Natural Alignment Beyond the Standard Model

Apostolos Pilaftsis

Department of Physics and Astronomy, University of Manchester,
Manchester M13 9PL, United Kingdom

Xmass TH Physics Workshop 2023
Athens Natl. Capodistrian U., 21-23 Dec 2023

Based on:

- P.S.B. Dev, AP, JHEP1412 (2014) 024
- AP, PRD93 (2016) 075012
- N. Darvishi, AP, PRD99 (2019) 115014; PRD101 (2020) 095008 . . .
- N. Darvishi, AP, J.-H. Yu, arXiv:2312.00882

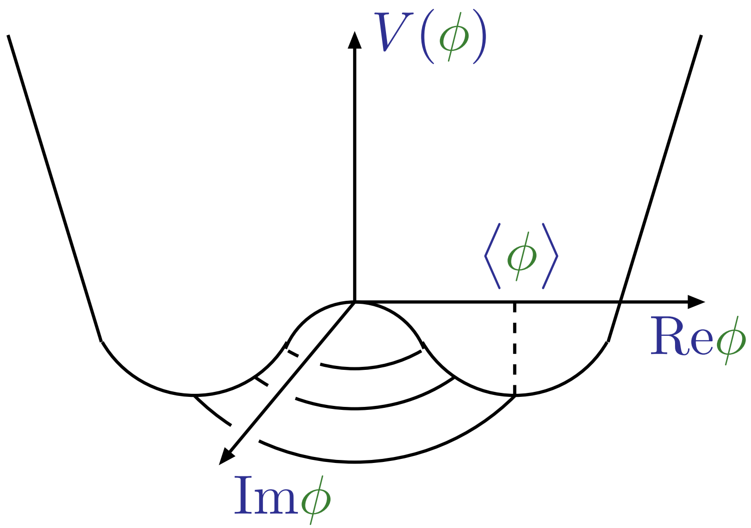
Outline:

- The Standard Theory of Electroweak Symmetry Breaking
- Brief history of symmetries for natural SM alignment
- SM alignment in the 2HDM and Beyond
- Quartic Coupling Unification
- Phenomenology at the LHC
- Summary

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in the SM: $\mathrm{SU}(3)_{\mathrm{colour}} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$

[P. W. Higgs '64; F. Englert, R. Brout '64.]



Higgs potential $V(\phi)$

$$V(\phi) \;=\; -m^2\phi^\dagger\phi \;+\; \lambda(\phi^\dagger\phi)^2\,.$$

Ground state:

$$\langle\phi\rangle \;=\; \sqrt{\frac{m^2}{2\lambda}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

carries weak charge, but no electric charge and colour.

Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

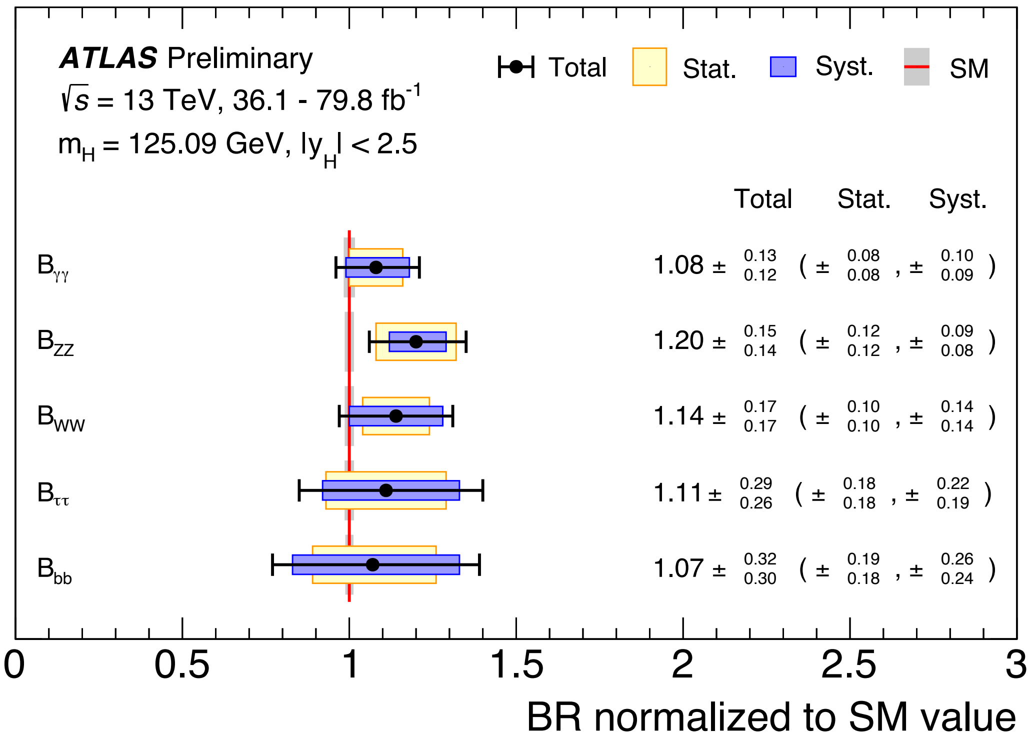
[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

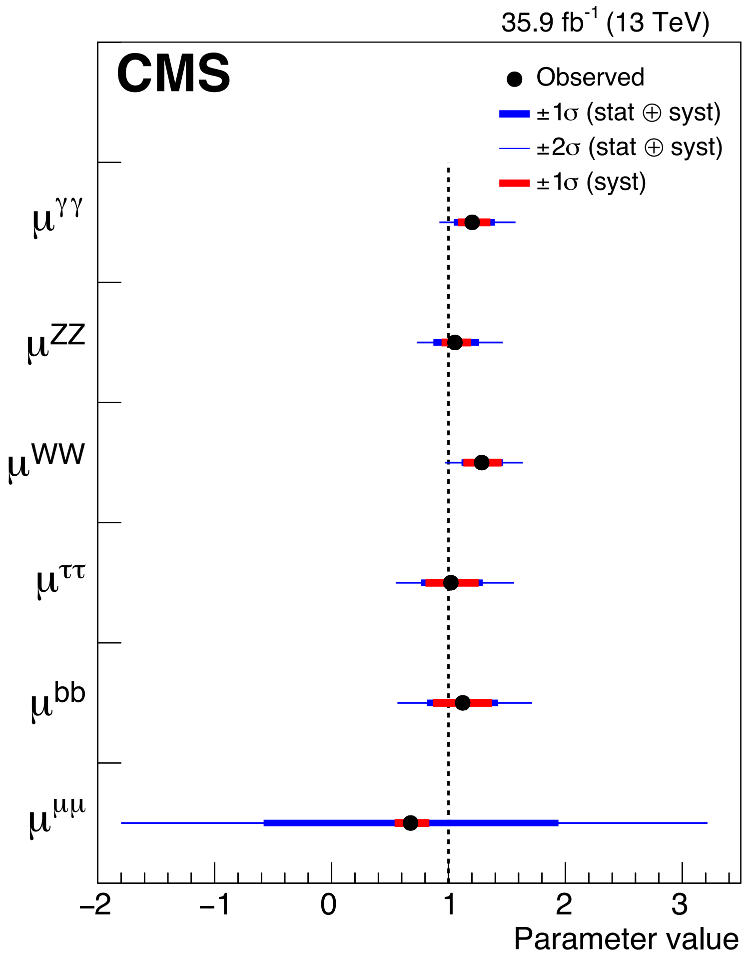
$$\mathbf{\Phi} \;\equiv\; (\phi\,,\, i\sigma^2\phi^*) \quad\mapsto\quad \mathbf{\Phi}' \;\equiv\; U_L\,\mathbf{\Phi}\,U_{\mathrm{C}}\,,$$

with $U_L \in \mathrm{SU}(2)_L$ and $U_{\mathrm{C}} \in \mathrm{SU}(2)_{\mathrm{C}}$, and $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_{\mathrm{C}}/\mathbb{Z}_2 \simeq \mathrm{S}\mathbb{O}(4)$.

On the SM Higgs-Boson at the LHC

ATLAS & CMS Results 2019:





• Brief history of symmetries for natural SM alignment

– Flavour unitarity of the CKM mixing matrix
[Gell-Mann, Levy '60; Cabbibo '63; Kobayashi, Maskawa '73]

– GIM mechanism to explain the smallness of the strangeness-changing interaction at the quantum level (requires the existence of the c-quark)
[Glashow, Iliopoulos, Maiani '70]

– Conditions for diagonal neutral currents in Z-boson interactions to quarks
[Paschos '77]

– Natural diagonal neutral currents in Z- **&** multi-Higgs-boson interactions to quarks
[Glashow, Weinberg '77]

– Renormalizable models with partial flavour non-conservation at tree level (GIM suppressed).
[Branco, Grimus, Lavoura '96]

– Yukawa alignment in the 2HDM broken by RG effects (no global symmetry protected)
[Pich, Tuzon '09]

– Natural Alignment Beyond the Standard Model
[Dev, Pilaftsis '14, AP '16 & this talk]

• SM Alignment in the 2HDM and Beyond

• 2HDM potential [TD Lee '73; AP, C Wagner '99; Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
\mathrm{V} \;=\; & -\mu_1^2(\phi_1^\dagger\phi_1) \;-\; \mu_2^2(\phi_2^\dagger\phi_2) \;-\; m_{12}^2(\phi_1^\dagger\phi_2) \;-\; m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 \;+\; \lambda_2(\phi_2^\dagger\phi_2)^2 \;+\; \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \;+\; \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 \;+\; \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \;+\; \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) \;+\; \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) \;+\; \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)\;.
\end{aligned}
$$

• Physical (CP-conserving) spectrum:

CP-even Higgs bosons $\boldsymbol{H}$ and $\boldsymbol{h}$; CP-odd scalar $\boldsymbol{a}$; charged scalars $\boldsymbol{h}^{\pm}$.

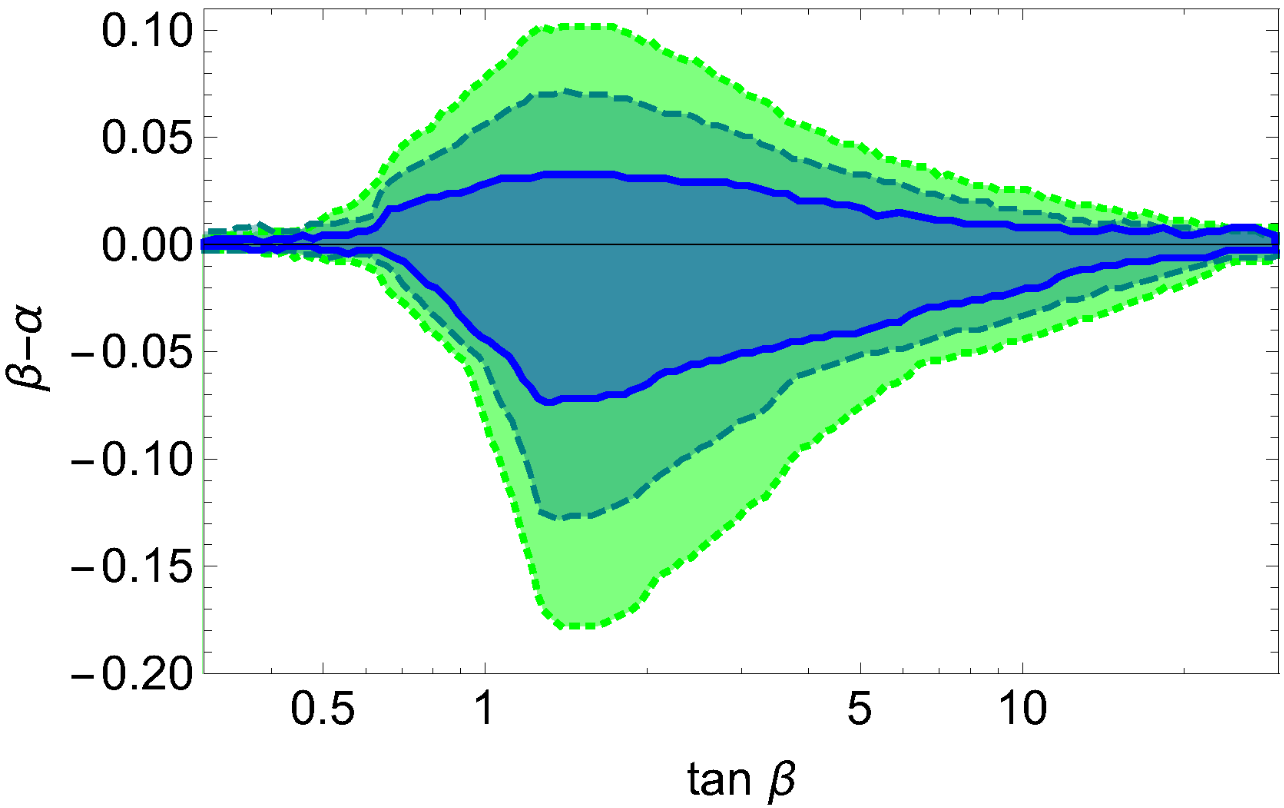
• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} \;=\; \cos(\beta-\alpha)\;, \qquad g_{hVV} \;=\; \sin(\beta-\alpha)\;,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

• **Global fit to SM mis-alignment**

[e.g. D. Chowdhury, O. Eberhardt, JHEP11 (2015) 052.]



Pheno favoured limit $\beta \to \alpha$: $g_{HVV} = \cos(\beta - \alpha) \;\to\; g_{H_{\rm SM}VV} = 1.$

- **SM Alignment** $\beta \to \alpha$:

(i) Decoupling: [Georgi, Nanopoulos '79; Gunion, Haber '03; Ginzburg, Krawczyk '05]

$$M_h^2 \;\simeq\; M_a^2 + \lambda_5 v^2 \;\simeq\; M_{h^\pm}^2 \;\gg\; v_{\mathsf{SM}}^2$$

$$M_H^2 \;\simeq\; 2\lambda_{\mathsf{SM}} v^2 - \frac{v^4 s_\beta^2 c_\beta^2}{M_a^2 + \lambda_5 v^2}\Big[s_\beta^2\,(2\lambda_2 - \lambda_{345}) - c_\beta^2\,(2\lambda_1 - \lambda_{345}) \;+\; \ldots\Big]^2$$

(ii) Fine-tuning: [Krawczyk et al. '99; Carena, Low, Shah, Wagner '14; Dev, AP '14]

$$\lambda_7 t_\beta^4 \;-\; (2\lambda_2 - \lambda_{345})t_\beta^3 \;+\; 3(\lambda_6 - \lambda_7)\,t_\beta^2 \;+\; (2\lambda_1 - \lambda_{345})t_\beta - \lambda_6 \;=\; 0$$

(iii) Natural SM alignment (independent of $M_{h^\pm}$ and t_β): [Dev, AP '14]

$$\lambda_1 = \lambda_2 = \frac{\lambda_{345}}{2} \quad (\text{with } \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5), \qquad \lambda_6 = \lambda_7 = 0$$

Symmetries:

- Sp(4): $\lambda_4 = \lambda_5 = 0 \;\to\; \lambda_{345} = \lambda_3$
- SU(2): $\lambda_5 = 0 \;\to\; \lambda_{345} = \lambda_{34} \equiv \lambda_3 + \lambda_4$
- SO(2)×$\mathcal{CP}$: $\lambda_{3,4,5} \;\neq\; 0$

References (*an incomplete list on SM Alignment in the 2HDM*)

- **On the SM Higgs basis (also Decoupling of FCNC Effects):**
  H. Georgi and D. V. Nanopoulos, Phys. Lett. B82 (1979) 95.

- **Alignment via Decoupling:**
  - J. F. Gunion, H. E. Haber, Phys. Rev. D67 (2003) 075019.
  - I. F. Ginzburg, M. Krawczyk, Phys. Rev. D72 (2005) 115013.

- **Alignment via Fine-tuning:**
  - P. H. Chankowski, T. Farris, B. Grzadkowski, J. F. Gunion, J. Kalinowski, M. Krawczyk, Phys. Lett. B496 (2000) 195.
  - A. Delgado, G. Nardini, M. Quiros, JHEP1307 (2013) 054.
  - M. Carena, I. Low, N. R. Shah, C. E. M. Wagner, JHEP1404 (2014) 015.

- **Natural Alignment without Decoupling and without Fine-tuning:**
  - P.S.B. Dev, AP, JHEP1412 (2014) 024.
  - B. Grzadkowski, O. M. Ogreid, P. Osland, Phys. Rev. D94 (2016) 115002.

References (*an incomplete list on symmetries in the 2HDM*)

- **Spontaneous CP Violation:** T. D. Lee, Phys. Rev. D8 (1973) 1226.
- **$\mathbf{Z}_2$ symmetry:** S. L. Glashow, S. Weinberg, Phys. Rev. D15 (1977) 1958.
- **Inert $\mathbf{Z}_2$ symmetry:** N. G. Deshpande, E. Ma, Phys. Rev. D18 (1978) 2574.
- **PQ U(1) symmetry:** R. D. Peccei, H. R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
- **Custodial $\mathbf{SU(2)}_L$-preserving symmetry:**
  P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov, Nucl. Phys. B173 (1980) 189.
- **Bilinear formalism:**
  M. Maniatis, A. von Manteuffel, O. Nachtmann, F. Nagel, EPJC48 (2006) 805;
  C. C. Nishi, Phys. Rev. D74 (2006) 036003.
- **$\mathbf{SU(2)}_L\otimes\mathbf{U(1)}_Y$-preserving symmetries: 6**
  I. P. Ivanov, Phys. Rev. D75 (2007) 035001;
  P. M. Ferreira, H. E. Haber, J. P. Silva, Phys. Rev. D79 (2009) 116004.
- **Hypercustodial $\mathbf{SU(2)}_L$-preserving symmetries: 7**
  R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.
- **On completeness and uniqueness of classification: $\mathbf{6+7=13}$**
  AP, Phys. Lett. B706 (2012) 465.

• Maximally Symmetric Two Higgs Doublet Model [P.S.B. Dev, AP '14]

$G_{\mathbf{\Phi}} = \mathrm{SU}(2)_L \otimes \mathrm{Sp}(4)/\mathrm{Z}_2 \simeq \mathrm{SU}(2)_L \otimes \mathrm{SO}(5).$

$$\mathrm{V} \;=\; -\,\mu^2\Big(|\Phi_1|^2+|\Phi_2|^2\Big)+\lambda\,\Big(|\Phi_1|^2+|\Phi_2|^2\Big)^2 \;=\; -\frac{\mu^2}{2}\,\mathbf{\Phi}^\dagger\,\mathbf{\Phi} + \frac{\lambda}{4}\,(\mathbf{\Phi}^\dagger\,\mathbf{\Phi})^2\,,$$

where [R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

$$\mathbf{\Phi} \;=\; \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}\,, \quad \text{with } \; U_L \in \; \mathrm{SU}(2)_L : \; \mathbf{\Phi} \;\mapsto\; \mathbf{\Phi}' \;=\; U_L\,\mathbf{\Phi}\,,$$

such that under **global field transformations**, [AP, Phys. Lett. B706 (2012) 465.]

$$\mathrm{Sp}(4): \;\; \mathbf{\Phi} \;\mapsto\; \mathbf{\Phi}' \;=\; U\,\mathbf{\Phi}\,, \quad \text{with } U \in \mathrm{U}(4) \;\;\&\;\; U\mathrm{C}U^{\mathsf{T}} = \;\mathrm{C} \equiv i\sigma^2 \otimes \sigma^0$$

$\mathrm{SU}(2)_L$ gauge kinetic terms remain invariant.

Breaking Effects: $-m_{12}^2\,\phi_1^\dagger\phi_2$, $\mathrm{U}(1)_Y$ coupling g', Yukawa couplings $\mathbf{Y}^{u,d}$.

• Natural Alignment Beyond the 2HDM

[AP '16]

For nHDM with $m < n$ inert scalar doublets, there are still **3** continuous alignment symmetries in the **field space of the *non*-inert sector**:

$$\text{(i)}\ \ \mathrm{Sp}(2N_H)\times\mathcal{D}; \qquad \text{(ii)}\ \ \mathrm{SU}(N_H)\times\mathcal{D}; \qquad \text{(iii)}\ \ \mathrm{SO}(N_H)\times\mathcal{CP}\times\mathcal{D},$$

where $N_H = n - m$, $\mathcal{D}$ acts on the inert sector *only*, and $\mathcal{CP}$ is the canonical CP: $\Phi_i(t,\mathbf{x}) \to \Phi_i^*(t,-\mathbf{x})$ (with $i = 1, 2, \ldots, N_H$).

Symmetry invariants:

$$\begin{aligned}
\text{(i)} \quad S &= \Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 + \ldots = \tfrac{1}{2}\,\boldsymbol{\Phi}^\dagger\,\boldsymbol{\Phi} \\
\text{(ii)} \quad D^a &= \Phi_1^\dagger\sigma^a\Phi_1 + \Phi_2^\dagger\sigma^a\Phi_2 + \ldots \\
\text{(iii)} \quad T &= \Phi_1\Phi_1^\mathsf{T} + \Phi_2\Phi_2^\mathsf{T} + \ldots
\end{aligned}$$

Symmetric part of the scalar potential:

$$V_{\mathrm{sym}} = -\mu^2\,S + \lambda_S\,S^2 + \lambda_D\,D^aD^a + \lambda_T\,\mathrm{Tr}\,(T\,T^*)\ .$$

Minimal Symmetry of Alignment: $\mathbf{Z_2^{EW}} \times \mathbf{Z_2^{I}}$.

• Quartic Coupling Unification in the MS-2HDM

[Dev, AP '14; N. Darvishi, AP '19]

Symmetry-breaking of Sp(4)/$Z_2 \sim$ SO(5):

- Soft breaking (e.g. through m_{12}^2):

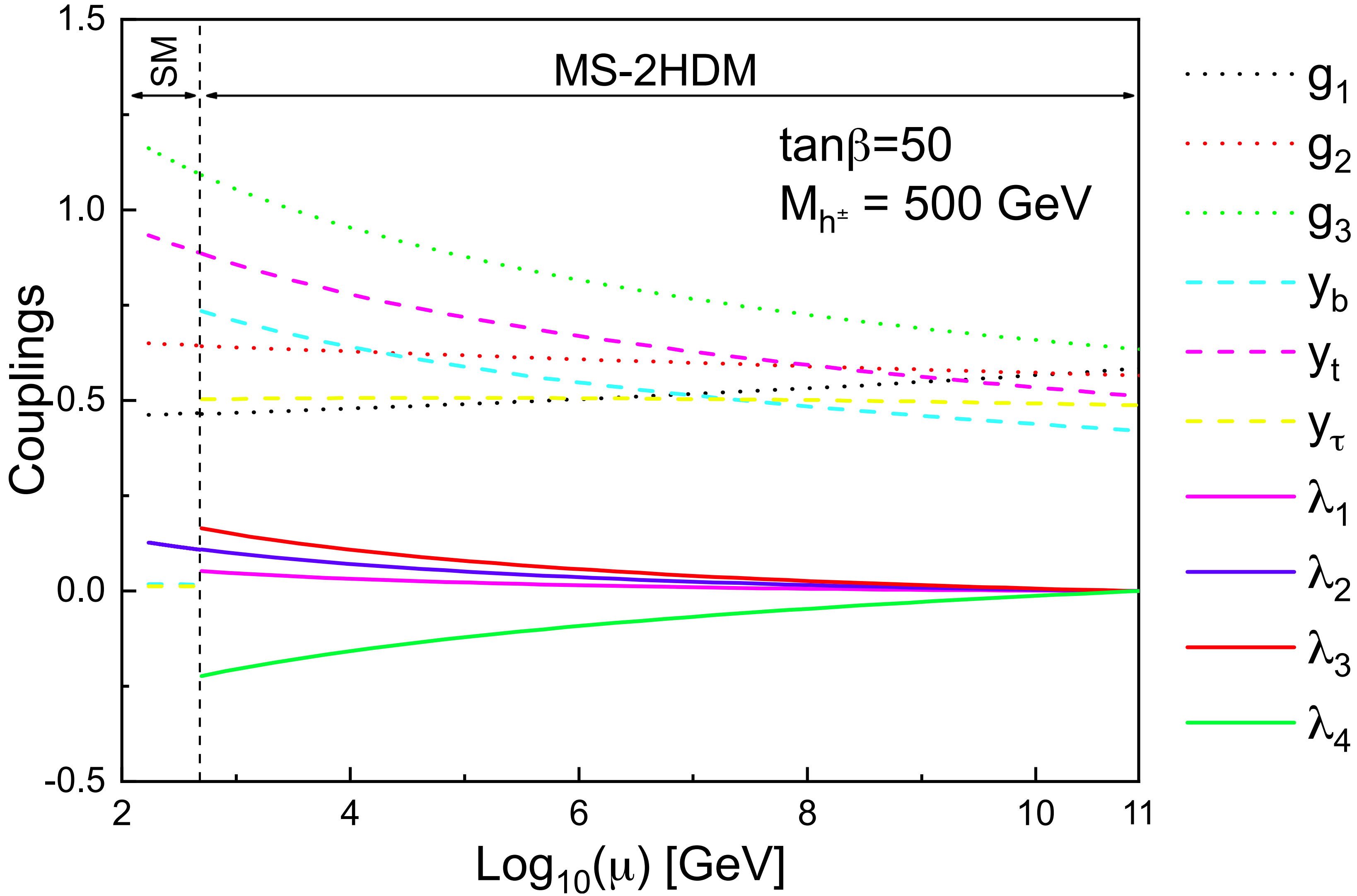
$$M_H^2 = 2\lambda_2 v^2, \qquad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}$$

Heavy Higgs spectrum is degenerate at tree level.

- Explicit breaking through RG running (two loops):

$$\begin{aligned} \mathrm{Sp}(4)/\mathrm{Z}_2 \otimes \mathrm{SU}(2)_L &\xrightarrow{g'\neq 0} \mathrm{SU}(2)_{\mathrm{HF}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ &\xrightarrow{\mathbf{Y}^{u,d}} \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\ &\xrightarrow[\langle\Phi_{1,2}\rangle]{m_{12}^2} \mathrm{U}(1)_{\mathrm{em}} \end{aligned}$$

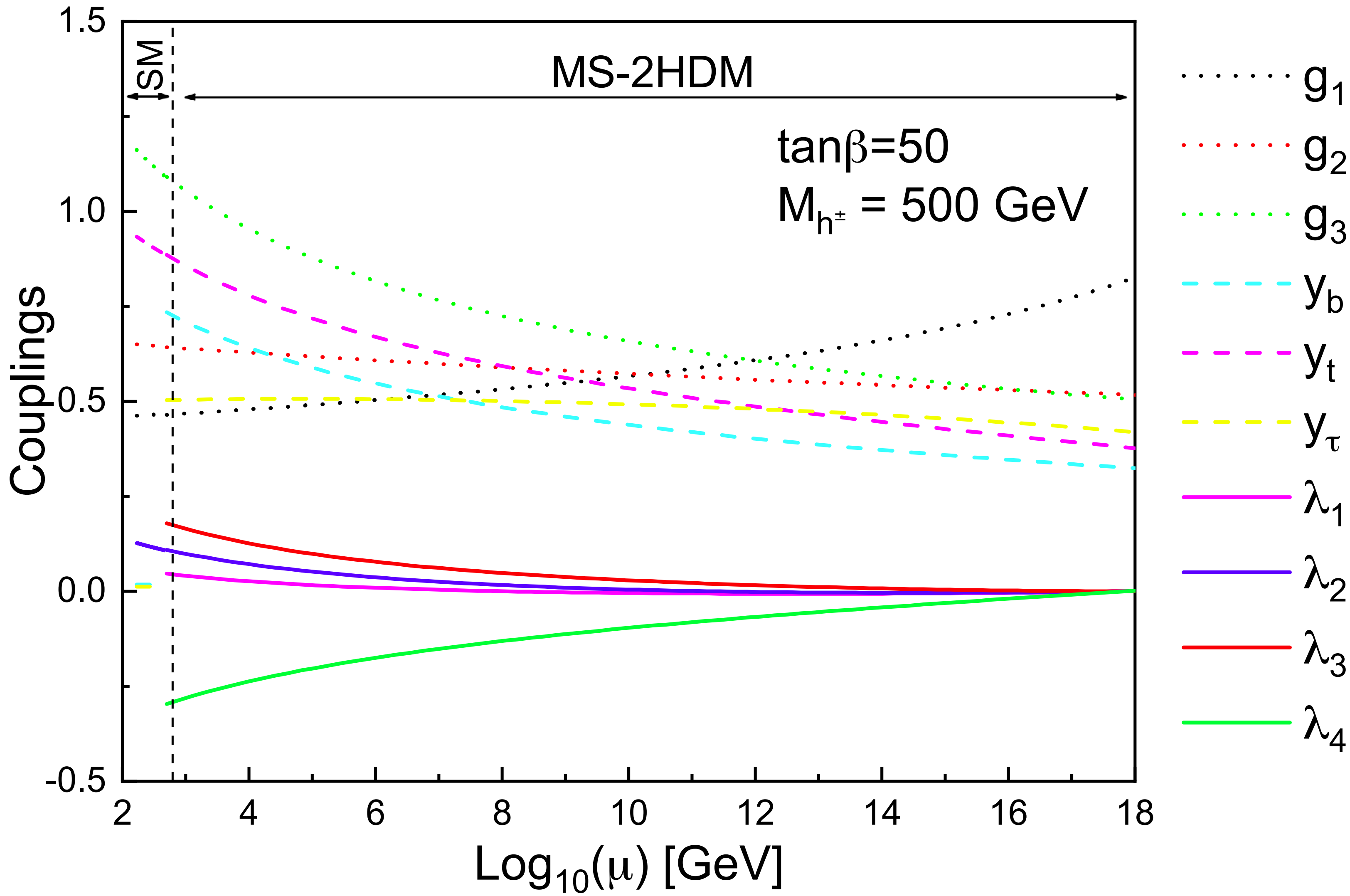
- **Quartic Coupling Unification (two loops)** [N. Darvishi, AP '19]



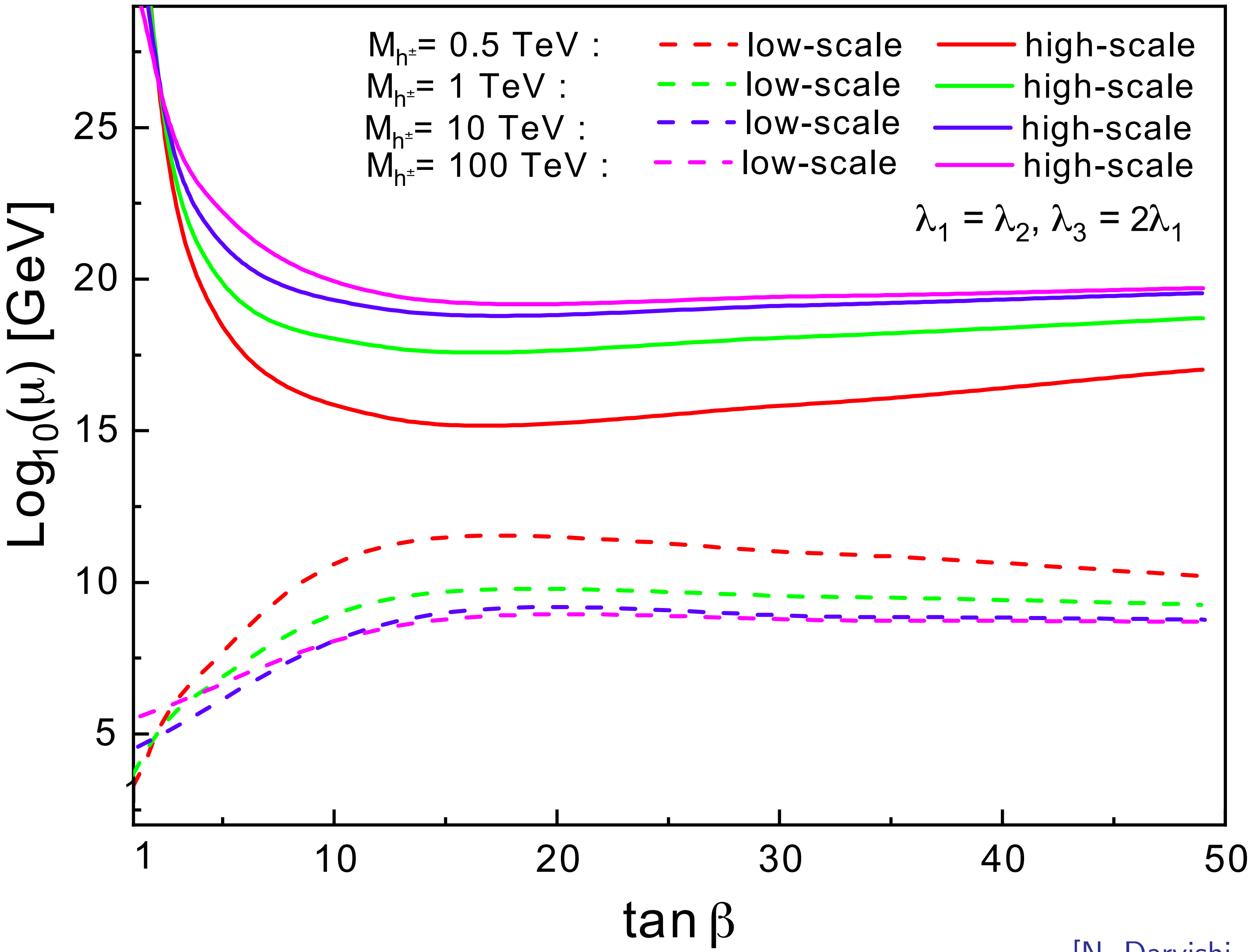
First conformal unification point: $\mu_X^{(1)} \sim 10^{11}$ GeV (of order PQ scale)

Second conformal unification point: $\mu_X^{(2)} \sim 10^{18}$ GeV (of order m_{Pl})

[N. Darvishi, AP '19]



– **Low- and high-scale quartic coupling unification: $\tan\beta$ vs $\mu_X^{(1,2)}$**



[N. Darvishi, AP '19]

– Misalignment in the MS-2HDM

CP-even mass matrix in Higgs basis:

$$\mathcal{M}_{\mathcal{S}}^2 \;=\; \begin{pmatrix} \widehat{A} & \widehat{C} \\ \widehat{C} & \widehat{B} \end{pmatrix} \quad \xrightarrow[\text{approx.}]{\text{seesaw}} \quad M_H^2 \simeq \widehat{A} - \frac{\widehat{C}^2}{\widehat{B}} \quad \& \quad M_h^2 \simeq \widehat{B} \gg \widehat{A},\, \widehat{C}$$

Light-to-heavy scalar mixing:

$$\theta_{\mathcal{S}} \;\equiv\; \frac{\widehat{C}}{\widehat{B}} \;=\; \frac{v^2 s_\beta c_\beta \big[s_\beta^2\,(2\lambda_2 - \lambda_{34}) - c_\beta^2\,(2\lambda_1 - \lambda_{34}) \big]}{M_a^2 + 2v^2 s_\beta^2 c_\beta^2\,(\lambda_1 + \lambda_2 - \lambda_{34})} \;\ll\; 1$$

Higgs couplings to $V = W, Z$:

$$g_{HVV} \;\simeq\; 1 \,-\, \frac{1}{2}\,\theta_{\mathcal{S}}^2\,, \qquad g_{hVV} \;\simeq\; -\,\theta_{\mathcal{S}}$$

Higgs couplings to quarks:

$$g_{Huu} \;\simeq\; 1 \,+\, t_\beta^{-1}\,\theta_{\mathcal{S}}\,, \qquad g_{Hdd} \;\simeq\; 1 \,-\, t_\beta\,\theta_{\mathcal{S}}\,,$$

$$g_{huu} \;\simeq\; -\,\theta_{\mathcal{S}} \,+\, t_\beta^{-1}\,, \qquad g_{hdd} \simeq -\,\theta_{\mathcal{S}} \,-\, t_\beta\,.$$

Misalignment predictions in the MS-2HDM with low- and high-scale quartic coupling unification, assuming $M_{h^\pm} = 500$ GeV.

[N. Darvishi, AP '19]

Couplings	ATLAS	CMS	$\tan\beta = 2$	$\tan\beta = 20$	$\tan\beta = 50$
$\vert g_{HZZ}^{\text{low-scale}}\vert$	[0.86, 1.00]	[0.90, 1.00]	0.9999	0.9999	0.9999
$\vert g_{HZZ}^{\text{high-scale}}\vert$			0.9981	0.9999	0.9999
$\vert g_{Htt}^{\text{low-scale}}\vert$	$1.31^{+0.35}_{-0.33}$	$1.45^{+0.42}_{-0.32}$	1.0049	1.0001	1.0000
$\vert g_{Htt}^{\text{high-scale}}\vert$			1.0987	1.0003	1.0001
$\vert g_{Hbb}^{\text{low-scale}}\vert$	$0.49^{+0.26}_{-0.19}$	$0.57^{+0.16}_{-0.16}$	0.9803	0.9560	0.9590
$\vert g_{Hbb}^{\text{high-scale}}\vert$			0.8810	0.9449	0.9427

$\longrightarrow$ Misalignment predictions consistent with experiment

• **Maximally Symmetric Three Higgs Doublet Model (MS-3HDM)**

[N. Darvishi, M. Masouminia, AP '21]

Breaking pattern:

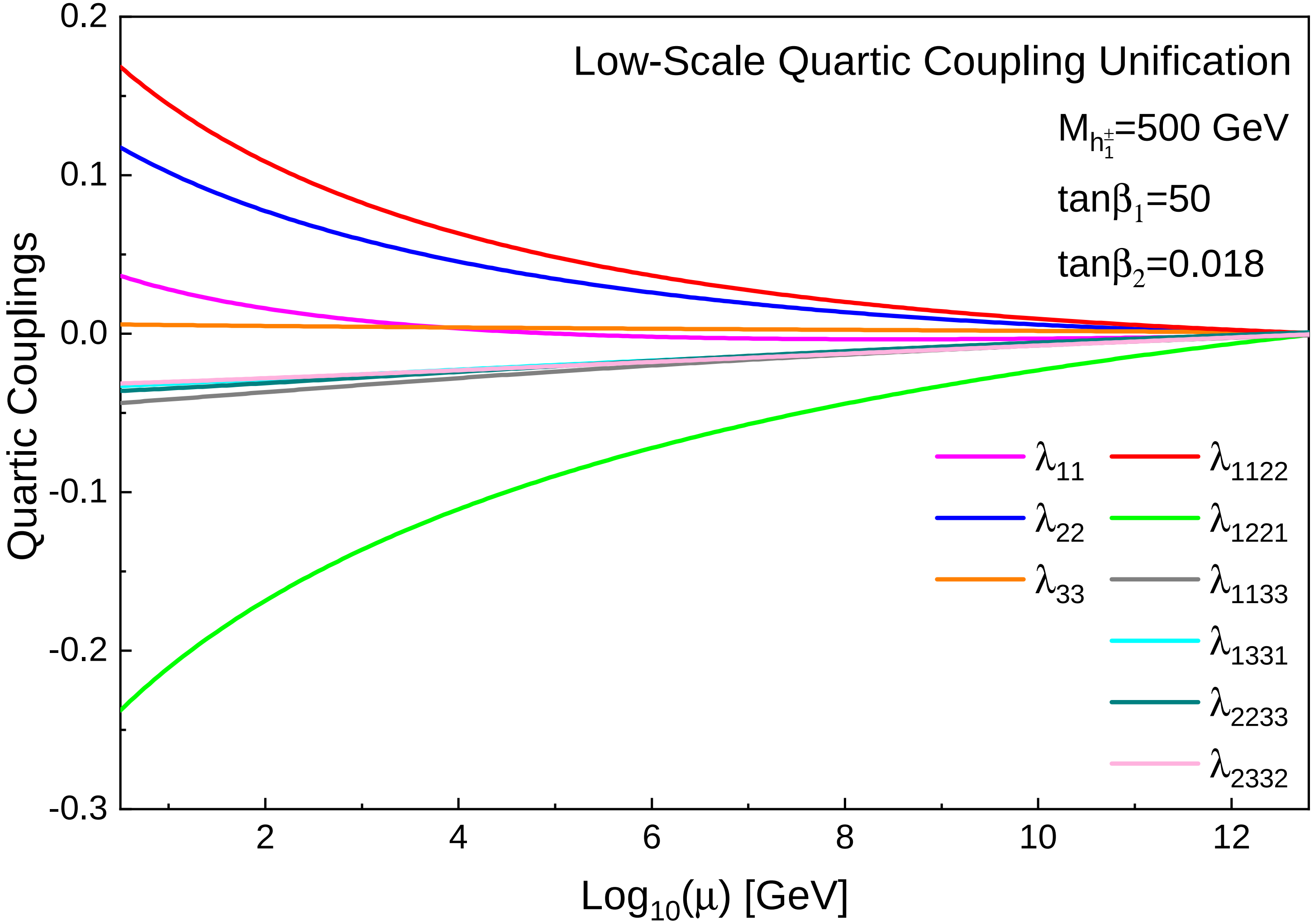
$$
\begin{aligned}
\mathrm{Sp}(6)/\mathrm{Z}_2 \otimes \mathrm{SU}(2)_L \quad &\xrightarrow{g' \neq 0} \quad \mathrm{SU}(3)_{\mathrm{HF}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow[\text{Type V}]{\mathbf{Y}^{u,d,e}} \quad \mathrm{U}(1)_{\mathrm{PQ}} \otimes \mathrm{U}(1)'_{\mathrm{PQ}} \otimes \mathrm{U}(1)_Y \otimes \mathrm{SU}(2)_L \\
&\xrightarrow[\text{soft } m^2_{ij}]{\langle \Phi_{1,2,3} \rangle} \quad \mathrm{U}(1)_{\mathrm{em}}
\end{aligned}
$$

– Quartic Coupling Unification in the MS-3HDM

[N. Darvishi, M. Masouminia, AP '21]

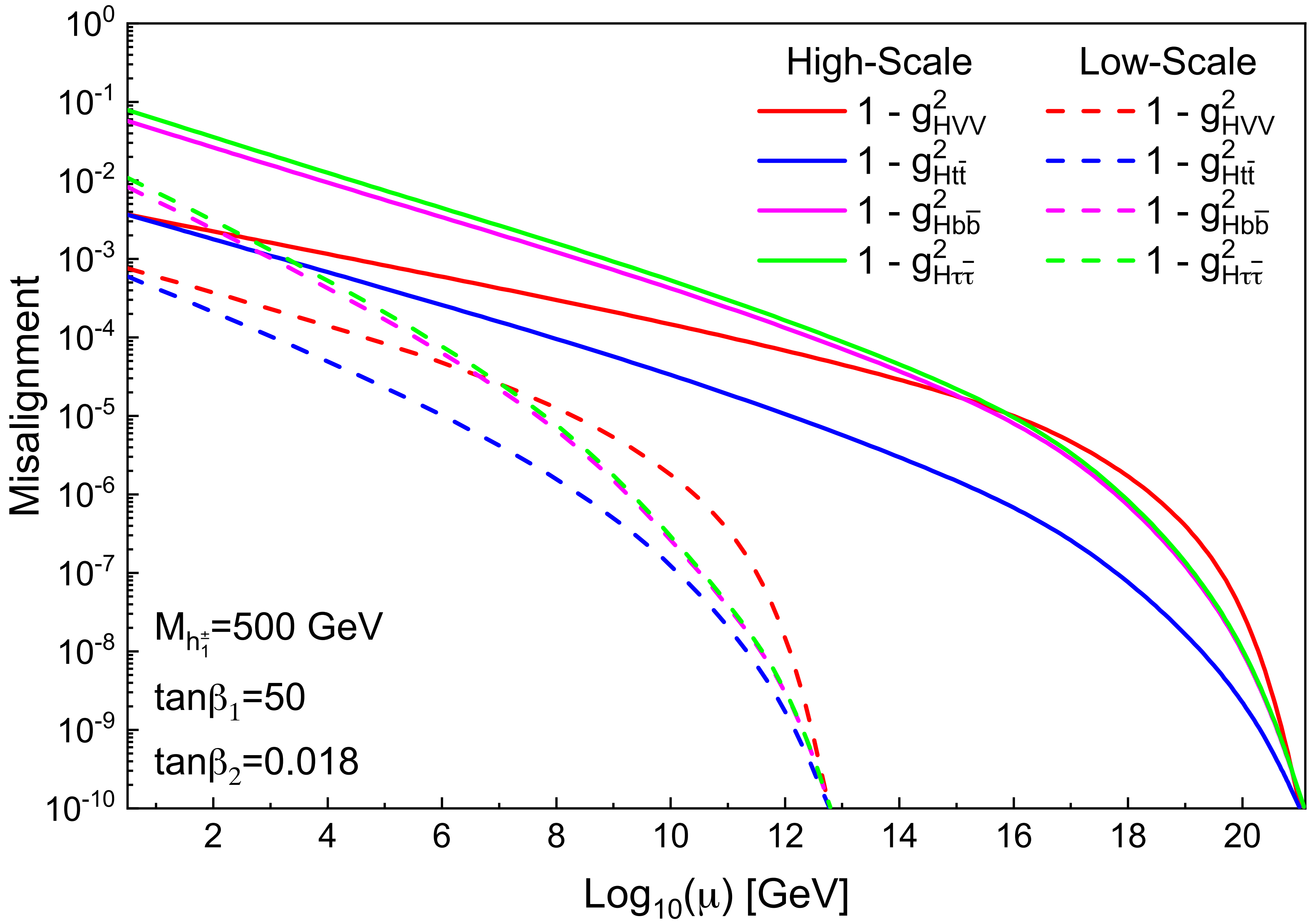
Input parameters: $\tan\beta_1 = v_2/v_1$, $\tan\beta_2 = v_3/\sqrt{v_1^2 + v_2^2}$,

$M_{h^\pm_{1,2}}$ and $h_1^\pm h_2^\mp$-mixing angle: σ

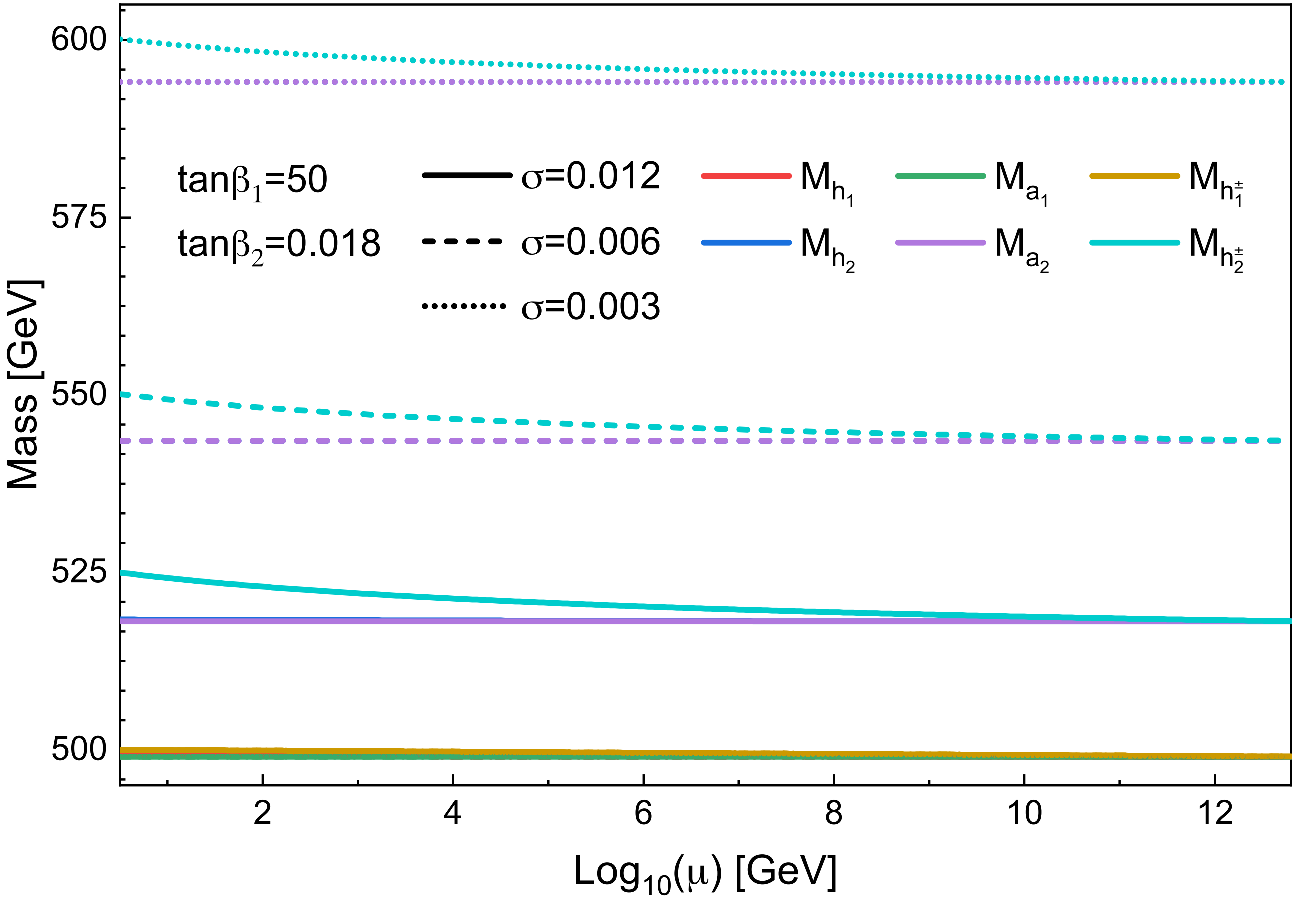


– Misalignment predictions in the MS-3HDM

[N. Darvishi, M. Masouminia, AP '21]



– Scalar Mass Spectrum in the MS-3HDM [N. Darvishi, M. Masouminia, AP '21]



Predictions:

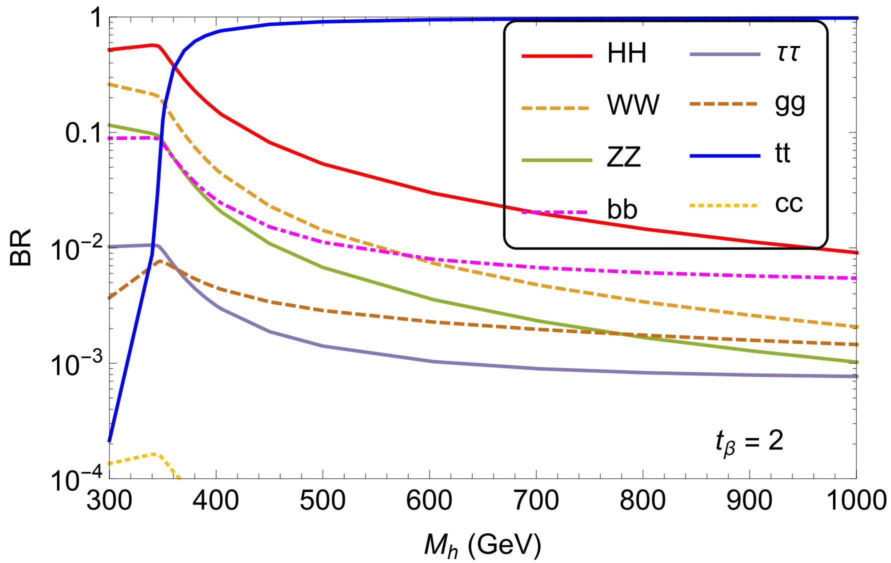
Alignment of masses: $M_{h_1} \sim M_{a_1} \sim M_{h_1^\pm} \qquad M_{h_2} \sim M_{a_2} \sim M_{h_2^\pm}$

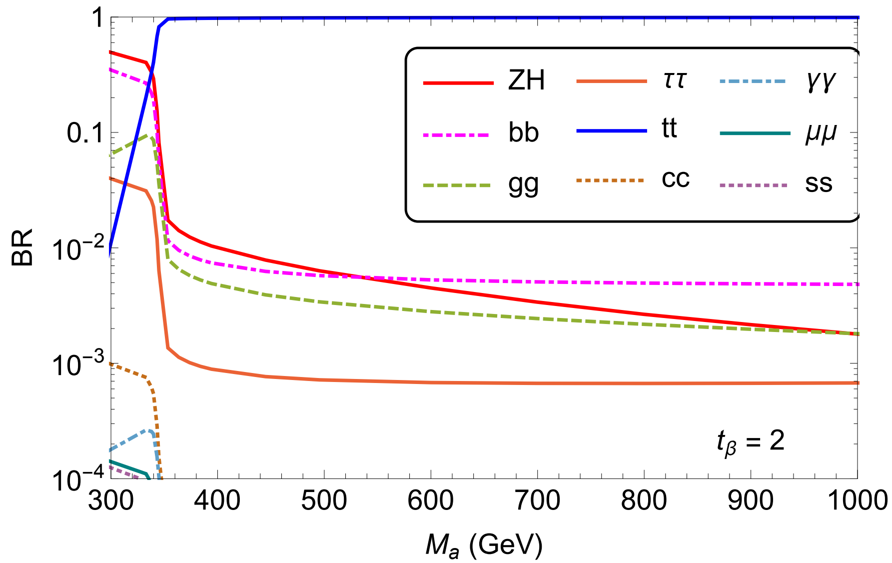
Alignment of all heavy-sector mixing angles in the Higgs basis: $\alpha \simeq \rho \simeq \sigma$

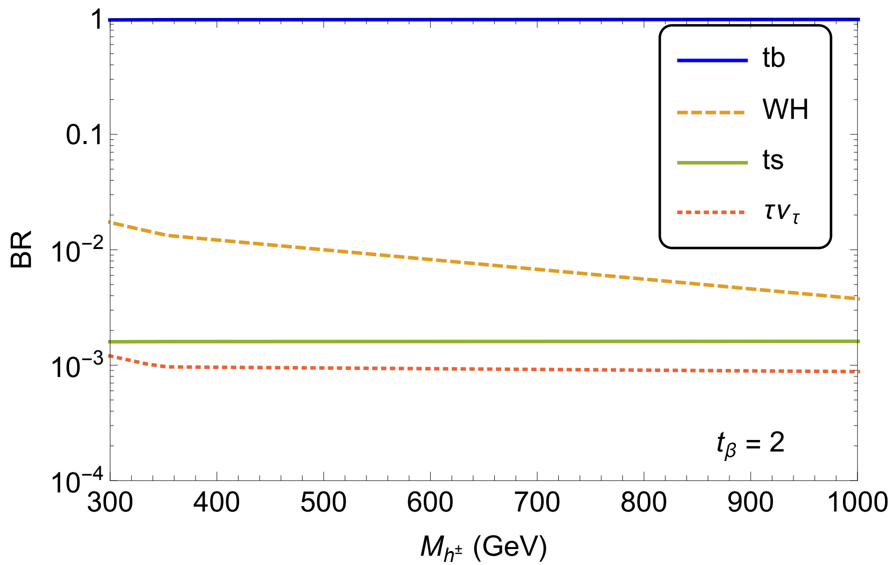
• Phenomenology at the LHC

– Branching ratios in the MS-2HDM

[Dev, AP '14]

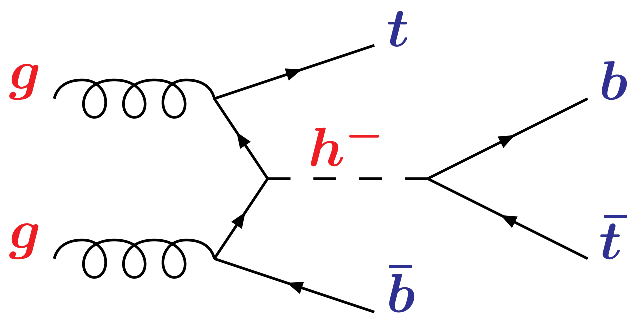


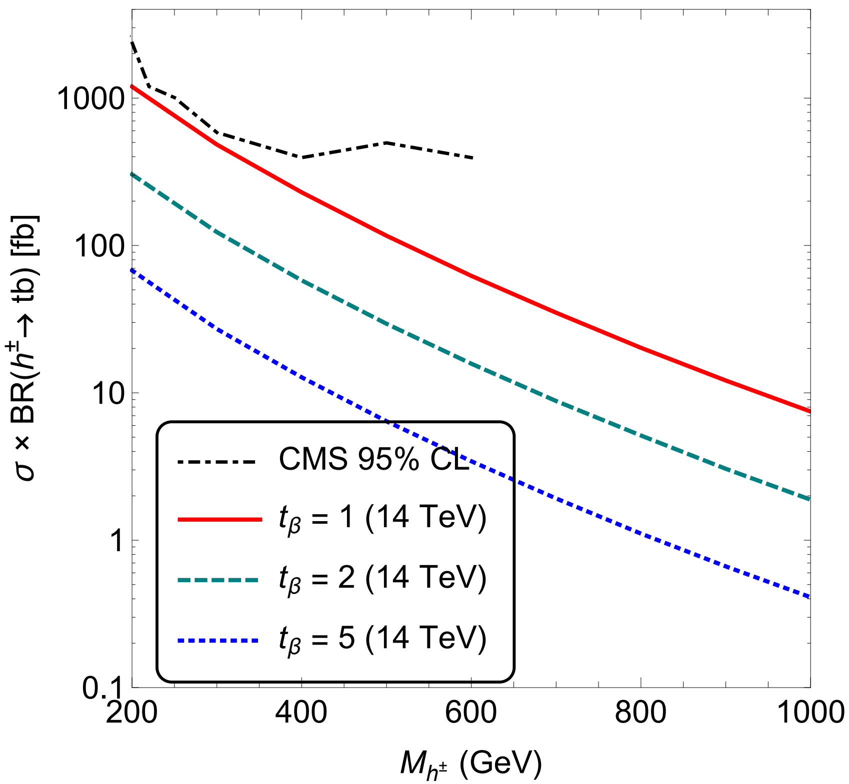




– Discovery channels for aligned Higgs doublets:

- $gg \to t\bar{b}h^- \to t\bar{b}\bar{t}b$ [Dev, AP '14]

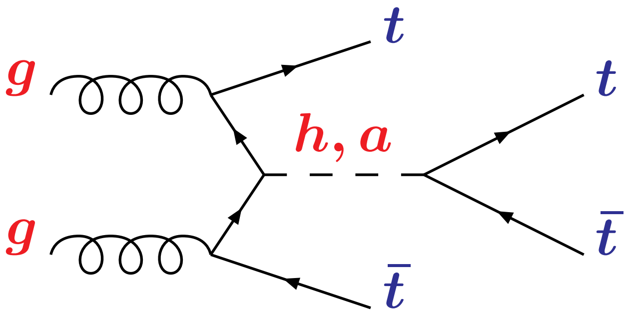


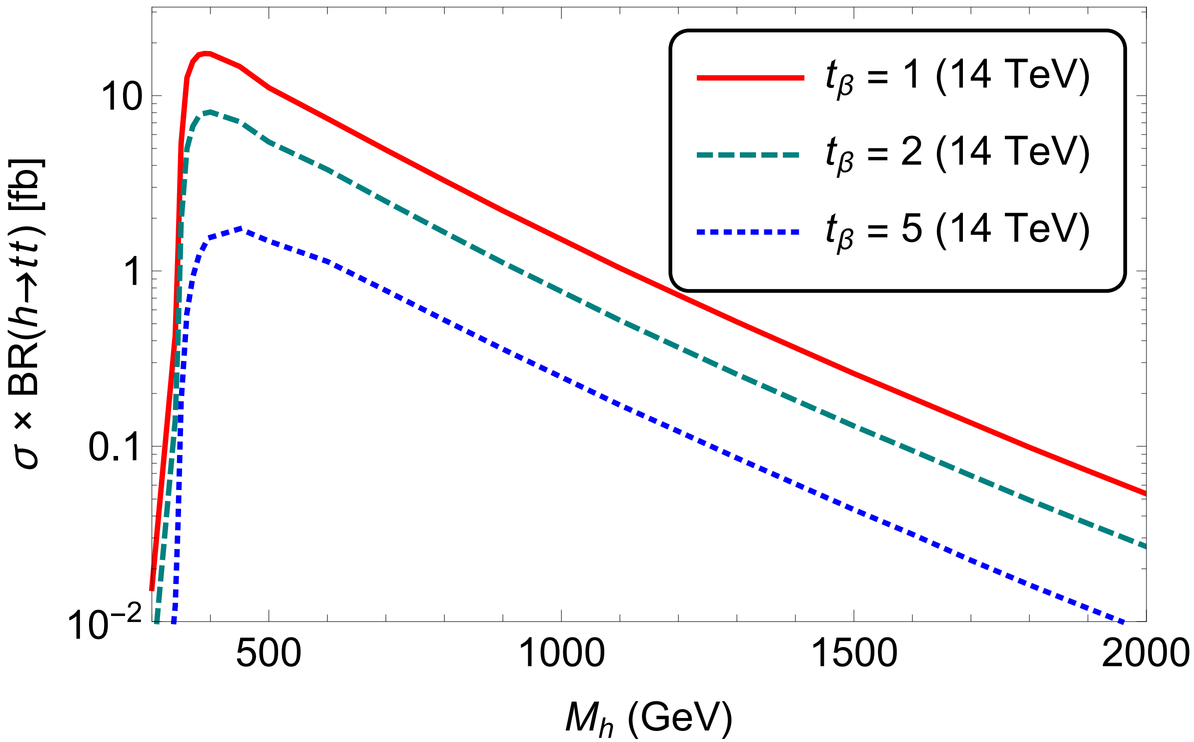


$$p_T^\ell > 20 \text{ GeV},$$
$$|\eta^\ell| < 2.5,$$
$$\Delta R^{\ell\ell} > 0.4,$$
$$M_{\ell\ell} > 12 \text{ GeV},$$
$$|M_{\ell\ell} - M_Z| > 10 \text{ GeV},$$
$$p_T^j > 30 \text{ GeV},$$
$$|\eta^j| < 2.4,$$
$$\not{E}_T > 40 \text{ GeV}.$$

- $gg \to t\bar{t}(h, a) \to t\bar{t}t\bar{t}$

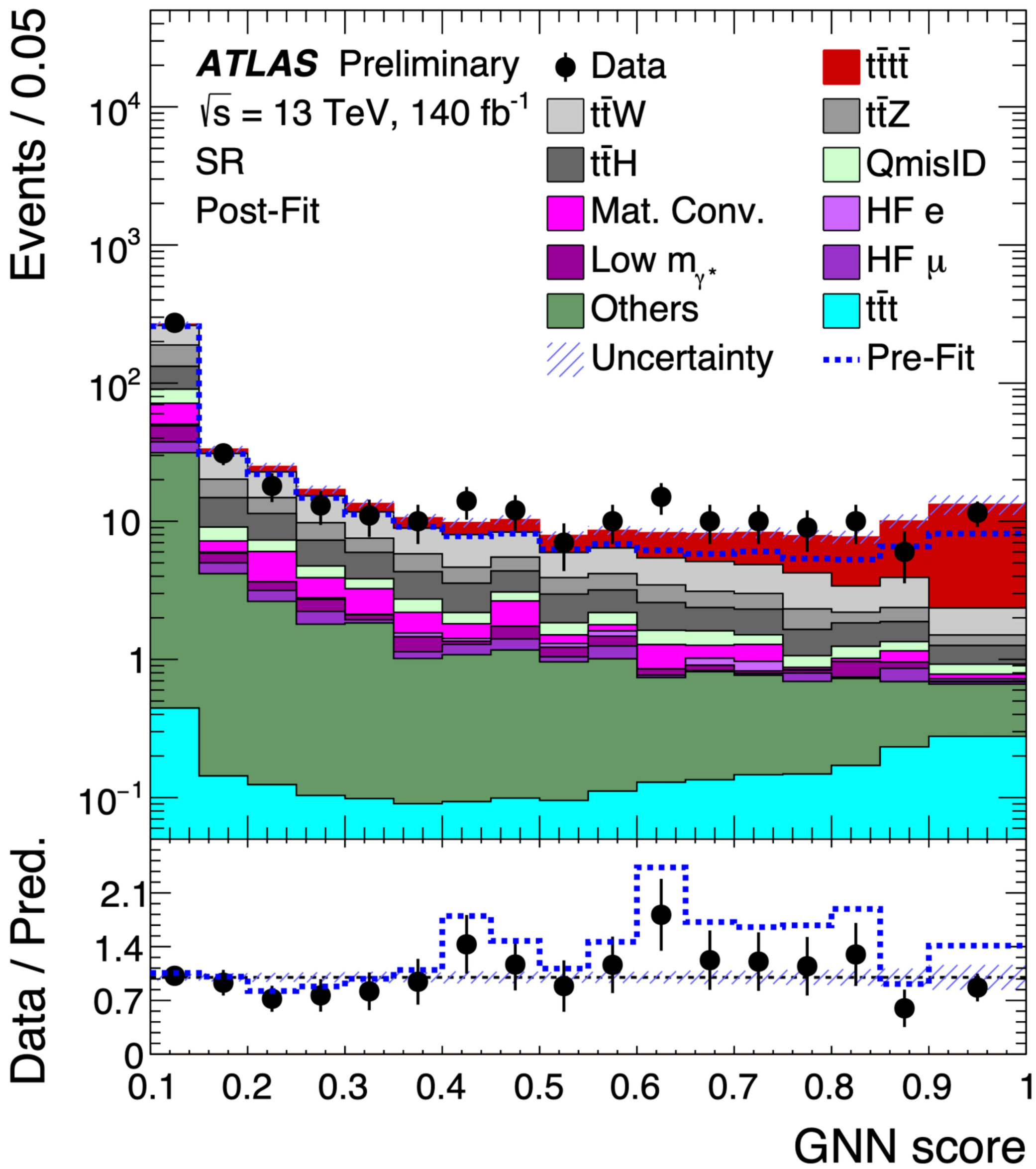
[Dev, AP '14]





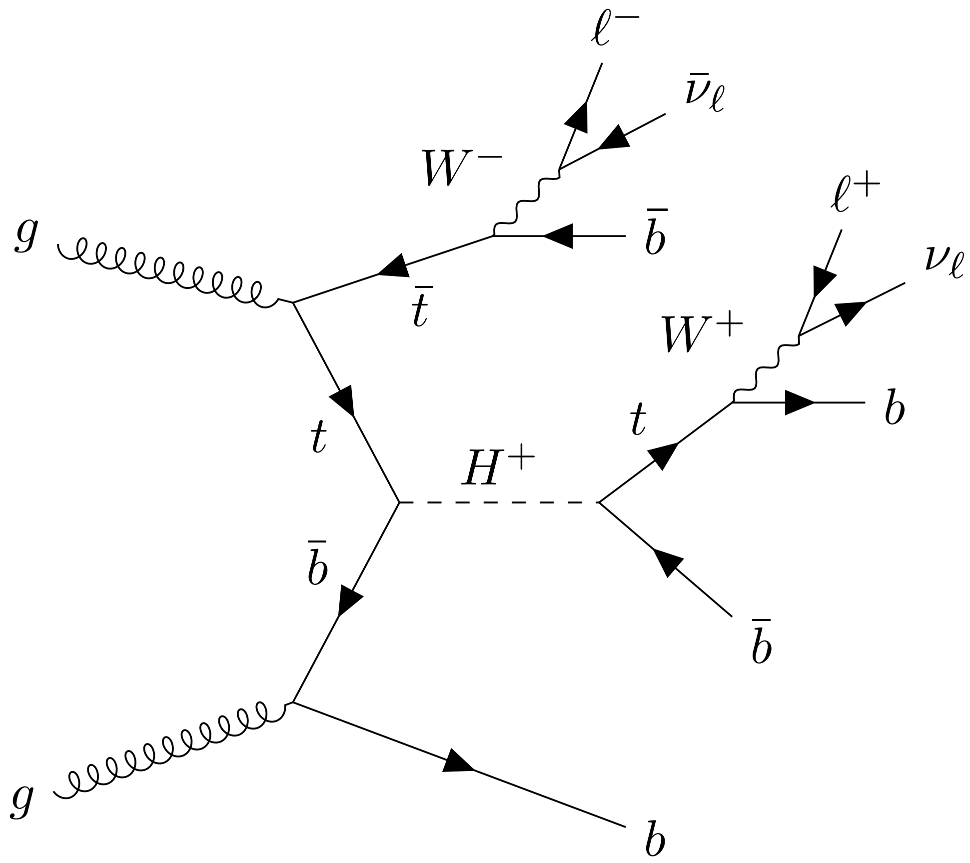
Observation of $t\bar{t}t\bar{t}$ with the ATLAS detector

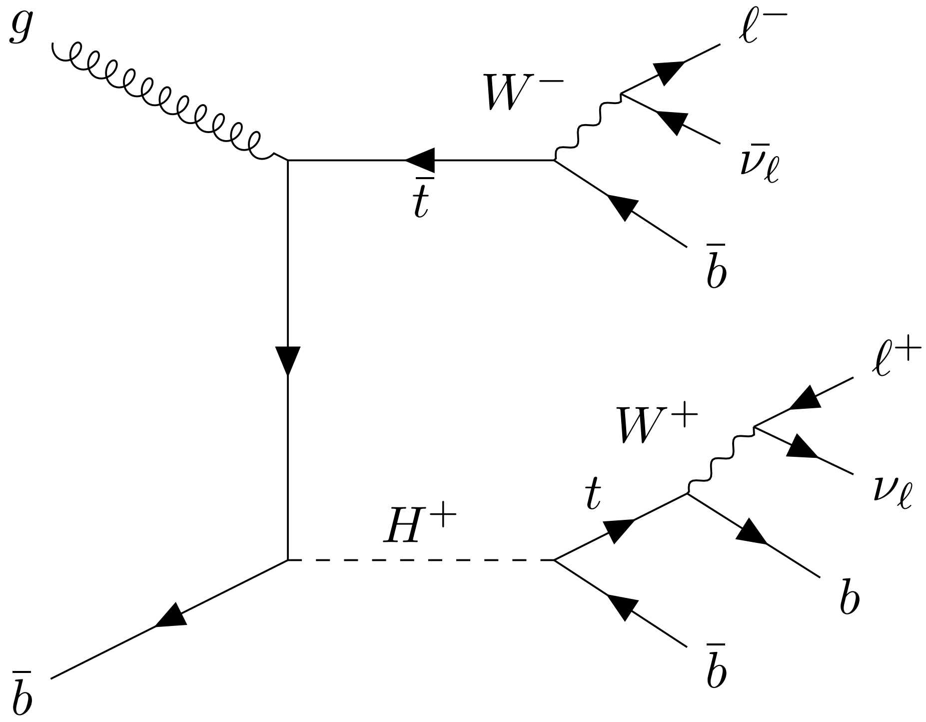
[ATLAS, arXiv:2303.15061]



– Realistic simulation analysis with a reconstruction BDT

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]



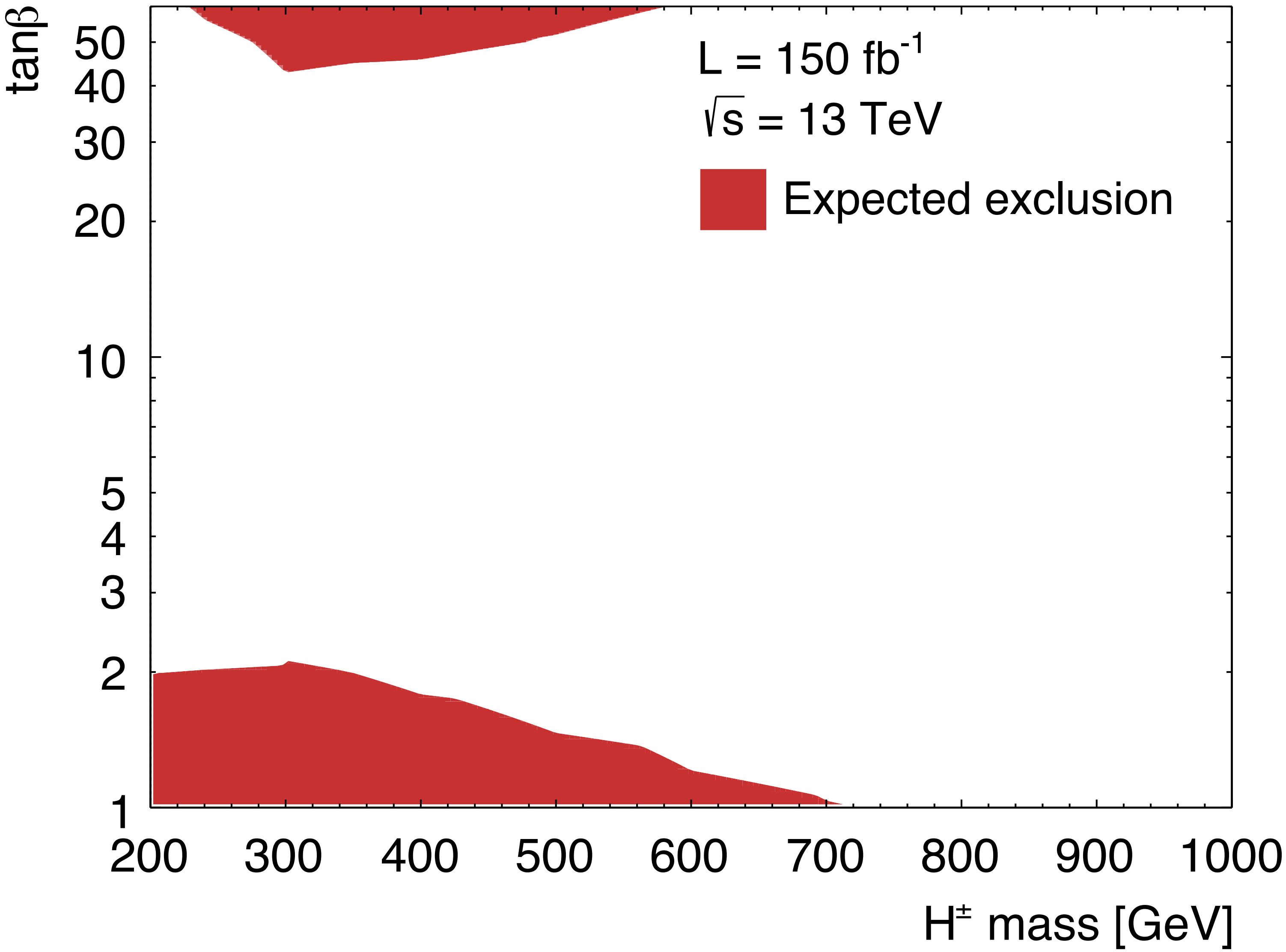


Reconstruction BDT trained on 57 observables:

- $\Delta R(b_i, l^a)$, $\Delta\eta(b_i, l^a)$, $\Delta\phi(b_i, l^a)$, $p_T^{b_i+l^a}$, $m(b_i, l^a)$, where $i = tH, t$ and $a = +, -$
- $\left|m(l^+, b_{tH}) - m(l^-, b_t)\right|$ and $\left|m(l^-, b_{tH}) - m(l^+, b_t)\right|$
- $p_T^{b_j}$, where $j = tH, H, t$
- $\Delta R(b_{tH}, b_k)$, $\Delta\eta(b_{tH}, b_k)$, $\Delta\phi(b_{tH}, b_k)$, $p_T^{b_{tH}+b_k}$, $m(b_{tH}, b_k)$, where $k = H, t$
- $\Delta R(t_{H^a}, b_H)$, $\Delta\eta(t_{H^a}, b_H)$, $\Delta\phi(t_{H^a}, b_H)$, $p_T^{t_{H^a}, b_H}$, $m(t_{H^a}, b_H)$, where $a = +, -$
- $\Delta R(t_{H^a}, t_c)$, $\Delta\eta(t_{H^a}, t_c)$, $\Delta\phi(t_{H^a}, t_c)$, where $(H^a, t_c) = (H^+, \bar{t})$ or (H^-, t)
- $m(H^a) - m(b_H)$, where $a = +, -$
- $m(H^+) - m(\bar{t})$ and $m(H^-) - m(t)$
- $p_T^{H^\pm + t_{\text{other}}}$
- $m(H^\pm, t_{\text{other}})$

– Results

[Emily Hanson, W. Klemm, R. Naranjo, Yvonne Peters, AP, PRD100 (2019) 035026]



• SUMMARY

- Symmetries for natural alignment *without* decoupling in multi-HDMs:

  (i) Sp$(2N_H)$ (ii) SU(N_H) (iii) SO$(N_H) \times \mathcal{CP}$

  $N_H > 1$: number of EWSB Higgs doublets

- Soft breaking $\longrightarrow$ minimal alignment symmetry: $\mathrm{Z}_2^{\mathrm{EW}} \times \mathrm{Z}_2^{\mathrm{I}}$

  $\rightarrow$ Naturally aligned heavy Higgs sector is $\mathrm{Z}_2^{\mathrm{EW}}$ odd.

- Quartic coupling unification for maximally symmetric nHDMs:

$$G_\Phi \;=\; \mathrm{SU}(2)_L \otimes \mathrm{Sp}(2n)/\mathrm{Z}_2 \quad \text{(here } n = 2, 3\text{)}.$$

  INPUT: $M_{h_i^\pm}$ & $\tan\beta_i \;\rightarrow\; \mu_X^{(1)} \sim 10^{11}$ GeV & $\mu_X^{(2)} \sim 10^{19}$ GeV.

  $\Rightarrow$ RG effects provide *definite* misalignment predictions for the heavy Higgs spectrum and for *all* H-couplings to SM particles.

- The $t\bar{t}t\bar{t}$ channel is a powerful probe for Naturally Aligned 2HDMs

Back-Up Slides

• Accidental Symmetries in 2HDM, 2HDMEFT, and multi-HDMs

• 2HDM potential

[TD Lee '73; AP, C Wagner '99;
Review: Branco, Ferreira, Lavoura, Rebelo, Sher, Silva '12.]

$$
\begin{aligned}
\mathrm{V} \;=\; & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{*2}(\phi_2^\dagger\phi_1) \\
& + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) \\
& + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)\,.
\end{aligned}
$$

• Physical spectrum (CP-conserving limit):

CP-even Higgs bosons $\boldsymbol{H}$ and $\boldsymbol{h}$; CP-odd scalar $\boldsymbol{a}$; charged scalars $\boldsymbol{h}^{\pm}$.

• Higgs coupling to gauge bosons $V = W, Z$:

$$
g_{HVV} \;=\; \cos(\beta-\alpha)\,, \qquad g_{hVV} \;=\; \sin(\beta-\alpha)\,,
$$

where $\tan\beta = \langle\phi_2\rangle/\langle\phi_1\rangle$ and α diagonalizes the CP-even mass matrix.

• Symmetries of the 2HDM Potential

[R. A. Battye, G. D. Brawn, AP, JHEP1108 (2011) 020.]

Introduce the SU(2)$_L$-covariant 8D complex field multiplet

$$\mathbf{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix}, \quad \text{with } U_L \in \text{SU(2)}_L : \mathbf{\Phi} \mapsto \mathbf{\Phi}' = U_L\,\mathbf{\Phi}\,.$$

$\mathbf{\Phi}$ satisfies the **Majorana constraint**

$$\mathbf{\Phi} = \mathrm{C}\,\mathbf{\Phi}^*\,,$$

where C is the **charge conjugation** 8D matrix

$$\mathrm{C} = \sigma^2 \otimes \sigma^0 \otimes \sigma^2 = \begin{pmatrix} \mathbf{0}_4 & \mathbf{1}_4 \\ -\mathbf{1}_4 & \mathbf{0}_4 \end{pmatrix} \otimes (-i\sigma_2)\,.$$

• The SO(1,5) Bilinear Formalism

Introduce the *null* 6-Vector

$$\mathrm{R}^A \;=\; \mathbf{\Phi}^\dagger\,\Sigma^A\,\mathbf{\Phi} \;=\; \begin{pmatrix} \phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2+\phi_2^\dagger\phi_1 \\ -i\left[\phi_1^\dagger\phi_2-\phi_2^\dagger\phi_1\right] \\ \phi_1^\dagger\phi_1-\phi_2^\dagger\phi_2 \\ \phi_1^\mathsf{T} i\sigma^2\phi_2-\phi_2^\dagger i\sigma^2\phi_1^* \\ -i\left[\phi_1^\mathsf{T} i\sigma^2\phi_2+\phi_2^\dagger i\sigma^2\phi_1^*\right] \end{pmatrix},$$

with $A=\mu,\ 4,\ 5$, and

$$\Sigma^\mu \;=\; \frac{1}{2}\begin{pmatrix} \sigma^\mu & \mathbf{0}_2 \\ \mathbf{0}_2 & (\sigma^\mu)^\mathsf{T} \end{pmatrix}\otimes\sigma^0\,,$$

$$\Sigma^4 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & i\sigma^2 \\ -i\sigma^2 & \mathbf{0}_2 \end{pmatrix}\otimes\sigma^0\,,\qquad \Sigma^5 \;=\; \frac{1}{2}\begin{pmatrix} \mathbf{0}_2 & -\sigma^2 \\ -\sigma^2 & \mathbf{0}_2 \end{pmatrix}\otimes\sigma^0\,.$$

• The 2HDM Potential in the SO(1,5) Formalism

$$V_{\mathrm{2HDM}} \;=\; -\frac{1}{2}\,M_A\,\mathsf{R}^A \;+\; \frac{1}{4}\,L_{AB}\,\mathsf{R}^A\,\mathsf{R}^B\;,$$

with

$$M_A \;=\; \big(\;\mu_1^2+\mu_2^2\,,\quad 2\mathrm{Re}(m_{12}^2)\,,\quad -2\mathrm{Im}(m_{12}^2)\,,\quad \mu_1^2-\mu_2^2\,,\quad 0\,,\quad 0\;\big)\;,$$

$$L_{AB} \;=\; \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) & 0 & 0 \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\;.$$

• Unitary Field Transformations:

[AP, Phys. Lett. B706 (2012) 465.]

$\mathsf{Sp}(4):\qquad \mathbf{\Phi}\;\mapsto\;\mathbf{\Phi}'\;=\;U\,\mathbf{\Phi}\,,\qquad$ with $\;U\in\mathsf{U}(4)\;$ <u>and</u> $\;UCU^{\mathsf{T}}=\mathsf{C}$

$\mathsf{SO}(5):\qquad \mathsf{R}^{\mathsf{I}}\;\mapsto\;\mathsf{R}'^{\mathsf{I}}\;=\;\mathsf{O}^{\mathsf{I}}{}_{\mathsf{J}}\,\mathsf{R}^{\mathsf{J}}\,,\qquad$ with $\;\mathsf{O}\in\mathsf{SO}(5)\;\subset\;\mathsf{SO}(1,5)$

$\Longrightarrow\qquad \mathsf{SO}(5)\;\sim\;\mathsf{Sp}(4)/\mathbf{Z}_2$

• Symmetry Breaking Scenarios and pseudo-Goldstone Bosons

[AP, Phys. Lett. B706 (2012) 465.]

No	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete Group Elements	Maximally Broken SO(5) Generators	Number of Pseudo-Goldstone Bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	$T^3,\ T^0$	–	T^3	1 (a)
✓ 5	$Z_2 \times [O(2)]^2$	$T^2,\ T^0$	D_{CP1}	T^2	1 (h)
✓ 6	$O(3) \times O(2)$	$T^{1,2,3},\ T^0$	–	$T^{1,2}$	2 $(h,\ a)$
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3,\ T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a,\ h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a,\ h^\pm)$
✓ 13	SO(5)	$T^{0,1,2,\dots,9}$	–	$T^{1,2,8,9}$	4 $(h,\ a,\ h^\pm)$

✓: Natural SM Alignment $\mapsto$ [Dev, AP, JHEP1412 (2014) 024.]

• Symmetries in 2HDMEFTs

[C Birch-Sykes, N Darvishi, Y Peters, AP, NPB960 (2020) 115171]

$$V_{\text{2HDMEFT}} = -\frac{1}{2} M_A \mathsf{R}^A + \frac{1}{4} L_{AB} \mathsf{R}^A \mathsf{R}^B + \frac{1}{\Lambda^2} K_{ABC} \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C + \frac{1}{\Lambda^4} Z_{ABCD} \mathsf{R}^A \mathsf{R}^B \mathsf{R}^C \mathsf{R}^D + \cdots$$

No. of couplings: $\boldsymbol{N^{(\text{dim}=2n)} = \frac{1}{6}(n+1)(n+2)(n+3)}$

$$N^{(\text{dim}\leq 4)} = 14, \quad N^{(\text{dim}\leq 6)} = 34, \quad N^{(\text{dim}\leq 8)} = 69, \quad \ldots, \quad N^{(\text{dim}\leq 20)} = 1000$$

Symmetry restrictions:

$$M_A [T^a]^{A'}_{\ A} = 0, \quad L_{A'B} [T^a]^{A'}_{\ A} + L_{AB'} [T^a]^{B'}_{\ B} = 0,$$

$$K_{A'BC} [T^a]^{A'}_{\ A} + K_{AB'C} [T^a]^{B'}_{\ B} + K_{ABC'} [T^a]^{C'}_{\ C} = 0,$$

$$Z_{A'BCD} [T^a]^{A'}_{\ A} + Z_{AB'CD} [T^a]^{B'}_{\ B} + Z_{ABC'D} [T^a]^{C'}_{\ C} + Z_{ABCD'} [T^a]^{D'}_{\ D} = 0\,,$$

where $T^a \in \mathfrak{g}$ are the generators of the symmetry subgroup $G \subseteq \mathsf{SO}(5)$.

No.	Symmetry	Non-zero parameters of Symmetric 2HDMEFT Potential	Dim
1	CP1	μ_1^2, μ_2^2, Re(m_{12}^2), λ_1, λ_2, λ_3, λ_4, Re(λ_5), Re(λ_6), Re(λ_7) $\kappa_1, \ldots, \kappa_6$, Re($\kappa_7, \ldots, \kappa_{13}$) $\zeta_1, \ldots, \zeta_9$, Re($\zeta_{10}, \ldots, \zeta_{22}$)	$D \geq 4$
2	Z_2	$\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6, \kappa_8, \kappa_9$ $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\ \zeta_6, \zeta_7, \zeta_8, \zeta_9, \zeta_{10}, \zeta_{13}, \zeta_{14}, \zeta_{15}, \zeta_{16}$	$D \geq 4$
3	Z_3	$\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6, \kappa_7$ $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6, \zeta_7, \zeta_8, \zeta_9, \zeta_{11}, \zeta_{12}$	$D \geq 6$
4	Z_4	$\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6$ $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6, \zeta_7, \zeta_8, \zeta_9, \zeta_{10}$	$D \geq 8$
5	CP2	$\mu_1^2 = \mu_2^2, \lambda_1 = \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 = -\lambda_7$ $\kappa_1 = \kappa_2, \kappa_3 = \kappa_4, \kappa_5 = \kappa_6, \kappa_8 = \kappa_9, \kappa_{11} = -\kappa_{12}$ $\zeta_1 = \zeta_2, \zeta_3, \zeta_4 = \zeta_5, \zeta_6, \zeta_7 = \zeta_8, \zeta_9,$ $\zeta_{10}, \zeta_{11} = -\zeta_{12}, \zeta_{13}, \zeta_{14} = \zeta_{15}, \zeta_{16}, \zeta_{17} = -\zeta_{18},$ $\zeta_{19} = -\zeta_{20}, \zeta_{21} = -\zeta_{22}$	$D \geq 4$
6	CP3	$\mu_1 = \mu_2, \lambda_1 = \lambda_2, \lambda_3, \lambda_4$ $\kappa_1 = \kappa_2, \kappa_3 = \kappa_4, \kappa_5 = \kappa_6, \kappa_7$ $\zeta_1 = \zeta_2, \zeta_3, \zeta_4 = \zeta_5, \zeta_6, \zeta_7 = \zeta_8, \zeta_9, \zeta_{11} = \zeta_{12}$	$D \geq 6$
7	CP4	$\mu_1 = \mu_2, \lambda_1 = \lambda_2, \lambda_3, \lambda_4$ $\kappa_1 = \kappa_2, \kappa_3 = \kappa_4, \kappa_5 = \kappa_6, \kappa_8 = -\kappa_9$ $\zeta_1 = \zeta_2, \zeta_3, \zeta_4 = \zeta_5, \zeta_6, \zeta_7 = \zeta_8, \zeta_9, \zeta_{10}, \zeta_{14} = -\zeta_{15}$	$D \geq 6$

8	$\mathrm{U(1)_{PQ}}$	$\mu_1^2,\ \mu_2^2,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4$ $\kappa_1,\ \kappa_2,\ \kappa_3,\ \kappa_4,\ \kappa_5,\ \kappa_6$ $\zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6,\ \zeta_7,\ \zeta_8,\ \zeta_9$	$D \geq 4$
9	$\mathrm{CP1} \otimes \mathrm{SO(2)_{HF}}$	$\mu_1^2 = \mu_2^2,\ \lambda_1 = \lambda_2,\ \lambda_3,\ \lambda_4,\ \mathrm{Re}(\lambda_5) = 2\lambda_1 - \lambda_{34}$ $\kappa_1 = \kappa_2,\ \kappa_3 = \kappa_4,\ \kappa_5 = \kappa_6,$ $\mathrm{Re}(\kappa_8) = \mathrm{Re}(\kappa_9) = \frac{1}{2}(3\kappa_1 - \kappa_3 - \kappa_5)$ $\zeta_1 = \zeta_2,\ \zeta_3,\ \zeta_4 = \zeta_5,\ \zeta_6,\ \zeta_7 = \zeta_8,\ \zeta_9,$ $\mathrm{Re}(\zeta_{10}) = -\frac{1}{4}\mathrm{Re}(\zeta_{13}) + \frac{1}{2}\mathrm{Re}(\zeta_{14}) - \frac{1}{4}\mathrm{Re}(\zeta_{16}),$ $\mathrm{Re}(\zeta_{13}) = \frac{1}{6}(4\zeta_1 + 2\zeta_3 - 4\zeta_4 - 4\zeta_6 + 2\zeta_7 - \zeta_9),$ $\mathrm{Re}(\zeta_{14}) = \mathrm{Re}(\zeta_{15}) = \frac{1}{2}(4\zeta_1 - \zeta_4 - \zeta_7),$ $\mathrm{Re}(\zeta_{16}) = \frac{1}{2}(4\zeta_1 - 2\zeta_3 + 2\zeta_4 - \zeta_9)$	$D \geq 4$
10	$\mathrm{SU(2)_{HF}}$	$\mu_1^2 = \mu_2^2,\ \lambda_1 = \lambda_2,\ \lambda_3,\ \lambda_4 = 2\lambda_1 - \lambda_3$ $\kappa_1 = \kappa_2,\ \kappa_3 = \kappa_4,\ \kappa_5 = \kappa_6 = 3\kappa_1 - \kappa_3$ $\zeta_1 = \zeta_2,\ \zeta_3,\ \zeta_4 = \zeta_5,\ \zeta_6 = 2\zeta_1 + \zeta_3 - 2\zeta_4,$ $\zeta_7 = \zeta_8 = 4\zeta_1 - \zeta_4,\ \zeta_9 = 4\zeta_1 - 2\zeta_3 + 2\zeta_4$	$D \geq 4$
11	$\mathrm{Sp(2)}_{\phi_1+\phi_2}$	$\mu_1^2,\ \mu_2^2,\ \mathrm{Re}(m_{12}^2),\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 = \mathrm{Re}(\lambda_5),\ \mathrm{Re}(\lambda_6),\ \mathrm{Re}(\lambda_7)$ $\kappa_1,\ \kappa_2,\ \kappa_3,\ \kappa_4,\ \kappa_5 = 2\mathrm{Re}(\kappa_8),\ \kappa_6 = 2\mathrm{Re}(\kappa_9),$ $\mathrm{Re}(\kappa_7) = \frac{1}{3}\mathrm{Re}(\kappa_{10}),\ \mathrm{Re}(\kappa_{11}),\ \mathrm{Re}(\kappa_{12}),\ \mathrm{Re}(\kappa_{13})$ $\zeta_1,\ \zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6 = 6\mathrm{Re}(\zeta_{10}) = \frac{3}{2}\mathrm{Re}(\zeta_{13}),$ $\zeta_7 = 2\mathrm{Re}(\zeta_{14}),\ \zeta_8 = 2\mathrm{Re}(\zeta_{15}),\ \zeta_9 = 2\mathrm{Re}(\zeta_{16}),$ $\mathrm{Re}(\zeta_{11}) = \frac{1}{3}\mathrm{Re}(\zeta_{17}),\ \mathrm{Re}(\zeta_{12}) = \frac{1}{3}\mathrm{Re}(\zeta_{18}),$ $\mathrm{Re}(\zeta_{19}),\ \mathrm{Re}(\zeta_{20}),\ \mathrm{Re}(\zeta_{21}),\ \mathrm{Re}(\zeta_{22})$	$D \geq 4$

12	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, Re(m_{12}^2), $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 =$ Re(λ_5), Re(λ_6)=Re(λ_7) $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2$Re$(\kappa_8) = 2$Re(κ_9), Re$(\kappa_7) = \frac{1}{3}$Re(κ_{10}), Re$(\kappa_{11}) =$ Re(κ_{12}), Re(κ_{13}) $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6$Re$(\zeta_{10}) = \frac{3}{2}Re(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2$Re$(\zeta_{14}) = 2$Re(ζ_{15}), $\zeta_9 = 2$Re(ζ_{16}) Re$(\zeta_{11}) =$ Re$(\zeta_{12}) = \frac{1}{3}Re(\zeta_{17}) = \frac{1}{3}Re(\zeta_{18})$, Re$(\zeta_{19}) =$ Re(ζ_{20}), Re$(\zeta_{21}) =$ Re(ζ_{22})	$D \geq 4$
13	$\mathrm{CP2} \otimes \mathrm{Sp}(2)_{\phi_1+\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3, $\lambda_4 =$ Re(λ_5), Re$(\lambda_6) = -$Re(λ_7) $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$, $\kappa_5 = \kappa_6 = 2$Re$(\kappa_8) = 2$Re(κ_9), Re$(\kappa_{11}) = -$Re(κ_{12}) $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$, $\zeta_6 = 6$Re$(\zeta_{10}) = \frac{3}{2}Re(\zeta_{13})$, $\zeta_7 = \zeta_8 = 2$Re$(\zeta_{14}) = 2$Re(ζ_{15}), $\zeta_9 = 2$Re(ζ_{16}) Re$(\zeta_{11}) = -$Re$(\zeta_{12}) = \frac{1}{3}Re(\zeta_{17}) = -\frac{1}{3}Re(\zeta_{18})$, Re$(\zeta_{19}) = -Re(\zeta_{20})$, Re$(\zeta_{21}) = -Re(\zeta_{22})$	$D \geq 4$
14	$\mathrm{U(1)_{PQ}} \otimes \mathrm{Sp}(2)_{\phi_1\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, λ_4 $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$, $\kappa_5 = \kappa_6$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$, ζ_6, $\zeta_7 = \zeta_8 = \frac{1}{2}\zeta_9$	$D \geq 4$
15	$\mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	μ_1^2, μ_2^2, λ_1, λ_2, λ_3 κ_1, κ_2, κ_3, κ_4 ζ_1, ζ_2, ζ_3, ζ_4, ζ_5	$D \geq 4$
16	$S_2 \otimes \mathrm{Sp}(2)_{\phi_1} \otimes \mathrm{Sp}(2)_{\phi_2}$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$, λ_3 $\kappa_1 = \kappa_2$, $\kappa_3 = \kappa_4$ $\zeta_1 = \zeta_2$, ζ_3, $\zeta_4 = \zeta_5$	$D \geq 4$
17	$\mathrm{Sp}(4)$	$\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2 = \frac{1}{2}\lambda_3$, $\kappa_1 = \kappa_2 = \frac{1}{3}\kappa_3 = \frac{1}{3}\kappa_4$ $\zeta_1 = \zeta_2 = \frac{1}{6}\zeta_3 = \frac{1}{4}\zeta_4 = \frac{1}{4}\zeta_5$	$D \geq 4$

• Symmetries of multi-HDM Potentials

[N Darvishi, AP, PRD101 (2020) 095008]

Prime bilinear invariants:

► Maximal block:

$$\begin{cases} \mathrm{Sp}(2n): & S_n = \Phi^+\Phi & \text{with } \Phi = \begin{pmatrix} \phi \\ i\sigma^2\phi^* \end{pmatrix} \\ \mathrm{SU}(n): & D_n^a = \phi^\dagger \sigma^a \phi & \text{and } \phi = (\phi_1, \phi_2, \cdots, \phi_n)^\mathsf{T} \\ \mathrm{SO}(n): & T_n = \phi\phi^\mathsf{T} & \end{cases}$$

► Minimal block:

$$\begin{cases} \mathrm{Sp}(2): & \begin{cases} S_{ii} = \phi_i^\dagger \phi_i & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_i^* \end{pmatrix} \\ S_{ij} = \phi_i^\dagger \phi_j + \phi_j^\dagger \phi_i & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_j^* \end{pmatrix} \& \begin{pmatrix} \phi_j \\ i\sigma^2\phi_i^* \end{pmatrix} \end{cases} \\ \mathrm{SU}(2)\times\mathrm{U}(1): & \begin{cases} D_{ij}^a = \phi_i^\dagger \sigma^a \phi_i + \phi_j^\dagger \sigma^a \phi_j = D_{ji}^a & \text{for } \begin{pmatrix} \phi_i \\ \phi_j \end{pmatrix} \\ D_{ij}'^a = \phi_i^\dagger \sigma^a \phi_i + (i\sigma^2\phi_j^*)\sigma^a(i\sigma^2\phi_j^*) & \text{for } \begin{pmatrix} \phi_i \\ i\sigma^2\phi_j^* \end{pmatrix} \end{cases} \\ \mathrm{SO}(2): & \left\{ T_{ij} = \phi_i\phi_i^\mathsf{T} + \phi_j\phi_j^\mathsf{T} = T_{ji} \quad \text{for } \begin{pmatrix} \phi_i \\ \phi_j \end{pmatrix} \right. \end{cases}$$

[AP, PRD93 (2016) 075012]

The symmetric potential $\rightarrow V_{\mathrm{sym}} = -\mu^2 S_n + \lambda_S S_n^2 + \lambda_D D_n^2 + \lambda_T T_n^2$

• Discrete Symmetries

[Earlier studies: Ivanov, Vdovin '12; V Keus et al '13; Ivanov, Varzielas '19, . . .]

→ **Generalized CP (GCP) transformations:**

$$\mathsf{GCP}[\phi_i] = G_{ij}\phi_j^* \qquad G_{ij} \in \mathsf{SU}(n)$$

→ **Abelian Discrete Symmetries:**

$$Z_2, \quad Z_3, \quad Z_4, \quad Z_2 \times Z_2, \quad Z_3 \times Z_3, \quad \cdots, \quad Z_n, \quad \cdots,$$

where $Z_n = \{1, \omega, \cdots, \omega^{(n-1)}\}$ with $\omega^n = 1$.

→ **Non-Abelian Discrete Symmetries**

- **Typical Non-Abelian Discrete Symmetries**

  – *Permutation group* $S_N \xrightarrow{\text{with order}} N!$

  – *Alternating group* $A_N \xrightarrow{\text{with order}} N!/2$

  – *Dihedral group* $D_N \xrightarrow{\text{isomorphic to}} Z_N \rtimes Z_2$

  – *Binary Dihedral group* $Q_{2N} \xrightarrow{\text{with order}} 4N$

  – *Tetrahedral group* $T_{N\,(\text{prime number})} \xrightarrow{\text{isomorphic to}} Z_N \rtimes Z_3$

  – *Dihedral-like groups*:

$$\Sigma(2N^2) \cong (Z_N \times Z'_N) \rtimes Z_2 \qquad \Delta(3N^2) \cong (Z_N \times Z'_N) \rtimes Z_3$$

$$\Sigma(3N^3) \cong Z_N \times \Delta(3N^2) \qquad \Delta(6N^2) \cong (Z_N \times Z'_N) \rtimes S_3$$

  – *Crystal-like groups* $\Sigma(M\phi)$, with $\phi = 1, 2, 3$:

$$\Sigma(60\phi), \quad \Sigma(168\phi), \quad \Sigma(36\phi), \quad \Sigma(72\phi), \quad \Sigma(216\phi), \quad \Sigma(360\phi)$$

No.	Symmetry	Non-zero parameters for 3HDM potentials
1	CP1	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), Re(m_{13}^2), Re(m_{23}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, Re(λ_{1212}), Re(λ_{1313}), Re(λ_{2323}), Re(λ_{1213}), Re(λ_{2113}), Re(λ_{1223}), Re(λ_{2123}), Re(λ_{1323}), Re(λ_{1332}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}), Re(λ_{2213}), Re(λ_{3313}), Re(λ_{1123}), Re(λ_{2223}), Re(λ_{3323})
2	Z_2	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{13}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1232}, λ_{1113}, λ_{2213}, λ_{3313} and H.c.$\}$
$2'$	Z_2'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{23}^2$, λ_{1212}, λ_{1313}, λ_{2323}, λ_{1213}, λ_{1123}, λ_{2223}, λ_{3323} and H.c.$\}$
3	$Z_2 \otimes Z_2'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1313}, λ_{2323} and H.c.$\}$
4	Z_3	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1213}$, λ_{1323}, λ_{2123} and H.c.$\}$
5	Z_4	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$, λ_{1323} and H.c.$\}$
$5'$	Z_4'	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1313}$, λ_{3212} and H.c.$\}$
6	^{a}U(1)	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1323}$ and H.c.$\}$
$6'$	^{b}U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{m_{12}^2$, λ_{1212}, λ_{1112}, λ_{2212}, λ_{3312}, λ_{1332} and H.c.$\}$
7	U(1) $\otimes$ U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}
8	$Z_2 \otimes$ U(1)$'$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, λ_{1331}, λ_{2332}, $\{\lambda_{1212}$ and H.c.$\}$
9	CP1 $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
10	CP1 $\otimes Z_2 \otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}, Re(λ_{1212})
11	U(1) $\otimes$ Sp(2)$_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}
12	CP2	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, Re($\lambda_{1313}$)=Re($\lambda_{2323}$), Re($\lambda_{1212}$), $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$

13	$\mathrm{CP2} \otimes \mathrm{Sp(2)}_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\mathrm{Re}(\lambda_{1212})$, $\{\lambda_{1112} = -\lambda_{2212}$ and H.c.$\}$
14	$\mathrm{SO(2)}_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$, $\mathrm{Re}(\lambda_{1313}) = \mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1212}) = 2\lambda_{11} - (\lambda_{1122} + \lambda_{1221})$,
15	D_3	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\lambda_{1331} = \lambda_{2332}$, $\{\lambda_{2131} = -\lambda_{1232}$, λ_{1323} and H.c.$\}$
16	D_4	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\{\lambda_{1212}$ and H.c.$\}$, $\lambda_{1331} = \lambda_{2332} = \mathrm{Re}(\lambda_{3231})$
17	$D_3 \otimes \mathrm{Sp(2)}_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}
18	$D_4 \otimes \mathrm{Sp(2)}_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}, $\mathrm{Re}(\lambda_{1212})$
19	$\mathrm{SO(2)}_{\phi_1,\phi_2} \otimes \mathrm{Sp(2)}_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212}) = \lambda_{11} - \frac{1}{2}\lambda_{1122}$
20	$\mathrm{SU(2)}_{\phi_1,\phi_2}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, $\lambda_{1122} = 2\lambda_{11} - \lambda_{1221}$, λ_{1221}, $\lambda_{1133} = \lambda_{2233}$, $\lambda_{1331} = \lambda_{2332}$
21	$\mathrm{SU(2)}_{\phi_1,\phi_2} \otimes \mathrm{Sp(2)}_{\phi_3}$	$\mu_1^2 = \mu_2^2$, μ_3^2, $\lambda_{11} = \lambda_{22}$, λ_{33}, $\lambda_{1122} = 2\lambda_{11} - \lambda_{1221}$, $\lambda_{1133} = \lambda_{2233}$, λ_{1221}
22	$\mathrm{Sp(2)}_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Re}(m_{13}^2)$, $\mathrm{Re}(m_{23}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212})$, $\lambda_{1331} = \mathrm{Re}(\lambda_{1313})$, $\lambda_{2332} = \mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1213}) = \mathrm{Re}(\lambda_{2113})$, $\mathrm{Re}(\lambda_{1223}) = \mathrm{Re}(\lambda_{2123})$, $\mathrm{Re}(\lambda_{1323}) = \mathrm{Re}(\lambda_{1332})$, $\mathrm{Re}(\lambda_{1112})$, $\mathrm{Re}(\lambda_{2212})$, $\mathrm{Re}(\lambda_{3312})$, $\mathrm{Re}(\lambda_{1113})$, $\mathrm{Re}(\lambda_{2213})$, $\mathrm{Re}(\lambda_{3313})$, $\mathrm{Re}(\lambda_{1123})$, $\mathrm{Re}(\lambda_{2223})$, $\mathrm{Re}(\lambda_{3323})$
23	$Z_2 \otimes \mathrm{Sp(2)}_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{13}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212})$, $\lambda_{1331} = \mathrm{Re}(\lambda_{1313})$, $\lambda_{2332} = \mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1113})$, $\mathrm{Re}(\lambda_{2213})$, $\mathrm{Re}(\lambda_{3313})$
23′	$Z_2' \otimes \mathrm{Sp(2)}_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, $\mathrm{Re}(m_{23}^2)$, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212})$, $\lambda_{1331} = \mathrm{Re}(\lambda_{1313})$, $\lambda_{2332} = \mathrm{Re}(\lambda_{2323})$, $\mathrm{Re}(\lambda_{1123})$, $\mathrm{Re}(\lambda_{2223})$, $\mathrm{Re}(\lambda_{3323})$
24	$Z_2 \otimes Z_2' \otimes \mathrm{Sp(2)}_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212})$, $\lambda_{1331} = \mathrm{Re}(\lambda_{1313})$, $\lambda_{2332} = \mathrm{Re}(\lambda_{2323})$
25	$Z_4 \otimes \mathrm{Sp(2)}_{\phi_1+\phi_2+\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, $\lambda_{1221} = \mathrm{Re}(\lambda_{1212})$

26	$(\mathrm{CP1} \rtimes S_2)\otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, Re(m_{12}^2), Re(m_{13}^2)=Re(m_{23}^2), $\lambda_{11}=\lambda_{22}$, λ_{33}, λ_{1122}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}= Re(λ_{1212}), λ_{1331}=λ_{2332}=Re(λ_{1313})=Re(λ_{2323}), Re(λ_{1323}) = Re(λ_{1332}), Re(λ_{1223}) = Re(λ_{2123})=Re(λ_{1213}) = Re(λ_{2113}), Re(λ_{3313}) = Re(λ_{3323}), Re(λ_{1112}) = Re(λ_{2212}), Re(λ_{3312}), Re(λ_{1113}) = Re(λ_{1123}) = Re(λ_{2213}) = Re(λ_{2223})
27	$D_4\otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221} =Re(λ_{1212})
28	$\mathrm{Sp}(2)_{\phi_1+\phi_2}\otimes \mathrm{Sp}(2)_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, Re(m_{12}^2), λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}, λ_{1221}= Re(λ_{1212}), Re(λ_{1112}), Re(λ_{2212}), Re(λ_{3312})
29	$\mathrm{Sp}(2)_{\phi_1\phi_2}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}, $\lambda_{1331}=\lambda_{2332}$
30	$\mathrm{Sp}(2)_{\phi_1\phi_2}\otimes \mathrm{Sp}(2)_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$, λ_{1221}
31	A_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{1212}=\lambda_{1313}=\lambda_{2323}$, and H.c.$\}$
32	S_4	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re$(\lambda_{1212})$ = Re(λ_{1313}) = Re(λ_{2323})
33	SO(3)	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re$(\lambda_{1212})$ = Re(λ_{1313}) = Re(λ_{2323}) $=2\lambda_{11}-(\lambda_{1122}+\lambda_{1221})$
34	$S_4\otimes \mathrm{Sp}(2)_{\phi_1+\phi_2+\phi_3}$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}=\frac{1}{2}\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$=Re$(\lambda_{1212})$ = Re(λ_{1313}) = Re(λ_{2323})
35	$\Delta(54)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, $\{\lambda_{1213}=\lambda_{2123}=\lambda_{3231}$ and H.c.$\}$
36	$\Sigma(36)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}$, Re$(\lambda_{1213})$ = Re(λ_{1323}) = Re(λ_{1232})= $\frac{3}{4}(2\lambda_{11}-\lambda_{1122}-\lambda_{1221})$
37	$\mathrm{Sp}(2)_{\phi_1}\otimes \mathrm{Sp}(2)_{\phi_2}\otimes \mathrm{Sp}(2)_{\phi_3}$	μ_1^2, μ_2^2, μ_3^2, λ_{11}, λ_{22}, λ_{33}, λ_{1122}, λ_{1133}, λ_{2233}
38	$\mathrm{Sp}(4)\otimes \mathrm{Sp}(2)_{\phi_3}$	$\mu_1^2=\mu_2^2$, μ_3^2, $\lambda_{11}=\lambda_{22}=\frac{1}{2}\lambda_{1122}$, λ_{33}, $\lambda_{1133}=\lambda_{2233}$
39	$\mathrm{SU}(3)\otimes \mathrm{U}(1)$	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}$, $\lambda_{1122}=\lambda_{1133}=\lambda_{2233}$, $\lambda_{1221}=\lambda_{1331}=\lambda_{2332}=2\lambda_{11}-\lambda_{1122}$
40	Sp(6)	$\mu_1^2=\mu_2^2=\mu_3^2$, $\lambda_{11}=\lambda_{22}=\lambda_{33}=\frac{1}{2}\lambda_{1122}=\frac{1}{2}\lambda_{1133}=\frac{1}{2}\lambda_{2233}$

• Vacuum Topology of the 2HDM

[R Battye, G Brawn, AP, JHEP08 (2011) 020.]

$\mathrm{G}_{\mathrm{HF/CP}}$	$\mathrm{H}_{\mathrm{HF/CP}}$	$\mathcal{M}_{\Phi}^{\mathrm{HF/CP}}$	Topological Defect
Z_2	$\mathbf{I}$	Z_2	Domain Wall
$\mathrm{U}(1)_{\mathrm{PQ}} \simeq S^1$	$\mathbf{I}$	S^1	Vortex
$\mathrm{SO}(3)_{\mathrm{HF}}$	$\mathrm{SO}(2)_{\mathrm{HF}}$	S^2	Global Monopole
$\mathrm{CP1} \simeq \mathrm{Z}_2$	$\mathbf{I}$	Z_2	Domain Wall
CP2 $= \mathrm{Z}_2 \otimes \Pi_2$	Π_2	Z_2	Domain Wall
$\mathrm{CP1} \otimes \mathrm{SO}(2)$	CP1	S^1	Vortex

• Energy density of the topological defect $\phi_{1,2}(\mathbf{r})$:

$$\mathcal{E}(\phi_1, \phi_2) \;=\; (\nabla\phi_1^\dagger)\cdot(\nabla\phi_1) + (\nabla\phi_2^\dagger)\cdot(\nabla\phi_2) + \; \mathrm{V}(\phi_1, \phi_2) \;+\; \mathrm{V}_0\,.$$

• Gradient flow approach to numerically find $\phi_{1,2}(\mathbf{r})$

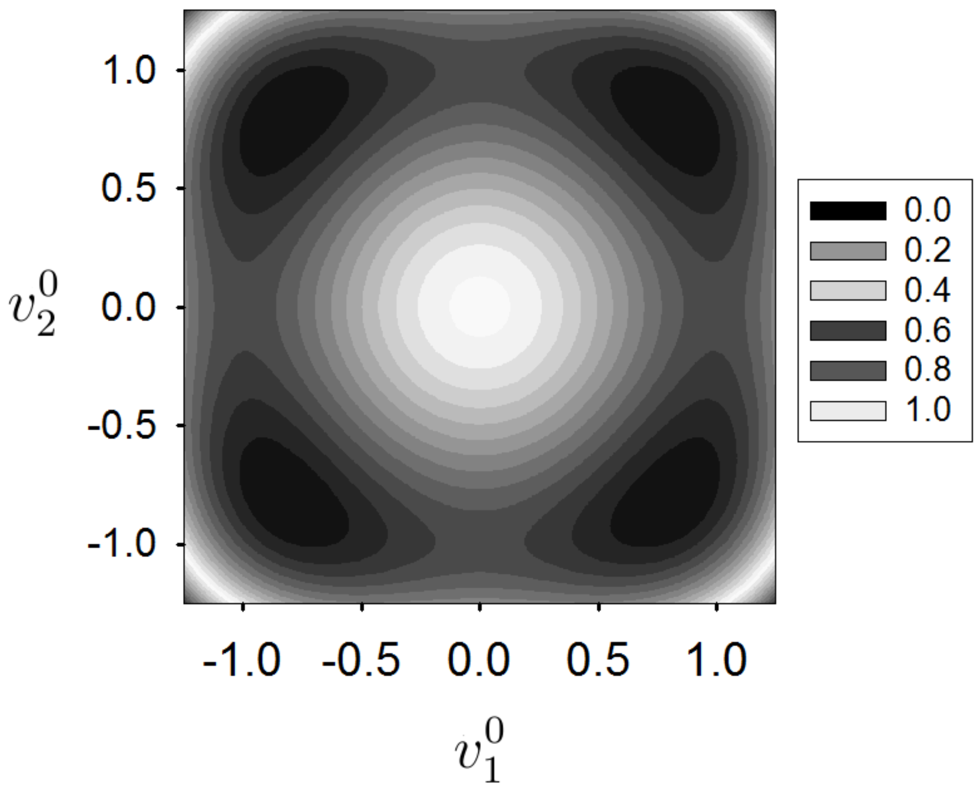
$$-\frac{\delta E[\phi_{1,2}]}{\delta\phi_{1,2}(\mathbf{r},\tau)} \;=\; \frac{\partial\phi_{1,2}(\mathbf{r},\tau)}{\partial\tau} \;\to\; 0\,, \quad \text{for } \tau \gg 1\,.$$

• Z_2 Domain Walls

$$\begin{pmatrix} v_1^0 \\ v_2^0 \end{pmatrix} \overset{U(1)_Y}{\longleftrightarrow} \begin{pmatrix} -v_1^0 \\ -v_2^0 \end{pmatrix}$$

$$Z_2 \updownarrow \qquad\qquad \updownarrow Z_2$$

$$\begin{pmatrix} v_1^0 \\ -v_2^0 \end{pmatrix} \overset{U(1)_Y}{\longleftrightarrow} \begin{pmatrix} -v_1^0 \\ v_2^0 \end{pmatrix}$$

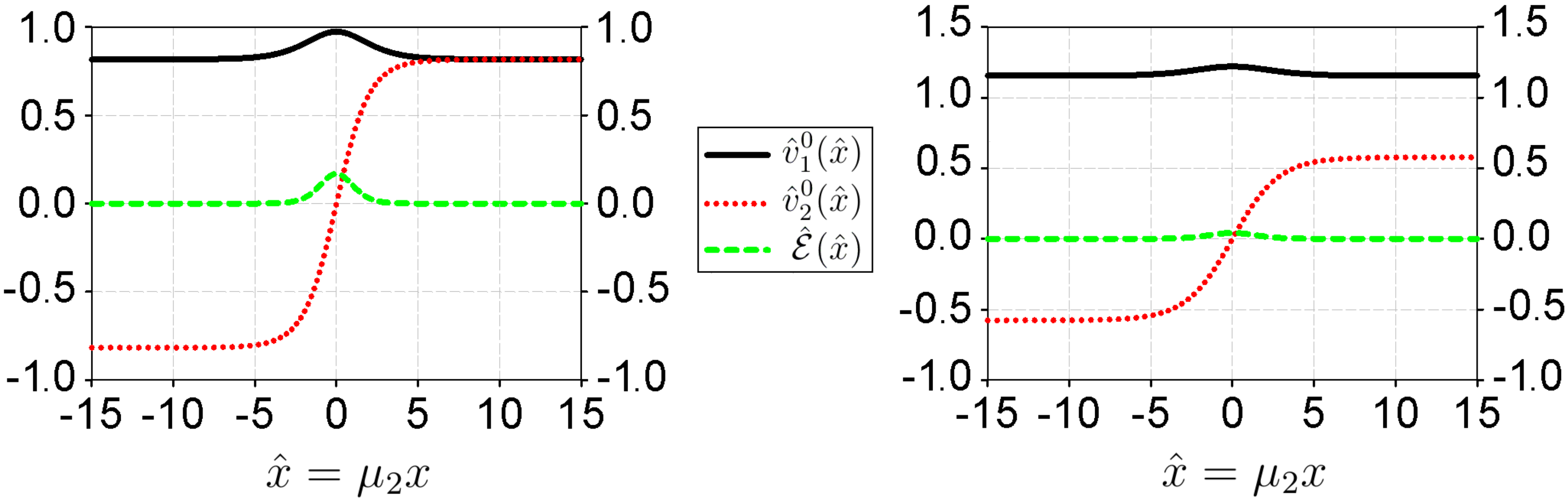


– Spatial profile of the Z_2 domain wall

[R Battye, G Brown, AP, JHEP08 (2011) 020.]

Introduce dimensionless quantities:

$$\hat{x} \;=\; \mu_2 x\;, \quad \hat{v}^0_{1,2}(\hat{x}) \;=\; \frac{v^0_{1,2}(\hat{x})}{\eta}\;, \quad \widehat{\mathrm{E}} \;=\; \frac{\lambda_2 \mathrm{E}}{\mu_2^3}\;, \quad \text{with } \eta \;=\; \frac{\mu_2}{\sqrt{\lambda_2}}\;.$$



• Charge-Violating Domain Walls in the 2HDM

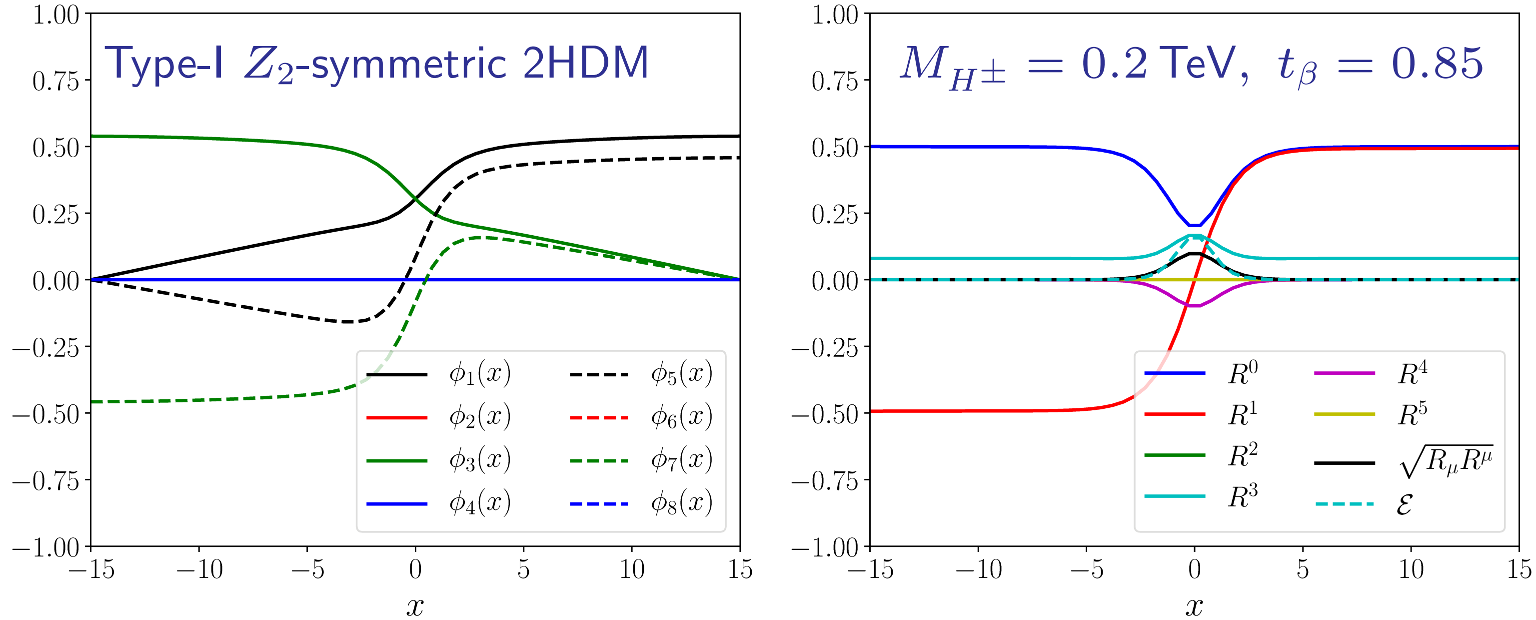
[R Battye, AP, D Viatic, JHEP2101 (2021) 105.
K.H. Law, AP, PRD105 (2022) 056007]

– Relatively gauge-rotated vacua at the boundaries:

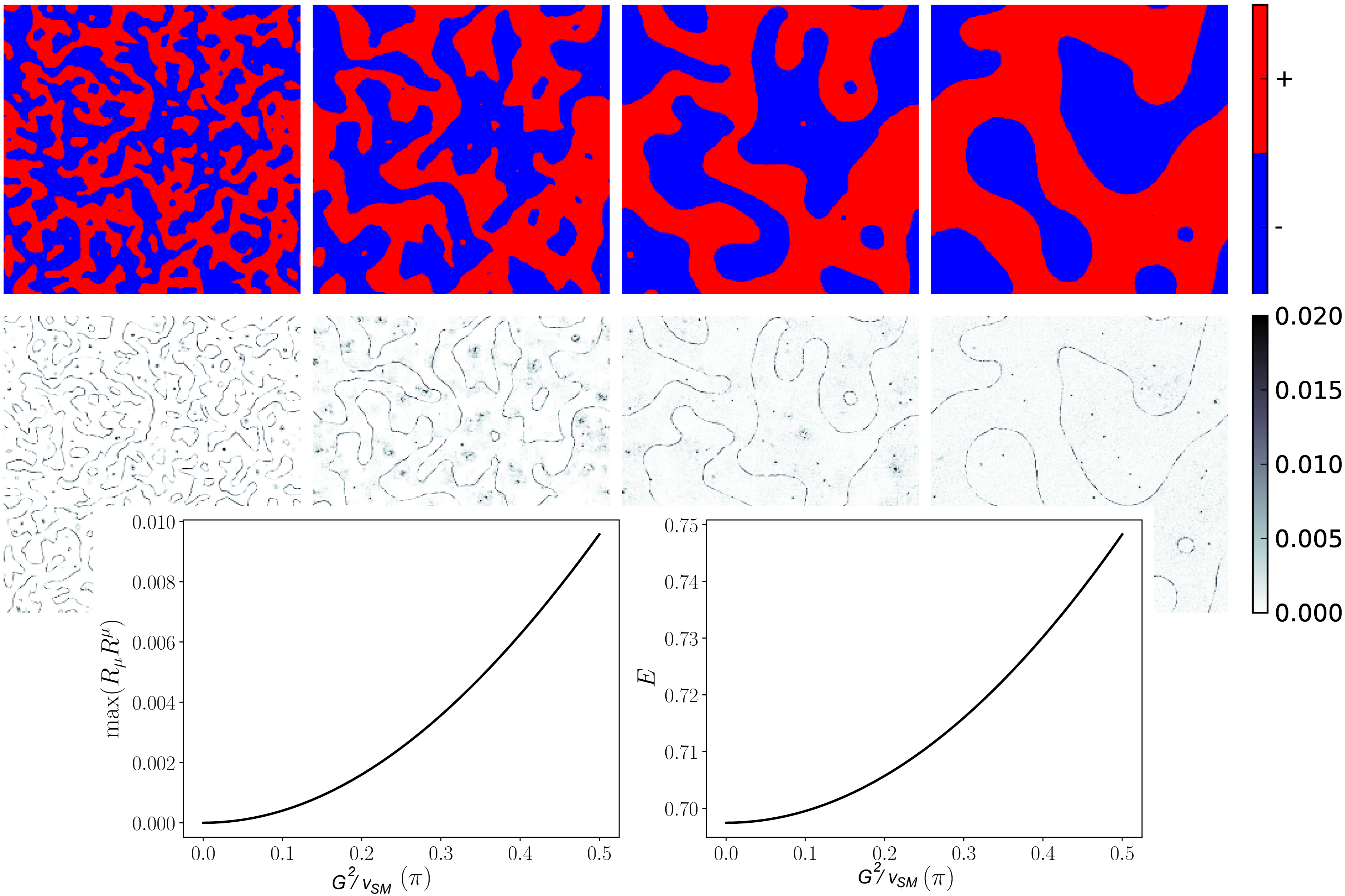
$$\Phi_1(-\infty) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(-\infty) = \frac{1}{\sqrt{2}}\begin{pmatrix} v_+ \to 0 \\ -v_2\, e^{-i\xi} \end{pmatrix},$$

$$\Phi_1(+\infty) = U(+\infty)\,\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2(+\infty) = U(+\infty)\,\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ +v_2\, e^{+i\xi} \end{pmatrix},$$

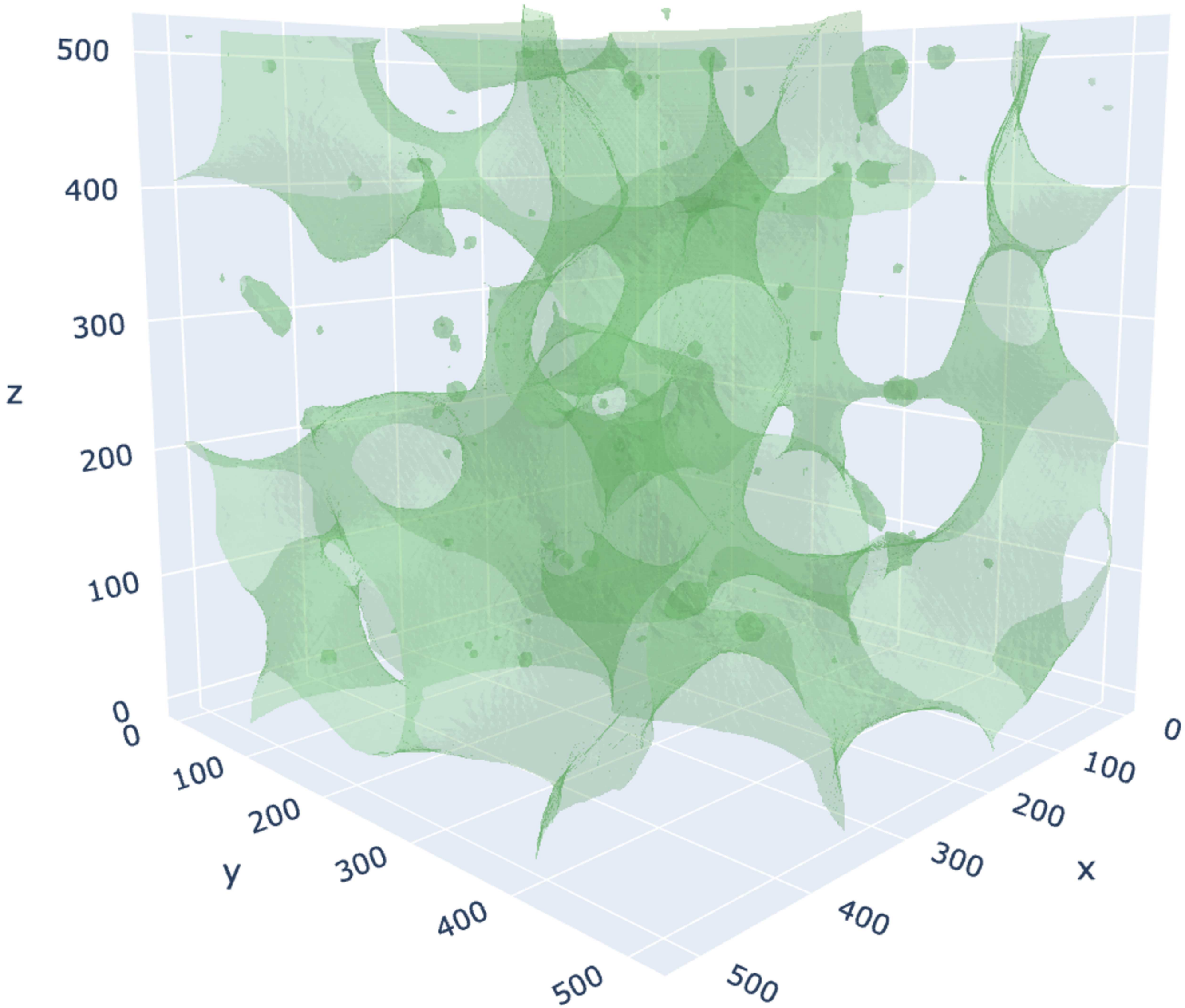
$$U(x) = e^{i\theta(x)}\,\exp\left(\frac{i\,G^i(x)}{v_{\rm SM}}\,\frac{\sigma^i}{2}\right), \qquad \text{with} \quad U(-\infty) = \mathbf{1}_2\,.$$



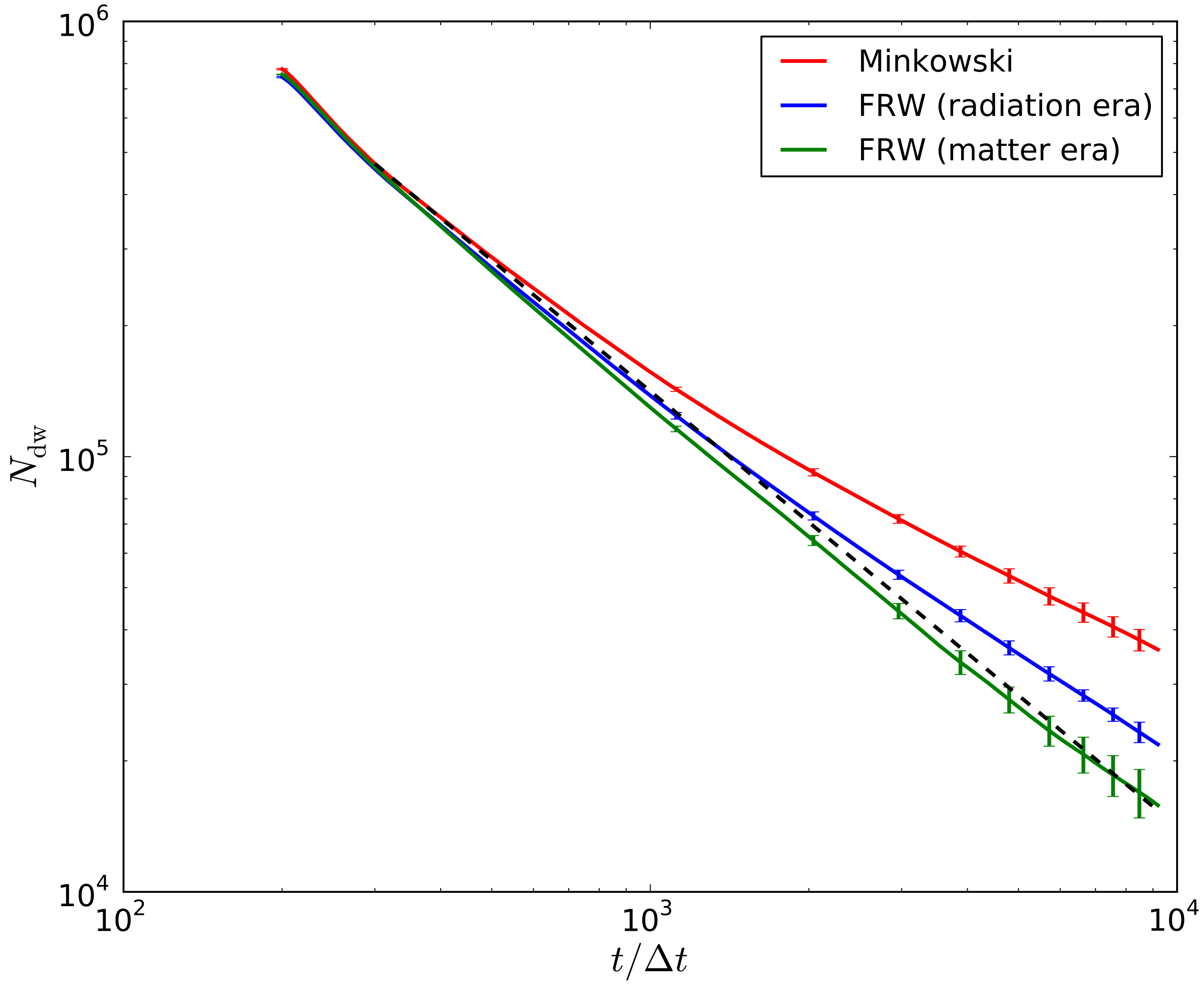
– 2D DW simulations in the Type-I Z_2-symmetric 2HDM



– 3D DW network in the Type-I Z_2-symmetric 2HDM



– Evolution of DW number $N_{\rm dw}$ in the Type-I Z_2-symmetric 2HDM



– QCD instantons in Type-II Z_2-symmetric 2HDM

[R Battye, AP, D Viatic, PRD102 (2020) 123536;
RD Peccei, HR Quinn '77]

$$V_{\rm inst} \sim \Lambda^4_{\rm QCD} \left[\left(\frac{\Phi_1^\dagger \Phi_2}{v^2_{\rm SM}} \right)^{n_{\sf G}} - \left(\frac{\Phi_1^\dagger \Phi_2\, e^{i\theta_{\sf QCD}}}{v^2_{\rm SM}} \right)^{n_{\sf G}} \right] \;+\; {\rm H.c.}$$

$$\lesssim \frac{\Lambda^4_{\rm QCD}}{v^2_{\rm SM}}\, s^2_\beta c^2_\beta \Big(1 - \cos\big(n_{\sf G}\theta_{\sf QCD}\big)\Big)\; \Phi_1^\dagger \Phi_2 \;+\; {\rm H.c.}\,,$$

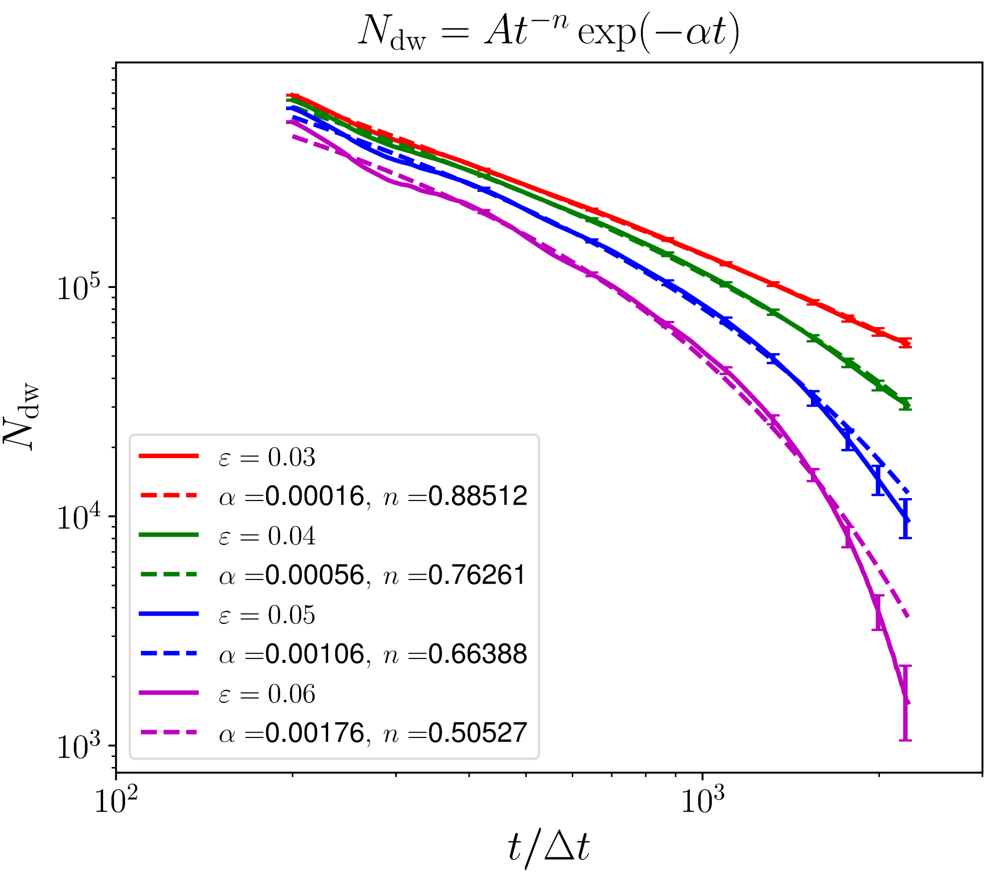
$$\Longrightarrow \quad \theta_{\sf QCD} \;\gtrsim\; \frac{10^{-11}}{\sin\beta\cos\beta}$$

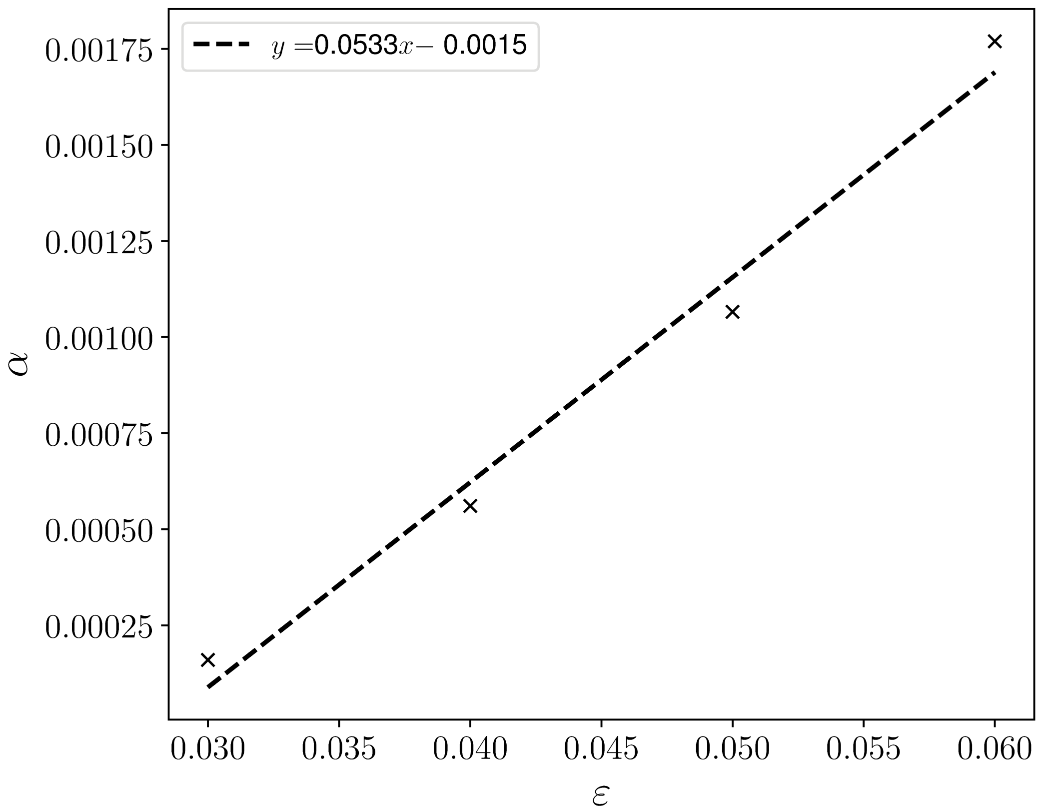
From neutron EDM limit: $\theta_{\sf QCD} \;\lesssim\; 10^{-11} - 10^{-10}$

Loose constraint: $\mathbf{0.3 \lesssim \tan\beta \lesssim 3}$

– Biased initial conditions in Z_2-symmetric 2HDMs

[R Battye, AP, D Viatic, PRD102 (2020) 123536]





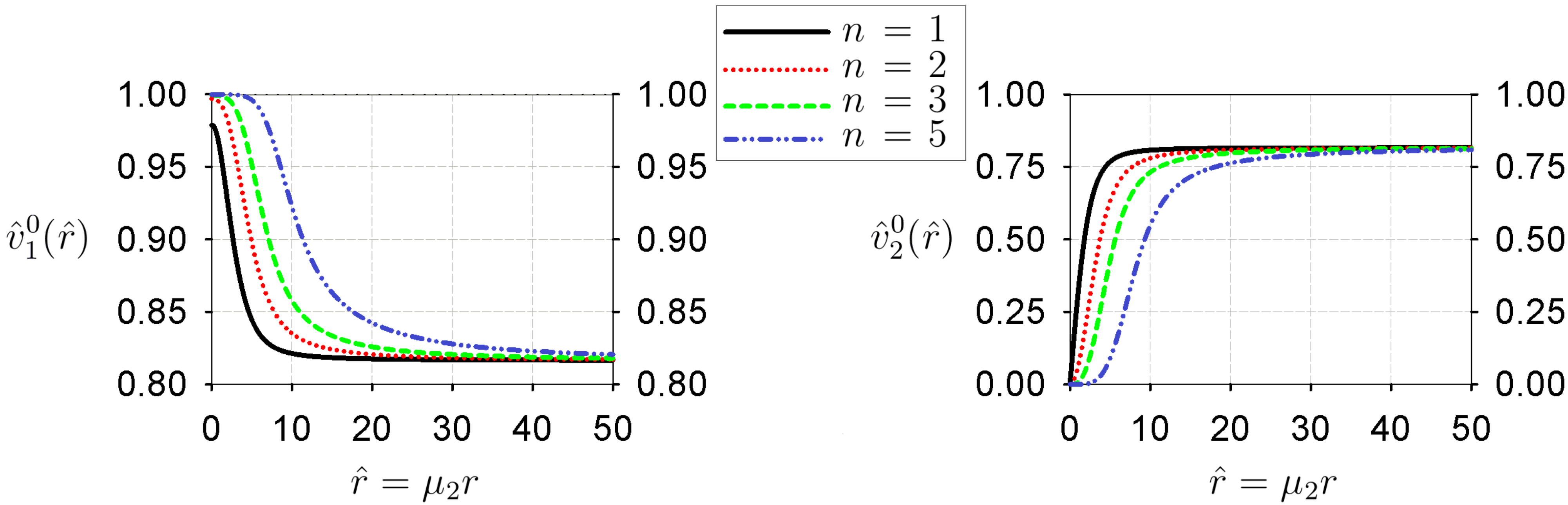
Avoidance of DW domination in the Universe:

$$\varepsilon \; > \; \frac{640\pi}{3}\frac{A\widehat{E}}{e}\left(\frac{v_{\rm SM}^{3/2}}{M_{\rm Pl}}\right)^2 \;\simeq\; 2.5\times 10^{-29} A\widehat{E}\ \text{GeV}, \quad \text{with } A,\ \widehat{E}\sim 1.$$

• Other Topological Defects from the 2HDM Potential

• **$U(1)_{PQ}$ Vortices** [R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1^0(r) \end{pmatrix}, \qquad \phi_2(r,\chi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2^0(r)e^{in\chi} \end{pmatrix}.$$

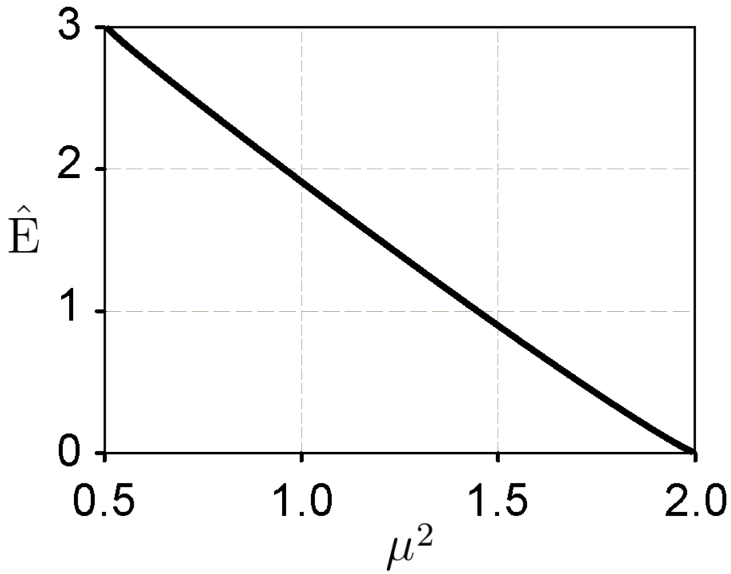


Energy dependence of the $\mathbf{U(1)_{\mathrm{PQ}}}$ Vortex

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

Energy per unit length:

$$\mathrm{E} \ = \ 2\pi \int_0^\infty r dr \ \ \mathcal{E}(\phi_1, \phi_2) \ ,$$



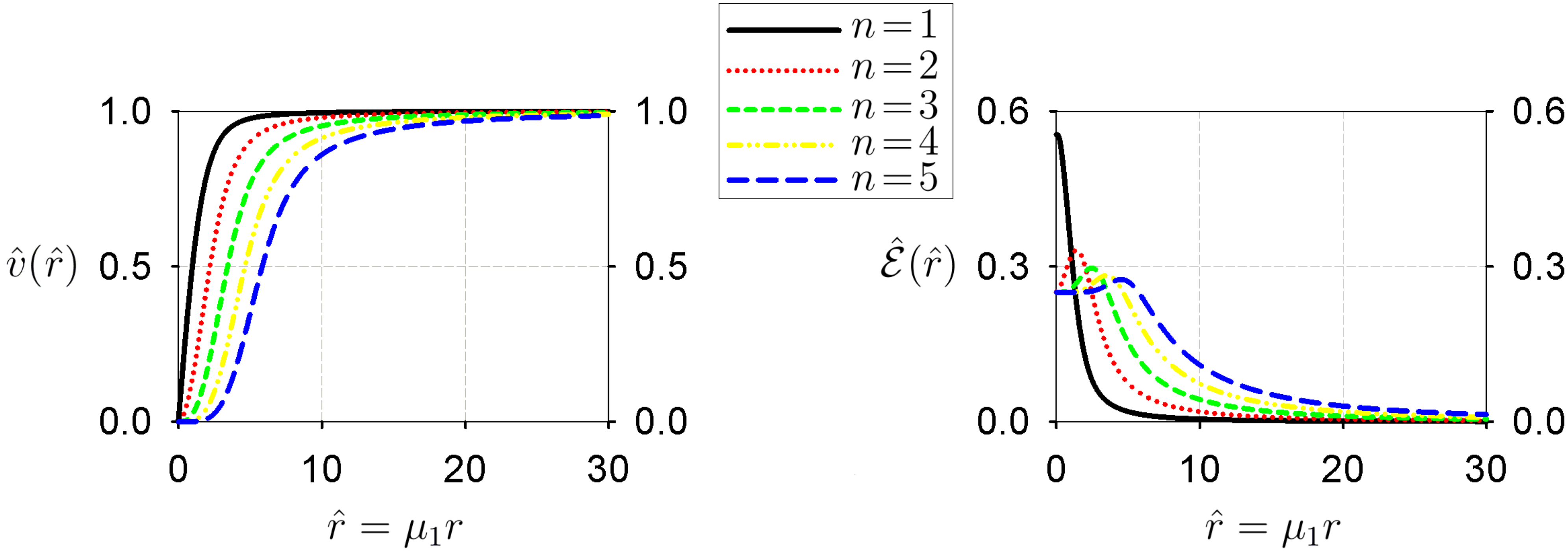
with

$$\mu^2 \ = \ \frac{\mu_1^2}{\mu_2^2} \ .$$

• CP3 Vortices

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

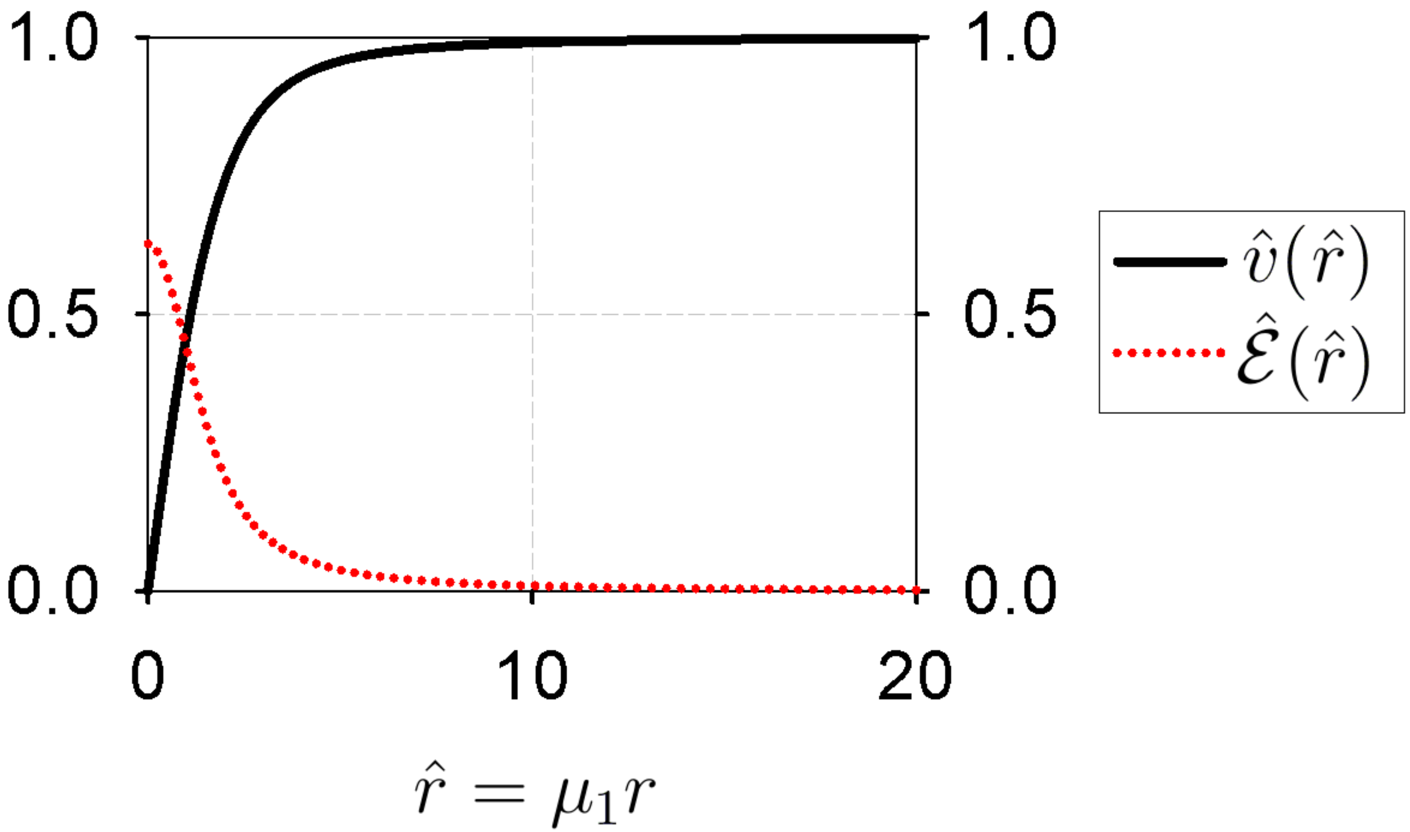
$$\phi_1(r,\chi) \ = \ \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v(r)\cos(n\chi) \end{array}\right), \qquad \phi_2(r,\chi) \ = \ \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ -v(r)\sin(n\chi) \end{array}\right).$$



• SO(3)$_{\rm HF}$ Global Monopole

[R. A. Battye, G. D. Brawn, A.P., JHEP08 (2011) 020.]

$$\phi_1(r,\chi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)\sin\chi \end{pmatrix}\,, \qquad \phi_2(r,\chi,\psi) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v(r)e^{i\psi}\cos\chi \end{pmatrix}\,.$$



• Natural Alignment Beyond the 2HDM

[AP '16]

– **nHDM** **potential** with m **inert scalar doublets:**

$$V_{n\mathrm{HDM}} \;=\; V_{\mathrm{sym}} \;+\; V_{\mathrm{inert}} \;+\; \Delta V_{\mathrm{soft}}\;,$$

– **3 continuous alignment symmetries** in the **field space of the active EWSB sector** $(N_H = n - m)$:

(i) $\mathrm{Sp}(2N_H)\times\mathcal{D}$ (ii) $\mathrm{SU}(N_H)\times\mathcal{D}$ (iii) $\mathrm{SO}(N_H)\times\mathcal{CP}\times\mathcal{D}$,

where $\mathcal{D}$ acts on the inert sector *only*.

– **Symmetry invariants:**

$$\text{(i)} \quad S \;=\; \Phi_1^\dagger\Phi_1 \;+\; \Phi_2^\dagger\Phi_2 \;+\; \ldots \;=\; \tfrac{1}{2}\,\mathbf{\Phi}^\dagger\,\mathbf{\Phi}$$

$$\text{(ii)} \quad D^a \;=\; \Phi_1^\dagger\sigma^a\Phi_1 \;+\; \Phi_2^\dagger\sigma^a\Phi_2 \;+\; \ldots$$

$$\text{(iii)} \quad T \;=\; \Phi_1\Phi_1^\intercal \;+\; \Phi_2\Phi_2^\intercal \;+\; \ldots$$

– **Symmetric part of the scalar potential:**

$$V_{\mathrm{sym}} \;=\; -\,\mu^2\,S \;+\; \lambda_S\,S^2 \;+\; \lambda_D\,D^aD^a \;+\; \lambda_T\,\mathrm{Tr}\,(T\,T^*)\;.$$

– **Inert part of the scalar potential:**

$$V_{\rm inert} \; = \; \widehat{m}^2_{\hat{a}\hat{b}}\, \widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}} \, + \, \lambda_{\hat{a}\hat{b}\hat{c}\hat{d}}\, (\widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}} \widehat{\Phi}_{\hat{d}}) \, + \, \lambda_{\hat{a}\hat{b}cd}\, (\widehat{\Phi}^\dagger_{\hat{a}} \widehat{\Phi}_{\hat{b}})(\Phi^\dagger_{c} \Phi_{d})$$

$$+\, \lambda_{a\hat{b}\hat{c}d}\, (\Phi^\dagger_{a} \widehat{\Phi}_{\hat{b}})(\widehat{\Phi}^\dagger_{\hat{c}} \Phi_{d}) \, + \, \Big[\lambda_{a\hat{b}c\hat{d}}\, (\Phi^\dagger_{a} \widehat{\Phi}_{\hat{b}})(\Phi^\dagger_{c} \widehat{\Phi}_{\hat{d}}) \, + \, {\rm H.c.} \Big]$$

$$\mathbf{Z_2^I} : \quad \Phi_a \, \to \, \Phi_a \quad (a = 1,\, 2, \dots, N_H)\, , \qquad \widehat{\Phi}_{\hat{b}} \, \to \, -\,\widehat{\Phi}_{\hat{b}} \quad (\hat{b} = \hat{1},\, \hat{2}, \dots, \widehat{m})$$

– **Soft-symmetry Breaking:**

$$\Delta V_{\rm soft} \; = \; m^2_{ab}\, \Phi^\dagger_a \Phi_b$$

– **Minimal Symmetry of Alignment in the Higgs basis:**

$$\mathbf{Z_2^{EW}} : \quad \Phi'_1 \to \Phi'_1 \, , \qquad \Phi'_{a'} \to -\,\Phi'_{a'} \qquad (a' = 2,\, 3, \dots, N_H)$$

where m^2_{ab} becomes diagonal.

$\Longrightarrow$ **Minimal Alignment Symmetry:** $\mathbf{Z_2^{EW}} \times \mathbf{Z_2^I}$

[AP '16]