Corrections to electroweak vacuum decay in metric-affine gravity

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I.D.G., H. Veermäe, Phys.Lett.B. (2023), 2305.07693

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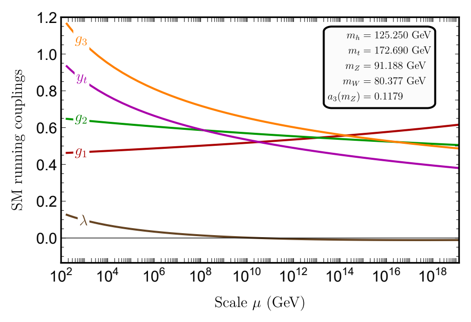
Contents

The running of the coupling constants

The **couplings**, which set the **strength** for the interactions, change their value if one probes smaller distances with **higher energies**. This is due to contributions of virtual particles that cause a **"running"** of the coupling with the energy scale.

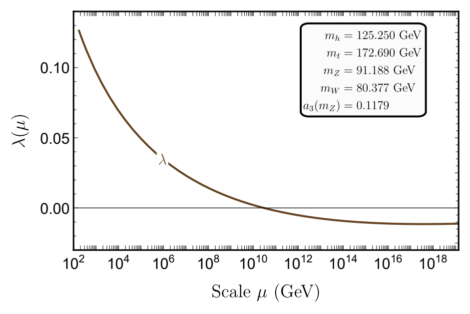


- U(1) coupling g_1
- SU(2) coupling g_2
- SU(3) coupling g_3
- Yukawa coupling y_t
- Higgs quartic coupling λ

Higgs metastability

In one-loop:

$$(4\pi)^2 \frac{d\lambda(\mu)}{d\ln\mu} = -6y_t^4 + \frac{27}{200}g_1^4 + \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2 g_2^2 + 24\lambda^2 + \lambda\left(12y_t^2 - 9g_2^2 + \frac{9g_1^2}{5}\right)$$

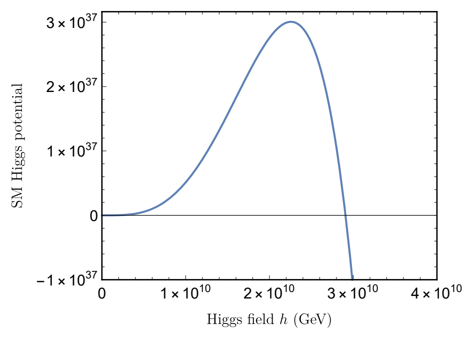


- As $m_t \uparrow$ the yukawa coupling y_t becomes larger.
- So, heavy top-quark masses (m_t) makes the quartic coupling $\lambda(\mu)$ negative at large energies.

Higgs metastability

Higgs potential:

$$V(h) \simeq \frac{\lambda(h)}{4} h^4$$ the Higgs mass term is negligible at large h .



Once the Higgs quartic coupling λ becomes negative the potential begins to decrease, opening the possibility of vacuum tunelling that could destroy the Universe.

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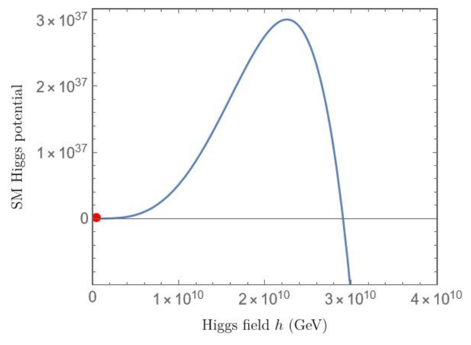

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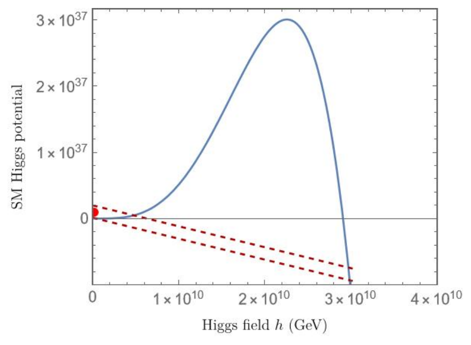

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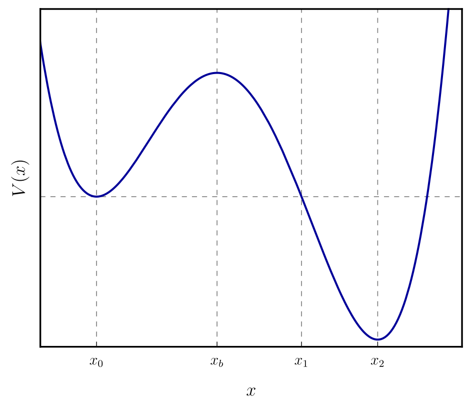
Decay of a metastable state in quantum mechanics



- $S = \int dt \left(\frac{1}{2} m \left(\frac{dx}{dt} \right)^2 - V(x) \right)$
- $x_0 \to x_2$ classically forbidden
- $x_0 \to x_2$ possible in QM
- Decay width $\Gamma \sim e^{-S_b}$
- $\boxed{S_b = 2 \int_{x_0}^{x_1} \sqrt{2mV(x)} dx}$

WKB approximation

The Euclidean trick

In Minkowski spacetime $(-,+,+,+)$, the action of a single particle is

$$S_M = \int dt \left(\frac{1}{2} m \left(\frac{dx}{dt} \right)^2 - V(x) \right) .$$

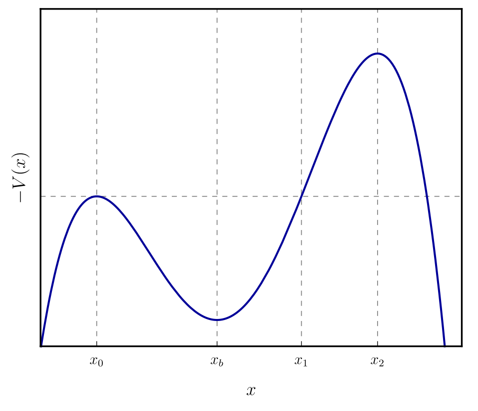
Suppose that we change variables to an imaginary time $\tau = it$, we have

$$S_M = i \int d\tau \left(\frac{1}{2} m \left(\frac{dx}{d\tau} \right)^2 + V(x) \right) = iS_E \, ,$$

where S_E is the Euclidean action.[1]

[1] In order to speed up notation we discard the subscript E from now on.

The Euclidean trick



Euclidean action

- $S = \int d\tau \left(\frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 + V(x) \right)$
- EOM: $m \frac{d^2 x}{d\tau^2} - \frac{\partial V}{\partial x} = 0$
- Integral: $\frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 - V(x) = 0$
- Bounce solution (x_B): $x_0 \to x_1 \to x_0$ at $\tau = -\infty \to 0 \to +\infty$
- $S(x_B) = \int_{-\infty}^{+\infty} d\tau \left(\frac{m}{2} \left(\frac{dx_B}{d\tau} \right)^2 + V(x_B) \right)$

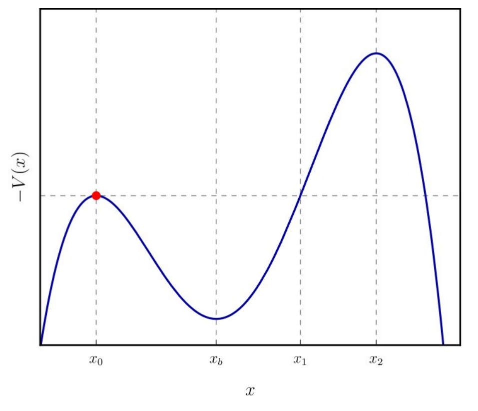
$$= 2 \int_{-\infty}^{0} 2V(x_B) d\tau$$

$$= 2 \int_{x_0}^{x_1} \sqrt{2mV(x_B)} dx_B$$

Same as before

$$d\tau = \sqrt{\frac{m}{2V(x_b)}} dx_B$$

The Euclidean trick



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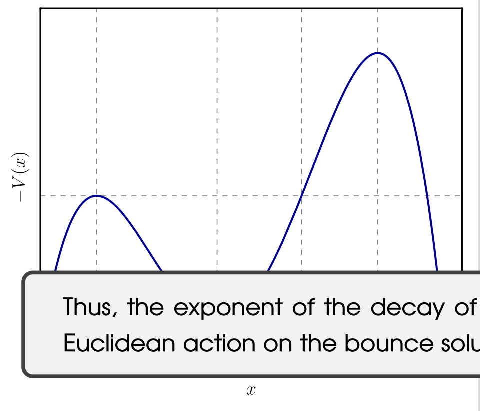

Euclidean action

- $S = \int d\tau \left(\frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 + V(x) \right)$
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- Integral: $\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2 - V(x) = 0$
- Bounce solution (x_B): $x_0 \to x_1 \to x_0$

Thus, the exponent of the decay of a metastable state, is equal to the Euclidean action on the bounce solution.

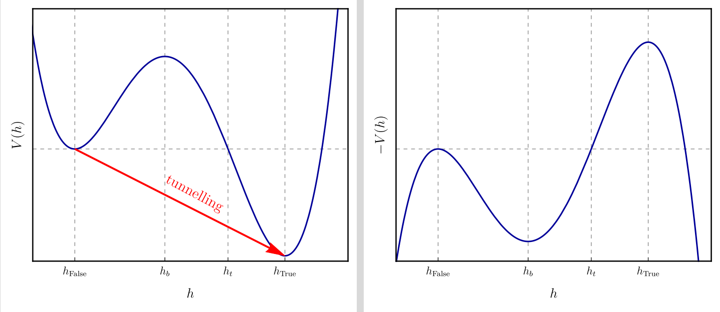
$$= 2\int_{-\infty} 2V(x_B)d\tau$$

$$= 2\int_{x_0}^{x_1} \sqrt{2mV(x_B)}dx_B$$

Same as before

$$d\tau = \sqrt{\frac{m}{2V(x_b)}}dx_B$$

Tunneling in QFT



The action in the Euclidean $4-D$ spacetime is

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu h)^2 + V(h) \right].$$

- The false vacuum is h_F with $V(h_F) = 0$
- The true vacuum is h_T with $V(h_T) < 0$

Tunneling in QFT

- 1st step: Find the bounce solution.
- 2nd step: Calculate the exponent of the decay rate $S(h_B)$
- Lowest action $\Rightarrow$ O(4) symmetry, i.e. $\mathrm{ds}^2 = \mathrm{dr}^2 + \mathrm{r}^2 \mathrm{d}\Omega_3^2$

(*Coleman, Glaser (1978)*)

- $h = h(r)$.

$$S = 2\pi^2 \int_0^\infty r^3 dr \left[\frac{1}{2} \left(\frac{dh}{dr} \right)^2 + V(h) \right].$$

From $\delta S = 0 \quad \forall \quad \delta h \Rightarrow$

$$\frac{d^2 h}{dr^2} + \frac{3}{r} \frac{dh}{dr} = \frac{dV(h)}{dh},$$

with

- $h(r)|_{r\to\infty} = h_F$
- $h'(r)|_{r=0} = 0$

If we include gravity, the Euclidean action becomes (*Coleman, De Luccia (1980)*)

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi G} + \frac{1}{2} \nabla_\mu h \nabla^\mu h + V(h) \right].$$

Metric-affine gravity

Riemann tensor $\mathcal{R}^{\alpha}{}_{\mu\beta\nu} = \partial_\beta \Gamma^{\alpha}{}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\sigma\beta}\Gamma^{\sigma}{}_{\nu\mu} - \Gamma^{\alpha}{}_{\sigma\nu}\Gamma^{\sigma}{}_{\beta\mu}$

- Γ is an independent connection
- $\Gamma^{\lambda}{}_{\mu\nu} \equiv \left\{ {}^{\lambda}_{\mu\nu} \right\} + C^{\lambda}{}_{\mu\nu}$

  $\left\{ {}^{\lambda}_{\mu\nu} \right\} \rightarrow$ Levi-Civita connection $C^{\lambda}{}_{\mu\nu} \rightarrow$ distortion tensor

$$\boxed{\mathcal{R}[g,\mathcal{C}] = R[g] + D_\mu C^{\mu}{}_{\nu}{}^{\nu} - D_\nu C^{\mu}{}_{\mu}{}^{\nu} + C^{\mu}{}_{\mu\lambda} C^{\lambda}{}_{\nu}{}^{\nu} - C^{\mu}{}_{\nu\lambda} C^{\lambda}{}_{\mu}{}^{\nu}}$$

- Torsion $T_{\rho\mu\nu} \equiv 2\Gamma_{\rho[\mu\nu]}$ ($T_{\rho\mu\nu} = 0 \rightarrow$ Palatini gravity)
- Non-metricity $Q_{\rho\mu\nu} \equiv \nabla_\rho g_{\mu\nu}$ ($Q_{\rho\mu\nu} = 0 \rightarrow$ Einstein-Cartan gravity)
- 21 additional scalars of mass dimension 2
- $\int \mathrm{d}^4x\sqrt{-g}\mathcal{R} \xrightarrow[\text{equivalent}]{C=0} \int \mathrm{d}^4x\sqrt{-g}R$

Gravitational corrections to Higgs vacuum decay

Absence of gravity

The Euclidean e.o.m. is

$$h''(r) + \frac{3}{r}h'(r) = \frac{\mathrm{dV(h)}}{\mathrm{dh}}\,,$$

combined with the boundary conditions

$$\lim_{r\to\infty} h(r) = h_F\,, \qquad h'(r)|_{r=0} = 0\,.$$

A solution for $\lambda < 0 =$ constant is

$$h_0(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2\mu}{1+\mu^2 r^2}$$ ← Fubini or Lee-Weinberg bounce

(*Fubini, 1976., Lee & Weinberg, 1986*)

$$S_0 = \frac{8\pi^2}{3|\lambda(\mu)|}$$ ← Euclidean action for the bounce

Gravitational corrections to Higgs vacuum decay

In metric formulation $\rightarrow$ (*Isidori, Rychkov, Strumia, Tetradis, 0712.0242., Branchina, Messina, Zappala, 1601.06963, Rajantie,Stopyra, 1606.00849, Salvio, Strumia, Tetradis, Urbano, 1608.02555*)

The most general action linear in the Riemann tensor and containing terms of at most dimension 4 has the form

$$S_M = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{\mathcal{R}}{2}f(h) + \frac{\widetilde{\mathcal{R}}}{2}\tilde{f}(h) - \frac{1}{2}(\partial h)^2 - V(h)\right],$$

where $V(h)$ is the Higgs potential and,

$$f(h) = 1/\kappa + \xi h^2, \qquad \tilde{f}(h) = \beta/\kappa + \tilde{\xi}h^2\,, \qquad \text{and} \qquad \kappa = 1/M_{\mathrm{Pl}}^2\,.$$

$$\mathcal{R} \equiv \mathcal{R}^{\mu\nu}{}_{\mu\nu}\,, \qquad \widetilde{\mathcal{R}} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}$$

$\uparrow$ Ricci scalar $\qquad$ $\uparrow$ Holst invariant

Gravitational corrections to Higgs vacuum decay

We express the Ricci scalar and the Holst invariant in terms of the metric Ricci scalar $R[g]$ and the distortion tensor,

$$\mathcal{R} = R + D_\mu C^{\mu}{}_{\nu}{}^{\nu} - D_\nu C^{\mu}{}_{\mu}{}^{\nu} + C^{\mu}{}_{\mu\lambda} C^{\lambda}{}_{\nu}{}^{\nu} - C^{\mu}{}_{\nu\lambda} C^{\lambda}{}_{\mu}{}^{\nu} ,$$

$$\widetilde{\mathcal{R}} = \epsilon^{\mu\nu\rho\sigma} \left(D_\mu C_{\rho\nu\sigma} + C_{\rho\mu\lambda} C^{\lambda}{}_{\nu\sigma} \right) ,$$

The Euclidean action is

$$\boxed{\begin{aligned} S_E = \int \mathrm{d}^4x \sqrt{g} \Big[& - \frac{R}{2} f + \frac{1}{2} (\partial h)^2 + V - \frac{i\tilde{f}}{2} \epsilon^{\mu\nu\rho\sigma} \left(D_\mu C_{\rho\nu\sigma} + C_{\rho\mu\lambda} C^{\lambda}{}_{\nu\sigma} \right) \\ & - \frac{f}{2} \left(D_\mu C^{\mu}{}_{\nu}{}^{\nu} - D_\nu C^{\mu}{}_{\mu}{}^{\nu} + C^{\mu}{}_{\mu\lambda} C^{\lambda}{}_{\nu}{}^{\nu} - C^{\mu}{}_{\nu\lambda} C^{\lambda}{}_{\mu}{}^{\nu} \right) \Big] \end{aligned}}$$

A variation gives

$$\frac{\delta S}{\delta C} = 0 \Rightarrow C_{\mu\nu\rho} = \frac{1}{2} \left(g_{\nu\mu} \partial_\rho X - g_{\nu\rho} \partial_\mu X - i \epsilon_{\mu\nu\rho\sigma} \partial^\sigma Y \right)$$

where

$$\boxed{f = e^X \cos(Y), \qquad \tilde{f} = e^X \sin(Y)}$$

Gravitational corrections to Higgs vacuum decay

Substituting the solution for the distortion in the action we obtain

$$S = \int \mathrm{d}^4x\sqrt{g}\left[-\frac{R}{2}f + \frac{1}{2}K(\partial h)^2 + V\right],$$

where the contribution of the independent connection is now fully captured by the kinetic function (*Rigouzzo, Zell, 2204.03003* full quadratic metric-affine action)

$$K = 1 + \frac{3}{2}\frac{f\tilde{f}'^2 - 2f'\tilde{f}\tilde{f}' - ff'^2}{f^2+\tilde{f}^2} \xrightarrow{\text{limits}} \begin{cases} 1 & \text{(metric)} \\ 1-(3/2)f'^2/f & \text{(Palatini)} \end{cases}$$

$$\tilde{f} = cf^2 - 1/(4c)\,, \quad \text{(metric)}$$

$$\tilde{f} = cf\,, \quad \text{(Palatini)}$$

(*Langvik, Ojanpera, Raatikainen, Rasanen 2007.12595* for Holst + Nieh-Yan term)

Gravitational corrections to Higgs vacuum decay

• More specifically, without loss of generality we can consider $\tilde{f}(h) = \beta/\kappa + \tilde{f}_1(h)$, where $\tilde{f}_1, f$ are arbitrary functions of h. If $|\beta|/\kappa \gg \tilde{f}_1, f$, then

$$K(h) = 1 - \frac{3\kappa}{\beta} f' \tilde{f}' + \mathcal{O}\left(\frac{\kappa}{\beta}\right)^2$$

and thus the metric-affine theory approaches the purely metric theory when

$$\boxed{|\beta| \to \infty}$$

• The Palatini formulation corresponds to $\beta = \tilde{\xi}/\xi$, of which $\beta = \tilde{\xi} = 0$ is only a special case.

Consequently, as $\boxed{\beta}$ ranges from $\boxed{-\infty \to \tilde{\xi}/\xi \to \infty}$, the metric formulation is continuously deformed to the Palatini one and back.

Gravitational corrections to Higgs vacuum decay

The Euclidean equations of motion are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= T_{\mu\nu} \,, \\ K(h)\Box h + \frac{f'(h)}{2} R + \frac{K'(h)}{2} (\partial h)^2 &= \frac{\mathrm{dV(h)}}{\mathrm{dh}} \,, \end{aligned}$$

where the effective energy momentum tensor is defined as

$$T_{\mu\nu} = \frac{1}{f(h)} \Bigg[K(h) D_\mu D_\nu h - g_{\mu\nu} \Bigg(\frac{1}{2} K(h) (\partial h)^2 + V(h) + f'(h) \Box h + f''(h)(\partial h)^2 \Bigg) + f'(h) D_\mu D_\nu h + f''(h) D_\mu h D_\nu h \Bigg] .$$

- D_μ is the covariant derivative constructed from the Levi-Civita connection.
- Euclidean $O(4)$-symmetric geometry $\mathrm{ds}^2 = \mathrm{dr}^2 + \rho^2(\mathrm{r}) \mathrm{d}\Omega_3^2$.

Gravitational corrections to Higgs vacuum decay

In this background the action simplifies to

$$S = 2\pi^2 \int \mathrm{d}r \rho^3 \left[-\frac{\mathrm{R}}{2}\mathrm{f(h)} + \mathrm{K(h)}\frac{\mathrm{h}'^2}{2} + \mathrm{V(h)} \right] .$$

The Ricci scalar is given by

$$R = \frac{-6\left(\rho^2 \rho'' + \rho \rho'^2 - \rho\right)}{\rho^3} .$$

Equations of motion

$$\rho'^2 = 1 + \frac{\rho^2}{3f}\left(\frac{1}{2}K h'^2 - V - 3\frac{\rho'}{\rho} f' h'\right) , \quad \leftarrow \text{Einstein equation}$$

$$h'' = -3\frac{\rho'}{\rho}h' + \frac{1}{K}\left(-\frac{1}{2}K' h'^2 + V' - \frac{R}{2}f'\right) . \quad \leftarrow \text{K-G equation}$$

Gravitational corrections to Higgs vacuum decay

Following (*Isidori et al, 0712.0242., Salvio et al, 1608.02555*) we perform a leading order expansion in the gravitational coupling κ:

$$
\begin{aligned}
h(r) &= h_0(r) + \kappa h_1(r) + \mathcal{O}(\kappa^2)\,, \\
\rho(r) &= r + \kappa \rho_1(r) + \mathcal{O}(\kappa^2)\,,
\end{aligned}
$$

The action at $\mathcal{O}(\kappa)$ is $S = S_0 + \kappa S_1 + \mathcal{O}(\kappa^2)\,,$

with

$$
S_0 = 2\pi^2 \int \mathrm{d}r\, r^3 \left(\frac{h_0'^2}{2} + V(h_0) \right) = \frac{8\pi^2}{3|\lambda|}
$$

and

$$
\begin{aligned}
S_1 =& 6\pi^2 \int \mathrm{d}r \Bigg[\left(r\rho_1'^2 + 2\rho_1\rho_1' + 2r\rho_1\rho_1'' \right) + \frac{K_1}{6} r^3 h_0^2 h_0'^2 \\
&+ r\xi h_0^2 \left(r\rho_1'' + 2\rho_1' \right) + r^2 \rho_1 \left(\frac{h_0'^2}{2} + V(h_0) \right) \Bigg].
\end{aligned}
$$

Gravitational corrections to Higgs vacuum decay

The leading order gravitational corrections to the kinetic function can be expressed as

$$K = 1 + \kappa K_1 h^2 + \mathcal{O}(\kappa^2)\,,$$

where K_1 is a dimensionless constant given by

$$\boxed{K_1 \equiv -6\frac{\xi^2 + 2\beta\xi\tilde{\xi} - \tilde{\xi}^2}{1+\beta^2}} \xrightarrow{\text{limits}} \begin{cases} 0 & \text{(metric)} \\ -6\xi^2 & \text{(Palatini)} \end{cases}$$

At order $\mathcal{O}(\kappa)$, the equation of motion is

$$\rho_1' = \frac{1}{6}r^2\left(\frac{1}{2}h_0'^2 - V(h_0) - 3f'(h_0)h_0'\right)\,,$$

independently of the shape of $\tilde{f}$. For the Fubini bounce, it is solved by

$$\rho_1 = \frac{1+6\xi}{3|\lambda|/\mu^2}\left(r\frac{\mu^2r^2-1}{(\mu^2r^2+1)^2} + \mu^{-1}\arctan(\mu r)\right)\,.$$

Gravitational corrections to Higgs vacuum decay

The final gravitationally corrected action is (*IDG, Veermäe, 2305.07693*)

Metric-affine with Holst

$$
\begin{aligned}
S &= S_0 + \kappa S_1 \\
&= \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\mu^2}{45\lambda^2(\mu)}\left((1+6\xi)^2 + 6K_1\right) \\
&= \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\mu^2}{45\lambda^2(\mu)}\left((1+6\xi)^2 - 36\frac{\xi^2 + 2\beta\xi\tilde{\xi} - \tilde{\xi}^2}{1+\beta^2}\right)
\end{aligned}
$$

Gravitational corrections to Higgs vacuum decay

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Metric

$$S = \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\mu^2}{45\lambda^2(\mu)}(1+6\xi)^2$$

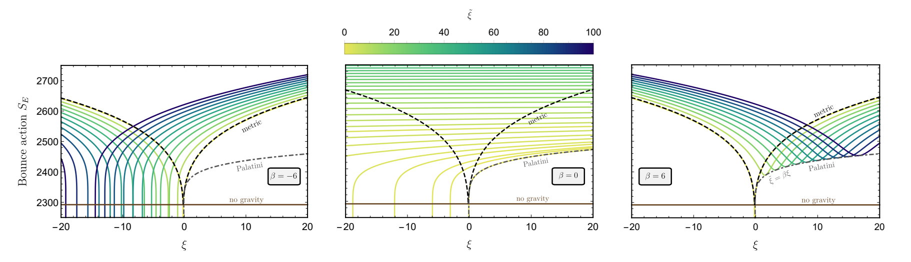
(*Salvio, Strumia, Tetradis, Urbano, 1608.02555*)

Palatini

$$S = \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\mu^2}{45\lambda^2(\mu)}(1+12\xi)$$

(*IDG, Karam, Pappas, 2212.03052*)

Gravitational corrections to Higgs vacuum decay



General case

- $S_1 > 0$ if $\beta\tilde{\xi} \geq 1/12$
- The limit $\beta \to \infty$ is already realized for $|\beta| = 6$
- For $\tilde{\xi} \gg 1$, the bounce action is typically enhanced when β and ξ have the opposite signs.
- In comparison to the metric case, for $|\xi| < |\tilde{\xi}|$, the stability is improved.

$\beta = 0$

- $S_1 \sim \left(1 + 12\xi + 36\tilde{\xi}^2\right)$, so $\tilde{\xi}$ improves the stability in comparison with Palatini.
- The positivity of S_1 implies

$$\xi \geq -1/12 - 3\tilde{\xi}^2 \,.$$

$\tilde{\xi} = 0$

- The region allowed by $S_1 > 0$ is

$$\left|\xi + \frac{1}{6} + \frac{1}{6\beta^2}\right| \geq \frac{\sqrt{1+\beta^2}}{6\beta^2} \,.$$

- For $\beta \ll 1$, $\xi \leq -1/(3\beta^2)$ or $\xi \geq -1/12$
- For $\beta \gg 1$, $|\xi + 1/6| \geq 1/(6\beta)$.

Summary

- Given the absence of new physics in the LHC data, the stability of the SM electroweak vacuum has become a more pressing issue.
- According to the current experimental values of the Higgs and top quark masses, the Standard Model (SM) Higgs potential, at high scales, becomes many orders of magnitude deeper than its value in the standard electroweak vacuum.
- Nevertheless, the expected life-time of the visible universe is much smaller than the life-time of our electroweak vacuum, therefore the non-occurrence of true vacuum bubbles is consistent.
- The gravity suppresses the electroweak vacuum decay less in the Palatini case than in the usual metric approach but still it improves on the vacuum metastability problem.
- The linear ξ-dependence of the gravitational correction indicates that the Palatini nonminimal coupling has to be larger than the characteristic value $-1/12$ in order to avoid a negative action.
- **A non-minimally coupled Holst term provides a class of models that continuously connects metric and Palatini gravity. We find that the limiting case of Palatini gravity displays the mildest improvement to vacuum stability.**

Thank you!

BACKUP SLIDES

The solution is

$$C_{\mu\nu\rho} = \frac{1}{2}\left(g_{\nu\mu}\partial_\rho X - g_{\nu\rho}\partial_\mu X - i\epsilon_{\mu\nu\rho\sigma}\partial^\sigma Y \right) , \quad (1)$$

where

$$f = e^X \cos(Y), \qquad \tilde{f} = e^X \sin(Y) . \quad (2)$$

This solution is metric compatible, i.e., $Q_{\rho\mu\nu} = -2C_{(\nu|\rho|\mu)} = 0$, and has torsion $T_{\rho\nu\mu} = 2C_{\rho[\nu\mu]}$. So, the theory is dynamically equivalent to the Einstein-Cartan theory. On the other hand, the particular solution (1) is not the general one because of the projective symmetry of the action, $C_{\rho\nu\mu} \to C_{\rho\nu\mu} + g_{\rho\mu}A_\nu$, which can be used to induce the non-metricity $Q_{\rho\mu\nu} = -2g_{\mu\nu}A_\rho$. In particular, the Palatini limit with $\tilde{f} = 0$ is obtained by choosing $A_\nu = \partial_\nu X/2$.

BACKUP SLIDES

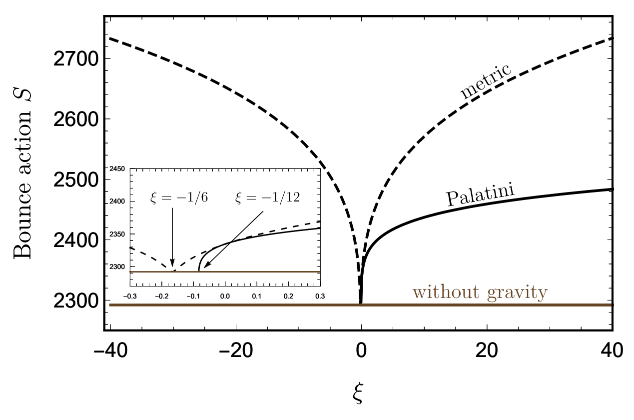
- Rescale $\rho_1 \to s\rho_1$
- Require $\mathrm{dS_k}/\mathrm{ds}|_{\mathrm{s=1}} = 0$

The gravitational action at $\mathcal{O}(\kappa)$ then acquires the form

$$S_1 = 6\pi^2 \int \mathrm{d}r \left[r\rho_1'^2 + \frac{K_1}{6} r^3 h_0^2 h_0'^2 \right] .$$

BACKUP SLIDES

$$S = \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\left(1 + 12\xi + 36\xi^2(1-\delta_F)\right)}{45R^2\lambda^2(\mu)}\,.$$



BACKUP SLIDES

The new boundary conditions are those of the "absence of gravity case", along with the gravity-imposed conditions

$$\boxed{\rho(0) = 0 \quad \text{and} \quad \rho(\infty) = r}$$

Adding these conditions, ensures that $r = 0$ is at the centre of the bounce and that a Minkowski-like background is laid out in the false vacuum.

BACKUP SLIDES

- At one-loop order

$$\boxed{\Gamma = \frac{B^2}{4\pi^2\hbar^2}\left|\frac{\det'(S''(h_B))}{\det(S''(h_F))}\right|^{-\frac{1}{2}}(1+\mathcal{O}(\hbar))\,e^{-B/\hbar}}$$

where $B = S(h_B) - S(h_f)$.

- The life time of the false vacuum (without gravity) is

$$\boxed{\tau = \frac{1}{\Gamma} \sim \frac{10^{550}}{H_0}}$$

1-loop corrections

tree-level $\Rightarrow$ $$h_0(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{R^2+r^2}$$

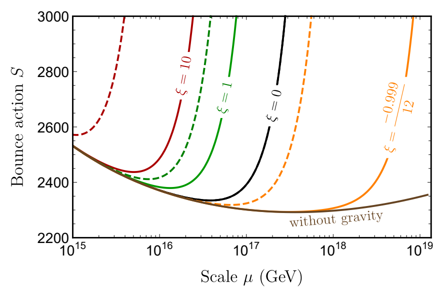
The one-loop corrections remove the tree-level ambiguity on the RGE scale μ by fixing it to be the scale $\mu = 1/R$ of the bounce

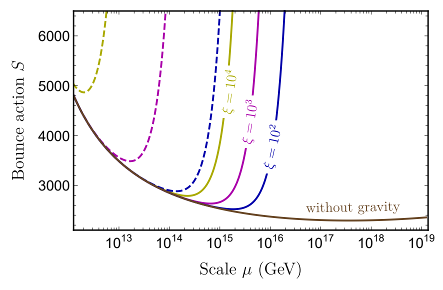
$$\boxed{h_0(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2\mu}{1+\mu^2 r^2}}$$

(*hep-ph/0104016 Isidori, Ridolfi & Strumia*)

BACKUP SLIDES

$$S = \frac{8\pi^2}{3|\lambda(\mu)|} + \frac{32\pi^2\kappa\left(1+12\xi+36\xi^2(1-\delta_F)\right)}{45R^2\lambda^2(\mu)}\,.$$

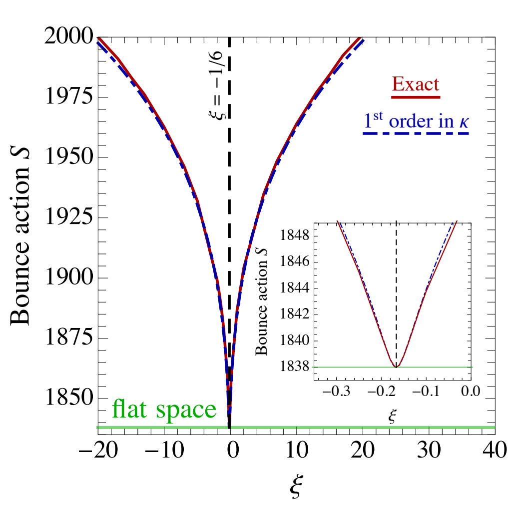




- For same ξ the S is extremized at higher scales in the **Palatini** case.
- $\mu^P_{\text{extr}}/\mu^m_{\text{extr}} \uparrow$ as $\xi \uparrow$
- E.g. $\mu^P_{\text{extr}}/\mu^m_{\text{extr}}|_{\xi=1} \sim \mathcal{O}(1)$, $\mu^P_{\text{extr}}/\mu^m_{\text{extr}}|_{\xi=10^4} \sim \mathcal{O}(10^2)$

BACKUP SLIDES

Numerical result from (*Salvio, Strumia, Tetradis, Urbano, 1608.02555*)



BACKUP SLIDES

When the underlying spacetime manifold has a boundary, the Gibbons - Hawking -York boundary term needs to be added to the Einstein - Hilbert action:

$$S_{GH} = -\frac{1}{16\pi G}\left(2\int_{\partial\mathcal{M}} dA\sqrt{-g_{ind}}\,K\right) \tag{3}$$

with

- $K = \nabla_\alpha \eta^\alpha$ is the extrinsic curvature
- g_{ind} is the induced metric on the boundary

The total action is

$$S = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\,R - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} dA\sqrt{-g_{ind}}\,K \tag{4}$$

BACKUP SLIDES

The Schwarzschild metric is of the form

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{5}$$

Consider the variable $\bar{\xi} = r - 2GM$ with $\bar{\xi} \ll 2GM$ and $\rho = \sqrt{8GM\bar{\xi}}$, then

$$ds^2 = \underbrace{-\frac{\rho^2}{16G^2M^2}dt^2 + d\rho^2}_{\text{space}(\rho,t)} + \underbrace{(2GM)^2 d\Omega^2}_{2-\text{sphere}}. \tag{6}$$

The metric of space (ρ, t) is written as

$$ds_2^2 = \underbrace{-\rho^2 k^2 dt^2 + d\rho^2}_{\text{Rindler coordinates}} \quad \text{with} \quad k = \frac{1}{4GM}. \tag{7}$$

BACKUP SLIDES

In the Euclidean space $\tau = it$, so

$$ds_2^2 = \rho^2 k^2 d\tau^2 + d\rho^2. \tag{8}$$

- The imaginary time τ is equivalent to the inverse of temperature $\beta = \dfrac{1}{T}$.
- The variable $\theta = k\tau$ must have a periodicity 2π so that no conical singularity appears in the geometry.

So

$$\left.\begin{aligned} \tau &= \frac{2\pi}{k} \\ \tau = \beta &= \frac{1}{T} \end{aligned}\right\} \Rightarrow T = \frac{k}{2\pi} \Rightarrow$$

$$T = \frac{1}{8\pi G M}, \tag{9}$$

which is the Hawking temperature.

BACKUP SLIDES

For the entropy we know that

$$dU = TdS_{ent}, \quad \text{with} \quad U = M. \tag{10}$$

So

$$S_{ent} = \int 8\pi GM\, dM = \frac{4\pi R_h^2}{4G} = \frac{A_h}{4G}. \tag{11}$$

where A_h is the horizon's surface.

BACKUP SLIDES

The partition function is

$$Z[\beta] = \int \mathcal{D}g \, \mathcal{D}\phi \, e^{-S_E[g,\phi]} \Rightarrow Z[\beta] \simeq exp\left(-S_E[g_{cl}]\right). \quad (12)$$

- Entropy $S_{ent} = (1 - \beta \, \partial_\beta) \ln \left(Z[\beta]\right).$
- Energy $U = -\partial \ln \left(Z[\beta]\right) / \partial_\beta.$

The Euclidean action is

$$S_E = -\frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{g} \, R - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} dA \sqrt{g_{ind}} \, K. \quad (13)$$

For a surface $r = R_0$ in Schwarzschild spacetime the extrinsic curvature is

$$K = \frac{2 - 3GM/r}{r\sqrt{1 - 2GM/r}}. \quad (14)$$

The Euclidean action becomes $S_E = -\frac{1}{8\pi G} \frac{1}{T} (8\pi R_0 - 6\pi R_h).$

BACKUP SLIDES

To adjust the divergence that occurs as $R_0 \to \infty$, we add an extra term in the action, i.e.

$$S_E = -\int_{\mathcal{M}} d^4x\,\sqrt{g}\,\frac{R}{16\pi G} - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} dA\,\sqrt{g_{ind}}\,K + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} dA\,\sqrt{g_{ind}}\,K_0$$

where K_0 is the extrinsic curvature of the same boundary of the Manifold $\partial\mathcal{M}$, embedded in flat spacetime.

The metric in the boundary is

$$ds^2_{bdry} = \left(1 - \frac{2GM}{R_0}\right) d\tau^2 + R_0^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{15}$$

Therefore if we embed it in flat space, the flat space metric will be

$$ds^2 = \left(1 - \frac{2GM}{R_0}\right) d\tau^2 + dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2). \tag{16}$$

BACKUP SLIDES

The extrinsic curvature is

$$K_0 = \frac{2}{r}\,. \tag{17}$$

The additional term is

$$\frac{1}{8\pi G}\int_{\partial\mathcal{M}} dA\,\sqrt{g_{ind}}\,K_0 = \frac{1}{8\pi G}\frac{1}{T}\left(8\pi R_0 - 4\pi R_h + \mathcal{O}\left(1/R_0^2\right)\right)\,. \tag{18}$$

The Euclidean action is

$$S_E = \frac{R_h}{4GT} = \frac{M}{2T}. \tag{19}$$

- Partition function $Z[\beta] = e^{-\frac{M}{2T}} = e^{-\frac{\beta^2}{16\pi G}}$.

BACKUP SLIDES

- Entropy

$$S_{ent} = (1 - \beta\,\partial_\beta)\, ln\,(Z[\beta]) = \frac{\beta^2}{16\pi G} = 4\pi G M^2. \quad \checkmark$$

- Energy

$$U = -\frac{\partial ln\,(Z[\beta])}{\partial_\beta} = \frac{\beta}{8\pi G} = M. \quad \checkmark$$

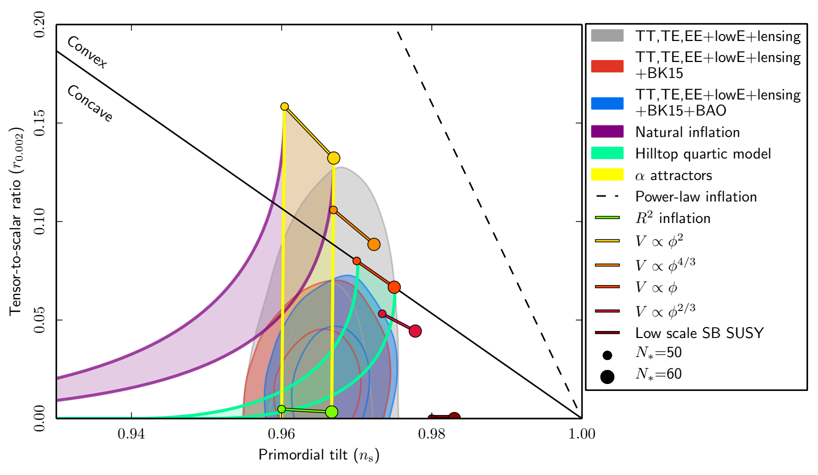
- Free energy

$$F = U - T\,S_{ent} = \frac{M}{2}.$$

Combining the above we see that

$$S_E = \frac{F}{T}\,. \tag{20}$$

Inflationary Observables (Planck 2018 1807.06211)



$n_s = 0.9649 \pm 0.0042,\ r < 0.036$ and $A_s = (2.10 \pm 0.03) \times 10^{-9}$

Metric vs. Palatini

- In **metric formulation**, the metric is the only dynamical degree of freedom and the connection is the Levi-Civita: $R_{\mu\nu} = R_{\mu\nu}\left(g, \partial g, \partial^2 g\right)$.
- In **Palatini formulation**, both the metric and the connection are independent dynamical degrees of freedom $\mathcal{R}_{\mu\nu} = \mathcal{R}_{\mu\nu}\left(\Gamma, \partial\Gamma\right)$.

$$S_J = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}A(\phi)g^{\mu\nu}\mathcal{R}_{\mu\nu}(\Gamma) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \tag{21}$$

Variation with respect to Γ gives

$$\Gamma^\lambda_{\alpha\beta} = \left\{{\lambda \atop \alpha\beta}\right\} + (1-\kappa)\left[\delta^\lambda_\alpha\partial_\beta\omega(\phi) + \delta^\lambda_\beta\partial_\alpha\omega(\phi) - g_{\alpha\beta}\partial^\lambda\omega(\phi)\right], \quad \omega(\phi) = \ln\sqrt{A(\phi)}$$

where $\kappa = 1$ in metric and $\kappa = 0$ in Palatini. Performing a Weyl rescaling

$$\tilde{g}_{\mu\nu} \equiv A(\phi)g_{\mu\nu} \quad \rightarrow \quad \sqrt{-g} = A^{-2}\sqrt{-\tilde{g}}, \quad R = A\left(1 - \kappa\times 6\,A^{1/2}\tilde{\nabla}^\mu\tilde{\nabla}_\mu A^{-1/2}\right)\tilde{R},$$

the action becomes

$$S_E^{Pal\ or\ metric} = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\left(\frac{1}{2}\tilde{R} - \frac{1}{2}\left(\frac{1}{A} + \kappa\times\frac{3}{2}\frac{A_{,\phi}}{A^2}\right)\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{V(\phi)}{A^2}\right). \tag{22}$$

Higgs Inflation: (0710.3755 Bezrukov & Shaposhnikov) (0803.2664 Bauer & Demir)

We consider the Higgs-like inflationary potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2\,, \qquad A(\phi) = 1 + \xi\phi^2\,. \tag{23}$$

Canonical field redefinition gives

$$\phi(\chi) \simeq \frac{1}{\sqrt{\xi}} \exp\left(\sqrt{\frac{1}{6}}\chi\right) \quad (\text{Metric})\,, \qquad \phi(\chi) = \frac{1}{\sqrt{\xi}} \sinh(\sqrt{\xi}\chi) \quad (\text{Palatini}) \tag{24}$$

The Einstein-frame potential in terms of χ can be expressed as

$$U(\chi) \simeq \frac{\lambda}{4\xi^2}\left(1 + \exp\left(-\sqrt{\frac{2}{3}}\,\chi\right)\right)^{-2}, \quad (\text{Metric})\,, \tag{25}$$

$$U(\chi) = \frac{\lambda}{4\xi^2} \tanh^4\left(\sqrt{\xi}\chi\right), \qquad (\text{Palatini}) \tag{26}$$

$$n_s \simeq 1 - \frac{2}{N_*} + \frac{3}{2N_*^2}, \qquad r \simeq \frac{12}{N_*^2}, \qquad A_s \simeq \frac{\lambda N_*^2}{72\pi^2\xi^2} \quad (\text{Metric})\,,$$

$$n_s \simeq 1 - \frac{2}{N_*} - \frac{3}{8\xi N_*^2}, \qquad r \simeq \frac{2}{\xi N_*^2}, \qquad A_s \simeq \frac{\lambda N_*^2}{12\pi^2\xi} \quad (\text{Palatini})\,.$$

UV cutoff

Scale of inflation: $H \sim \frac{\sqrt{\rho}}{3M_{\rm Pl}} \sim \frac{M_{\rm Pl}}{\xi}$

- $\Lambda_{UV}^{Metric} \sim \frac{M_{\rm Pl}}{\xi} \sim H$
- $\Lambda_{UV}^{Palatini} \sim \frac{M_{\rm Pl}}{\sqrt{\xi}} > H$

(0903.0355 Barbon & Espinosa)
(1012.2900 Bauer & Demir)
(2001.09088 Shaposhnikov, Shkerin & Zell)
(2007.04111 McDonald)
(2106.09390 Antoniadis, Guillen & Tamvakis)

Palatini inflation with an $\mathcal{R}^2$ term (1810.05536 Enckell et al)

1810.10418 Antoniadis et al, 1901.01794 Tenkanen, 1911.11513 I.D.G & A.B. Lahanas
2012.06831 Dimopoulos et al

Action

$$S_J = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{A(\phi)}{2}\mathcal{R} + \frac{\alpha}{2}\mathcal{R}^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]. \tag{27}$$

Introducing an auxiliary field we eliminate the R^2 term and after a Weyl rescaling we obtain

$$S_E = \int d^4x\sqrt{-g}\left(\frac{R}{2} + K(\phi)\,X + L(\phi)\,X^2 - U(\phi)\right), \tag{28}$$

with $X = -1/2\partial_\mu\phi\partial^\mu\phi$ and $K(\phi) = \dfrac{A(\phi)}{A^2(\phi)+8\alpha V(\phi)}$, $L(\phi) = \dfrac{2\alpha}{A^2(\phi)+8\alpha V(\phi)}$,

$$U(\phi) = \frac{V(\phi)}{A^2(\phi)+8\alpha V(\phi)}.$$

- Equation of motion

$$(K + 3L\dot\phi^2)\ddot\phi + 3H(K + L\dot\phi^2)\dot\phi + U'(\phi) + \frac{1}{4}(2K' + 3L'\dot\phi^2)\dot\phi^2 = 0\,, \tag{29}$$

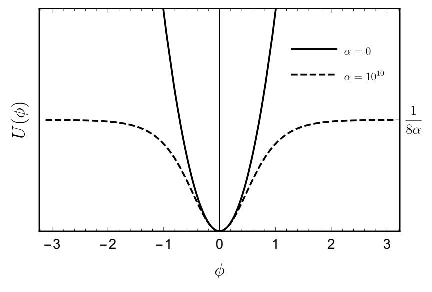
- Speed of sound

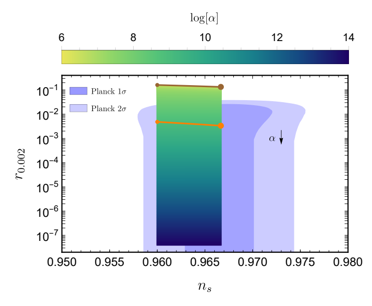
$$c_s^2 = \frac{\partial p/\partial X}{\partial\rho/\partial X} = \frac{1 + L\,\dot\phi^2/K}{1 + 3L\,\dot\phi^2/K}\,, \tag{30}$$

Palatini inflation with an $\mathcal{R}^2$ term

As a result, the tensor-to-scalar ratio becomes

$$r = 16\epsilon_U = \frac{\bar{r}}{1+8\alpha U} = \frac{\bar{r}}{1+12\pi^2 A_s \bar{r}\alpha} . \tag{31}$$





Inflationary Observables

The scalar $(\mathcal{P}_\zeta)$ and tensor $(\mathcal{P}_T)$ power spectrum is

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} , \;\; A_s = \frac{1}{24\pi^2}\frac{V(\phi_*)}{\epsilon_V(\phi_*)} , \;\; \mathcal{P}_T = 8\left(\frac{H}{2\pi}\right)^2 \simeq \frac{2V}{3\pi^2} \tag{32}$$

Spectral tilt (n_s) and tensor-to-scalar ratio (r)

$$n_s - 1 \equiv \frac{\mathrm{d}\ln\mathcal{P}_\zeta(k)}{\mathrm{d}\ln k} \simeq -6\epsilon_V + 2\eta_V , \;\; r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \simeq 16\epsilon_V \tag{33}$$

We have used the potential slow-roll parameters:

$$\epsilon_V = \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 , \qquad \eta_V = \frac{V''(\phi)}{V(\phi)} . \tag{34}$$

Number of e-folds

$$N(\phi) = \int_t^{t_{\rm end}} H\mathrm{d}t = \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_H}} \approx \int_{\phi_{\rm end}}^{\phi} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}} \simeq 50 - 60 \tag{35}$$

Levi-Civita identity

$$\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\beta\lambda\eta} = (-1)^t \left(\delta^\mu_\lambda \delta^\nu_\eta - \delta^\mu_\eta \delta^\nu_\lambda\right) \tag{36}$$

- $t = 0$ in Euclidean
- $t = 1$ in Minkowski