Two dimensional black holes and matrix quantum mechanics

Olga Papadoulaki

Perimeter Institute

Work in collaboration with P. Betzios

arXiv:1711.04369, arXiv:2210.11484

and arXiv:2312.02257 w. P. Betzios and N. Gaddam

Xmas Theoretical Physics Workshop @Athens 2023

Microscopic Models for Black Holes

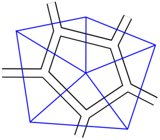
- Black Holes constitute some of the most fascinating but confusing objects in (Quantum) Gravity
- Pertinent questions: The Information Paradox, Microstructure, Interior, Nature of their Singularities
- It is possible that their complete understanding will involve "a complete theory of Quantum Gravity"
- It is reasonable to attack and resolve these questions in lower dimensions: Even in 2d there do exist manifolds with similar characteristics - horizons and singularities
- Many efforts to construct microscopic models for Black Holes that are "solvable" (at large-N) `[various authors ...]`
- We would like to have such a model in some version of string theory (our best bet for a theory of QGR) - the Black Hole background should be the target space of the string

The scope of this talk is to describe such models in 2-dim string theories that are dual to MQM models

Matrix Models and large N

- In the 't Hooft limit $N_c \to \infty$ QCD becomes a theory of large random matrices
- In the Feynman graphs of gluons the matrix structure is represented in the propagators having double lines where each line carries one of the two indices of the matrix (these are the ribbon graphs)

- Probability amplitudes can be formulated as sums over Ribbon graphs, which describe the dynamics of surfaces



- This was the inspiration for studying surfaces using random matrices

`Brezin-Itzykson-Parisi-Zuber`

Matrix Quantum Mechanics (MQM)

Reviews by: [Kazakov, Ginsparg-Moore, Klebanov, Martinec,...]

- MQM (gauged) is a $0+1$ dimensional quantum mechanical theory of $N \times N$ Hermitian matrices $M(t)$ and a non dynamical gauge field $A(t)$

$$S = \int_{t_{in}}^{t_f} dt \, \mathrm{Tr} \left(\frac{1}{2} (D_t M)^2 - V(M) \right)$$

- Diagonalise $M(t) = U(t)\Lambda(t)U^\dagger(t)$, with $\Lambda(t)$ diagonal and $U(t)$ unitary $\Rightarrow$ Presence of a Vandermonde determinant $\Delta(\Lambda) = \prod_{i<j}(\lambda_i - \lambda_j)$ i.e. The Hamiltonian is

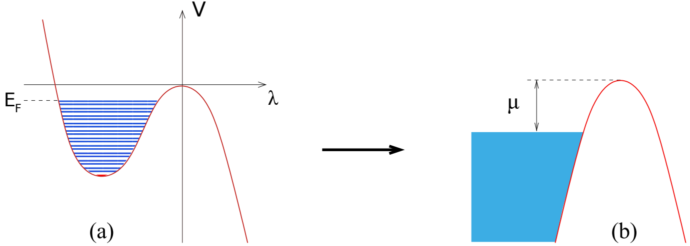
$$H = -\frac{1}{2\Delta^2(\lambda)} \frac{d}{d\lambda_i} \Delta^2(\lambda) \frac{d}{d\lambda_i} + \sum_{i<j} \frac{J_{ij} J_{ji}}{(\lambda_i - \lambda_j)^2} + V(\lambda_i) \; ,$$

- J_{ij} are "momenta" conjugate to $SU(N)$ rotations
- Gauged model: Impose the Gauss-law constraint $\delta S / \delta A = i[M, \dot{M}] \sim J = 0$ (*singlet sector projection*)
- Redefining the wavefunction as $\tilde{\Psi}(\lambda) \equiv \Delta(\lambda)\Psi(\lambda)$, the Schrödinger equation describes N non interacting fermions in a potential $V(\lambda)$

Double scaling limit in MQM

[Kazakov-Migdal...]

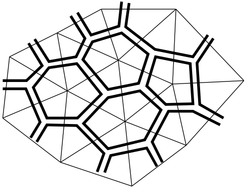
- Consider an initial state where the energy levels are populated up to some Fermi energy E_F below the top of the barrier, and send $\hbar \to 0$, $N \to \infty$ ("WKB").
- We focus near the quadratic maximum of the potential. We hold $\mu = -E_F/\hbar$ fixed in the limit
- The system is just quantum mechanics of free fermions in an inverted harmonic oscillator potential, with states filled up to $-\mu < 0$



Connection with the quantum gravity path integral

The connection with 2d random surfaces (string theory) is via this double scaling limit, where we tune our system near criticality.

- The double scaling limit produces smooth surfaces out of the Matrix fat-graphs while at the same time keeping all higher genera. The genus expansion is (roughly) in terms of $g_{st} \sim 1/\mu$



- The matrix eigenvalues λ introduce an emergent coordinate ϕ
- An archetypal form of Holography: "Geometry from the Matrix Eigenvalues"
- The QG theory is called $c = 1$ *Liouville theory*. The target space is $2d$: a time direction t and an emergent space direction ϕ. The background is asymptotically flat and contains a linear dilaton and exponential tachyon

$$S_L = \frac{1}{4\pi} \int d^2\sigma \sqrt{\hat{g}} \left[\hat{g}^{ab} (\partial_a X \partial_b X + \partial_a \phi \partial_b \phi) + 2\hat{R}\phi + 4\pi\mu e^{2\phi} \right]$$

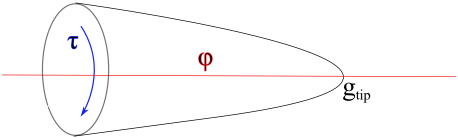
- What about other string backgrounds? Black holes?

(Euclidean) 2d black hole

Elitzur-Forge-Rabinovici, Mandal-Sengupta-Wadia

- (ST) low energy effective action: $S = \int d^2x\sqrt{G}e^{-2\Phi}(-R - 4\nabla\Phi^2 - \frac{8}{\alpha'})$
- $2d$ "Cigar" solution ($-\infty < \phi < 0$ and $\phi = 0$ is the tip of the cigar)

$$ds^2 = (1 - e^{2Q\phi})d\tau^2 + \frac{d\phi^2}{1 - e^{2Q\phi}}\,, \quad \Phi = \Phi_0 + Q\phi\,,\ Q^2 = 4/\alpha'$$



- $g_{tip} = e^{\Phi_0}$ is a parameter of the solution
- The weak string coupling region is at the "boundary of the cigar"
- The string coupling becomes strongest near the tip
- It has a fixed temperature

- The entropy and mass of the black hole scale as [Gibbons, Nappi, Kazakov-Tseytlin]

$$S \sim M \sim \frac{1}{g_{tip}^2} \sim e^{-2\Phi_0}$$

Exact (CFT) $2d$ black hole background

Witten, Dijkgraaf-Verlinde^2, ...

- The Euclidean black hole worldsheet CFT can be described using a WZW coset model, with the target space coset being

$$H_3^+/U(1)\,, \qquad H_3^+ = \frac{SL(2,C)}{SU(2)}$$

- The compactification radius of the Euclidean black hole is $R^2 = k\alpha'$, with k being also the level of the associated $SL(2,R)_k/U(1)$ WZW model
- The $SL(2,R)_k$ algebra dictates the central charge $c_{\mathsf{cigar}} = \frac{3k}{k-2} - 1$, and the spectrum of primaries for the coset CFT
- Conformal invariance of the worldsheet theory requires $k = 9/4$
- In order to change the radius we need to append to this model additional degrees of freedom of an "internal" CFT

FZZ duality and the black-hole string transition

- FZZ duality: coset CFT is dual to *Sine-Liouville theory*, Fateev-Zamolodchikov[2]

- Sine-Liouville:
$L_{SL} = \frac{1}{4\pi}\Big((\partial x)^2 + (\partial\phi)^2 + Q\hat{R}\phi + \xi e^{b\phi}\cos R\,(x_L - x_R)\Big)$

- Matches with the coset when the radius of x is $R = \sqrt{k}$ and

$$c_{\text{cigar}} = c_{SL} = 2 + 6Q^2 \;\Rightarrow\quad Q^2 = \frac{1}{k-2}\,,\quad b = \sqrt{k-2} = \frac{1}{Q}$$

- The asymptotic weakly coupled region is $\phi \to -\infty$

- The strongly coupled region is for $\phi \to \infty$ near the potential wall

- The duality is a strong-weak duality

- For small radii, the black hole is better described in terms of a condensate of winding-strings, while for large radii the black hole description is the simplest ($Q \to \infty\,, k \to 2$ vs. $k \to \infty\,, Q \to 0$)

- A transition between such two behaviours is usually termed the black hole/string transition, Susskind, Horowitz-Polchinski, Sen, Kutasov ...

Sine-Gordon coupled to $2d$ QG

[Kazakov-Kostov-Kutasov, Moore]

- Deform the linear dilaton background ($c = 1$ Liouville/MQM) including in the Lagrangian the first winding operators - $\xi(\mathcal{T}_{+R} + \mathcal{T}_{-R})$

$$L_d = \frac{1}{4\pi}\left((\partial x)^2 + (\partial\phi)^2 + 2R\phi + \mu e^{2\phi} + \xi e^{(2-R)\phi} \cos R\,(x_L - x_R)\right)$$

  This is a Sine-Gordon model coupled to $2d$ quantum gravity (μ-term)

- Approach the $\xi \to \infty$, $\mu \to 0$ region of parameters (SL-point)
- The target space winding modes correspond to vortices on the worldsheet
- What are these winding/vortex deformations in Matrix Quantum Mechanics?

Matrix Model with Wilson loop/winding deformations

[Kazakov-Kostov-Kutasov]

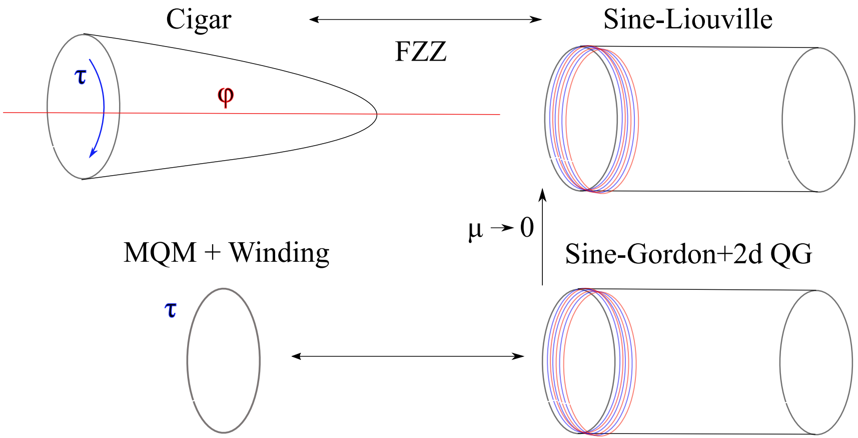
- The winding modes correspond to the inclusion of (backreacting) Wilson loops in the original gauged matrix model
- The mapping is $\mathcal{T}_{nR} \leftrightarrow \mathrm{Tr}P\left[e^{i\oint A}\right]^n = \mathrm{Tr}U^n$ between Liouville winding and matrix model operators
- A quantity that one can describe on both sides of the duality is the general winding/vortex perturbed free energy

$$F(\mu, R; t) = \left\langle e^{\sum_n t_n(\mathcal{T}_{nR} + \mathcal{T}_{-nR})} \right\rangle_c = \log \left\langle e^{\sum_n \tilde{t}_n \left(\mathrm{Tr}U^n + \mathrm{Tr}U^{-n}\right)} \right\rangle_U^{MQM}$$

(the average contains only the connected contributions of the winding mode correlators)

- The `Kazakov-Kostov-Kutasov` model dual to SG/SL theory has only the first winding modes $t_1 = t_{-1} = \xi$ turned on

The duality web

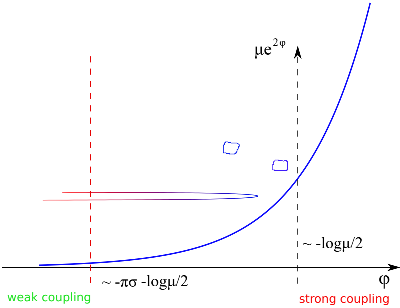


- Caveat I: From the gravity side it seems that we cannot change the radius of the cigar, while in the MQM/SG/SL description we can tune it
- Caveat II: These dualities are inherently Euclidean. Lorentzian description for the deformation? (long-strings...)

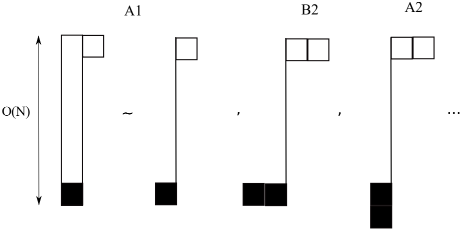
Non-singlet sectors in MQM - Long strings in Liouville

- The singlet sector of gauged Matrix Quantum Mechanics (MQM) cannot describe black holes [Martinec, Karczmarek-Maldacena-Strominger ...]

- Proposals that black holes could exist in the non-singlet sector of MQM [Kazakov-Kostov-Kutasov, Klebanov, ...]
- Adjoint representation: related to a long folded string that extends along Liouville [Maldacena, ...]
- Long strings are also related to the presence of FZZT branes (retracted at the weak coupling region)



- States containing n folded strings ⇒ Irreps with a Young-Tableaux of n-boxes and n-anti-boxes [Maldacena]
- Can we keep $SU(N)$ gauged and still have long strings?



A dynamical "N-ZZ N_f-FZZT" matrix model

[Betzios-OP ('17), see also the Spin-Calogero models [Polychronakos]]

- Start with the (gauged) MQM action (N - ZZ/$D0$ branes)
- Introduce open strings between N-ZZ and N_f-FZZT branes, by adding $N_f \times N$ bi-fundamental fields $\chi_{\alpha i}$, $\psi_{\alpha i}$ ("quarks")

$$S_f = \int dt \sum_{\alpha}^{N_f} \mathrm{Tr}\left(i\psi_\alpha^\dagger D_t \psi_\alpha - m_\alpha \psi_\alpha^\dagger \psi_\alpha + i\chi_\alpha^\dagger D_t^* \chi_\alpha - m_\alpha \chi_\alpha^\dagger \chi_\alpha \right) \,,$$

- The $SU(N)$ Gauss' law constraint becomes

$$: i[M \,, \dot{M}]_{ij} :=: J_{ij} := \sum_{\alpha}^{N_f} \left[\psi_{\alpha j}^\dagger \psi_{\alpha i} - \chi_{\alpha i}^\dagger \chi_{\alpha j} \right]$$

- The Hamiltonian is now

$$\hat{H} = \sum_i^N -\frac{1}{2}\frac{\partial^2}{\partial \lambda_i^2} + V(\lambda_i) + \frac{1}{2}\sum_{i \neq j} \frac{J_{ij} J_{ji}}{(\lambda_i - \lambda_j)^2} + \sum_{i,\alpha}^{N,N_f} m_\alpha \psi_{\alpha i}^\dagger \psi_{\alpha i} + m_\alpha \chi_{\alpha i}^\dagger \chi_{\alpha i}$$

- The bi-fundamentals thus "feed" non-trivial representations (J_{ij})

Limit of [Kazakov-Kostov-Kutasov] Matrix Model

- The canonical partition function is expressed in terms of holonomy/Wilson loop zero modes ($U = \mathrm{Tr} P e^{i \oint A}$)

$$Z_N^{(N_f)} \sim \int_{U(N)} \mathcal{D}U \, \frac{\exp\left[N_f \sum_{l=1} \frac{(-1)^{l+1}}{l} e^{-\beta m} \left(\mathrm{Tr} U^l + \mathrm{Tr} U^{-l}\right)\right]}{\exp\left(\sum_l \frac{q^l}{l} \mathrm{Tr} U^l \mathrm{Tr}(U^{-1})^l\right)}$$

- Take a double scaling limit ("heavy/quenched quarks")

$$N_f \to \infty, \quad m \to \infty, \quad \text{with} \quad N_f e^{-\beta m} = \tilde{t}, \quad \text{finite}$$

- The only surviving winding modes in this case: $\exp\left(\tilde{t} \mathrm{Tr} U + \tilde{t} \mathrm{Tr} U^\dagger\right)$, are identical to those studied in the matrix model of [Kazakov-Kostov-Kutasov]
- We can therefore obtain the later by taking a limit of a model that does admit a Lorentzian description and has a natural Liouville (FZZT brane - long string) interpretation

Partition function - Microstates from representations

[Betzios-OP ('22)]

- The grand canonical partition function can be expanded as a statistical sum over representations/partitions (labelled by λ)

$$\mathcal{Z}_{MQM}(\mu, R; \tilde{t}) = \sum_{\lambda} s_\lambda(t_+) s_\lambda(t_-) \langle \lambda | \mathbf{G}(\mu, R) | \lambda \rangle$$

- The operator $\mathbf{G}(\mu, R)$ is the "T-dual of the S-matrix" and incorporates all the MQM dynamics
- In this formalism the microstates of the winding condensate/black hole are manifest ($GL(\infty)$ representations/partitions)
- Our idea is to define a coarse graining/thermodynamic limit, considering large Young diagrams, that acquire a "continuous limiting shape"

  Many inequivalent representations have the same "limiting shape"

Plancherel measure

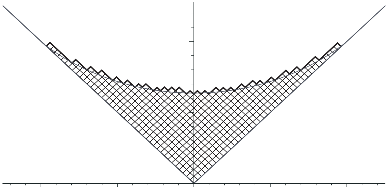
- Turn on only the first winding mode $t^+ = t^- = \xi$, $\Rightarrow$ the Schur measure specialises to "Plancherel measure" on partitions of n

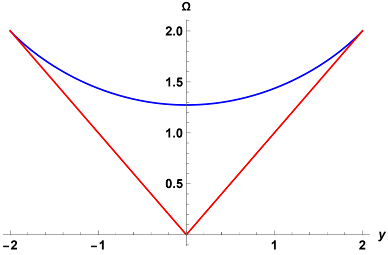
$$\mathfrak{M}_\lambda(\xi) = \sum_{n=0}^{\infty} \frac{\xi^{2n}}{n!} M_n(\lambda)\delta(|\lambda| - n)\,, \quad M_n(\lambda) = \frac{(\dim \lambda)^2}{n!}$$

- As the size of the partitions goes to infinity $n \to \infty$, the Plancherel measure exhibits a Cardy-like growth

$$\lim_{n\to\infty} M_n(\lambda) \sim \exp\left(2\sqrt{n}\right)$$

and concentrates to a universal limiting Young diagram shape the *Vershik-Kerov-Logan-Shepp limiting shape* $\mathbf{\Omega}$





The limiting shape for the complete partition function

- Our limiting shape should be found self consistently by including the contribution of the amplitude $\langle\lambda|\mathbf{G}|\lambda\rangle$ together with the Schur measure
- Solve a minimization problem in the space of highest weights [Douglas-Kazakov, ...] $\Rightarrow$ determine the resolvent and leading shape
- The leading shapes (for $R < 2$) are found to be

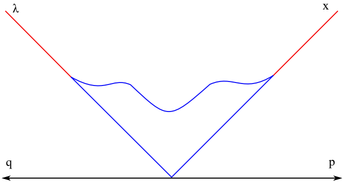


Figure: Left: large ξ_{eff} (small g_s) Right: small ξ_{eff} (large g_s). There is a third order phase transition between the two behaviours

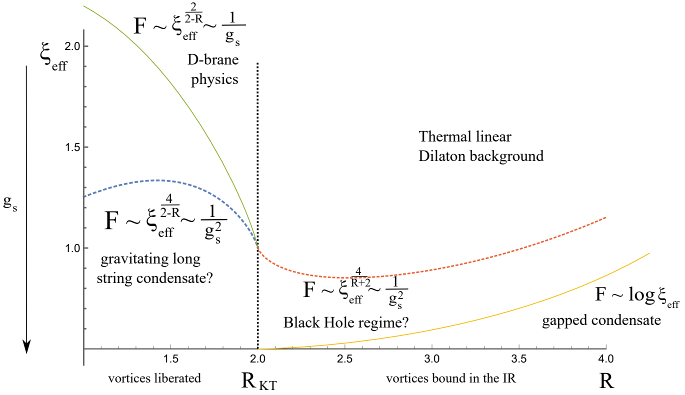
- The genus zero free energy near the transition (right fig.) behaves as

$$\mathcal{F}_0 \simeq -\frac{(2-R)^2}{8}\xi_{eff}^{\frac{4}{2-R}} \sim -\frac{(2-R)^2}{8}\frac{1}{g_s^2}$$

and coincides with the result of [Kazakov-Kostov-Kutasov, Tseytlin]

The phase diagram

Generalising the findings of [Moore, Kazakov-Kostov-Kutasov]



- The dashed lines signal phase transitions
- $T_{KT} \sim 1/R_{KT}$ is the Kosterlitz-Thouless temperature, above which worldsheet vortices get liberated and proliferate
- The continuous lines are regimes of different behaviour (cross-over)

Future Directions

- Analysis of observables that can distinguish a black hole from a gravitating long string condensate (absorption, QNMs, chaos?)
- The $2d$ black hole appears in higher dimensional (non-supersymmetric) asymptotically flat black holes taking a large D limit
  [Emparan-Grumiller-Tanabe ...]
- Can we formulate a matrix model for these black holes? Any connection with the present work?
- Understand the physics of the Black Hole/String transition from a microscopic model, and any possible relation to the wormhole case
- Coarse graining in the space of Young diagrams - Other applications? (Wormholes [Betzios-Kiritsis-OP (21)])
- Supersymmetric examples [Betzios-Gaddam-OP (23)]

Thank You!

Solutions of Type 0A

$$S^E_{eff.} = \int d^2x\sqrt{G}e^{-2\phi}\left(-R - 4(\nabla\phi)^2 - \frac{8}{\alpha'}\right) + \frac{(2\pi)\alpha'}{2}\int d^2x\sqrt{G}F^{\mu\nu}F_{\mu\nu}$$

- We can find the following types of solutions:
- Global AdS_2 solution (wormhole) with constant dilaton and non trivial flux
- Charged asymptotically flat 2d black holes with running dilaton
- Near extremal asymptotically flat 2d black holes with an AdS_2 near horizon region
- By gluing two asymptotically flat near extremal black holes we can produce asymptotically flat wormholes
- The wormhole throat size is of string scale; in order to trust it we need a worldsheet description in terms of a WZW coset model or a matrix model description, as in the uncharged case

Type 0A MQM

- The MQM model for the 0A case is in terms of a rectangular complex matrix

- The Schroedinger equation is

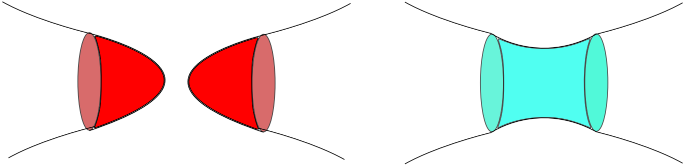
$$\left[\sum_i \left(-\frac{1}{2}\frac{\partial^2}{\partial y_i^2} + \frac{q^2 - 1/4}{y_i^2} - \frac{1}{4\alpha'}y_i^2\right) + \frac{1}{2}\sum_A \sum_{i\neq j} \frac{\tilde{T}_{ij}^A \tilde{T}_{ji}^A}{(y_i - y_j)^2}\right]\tilde{\Psi} = E\tilde{\Psi}$$

- $\tilde{T}_{ij}^A$ are appropriate $U(N)\times U(N)$ generators (A is the group index)

- q measures the charge (flux) of the background

- This extended model has classical entropy thus it is a viable candidate to describe charged black holes and wormholes unlike the singlet sector

Wormhole/ Black Hole Transition



- The transition depends on the difference of the effective electrostatic mass between the two backgrounds $E_{static} = Q\Phi$
- When it is negative the bound wormhole background is dominant. It is a first order phase transition
- To study further quantum and α' corrections, and the fate of the phase transition when including such corrections, we must resort to the more powerful worldsheet CFT or matrix model constructions