From fluctuating gravitons to Lorentzian quantum gravity and scattering amplitudes

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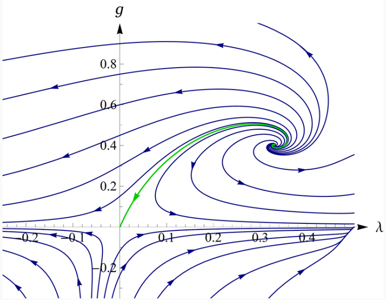
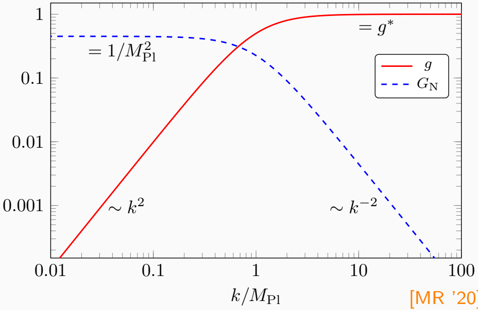
UNIVERSITY OF SUSSEX

Asymptotically safe quantum gravity

QG could be non-perturbatively renormalisable via an interacting UV FP [Weinberg '76]



[Reuter '96; Reuter, Saueressig '01]



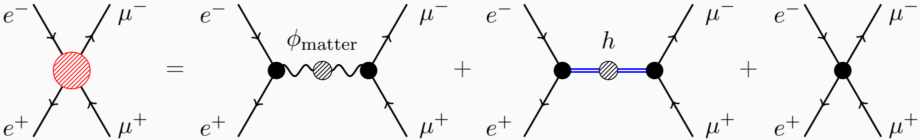
[MR '20]

Predictivity: number of free parameters = dimension of UV critical hypersurface
[Denz, Pawlowski, MR '16; Falls, Ohta, Percacci '20; Kluth, Litim '20; Knorr '21; . . .]

Unitarity: Positivity and finiteness of spectral functions and scattering amplitudes
[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; . . .]

Coupling to matter: Existence of fixed point with matter, Landscape of asymptotic safety
[Meibohm, Pawlowski, MR '15; Eichhorn, Held '18; Smirnov, MR '19; Eichhorn, Schiffer '22; . . .]

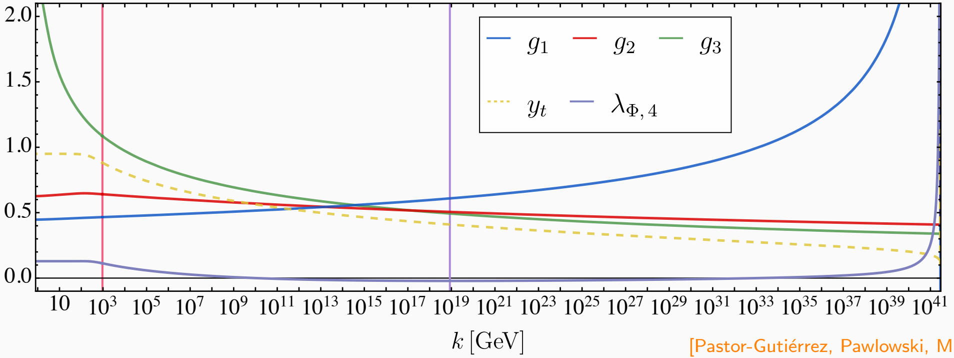
Towards scattering amplitudes



Need:

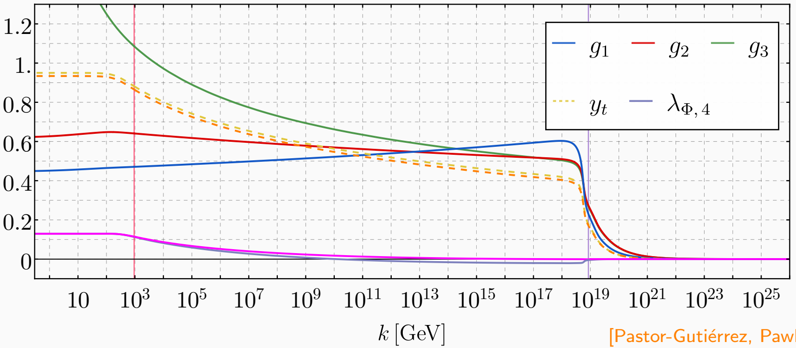
- a UV-IR trajectory of the SM with gravity
- well-behaved propagators without ghost or tachyonic instabilities
- access to correlation functions on Lorentzian signature at time-like momenta

Standard Model without Gravity



[Pastor-Gutiérrez, Pawlowski, MR '22]

The Asymptotically Safe Standard Model



[Pastor-Gutiérrez, Pawlowski, MR '22]

- g_1 relevant ✓ [Folkerts at al '11; Eichhorn, Versteegen '17; Christiansen, Litim, Pawlowski, MR '17; . . .]
- y relevant ? [Eichhorn, Held '17; Pastor-Gutiérrez, Pawlowski, MR '22; . . .]
- irrelevant λ predicts m_H/m_t [Shaposhnikov, Wetterich '12]

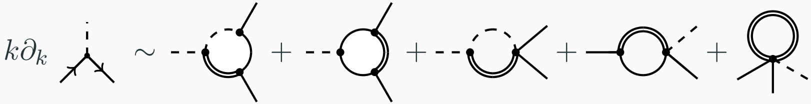
  Small mismatch between prediction and measurement [Pastor-Gutiérrez, Pawlowski, MR '22]

Precision computation of the Yukawa coupling

[de Brito, MR, Schiffer (in prep)]

- Leading order: $y\sqrt{|g|}\phi\bar{\psi}\psi$ + wave function renormalisations

  Contains strong regulator dependence [Eichhorn, Held '17; Pastor-Gutiérrez, Pawlowski, MR '22]



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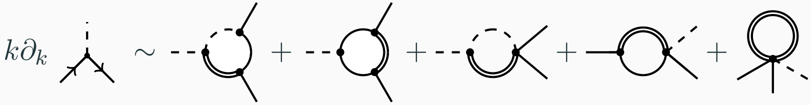
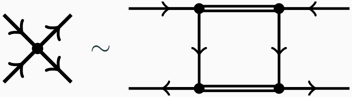


- NLO (dim-6 operators): $y_R\sqrt{|g|}R\phi\bar{\psi}\psi + \dots$

- NNLO (dim-8 operators): $y_{R^2}\sqrt{|g|}R^2\phi\bar{\psi}\psi + y_{C^2}\sqrt{|g|}C^2_{\mu\nu\rho\sigma}\phi\bar{\psi}\psi + \dots$

- Resummation due to gravity induced fixed point value

$k\partial_k$ 

Small fixed point value $\longrightarrow$ highly suppressed

Resummation of higher order operators

[de Brito, MR, Schiffer (in prep)]

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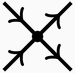
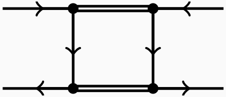
$k\partial_k$  $\sim$ 

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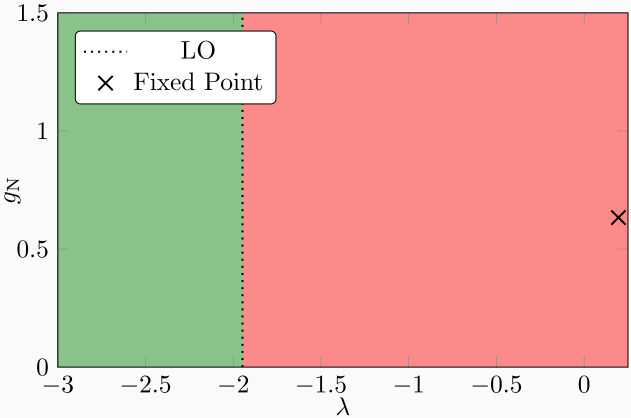
- Resummation via the stability matrix (leading G_N, $\Lambda = 0$)

$$M = -\frac{\partial \beta_{y_i}(\vec{y})}{\partial y_j}\bigg|_{\vec{y}=0} = \begin{pmatrix} -0.21\,G_\mathrm{N} & -0.14\,G_\mathrm{N} & -0.24\,G_\mathrm{N} & 0.41\,G_\mathrm{N} \\ 1.1\,G_\mathrm{N} & 2+0.74\,G_\mathrm{N} & 0.029\,G_\mathrm{N} & -0.20\,G_\mathrm{N} \\ -1.2\,G_\mathrm{N} & 0.27\,G_\mathrm{N} & 4+1.8\,G_\mathrm{N} & 0.44\,G_\mathrm{N} \\ -0.31\,G_\mathrm{N} & -0.33\,G_\mathrm{N} & 0.48\,G_\mathrm{N} & 4+0.40\,G_\mathrm{N} \end{pmatrix}$$

Relevance of the Yukawa coupling in G-Λ plane



[de Brito, MR, Schiffer *(in prep)*]

Green region: Yukawa relevant → finite Yukawa couplings in IR

Red region: Yukawa irrelevant → vanishing Yukawa couplings in IR

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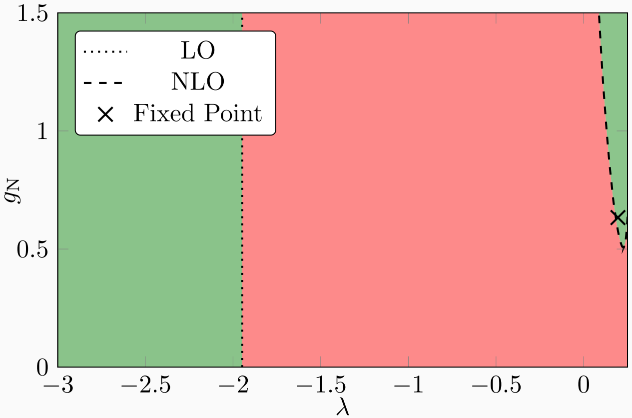

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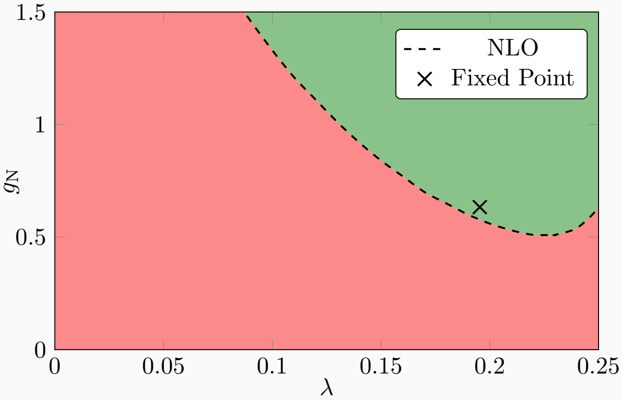

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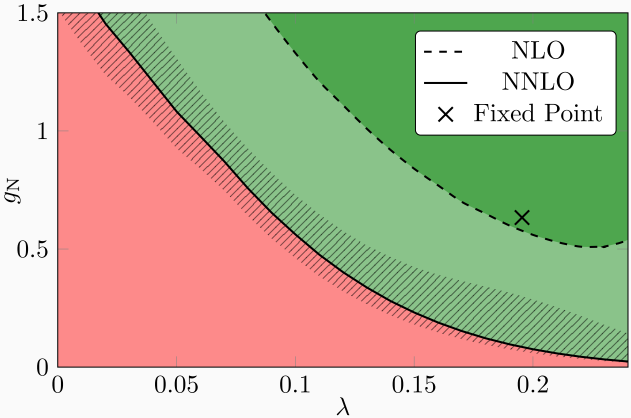

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Error estimate from N^3LO contributions

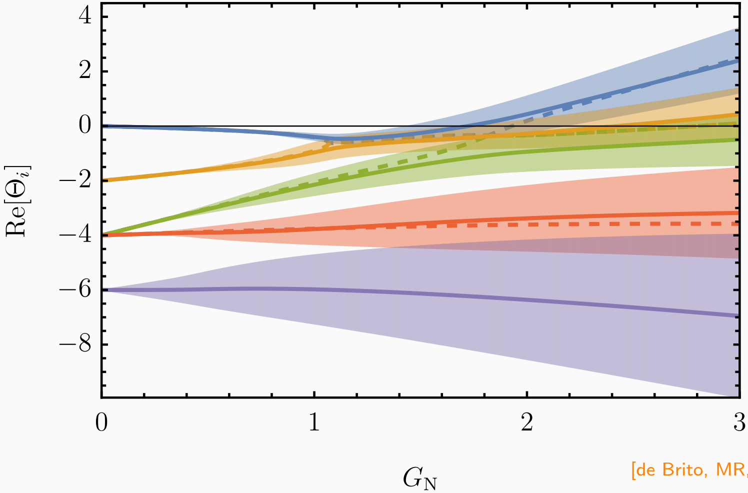
[de Brito, MR, Schiffer (in prep)]

- Stability matrix at N^3LO

$$M = \begin{pmatrix} -0.21\,G_N & -0.14\,G_N & -0.24\,G_N & 0.41\,G_N & 0 \\ 1.1\,G_N & 2+0.74\,G_N & 0.029\,G_N & -0.20\,G_N & \#G_N \\ -1.2\,G_N & 0.27\,G_N & 4+1.8\,G_N & 0.44\,G_N & \#G_N \\ -0.31\,G_N & -0.33\,G_N & 0.48\,G_N & 4+0.40\,G_N & \#G_N \\ \#G_N & \#G_N & \#G_N & \#G_N & 6+\#G_N \end{pmatrix}$$

- Assumption: all # of same size than lower-order coefficients
- Estimate uncertainty by averaging over 10^6 random generated N^3LO contributions

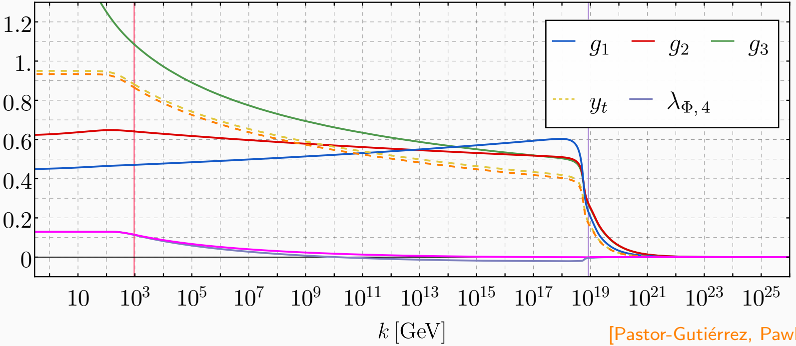
Relevance of the Yukawa coupling at $\Lambda = 0$



[de Brito, MR, Schiffer *(in prep)*]

For $\Lambda = 0$, the Yukawa coupling becomes relevant at $G_{\mathrm{N}} \sim 1.4 - 2.1$

The Asymptotically Safe Standard Model



[Pastor-Gutiérrez, Pawlowski, MR '22]

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Lorentzian Signature

- Callan-Symanzik cutoff preserves causality and Lorentz invariance

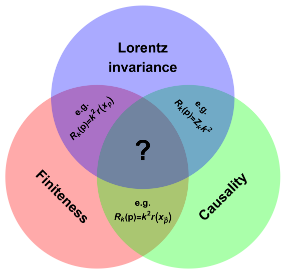
$$R_k = Z_\phi\, k^2$$

- Finite flow equation with counterterms

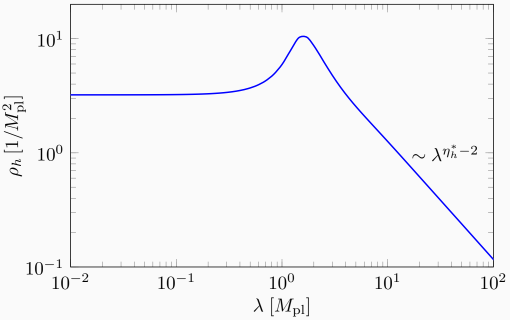
$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\, \mathcal{G}_k\, \partial_t R_k - \partial_t S_{\mathrm{ct},k}}$$

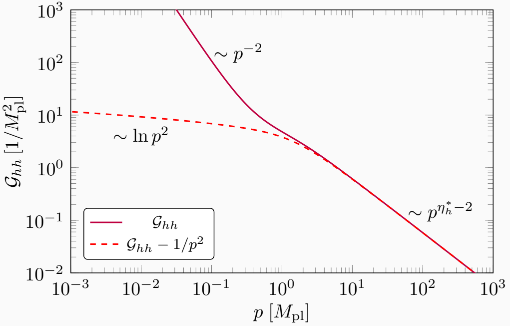
- Dim reg of UV divergences in $d = 4 - \varepsilon$ possible
- Use spectral function in flow diagrams

$$\partial_t \rho_h \propto \quad [\text{diagram}] \quad + \ldots \qquad \text{with} \qquad \mathcal{G}_h(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}$$



Graviton spectral function

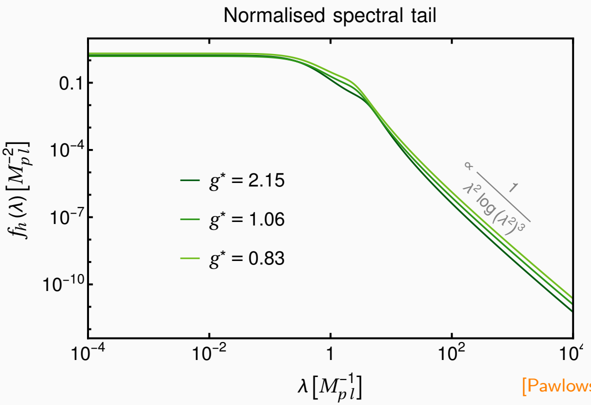




[Fehre, Litim, Pawlowski, MR '21]

- Massless graviton delta-peak with positive multi-graviton continuum
- No ghosts and no tachyons $\longrightarrow$ no indications for unitarity violation
- Good agreement with reconstruction results and EFT [Bonanno, Denz, Pawlowski, MR '21]
- Approximation: neglect feedback from multi-graviton continuum

Graviton spectral function with full feedback

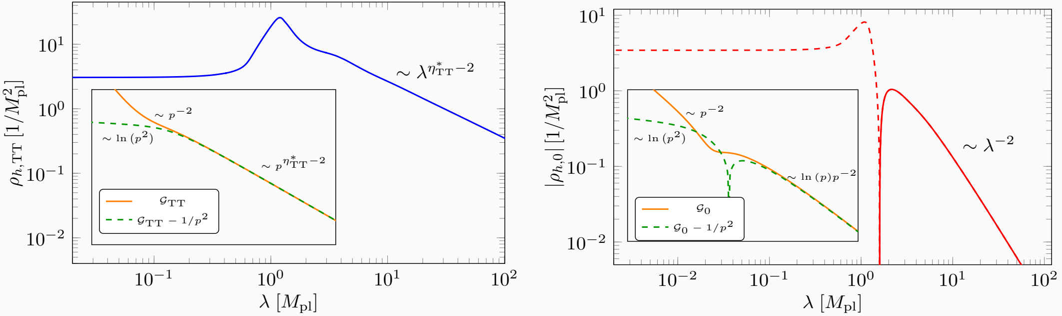


[Pawlowski, MR, Wessely *(in prep)*]

- Fully converged spectral function including feedback from multi-graviton continuum
- On-shell renormalisation

See poster from Jonas Wessely for more details

Graviton spectral function – TT and scalar graviton mode



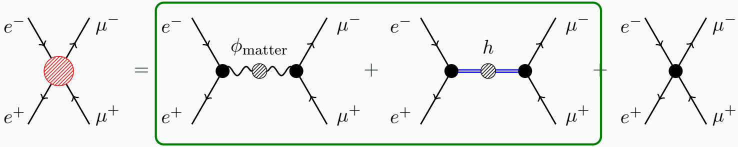
[Assant, Litim, MR *(in prep)*]

- Coupled system of transverse-traceless and scalar graviton mode
- First direct computation of form factors

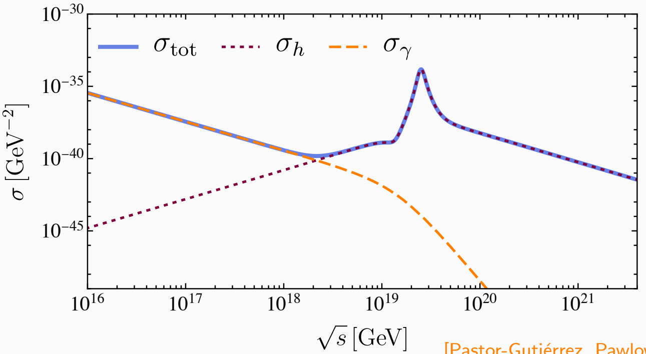
$$f_{C/R}(p^2) \sim \frac{1}{p^2} + \left(p^4 \int \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho_{h,\mathrm{tt}/0}(\lambda^2)}{p^2 - \lambda^2}\right)^{-1}$$

See poster from Gabriel Assant for more details

Towards graviton-mediated scattering cross-sections



Approximation: background propagator



[Pastor-Gutiérrez, Pawlowski, MR, Ruisi *(in prep)*]

Summary – Towards scattering amplitudes in asymptotically safe gravity

- Inclusion of higher-order operators makes the Yukawa coupling relevant
- Connecting UV-IR trajectory to the Standard Model ($+\varepsilon$)
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Well-behaved spectral functions without ghost or tachyonic instabilites
- First steps towards scattering processes and unitarity

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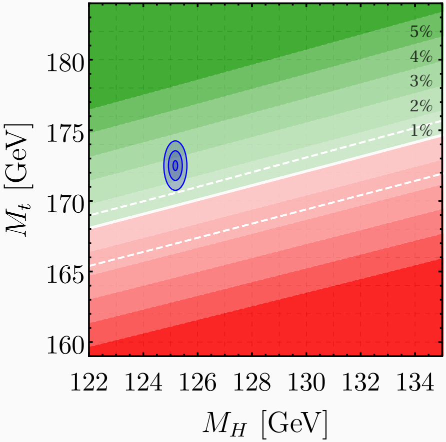
Thank you for your attention!

Back-up slides

Higgs vs top mass in the asymptotically safe Standard Model



[Pastor-Gutiérrez, Pawlowski, MR '22]

- Predicted Higgs mass of 125 GeV
  [Shaposhnikov, Wetterich '12]
- Small mismatch between predicted and measured Higgs-top mass ratio in pure SM
- Can be fixed with BSM physics, e.g., dark matter [MR, Smirnov '19]

Källén-Lehmann spectral representation

[Källén '52; Lehmann '54]

$$G(q^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon\to 0} \operatorname{Im} G(\omega^2 + i\varepsilon)$$