Asymptotic Safety in Generalized Proca Theories

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Outline

1) Generalized Proca Theories (GPTs);
2) Asymptotic Safety in GPTs;
3) Setup;
4) Results;
5) Conclusions.

Generalized Proca Theories

- Motivations:
  - *Cosmology* $\longrightarrow$ *Dynamical Dark Energy* [1];
- Derivative self-interactions of the vector field are included in the Lagrangian in flat spacetime [2] to have:
  - *Second order equation of motion;*
  - *Longitudinal mode as a Galileon.*
- Constraints from:
  - *Data (e.g.* [3]*);*
  - *EFT constraints (e.g.* [4]*).*

[1] L. Heisenberg, "A systematic approach to generalisations of General Relativity and their cosmological implications", Physics Reports Volume 796, 15 March 2019, Pages 1-113.
[2] L. Heisenberg, "Generalization of the Proca Action", JCAP 1405 (2014) 015.
[3] A. De Felice et al., " Observational constraints on generalized Proca theories ", Phys. Rev. D 93, 104016 (2016).
[4] C. de Rham et al., "Positivity bounds in vector theories", JHEP12 (2022) 086.

Generalized Proca Theories

- Galileon Theory [1]:
  - *Higher order derivatives in the scalar field;*
  - *Invariance under Galilean transformations and shift;*
  - *Second order equation of motion.*

$$\mathcal{L}_{\text{Gal}} = \sum_{i=1}^{5} c_i \mathcal{L}_i \quad (1)$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= (\partial\pi)^2 \\
\mathcal{L}_3 &= (\partial\pi)^2 \Box\pi \\
\mathcal{L}_4 &= (\partial\pi)^2 \left[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right] \\
\mathcal{L}_5 &= (\partial\pi)^2 \left[(\Box\pi)^3 - 3\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3\right]
\end{aligned} \quad (2)$$

[1] L. Heisenberg, "A systematic approach to generalisations of General Relativity and their cosmological implications", Physics Reports Volume 796, 15 March 2019, Pages 1-113.

Generalized Proca Theories

- Generalized Proca Theories (flat spacetime) [2]:

$$\mathcal{L}_{\text{gen.Proca}} = -\frac{1}{4} F_{\mu\nu}^2 + \sum_{n=2}^{5} \alpha_n \mathcal{L}_n \tag{3}$$

$$\begin{aligned}
\mathcal{L}_2 &= f_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu}) \\
\mathcal{L}_3 &= f_3(A^2) \;\; \partial \cdot A \\
\mathcal{L}_4 &= f_4(A^2) \;\; \left[(\partial \cdot A)^2 - \partial_\rho A_\sigma \partial^\sigma A^\rho\right] + c_2 \tilde{f}_4(A^2) F^2 \\
\mathcal{L}_5 &= f_5(A^2) \;\; \left[(\partial \cdot A)^3 - 3(\partial \cdot A)\partial_\rho A_\sigma \partial^\sigma A^\rho + 2\partial_\rho A_\sigma \partial^\gamma A^\rho \partial^\sigma A_\gamma\right] \\
&\quad + d_2 \tilde{f}_5(A^2) \tilde{F}^{\mu\alpha} \tilde{F}^\nu_\alpha \partial_\mu A_\nu \\
\mathcal{L}_6 &= e_2 f_6(A^2) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \partial_\alpha A_\mu \partial_\beta A_\nu \, ,
\end{aligned} \tag{4}$$

[2] L. Heisenberg, ”Generalization of the Proca Action”, JCAP 1405 (2014) 015.

Generalized Proca Theories

- Partial derivatives (flat spacetime) ⟶ Covariant derivatives (curved spacetime) + non-minimal couplings [2];
- Longitudinal mode ⟶ Hordenski interactions.

$$\mathcal{L}^{\text{curved}}_{\text{gen.Proca}} = -\frac{1}{4}F^2_{\mu\nu} + \sum_{n=2}^{5} \beta_n \mathcal{L}_n \tag{5}$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu}) \\
\mathcal{L}_3 &= G_3(X)\nabla_\mu A^\mu \\
\mathcal{L}_4 &= G_4(X)R + G_{4,X}\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right] \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}\Big[(\nabla \cdot A)^3 \\
&\quad + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3(\nabla \cdot A)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\Big] \\
&\quad - g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta \\
\mathcal{L}_6 &= G_6(X)\mathcal{L}^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,X}}{2}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu
\end{aligned} \tag{6}$$

with $X = -A_\mu A^\mu/2$

[2] L. Heisenberg, ”Generalization of the Proca Action”, JCAP 1405 (2014) 015.

Asymptotic Safety of GPTs

- GPTs lack constraints from fundamental physics;
  - *Idea: Constrain Wilson coefficients from fundamental physics ↔ Determine the AS landscape [5,6].*
- Asymptotic Safety for GPTs:
  1. *Investigation of the possible UV completion of GPTs;*
  2. *Study the AS landscape from the FP.*
- Important for:
  - ***Positivity bounds**;*
  - *Cosmological applications.*

[5] I. Basile, A. Platania, "Asymptotic Safety: Swampland or Wonderland?", Universe 2021, 7, 389.
[6] B. Knorr, A. Platania, " Unearthing the intersections: positivity bounds, weak gravity conjecture, and asymptotic safety landscapes from photon-graviton flows", arXiv:2405.08860.

Setup

- Truncation:

$$\mathcal{L} = \frac{1}{16\pi G}\left[2\Lambda - R\right] - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\,G_2\,A^2 + G_{4,1}\left[A^2\,R + (\nabla_\mu A^\mu)^2 - (\nabla_\mu A_\nu)(\nabla^\nu A^\mu)\right]$$
$$+ G_{4,2}\left[A^4\,R + 2\,A^2\,(\nabla_\mu A^\mu)^2 - 2\,A^2(\nabla_\mu A_\nu)(\nabla^\nu A^\mu)\right] \quad (7)$$

- Minimal setup to compare with positivity bounds [4] (non-minimal couplings, 4th order in *A*), but minimizing difficulties:
  - ***Background field method*** *(linear split);*
  - ***Background field approximation*** *(Einstein manifold, constant Proca field);*
  - ***Harmonic gauge***.

[4] C. de Rham et al., ”Positivity bounds in vector theories”, JHEP12 (2022) 086.

Results

Einstein-Hilbert

$$\mathcal{L}_{EH} = \frac{1}{16\pi G}\left[2\Lambda - R\right]$$

Fixed Point		Critical Exponents	
g^*	λ^*	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$
0.645	0.194	$2.413 - i\, 2.612$	$2.413 + i\, 2.612$

Table 1: NGFP for A^0 truncation in Generalized Proca Theories and corresponding critical exponents.

Einstein-Hilbert + free Proca field

$$\mathcal{L}_{EH+G_2A^2} = \frac{1}{16\pi G}\left[2\Lambda - R\right] - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}G_2\, A^2$$

Fixed Points			Critical Exponents		
g^*	λ^*	g_2^*	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$
0.746	0.159	0.0457	$2.180 - i\, 2.153$	$2.180 + i\, 2.153$	1.041

Table 2: NGFP for Einstein-Hilbert plus free Proca field theory and corresponding critical exponents.

Results

- **GPT – second order**

$$\mathcal{L}_2 = \frac{1}{16\pi G}\left[2\Lambda - R\right] - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}G_2\,A^2 + G_{4,1}\left[A^2\,R + (\nabla_\mu A^\mu)^2 - (\nabla_\mu A_\nu)(\nabla^\nu A^\mu)\right]$$

Fixed Point			
g^*	λ^*	g_2^*	$g_{4,1}^*$
0.734	0.136	0.0897	0.229
Critical Exponents			
$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$
$2.103 - i\,1.655$	$2.103 + i\,1.655$	$2.664 - i\,1.468$	$2.664 + i\,1.468$

Table 3: NGFP for truncation up to A^2 in Generalized Proca Theories and corresponding critical exponents.

Results

- **GPT – fourth order**

$$\mathcal{L} = \frac{1}{16\pi G}\left[2\Lambda - R\right] - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}G_2\,A^2 + G_{4,1}\left[A^2\,R + (\nabla_\mu A^\mu)^2 - (\nabla_\mu A_\nu)(\nabla^\nu A^\mu)\right]$$

$$+ G_{4,2}\left[A^4\,R + 2\,A^2\,(\nabla_\mu A^\mu)^2 - 2\,A^2(\nabla_\mu A_\nu)(\nabla^\nu A^\mu)\right]$$

Fixed Points				
g^*	λ^*	g_2^*	$g_{4,1}^*$	$g_{4,2}^*$
0.711	0.131	0.0354	0.297	−3.933
Critical Exponents				
$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{\theta_5}$
$2.239 - i\,1.662$	$2.239 + i\,1.662$	3.506	7.206	−7.093

Table 4: NGFP for truncation up to A^4 in Generalized Proca Theories and corresponding critical exponents.

Plots

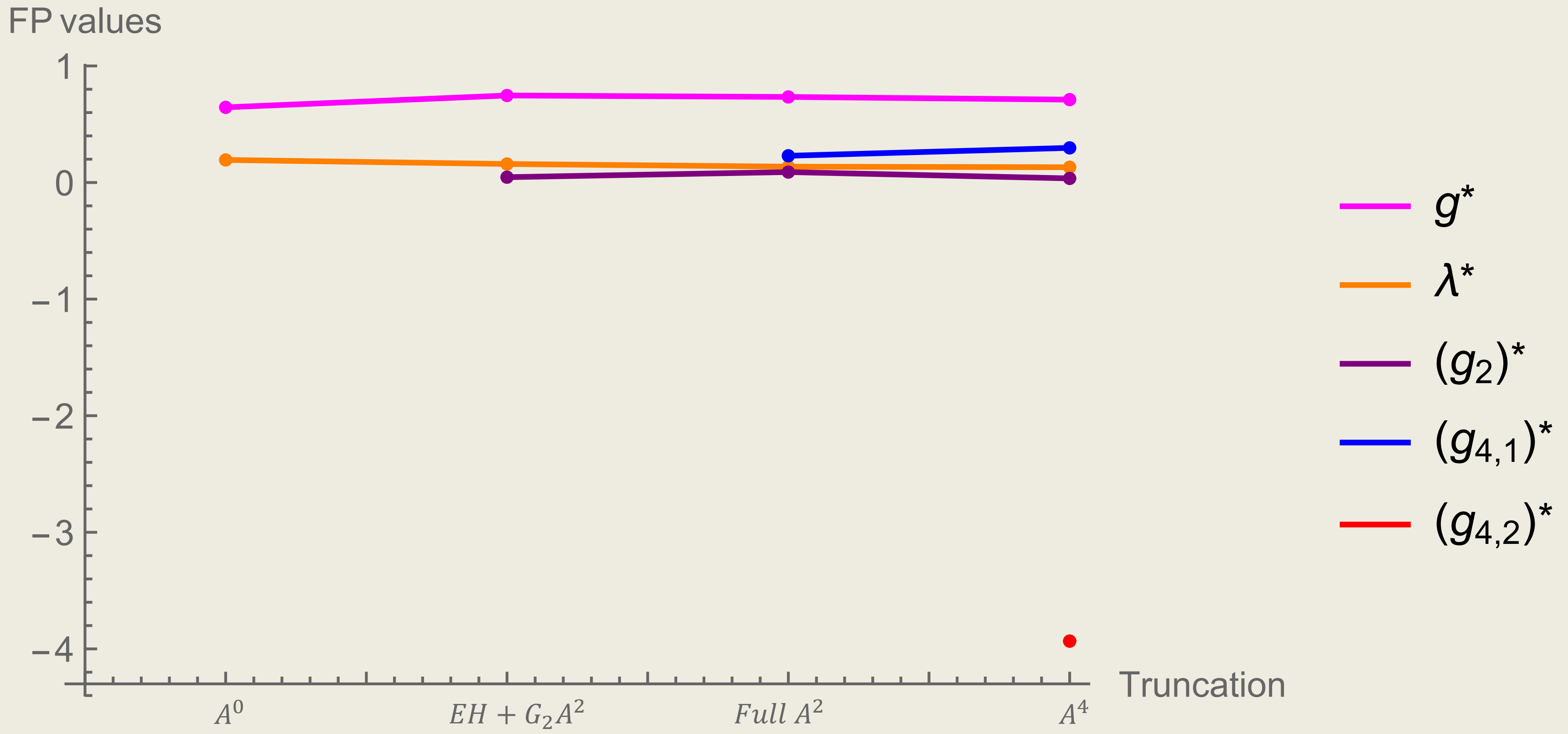

Fig. 1: Fixed point values of dimensionless couplings vs the truncation considered.

Fixed Points				
g^*	λ^*	g_2^*	$g_{4,1}^*$	$g_{4,2}^*$
0.711	0.131	0.0354	0.297	−3.933

Plots

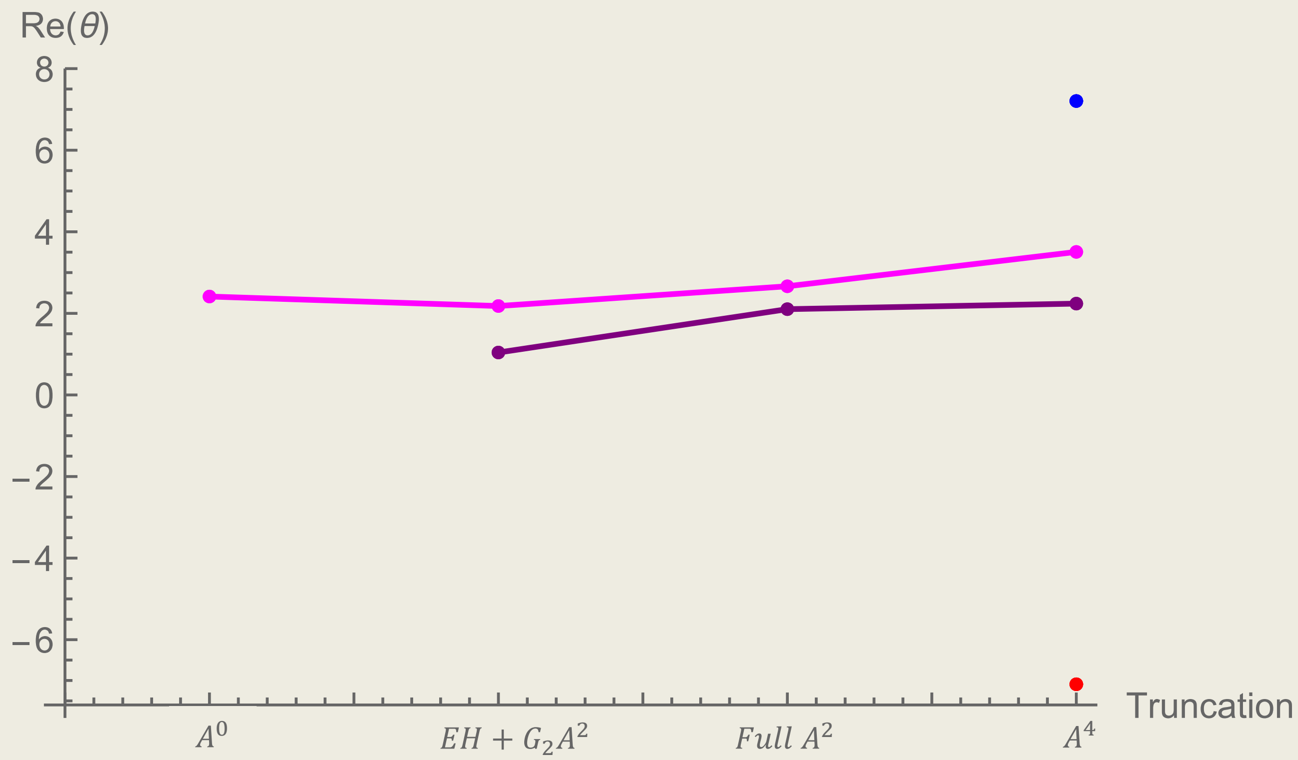

Fig. 2: Real part of the critical exponents for each stable NGFP vs the truncation considered.

Critical Exponents				
$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{\theta_5}$
$2.239 - i\,1.662$	$2.239 + i\,1.662$	3.506	7.206	-7.093

Conclusions

- Generalized Proca Theories:
  - *Possible expanation for Dynamical Dark Energy* [1];
  - *Theoretically and experimentally constrained;*
  - *Lack constraints on Wilson coefficients from fundamental physics* $\longrightarrow$ *Asymptotic Safety?*
- Properties of fixed points in GPT analyzed:
  - *Four relevant directions* $\longrightarrow$ *less predictivity than usual;*
  - *Slower convergence than usual* $\longrightarrow$ *need for higher-order terms?*
- Next steps:
  - *Adding more terms in the Lagrangian;*
  - *Using the FP to determine Wilson coefficients in the AS landscape of GPTs (as in* [5, 6]*).*

[1] L. Heisenberg, "A systematic approach to generalisations of General Relativity and their cosmological implications", Physics Reports Volume 796, 15 March 2019, Pages 1-113.
[5] I. Basile, A. Platania, "Asymptotic Safety: Swampland or Wonderland?", Universe 2021, 7, 389.
[6] B. Knorr, A. Platania, " Unearthing the intersections: positivity bounds, weak gravity conjecture, and asymptotic safety landscapes from photon-graviton flows", arXiv:2405.08860.

Thank you for your attention!