Anisotropic scale-separated AdS flux vacua

George Tringas

Laboratoire d'Annecy-le-Vieux de Physique Théorique (LAPTh)

December 22, 2023

Xmas Theoretical Physics Workshop @ Athens

Based on

1. **F. Farakos, M. Morittu and G. T, “On/off scale separation,” [2304.14372].**

2. **G. T “Anisotropic scale-separated AdS_4 flux vacua,” [2309.16542].**

Introduction

Motivation

- String theory comes with:
  - Extra dimensions (lives in 10d)
  - Supersymmetry
  - Moduli fields
- Effective perspective: We don't observe any of the above!
- The possibility of constructing effective theories and making the extra dimensions unobservable turns out to be a rare feature.
- We call this feature ***scale separation***

Our scope

- We investigate further *classical* scale-separated effective theories.
- Exploit flux and scaling freedom to create anisotropic internal spaces.
- Test them against the distance conjecture.

EFT from string theory

An elementary starting point is the construction of vacua.

d-dimensional scalar field action:

$$S = \int d^{\mathsf{d}}x \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + V(\phi)\right), \qquad \Lambda = V(\phi_0)$$

- In our universe: $d = 4$, $\Lambda > 0$.
- (quasi) de-Sitter from string theory?

  U. H. Danielsson, T. Van Riet "What if string theory has no de Sitter vacua?," [1804.01120]. ,
  D. Andriot "Open problems on classical de Sitter solutions," [1902.10093].

Moduli stabilization

Compactification : Method for constructing an EFT at low energies

$$M_{10} = M_d \times X^{10-d}$$

where X^{10-d} the "internal" compact space.

Fields appear as **moduli**: do not have scalar potential

- We give them mass, stabilized at VEV
- Use **classical** ingredient, fluxes:

$$F_p^{\mathrm{bg},i} = e^i \, \mathrm{d}y^1 \wedge \cdots \wedge \mathrm{d}y^p \, .$$

- String theory conditions: fluxes should be quantized, e.g.

$$e^i \sim \int_{\Sigma_4} F_4^{bg,i} = (2\pi\sqrt{\alpha'})^3 \times N \, , \qquad \text{with} \qquad N \in \mathbb{Z}$$

Scale separation

Set the internal space coordinates to be periodical

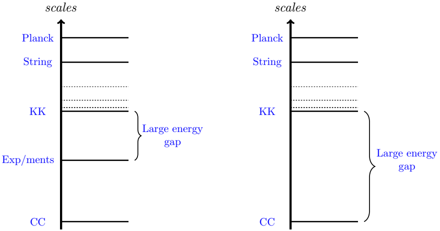
$$\Phi(x^\mu, y^m) = \Phi(x^\mu, y^m + 1)$$

E.g. for compactification on a circle with radius R

$$\Phi^D(x^\mu, y) = \sum_n \phi_n^{D-1}(x^\mu) e^{\frac{iyn}{R}} \quad \rightarrow \quad \left(\partial_\mu \partial^\mu - \left(\frac{n}{R}\right)^2\right)\phi_n^{D-1} = 0$$

- The *lower-dimensional* EFT is valid for energies much lower than the compactification scale.
- Qualitative condition to estimate whether there is a large energy gap between extra dimensional states and the vacuum energy of the EFT

$$\frac{\langle V \rangle}{m_{\mathsf{KK}}^2} \equiv \frac{L_{\mathsf{KK}}^2}{L_\Lambda^2} \ll 0\,,$$



Classical regime

String theory solution can be described by 10d supergravities when we are at large volume and weak coupling.

- String coupling to be small

$$g_s = e^{\langle \phi \rangle} < 1\,.$$

- Large radii : Much larger compared to string length

$$r_i \gg l_s\,.$$

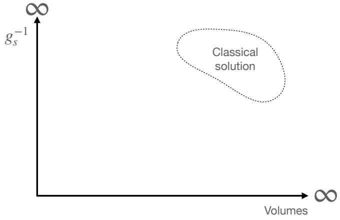


Image inspired by: Van Riet, Zaccorato [2305.01722]

Classical scale-separated AdS vacua

The 4d solution: $AdS_4 \times X^6$ DeWolfe, Giryavets, Kachru, Taylor [0505160].
The 3d solution: $AdS_3 \times X^7$ F.Farakos, Van Riet, G.T [2005.05246].

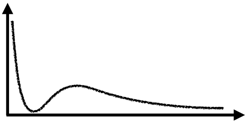
Common ingredients and features:

- Originate from **massive type IIA** supergravity with ***smeared*** O6 planes.
- Both give $\mathcal{N} = 1$ 4d and 3d supergravity respectively.
- Both contain an unbounded flux $N \to \infty$
- For $N \to \infty$ they exhibit scale separation and classical solution

Other attempts: Tsimpis [1206.5900], Petrini, Solard, Van Riet [1308.1265], D.Lüst,Tsimpis [2004.07582], Marchesano, Quirant [1908.11386], Cribiori, Junghans, Van Hemelryck, Van Riet, Wrase [2107.00019], Van Hemelryck [2207.14311], Carrasco, Coudarchet, Marchesano, Prieto [2309.00043].

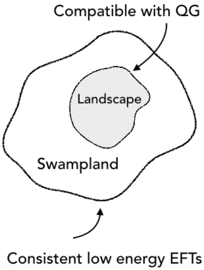
Why AdS?

- Is it possible to find solutions with such characteristics from string theory?
- SUSY AdS are well controlled : Starting point for dS constructions (e.g. KKLT).

Swampland conjectures

Effective field theory space:



- Conjectures disfavor the existence of such constructions, e.g., Gautason, Van Hemelryck, Van Riet [1810.08518], D. Lüst, Palti, Vafa [1906.05225]
- We will test the distance conjecture Ooguri, Vafa [0605264].

"*Infinite tower of states become exponentially light at large field distances* Δ"

$$m_f(\phi_f) \sim m_i(\phi_i) e^{-\gamma \Delta(\phi_i, \phi_f)}, \qquad \gamma \sim \mathcal{O}(1).$$

AdS flux vacua from type IIA

Massive type IIA supergravity

The bosonic type IIA action (p=0,2,4,6) in the string frame

$$S_{\text{IIA}} = \frac{1}{2\kappa_{10}^2} \int \mathsf{d}^{10}X\sqrt{-G}\left[e^{-2\phi}\left(R_{10} + 4\partial_M\phi\partial^M\phi - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}\sum_p |F_p|^2\right]$$

Op-planes : Span $p+1$ dimensions of the 10d space and wrap internal cycles

$$S_{\text{O}p} = -\mu_{\text{O}p}\int \mathsf{d}^{p+1}\xi\, e^{-\phi}\sqrt{-\mathsf{det}\,(g_{p+1})} \;+\; \mu_{\text{O}p}\int C_{p+1}\,,$$

Relevant Bianchi identity:

$$\mathsf{d}F_p = H_3 \wedge F_{p-2} + \mu_{\text{O}(8-p)}\, j_{p+1} \quad \xrightarrow{\int_{\Sigma_{p+1}}} \quad h_3\, f_{p-2} \sim -\mu_{\text{O}(8-p)}$$

Cancel the tadpole properly but also leave the flux of F_4 unconstrained.

From G2-manifold to Toroidal orbifold

G2-manifolds are characterized by the **three-form**

$$\Phi = e^{127} - e^{347} - e^{567} + e^{136} - e^{235} + e^{145} + e^{246} \,,$$

We choose the internal space X_7 to be a **seven-torus** with the orbifold Γ:

$$X_7 = \frac{T^7}{\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2}$$

with specific $\mathbb{Z}_2$ involutions, see Joyce *J.Diff Geom.* 43.

$$\begin{aligned}
\Theta_\alpha :\; y^m &\to (-y^1, -y^2, -y^3, -y^4, y^5, y^6, y^7)\,, \\
\Theta_\beta :\; y^m &\to (-y^1, -y^2, y^3, y^4, -y^5, -y^6, y^7)\,, \\
\Theta_\gamma :\; y^m &\to (-y^1, y^2, -y^3, y^4, -y^5, y^6, -y^7)\,,
\end{aligned}$$

The vielbein of the torus $e^m = r^m \mathrm{d}y^m$

$$\Phi = s^i(x)\Phi_i \,, \qquad \Phi_i = \left(\mathrm{d}y^{127}, -\mathrm{d}y^{347}, -\mathrm{d}y^{567}, \mathrm{d}y^{136}, -\mathrm{d}y^{235}, \mathrm{d}y^{145}, \mathrm{d}y^{246}\right) \,,$$

where the s^i are the **metric moduli** related to the seven-torus **radii** r^m

$$e^{127} = s^1\Phi_1 \;\to\; s^1 = r^1 r^2 r^7 \;, \text{ etc.}$$

Orientifolds

Target space involutions for the sources (fixed points)

$$\sigma_{\text{O2}}: \; y^m \to -y^m \,, \qquad \sigma_{\text{O6}_i} : \sigma_{\text{O2}}\Gamma \,.$$

In total we have 7 different directions for O6-planes

	y^1	y^2	y^3	y^4	y^5	y^6	y^7
O6_α	$\otimes$	$\otimes$	$\otimes$	$\otimes$	–	–	–
O6_β	$\otimes$	$\otimes$	–	–	$\otimes$	$\otimes$	–
O6_γ	$\otimes$	–	$\otimes$	–	$\otimes$	–	$\otimes$
$\text{O6}_{\alpha\beta}$	–	–	$\otimes$	$\otimes$	$\otimes$	$\otimes$	–
$\text{O6}_{\beta\gamma}$	–	$\otimes$	$\otimes$	–	–	$\otimes$	$\otimes$
$\text{O6}_{\gamma\alpha}$	–	$\otimes$	–	$\otimes$	$\otimes$	–	$\otimes$
$\text{O6}_{\alpha\beta\gamma}$	$\otimes$	–	–	$\otimes$	–	$\otimes$	$\otimes$

Table: Localized positions "-" and warped directions $\otimes$ in the internal space.

We get 3d N=1 minimal effective supergravity :

Type IIA supercharges : **32** $\xrightarrow{\Gamma \text{ orbifold}}$ **4** $\xrightarrow{\text{O2-plane}}$ **2** real

The 3d effective theory

The 3d bosonic effective action has the form

$$e^{-1}\mathcal{L} = \frac{1}{2}R_3 - \frac{1}{4}(\partial x)^2 - \frac{1}{4}(\partial y)^2 - \frac{1+\delta_{ij}}{4\tilde{s}^i\tilde{s}^j}\partial\tilde{s}^i\partial\tilde{s}^j - V(x,y,\tilde{s}^i)$$

The scalar potential in 3d supergravity is given by

$$V(x,y,\tilde{s}^i) = G^{IJ}\partial_I P\partial_J P - 4P^2$$

For Kähler see: Beasley, Witten [0203061]

We find superpotential P which gives the 3d effective potential:

$$P = \frac{e^y}{8}\left[e^{\frac{x}{\sqrt{7}}}\int \tilde{\star}\Phi\wedge H_3\,\mathrm{vol}(\tilde{X}_7)^{-\frac{4}{7}} + e^{-\frac{x}{\sqrt{7}}}\int \Phi\wedge F_4\,\mathrm{vol}(\tilde{X}_7)^{-\frac{3}{7}}\right] + \frac{F_0}{8}\,e^{\frac{1}{2}y-\frac{\sqrt{7}}{2}x}$$

The fluxes H_3 and F_4 are expanded on the Φ_i and Ψ_i basis

$$H_3 = \sum_{i=1}^{7} h^i\Phi_i\,, \qquad F_4 = \sum_{i=1}^{7} f^i\,\Psi_i\,, \qquad i = 1,\ldots,7\,.$$

Tadpole cancellation – Flux ansatz

The relevant tadpole:

$$0 = \int_7 (F_{4,q} + F_{4,f}) \wedge H_3 + \int_7 \Big(N_{\text{O2}}\mu_{\text{O2}} + \underbrace{N_{\text{D2}}}_{2^4} \mu_{\text{D2}} \Big) j_7$$

We will use the following flux ansatz:

Flux	anisotropic ansatz
h_3^i	$h\,(1,1,1,1,1,1,0)$
$f_{4,q}^i$	$q\,(0,0,0,0,0,0,-1)$
$f_{4,f}^i$	$f\,(-1,-1,-1,-1,-1,+5,0)$

For isotropic ansatz, see: F.Farakos, Van Riet, G.T [2005.05246].

- The tadpole is canceled while the fluxes "f" and "q" remain unconstrained:

$$\int_7 H_3 \wedge F_{4,q} = 0 \times (-q) = 0\,, \qquad \int_7 H_3 \wedge F_{4,f} = -5hf + 5hf = 0\,.$$

Scaling of the fluxes – Detailed balance

Equations of motion *naively* have the form:

$$(\text{flux}_1) \times (\text{radii}_1) \times (\text{radii}_2) \;-\; (\text{flux}_2) \times (\text{radii}_2) \times (\text{radii}_3) \;+\; \cdots = 0$$

see also Petrini, Solard, Van Riet [1308.1265]

Method: Assume the fluxes and the fields having the following scaling:

$$f \sim N\,, \quad q \sim N^Q\,, \quad e^y \sim N^Y\,, \quad e^x \sim N^X\,, \quad \tilde{s}^a \sim N^S\,.$$

Their scaling becomes:

$$Y = -\frac{9}{2} - 7S\,, \quad X = \frac{\sqrt{7}}{2}(1+2S)\,, \quad Q = 1 + 7S\,.$$

We have created anisotropic scaling to T^7 radii :

$$\{r_i^2\}_{i=1,3,5,7} \sim N^{\frac{7+11S}{8}} \times N^{+3S}\,,$$
$$\{r_i^2\}_{i=2,4,6} \sim N^{\frac{7+11S}{8}} \times N^{-2S}\,,$$

Classical regime and scale separation

Large radii: $$r_i^2 \gg 1 \ \rightarrow \ -\frac{1}{5} < S < \frac{1}{3}$$

Weak coupling: $$g_s = e^{\phi} \sim N^{-\frac{3+7S}{4}} < 0 \ \rightarrow \ S > -\frac{3}{7}\,.$$

Scale separation:
$$\{r_i\}_{i=1,3,5,7}: \qquad \frac{L^2_{\mathsf{KK},i}}{L^2_\Lambda} \sim N^{-1}$$
$$\{r_i\}_{i=2,4,6} \quad : \qquad \frac{L^2_{\mathsf{KK},i}}{L^2_\Lambda} \sim N^{-1-7S}\,,$$

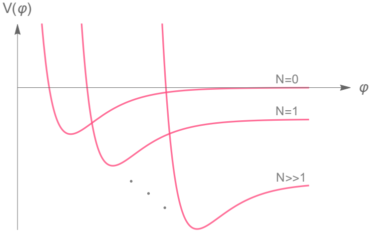
"on-off"

- Large volume, Weak coupling, **Scale separation** : $S = 0$
- Large volume, Weak coupling, **broken-Scale separation** : $-\frac{1}{5} < S \leq -\frac{1}{7}$

Landscape of 4d vacua

The effective 4d scalar potential scales in the following way

$$\langle V \rangle = -4P^2 \sim N^{-4-7S}$$



For different values of N we get a landscape of disconnected vacua.

Moduli stabilization

The supersymmetric equations reduce to the following system:

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial \tilde{s}^a} = 0 \quad \Rightarrow \quad \begin{cases} 0 &= c - a\sigma^5 + 5\sigma^5\tau^7 \,, \\ 0 &= c - a\sigma^4\tau - \sigma^6\tau \,, \\ 0 &= -3b + 2a\left(\frac{5}{\sigma} + \frac{1}{\tau}\right) \,, \\ 0 &= \frac{b}{2} + a\left(\frac{5}{\sigma} + \frac{1}{\tau}\right) + \left(-5\sigma + 5\tau - \frac{c}{\sigma^5\tau}\right) \,. \end{cases}$$

where $\quad c = \frac{q}{f}\,.$

the system is solved for $\quad a = \frac{h}{f} e^{\frac{2x}{\sqrt{7}}} \,, \quad b = \frac{m_0}{f} e^{-\frac{y}{2} - \frac{5x}{2\sqrt{7}}}\,.$

c	a	b	$\langle \tilde{s}^a \rangle = \sigma$	$\langle \tilde{s}^6 \rangle = \tau$
10^{-1}	0.298843	2.44476	0.884523	0.151095
10^{-3}	0.0801704	1.26626	0.458136	0.078259
10^{-6}	0.0111396	0.472009	0.170775	0.0291718
10^{-9}	0.00154785	0.175946	0.0636578	0.0108741

Interpolation between vacua

Methodology

- We have constructed a landscape of (non)-scale-separated vacua.
- To interpolate between vacua : Introduce a space-filling prode D4-brane G. Shiu, F. Tonioni, V. Van Hemelryck, T. Van Riet [2212.06169].
- The D4-brane is codimension-one in the six directions transverse to Ψ_7 filled with flux $q \sim N^Q$.

	t	x	z	y^2	y^4	y^6	y^1	y^3	y^5	y^7
D4	$\otimes$	$\otimes$	$\otimes$	$\circledast$	$\circledast$	$-$	$-$	$-$	$-$	$-$
Φ_7	$-$	$-$	$-$	$\otimes$	$\otimes$	$\otimes$	$-$	$-$	$-$	$-$
$F_{4,q} \sim q\Psi_7$	$-$	$-$	$-$	$-$	$-$	$-$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

Table: A D4-brane fills the AdS_3 and wraps 2-cycles inside the 3-cycle Φ_7.

$$\mathrm{d}F_{4,q} = Q_{\mathrm{D4}}\delta(\psi - \psi_0)\mathrm{d}\psi \wedge \Psi_7 \quad \rightarrow \quad F_{4,q} \sim \left(N^Q + \theta(\psi - \psi_0)\Big|_{\psi_1 < \psi_0}^{\psi_2 > \psi_0} \right) \Psi_7$$

The D4-brane induces a change to the flux $F_{4,q} \sim N^Q$ flux on either side of the brane.

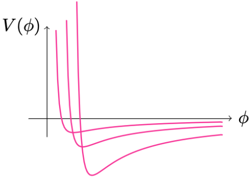
Open string modulus

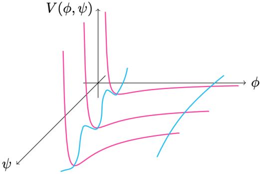
Let ψ depend on the external coordinates:

$$\mathrm{d}s^2_{\mathrm{D4}} = \left(e^{2\alpha v} g^{(3)}_{\mu\nu} + s_7^{\frac{2}{3}} R^2 \partial_\mu \psi \partial_\nu \psi\right) \mathrm{d}x^\mu \mathrm{d}x^\nu + s_7^{\frac{2}{3}} \left(\mathrm{d}R^2 + R^2 (\sin\psi)^2 \mathrm{d}\omega^2\right)$$

The field metric and potential for ψ are found to be:

$$g_{\psi\psi} = 2ck\tilde{s}_7^{\frac{4}{3}} e^{\frac{\phi}{4} - 3\alpha v} \sin\psi\,, \qquad V(\psi) \supset \frac{\mu_{\mathrm{D4}}}{8} e^{\frac{\phi}{4} - 21\beta v} s_7^{2/3} \sin\psi$$





Scalar potentials with discrete choice of fluxes are connected through ψ direction.

see also, Shiu, Tonioni, Van Hemelryck, Van Riet [2311.10828]

Measuring the distance

Geodesic distance: $$\Delta = \int_{\xi=0}^{\xi=1} \mathrm{d}\xi \sqrt{g_{AB} \frac{\mathrm{d}\phi^A}{\mathrm{d}s} \frac{\mathrm{d}\phi^B}{\mathrm{d}s}}$$

The scalar field space metric components: metric moduli, universal moduli and the new modulus.

- $\xi = 0$: non-scale-separated regime $S \leq -1/7$
- $\xi = 1$: scale-separated regime $S = 0$, i.e. $F_{4,f} = F_{4,q}$

After redefinitions we identify a $\mathbb{H}^2 \times \mathbb{R}^3$ space:

$$\Delta \sim \int_0^1 \mathrm{d}\xi \sqrt{\frac{1}{h_2^2}\left[\left(\frac{\mathrm{d}h_1}{d\xi}\right)^2 + \left(\frac{\mathrm{d}h_2}{d\xi}\right)^2\right] + \left(\frac{\mathrm{d}u_2}{d\xi}\right)^2 + \left(\frac{\mathrm{d}u_3}{d\xi}\right)^2 + \left(\frac{\mathrm{d}u_4}{d\xi}\right)^2}$$

We measure the distance parameter to be (for $N \sim 10^5$)

$$m_{\mathsf{KK}}(\xi = 1) \sim m_{\mathsf{KK}}(\xi = 0)\, e^{-\gamma\Delta} \quad \rightarrow \quad \gamma \sim 0.13$$

Conclusion

Conclusion slide

- We discussed AdS SUSY vacua with moduli stabilization and flux quantization.
- We exploited the flux and scaling freedom and canceled the tadpoles and created **anisotropy** to the scaling of the internal space.
- We constructed new vacua with scale separation and broken scale separation while remaining in the supergravity regime in the 3d case.
- The anisotropic 4d cannot support this feature: scale separation breaks always outside the classical regime.
- Introduced a D4 to interpolate between those vacua and verified the distance conjecture. The anisotropic cases exhibit better agreement with the distance conjecture compared to the isotropic ones.

Thank you!

Appendix – Backup slides

Smearing approximation

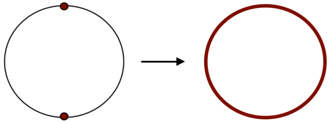
Smearing approximation

- Replace the singular density function with a regular one

$$\delta_{9-p} \;\rightarrow\; j_{9-p}$$

- Local sources are distributed globally all over the cycles

Image:Tomasiello's talk



- Slowly varying dilaton and warp factor, harmonic cycles, Ricci flat internal space

$$\phi(y) \approx \phi\,, \qquad w(y) \approx w\,, \qquad \mathrm{d}F_p = \mathrm{d}\star F_p = 0\,, \qquad R_{mn} = 0$$

- Fields ignore local backreaction : Not exact field profile.
- Simplifies the equations of motion and the potential

Anisotropic AdS_4 flux vacua

AdS_4 with scale-separation

Massive type IIA supergravity on the singular Calabi-Yau limit

$$X^6 = \frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

DeWolfe, Giryavets, Kachru, Taylor [0505160].

- Internal space metric ansatz

$$\mathrm{d}s_6^2 = \sum_{i=1}^{3} v_i(x) \left((\mathrm{d}y^{2i-1})^2 + (\mathrm{d}y^{2i})^2 \right)$$

- Fluxes

$$F_4 = e_i \tilde{w}^i \,, \quad H_3 = -p\beta_0 \,, \quad F_0 = m_0 \,.$$

Relevant Bianchi identity and equations of motions :

$$0 = H_3 \wedge F_0 + \mu_{\mathrm{O6}} \sum j_{\beta_i} \quad \xrightarrow{\int_{\Sigma_3}} \quad p\, m_0 = \pm\{1,2\}$$

$$0 = H_3 \wedge \star_6 F_4 \quad \rightarrow \quad \text{5-form in CY}\,.$$

The flux N of F_4 is unconstrained!

Scalings

Quantities expressed in terms of the F_4 scalings:

$$F_4^{(1)} \sim N^{f_1}\,, \qquad F_4^{(2)} \sim N^{f_2}\,, \qquad F_4^{(3)} \sim N^{f_3}\,.$$

- String coupling

$$e^{\phi} \sim N^{-\frac{1}{4}(f_1+f_2+f_3)}\,.$$

- Subvolumes

$$v_1 \sim N^{\frac{1}{2}(-f_1+f_2+f_3)}\,, \qquad v_2 \sim N^{\frac{1}{2}(f_1-f_2+f_3)}\,, \qquad v_3 \sim N^{\frac{1}{2}(f_1+f_2-f_3)}\,.$$

- Separation of scales?

$$\frac{L^2_{\mathsf{KK}_i}}{L^2_{\mathsf{AdS}}} \sim N^{-f_i}\,, \qquad i = 1,2,3.$$

New flux solutions

1. Scale separation, weak coupling & large volume

$$\upsilon_i \gg 1\,, \qquad \text{vol} \gg 1\,, \qquad e^{\phi} < 1\,, \qquad \frac{L^2_{\mathsf{KK}_i}}{L^2_{\mathsf{AdS}}} \ll 1\,,$$

as long as

$$f_1 > 0\,, \qquad 0 < f_2 \leq f_1\,, \qquad f_1 - f_2 < f_3 < f_1 + f_2\,.$$

e.g. for $f_1 = f_2 = 2$ and $f_3 = 3$

$$\upsilon_1 \sim N^{3/2}\,, \qquad \upsilon_2 \sim N^{3/2}\,, \qquad \upsilon_3 \sim N^{1/2}\,.$$

2. Scale separation, weak coupling & one small (shrinking) subvolume

$$\upsilon_i < 1\,, \qquad \upsilon_j \gg 1\,, \qquad \text{vol} \gg 1\,, \qquad e^{\phi} < 1\,, \qquad \frac{L^2_{\mathsf{KK}_i}}{L^2_{\mathsf{AdS}}} \ll 1\,.$$

3. Scale separation, weak coupling & small constant subvolumes
4. Broken scale separation, weak coupling & one small subvolume