{cases}$$

with $\eta \neq 0 \Longrightarrow$ associated *e.g.* correlations of stable membrane (e.g. graphene monolayer)

• a big challenge: computing η associated with scaling properties of graphene

RG approaches to pure membranes

One-loop perturbative approach of the crumpling-to-flat transition

(Paczuski, Kardar and Nelson (89))

$$F[\partial_\mu \mathbf{r}] = \int d^D\mathbf{x}\ \frac{\kappa}{2}(\Delta \mathbf{r})^2 + \lambda(\partial_a \mathbf{r}.\partial_b \mathbf{r})^2 + \mu(\partial_a \mathbf{r}.\partial_a \mathbf{r})^2$$

$\Longrightarrow$ perturbative expansion in λ and μ

β-functions in $D = 4 - \epsilon$ at one-loop order:

$$\begin{aligned}\partial_t \lambda &= -\epsilon\lambda + \frac{1}{8\pi^2}\left(\left(\frac{d}{3} + \frac{65}{12}\right)\lambda^2 + 6\mu\lambda + \frac{4}{3}\mu^2\right)\\ \partial_t \mu &= -\epsilon\mu + \frac{1}{8\pi^2}\left(\left(\frac{21}{16}\right)\lambda^2 + \frac{21}{2}\mu\lambda + (4d+5)\mu^2\right)\end{aligned}$$

$\Longrightarrow$ fluctuation induced 1^{st} order for $d < d_c \simeq 218.2$ near $D = 4$

One-loop perturbative approach of the flat phase

(Aronovitz, Golubović and Lubensky (88); Guitter, David, Leibler and Peliti (89))

$$F \simeq \int d^D\mathbf{x} \ \left[\frac{\kappa}{2}(\Delta h)^2 + \lambda g_{ab}^2 + \mu g_{aa}^2\right]$$

$\Longrightarrow$ perturbative expansion in λ and μ

β-functions in $D = 4 - \epsilon$ at one-loop order:

$$\begin{aligned} \partial_t \mu &= (-\epsilon + 2\eta)\mu + \frac{d_c \mu^2}{96\pi^2} \\ \partial_t \lambda &= (-\epsilon + 2\eta)\lambda + \frac{d_c(6\lambda^2 + 6\lambda\mu + \mu^2)}{96\pi^2} \end{aligned}$$

and $\eta = \frac{5\mu(\lambda+\mu)}{(2\mu+\lambda)}$ $\Longrightarrow$ fixed point P_4 with $\eta_4(\epsilon = 2, d = 3) = 0.96$

far from MC predictions: $\eta = 0.85$

Thus two great questions:

- nature of the phase transition of $D = 2, d = 3$, *i.e.* physical, membranes ?

- flat phase properties – η – of physical membranes ?

while all computations are performed near $D = 4$

Non perturbative RG

$\Longrightarrow$ use of a non perturbative RG approach (Kownacki and D.M. (08))

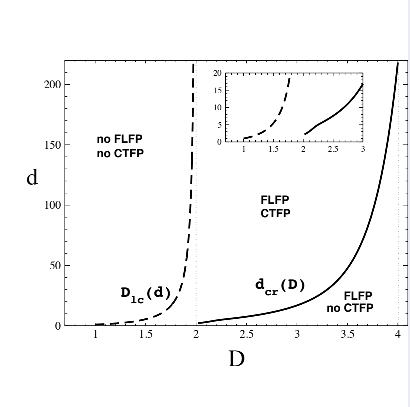
- Effective action $\Gamma_k[\partial_\mu \mathbf{r}]$ for membranes:

$$\Gamma_k\left[\partial_\mu \mathbf{r}\right] = \int d^D\mathbf{x}\ \frac{\kappa}{2}\left(\Delta \mathbf{r}\right)^2 + \lambda\left(\partial_a \mathbf{r}.\partial_b \mathbf{r} - \delta_{ab}\right)^2 + \mu\left(\partial_a \mathbf{r}.\partial_a \mathbf{r} - \delta_{aa}\right)^2$$

$\Longrightarrow$ Wetterich equation

$\Longrightarrow$ unified treatment of crumpling-to-flat transition and flat phase

Results:



- crumpled-to-flat transition: $d_c(D=2) \sim 3$ and strong dependence with respect to the ansatz (powers of $\partial \mathbf{r}$)
- flat phase $\eta = 0.85$ that compares very well to Monte Carlo $\eta = 0.85(1)$ (Los, Katsnelson, Yazyev, Zakharchenko and Fasolino (09))

Striking facts : in the flat phase (only):

- no corrections at orders $\varphi^4 \sim (\partial \mathbf{r})^4$!
  (Essafi, Kownacki and D.M. (14))
- no quantitative corrections at all orders in ∂^{2p} !
  $\eta = 0.849$ (Braghin and Hasselmann (10)) compared to $\eta = 0.85$

$\Longrightarrow$ extreme stability of the approach

Question: structure and properties of the perturbative theory at higher orders in λ and μ ?

Polymerized membranes at two and three loop order

Polymerized membranes at two-loop and three loop order

(Coquand, D.M. and Teber (20), Metayer, D.M. and Teber (22))

$$S[\mathbf{h},\mathbf{u}] = \int \mathrm{d}^D x \left\{ \frac{\kappa}{2} (\Delta \mathbf{h})^2 + \lambda\, g_{ab}^2 + \mu\, g_{aa}^2 \right\}$$

with the metric tensor: $g_{ij} = \frac{1}{2}(\partial_i \mathbf{r}.\partial_j \mathbf{r} - \delta_{ij})$ given by:

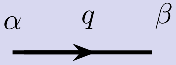
$$g_{ij} \simeq \frac{1}{2}\left[\partial_i u_j + \partial_j u_i + \partial_i \mathbf{h}.\partial_j \mathbf{h}\right] .$$

- not so simple: derivative field theory
  $\Longrightarrow$ momentum dependent vertices

good news: in the flat phase it is sufficient to renormalize the propagators

height-field propagator:

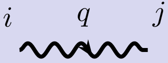
$$G_h^{\alpha\beta}(q) = \frac{\delta^{\alpha\beta}}{\kappa q^4} =$$



phonon-field propagator:

$$G_u^{ij}(q) = \frac{1}{\mu q^2} P_T^{ij}(q) + \frac{1}{(\lambda + 2\mu) q^2} P_L^{ij}(q)$$

$$=$$



with $P_T^{ij}(q) = \delta_{ij} - \frac{q_i q_j}{q^2}$ and $P_L^{ij}(q) = \frac{q_i q_j}{q^2}$

Results

36 years after Aronovitz et al.:

- the non-trivial stable fixed point P_4 controls the flat phase with remarkable, rapidly decreasing, series

$$\begin{cases} \eta_{3L} & = 0.4800\,\epsilon - 0.01152\,\epsilon^2 - 0.00334\,\epsilon^3 \\ \eta_{\text{NPRG re-expanded}} & = 0.4800\,\epsilon - 0.00918\,\epsilon^2 - 0.00333\,\epsilon^3 \end{cases}$$

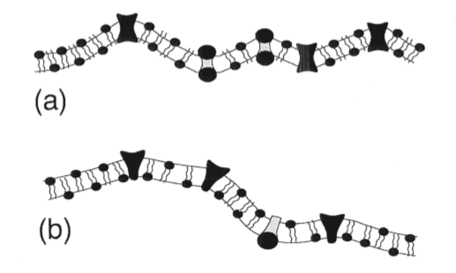
- a rapidly converging exponent η: in $D = 2$ (*i.e.* ϵ=2) and $d = 3$:

– 1L : $\eta = 0.96$
– 2L : $\eta = 0.9139$
– 3L : $\eta = 0.8872$
– 4L : $\eta = 0.8760$ (Pikelner (22))
to be compared to NPRG $\eta = 0.85$ *

RG approaches to disordered membranes

Disorder in membranes: imperfect polymerization, vacancies, impurities etc $\Longrightarrow$ "defects"

- isotropic defects $\Longrightarrow$ elastic disorder (a)
- anisotropic defect $\Longrightarrow$ curvature disorder (b)



Free energy:

$$\Gamma[\mathbf{r}] = \int \mathrm{d}^D x \left\{ \frac{\kappa}{2}\left(\Delta \mathbf{r} - \frac{\mathbf{c}(\mathbf{x})}{\kappa}\right)^2 + \lambda\Big(\partial_a \mathbf{r}.\partial_b \mathbf{r} - \delta_{ab}(1+2\,m(\mathbf{x}))\Big)^2 + \mu\Big(\partial_a \mathbf{r}.\partial_a .\mathbf{r} - \delta_{aa}(1+2\,m(\mathbf{x}))\Big)^2 \right\}$$

with $\mathbf{c}(\mathbf{x})$ and $m(\mathbf{x})$ Gaussian random fields coupled to curvature and metric

- average over (quenched) disorder using replica trick:

$$F = \overline{\log Z} = \lim_{n\to 0} \frac{Z^n - 1}{n}$$

$\Longrightarrow$ effective action with interacting replica : A,B

$$\Gamma[\mathbf{r}] = \int \mathrm{d}^d x \sum_A \left\{ \frac{\overline{\kappa}}{2} (\Delta \mathbf{r}^A)^2 + \overline{\lambda} \left(\partial_a \mathbf{r}^A . \partial_b \mathbf{r}^A - \delta_{ab} \right)^2 + \overline{\mu} \left(\partial_a \mathbf{r}^A . \partial_a \mathbf{r}^A - \delta_{aa} \right)^2 \right\}$$

$$- \frac{\overline{\Delta}_\kappa}{2} \sum_{A,B} \Delta \mathbf{r}^A . \Delta \mathbf{r}^B$$

$$- \overline{\Delta}_\lambda \sum_{A,B} \left(\partial_a \mathbf{r}^A . \partial_b \mathbf{r}^A - \delta_{ab} \right) \left(\partial_a \mathbf{r}^B . \partial_b \mathbf{r}^B - \delta_{ab} \right)$$

$$- \overline{\Delta}_\mu \sum_{A,B} \left(\partial_a \mathbf{r}^A . \partial_a \mathbf{r}^A - \delta_{aa} \right) \left(\partial_b \mathbf{r}^B . \partial_b \mathbf{r}^B - \delta_{bb} \right)$$

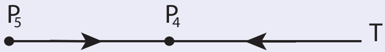
with $\overline{\Delta}_\kappa, \overline{\Delta}_\lambda, \overline{\Delta}_\mu$ disorder variances *

Weak coupling approach of the flat phase

- one-loop, weak coupling, analysis in $D = 4 - \epsilon$ (Morse and Lubensky (92))

$\Longrightarrow$ stability of the disorder-free fixed point P_4

$\Longrightarrow$ new zero-T, disordered fixed point P_5 but unstable

disorder 

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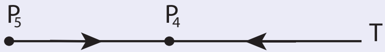
  disorder

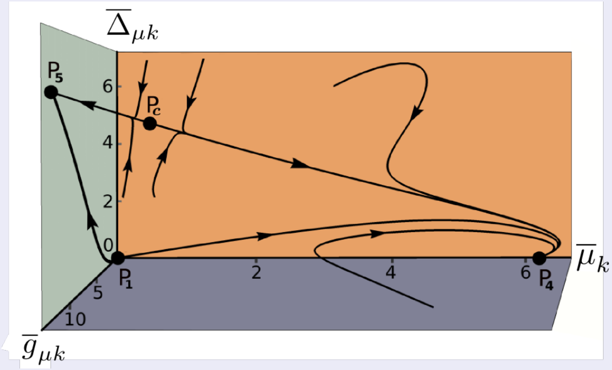


- disappointing ... and not expected (at least by me)

NPRG approach of the flat phase

- Functional RG approach (Coquand, Essafi, Kownacki and D.M. (17))

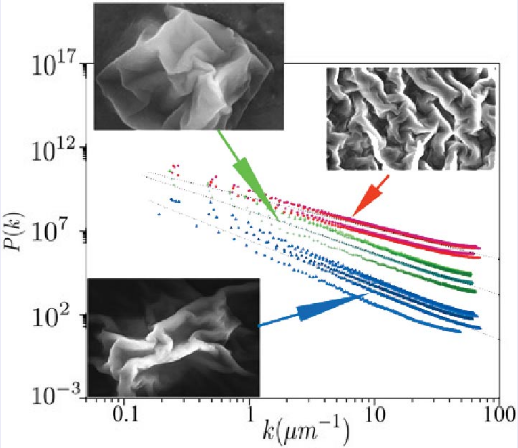
$\Longrightarrow$ new *critical fixed point* P_c between P_4 and P_5



$\longrightarrow$ Temperature

Three scaling behaviours associated with P_5, P_4 and P_c observed in partially fluid polymerized membranes

(S. Chaieb, V.K. Natrajan and A. A. El-rahman (06))



Question:

- could P_c just be an artifact of the NPRG ?

  … as P_c not seen via Self-Consistent Screening Approximation ($\sim$ 2P.I.)
  (Le Doussal and Radzihovsky (18))

Disordered membranes at two and three loop order

Disordered membranes at two-loop and three loop order

(Metayer and Mouhanna (22))

$$S = \int d^D x \left\{ \frac{\widetilde{\kappa}_{AB}}{2} \Delta \mathbf{h}^A(\mathbf{x}) \Delta \mathbf{h}^B(\mathbf{x}) + \widetilde{\lambda}_{AB}\, g^A_{ab}(\mathbf{x}) g^B_{ab}(\mathbf{x}) + \widetilde{\mu}_{AB}\, g^A_{aa}(\mathbf{x}) g^B_{bb}(\mathbf{x}) \right\}$$

with: $g^A_{ij} \simeq \frac{1}{2}\left[\partial_i u^A_j + \partial_i u^A_j + \partial_i \mathbf{h}^A . \partial_j \mathbf{h}^A\right]$ and with generalized coupling constants

$$\begin{cases} \widetilde{\kappa}^{AB} & = \widetilde{\kappa}\, \delta^{AB} - \widetilde{\Delta}_\kappa\, J^{AB} \\ \widetilde{\mu}^{AB} & = \widetilde{\mu}\, \delta^{AB} - \widetilde{\Delta}_\mu\, J^{AB} \\ \widetilde{\lambda}^{AB} & = \widetilde{\lambda}\, \delta^{AB} - \widetilde{\Delta}_\lambda\, J^{AB} \end{cases}$$

where $J^{AB} \equiv 1\ \forall\, A, B$.

Disordered membranes at two and three loop order

- a fixed point P_c of order ϵ^2 found !
- Very proximity between three-loop and NPRG !

$$\begin{cases} \eta_{3L} & = 0.42857\,\epsilon - 0.03695\,\epsilon^2 - 0.01191\,\epsilon^3 \\ \eta_{\text{NPRG re-expanded}} & = 0.42857\,\epsilon - 0.03621\,\epsilon^2 - 0.01318\,\epsilon^3 \end{cases}$$

- a rapidly converging exponent η_c: in $D = 2$ (*i.e.* ϵ=2) and $d = 3$:

– 1L : $\eta_c = 0.8571$
– 2L : $\eta_c = 0.7093$
– 3L : $\eta_c = 0.6140$
to be compared to NPRG $\eta_c = 0.490$ and experiment $\eta_c = 0.492$

Crumpled-to-flat transition in membranes

- Weak coupling analysis in $D = 4 - \epsilon$ at one-loop (Paczuski Kardar and Nelson (92))

  $\Longrightarrow$ first order transitions below $d_c \simeq 218.20$

- and beyond one loop ?

  $\Longrightarrow$ limited physical interest (?)

  $\Longrightarrow$ but a challenge: to tackle with the derivative $O(N)$ model : $\boldsymbol{\varphi} \longrightarrow \partial_a \boldsymbol{\varphi}$

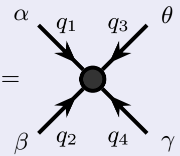
$$F\left[\partial_a \mathbf{r}\right] = \int d^D\mathbf{x}\ \frac{\kappa}{2}\left(\partial_a\partial_a\boldsymbol{\varphi}\right)^2 + \ \lambda\left(\partial_a\boldsymbol{\varphi}.\partial_b\boldsymbol{\varphi}\right)^2 + \ \mu\left(\partial_a\boldsymbol{\varphi}.\partial_a\boldsymbol{\varphi}\right)^2$$

rem: scalar case treated by Safari, Stergiou,Vacca and Zanusso (22)

Crumpled-to-flat transition in membranes

4-point vertex: strong momentum dependence: a nightmare

$$
\begin{aligned}
W_{\alpha\beta\gamma\theta}(q) \underset{q_1+q_2=q}{=} \frac{1}{24}\Big\{ \lambda\Big[& (q_1.q_2)(q_3.q_4)\delta_{\alpha\beta}\delta_{\gamma\theta} \\
& + (q_1.q_3)(q_2.q_4)\delta_{\alpha\gamma}\delta_{\beta\theta} \\
& + (q_1.q_4)(q_2.q_3)\delta_{\alpha\theta}\delta_{\beta\gamma}\Big] \\
+ \mu\Big[& \big((q_1.q_3)(q_2.q_4) + (q_1.q_4)(q_2.q_3)\big)\delta_{\alpha\beta}\delta_{\gamma\theta} \\
& + \big((q_1.q_4)(q_2.q_3) + (q_1.q_2)(q_3.q_4)\big)\delta_{\alpha\gamma}\delta_{\beta\theta} \\
& + \big((q_1.q_2)(q_3.q_4) + (q_1.q_3)(q_2.q_4)\big)\delta_{\alpha\theta}\delta_{\beta\gamma}\Big]\Big\}
\end{aligned}
$$

$=$ 

- Auxiliary field method $\Longrightarrow$ introducing D auxiliary d-components fields $\{\boldsymbol{A}_i\}$, $i = 1 \ldots D$ in place of the derivative fields $\partial_i \boldsymbol{\varphi}$ (Delzescaux, Duclut, D.M., Tissier (23))

$$Z = \int \mathcal{D}\boldsymbol{\varphi} \prod_{i=1}^{D} \mathcal{D}\boldsymbol{A}_i \; \delta(\boldsymbol{A}_i - \partial_i \boldsymbol{\varphi}) \; e^{-S[\{\boldsymbol{A}_i\}]} \; .$$

- δ-constraint raised with D auxiliary d-components fields $\{\boldsymbol{B}_\beta\}$:

$$Z = \int \mathcal{D}\boldsymbol{\varphi} \prod_{i,j=1}^{D} \mathcal{D}\boldsymbol{A}_i \mathcal{D}\boldsymbol{B}_j \; e^{-S[\{\boldsymbol{A}_i\}]} \; e^{-i \int d^D x \, \boldsymbol{B}_i . (\boldsymbol{A}_i - \partial_i \boldsymbol{\varphi})}$$

- $\{\boldsymbol{B}_i\}$ and $\{\boldsymbol{\partial_i \varphi}\}$ appear linearly $\Longrightarrow$ no renormalization
- only the auxiliary fields $\{\boldsymbol{A}_i\}$ renormalize nontrivially.
- propagator of the $\{\boldsymbol{A}_i\}$-fields given by $P^{\|}_{ij}/\boldsymbol{p}^2 + r)$

$$\beta_\lambda(\lambda,\mu) = -\epsilon\lambda + c_1\big((6d+7)\lambda^2 + 2(3d+17)\lambda\mu + (d+15)\mu^2\big)$$
$$- \frac{c_1^2}{6}\big((69d+52)\lambda^3 + (54d^2-16d+541)\lambda^2\mu$$
$$+ (36d^2+281d-110)\lambda\mu^2 + (6d^2+112d-95)\mu^3\big)$$

$$\beta_\mu(\lambda,\mu) = -\epsilon\mu + c_1\big(\lambda^2 + (d+21)\mu^2 + 10\lambda\mu\big)$$
$$+ \frac{c_1^2}{12}\big((96d+55)\lambda^3 + (470d+289)\lambda^2\mu$$
$$+ (146d+421)\lambda\mu^2 + (-212d+475)\mu^3\big)$$

$$\eta(\lambda,\mu) = \frac{(d+2)(\lambda+2\mu)}{3(32\pi^2)^3} \times \big((2d+3)\lambda^2 + 2(d+9)\lambda\mu + (d+19)\mu^2\big)$$

$\Longrightarrow$ very easily: $d_c(\epsilon) = 218.20 - 448.25\,\epsilon + \mathcal{O}(\epsilon^2)$

- recently extended to disordered membranes
  φ (Delzescaux, D.M., Tissier (23))

Conclusion

- membranes display a very rich physics:

  – pure systems due to (hidden) long range interactions

  – disordered systems: new fixed points, new phases

  • in the flat phase $\Longrightarrow$ glassy phase in graphene ?
  • in the crumpling-to-flat transition $\Longrightarrow$ rich RG flow diagram
  (Delzescaux, D.M., Tissier (23))

- technically the flat phase provides a unusual situation:

  – NPRG and perturbative approaches are particularly successful
  $\Longrightarrow$ all the more striking that the theory displays scale invariance without conformal invariance
  (Mauri and Katsnelson (21), Gimenez-Grau, Nakayama and Rychkov (23))

- this should be understood