

# On non-supersymmetric string models

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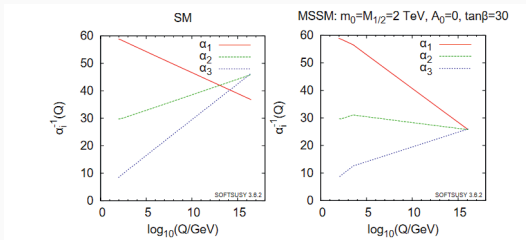
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J. R., I. Florakis and K. Violaris-Gkountonis, [arXiv:1608.04582 \[hep-th\]](#), [arXiv:1703.09272 \[hep-th\]](#), [arXiv:2110.06752 \[hep-th\]](#), [arXiv:2206.09732 \[hep-th\]](#), [arXiv:2310.10438 \[hep-th\]](#).

# Introduction

The Standard Model of particle physics is very successful in interpreting experimental results as of today. However, the Standard Model is not considered as a complete theory as it leaves a number of unanswered questions including the hierarchy problem, neutrino masses, dark matter and does not include gravity.

Supersymmetry (SUSY) is an extensively studied compelling extension of the Standard Model that can help resolving some of these issues, e.g. Stabilization of the weak scale



It can also lead to coupling unification.

SUSY GUTs combine the ideas of SUSY and unification. Moreover, they can explain charge quantization, accommodate family structure, neutrino masses and mixings, baryon asymmetry and provide a dark matter candidates.

Well studied SUSY GUTs include  $SO(10)$ ,  $SU(5)$ , flipped  $SU(5)$ , Pati–Salam  $SU(4) \times SU(2)_L \times SU(2)_R$  models.

However, the longevity of the proton,  $\tau_p > 10^{34}$  years, has become a serious challenge for SUSY GUTs.

# String Theory

String theory is our best candidate for a consistent theory of quantum gravity that can incorporate gauge interactions including the Standard Model of Particle Physics.

In the framework of string theory the point particles are replaced with extended objects called strings.



point particle

open string

closed string

Particles correspond to vibrations of a string.

# The size and the scale of strings

Assuming string theory describes quantum gravity it must involve three fundamental constants:

$c$  : the speed of light

$\hbar$  : Planck's constant

$G$  : Newton's gravitational constant

From dimensional analysis there is a unique way to form a length

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-33} \text{cm (Planck's length) ,}$$

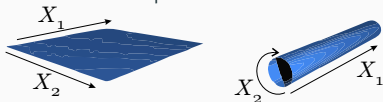
and a mass

$$m_P = \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19} \text{GeV}/c^2 \text{ (Planck's mass) .}$$

# String Phenomenology

The appearance of both gravitational and gauge degrees of freedom in the spectrum makes string theory a candidate theory for the unification of all interactions.

Of course string theory requires  $D = 9 + 1 = 10$  space-time dimensions. Going down to  $3+1=4$  dimensions requires **compactification** of the 6 spacial dimensions.



**String phenomenology** focuses on the construction and study of phenomenological features of string derived gauge models. These include extensions of the SM or GUTs that comprise the SM. The research in this field has yielded low energy effective models with realistic characteristics. These include the  $SU(3)^3$ ,  $SU(5) \times U(1)$  and Pati-Salam  $SU(4) \times SU(2)_R \times SU(2)_L$  models. All these models exhibit  $N = 1$  space-time supersymmetry.

# Non-supersymmetric strings

Space-time supersymmetry is not required for consistency in string theory.

From the early days of the heterotic strings in 10D it was known that they comprise both the supersymmetric  $E_8 \times E_8$  and  $SO(32)$  models and several non-supersymmetric models: e.g.  $SO(32)$ ,  $U(16)$ ,  $SO(8) \times SO(24)$ ,  $SO(16) \times SO(16)$ ,  $(E_7 \times SU(2))^2$ . Out of them only  $SO(16) \times SO(16)$  is tachyon free.

However, non-supersymmetric string models face serious challenges:

- Tachyon instabilities.
- Cosmological constant problems.

Recent developments provide some interesting solutions to these issues. I will restrict here in the framework of weakly coupled heterotic strings.

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\* see e.g. S. Abel, K. R. Dienes and E. Mavroudi (2015,2017) , J. R. and I. Florakis (2016,2017) , Y. Sugawara, T. Wada (2016) , A. Lukas, Z. Lalak and E. E. Svanes (2015) , S.G. Nibbelink, O. Loukas, A. Mütter, E. Parr, P. K. S. Vaudrevange (2017), Faraggi et al (2020) , T. Coudarchet, E. Dudas, H. Partouche (2021)...

# Coordinate dependent compactifications

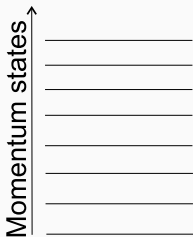
Supersymmetry may be spontaneously broken within a string theory setup admitting an exact worldsheet description via coordinate-dependent compactifications which essentially realise the stringy analogue of the Scherk-Schwarz mechanism. A (minimal) implementation of a stringy Scherk-Schwartz mechanism requires an extra dimension  $X^5$  and a conserved charge  $Q$ . Upon compactification

$$\Phi(X^5 + 2\pi R) = e^{iQ} \Phi(X^5)$$

we obtain a shifted tower of Kaluza-Klein states for charged fields, starting at

$$M_{KK} = \frac{|Q|}{2\pi R}$$

$$\Phi(X^5) = e^{\frac{iQX^5}{2\pi R}} \sum_{n \in \mathbb{Z}} \Phi_n e^{inX^5/R}$$





## Coordinate dependent compactifications

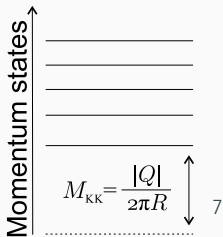
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# Coordinate dependent compactifications

Choosing

$$Q = \text{Fermion Number}$$

Leads to different masses for fermions-bosons (lying in the same supermultiplet) and thus to spontaneous breaking of supersymmetry.

SUSY breaking related to the compactification radius associated to the Scherk–Schwartz mechanism

$$m_{3/2} \sim \frac{1}{R}$$

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see e.g.

J. Scherk and J. H. Schwarz (1978,1979) , R. Rohm (1984) , C. Kounnas and M. Porrati (1988) , S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner (1989) , C. Kounnas and B. Rostand (1990) , C. Kounnas, H. Partouche (2017)

# Gravitino mass

We focus on compactifications of the six internal dimensions in three separate two-tori parametrised by the  $T^{(i)}, U^{(i)}$ ,  $i = 1, 2, 3$  moduli. For simplicity, we will assume that the Scherk–Schwartz mechanism is realised utilising the first torus  $T^{(1)} = T_1^{(1)} + iT_2^{(1)}, U^{(1)} = U_1^{(1)} + iU_2^{(1)}$ .

At tree level the **gravitino receives a mass**, e.g.

$$m_{3/2} = \frac{|U^{(1)}|}{\sqrt{T_2^{(1)} U_2^{(1)}}} = \frac{1}{R_1},$$

for a square torus:  $T = \imath R_1 R_2, U = \imath R_2 / R_1$ .

Moreover, at tree level, all  $T^{(i)}, U^{(i)}$  **moduli remain massless**.

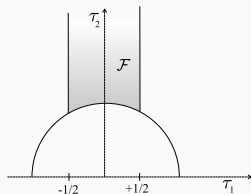
At  $R_1 \rightarrow \infty$  we have  $m_{3/2} = 0$  and the supersymmetry is restored.

# One loop potential

The effective potential at one loop, as a function moduli  $t_l = T^{(i)}, U^{(i)}$ , is obtained by integrating the string partition function  $Z(\tau_1, \tau_2; t_l)$  over the worldsheet torus  $\Sigma_1$

$$V_{\text{one-loop}}(t_l) = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3} Z(\tau, \bar{\tau}; t_l),$$

where  $\mathcal{F}$  is the fundamental domain.



For given values of the moduli

$$Z = \sum_{\substack{n \in \mathbb{Z}/2 \\ n \geq -1/2}} \sum_{m \in \mathbb{Z}} Z_{n,m} q_r^n q_i^m = \sum_{\substack{n \in \mathbb{Z}/2 \\ n \geq -1/2}} \left[ \sum_{m=-[n]-1}^{[n]+2} Z_{n,m} q_i^m \right] q_r^n.$$

where  $q_r = e^{-2\pi\tau_2}$  and  $q_i = e^{2\pi i\tau_1}$ .

# Partition function

The string partition function can be recast in the form

$$\begin{aligned}
 Z = & \frac{1}{\eta^2 \bar{\eta}^2} \frac{1}{2^4} \sum_{\substack{h_1, h_2, H, H' \\ g_1, g_2, G, G'}} \frac{1}{2^3} \sum_{\substack{a, k, \rho \\ b, \ell, \sigma}} \frac{1}{2^3} \sum_{\substack{H_1, H_2, H_3 \\ G_1, G_2, G_3}} (-1)^{a+b+HG+H'G'+\Phi} \\
 & \times \frac{\vartheta \left[ \begin{smallmatrix} a \\ b \end{smallmatrix} \right]}{\eta} \frac{\vartheta \left[ \begin{smallmatrix} a+h_1 \\ b+g_1 \end{smallmatrix} \right]}{\eta} \frac{\vartheta \left[ \begin{smallmatrix} a+h_2 \\ b+g_2 \end{smallmatrix} \right]}{\eta} \frac{\vartheta \left[ \begin{smallmatrix} a-h_1-h_2 \\ b-g_1-g_2 \end{smallmatrix} \right]}{\eta} \\
 & \times \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k \\ \ell \end{smallmatrix} \right]^3}{\bar{\eta}^3} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k+H' \\ \ell+G' \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k-H' \\ \ell-G' \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k+h_1 \\ \ell+g_1 \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k+h_2 \\ \ell+g_2 \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} k-h_1-h_2 \\ \ell-g_1-g_2 \end{smallmatrix} \right]}{\bar{\eta}} \\
 & \times \frac{\bar{\vartheta} \left[ \begin{smallmatrix} \rho+H' \\ \sigma+G' \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} \rho-H' \\ \sigma-G' \end{smallmatrix} \right]}{\bar{\eta}} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} \rho \\ \sigma \end{smallmatrix} \right]^2}{\bar{\eta}^2} \frac{\bar{\vartheta} \left[ \begin{smallmatrix} \rho+H \\ \sigma+G \end{smallmatrix} \right]^4}{\bar{\eta}^4} \\
 & \times \frac{\Gamma_{2,2}^{(1)} \left[ \begin{smallmatrix} H_1 \\ G_1 \end{smallmatrix} \middle| \begin{smallmatrix} h_1 \\ g_1 \end{smallmatrix} \right] (T^{(1)}, U^{(1)})}{\eta^2 \bar{\eta}^2} \frac{\Gamma_{2,2}^{(2)} \left[ \begin{smallmatrix} H_2 \\ G_2 \end{smallmatrix} \middle| \begin{smallmatrix} h_2 \\ g_2 \end{smallmatrix} \right] (T^{(2)}, U^{(2)})}{\eta^2 \bar{\eta}^2} \frac{\Gamma_{2,2}^{(3)} \left[ \begin{smallmatrix} H_3 \\ G_3 \end{smallmatrix} \middle| \begin{smallmatrix} h_1+h_2 \\ g_1+g_2 \end{smallmatrix} \right] (T^{(3)}, U^{(3)})}{\eta^2 \bar{\eta}^2},
 \end{aligned}$$

where  $T^{(i)} = T_1^{(i)} + iT_2^{(i)}$ ,  $U^{(i)} = U_1^{(i)} + iU_2^{(i)}$  are the moduli of the three two tori,  $\eta(\tau)$  is the Dedekind eta function and  $\vartheta \left[ \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix} \right](\tau)$  stand for the Jacobi theta functions.

## Twisted/shifted lattices

The Scherk–Schwarz breaking is implemented utilising orbifold shifts parametrised by  $G_i, H_i, i = 1, 2, 3$

$$\Gamma_{2,2}^{[H_i|h]}(T, U) = \begin{cases} \left| \frac{2\eta^3}{\vartheta[1-h]} \right|^2 & , (H_i, G_i) = (0, 0) \text{ or } (H_i, G_i) = (h, g) \\ \Gamma_{2,2}^{\text{shift}[H_i]}(T, U) & , h = g = 0 \\ 0 & , \text{otherwise} \end{cases}$$

$$\Gamma_{2,2}^{\text{shift}[H_i]}(T, U) = \sum_{\substack{m_1, m_2 \\ n_1, n_2}} (-1)^{G_i(m_1+n_2)} q^{\frac{1}{4}|P_L|^2} \bar{q}^{\frac{1}{4}|P_R|^2} ,$$

with

$$P_L = \frac{m_2 + \frac{H_i}{2} - Um_1 + T(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2 U_2}} ,$$
$$P_R = \frac{m_2 + \frac{H_i}{2} - Um_1 + \bar{T}(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2 U_2}} .$$

## One loop potential: Large volume limit

The asymptotic behaviour of the one loop potential is

$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T, U) = -\frac{(n_B - n_F)}{2^4 \pi^7 T_2^2} \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^3}{|m_1 + \frac{1}{2} + U m_2|^6} + \mathcal{O}\left(e^{-\sqrt{2\pi T_2}}\right)$$

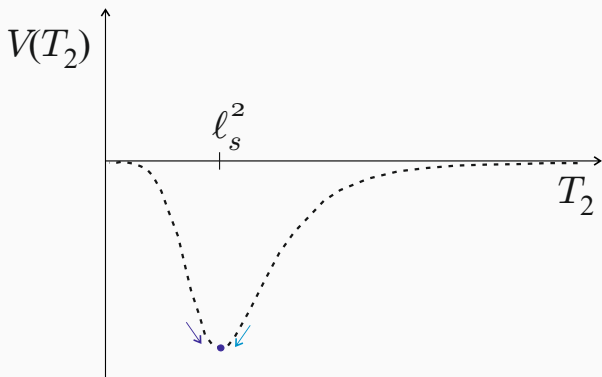
$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T, U) = \xi(U) \frac{(n_B - n_F)}{T_2^2} + \text{exponentially suppressed}$$

where  $\xi(U)$  does not depend on the radius and  $T_2 = R^2$  for a square torus. The integers  $n_B, n_F$  stand for the **number of massless** bosonic and fermionic degrees of freedom respectively.

Cosmological constant is exponentially small in the large  $R$  limit for models with fermion-boson degeneracy  $n_B = n_F$  or **super-no-scale models**, termed so by Costas Kounnas an excellent physicist and great friend who passed away last year.

## Shape of the potential

The self-dual points under the duality symmetry  $T_2 \rightarrow 1/T_2$  correspond to extrema of the potential  $V(T_2) = V(\ell_s^2/T_2)$ . Typically, the potential is of the form

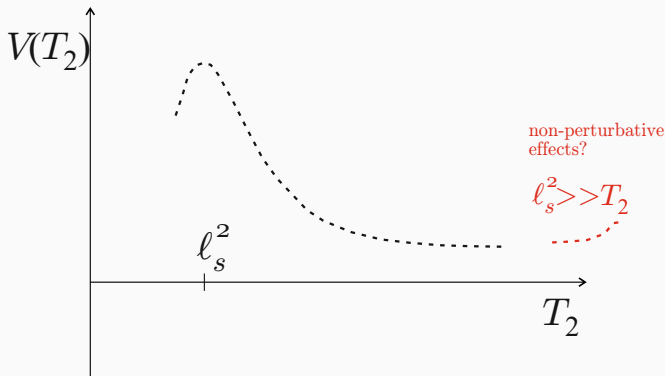


This leads to SUSY breaking at the string scale.



# Shape of the potential

It has been shown that there exist models with potentials of the form



This brings the possibility of supersymmetry breaking at a low scale provided a mechanism for modulus stabilization at  $T_2 \gg l_s^2$ .

## Realistic non-supersymmetric vacua ?

We consider a big class of  $\mathbb{Z}_2 \times \mathbb{Z}_2$  non-supersymmetric heterotic string vacua with SUSY broken spontaneously via the Scherk–Schwartz mechanism. Study their chirality, spectra and moduli potential.

To this end, we utilise both the [free fermionic formulation](#) and [orbifold formulation](#) of the heterotic string. In the former we have full control of the spectrum in the latter we have explicit moduli dependence. To interpolate between the two formalisms we use a detailed map recently reported in [J.R. and I. Florakis, arXiv:2310.10438 \[hep-th\]](#) .

# Free fermionic formulation of the heterotic string

The heterotic string is a hybrid construction that combines the 10-dimensional superstring with the 26-dimensional bosonic string.

In the free fermionic formulation of the heterotic string all world-sheet bosonic coordinates are fermionized (except the ones associated with 4D space-time). [World-sheet supersymmetry](#) is preserved as it is [non-linearly realized](#) among left moving fermions. In the standard notation the fermionic coordinates in the light-cone gauge are:

$$\begin{array}{l} \text{Left:} \\ \text{Right:} \end{array} \quad \begin{array}{l} \psi^\mu, \chi^{1,\dots,6}, \\ \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \end{array} \left| \begin{array}{l} y^{1,\dots,6}, \omega^{1,\dots,6} \\ \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8} \end{array} \right.$$

In this framework a model is defined by a set of [basis vectors](#) which encode the parallel transport properties of the fermionic fields along the non-contractible loops of the world-sheet torus, and a set of [phases](#) associated with generalised GSO projections (GGSO).

# The non-supersymmetric Pati–Salam model

Based on “Lepton Number as the Fourth Color”, J. C. Pati and A. Salam (1974)

Gauge symmetry :  $SU(4) \times SU(2)_L \times SU(2)_R$

SM Fermions:

$$F_L(\mathbf{4}, \mathbf{2}, \mathbf{1}) = Q(\mathbf{3}, \mathbf{2}, -1/6) + L(\mathbf{1}, \mathbf{2}, 1/2),$$

$$\bar{F}_R(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = u^c(\bar{\mathbf{3}}, \mathbf{1}, 2/3) + d^c(\bar{\mathbf{3}}, \mathbf{1}, -1/3) + e^c(\mathbf{1}, \mathbf{1}, -1) + \nu^c(\mathbf{1}, \mathbf{1}, 0)$$

Extra triplets:  $(\mathbf{6}, \mathbf{1}, \mathbf{1})$

Pati-Salam Higgs scalars:  $H(\mathbf{4}, \mathbf{1}, \mathbf{2})$

SM Higgs scalars:

$$h(\mathbf{1}, \mathbf{2}, \mathbf{2}) = H_u \left( \mathbf{1}, \mathbf{2}, +\frac{1}{2} \right) + H_d \left( \mathbf{1}, \mathbf{2}, -\frac{1}{2} \right)$$

# Pati–Salam string models

A class of Pati-Salam models can be generated by the basis

$$\beta_1 = \mathbf{1} = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\},$$

$$\beta_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$\beta_3 = T_1 = \{y^{12}, \omega^{12} | \bar{y}^{12}, \bar{\omega}^{12}\},$$

$$\beta_4 = T_2 = \{y^{34}, \omega^{34} | \bar{y}^{34}, \bar{\omega}^{34}\}$$

$$\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\},$$

$$\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^1\},$$

$$\beta_7 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^2\},$$

$$\beta_8 = z_1 = \{\bar{\phi}^{1,\dots,4}\}, \beta_9 = z_2 = \{\bar{\phi}^{5,\dots,8}\}, \beta_{10} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\},$$

and a set of  $10(10 - 1)/2 + 1 = 46$  GGSO phases  $c \begin{bmatrix} \beta_i \\ \beta_j \end{bmatrix} = \pm 1$ .

This class comprises  $2^{46} \approx 7 \times 10^{13}$  models.

Gauge group:

$$G = SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$$

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$$\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\},$$

$$\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^1\},$$

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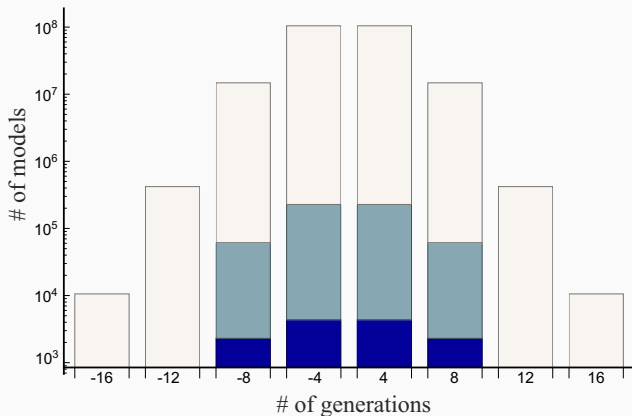
$$G = SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$$

# Phenomenological criteria

- (a) Absence of physical tachyons in the string spectrum
- (b) Existence of complete chiral fermion generations
- (c) Existence of Pati–Salam and SM symmetry breaking scalar Higgs fields
- (d) Absence of observable gauge group enhancements
- (e) Vector-like fractionally charged exotic states
- (f) Consistency with the Scherk–Schwarz SUSY breaking
- (g) Compliance with the super-no-scale condition, that is translated to equality of the fermionic and bosonic degrees of freedom

# Phenomenologically promising Pati–Salam string models

A comprehensive computer scan over the full parameter space ( $1.7 \times 10^{10}$  models) yields

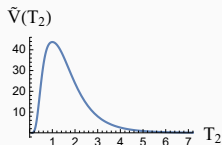


Light shaded bars: (a)-(c)  $2.4 \times 10^8$  models, Medium shaded bars  
(a)-(g)  $5.6 \times 10^5$  models, Dark shading bars:  $1.4 \times 10^4$  models

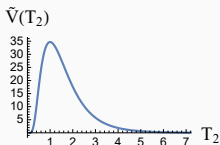


# One-loop potentials

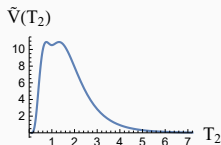
A1: 1536 models



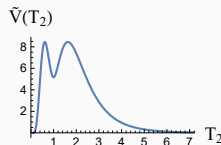
A2: 1536 models



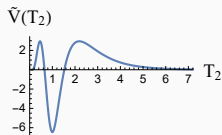
B1: 8448 models



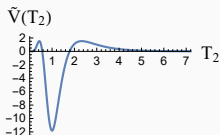
B2: 1792 models



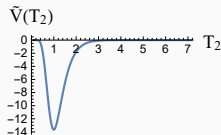
C1: 75264 models



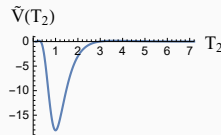
C2: 71936 models



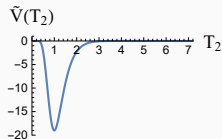
C3: 6272 models



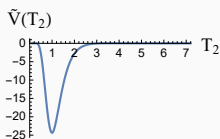
C4: 3840 models



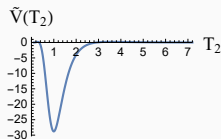
C5: 68096 models



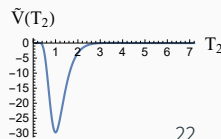
C6: 3584 models



C7: 8448 models



C8: 28032 models



## Three-generation models

Three generation models in the context of  $\mathbb{Z}_2 \times \mathbb{Z}_2$  orbifolds can be obtained only utilising real fermions. To this end, we introduce additional vectors that separate internal fermions

$$v_1 = \mathbb{1} = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\},$$

$$v_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\}, \quad v_{2+i} = e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, \quad i = 1, \dots, 6,$$

$$v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^3, y^4, y^5, y^6 | \bar{y}^3, \bar{y}^4, \bar{y}^5, \bar{y}^6, \bar{\psi}^{1,\dots,5}, \bar{\eta}^1\},$$

$$v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^1, y^2, y^5, y^6 | \bar{y}^1, \bar{y}^2, \bar{y}^5, \bar{y}^6, \bar{\psi}^{1,\dots,5}, \bar{\eta}^2\},$$

$$v_{11} = z_1 = \{\bar{\phi}^{1,\dots,4}\}, \quad v_{12} = z_2 = \{\bar{\phi}^{5,\dots,8}\}, \quad v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\},$$

For generic choices of the GGSO projections this class comprises a huge number of  $2^{\frac{13(13-1)}{2}+1} \sim 6 \times 10^{23}$  heterotic string models that exhibit

$$\mathcal{G} = SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$$

gauge symmetry.

# Large volume formula - Super-no-scale constraints

Depending on the implementation of the Scherk–Schwarz mechanism **new constraints arise**. In the simplest case, the large volume limit of the effective potential can be expressed as follows

$$V_{\text{eff}} = \lim_{T_2 \rightarrow \infty} -\frac{63}{2(2\pi)^4 T_2^2} \left[ \frac{1}{2} \sum_{H_2, G_2 \in \mathbb{Z}_2} (-1)^{H_2} C \left[ \begin{smallmatrix} 0, & H_2 \\ 1, & G_2 \end{smallmatrix} \right] E_{\infty}^*(3; U) \right. \\ \left. + \frac{1}{8} \sum_{G_2 \in \mathbb{Z}_2} C \left[ \begin{smallmatrix} 0, & 1 \\ 1, & G_2 \end{smallmatrix} \right] E_{\infty}^*(3; 2U) \right] + \dots$$

where  $E_{\infty}^*(s; z)$  is the zero weight, completed, non-holomorphic Eisenstein series and the ellipses denote exponentially suppressed terms.

A careful examination of this formula, shows that in order to obtain an exponentially suppressed contribution at large  $T_2$ , the coefficients of both Eisenstein series vanish independently.

## Super-no-scale conditions

The last constraint can be expressed as

$$\Sigma(H_2) \equiv \frac{1}{4} \sum_{G_1, G_2=0,1} C \left[ \begin{smallmatrix} 0 \\ G_1, G_2 \end{smallmatrix}, H_2 \right] \quad , \quad H_2 = 0, 1$$

The first condition

$$\Sigma(0) = n_B - n_F = 0$$

is associated with the full massless spectrum of the theory accompanied with a tower of states that tend to become massless at the limit  $T_2 \rightarrow \infty$ ,  $M^2(\Gamma_{2,2}^{(1)}) = |m_2 - Um_1|^2/T_2U_2$ . This is the “super no-scale” condition known in the string literature. The second condition

$$\Sigma(1) = 0$$

is non-trivial. It refers to a subset of massive states, arising from shifted lattice, that also become massless at the  $T_2 \rightarrow \infty$  limit,  $M^2(\Gamma_{2,2}^{(1)}) = |m_2 + \frac{1}{2} - Um_1|^2/T_2U_2$ .

## Three-generation models

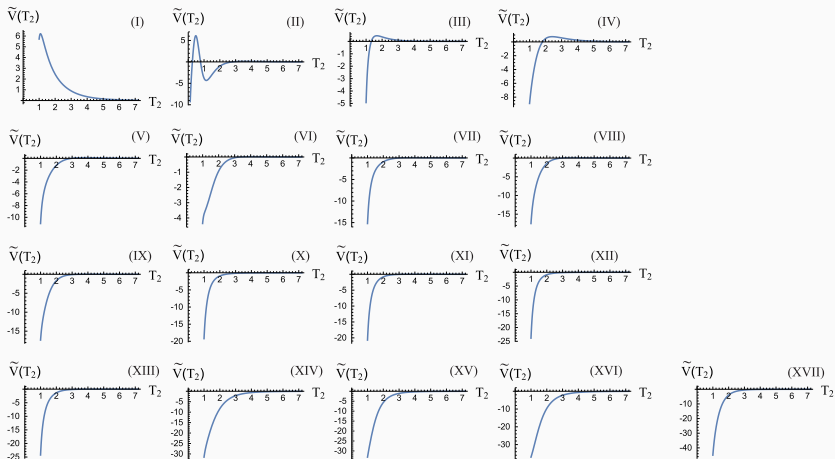
We have performed a detailed investigation of the model parameter space utilising a computer assisted two-stage scan procedure: we first perform a (random) scan and identify  $SO(10)$  configurations compatible with our search criteria. Next, we consider all possible offspring Pati–Salam models generated by each (fertile)  $SO(10)$  configuration and the related GGSO projection phases and check their compatibility with the aforementioned criteria.

In practice, this method allows us to effectively scan a big sample of  $8.1 \times 10^{12}$  models (almost one model in  $10^4$ ) of the full parameter space in about 10 days on a DELL PowerEdge R630 workstation with 32 GB of memory. It turns out that  $8.8 \times 10^6$  models fulfil our phenomenological criteria. Out of these, about 0.1% meet the first super no-scale constraint  $\Sigma(0) = 0$ , while around 21% meet the second super no-scale constraint  $\Sigma(1) = 0$ .

Altogether, we have identified 174 three generation Pati–Salam models that comply with all requirements (one in 50 billions).

# One-loop potential (3 generation models)

Based on the analysis of their partition functions the one-loop effective potentials of the 3-generation models fall into 17 distinct classes



The rescaled one-loop effective potential  $\tilde{V}(T_2) = 2(2\pi)^4 V(T_2)$  for each of the 17 classes of 3-generation models satisfying all requirements.

# Conclusions

We have shown the existence of non-supersymmetric heterotic string models with Pati-Salam gauge symmetry exhibiting interesting phenomenological characteristics:

- Chiral spectra with 3 generations, Pati-Salam and Standard Model breaking Higgs scalars. Vector-like fractional charge exotics.
- SUSY breaking via the Scherk-Schwarz mechanism at scales  $m_{3/2} \sim 1/R$  that could be much smaller than  $M_{string}$ .
- Fulfil the  $n_B = n_F$  **super-no-scale** requirement that leads to exponentially small (and possibly positive) vacuum energy at the large volume limit.
- That has to be supplemented in the case of real fermions, necessary to obtain 3 generations at least in the case of  $\mathbb{Z}_2 \times \mathbb{Z}_2$  models, by an additional model dependent constraints.

These developments pave the way to the construction of a non-supersymmetric Standard Model from weakly coupled heterotic string theory.

Thank you



## Appendix: Non-holomorphic Eisenstein series

$$E_{\infty}^*(s; z) = \frac{1}{2} \zeta^*(2s) \sum_{\substack{c, d \in \mathbb{Z} \\ (2c, d) = 1}} \frac{(\operatorname{Im} z)^s}{|2cz + d|^{2s}}, \quad \zeta^*(s) = \pi^{-s/2} \Gamma(s/2) \zeta(s).$$

where  $(2c, d) = 1$  restricts the summation to coprime pairs.