Non-Supersymmetric Strings and their (In)Stabilities

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(Two reviews with J. Mourad : 2107.04064, 1711.11494)

Recently: 2305.09587, 2309.04026, 2309.05268

SCUOLA NORMALE SUPERIORE

INFN

Alexander von Humboldt Stiftung/Foundation

XMAS THEORETICAL PHYSICS WORKSHOP
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The (SUSY) 10D-11D Zoo

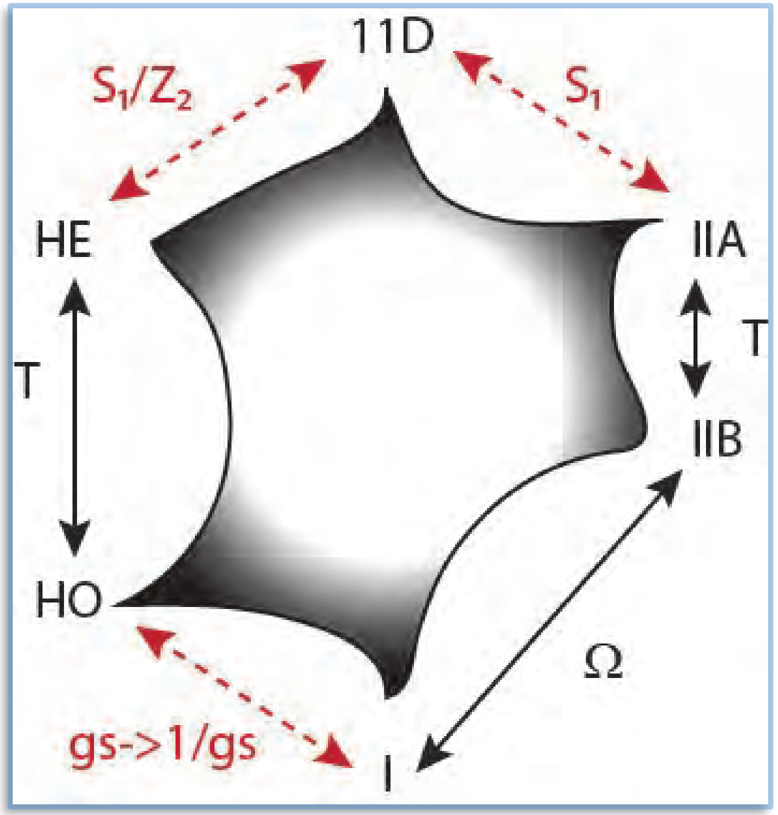
- Highest point of (SUSY) String Theory

BUT:

- Exhibits dramatically our limitations
- Perturbative → Solid arrows
- [10&11D supergravity → Dashed arrows]

(Witten, 1995)

- SUSY: stabilizes these 10D Minkowski vacua



BROKEN SUSY ?

Vacuum Energy in Field Theory

- BOSE (FERMI) OSCILLATOR: $$H = \frac{p^2}{2m} + \frac{1}{2}\, m\,\omega^2\, q^2 \;\longrightarrow\; \mathcal{E}_0 = (-)\frac{\hbar\,\omega}{2}$$

- QUANTUM FIELD THEORY: $$\frac{\mathcal{E}_0}{V} = \sum_i (-1)^{F_i} \int \frac{d^d\mathbf{k}}{(2\pi)^d}\, \frac{\hbar\, c}{2}\, \sqrt{\mathbf{k}^2 + m_i^2}$$

- THE COSMOLOGICAL CONSTANT ISSUE: $$\frac{\mathcal{E}_0}{V} \sim \frac{\mathcal{E}_{Pl}}{V_{Pl}} \sum_i (-1)^{F_i} + \ldots$$

(Zeldovich, 1968)

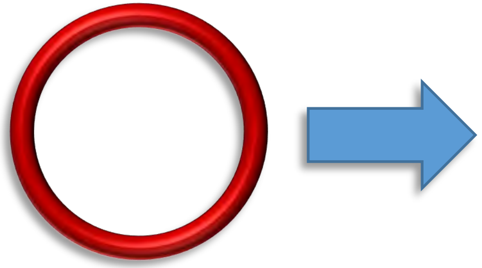
- (Exact, GLOBAL) SUPERSYMMETRY removes problem

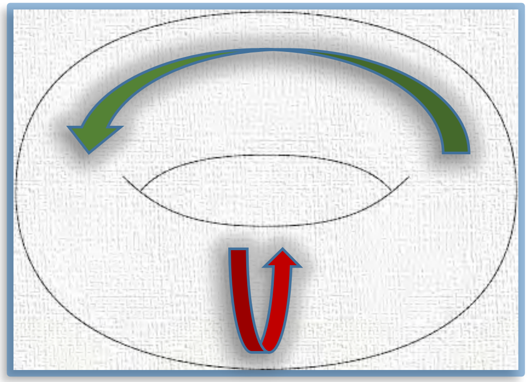
(Zumino, 1975)

Vacuum Energy and String Theory

VACUUM ENERGY: DETERMINES CONSISTENT STRING SPECTRA

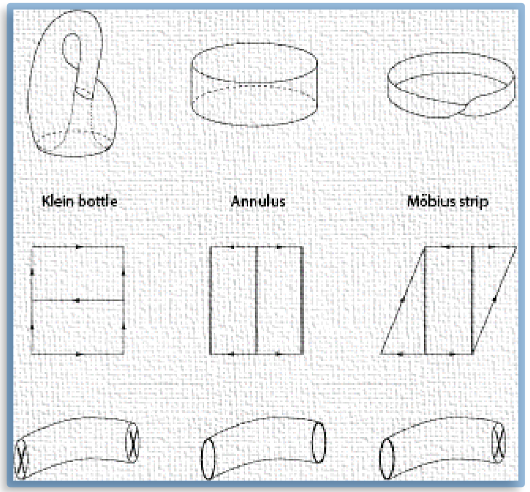
- VACUUM ENERGY & "CLOSED" STRINGS:
  - ❖ MODULAR INVARIANCE !



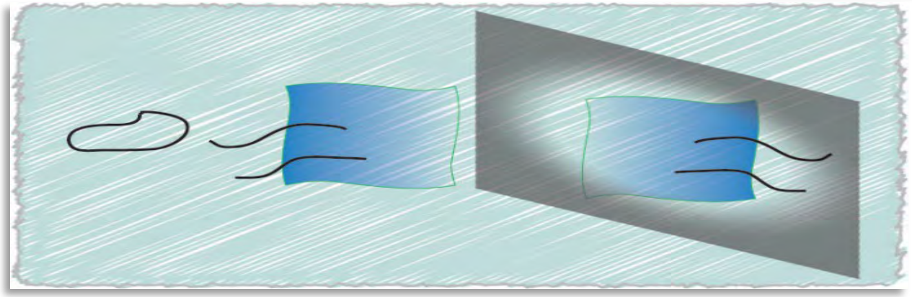


- ORIENTIFOLDS: MORE SUBTLE, OPEN AND CLOSED STRINGS

(AS, 1987)
[+Bianchi, Pradisi, 1988-96]
[+ Fioravanti, 1992]
[+Stanev, 1994-96]



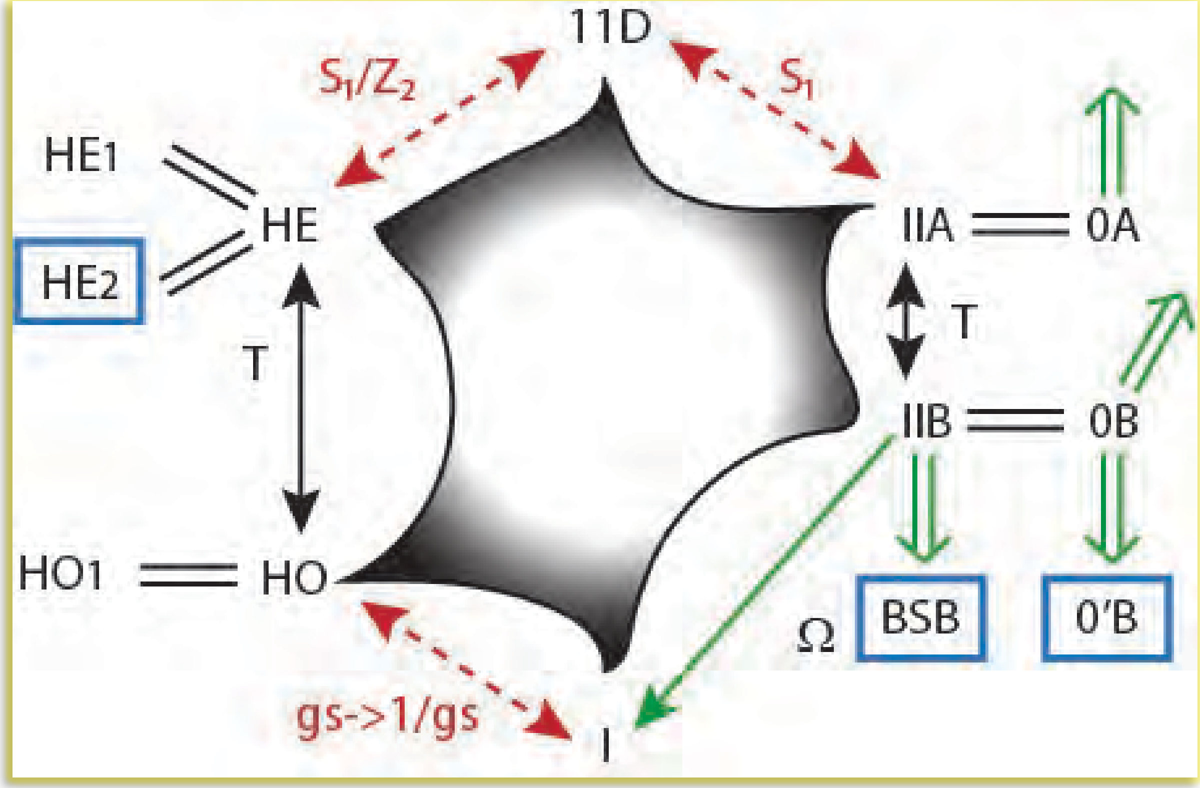
- VACUUM HOSTS (EXTENDED) SOLITONS



(Polchinski, 1995)

- OK WITH SUPERSYMMETRY (NO BACK-REACTION) !

The 10D-11D Zoo



- Non-SUSY closed & orientifolds (Seiberg, Witten, 1986) (Bianchi, AS, 1990)

+ 3 non-SUSY non-tachyonic strings

- SO(16)xSO(16) (Dixon, Harvey, 1986) (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)
- O'B U(32) (AS, 1995)
- [BSB: Usp(32)] (Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

BUT: Tadpole potential

- Compactifications? Stability?
- [NON-PERTURBATIVE LINKS?]

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - Te^{-\phi} + \ldots\right\}$$

VACUUM ENERGY → "TADPOLE POTENTIAL"

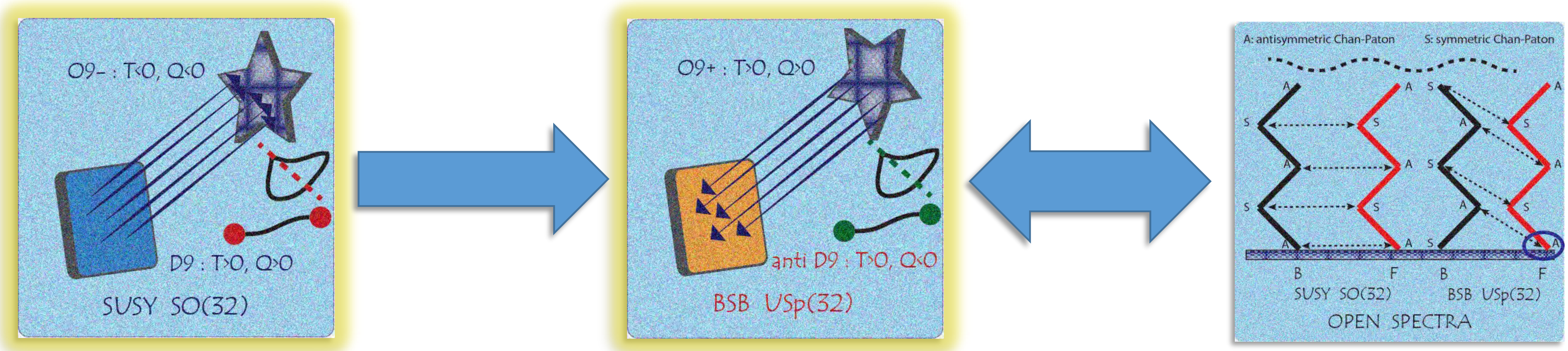
- Expansion in powers of $\alpha' R$
- Expansion in powers of $g_s = e^{\phi}$

Brane SUSY Breaking

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

- ❖ Non-linear SUSY: ∃ goldstino!
- ❖ NO TACHYONS

(Dudas, Mourad, 2000)
(Pradisi, Riccioni, 2001)



Non-linear SUSY: a striking feature in D=10!

SUMMARIZING

String Theory: determined by "vacuum Energy"

NO SUSY → Typically tachyonic modes

- BUT: 3 D=10 non-SUSY non-tachyonic strings

- SO(16)xSO(16) heterotic
- O'B U(32) orientifold (no SUSY)
- Usp(32) orientifold (non-linear SUSY)

(Dixon, Harvey, 1987)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1987)

(AS, 1995)

(Sugimoto, 1999, Antoniadis, Dudas, AS, 1999)

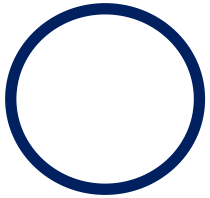
Vacuum modified (Tadpole potential)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2-T\,e^{-\phi}+\ldots\right\}$$

How to deal with broken SUSY? What can we learn?

Basic Compactifications
(Dudas-Mourad Vacua)

9D Kaluza-Klein Circle Compactification



$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2\right\}$$

SPONTANEOUS COMPACTIFICATION: circles of ARBITRARY radius R

- g_{MN} yields the following massless modes: $g_{\mu\nu}$, A_μ , φ
- The radius R of the circle is ARBITRARY (modulus) : (marginally) STABLE [non-perturbative instability]

(Witten, 1982)

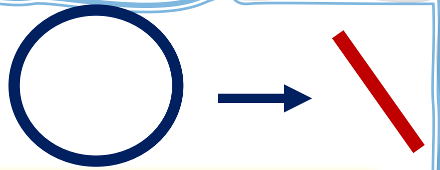
- Calabi-Yau manifolds, central to (supersymmetric) String Theory, are a notable generalization

9D Dudas-Mourad Vacua

(Dudas, and Mourad, 2000, 2001)

$$S = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

T: DRIVES the compactification



- ❖ SPONTANEOUS COMPACTIFICATIONS: INTERVALS, FINITE length $\sim \frac{1}{\sqrt{T}}$
- ❖ [Usp(32) and U(32), similar but more complicated for SO(16) x SO(16)] :
- ❖ FINITE 9D Planck mass & gauge coupling
- At ends: g_s→ (∞,0) & curvature diverges
- BUT: T-independent symptotics

$$e^{\phi} = e^{u+\phi_0}\,u^{\frac{1}{3}}$$
$$ds^2 = e^{-\frac{u}{6}}\,u^{\frac{1}{18}}\,dx^2 + \frac{2}{3\,T\,u^{\frac{3}{2}}}\,e^{-\frac{3}{2}(u+\phi_0)}\,du^2$$

$$u \to 0\ :\ ds^2 \sim (\mu_0\,\xi)^{\frac{2}{9}}\,dx^2 + d\xi^2\ ,\quad e^{\phi} \sim (\mu_0\,\xi)^{\frac{4}{3}}$$
$$u \to \infty\ :\ ds^2 \sim [\mu_0\,(\xi_m-\xi)]^{\frac{2}{9}}\,dx^2 + d\xi^2\ ,\quad e^{\phi} \sim [\mu_0\,(\xi_m-\xi)]^{-\frac{4}{3}}$$

- EXTENSION: $V_E = T\,e^{\frac{3}{2}\,\phi} \longrightarrow V_E = T\,e^{\gamma\,\phi}$ The orientifold value (γ=3/2) is "CRITICAL" !
  - ARE LARGE values of g_s INEVITABLE in non-SUSY compactifications?
  - STABILITY ?

$$V_S \sim e^{-\phi}$$
$$V_E = e^{\frac{3}{2}\,\phi}$$

Dudas-Mourad Vacua : Stability , I

(Basile Mourad, AS, 2018)

❖ AdS x S with tadpole potential : UNSTABLE
❖ Dudas-Mourad: STRONG COUPLING but STABLE !

• SETUP for Scalar perturbations: $ds^2 \;=\; e^{2\Omega(z)}\left[(1+A)\,dx^\mu\,dx_\mu \;+\; (1-7A)\,dz^2\right]$,

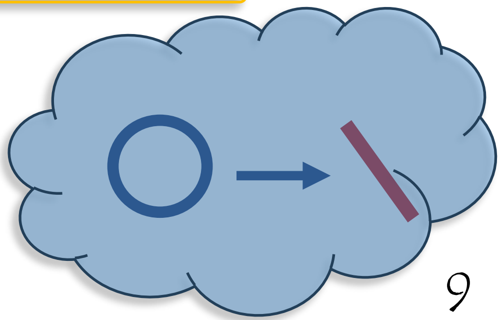
$$A'' + A'\left(24\,\Omega' \;-\; \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) \;+\; A\left(m^2 \;-\; \frac{7}{4}\,e^{2\Omega}\,V \;-\; 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) \;=\; 0$$

❖ Schrödinger-like form:

$$m^2\,\Psi \;=\; \left(b \;+\; \mathcal{A}^\dagger\,\mathcal{A}\right)\Psi$$

$$\mathcal{A} \;=\; \frac{d}{dr} \;-\; \alpha(r)\;, \qquad \mathcal{A}^\dagger \;=\; -\,\frac{d}{dr} \;-\; \alpha(r)\;, \qquad b \;=\; \frac{7}{2}\,e^{2\Omega}\,V\,\frac{1}{1\;+\;\frac{9}{4}\,\alpha_O\,y^2} \;>\; 0$$

BUT: Boundary Conditions !



Dudas-Mourad Vacua : Stability , II

(Mourad, AS, 2023)

- Self-adoint extensions (boundary conditions) → complete sets of modes, BUT positivity?

$$-\frac{d^2\,\Psi(z)}{d\,z^2} + V(z)\,\Psi(z) = m^2\,\Psi(z)$$

$$V(z) \sim \frac{\mu^2 - \frac{1}{4}}{z^2}\,, \qquad V(z) \sim \frac{\mu^2 - \frac{1}{4}}{(z_m - z)^2}$$

$$\left(\,\mathcal{H} = \mathcal{A}\,\mathcal{A}^\dagger\,\right)$$

- The potentials for the MODES are CLOSELY APPROXIMATED by:

$$V_\pm = \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 - \frac{1}{4}\right)}{\sin^2\left(\frac{\pi\,z}{z_m}\right)} - \left(\frac{1}{2} \pm \mu\right)^2\right]$$

Legendre functions

- The possible self—adjoint extensions depend on μ

  a) μ ≥ 1 : UNIQUE b.c. → Scalar Modes

  b) μ < 1 : b.c. ∈ SL(2,R) x U(1) → [indep.: AdS_3 boundary (θ_1,θ_2)] → Tensor & Vector Modes

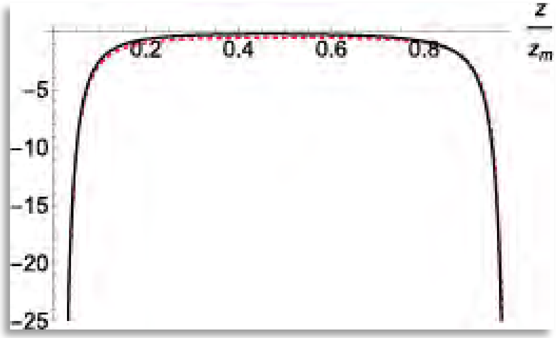
STABILITY ANALYSIS: can rely on EXACT Legendre eigenvalue equation

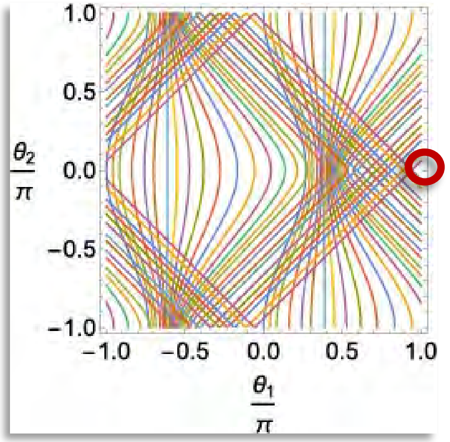
Dudas-Mourad Vacua : Stability , II

(Mourad, AS, 2023)

$$V_{\pm} \;=\; \left(\frac{\pi}{z_m}\right)^2 \left[\frac{\left(\mu^2 \;-\; \frac{1}{4}\right)}{\sin^2\left(\frac{\pi\, z}{z_m}\right)} \;-\; \left(\frac{1}{2} \pm \mu\right)^2\right]$$

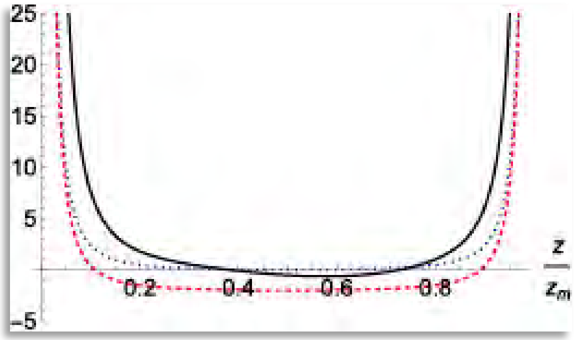
Tensor Modes (μ=0)





UNIQUE stable b.c. (π,0)
(MASSLESS graviton)

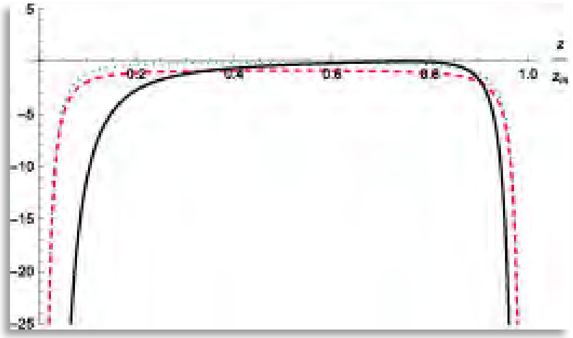
Scalar Modes (μ=1)

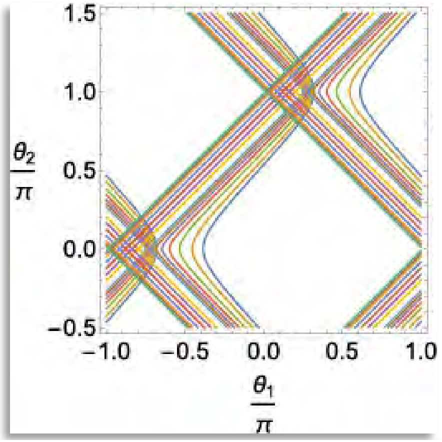


UNIQUE b.c.
[up to vertical adjustment]
(MASSIVE scalar)

$$m^2 \;\simeq\; \left(\frac{\pi}{z_m}\right)^2 \left[n(n+1) \;-\; \frac{7}{8}\right] \;,\; n = 1, 2, \ldots$$

Vector Modes (μ=1)





WIDE stability regions
(MASSLESS or MASSIVE vector)

AdS x S Flux Vacua with Tadpoles

(Gubser, Mitra, 2002)
(Mourad, AS, 2016)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{12}\,\mathcal{H}_3^2 - \frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi} + \ldots\right\}$$

- A WIDELY STUDIED OPTION FOR T=0 (AdS/CFT)
- Dilaton Eq: orientifolds NEED H_3 fluxes, SO(16)xSO(16) H_7 fluxes
- Eqs. determine $AdS_3 \times S^7$ (orientifolds) & $AdS_7 \times S^3$ (SO(16)xSO(16)
- ❖ WIDE REGIONS where the two couplings $\alpha' R$ and $g_s = e^{\phi}$ are SMALL
- ❖ (H_3 or H_7) FLUXES SUPPORT THESE SYMMETRIC COMPACTIFICATIONS
- ❖ BUT: unstable modes are present (Basile Mourad, AS, 2018)

Cosmology:
The Climbing Scalar

Cosmology: "Critical" Potential & Climbing Scalar

What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \quad \longrightarrow \quad \ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- IF V does not vanish : a convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$V e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

$$m\ddot{x} + b\dot{x} = f$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ Quadratic potential? Far away from origin *(Linde, 1983)*

❖ Exponential potential? YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{2\gamma\varphi}$: Climbing & Descending Scalars

(Halliwell, 1987;…, Dudas and Mourad, 2000; Russo, 2004; Dudas, Kitazawa, AS, 2010)

- $\gamma < 1$? Both signs of speed

a. "Climbing" solution (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

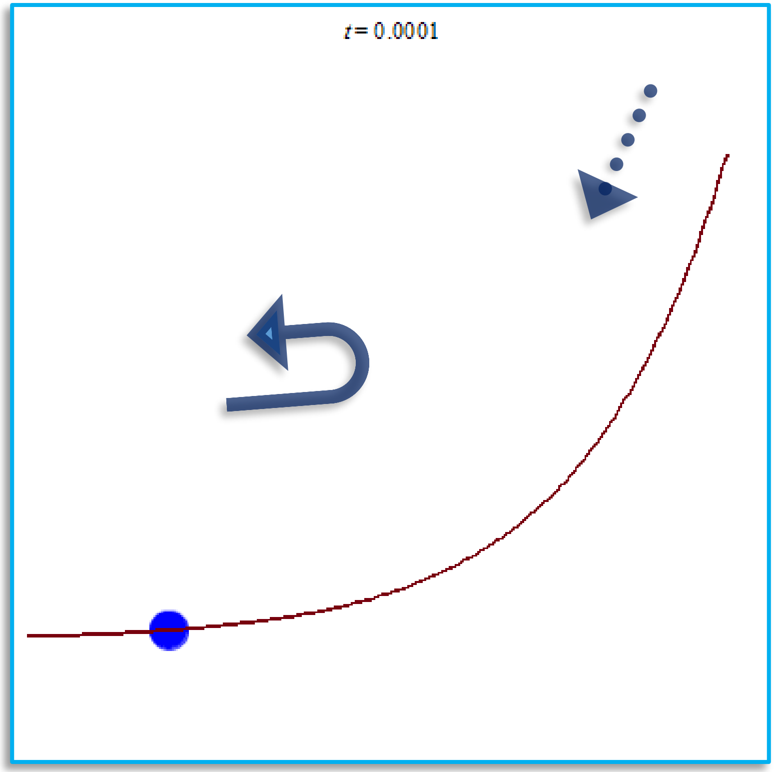
b. "Descending" solution (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

Limiting τ- speed (LM attractor):

$$\mathbf{v}_{\mathbf{lim}} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

(Lucchin and Matarrese, 1985)



$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond

$$V_S \sim e^{-\phi}$$
$$V_E = e^{\frac{3}{2}\phi}$$
$$\left(V_E = e^{2\varphi}\right)$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

$$\ddot{\varphi} + \dot{\varphi}\,|\dot{\varphi}| + \gamma\,\dot{\varphi}^2 \simeq 0 \longrightarrow \dot{\varphi} = \frac{C}{t}$$

$$|\mathbf{C}| = \frac{\mathbf{1}}{\mathbf{1}+\epsilon\gamma}\,, \quad \epsilon = \pm 1$$

$$V = T e^{2\gamma\varphi}$$

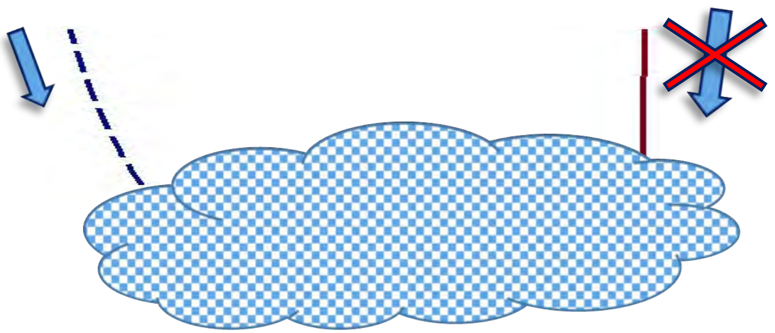
Cosmology: Climbing Scalar as Trigger of Inflation?

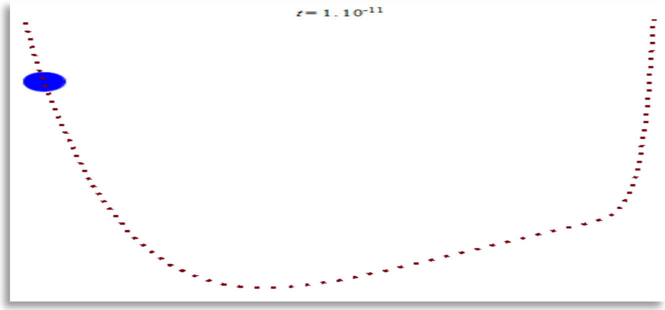
CLIMBING & SLOW-ROLL ? General with (super)critical Exponential, e.g. + Starobinsky

$$V(\phi) = T\, e^{2\varphi} + v(\phi)$$

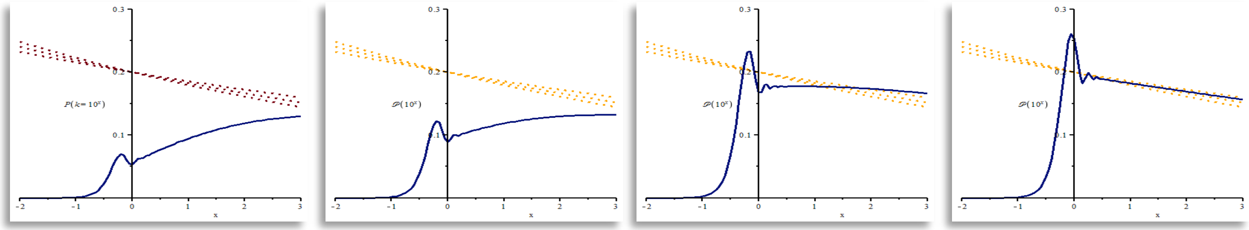
$$e.g. \;\; v(\phi) = v_0 \left(1 - e^{-\frac{2}{3}\varphi}\right)^2$$

(Dudas, Kitazawa, Patil, AS, 2013)
(Kitazawa, AS, 2014)



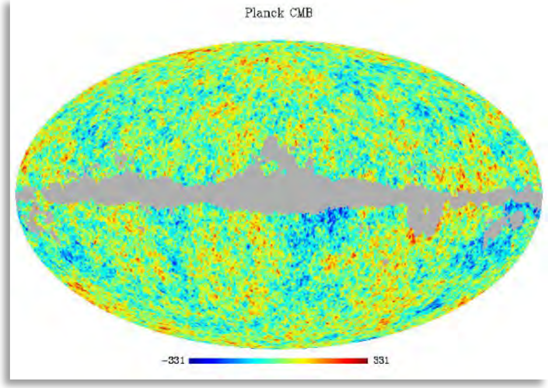


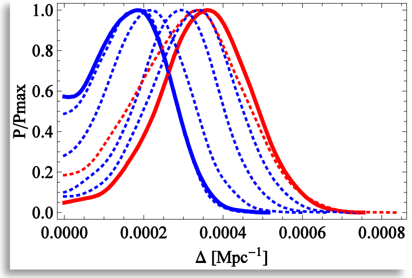
Damps low end of primordial power spectrum → POSSIBLY: damping of first CMB multipoles (cfr. lack-of-power)
[+ enhanced tensor-to-scalar ratio & (it seems) enhanced non-gaussianity at the transition]



$$P(k) \sim k^{3-3\nu} \longrightarrow P(k) \sim \frac{k^3}{[k^2 + \Delta^2]^\nu}$$

[Corrects Chibisov-Mukhanov tilt by Δ]





$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

$$\Delta_{infl} \sim 10^{12} - -10^{14}\; GeV \;\; \text{for} \;\; N \sim 60$$

[**RED** : + 30−degree extended mask]

(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)
(+ Lattanzi, 2017)

Climbing Scalars : Instability of Istotropy

(Basile, Mourad, AS, 2018)

- COSMOLOGY : the issue is the time evolution of perturbations
- INITIALLY (large η) V is negligible: tensor perturbations evolve as

$$h''_{ij} + \frac{1}{\eta} h'_{ij} + \mathbf{k}^2 h_{ij} = 0$$

$$h_{ij} \sim A_{ij} J_0(k\eta) + B_{ij} Y_0(k\eta) \quad (\mathbf{k} \neq 0)$$

$$h_{ij} \sim A_{ij} + B_{ij} \log\left(\frac{\eta}{\eta_0}\right) \quad (\mathbf{k} = 0)$$

- NOTE: logarithmic growth for k=0 (instability of isotropy) !!
- RESONATES with (Kim, Nishimura, Tsuchiya, 2018) (Anagnostopoulos, Auma, Ito, Nishimura, Papadoulis, 2018)

(HINT of) Dynamical origin of compactification ?

Extensions

Warped M_{p+1} x I x T_{8-p}

(Mourad, AS, 2021)

$$ds^2 = e^{2A(r)}dx^2 + e^{2B(r)}dr^2 + e^{2C(r)}dy^2 \ : \qquad B = (p+1)A + (D-p-2)C$$

Harmonic gauge

General H-T System:

$$\begin{aligned}
A'' &= -\frac{T}{8}\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} + \frac{(7-p)}{16}\, e^{2\left[\beta_p\,\phi+(p+1)\,A\right]}\, H_{p+2}^2 \,, \\
C'' &= -\frac{T}{8}\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} - \frac{(p+1)}{16}\, e^{2\left[\beta_p\,\phi+(p+1)\,A\right]}\, H_{p+2}^2 \,, \\
\phi'' &= T\,\gamma\, e^{2\left[(p+1)A+(8-p)C+\frac{\gamma}{2}\,\phi\right]} + \beta_p\, e^{2\left[\beta_p\,\phi+(p+1)\,A\right]}\, H_{p+2}^2
\end{aligned}$$

+ curvature terms: setup for T-deformed (spherical) black holes or branes

Extensions: Warped $M_{p+1} \times I \times T_{8-p}$

(Mourad, AS, 2021,-23)

$$ds^2 = e^{2A(r)} dx^2 + e^{2B(r)} dr^2 + e^{2C(r)} dy^2 \;:\qquad B = (p+1)A + (D-p-2)C$$

$$S = \frac{1}{2\,(\alpha')^{\frac{D-2}{2}}} \int d^D x \sqrt{-G} \Big\{ e^{-2\phi} \Big[R + 4(\partial\phi)^2 \Big] - T\, e^{\gamma_S \phi} - \frac{1}{2\,(p+2)!}\, e^{-2\beta_S \phi}\, \mathcal{H}^2_{p+2} \Big\}$$

Harmonic gauge

MORE GENERAL SETTING: warped $M_{p+1} \times I \times T_{8-p}$

(relatively simple, yet quite rich)

- INTERNAL T_{8-p} : can be tuned (see e.g. Calabi-Yau), NOT POSSIBLE in AdS x S !
- UPPER BOUNDS on g_s ? (OK) On curvatures ? (NOT OK)
- SHARP CHANGE of behavior across "critical" orientifold value (T-independent aymptotics for $\gamma \leq 3/2$)

- ∃ D=4 solutions with constant g_s with supersymmetry recovered at one end (Effective BPS orientifold) !
- ∃ solutions with tadpole & flux where the tadpole is NEVER dominant

Effective D=4 Orientifolds, I

(Mourad, AS, 2022)

One can "explore" the interval of constant-g_s 4D vacua with a probe brane :

$$\frac{S}{V_3} = -T_3 \int dt\, e^{4A(r(t))} \sqrt{1 - e^{2(B-A)(r(t))}\, \dot{r}(t)^2} + q_3 \int b[r(t)]\, dt$$

$$b(r) = -\frac{1}{4\,\rho\, H}\left[\coth\left(\frac{r}{\rho}\right) - 1\right]$$

$$E = \frac{T_3\ e^{4A(r(t))}}{\sqrt{1 - e^{2(3A+5C)(r(t))}\, \dot{r}(t)^2}} - q_3\, b$$

The probe brane feels the potential below:

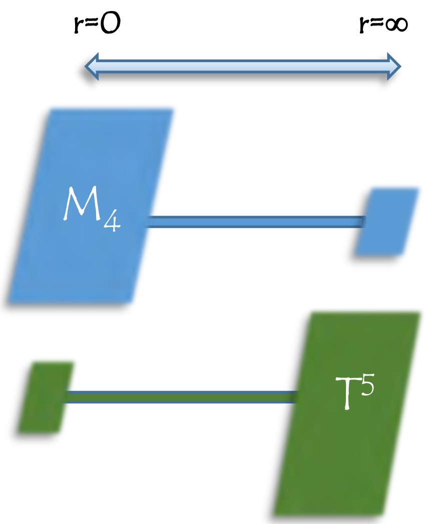
$$V(r) = T_3 e^{4A} - q_3 b = \frac{1}{2\,|H|\,\rho}\left[\frac{T_3}{\sinh\left(\frac{r}{\rho}\right)} + \frac{q_3\,\mathrm{sign}(H)}{2}\left(\coth\left(\frac{r}{\rho}\right) - 1\right)\right]$$

$$\mathbf{V} \sim \boxed{\frac{\mathbf{1}}{\mathbf{r}}\left[\mathbf{T_3} + \frac{\mathbf{q_3}}{\mathbf{2}}\,\mathrm{sign}\,(\mathbf{H})\right]}$$

BPS r=0 endpoint !

Effective D=4 Orientifolds, II

(Mourad, AS, 2309.05268)



Einstein action with York-Gibbons-Hawking term & its variation :

$$
\begin{aligned}
\mathcal{S}_{grav} &= \frac{1}{2\,k_{10}^2}\int_{\mathcal{M}} d^9x\,dr\,\sqrt{-\tilde{g}}\,\mathcal{N}\left[\tilde{R} + \mathcal{K}_{mn}\,\mathcal{K}_{pq}\left(\tilde{g}^{mn}\,\tilde{g}^{pq} - \tilde{g}^{mp}\,\tilde{g}^{nq}\right)\right] \\
\tilde{g}_{mn} &= g_{mn}\,, \qquad \mathcal{N}_m = g_{mr}\,, \qquad \mathcal{N}^2 + \mathcal{N}^m\,\mathcal{N}_m = g_{rr}\,, \qquad \mathcal{K}_{mn} = \frac{1}{2\,\mathcal{N}}\left(\partial_r\,\tilde{g}_{mn} - \tilde{D}_{(m}\mathcal{N}_{n)}\right) \\
\delta\,\mathcal{S} &= \frac{1}{2\,k_{10}^2}\int_{\mathcal{M}} d^{10}x\,\sqrt{-\tilde{g}}\,\mathcal{N}\,\delta\,g^{MN}\left(G_{MN} - T_{MN}\right) \\
&+ \boxed{\frac{1}{2\,k_{10}^2}\lim_{r^\star\to 0,\,R^\star\to\infty}\int d^9x\,\sqrt{-\tilde{g}}\,\delta\tilde{g}^{mn}\left(\mathcal{K}_{mn} - \tilde{g}_{mn}\,\mathcal{K}\right)\Big|_{r^\star}^{R^\star}} \\
G_{mn} &- T_{mn} + \frac{\delta(r-R^\star)}{\mathcal{N}}\left[\mathcal{K}_{mn} - \tilde{g}_{mn}\,\mathcal{K}\right] - \frac{\delta(r-r^\star)}{\mathcal{N}}\left[\mathcal{K}_{mn} - \tilde{g}_{mn}\,\mathcal{K}\right] = 0\,,
\end{aligned}
$$

∃ CONTACT TERMS corresponding to an effective BPS O3 orientifold at r=0:

$$
\begin{aligned}
G_{\mu\nu} &- T_{\mu\nu} + H\,\tilde{g}_{\mu\nu}\sqrt{-\det\,\tilde{g}_{\mu\nu}}\;\delta(z-z^\star) = 0 \\
\mathcal{S}_T &= \frac{H}{k_{10}^2}\int d^9x\,\sqrt{-\det\,\tilde{g}_{\mu\nu}}\,\Big|_{z^\star}
\end{aligned}
$$

$$
\boxed{T = -\,Q = \frac{\Phi}{k_{10}^2}}
$$

Summarizing

- **Dudas-Mourad vacua:** NEW PARADIGM with broken SUSY, internal BOUNDARIES !
- Strong coupling BUT stable 9D compactifications : (self-adjoint) Schrödinger-like systems
- INTRIGUING instability of isotropy (k=0) in "climbing scalar" Cosmology : 4D by accident?
- COSMOLOGY: climbing & inflation → (lack-of-power [HIGHER tens.-to-scal. ratio] [[& non-Gaussianities?]

- **More general** (warped) compactifications with fluxes:

❖ STABILITY: OK, internal scales NOT FIXED in Minkowski_4 [cfr AdS x S !]

❖ Can determine precisely tension & charge of effective SUSY orientifold at end

❖ [cfr: work on "Dynamical Cobordism"]

(Vafa et al)
(Uranga et al)
(Blumehagen, Font et al)

There is room for stable non-SUSY 4D Minkowski string vacua !

Thank You