## Polarization studies for <br> EIC Electron Storage Ring and FCC-ee

EIC ESR polarization:

- Assessing needed polarization in the EIC ESR
- Results for unperturbed and perturbed optics
- $\sigma_{y}^{*}$ knobs

FCC-ee polarization:

- Polarization wigglers.
- Some simulation results.

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## Introduction to EIC $e^{-}$polarization

The electron-ion collider (EIC) aims to collide polarized electrons with a variety of polarized hadron beams at various CM energies.

Experiments require

- $\boldsymbol{p}$ and $\boldsymbol{e}^{-}$average polarization $\gtrsim 70 \%$
- Longitudinal polarization at the IP with both helicity within the same store
- Energy
- protons: between 41 and 275 GeV
- electrons: between 5 and 18 GeV


Hadron beams will to large extent exploit the already existing BNL facilities.
The $e^{-}$storage ring will be accommodated inside the RHIC tunnel together with the Rapid Cycling Synchrotron.

Sokolov-Ternov effect tends to polarize the ESR $e^{-}$upwards.

- A full energy injector is needed for filling the ring with up and down polarized bunches.
- The polarization is turned into the longitudinal direction at the IP by pair of spin rotators.

S-T effect may impact the bunch polarization, especially at high energy.
Polarization builds-up exponentially:

$$
P(t)=P_{\infty}\left(1-\mathrm{e}^{-t / \tau_{p}}\right)+P(0) \mathrm{e}^{-t / \tau_{p}}
$$

From Derbenev-Kondratenko expressions:
asymptotic polarization (unknown)

$$
\begin{gathered}
\frac{1}{\tau_{p}} \simeq \frac{1}{\tau_{\mathrm{BKS}}}+\frac{1}{\tau_{\boldsymbol{d}}} \quad \text { and } \quad P_{\infty} \simeq \frac{\tau_{p}}{\tau_{\mathrm{BKS}}} P_{\mathrm{BKS}} \\
\text { diffusion time (unknown) }
\end{gathered}
$$

- $P_{\mathrm{BKS}}$ and $\tau_{\mathrm{BKS}}$ (Baier-Katkov-Strakhovenko generalization of Sokolov-Ternov quantities) are known for the nominal lattice.
- $\tau_{d}$ and thus $P_{\infty}$ are interconnected and depend on actual machine.

Expected $\boldsymbol{P}(0)$ from RCS is $\pm 85 \%$.

- esr optics at 9.8 GeV (Version-5.3)
$-\boldsymbol{P}_{\mathrm{BKS}} \simeq 80.8 \%$
$-\tau_{\mathrm{BKS}} \simeq 704 \mathrm{~min}$
- esr optics at 18 GeV (Version-5.2)
$-\boldsymbol{P}_{\mathrm{BKS}} \simeq 82.7 \%$
$-\tau_{\mathrm{BKS}} \simeq 35.5 \mathrm{~min}$
At high energy a small $\boldsymbol{P}_{\infty}$ means fast depolarization (even for the upward polarized bunches!)





## Expected polarization for v5.6

 Unperturbed ring


Resonances related to the longitudinal motion are
v5.6 1 IP - unperturbed
 very strong and limit polarization.

- The IR optics is not spin-matched for longitudinal motion.
- $\hat{n}_{0}$ being not vertical in the arcs, away from $\nu_{s}=40.5$, results in spin diffusion for longitudinal motion in the arcs too.


## Polarization in presence of misalignments

In these simulations, the orbit correction scheme in the arcs is as in HERA-e (scheme changed recently for using APS dual plane correctors).
For the IR region

- one BPM (dual plane reading) close to each quadrupole;
- one horizontal and one vertical corrector close to each quadrupole.

All together: $271 \mathrm{CHs}, 242 \mathrm{CVs}$ and 242 BPMs .
The correctors at low $\boldsymbol{\beta}$ or at small phase advance are disabled.
Assumed quadrupole misalignments:

$$
\begin{array}{ccc}
\hline \delta x^{Q}[\mu \mathrm{~m}] & \delta y^{Q}[\mu \mathrm{~m}] & \delta \psi^{\boldsymbol{Q}}[\boldsymbol{\mu \mathrm { rad } ]} \\
200 & 200 & 200
\end{array}
$$

The BPMs are integrated in the quadrupoles with added random residual misalignments:

$$
\begin{array}{ccc}
\hline \delta x^{M}[\mu \mathrm{~m}] & \delta y^{M}[\mu \mathrm{~m}] & \delta \psi^{M}[\mu \mathrm{rad}] \\
20 & 20 & 20
\end{array}
$$

and $1 \%$ calibration errors.

## Seed $15^{a}$

|  | $\boldsymbol{q}_{\boldsymbol{x}} / \boldsymbol{q}_{\boldsymbol{y}}$ | $\boldsymbol{x}_{\boldsymbol{r m s}}$ | $\boldsymbol{y}_{\boldsymbol{r} \boldsymbol{m s}}$ | $\boldsymbol{\Delta} \boldsymbol{D}_{\boldsymbol{x}}$ | $\boldsymbol{\Delta} \boldsymbol{D}_{\boldsymbol{y}}$ | $\boldsymbol{\epsilon}_{\boldsymbol{x}}$ | $\boldsymbol{\epsilon}_{\boldsymbol{y}}$ | $\left\|\boldsymbol{C}^{-}\right\|$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $[\mathrm{mm}]$ | $[\mathrm{mm}]$ | $[\mathrm{m}]$ | $[\mathrm{m}]$ | $[\mathrm{nm}]$ | $[\mathrm{nm}]$ |  |
| before c.o. corr. | $.20 / .27$ | 24.7 | 15.9 | 10.9 | 6.5 | - | - | - |
| after, no BPMs errors | $.16 / .22$ | 0.56 | 0.17 | 0.031 | 0.026 | 26.0 | 0.06 | 0.003 |
| after, with BPMs errors | $.16 / .22$ | 0.67 | 0.24 | 0.083 | 0.007 | 27.9 | 0.19 | 0.009 |
| SITF | $.15 / .23$ | 0.66 | 0.24 | - | - | 26.2 | 0.18 | - |

## Seed 13

|  | $\boldsymbol{q}_{\boldsymbol{x}} / \boldsymbol{q}_{\boldsymbol{y}}$ | $\boldsymbol{x}_{\boldsymbol{r m s}}$ | $\boldsymbol{y}_{\boldsymbol{r m s}}$ | $\boldsymbol{\Delta} \boldsymbol{D}_{\boldsymbol{x}}$ | $\boldsymbol{\Delta} \boldsymbol{D}_{\boldsymbol{y}}$ | $\boldsymbol{\epsilon}_{\boldsymbol{x}}$ | $\boldsymbol{\epsilon}_{\boldsymbol{y}}$ | $\left\|\boldsymbol{C}^{-}\right\|$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $[\mathrm{mm}]$ | $[\mathrm{mm}]$ | $[\mathrm{m}]$ | $[\mathrm{m}]$ | $[\mathrm{nm}]$ | $[\mathrm{nm}]$ |  |  |  |
| before c.o. corr. | $.20 / .27$ | 6.1 | 7.3 | 2.2 | 2.6 | - | - | - |
| after, no BPMs errors | $.16 / .22$ | 0.35 | 0.12 |  |  | 26.4 | 0.19 |  |
| after, with BPMs errors | $.16 / .22$ | 0.40 | 0.21 | 0.013 | 0.021 | 26.9 | 0.24 | 0.010 |
| SITF | $.16 / .22$ | 0.40 | 0.20 | - | - | 26.1 | 0.24 | - |

[^0]v5.6 - seed 15 with added BPMs errors
.16/.22/.046

$\mathrm{a}^{*} \gamma$
v5.6-seed 13 with added BPMs errors
.16/.22/.046

v5.6-seed 15 with added BPMs errors 16/.22/.046

v5.6-seed 13 with added BPMs errors .16/.22/.046


Using harmonic bumps for maximizing $\boldsymbol{P}$.
v5.6-seed 15
with added BPMs errors
.16/.22/.046

v5.6-seed 13
with added BPMs errors
.16/.22/.046
with Harm.Bumps(Pol.)

v5.6 - seed 15 with added BPMs errors 16/.22/.046

v5.6-seed 13
with added BPMs errors
.16/.22/.046
with Harm.Bumps(Pol.)


## SITROS tracking.

|  | $\boldsymbol{\sigma}_{\boldsymbol{x}}(\boldsymbol{\mu \mathrm { m }})$ | $\boldsymbol{\sigma}_{\boldsymbol{y}}(\boldsymbol{\mu \mathrm { m } )}$ | $\boldsymbol{\sigma}_{\boldsymbol{\ell}}(\mathrm{mm})$ |
| :---: | :---: | :---: | :---: |
| Analytic | 112.3 | 7.0 | 8.733 |
| Tracking | 64.4 | 4.7 | 8.751 |


v5.6-seed 13 with BPMs errors

16/.22/.046
hb for maximizing $P$


## ifm

## 00000000000

## The $\sigma_{y}^{*}$ problem

For reproducing the ideal Sokolov-Ternov conditions for maximum polarization, $\epsilon_{y}$ should be as small as possible.

From beam-beam simulations, supported by HERA experience, the proton and electron beam sizes at the IP must be matched: for the 18 GeV case the vertical beam size at the IP should be $\approx 10 \mu \mathrm{~m}$.

This can be realized by ${ }^{\text {a }}$

- Local betatron coupling at the IP (it could be embedded into the experiment solenoid correction): good for polarization but simulations indicate beam-beam is unmanageable.
v5.2 1 IP - local IP coupling
$\sigma_{\mathrm{y}}^{*}=12 \mu \mathrm{u}$

$a^{\star} \gamma$

[^1]- A dispersion bump via vertical dipoles in a convenient straight section (possibly a dedicated spin matched one). It requires extra magnets, it is not tunable unless the beam pipe is large enough or the
 magnets are movable.
- Long vertical orbit bump in the ring arcs: the vertical offset in the sextupoles creates betatron coupling. Simplest solution, but with large impact on polarization.

$a^{*} y$
v5.2 1 IP with long arc bump



## Adding long arc bump CV106-CV126 to the corrected machines.

v5.6-seed 15
with added BPMs errors .16/.22/.046 with Harm.(P)+long bump

v5.6-seed 13
with added BPMs errors .16/.22/.046 with Harm.Bumps(Pol.)+long bump

$a^{*} \gamma$
v5.6-seed 15
with added BPMs errors
.16/.22/.046
with Harm.(P)+long bump

v5.6-seed 13 with added BPMs errors 16/.22/.046 with Harm.Bumps(Pol.)+long bump

$a^{*} \gamma$

## SITROS tracking.

|  | $\sigma_{\boldsymbol{x}}(\boldsymbol{\mu \mathrm { m }})$ | $\boldsymbol{\sigma}_{\boldsymbol{y}}(\boldsymbol{\mu \mathrm { m } )}$ | $\boldsymbol{\sigma}_{\ell}(\mathrm{mm})$ |
| :---: | :---: | :---: | :---: |
| Analytic | 109.5 | 12.7 | 8.704 |
| Tracking | 63.2 | 8.0 | 8.744 |


|  | $\sigma_{x}(\mu \mathrm{~m})$ | $\sigma_{y}(\mu \mathrm{~m})$ | $\sigma_{\ell}(\mathrm{mm})$ |
| :---: | :---: | :---: | :---: |
| Analytic | 112.4 | 14.6 | 8.413 |
| Tracking | 73.6 | 9.7 | 8.462 |

v5.6-seed 15 with BPMs errors .16/.22/.046

v5.6-seed 13
with added BPMs errors .16/.22/.046 with Harm.Bumps(Pol.)+long bump


## Summary and Outlook for EIC ESR polarization

Although the polarization for the new optics design is relatively low, even for the unperturbed ring, the goal of $<70>\%$ polarization can be met with the baseline injection rate ( 2 bunches/s for filling esr with 290 bunches at 18 GeV ).

But...

- Larger statistics needed.
- It must be still evaluated the impact on polarization of
- Dipoles roll;
- Beam-beam effects.

With $P_{\infty} \approx 27 \%$ the "safety" margin is not very large.

| $\boldsymbol{P}_{\boldsymbol{\infty}}$ | $\boldsymbol{P}(\mathbf{0})$ | $<\boldsymbol{P}>_{\mathbf{3 . 4}} \mathbf{4}^{\prime}$ | $\boldsymbol{P}(\mathbf{3 . 4} \mathbf{)}$ | $\boldsymbol{P}(\mathbf{0})$ | $<\boldsymbol{P}>_{\boldsymbol{7}^{\prime}}$ | $\boldsymbol{P}\left(\mathbf{7}^{\prime}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +27 | -85 | -70 | -56 | +85 | +70 | +59 |

However the injection rate may be incremented by a factor 2 if needed.

## Introduction to polarization in the FCCee

- Resonant de-polarization has been proposed for accurate beam energy calibration at 45 and 80 GeV beam energy.
It relies on the relationship $\nu_{\text {spin }}=\boldsymbol{a} \boldsymbol{\gamma}^{\text {a }}$.
- $5 \%-10 \%$ beam polarization is estimated to be enough for the purpose of energy calibration.
- Beam polarization is obtained "for free" through Sokolov-Ternov effect.

Asymptotic polarization and build-up rate

$$
\begin{gathered}
P_{\infty}=92.3 \% \\
\tau_{p}^{-1}=\frac{5 \sqrt{3}}{8} \frac{r_{e} \gamma^{5} \hbar}{m_{0} C} \oint \frac{d s}{|\rho|^{3}}
\end{gathered}
$$

For FCCee ( $\rho \approx 10424 \mathrm{~m}$ )

| $\boldsymbol{E}$ |  |  |
| :---: | :---: | :---: |
| $(\mathrm{GeV})$ | $\boldsymbol{\tau}_{\text {pol }}$ <br> $(\mathrm{h})$ | $\boldsymbol{\tau}_{\mathbf{1 0 \%}}$ <br> h |
| 45 | 256 | 29 |
| 80 | 14 | 1.6 |

[^2]
## Polarization wigglers

At $45 \mathrm{GeV} \tau_{p}$ is reduced by introducing wigglers, a chain of horizontal bending magnets with alternating field sign.
Polarization rate is dominated by the wigglers $\left(\left|\rho_{w}\right| \ll\left|\rho_{d}\right|\right)$ :

$$
\tau_{p}^{-1}=F \gamma^{5}\left[\int_{d i p} \frac{d s}{\left|\rho_{d}\right|^{3}}+\int_{w i g} \frac{d s}{\left|\rho_{w}\right|^{3}}\right] \quad F \equiv \frac{5 \sqrt{3}}{8} \frac{r_{e} \hbar}{m_{0} C}
$$

Polarization:

$$
P_{\infty}=\frac{8}{5 \sqrt{3}} \frac{\oint d s \frac{\hat{B} \cdot \hat{n}_{0}}{\left.| |\right|^{3}}}{\oint d s \frac{1}{\left.|\rho|\right|^{3}}} \propto \tau_{p}\left[\int_{d i p} d s \frac{\hat{B}_{d} \cdot \hat{n}_{0}}{\left|\rho_{d}\right|^{3}}+\int_{w i g} d s \frac{\hat{B}_{w} \cdot \hat{n}_{0}}{\left|\rho_{w}\right|^{3}}\right]
$$

$\hat{\boldsymbol{n}}_{0} \equiv \hat{\boldsymbol{y}}$ in a perfectly planar ring. Constraints:

- Orbit unperturbed outside the wigglers
$-x^{\prime}=0$ outside the wiggler $\Rightarrow \int_{w i g} d s B_{w}=0 \quad$ (vanishing field integral)
$-x=0$ outside the wiggler $\Rightarrow \int_{w i g} d s s B_{w}=0 \quad$ (true for symmetric field)
- $P$ large $\Rightarrow \int_{w i g} d s B_{w}^{3}$ must be large

LEP polarization wigglers (J. M. Jowett).


$$
\leftrightarrow \mathrm{L}_{-} / \mathrm{R} \leftrightarrow \mathrm{~L}_{+}-\propto \mathrm{L}_{-} / \mathrm{R} \mapsto
$$

$$
\int_{w i g} d s \frac{1}{\rho_{w}^{3}}=\frac{L_{+}}{\rho_{+}^{3}}\left(1-\frac{1}{N^{2}}\right) \quad N \equiv L_{-} / L_{+}=B_{+} / B_{-}
$$

$N$ should be large for keeping polarization high!

Using more than one period for smaller impact on $\boldsymbol{\epsilon}_{\boldsymbol{x}}$ keeping $\boldsymbol{\sigma}_{\boldsymbol{E}}$ for the same $\boldsymbol{\tau}_{\boldsymbol{p}}$.

8 wigglers with $\boldsymbol{B}^{+} \simeq 0.57 \mathrm{~T}$ :

- $\boldsymbol{\tau}_{\mathbf{1 0 \%}} \simeq 2.7 \mathrm{~h}$
- $\sigma_{\boldsymbol{E}}=50 \mathrm{MeV}$
- For the (obsolete) $90 / 90 \mathrm{deg}$ optics $\boldsymbol{\epsilon}_{\boldsymbol{x}}$ increases from 90 pm to 120 pm with 3 periods.


Trajectory $\left(\boldsymbol{B}^{+}=0.7 \mathrm{~T}\right)$

## Polarization simulations for FCC-ee in presence of misalignments

 FCC- $e^{ \pm}$design relies on ultra-flat beams|  | $\boldsymbol{Z}$ | $\boldsymbol{W} \boldsymbol{W}$ |
| :---: | :---: | :---: |
| Beam energy $[\mathrm{GeV}]$ | 45.6 | 80 |
| FODO | $60^{\mathbf{0}} / 60^{\mathbf{0}} /$ | $60^{\mathbf{0}} / 60^{\mathbf{0}}$ |
| $\boldsymbol{\epsilon}_{\boldsymbol{x}}[\mathrm{nm}]$ | 0.27 | 0.84 |
| $\boldsymbol{\epsilon}_{\boldsymbol{y}}[\mathrm{pm}]$ | 1 | 1.7 |
| $\boldsymbol{\beta}_{\boldsymbol{x}}^{*}[\mathrm{~m}]$ | 0.15 | 0.2 |
| $\boldsymbol{\beta}_{\boldsymbol{y}}^{*}[\mathrm{~mm}]$ | 0.8 | 1 |
| $\boldsymbol{\sigma}_{\boldsymbol{x}}^{*}[\boldsymbol{\mu \mathrm { m }}]$ | 6.4 | 13 |
| $\boldsymbol{\sigma}_{\boldsymbol{y}}^{*}[\mathrm{~nm}]$ | 28 | 41 |

(CDR, 2018)

## Chromaticity and response to quads misalignments

| Optics |  | $\boldsymbol{\xi}_{\boldsymbol{x}}$ | $\boldsymbol{\xi}_{\boldsymbol{y}}$ |
| :---: | :---: | :---: | :---: |
| 45 GeV | all sexts off | -361 | -1540 |
|  | IR setxs off | +3.5 | -1230 |
| 80 GeV | all sexts off | -359 | -1331 |
|  | IR setxs off | +3 | -1017 |


| Optics |  | $\left\langle\boldsymbol{y}_{\boldsymbol{r m s}}>/ \boldsymbol{\delta} \boldsymbol{z}_{\boldsymbol{r m s}}^{\boldsymbol{Q}}\right.$ |
| :---: | :---: | :---: |
| 45 GeV | all quads | 665 |
|  | w/o IRs quads | 124 |
| 80 GeV | all quads | $\boldsymbol{F}_{\boldsymbol{y}}$ |
|  | w/o IRs quads | 492 |
|  | 127 |  |

Large impact of strong IRs quads.
Additional related problems:

- Beam offsets in the strong IRs sextupoles, produce tune shift and betatron coupling.
- Small offsets of the IRs quads may lead to an anti-damped machine.

Simulations in presence of misalignments optics from T.Charles (2019) ${ }^{a}$

|  | IR Quads | other Quads | Sexts |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{\delta} \boldsymbol{x}(\boldsymbol{\mu \mathrm { m }})$ | 50 | 100 | 100 |
| $\boldsymbol{\delta} \boldsymbol{y}(\boldsymbol{\mu \mathrm { m }})$ | 50 | 100 | 100 |
| $\boldsymbol{\delta} \boldsymbol{\theta}(\boldsymbol{\mu} \mathrm{rad})$ | 50 | 100 | 100 |

- 1594 HBPMs, 1594 VBPMs, 1594 CVs and 1600 CHs. Orbit corrected down to few tens of microns.
- Tune shift and coupling corrected by 1204 normal + 1204 skew thin lenses quadrupoles SITROS can't treat thin lenses $\rightarrow$ replaced by 5 mm long quads, in lack of more space. Code edited for dropping
- magnets shorter than 10 mm in emittance and damped transport matrix calculation;
- quadrupole component of misaligned sextupoles in the closed orbit calculation (for compatibility with MADX).

The substitution did not work always well, even within MADX.

[^3]45 GeV optics with 8 wigglers.
Seed 13, $\boldsymbol{x}_{\text {rms }}=23 \boldsymbol{\mu}$

$$
\boldsymbol{y}_{r m s}=21 \mu \mathrm{~m}
$$




## Seed $13,80 \mathrm{GeV}$ optics w/o wigglers.



80 GeV with $\mathrm{Q}_{\mathrm{x}}=0.14, \mathrm{Q}_{\mathrm{y}}=0.22, \mathrm{Q}_{\mathrm{s}}=0.049$ seed 13 , no tapering



Yi Wu has taken over the FCCee polarization simulations for her PhD at EPFL. She uses Bmad (D. Sagan).

Using 4 bumps which are optimized at $45.82 \mathrm{GeV}(a \gamma=103.983)$



## Summary for FCCee polarization

- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
- At 45 GeV wigglers are required to get $\tau_{10 \%} \approx 2-3 \mathrm{~h}$.

They do not harm polarization, but the extra synchrotron radiation set some "boundary conditions" to operation.

- $\boldsymbol{P}_{\infty}$ depends on how well is the machine aligned/corrected, requirements becoming stricter at high energy.
- Extremely well corrected orbit/optics is required for a large chromatic machine with $\boldsymbol{\beta}_{y}^{*}=0.8-1 \mathrm{~mm}$ as FCC-ee to work and meet required performance. * This benefits also polarization.

The real challenge: achieving the extreme precision ( 4 KeV at 45 GeV and 250 KeV at 80 GeV ) required by the experiments.

Thanks!

## EXTRA SLIDES

## FCCee Polarization Operation

At 45 GeV operation is dictated by

- need of wigglers, which cannot be operated with a full machine;
- time needed to reach the minimum polarization useful for resonant depolarization;
- lifetime of the bunches used for energy calibration.

Lifetime of colliding bunches is about 1 h (Bhabha limited). Exhausted colliding bunches are replenished by top-up injection, but the time needed to reach $10 \%$ polarization $\mathrm{w} / \mathrm{o}$ wigglers is 29 hours.

- Some $100-200$ non colliding bunches will be used for monitoring the beam energy with $N_{b} \approx 1 \mathrm{e} 10$, their lifetime being Touschek effect limited.
- The low intensity non-colliding (unpolarized) bunches are injected first and wigglers are turned on until their polarization reaches $\approx 5-10 \%$
* The extra SR is tolerable at low beam intensity.

T. Tydecks courtesy

Energy may vary during the physics run due to changes of

- main dipole field,
- horizontal orbit and corrector settings,
- machine circumference (geological, tides etc),
- phasing drifts between RF stations.

Some effects may be mitigated by feed-backs.
$e^{ \pm}$(double ring!) energy should be monitored $\approx$ each 10 minutes targeting a different bunch. With 100 bunches and $10^{\prime}$ interval there are 17 h during which newly injected non-colliding bunches can reach $5-10 \%$ polarization, $\mathrm{w} / \mathrm{o}$ wigglers.

Operation is much relaxed at 80 GeV where the Touschek lifetime is larger for the same $N_{b}$ and wigglers are not needed.

## From depolarizing frequency to CM energy

Depolarization occurs when spin precession and RF field frequency are in resonance

$$
\begin{gathered}
\nu_{s p i n}=\frac{f_{e x c}}{f_{r e v}}+k \\
\nu_{s p i n}=a \gamma \quad \rightarrow E_{b e a m}=\left[k \pm \frac{f_{e x c}}{f_{r e v}}\right] \frac{E_{0}}{a}
\end{gathered}
$$

It must be proven that the required calibration precision can be reached: a careful review of all possible biases is needed (see Amsterdam FCC week contributions by A.Bogomyagkov and T.Tydecks), keeping in mind that energy measured by resonant depolarization is the average beam energy over many turns.

Different kinds of problems:

- Time varying effects (discussed before) call for frequent monitoring and development of models for interpolation.
- The relationship $\nu_{\text {spin }}=\boldsymbol{a} \boldsymbol{\gamma}$ does not hold always!
- Experiment solenoids; known (and measurable): negligible for FCCee.
- Vertical closed orbit (2d order effect). Tune shift (Yokoya, Barber):
$\Delta \nu_{s}^{(2)}=\frac{1}{4 \pi} R^{2}(a \gamma+1)^{2} \Im\left[\frac{1}{e^{-i 2 \pi \nu_{s}^{0}}-1} \int_{0}^{2 \pi} d \theta h^{*}(\theta) y_{c o}^{\prime \prime} \int_{\theta}^{\theta+2 \pi} d \theta^{\prime} h\left(\theta^{\prime}\right) y_{c o}^{\prime \prime}\right]$
with

$$
h(\theta)=\left(\hat{m}_{0}+i \hat{l}_{0}\right) \cdot \hat{x} \quad \text { and } y^{\prime \prime}=-K\left(y-\delta_{y}^{Q}\right)+\left(\frac{\Delta B}{B \rho}\right)_{c o r}
$$

Evaluation over 10 seeds, resorting to approximated expression by R. Assmann et al.


- Electric fields (term $\overrightarrow{\boldsymbol{\beta}} \times \overrightarrow{\mathcal{E}}_{\boldsymbol{R} F}$ in BMT-equation): negligible spin tune shift for a well corrected orbit.
- The average beam energy can be azimuth dependent (SR, RF phasing, wake-fields)
- The relationship between CM energy and beams average energy may be distorted.
- CM energy with a crossing angle: $\sqrt{s}=\mathbf{2} \sqrt{\boldsymbol{E}^{+} \boldsymbol{E}^{-}} \boldsymbol{\operatorname { c o s }}(\alpha / \mathbf{2})$. Dimuon events in the experiment detector allow high precision measurement of the crossing angle (P. Janot).
- An offsets, $\boldsymbol{u}_{\mathbf{0}}$, between the colliding beams change the actual CM energy if the dispersion of the two rings has opposite sign. Evaluation for FCCee (T. Tydecks):

| assuming: $\sigma_{x}=6.4 \mu \mathrm{~m}, \sigma_{y}=28 \mathrm{~nm}, \sigma_{D_{x}}=0.1 \mathrm{~mm}, \sigma_{D_{y}}=1.0 \mu \mathrm{~m}$ |  |  |  |
| :--- | :--- | :--- | :--- |
| $\frac{u_{0}}{\sigma_{u}}$ | 0.1 | 0.5 | 1.0 |
| $\Delta E_{c m}\left(D_{x}\right) / \mathrm{MeV}$ | 0.12 | 0.59 | 1.18 |
| $\Delta E_{c m}\left(D_{y}\right) / \mathrm{MeV}$ | 0.28 | 1.42 | 2.84 |

A relatively large effect which calls for frequent position scans.


[^0]:    ${ }^{\text {a }}$ Orbit misalignment and correction with MAD-X.

[^1]:    ${ }^{\text {a }} \mathrm{nb}$ : examples refer to $\mathbf{v} 5.2$ with $Q_{x}=49.12, Q_{y}=43.10$

[^2]:    ${ }^{\mathrm{a}} \boldsymbol{a}=$ gyromagnetic anomaly

[^3]:    ${ }^{\text {a }}$ Optics and misalignment simulations and corrections are still in evolution!

