ETH zürich

Different flavours of fermion quadrupling condensates in magic-angle twisted bilayer graphene

Dr. Ilaria Maccari
SNSF Postdoctoral Fellow, ETH Zürich

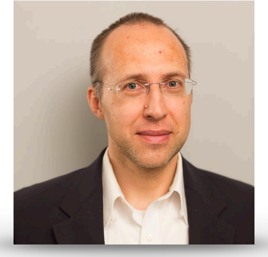
ERG 2024 - Les Diablares

26.09.2024

Acknowledgements

Today's talk

Theory



Egor Babaev
(KTH Stockholm)

Julien Garaud
(University of Tours)

Johan Carlström
(Stockholm University)

Daniel Weston
(KTH Stockholm)

Experiments

μSR/NQR
Vadim Grinenko, Rajib Sarkar, Hans-Henning Klauss,
TU Dresden, Germany/Shanghai Jiao Tong University, China

$Ba_{1-x}K_xFe_2As_2$ single crystals
Kunihiro Kihou and Chul-Ho Lee
AIST, Tsukuba, Japan

AC specific heat and ultrasound
Tino Gottschall, Gorbunov, Denis, Sergei Zherlitsyn, Jochen Wosnitza
HZDR, Dresden-Rossendorf, Germany

AC microcalorimetry
Andreas Rydh,
Stockholm University, Sweden

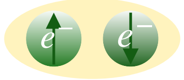
STM Measurements
Quanxin Hu, Yu Zheng, Fashi Yang, Yongwei Li, Chi-Ming Yim
Shanghai Jiao Tong University, China

X-Ray measurements
Ilya Shipulin, Yongwei Li
TU Dresden, Germany / Shanghai Jiao Tong University, China

Thermoelectrical transport
Federico Caglieris, Ilya Shipulin, Nadia Stegani, Christoph Wuttke, Christian Heß
IFW Dresden, Germany / CNR-Spin Genova, Italy

Beyond the pairing paradigm: fermion quadrupling condensates

Cooper's pair



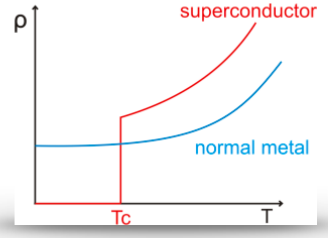
Ginzburg-Landau theory

Complex order parameter:

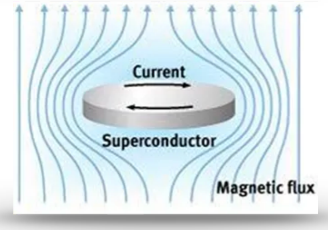
Cooper pairs density

$$\psi(x) = |\psi(x)|\, e^{i\phi(x)}$$

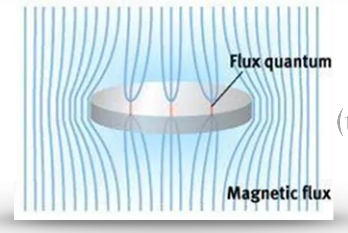
$$f = \frac{1}{2}\left|\left(\vec{\nabla} - ie\vec{A}\right)\psi\right|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\mathbf{B}^2}{2}$$



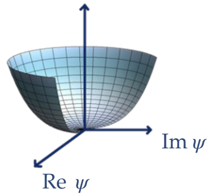
Zero Resistance



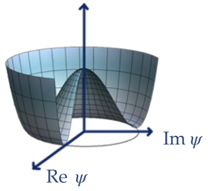
Meissner effect (low magnetic fields)



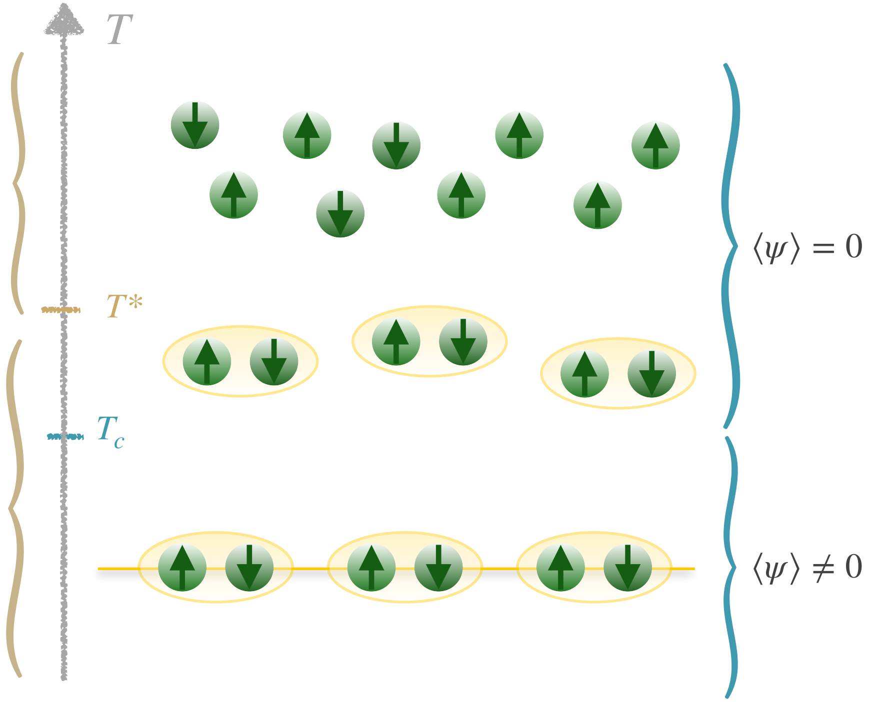
Magnetic flux quantisation (type-II SCs, higher magnetic fields)



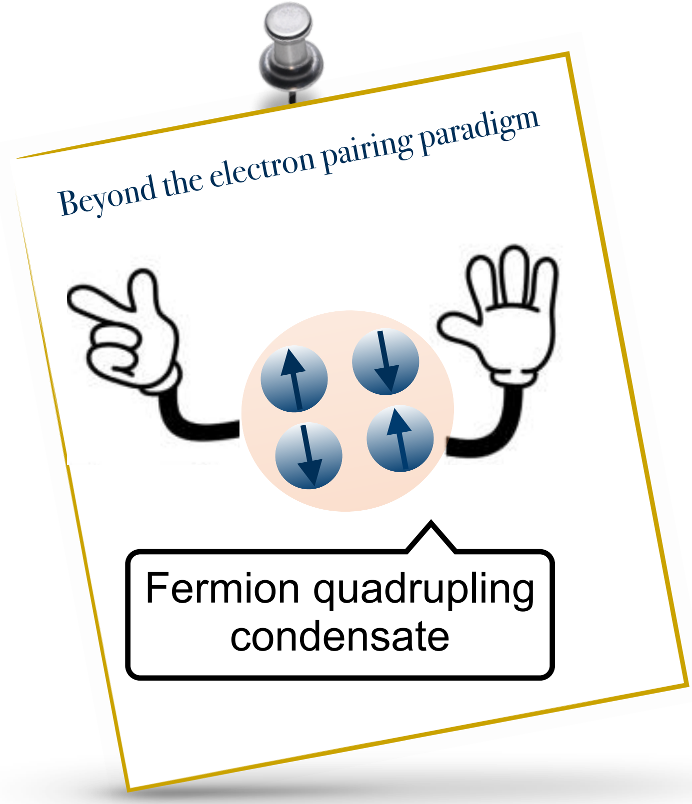
$|\psi| = 0$



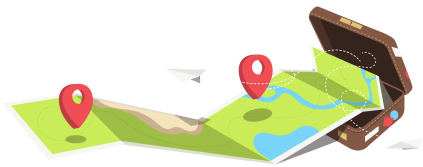
$|\psi| \neq 0$



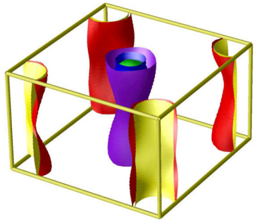
Spontaneous breaking of the U(1) gauge symmetry



How can we go beyond the pairing paradigm?

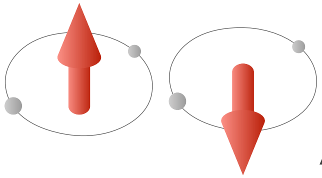


1. Multicomponent Superconductors



I.I. Mazin, J. Schmalian, Physica C 469 (2009) 614.

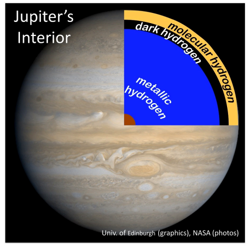
Multi-band SCs



Single-band SCs unconventional pairing

p-wave chiral SC:

$\Delta(\vec{k}) = \Delta_x \sin(k_x a) + \Delta_y \sin(k_y a)$



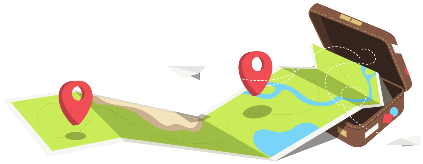
Mixture of charged condensates

(e.g. liquid metallic hydrogen)

1. $\psi_{i=1,2}(x) = |\psi_i(x)|\, e^{i\phi_i(x)}$ Two components:

Component label

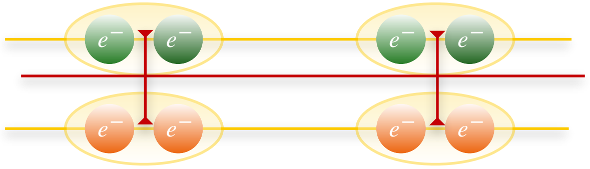
How can we go beyond the pairing paradigm?

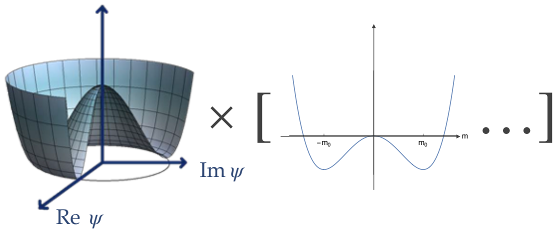


1. Multicomponent Superconductors

2. Multiple broken symmetries

Inter-component interaction





SC state breaking multiple symmetries in the GS

1. $\psi_{i=1,2}(x) = |\psi_i(x)|\, e^{i\phi_i(x)}$ Two components:

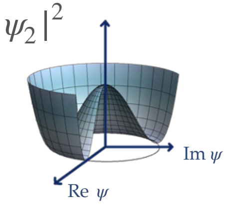
Component label

2. Simplest case: two components interacting only via the EM field

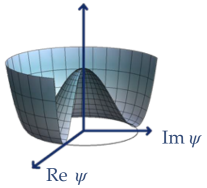
$$f = \frac{1}{2}\left|\left(\vec\nabla - ie\vec A\right)\psi_1\right|^2 + \frac{1}{2}\left|\left(\vec\nabla - ie\vec A\right)\psi_2\right|^2 + \sum_{i=1,2}\left[\alpha_i|\psi_i|^2 + \frac{\beta_i}{2}|\psi_i|^4\right] + \frac{\mathbf{B}^2}{2}$$

$$f = \frac{|\psi_1|^2|\psi_2|^2}{2\rho^2}\left[\vec\nabla(\phi_1 - \phi_2)\right]^2 + \frac{1}{2\rho^2}\left[|\psi_1|^2\vec\nabla\phi_1 + |\psi_2|^2\vec\nabla\phi_2 - e\rho^2\vec A\right]^2 + \sum_{i=1,2}\left[\alpha_i|\psi_i|^2 + \frac{\beta_i}{2}|\psi_i|^4\right] + \frac{\mathbf{B}^2}{2}$$

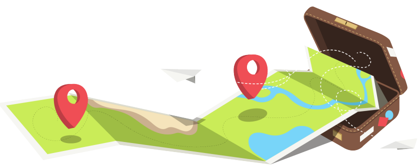
$$\rho^2 = |\psi_1|^2 + |\psi_2|^2$$



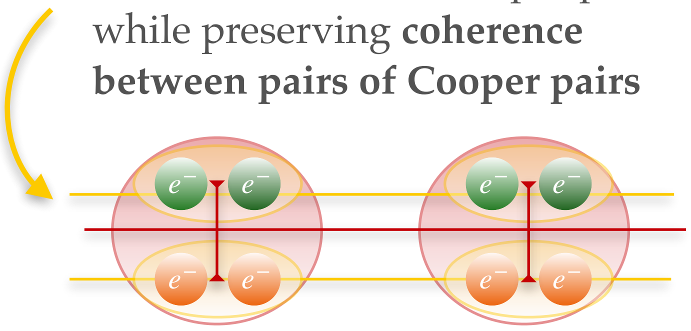
$U(1)$ Neutral mode $\times$ $U(1)$ charged mode



How can we go beyond the pairing paradigm?

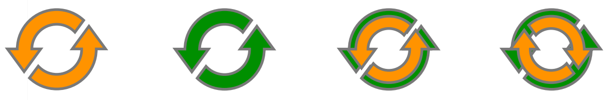


1. Multicomponent Superconductors
2. Multiple broken symmetries
3. **"Something"** destroying coherence between Cooper pairs while preserving **coherence between pairs of Cooper pairs**



Composite vortices

$(Q_1\,,\,Q_2)$ with: $Q_i = \frac{1}{2\pi}\oint d\vec{l}\;\vec{\nabla}\phi_i$



$(0\,,\,1)$ $(1\,,\,0)$ $(1\,,\,1)$ $(1\,,\,-1)$

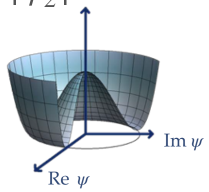
1. $\psi_{i=1,2}(x) = |\psi_i(x)|\,e^{i\phi_i(x)}$ Two components: e^- e^- e^- e^-

Component label

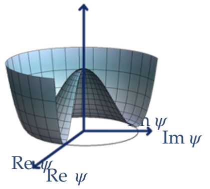
2. Simplest case: two components interacting only via the EM field

$$f = \frac{|\psi_1|^2|\psi_2|^2}{2\rho^2}\left[\vec{\nabla}(\phi_1-\phi_2)\right]^2 + \frac{1}{2\rho^2}\left[|\psi_1|^2\vec{\nabla}\phi_1 + |\psi_2|^2\vec{\nabla}\phi_2 - e\rho^2\vec{A}\right]^2 + \sum_{i=1,2}\left[\alpha_i|\psi_i|^2 + \frac{\beta_i}{2}|\psi_i|^4\right] + \frac{\mathbf{B}^2}{2}$$

$$\rho^2 = |\psi_1|^2 + |\psi_2|^2$$



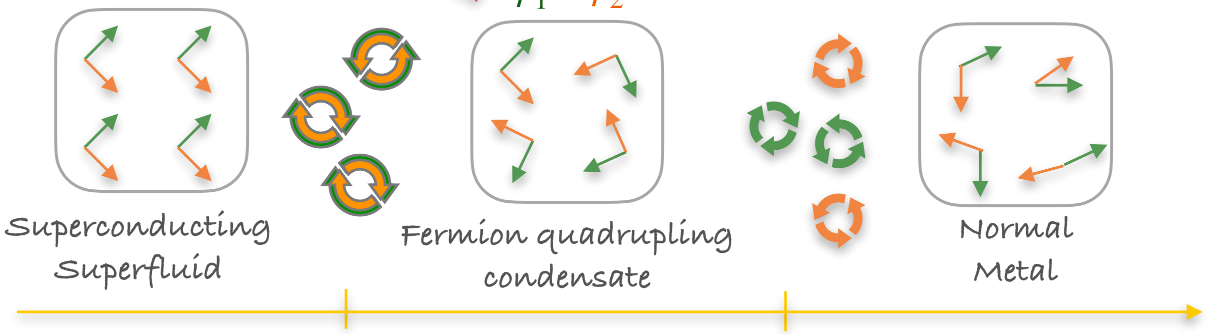
$U(1)$ Neutral mode $\times$ $U(1)$ Charged mode



3.

Rigid phase difference

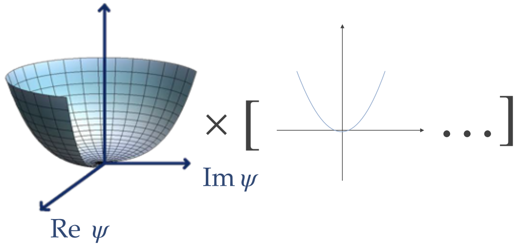
$\phi_1 - \phi_2$



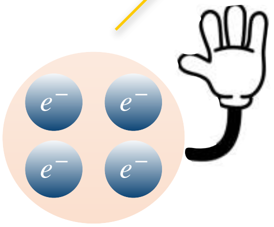
$\langle\psi_{1,2}\rangle \neq 0$	T_c^{SC}	$\langle\psi_{1,2}\rangle = 0$ $\langle\psi_1\psi_2^* + \psi_1^*\psi_2\rangle \neq 0$	T_c^{SF}	$\langle\psi_{1,2}\rangle = 0$ $\langle\psi_1\psi_2^* + \psi_1^*\psi_2\rangle = 0$	T

Beyond the pairing paradigm: fermion quadrupling condensates

T

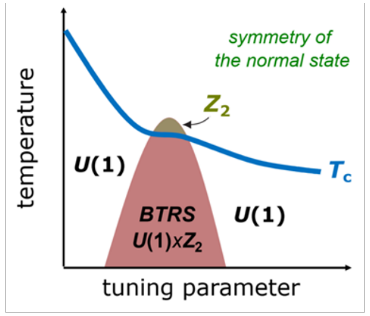


Fermion quadrupling condensates as vestigial phases



Above T_c

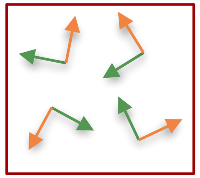
non-SC fermion quadruplets



V. Grinenko, D. Weston, F. Caglieris, C. Wuttke, C. Hess, T. Gottschall, **I. Maccari**, et al., *Nat. Physics 17, 1254–1259 (2021)*

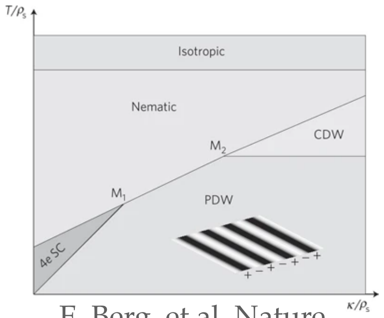
Typically associated with a phase difference rigidity

$$\langle \psi_i \psi_j^* + \psi_i^* \psi_j \rangle \neq 0$$



Below T_c

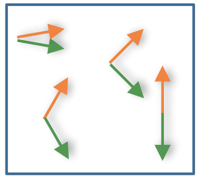
Charge-4e SC

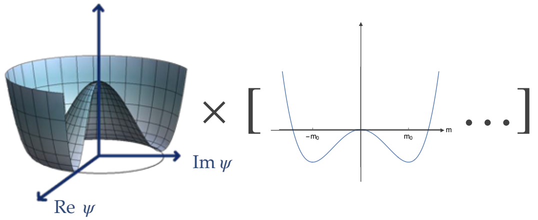


E. Berg, et al. Nature Physics 5, 830 (2009)

Typically associated with a phase sum rigidity

$$\langle \psi_i^* \psi_j^* + \psi_i \psi_j \rangle \neq 0$$



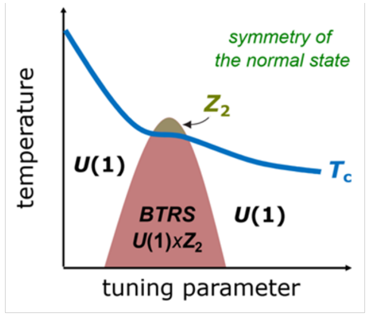


Multiple broken symmetries

Beyond the pairing paradigm: fermion quadrupling condensates

Above T_c

non-SC fermion quadruplets



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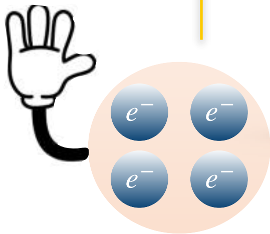
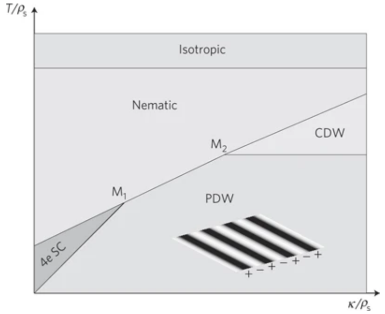


Below T_c

Charge-4e SC



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First experimental observation in a multi band iron-based SC

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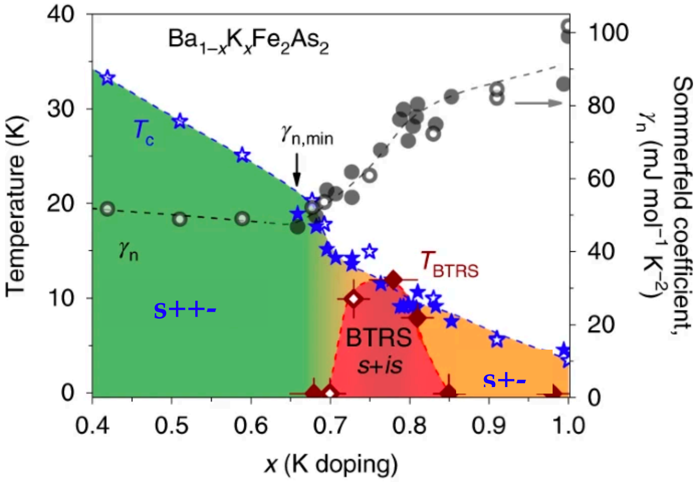
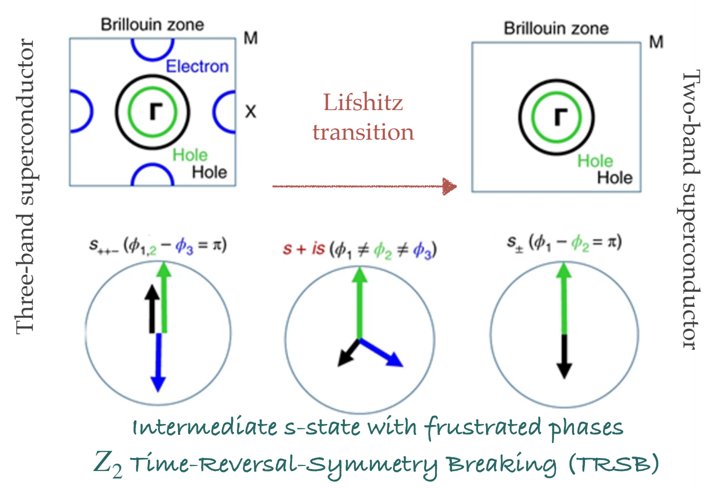
V. Grinenko et al., Phys. Rev. B 95, 214511 (2017), V. Grinenko et al., *Nat. Phys.* 16, 789 - 794 (2020).





Interband tunneling of Cooper Pairs

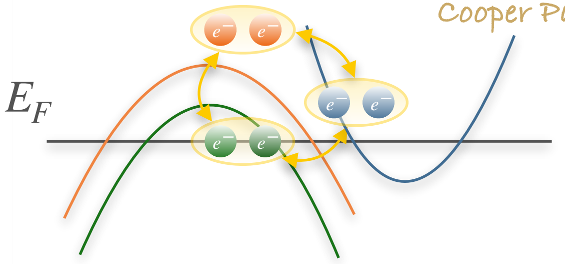


Illustration of a three-band SC

$$f = \frac{1}{2}\sum_i \left|\left(\vec{\nabla} - ie\vec{A}\right)\psi_i\right|^2 + \sum_i \left[\alpha_i|\psi_i|^2 + \frac{\beta_i}{2}|\psi_i|^4\right] +$$

$$+\sum_{i>j}\eta_{ij}|\psi_i||\psi_j|\cos(\phi_i - \phi_j) + \frac{\mathbf{B}^2}{2};$$

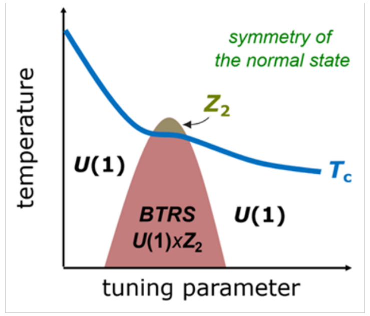
Josephson intercomponent coupling

$$\propto \frac{1}{2}[\psi_i\psi_j^* + \psi_i^*\psi_j]$$

Beyond the pairing paradigm: fermion quadrupling condensates

Above T_c

non-SC fermion quadruplets



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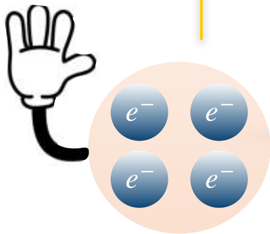
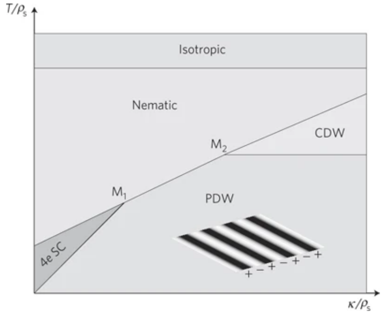


Below T_c

Charge-4e SC



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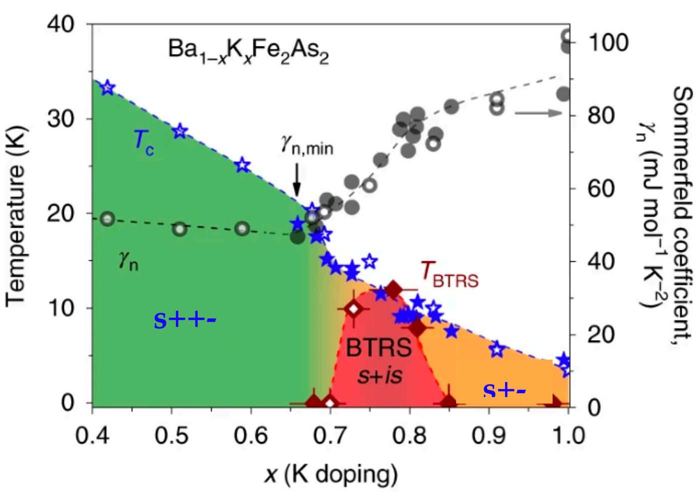
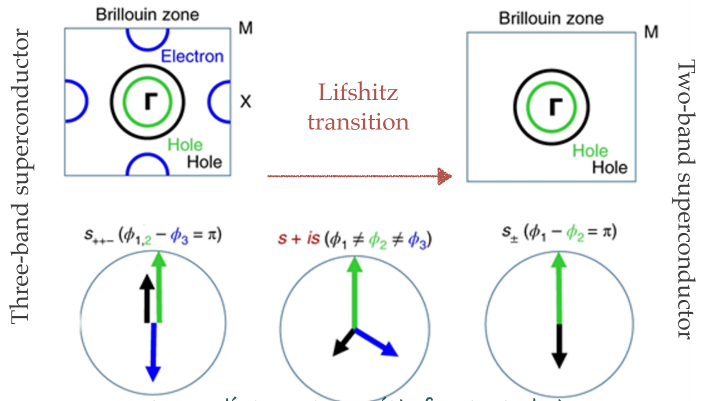




Intermediate s-state with frustrated phases

Z_2 Time-Reversal-Symmetry Breaking (TRSB)

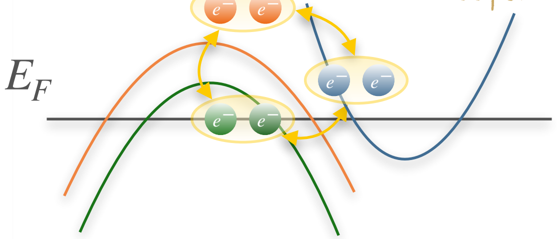
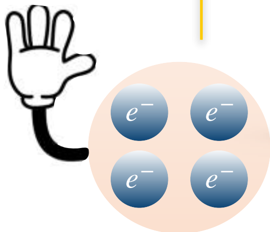


Illustration of a three-band SC

$$f = \frac{1}{2\rho^2}\left[\left(\sum_{i=1,2,3}|\psi_i|^2\vec{\nabla}\phi_i\right) - e\rho^2\vec{A}\right]^2 + \sum_{i>j}\frac{|\psi_i|^2|\psi_j|^2}{2\rho^2}\left[\vec{\nabla}(\phi_i-\phi_j)\right]^2 + \sum_{i>j}\eta_{ij}|\psi_i||\psi_j|\cos(\phi_i-\phi_j) + \sum_i V(|\psi_i|^2,|\psi_i|^4) + \frac{\mathbf{B}^2}{2};$$

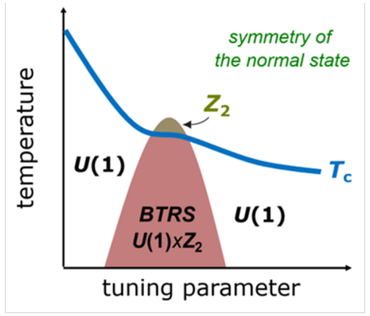
Beyond the pairing paradigm: fermion quadrupling condensates

Above T_c



Below T_c

non-SC fermion quadruplets



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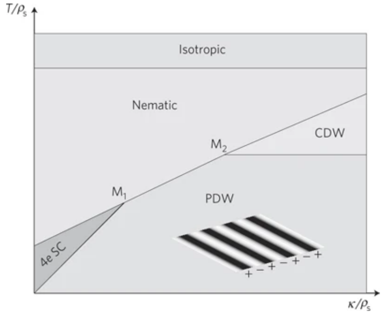
Charge-4e SC



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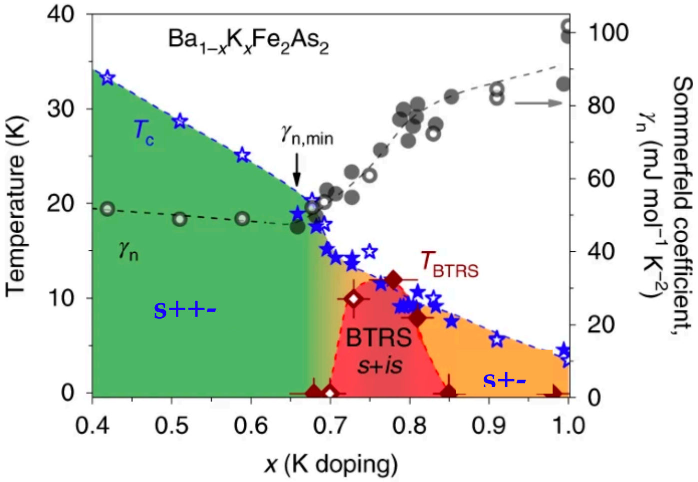
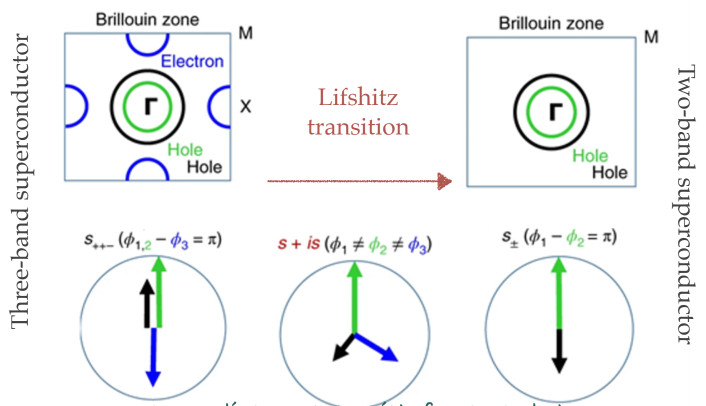
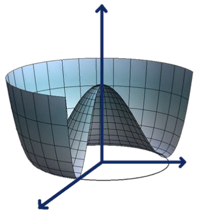
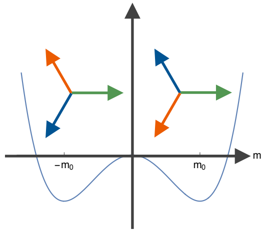




The ground state breaks a total $U(1) \times Z_2$ symmetry

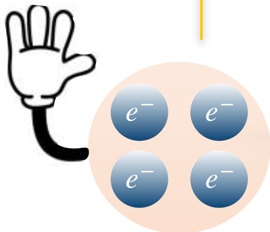




$$f = \frac{1}{2\rho^2}\left[\left(\sum_{i=1,2,3} |\psi_i|^2 \vec{\nabla}\phi_i\right) - e\rho^2 \vec{A}\right]^2 + \sum_{i>j} \frac{|\psi_i|^2|\psi_j|^2}{2\rho^2}\left[\vec{\nabla}(\phi_i - \phi_j)\right]^2 + \sum_{i>j} \eta_{ij}|\psi_i||\psi_j|\cos(\phi_i - \phi_j) + \sum_i V(|\psi_i|^2, |\psi_i|^4) + \frac{\mathbf{B}^2}{2};$$

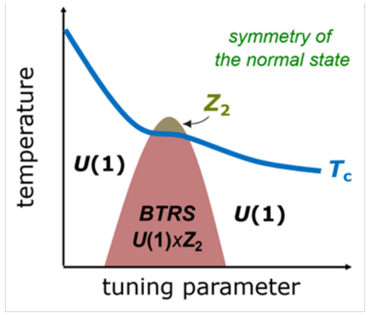
Beyond the pairing paradigm: fermion quadrupling condensates

Above T_c



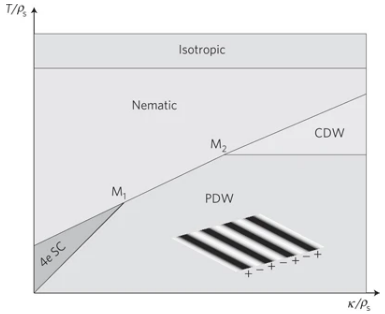
Below T_c

non-SC fermion quadruplets



V. Grinenko, D. Weston, F. Caglieris, C. Wuttke, C. Hess, T. Gottschall, **I. Maccari**, et al., *Nat. Physics 17, 1254–1259 (2021)*

Charge-4e SC



E. Berg, et al. Nature Physics 5, 830 (2009)

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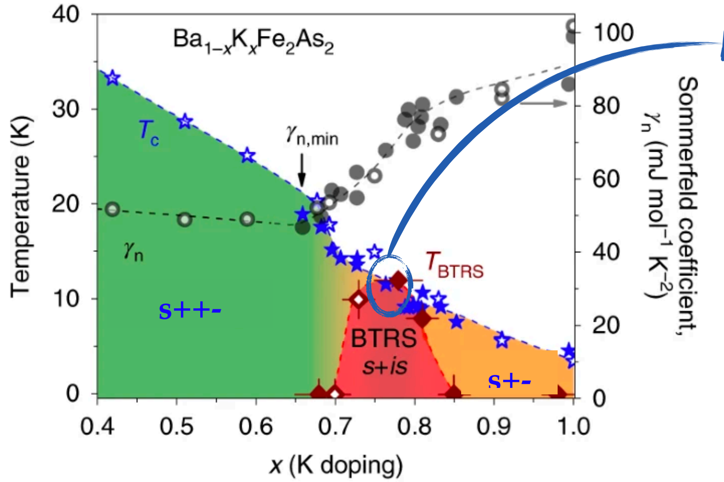
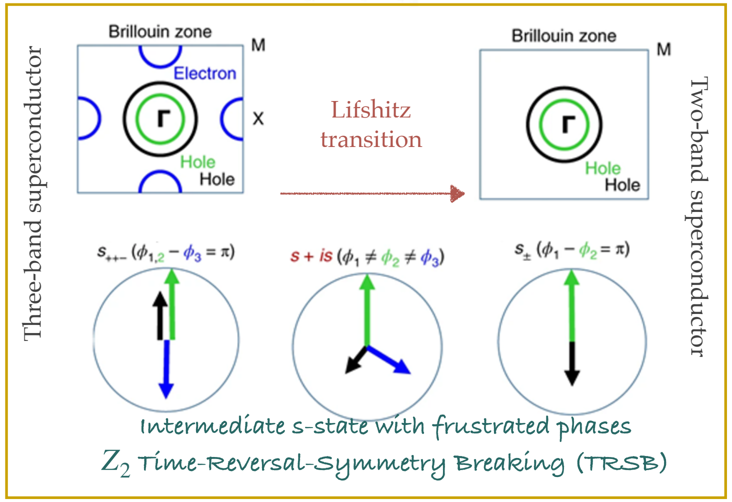
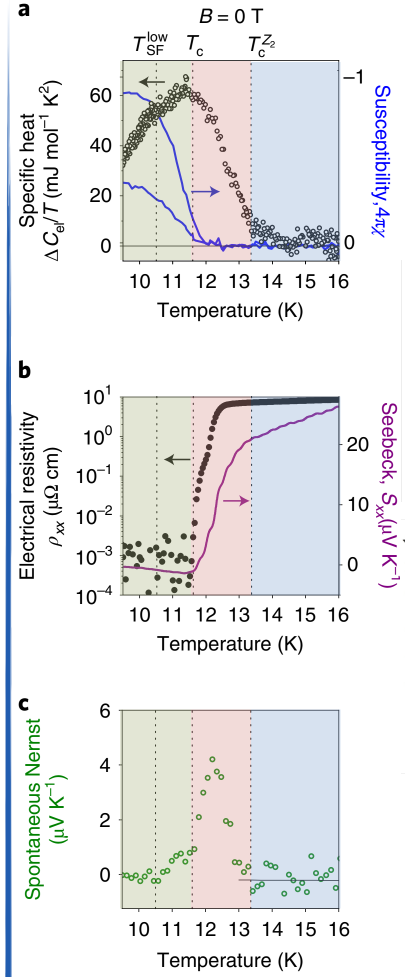
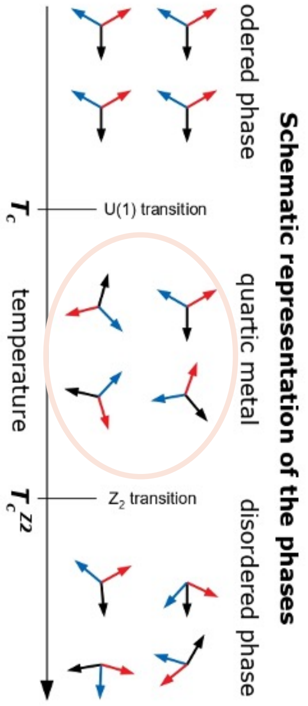
V. Grinenko, D. Weston, F. Caglieris, C. Wuttke, C. Hess, T. Gottschall, **I. Maccari**, et al., *Nature Physics 17, 1254–1259 (2021)*

V. Grinenko et al., Phys. Rev. B 95, 214511 (2017), V. Grinenko et al., Nat. Phys. 16, 789 - 794 (2020).









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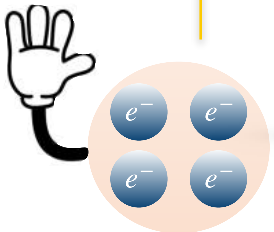
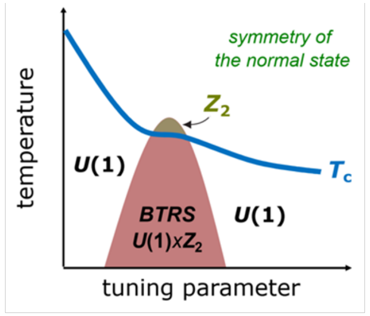


Above T_c

non-SC fermion quadruplets



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Below T_c

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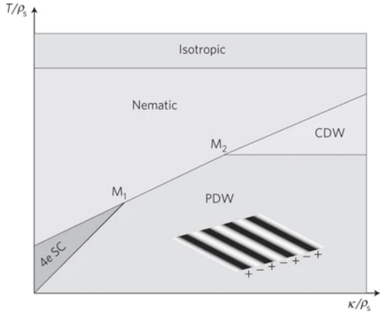

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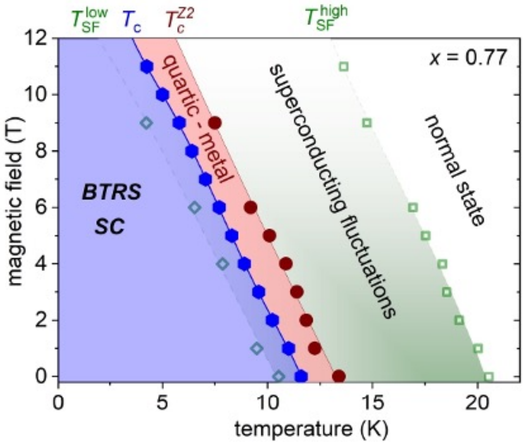
Nature Physics 17, 1254–1259 (2021)

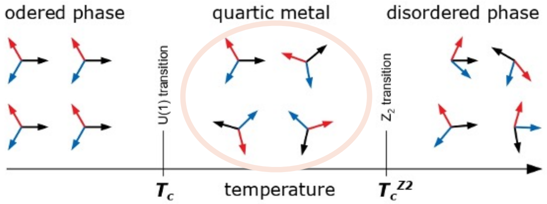
Main experimental evidences

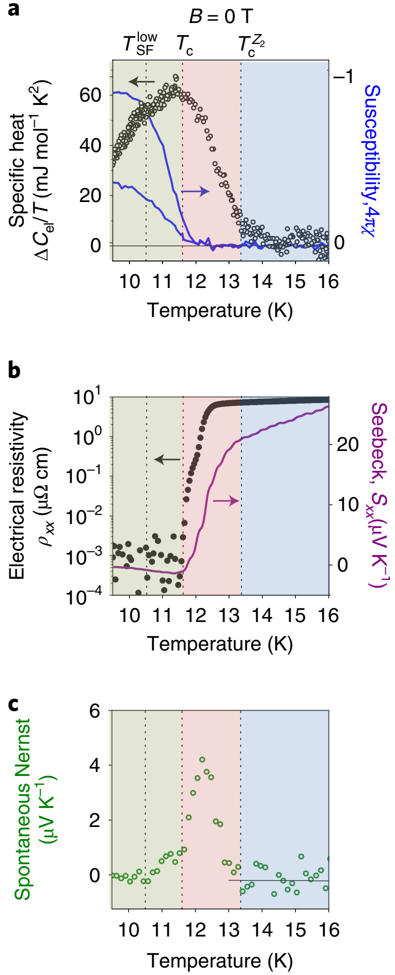
- ✳ Muon-spin relaxation measurements
- ✳ Spontaneous Nernst effect
- ✳ Ultrasound measurements
- ✳ Two peaks in the specific heat

Experimental phase diagram of $Ba_{1-x}K_xFe_2As_2$



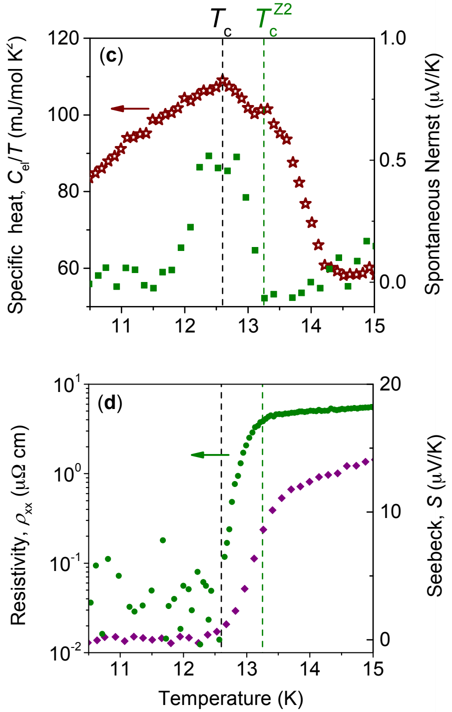
Schematic representation of the phases





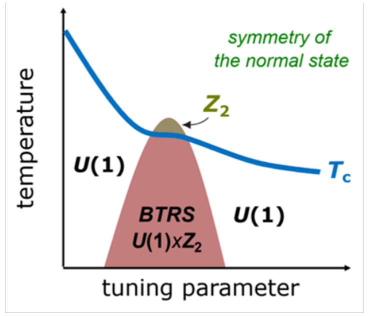
Calorimetric signatures

I. Shipulin, N. Stegani, **I. Maccari**, et al.,
Nature Communications **14**, 6734 (2023)



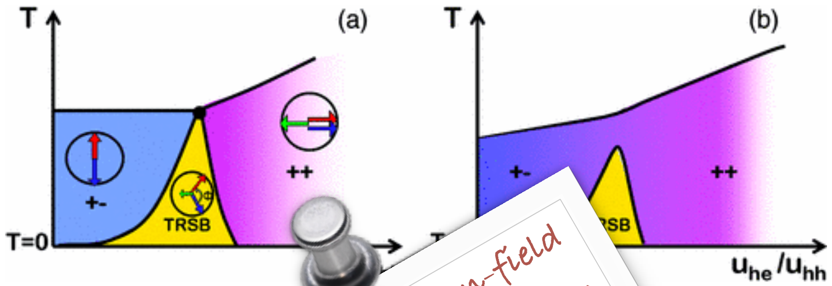
Monte Carlo results on an effective 3D multicomponent model

The role of different intercomponent interactions



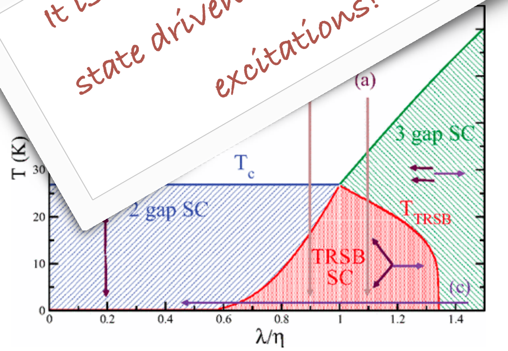
I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**

- Mean-field models



S. Maiti and A. V. C…

It is a beyond-mean-field state driven by topological excitations!



M. Marciani et al, Phys. Rev. B **88**, 214508 (2013)

Increasing x= hole doping

- Renormalization group approach

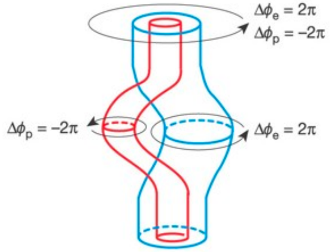
✓ It accounts for the proliferation of topological phase excitations.

✦ You need to properly account for the coupling between the different topological defects.

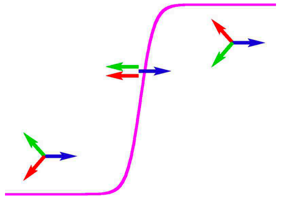
- Monte Carlo numerical simulations

✓ They allow for a systematic and unbiased study of the proliferation of different topological defects and their interactions.

Vortices



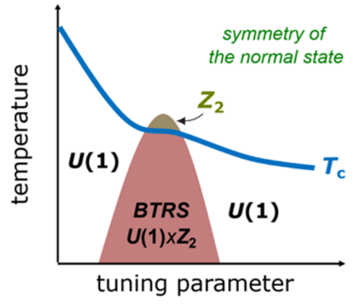
Domain Walls



Monte Carlo results on an effective 3D multicomponent model

The role of different intercomponent interactions

I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**



Starting point: Three-component GL model with phase frustration

$$f = \frac{1}{2}(\nabla \times \mathbf{A})^2 + \sum_i \frac{1}{2}|(\nabla + ie\mathbf{A})\psi_i|^2 + a_i|\psi_i|^2 + \frac{b_i}{2}|\psi_i|^4 + \sum_{i<j}\eta_{ij}|\psi_i||\psi_j|\cos(\phi_i - \phi_j)$$

At zero external magnetic field: any BTRS quadrupling fermionic condensate in the type-II regime ($e \to 0$)

What are we missing?

+Andreev-Bashkin interaction

A. F. Andreev and E. P. Bashkin, Sov. Phys. JETP 42, 164 (1975); A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975).

$$f = \frac{1}{2}(\nabla \times \mathbf{A})^2 + \sum_i \frac{1}{2}|(\nabla + ie\mathbf{A})\psi_i|^2 + a_i|\psi_i|^2 + \frac{b_i}{2}|\psi_i|^4 + \sum_{i<j}\eta_{ij}|\psi_i||\psi_j|\cos(\phi_i - \phi_j) - \nu\sum_{j>i}\vec{J}_j \cdot \vec{J}_i$$

Mapping to a two-component London effective model

J. Garaud, M. Silaev, and E. Babaev, Physica C 533, 63 (2017)

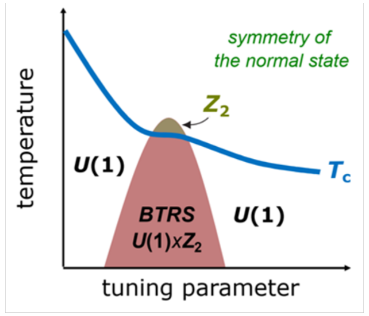
$$f = \frac{1}{2}\left(\nabla \times \mathbf{A}\right)^2 + \sum_{i=1,2}\frac{\rho_i}{2}\left(\nabla\phi_i - e\mathbf{A}\right)^2 - \nu\left(\nabla\phi_1 - e\mathbf{A}\right)\cdot\left(\nabla\phi_2 - e\mathbf{A}\right) + \eta_2\cos[2(\phi_1 - \phi_2)]$$

Phase frustration via a biquadratic Josephson coupling

Monte Carlo results on an effective 3D multicomponent model

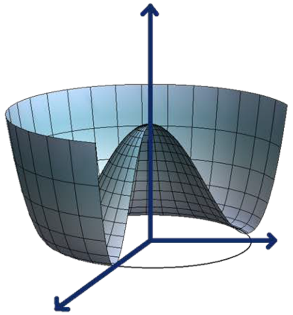
The role of different intercomponent interactions

I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**

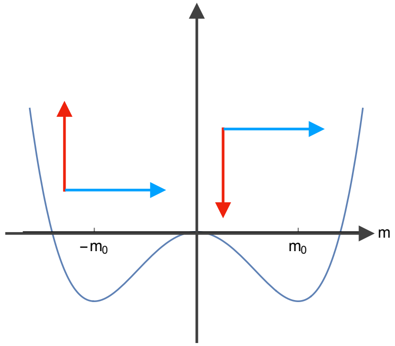


$$f = \frac{1}{2}\left(\nabla \times \mathbf{A}\right)^2 + \sum_{i=1,2} \frac{\rho_i}{2}\left(\nabla\phi_i - e\mathbf{A}\right)^2 - \nu\left(\nabla\phi_1 - e\mathbf{A}\right)\cdot\left(\nabla\phi_2 - e\mathbf{A}\right) + \eta_2\cos[2(\phi_1 - \phi_2)]$$

Phase frustration via a biquadratic Josephson coupling

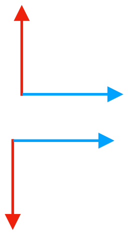


$U(1)$



Z_2

Two possible chiralities:



$(\rho_1, \rho_2 e^{i\frac{\pi}{2}})$

$(\rho_1, \rho_2 e^{\frac{-i\pi}{2}})$

$\rho_1 = \rho_2 = 1$

$$f = \frac{1-\nu}{4}\left(\nabla\phi_1 + \nabla\phi_2 - 2e\mathbf{A}\right)^2 + \frac{1+\nu}{4}\left(\nabla\phi_1 - \nabla\phi_2\right)^2 + \eta_2\cos[2(\phi_1 - \phi_2)] + \frac{1}{2}(\nabla \times \mathbf{A})^2$$

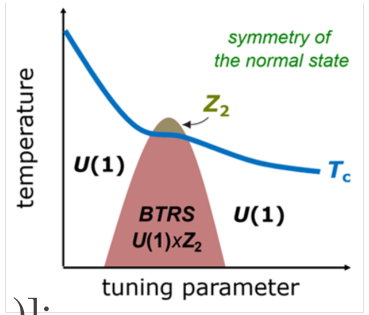
Key intercomponent interactions:

- e: coupling with the EM field → Increasing e decreases the cost of (1,1) vortices
- η_2: biquadratic Josephson coupling → Increasing η_2 increases the cost of domain walls
- ν: : Current-current interaction → Increasing ν decreases the relative cost of phase sum fluctuations

Monte Carlo results on an effective 3D multicomponent model

The role of different intercomponent interactions

I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**



Extreme type-II limit: $\lambda \to \infty \longrightarrow e = 0$

$$f = \frac{1-\nu}{4}\left(\nabla\phi_1 + \nabla\phi_2\right)^2 + \frac{1+\nu}{4}\left(\nabla\phi_1 - \nabla\phi_2\right)^2 + \eta_2 \cos[2(\phi_1 - \phi_2)];$$

Helicity modulus sum (superfluid stiffness)

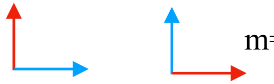
$$\Upsilon_+^\mu = \frac{1}{L^2}\frac{\partial^2 F(\{\phi_i'\})}{\partial \delta_\mu^2}\bigg|_{\delta_\mu=0}$$

$$\begin{pmatrix}\phi_1'(\mathbf{r})\\ \phi_2'(\mathbf{r})\end{pmatrix} = \begin{pmatrix}\phi_1(\mathbf{r}) + \delta\cdot\mathbf{r}_\mu\\ \phi_2(\mathbf{r}) + \delta\cdot\mathbf{r}_\mu\end{pmatrix}.$$

Fermion quadrupling condensate with TRSB

T_c ⏟ $T_c^{Z_2}$

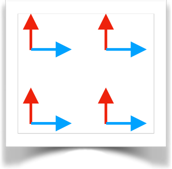
Z_2 Ising parameter

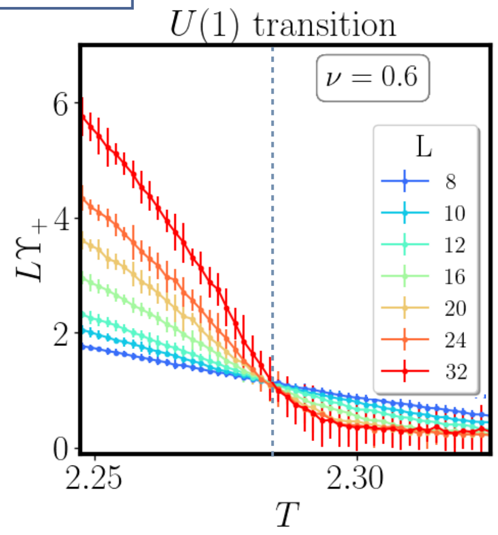


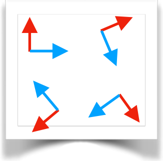
$[\phi_1 - \phi_2]_{-\pi,\pi} > 0$ $\quad$ $[\phi_1 - \phi_2]_{-\pi,\pi} < 0$

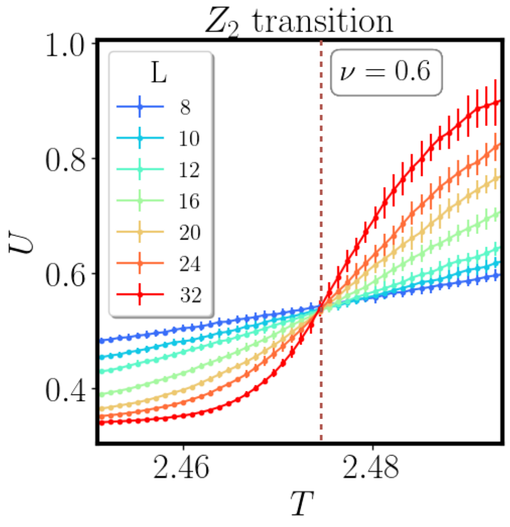
Binder cumulant

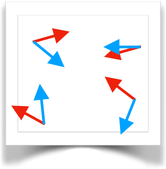
$$U = \frac{\langle m^4\rangle}{3\langle m^2\rangle^2}$$











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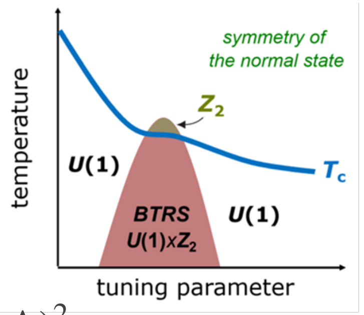
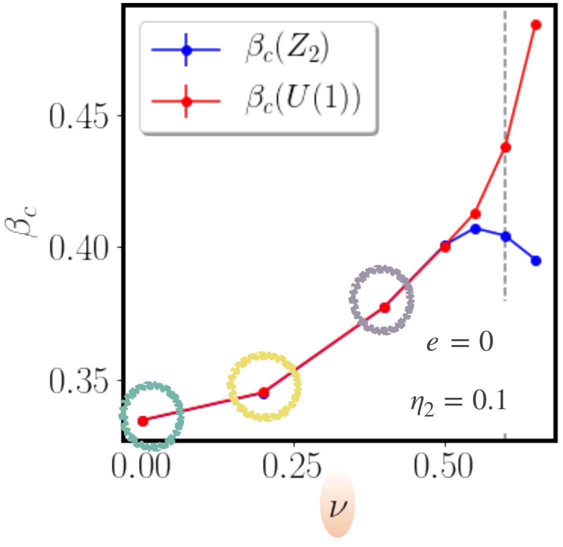
I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**

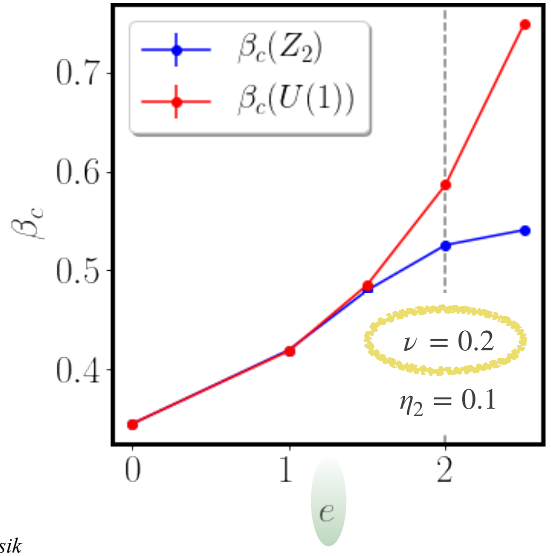


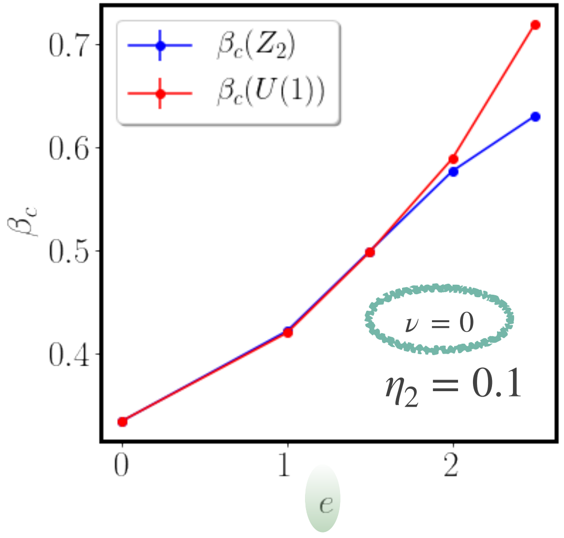
$$\rho_1 = \rho_2 = 1 \qquad f = \frac{1-\nu}{4}\left(\nabla\phi_1 + \nabla\phi_2 - 2e\mathbf{A}\right)^2 + \frac{1+\nu}{4}\left(\nabla\phi_1 - \nabla\phi_2\right)^2 + \eta_2\cos[2(\phi_1-\phi_2)] + \frac{1}{2}(\nabla\times\mathbf{A})^2$$



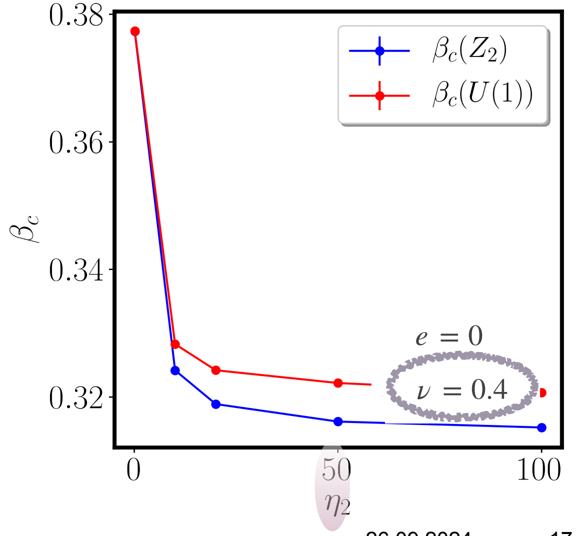
It decreases the relative cost of phase sum fluctuations

It decreases the cost of (1,1) vortices





It increases the cost of domain walls



Beyond the pairing paradigm: fermion quadrupling condensates

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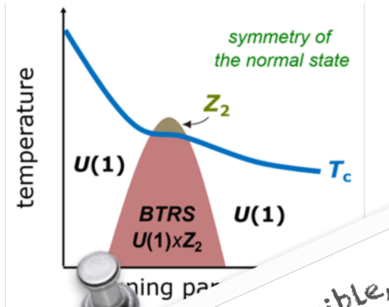
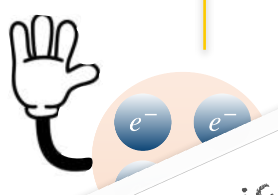
non-SC fermion quadruplets



V. Grinenko …

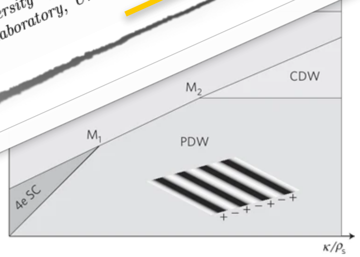


Bosonic counterparts are also possible, where $\langle b_i \rangle = 0$ but $\langle b_i^\dagger b_j \rangle \neq 0$

Observation of counterflow superfluidity in a two-component Mott insulator

Yong-Guang Zheng,[1,2,3,*] An Luo,[1,2,*] Ying-Chao Shen,[1,2] Ming-Gen He,[1,2] Zi-Hang Zhu,[1,2] Ying Liu,[1,2] Wei-Yong Zhang,[1,2] Hui Sun,[1,2] Youjin Deng,[1,2,3] Zhen-Sheng Yuan,[1,2,3] and Jian-Wei Pan[1,2,3]

[1] Hefei National Research Center for Physical Sciences at the Microscale and School of Physical Sciences, University of Science and Technology of China, Hefei 230026, China
[2] CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China
[3] Hefei National Laboratory, University of Science and Technology of China, Hefei 230088, China
(Dated: March 7, 2024)



E. Berg, et al. Nature Physics 5, 830 (2009)

First experimental observation in a multi band iron-based SC

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Nat. Physics 17, 1254–1259 (2021)

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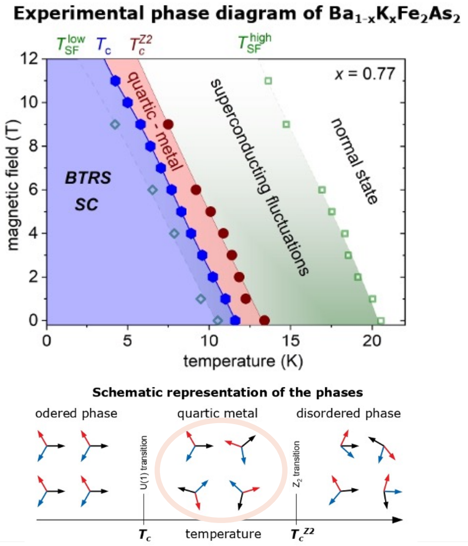
- Spontaneous Nernst effect
- Ultrasound measurements
- peaks in the specific heat



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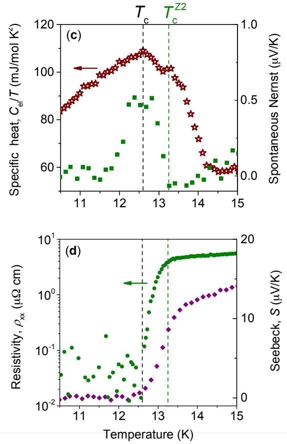
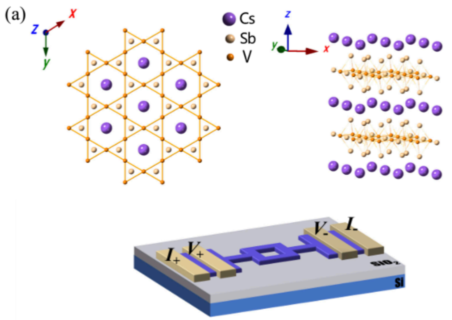
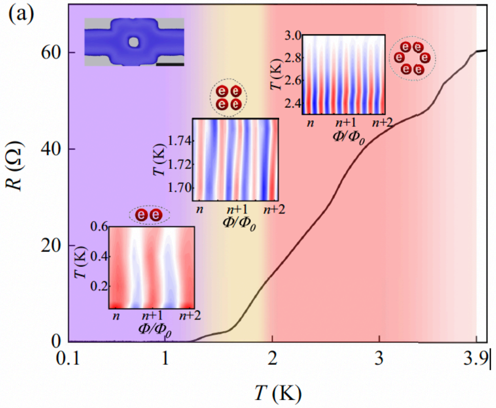
I. Shipulin, N. Stegani, **I. Maccari**, et al.,
Nature Communications **14**, 6734 (2023)



First experimental signatures in a Kagome SC ring device





Main experimental evidence:

- h/4e and h/6e quantization of magnetic flux in magnetoresistance oscillations.

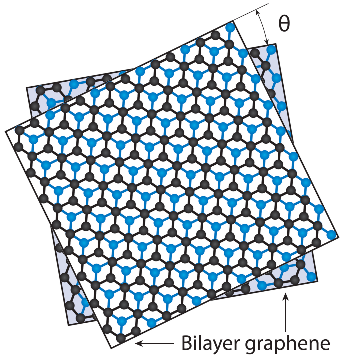
J. Ge et al. , Phys. Rev. X 14, 021025 (2024)

Hunting fermion quadruplets: the case of twisted bilayer graphene

I. Maccari , J. Carlström and E. Babaev, ***Phys. Rev. B 107 (6), 064501 (2023)***

MATBG promising candidate
to stabilize fermion quadrupling condensates

1. 2D $\longrightarrow$ more phase fluctuations
2. Single-band SC unconventional ground state (multicomponent)



Picture from:
X. Liu et al., Nature 583, 221 (2020)

Proposed pairing ad $n=-2$:
$d \pm id \rightarrow (\Delta_{E_1}, \Delta_{E_2})$

Key inter-component potential term

$$V(\Delta_{E_1}, \Delta_{E_2}) = 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1-\phi_2))-1\right]$$

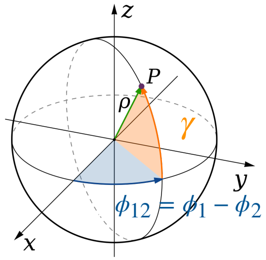
[D. V. Chichinadze, L. Classen, A. Chubokov, **PRB 101, 224513 (2020)**]

$$f = \frac{1}{2}(\vec{\nabla}\times\vec{A})^2 + \sum_{i=1,2}\left[\frac{1}{2}|(\vec{\nabla}-ie\vec{A})\Delta_i|^2\right] + V(\Delta_1,\Delta_2);$$

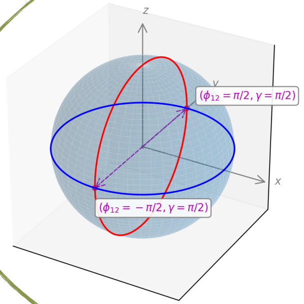
$$V(\Delta_1,\Delta_2) = 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1-\phi_2))-1\right]$$

$$\begin{cases}\Delta_1 = |\Delta_1|e^{i\phi_1};\\ \Delta_2 = |\Delta_2|e^{i\phi_2};\\ |\Delta_1|^2+|\Delta_2|^2 = \rho^2 = 1.\end{cases} \longrightarrow \begin{cases}\Delta_1 = |\cos(\frac{\gamma}{2})|e^{i\phi_1};\\ \Delta_2 = |\sin(\frac{\gamma}{2})|e^{i\phi_2};\end{cases}$$

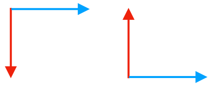
$$V(\Delta_1,\Delta_2) = \frac{K}{2}\sin^2(\gamma)\left[\cos(2(\phi_1-\phi_2))-1\right]$$



$K>0$



SC + BTRS

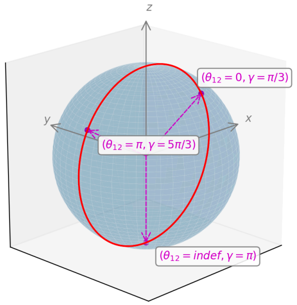


$U(1)\times Z_2$

$K<0$

Nematic SC

$U(1)\times Z_3$



$$V_6(\Delta_1,\Delta_2) = \frac{\lambda}{2}\left[(\Delta_1-i\Delta_2)^3(\Delta_1^*-i\Delta_2^*)^3 + c.c.\right]$$

To be added to lift the accidental degeneracy

Two complementary regions of parameter space describing a chiral ($K>0$) and a nematic ($K<0$) SC

Hunting fermion quadruplets: the case of twisted bilayer graphene

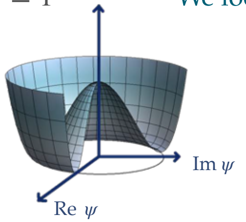
I. Maccari , J. Carlström and E. Babaev, ***Phys. Rev. B 107 (6), 064501 (2023)***

Large-scale Monte Carlo simulations $K > 0$ $\quad e = 0$

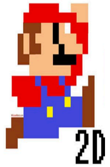
$$f = \frac{1}{2\rho^2}\left[|\Delta_1|^2\vec{\nabla}\phi_1 + |\Delta_2|^2\vec{\nabla}\phi_2\right]^2 + \frac{|\Delta_1|^2|\Delta_2|^2}{2\rho^2}\left[\vec{\nabla}(\phi_1-\phi_2)\right]^2 + \frac{1}{2}\left[(\vec{\nabla}|\Delta_1|)^2 + (\vec{\nabla}|\Delta_2|)^2\right] + 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1-\phi_2)) - 1\right]$$

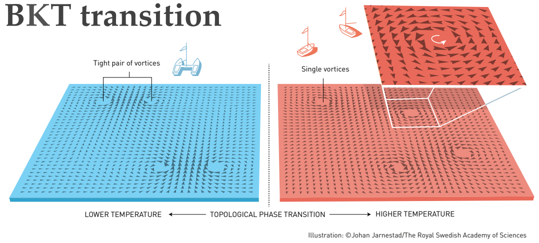
$\rho^2 = |\Delta_1|^2 + |\Delta_2|^2 = 1$

*We looked at the limit of very large penetration length, so we neglect the coupling to the vector potential



BKT transition

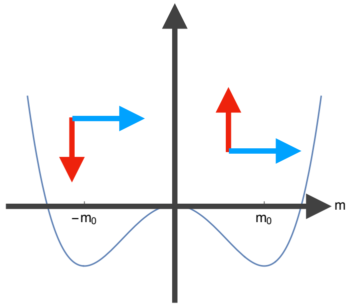




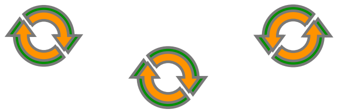
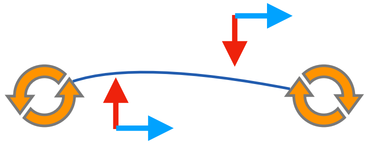
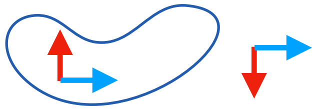
T_{BKT}

$K > 0$

Two possible chiralities: $\phi_1 - \phi_2 = \pm\dfrac{\pi}{2}$



T_{Z_2} **TRSB transition**

$T_{BKT} < T_c^{Z_2}$?	$T_{BKT} = T_c^{Z_2}$?	$T_c^{Z_2} < T_{BKT}$?
SC vortices	Fractional vortices + DW string	Z_2 DW
		
Trigger BKT transition	Trigger BKT **and** Z_2 transition	Trigger Z_2 transition

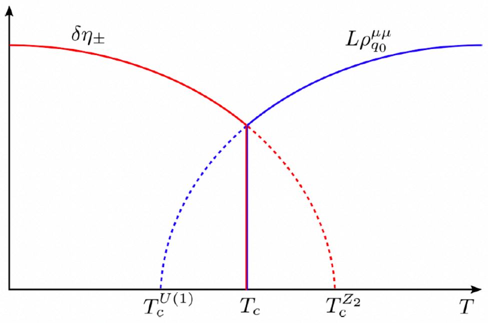
Stabilising fermion quadruplets is not easy…

Chiral and nematic SC in 3D

- ✳ "Absence of Ginzburg-Landau mechanism for vestigial order in the normal phase above a two-component superconductor"
  P. T. How et al., Phys. Rev. B 107, 104514 (2023)

- ✳ "First order SC phase transition in a chiral p+ip system"
  H. H. Haugen et al., Phys. Rev. B 104, 104515 (2021).

Preemptive first-order phase transition



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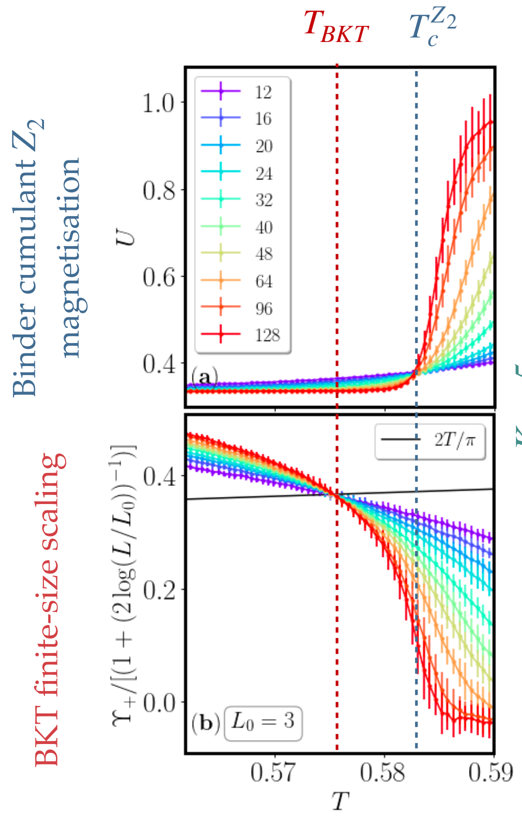
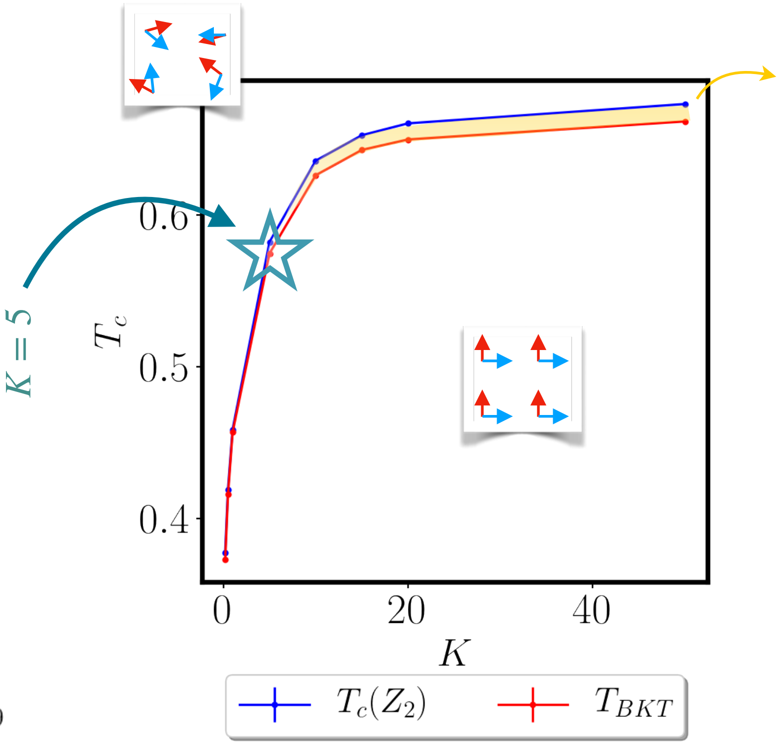
$$f = \frac{1}{2\rho^2}\left[|\Delta_1|^2\vec{\nabla}\phi_1 + |\Delta_2|^2\vec{\nabla}\phi_2\right]^2 + \frac{|\Delta_1|^2|\Delta_2|^2}{2\rho^2}\left[\vec{\nabla}(\phi_1-\phi_2)\right]^2 + \frac{1}{2}\left[(\vec{\nabla}|\Delta_1|)^2 + (\vec{\nabla}|\Delta_2|)^2\right] + 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1-\phi_2))-1\right]$$

$\rho^2 = |\Delta_1|^2 + |\Delta_2|^2 = 1$

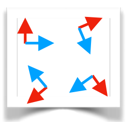
*We looked at the limit of very large penetration length, so we neglect the coupling to the vector potential

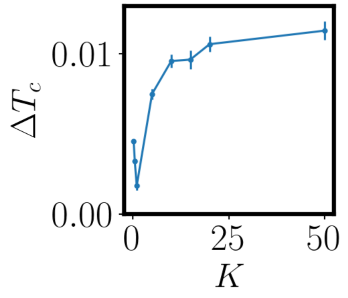




$\langle\psi_{1,2}\rangle = 0$

$\langle\psi_1\psi_2^* + \psi_1^*\psi_2\rangle \neq 0$



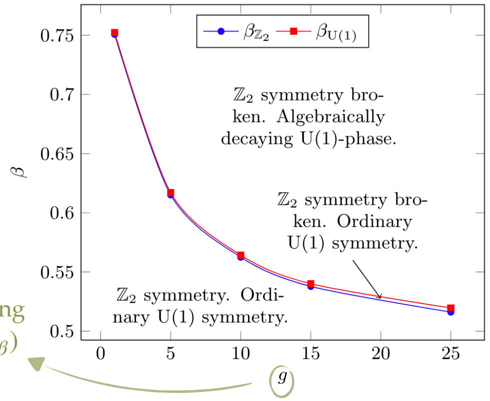


$\Delta T_c = T_c^{Z_2} - T_{BKT}$

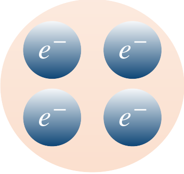
In other models this phase was found only with very strong Josephson coupling

- 2D three-band model with phase frustration

Josephson coupling $\propto g\cos(\phi_\alpha - \phi_\beta)$



T. A. Bojesen, E. Babaev, and A. Sudbø, Phys. Rev. B 88, 220511 (R) (2013).



Fermionic quadrupling condensate for all finite values of K considered!!

Promising candidate to observe fermion quadruplets above T_c!

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$\rho^2 = |\Delta_1|^2 + |\Delta_2|^2 = 1$

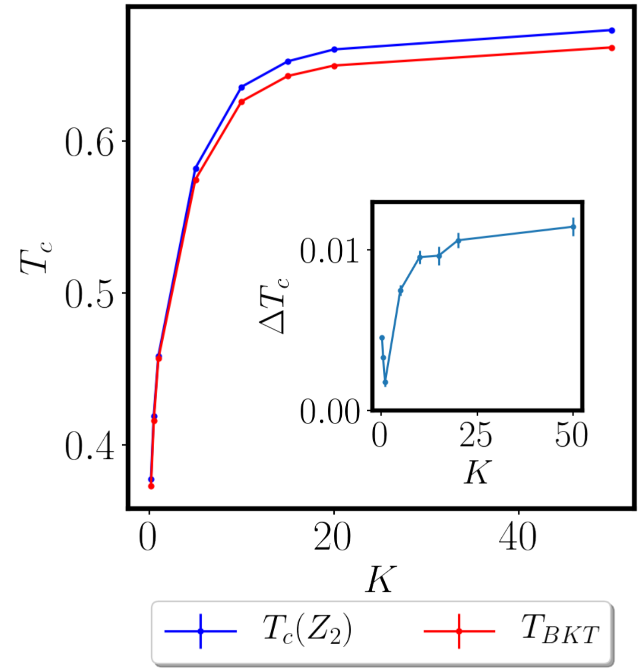
*We looked at the limit of very large penetration length, so we neglect the coupling to the vector potential

Great, but…

- Why do vortices proliferate always at lower temperatures than domain walls?
- At very small values of K, finite-size effects become prominent, are the two transition still split apart in the thermodynamic limit?
- What is the role of relative density fluctuations?

RG approaches could be crucial to shed light on these main questions

- Related, but different model previously studied via RG:
  G. Bighin et al, PRL 123, 100601 (2019)
  Coupled XY models studied with an interlayer coupling $\propto \cos(\phi_1 - \phi_2)$



Working in progress with
Benjamin Liégeois,
Chitra Ramasubramanian,
and Nicolò Defenu

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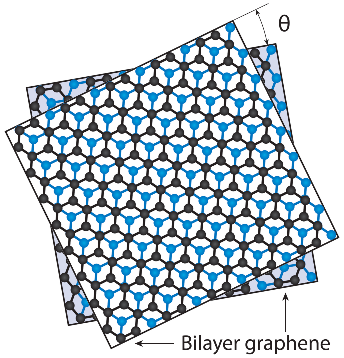
I. Maccari , J. Carlström and E. Babaev, ***Phys. Rev. B 107 (6), 064501 (2023)***

I. Maccari , J. Carlström and E. Babaev, ***In preparation* (2024)**

MATBG promising candidate
to stabilize fermion quadrupling condensates

1. 2D $\longrightarrow$ more phase fluctuations
2. Single-band SC unconventional ground state (multicomponent)



Picture from:
X. Liu et al., Nature 583, 221 (2020)

Proposed pairing ad $n = -2$:

$d \pm id \rightarrow (\Delta_{E_1}, \Delta_{E_2})$

Key inter-component potential term

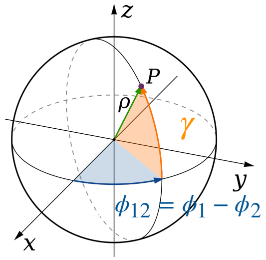
$$V(\Delta_{E_1}, \Delta_{E_2}) = 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1 - \phi_2)) - 1\right]$$

[D. V. Chichinadze, L. Classen, A. Chubokov, **PRB 101, 224513 (2020)**]

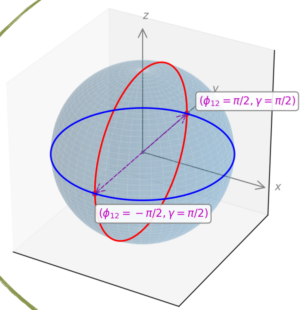
$$f = \frac{1}{2}(\vec{\nabla} \times \vec{A})^2 + \sum_{i=1,2}\left[\frac{1}{2}|(\vec{\nabla} - ie\vec{A})\Delta_i|^2\right] + V(\Delta_1, \Delta_2); \quad V(\Delta_1, \Delta_2) = 2K|\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1 - \phi_2)) - 1\right]$$

$$\begin{cases} \Delta_1 = |\Delta_1|e^{i\phi_1}; \\ \Delta_2 = |\Delta_2|e^{i\phi_2}; \\ |\Delta_1|^2 + |\Delta_2|^2 = \rho^2. \end{cases} \longrightarrow \begin{cases} \Delta_1 = |\cos(\frac{\gamma}{2})|e^{i\phi_1}; \\ \Delta_2 = |\sin(\frac{\gamma}{2})|e^{i\phi_2}; \end{cases}$$

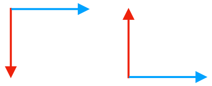
$$V(\Delta_1, \Delta_2) = \frac{K}{2}\sin^2(\gamma)\left[\cos(2(\phi_1 - \phi_2)) - 1\right]$$



$K > 0$



SC + BTRS

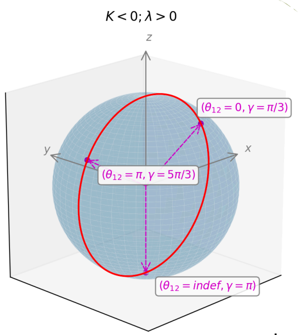


$U(1) \times Z_2$

$K < 0$

Nematic SC

$U(1) \times Z_3$



$$V_6(\Delta_1, \Delta_2) = \frac{\lambda}{2}\left[(\Delta_1 - i\Delta_2)^3(\Delta_1^* - i\Delta_2^*)^3 + c.c.\right]$$

To be added to lift the accidental degeneracy

Two complementary regions of parameter space describing a chiral (K>0) and a nematic (K<0) SC

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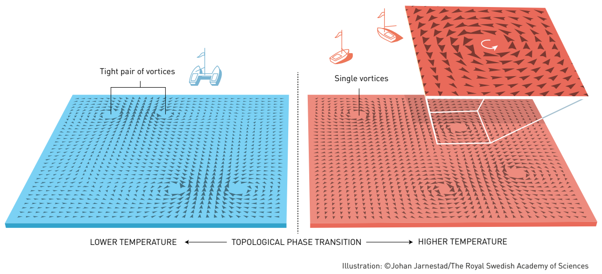
$$-2|K||\Delta_1|^2|\Delta_2|^2\left[\cos(2(\phi_1-\phi_2)) - 1\right] + \lambda\left[\cos(3\gamma) + 3\cos(\gamma)\sin^2(\gamma)\sin^2((\phi_1-\phi_2))\right]$$

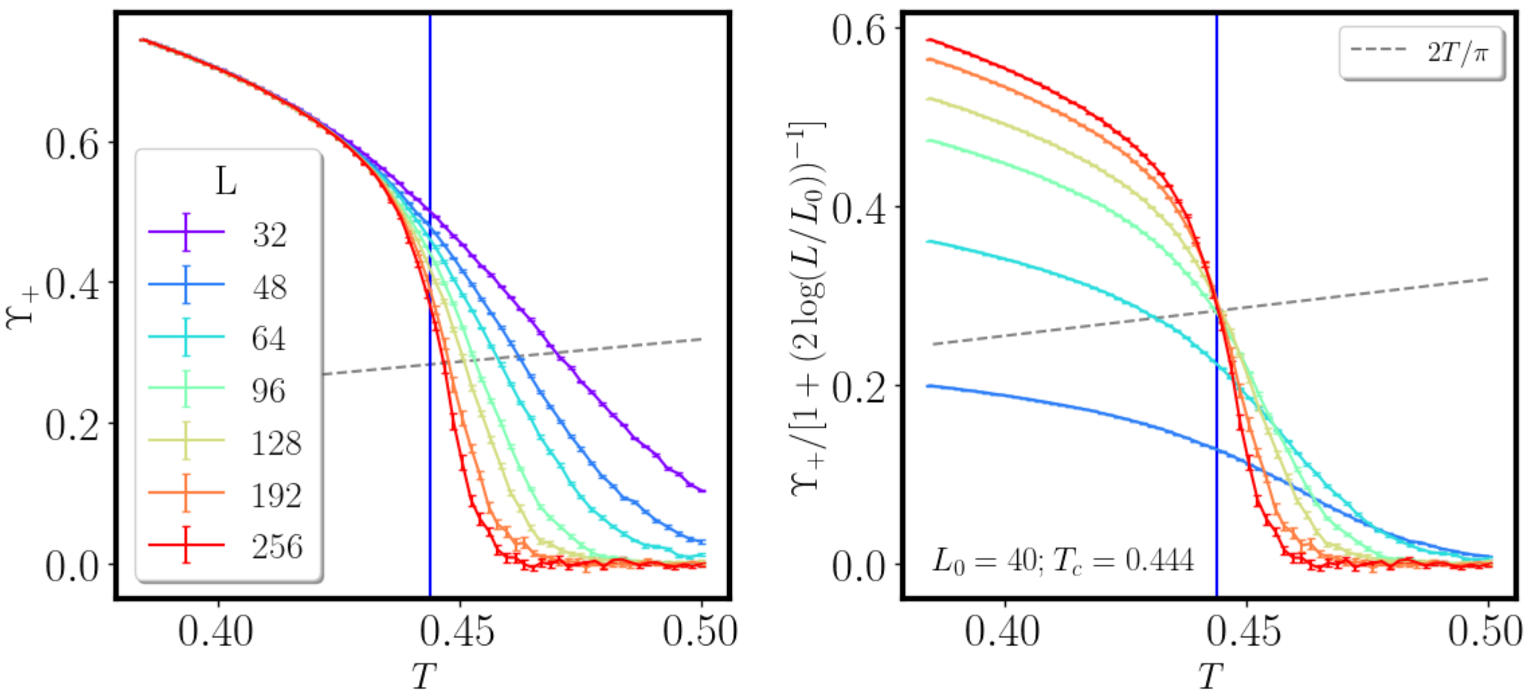
$$\gamma = 2\arctan(|\Delta_2|/|\Delta_1|)$$

BKT transition



T_{BKT}

$K = -1; \lambda = 1$



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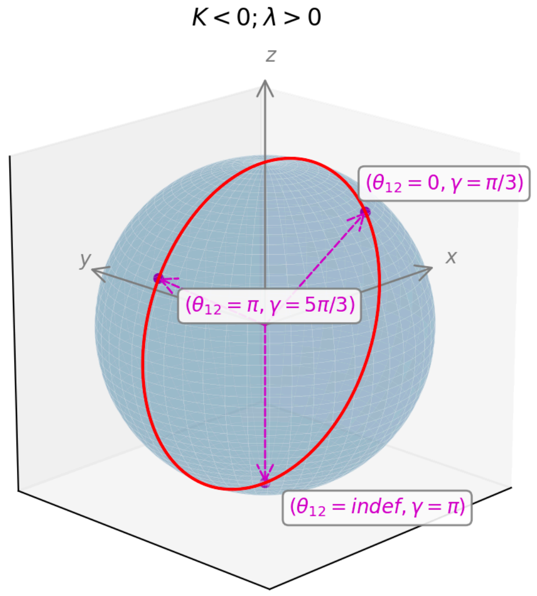
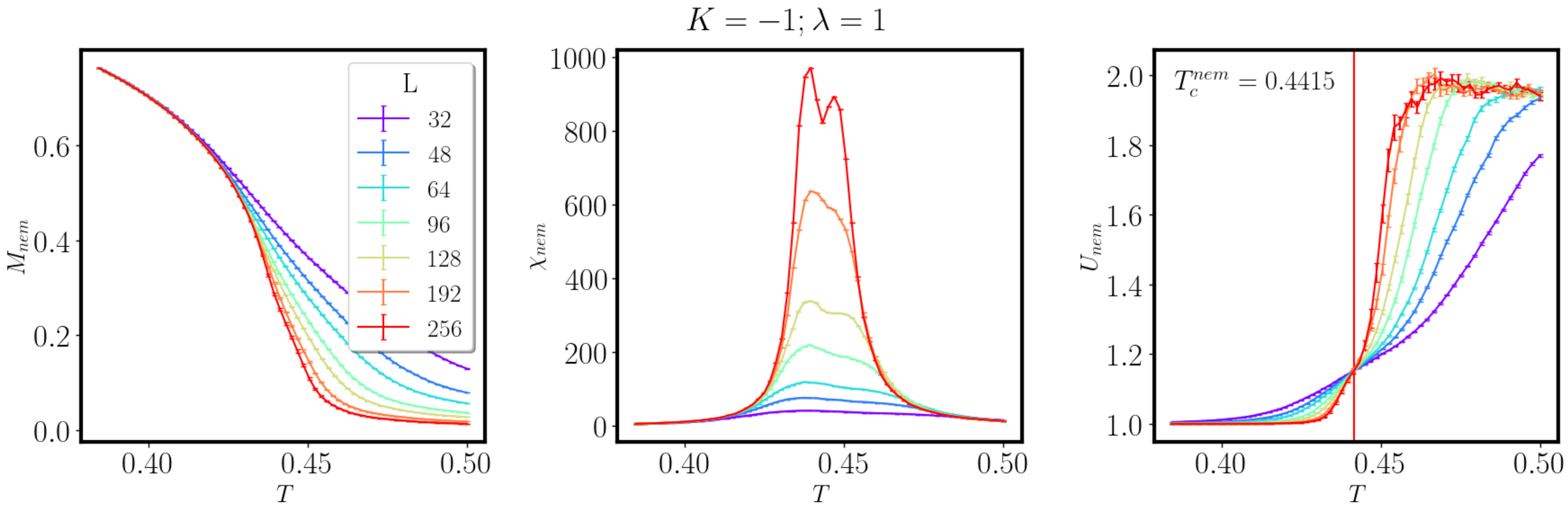
$$\gamma = 2\arctan(|\Delta_2|/|\Delta_1|)$$

Nematic Z_3 phase transition

$$M^2_{nematic} = \frac{1}{N^2}\left[\left(\sum_i \sin(\gamma_i)\cos(\theta_{12,i})\right)^2 + \left(\sum_i \sin(\gamma_i)\sin(\theta_{12,i})\right)^2 + \left(\sum_i \cos(\gamma_i)\right)^2\right]$$





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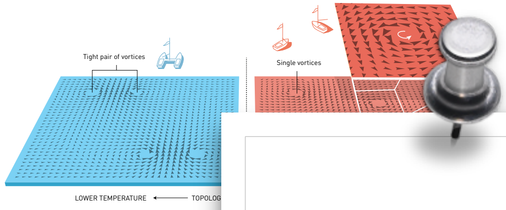
BKT transition

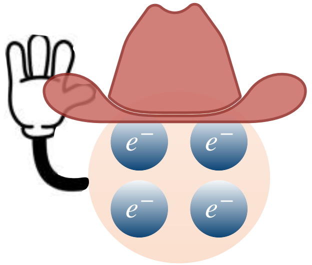


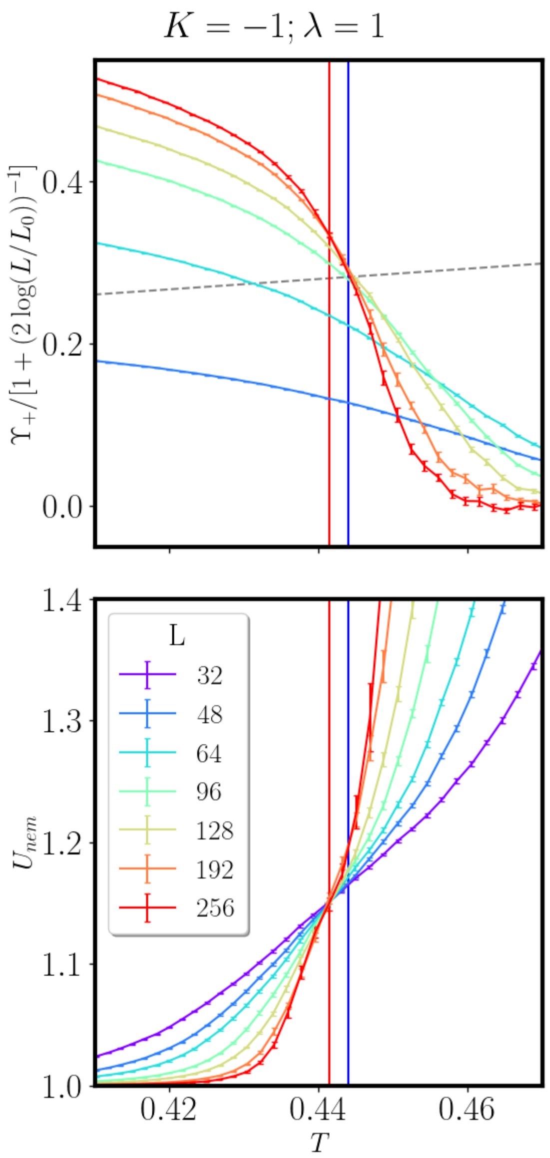
Nematic Z_3 phas
transition

These preliminary results seem to suggest:

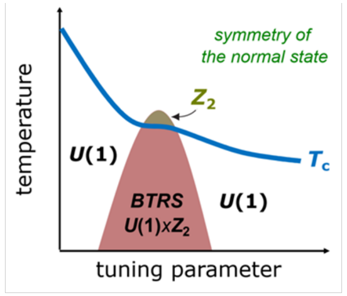
$$T_{BKT} > T_c^{Z_3}$$

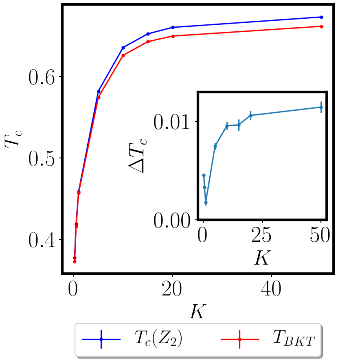
Charge-4e SC phase?





Conclusions and future directions





- Fermion quadrupling condensates are novel fascinating states of matter so far observed only in two real materials.
  - ★ V. Grinenko, D. Weston, F. Caglieris, C. Wuttke, C. Hess, T. Gottschall, **I. Maccari**, et al., **Nature Physics 17, 1254–1259 (2021)**
  - ★ I. Maccari and E. Babaev, **Phys. Rev. B 105, 214520 (2022)**
  - ★ I. Shipulin, N. Stegani, **I. Maccari**, et al., **Nature Communications 14, 6734 (2023)**
- Twisted-bilayer graphene may be a promising candidate for the observation of fermion quadruplets above T_c
  - ★ **I. Maccari** , J. Carlström and E. Babaev, **Phys. Rev. B 107 (6), 064501 (2023)**
  - ★ **I. Maccari** , J. Carlström and E. Babaev, **In preparation (2024)**

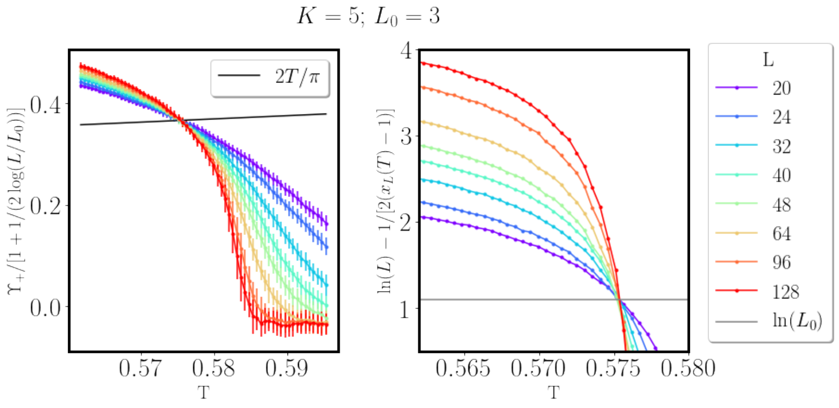
➡How to engineer materials in such a way as to observe higher order fermionic condensates: what symmetries are important? what interactions?

➡How to manipulate the nucleation of specific topological excitations?

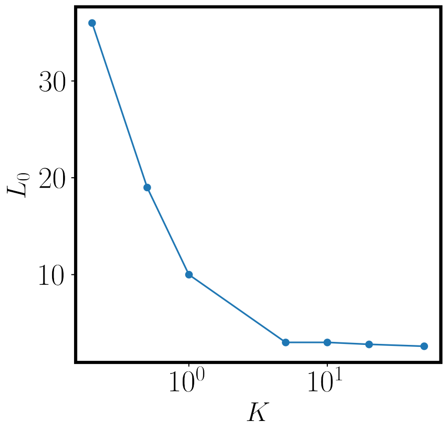
➡What are the new emergent properties of these states?

Thank you for your attention!

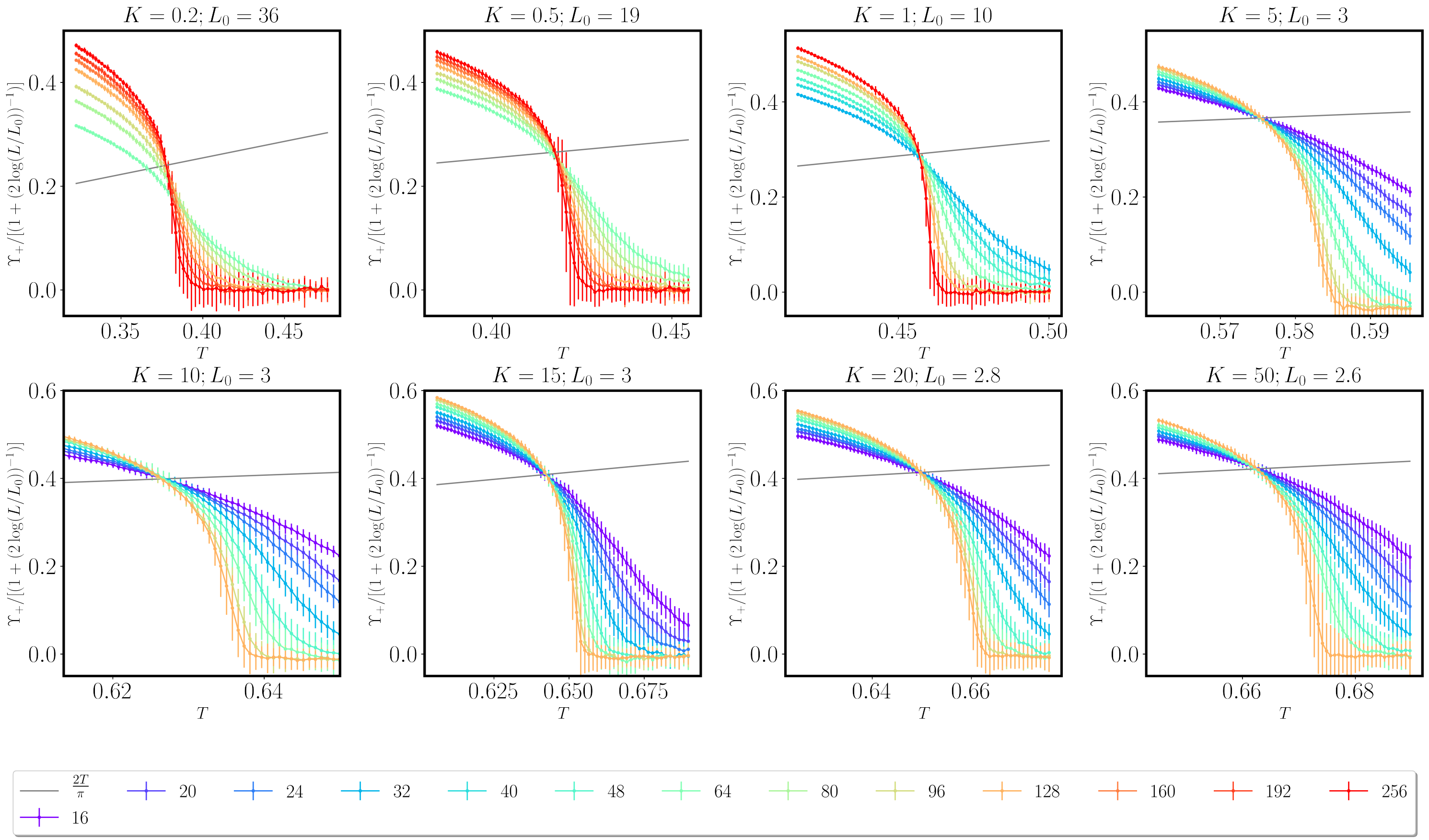
Backup slides



$$\Upsilon_+(\infty, T_{BKT}) = \frac{\Upsilon_+(L, T_{BKT})}{1 + (2\log(L/L_0))^{-1}},$$



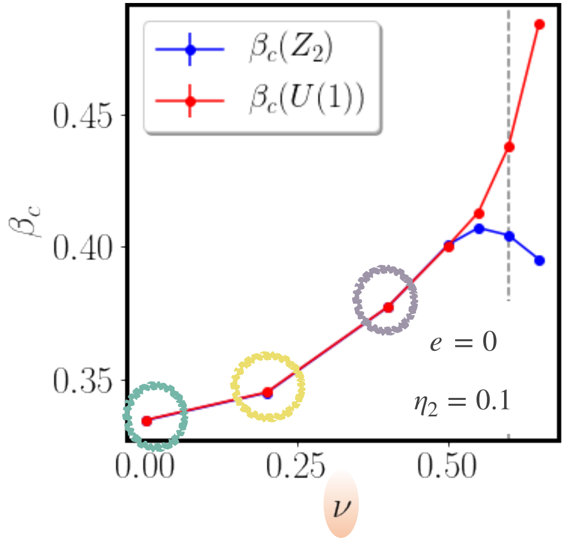
I. Maccari , J. Carlström and E. Babaev, ***Phys. Rev. B 107 (6), 064501 (2023)***



Backup slides

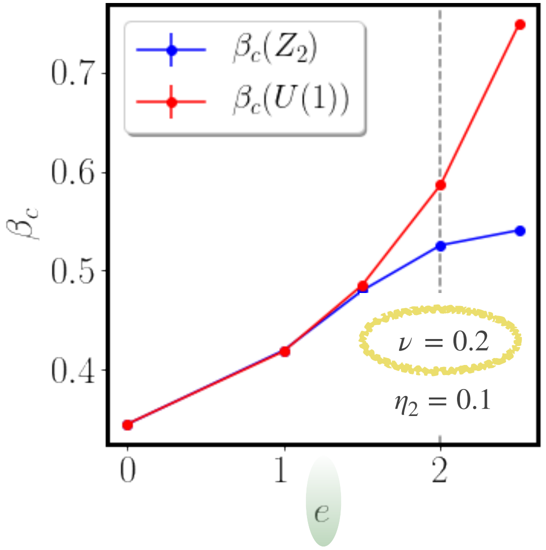
I. Maccari and E. Babaev **Phys. Rev. B 105, 214520 (2022)**

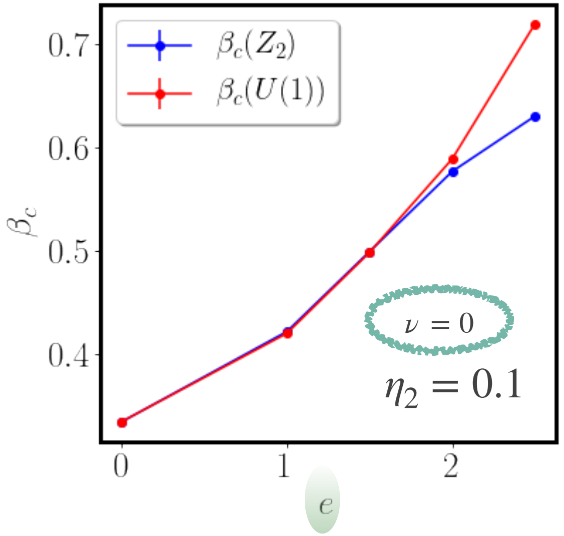
$$\rho_1 = \rho_2 = 1 \qquad f = \frac{1-\nu}{4}\left(\nabla\phi_1 + \nabla\phi_2 - 2e\mathbf{A}\right)^2 + \frac{1+\nu}{4}\left(\nabla\phi_1 - \nabla\phi_2\right)^2 + \eta_2 \cos[2(\phi_1 - \phi_2)] + \frac{1}{2}(\nabla\times\mathbf{A})^2$$



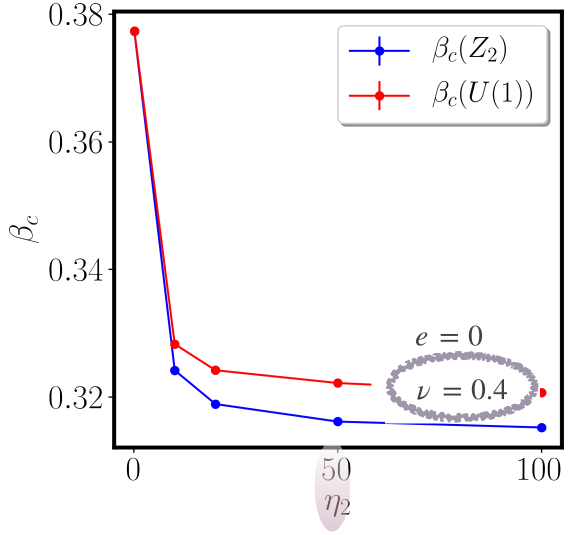
It decreases the relative cost of phase sum fluctuations

It decreases the cost of (1,1) vortices



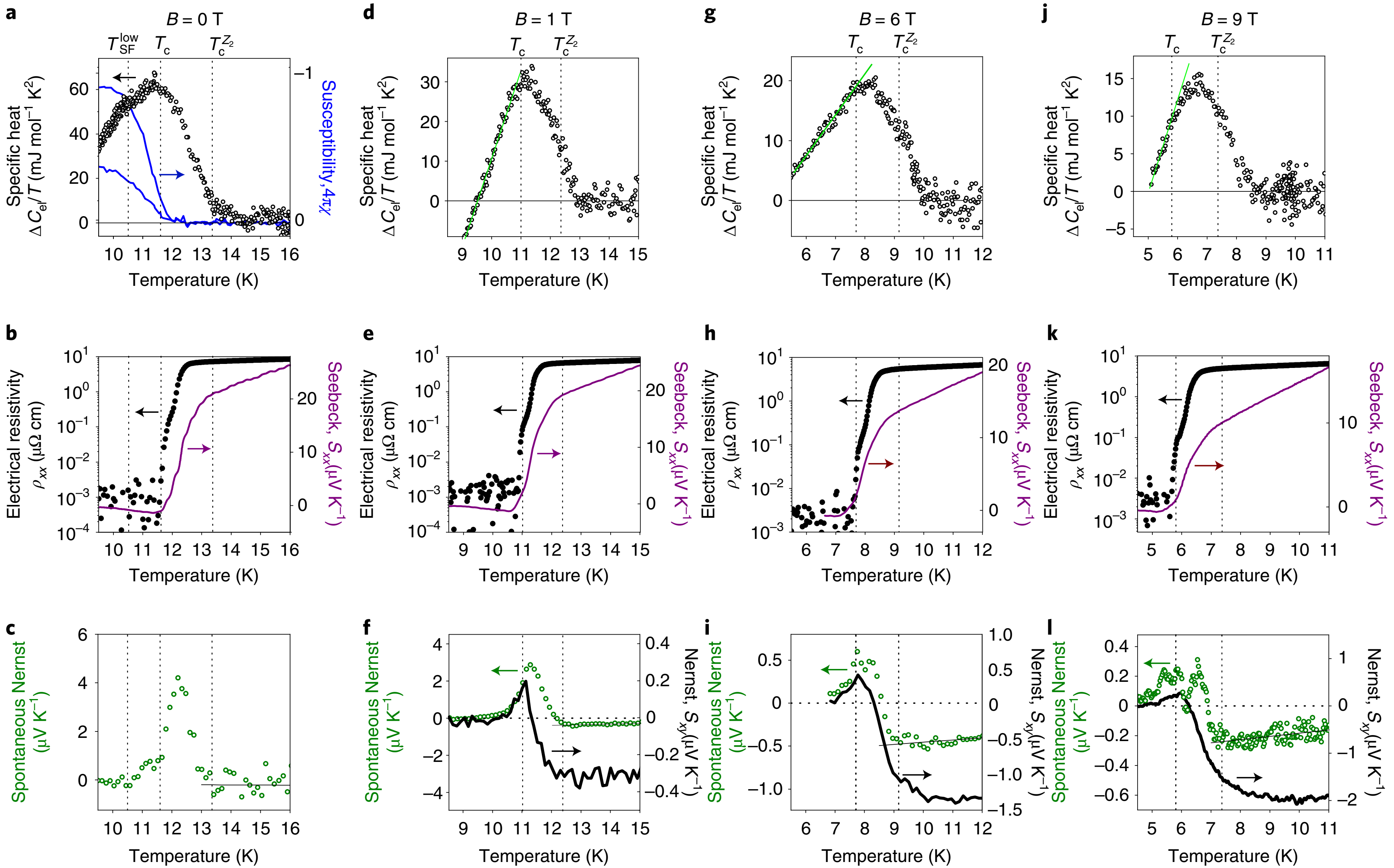


It increases the cost of domain walls



Backup slides

V. Grinenko, et al., *Nat. Phys.* 17, 1254 - 1259 (2021).



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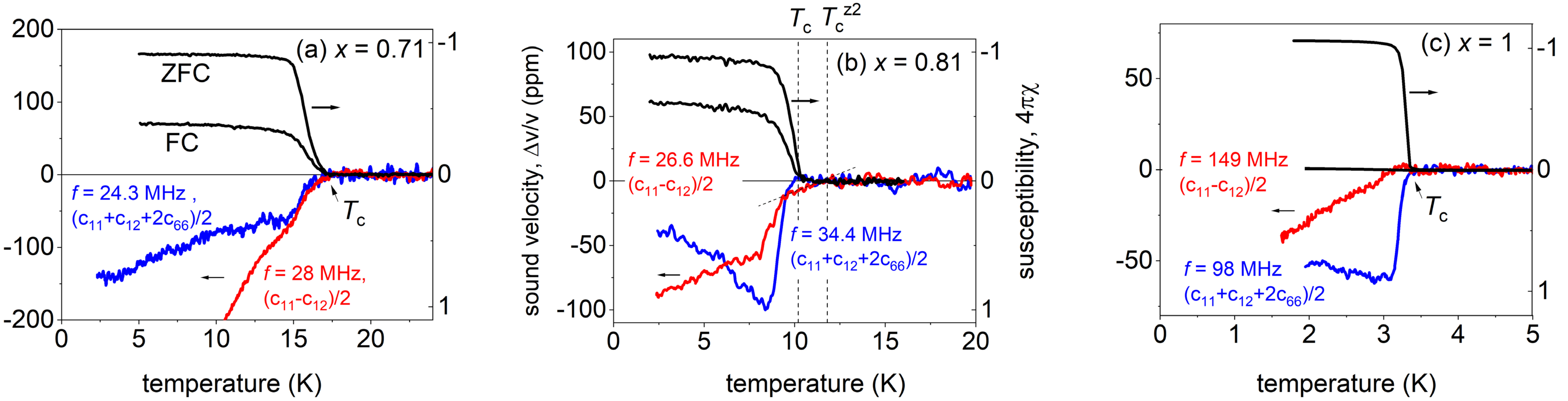
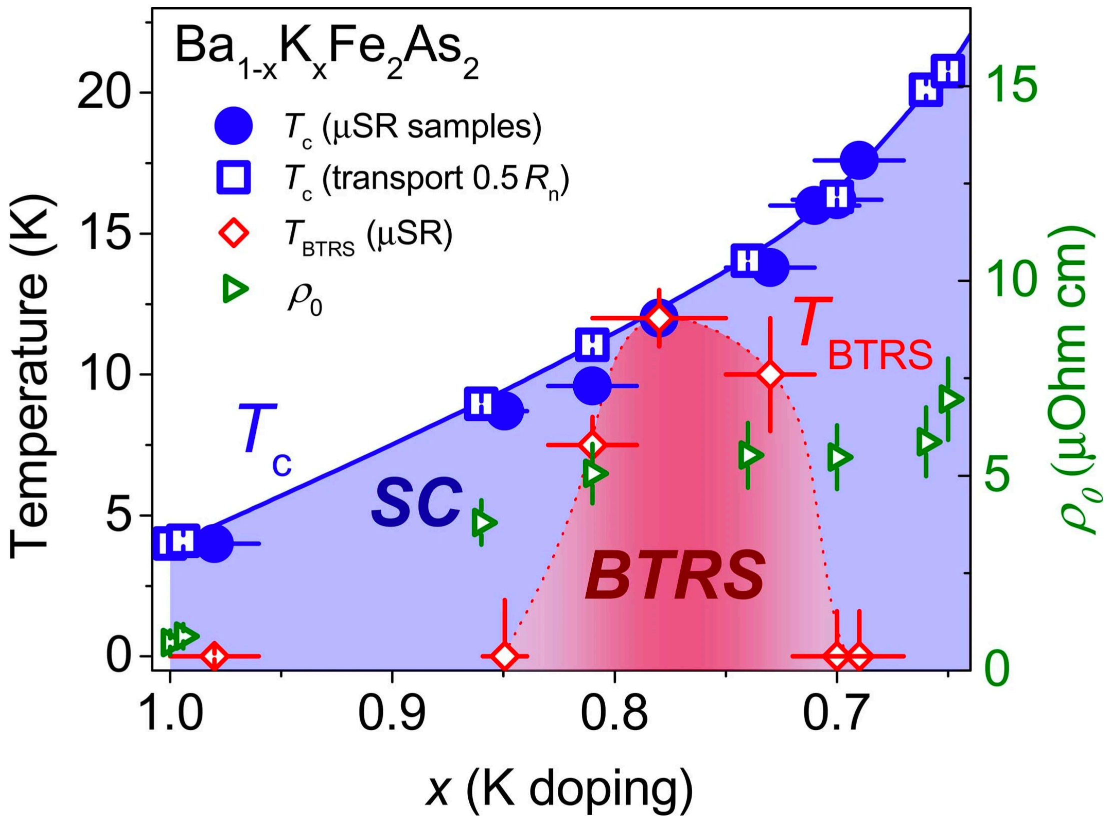
V. Grinenko, et al., *Nat. Phys.* 17, 1254 - 1259 (2021).



Extended Data Fig. 6 | Ultrasound data. Temperature dependence of the relative change of the sound velocity (left) for the longitudinal $(c_{11}+c_{12}+2c_{66})/2$ and transverse $(c_{11}-c_{12})/2$ acoustic modes with subtracted background (see Fig. S3 in the supplementary information) compared with temperature dependence of the magnetic susceptibility in $B||ab=0.5$ mT (right) measured after cooling in zero magnetic field (ZFC) and cooling in the applied field (FC). The data for the samples with $x=0.71$, 0.81, and 1 are shown in panels **(a)**, **(b)**, and **(c)**, respectively. The sample with $x=0.81$ exhibits a kink in the velocity of the transverse acoustic mode at a position which agrees with the onset of the broad feature in the specific heat shown in Extended Data Fig. 8 for the same sample. The position of the kink agrees with the position of the Z_2 transition indicated by the thermal transport experiments for the sample with $x\approx0.8$ (Fig. 2 in the main text).

Backup slides

V. Grinenko, et al., *Nat. Phys.* 17, 1254 - 1259 (2021).



The residual resistivity ρ_0 very weakly dependent on doping

$$\rho(T) = \rho_0 + A_{FL}T^2$$

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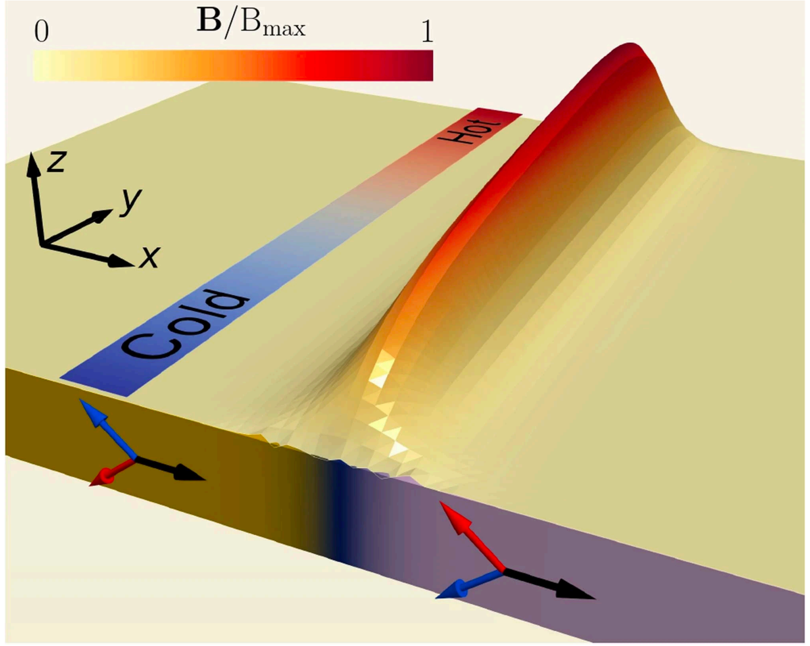
$$F = \frac{\mathbf{J}^2}{2e^2\varrho^2} + \frac{1}{2}\mathbf{B}^2 + \sum_i \frac{1}{2}(\nabla|\psi_i|)^2 + a_i|\psi_i|^2 + \frac{b_i}{2}|\psi_i|^4 + \sum_{i<j} \frac{|\psi_i|^2|\psi_j|^2}{\rho^2}\left(\frac{[\nabla(\phi_i-\phi_j)]^2}{2} + \frac{\eta_{ij}\rho^2\cos(\phi_i-\phi_j)}{|\psi_i||\psi_j|}\right).$$

$$\mathbf{J} \equiv \sum_i e\ \mathrm{Im}\left(\psi_i^* D\psi_i\right)$$

$$B_k = \epsilon_{lmk}\partial_l A_m = \epsilon_{lmk}\partial_l\left(\frac{J_m}{e^2|\Psi|^2}\right) + \epsilon_{lmk}\frac{i}{e|\Psi|^4}\left[|\Psi|^2\partial_l\Psi^\dagger\partial_m\Psi + \Psi^\dagger\partial_l\Psi\partial_m\Psi^\dagger\Psi\right]$$

Spontaneous Magnetic Fields



[J. Garaud and E. Babaev., **PRL 129, 087602 (2022)**]

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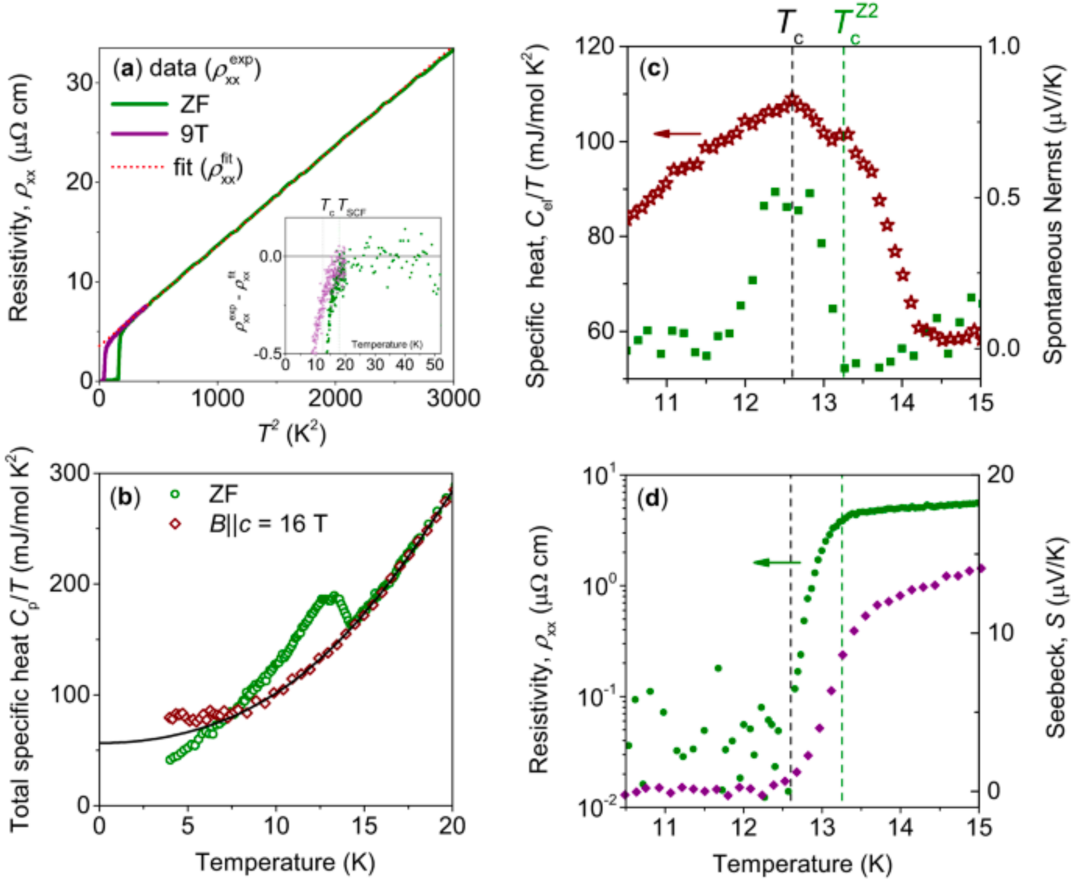
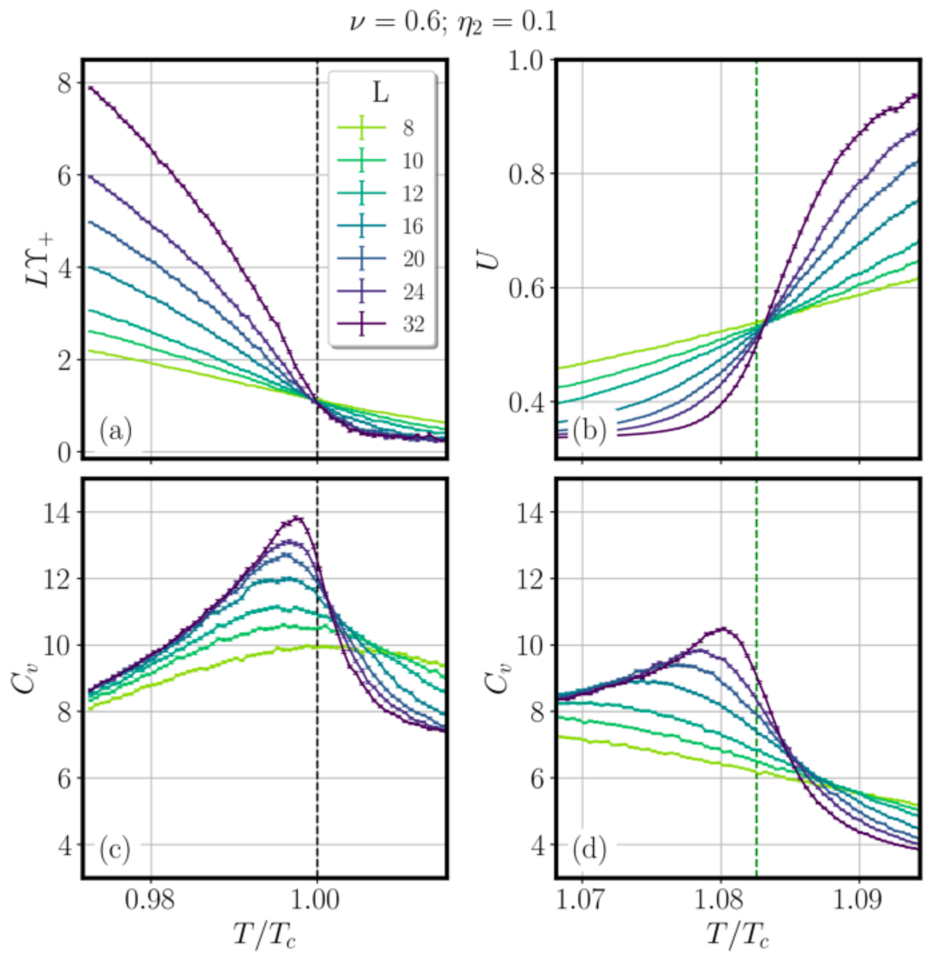
I. Shipulin, N. Stegani, **IM** et al. *Nat. Comm. 14, 6734 (2023)*





No CDW evidences from STM data

$T = 300$ mK

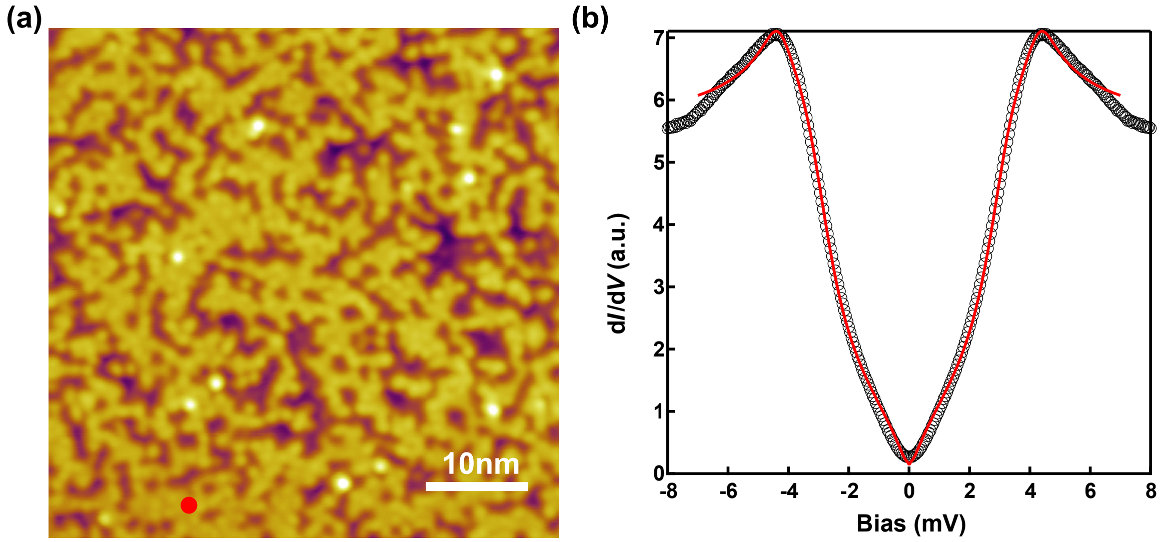
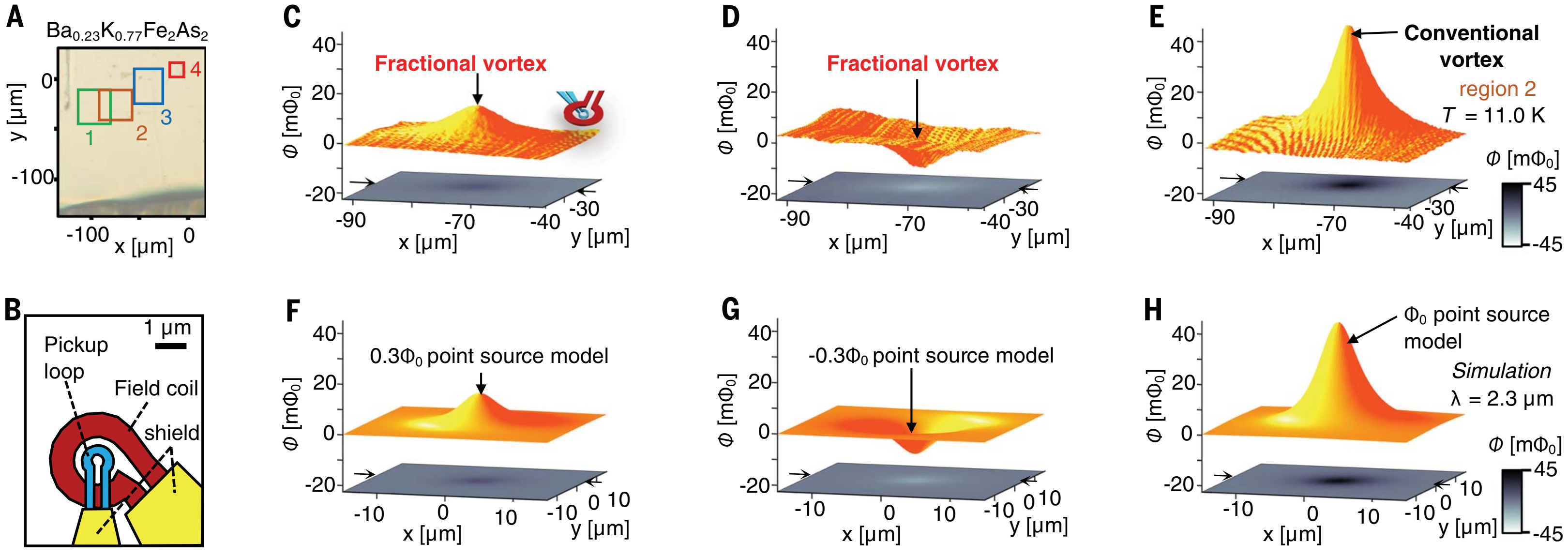


Figure S3. STM data for the sample S_{NP} **(a)** STM Topograph recorded from the disordered surface of a $Ba_{1-x}K_xFe_2As_2$ sample with $x = 0.77$ (image size: $(50 \times 50)\mathrm{nm}^2$, $V_b = 80\mathrm{mV}$, $I_t = 100\mathrm{pA}$). The disordered surface was likely caused by the mixture of Ba atoms and K atoms.**(b)** $\mathrm{d}I/\mathrm{d}V - V$ spectrum taken from a defect-free position (the red point) on the surface shown in (**a**) (Spectroscopic set-point: $V_s = 10\mathrm{mV}$, $I_s = 200\mathrm{pA}$, amplitude of bias modulation used $V_{\mathrm{mod}} = 0.25\mathrm{mV}$), showing a "V"-shaped superconducting gap. The black circle was experiment result. Red solid curve was fitting result using double-gap Dynes equation with a larger gap ($\Delta_2 = 3.1\mathrm{meV}$) and a smaller gap with node($\Delta_1 = 2.1\mathrm{meV}$). There is no "CDW" gap feature in the spectrum.

I. Shipulin, N. Stegani, **IM** et al.
Nat. Comm. 14, 6734 (2023)

SC vortices carrying a temperature-dependent fractional quantum flux

Yusuke Iguchi et al. **Science 380**, 1244 - 1247 (2023)



$T = 11\ \mathrm{K} < T_C$

Observation of vortices carrying fractional quantum fluxes

$$\oint \vec{J}_s \vec{dl} = -e\rho^2 \oint \left(\hbar \frac{|\psi_1|^2}{\rho^2} \vec{\nabla}\phi_1 - \frac{2e}{c}\vec{A} \right) \vec{dl} = 0$$

$$\hbar \frac{|\psi_1|^2}{\rho^2} 2\pi N = \frac{2e}{c}\Phi \implies \Phi = \Phi_0 \frac{|\psi_1|^2}{\rho^2} N$$

The fractional quantum flux shows a temperature dependence, that cannot be explained just in terms of $\lambda_L(T)$. Decreasing the temperature also the fractional quantum flux decreases.