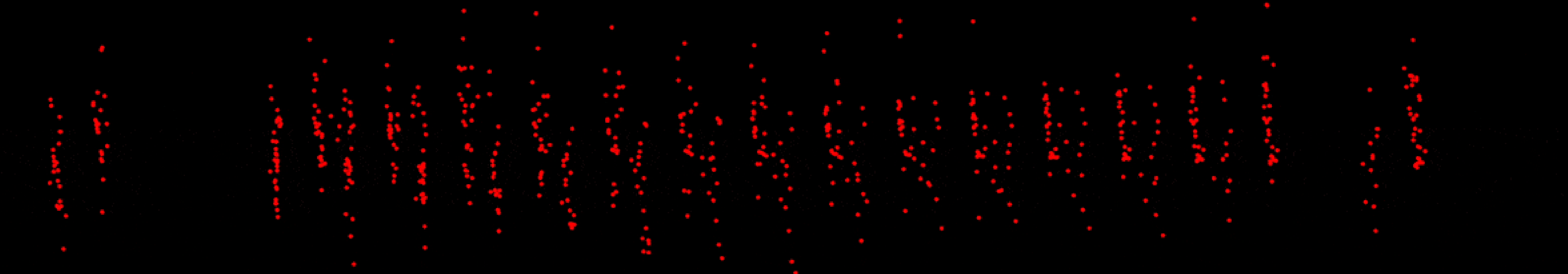
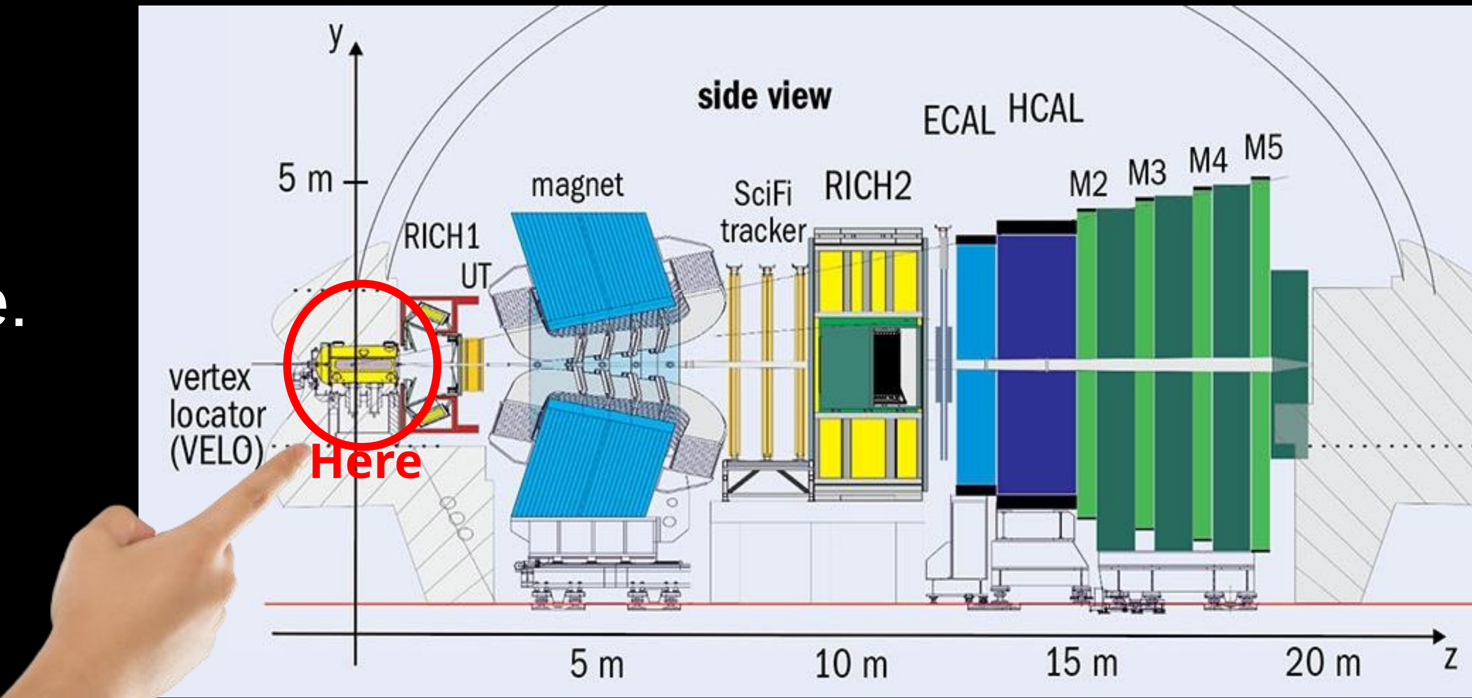
Track reconstruction with a quantum algorithm

Davide Nicotra

Connecting dots in the LHCb VELO



- Charged particles leave **hits** in the VELO.
- Reconstruction of the **tracks** crucial step of the LHCb pipeline.
- The **most computationally expensive** algorithm at trigger level.



Quantum Computing

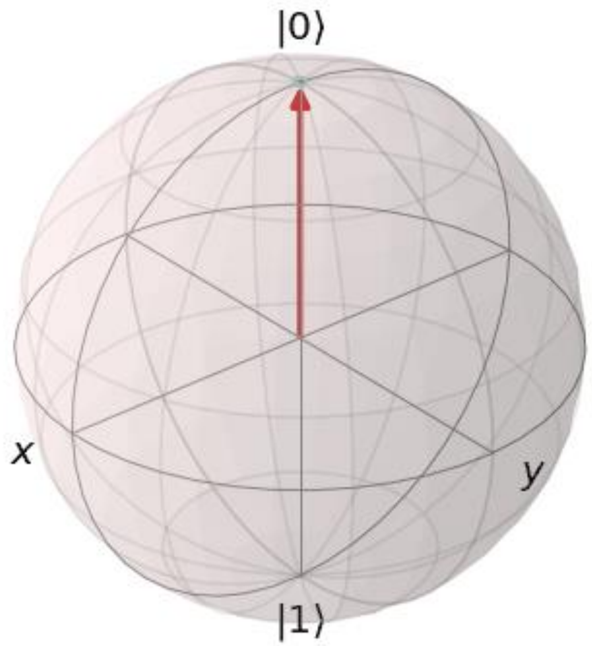
- Using a system of **qubits** to perform calculations

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

...

$$n - qubits \rightarrow 2^n$$



- A replacement for CPUs and GPUs
- The ultimate parallel computing tool (entanglement)
- Speed-up for any algorithm

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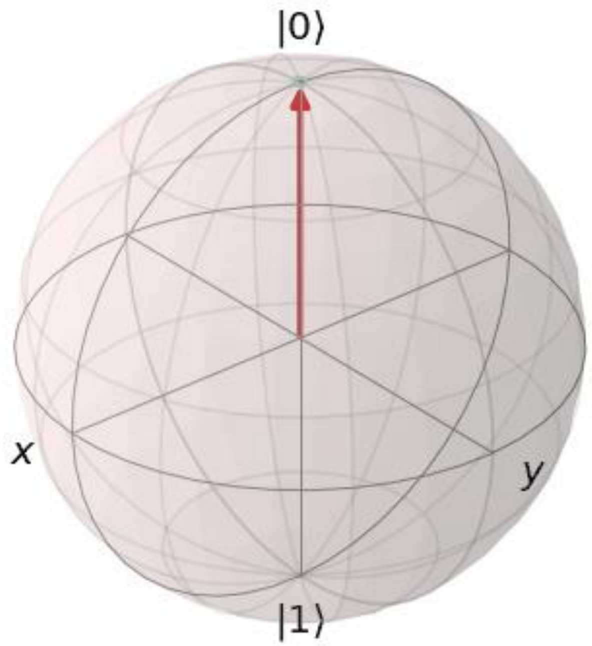

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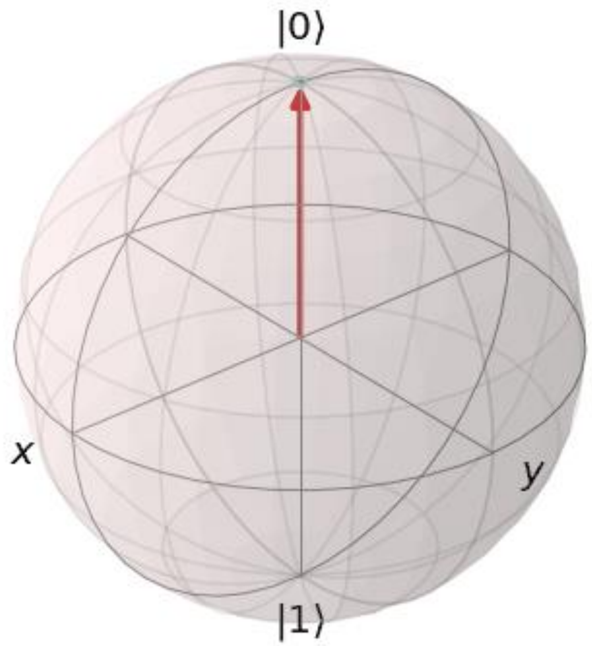
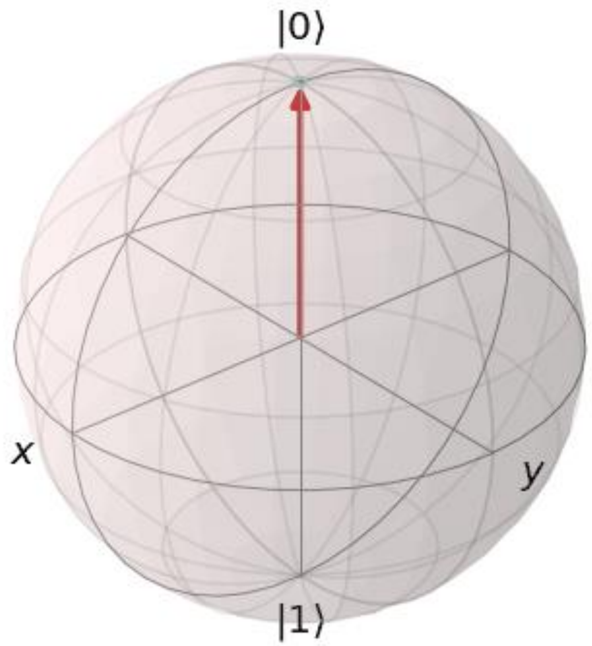


Quantum Computing



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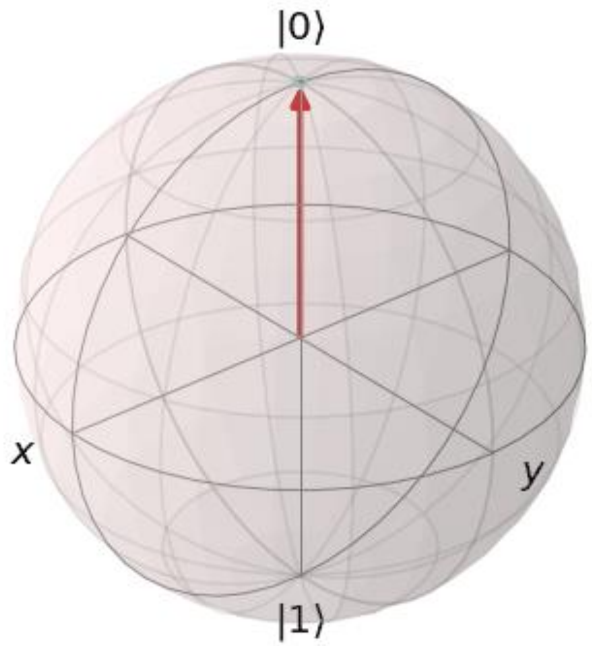

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- **A highly specific tool to solve certain kind of problems (hopefully) with a speed-up**

A quantum algorithm for tracking

From Hits to an Ising-like Hamiltonian

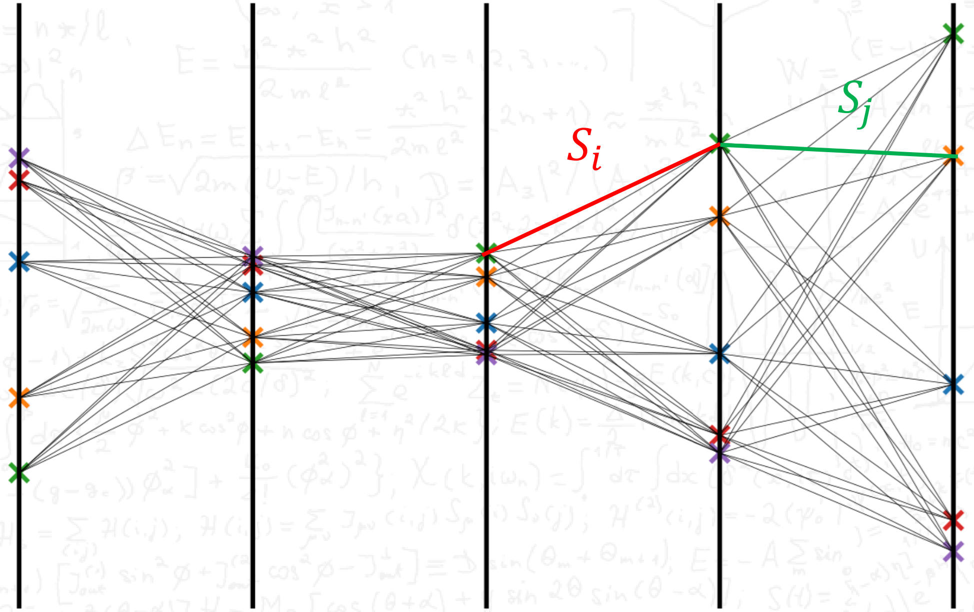
- *Inspired by* Denby-Peterson[1] model
- **Candidate doublets** constructed

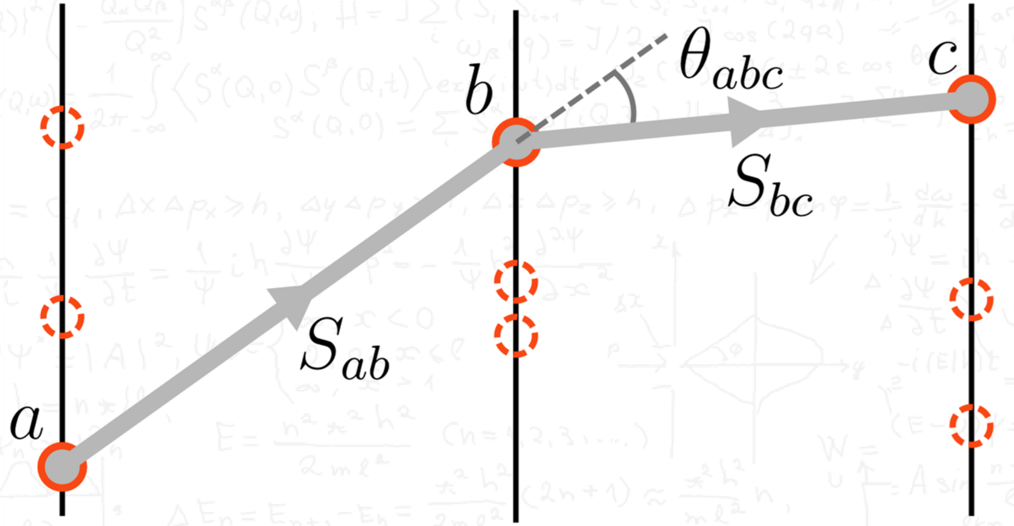
$$N = (\#Layers - 1) \times \#Particles^2$$

$$\mathcal{H} = -\frac{1}{2}\sum_{ij} A_{ij} S_i S_j + \sum_i b_i S_i \quad S_i \in \{0,1\}$$

MATH

$$\mathbf{A}\mathbf{S} = \mathbf{b}$$



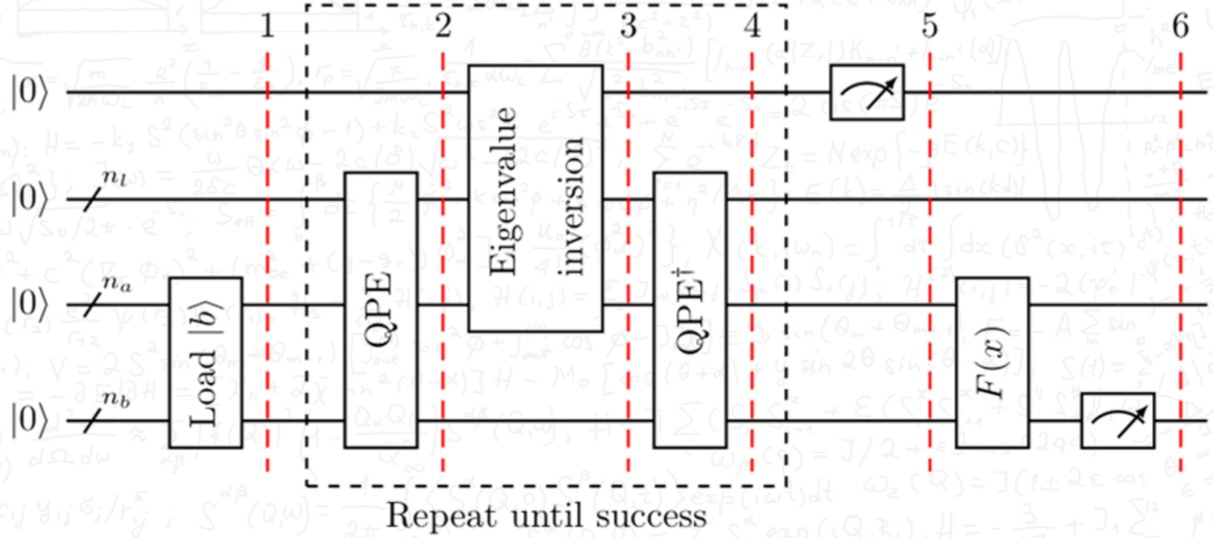


A quantum algorithm for tracking

Quantum Algorithm for Systems of Linear Equations

$$AS = b$$

- Also called Harrow-Hassadim-Lloyd (HHL) algorithm
  1. State preparation (loading $|b\rangle$)
  2. Quantum Phase Estimation
  3. Inversion
  4. Quantum Phase Estimation †
  5. Measurement



- Under certain conditions
  - **Classical** $O(N)$
  - **Quantum** $O(\log N)$
- **Exponential speed-up!**

Quantum Phase Estimation

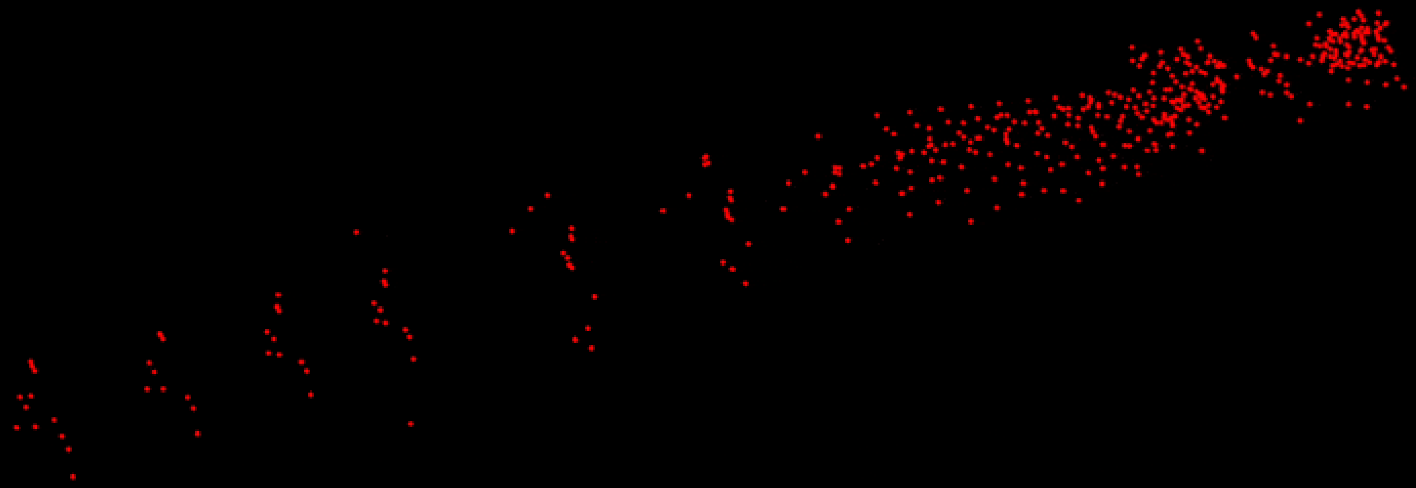
- The core of many exponentially faster algorithms, like **Shor's algorithm** that breaks RSA
- QPE is applied to e^{iA} (**Hamiltonian simulation**)

Pros and Cons of HHL

- ✓ **Exponential scaling**. **100** qubits can accommodate $\mathbf{10^{15}}$ **hits**
  - IBM has recently presented a **433-qubit** machine
- HHL requires 2 efficient subroutines
  - ✓ **State preparation**
    Solved by carefully designing the Hamiltonian
  - ✗ **Hamiltonian simulation**
    - Theoretical works suggest that **can be simulated efficiently**
    - An explicit implementation is **not available yet**
    - Very deep quantum circuits **hard to simulate**, **impossible to run** (noise)
- ✗ **The read-out issue**
  - Reading out the solution vector will **void the advantage**
  - **Quantum post-processing** of the final state is required

Does it work?

LHCb Simulated Event $B_s \rightarrow \phi\phi$
1 collision event
Half of the VELO



To validate our new Hamiltonian approach, HHL has been replaced with a **classical linear solver**

LHCb events on quantum simulator/hardware **is not possible yet**.

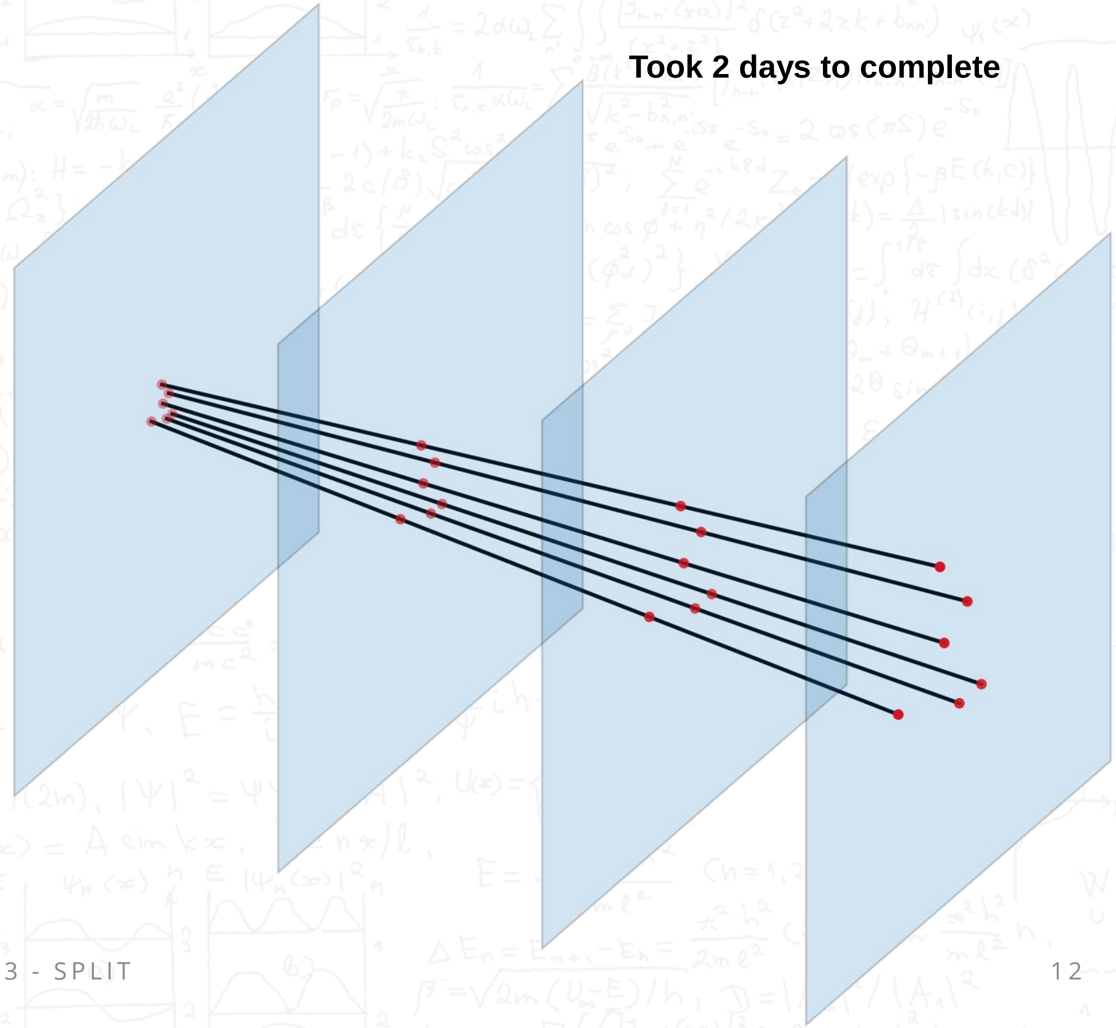
- Circuit **too deep**
- Hardware **too noisy**

Tracking performances

1. **Test the HHL algorithm**
   Smaller events with quantum simulator

2. **Validate the Hamiltonian**
   LHCb simulated events with a classical solver

6 particles – 4 layers detector
Qiskit **quantum simulation**



Thank you

Questions?

Conclusions

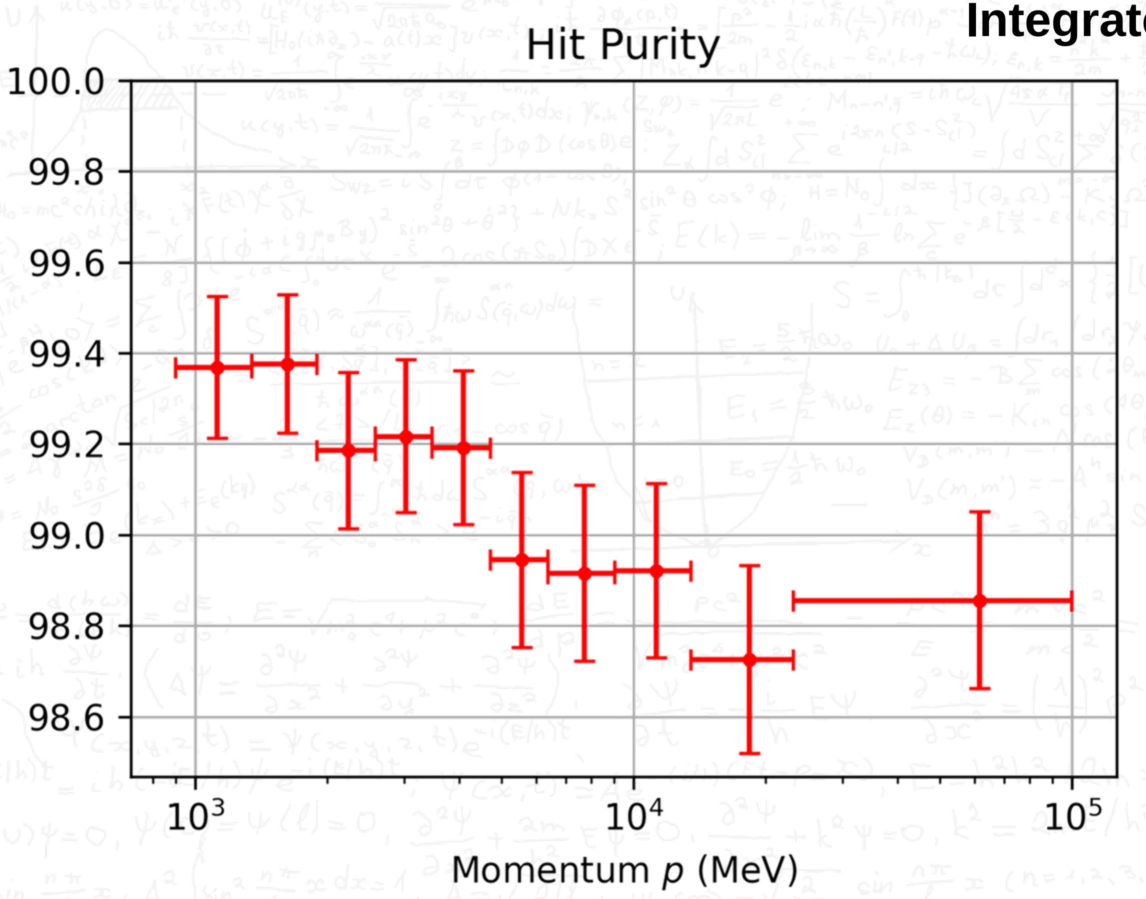
- ✓ **New** track reco algorithm based on Ising-like Hamiltonian
  - Tested and validated classically on LHCb simulated events
  - Good performance!
- ✓ **First** track reco algorithm based on the HHL quantum algorithm
  - Tested on smaller events
- 🚧 Still many **open challenges**
  - Efficient **Hamiltonian simulation**
  - Quantum **Post-processing**
  - **Hardware** implementation

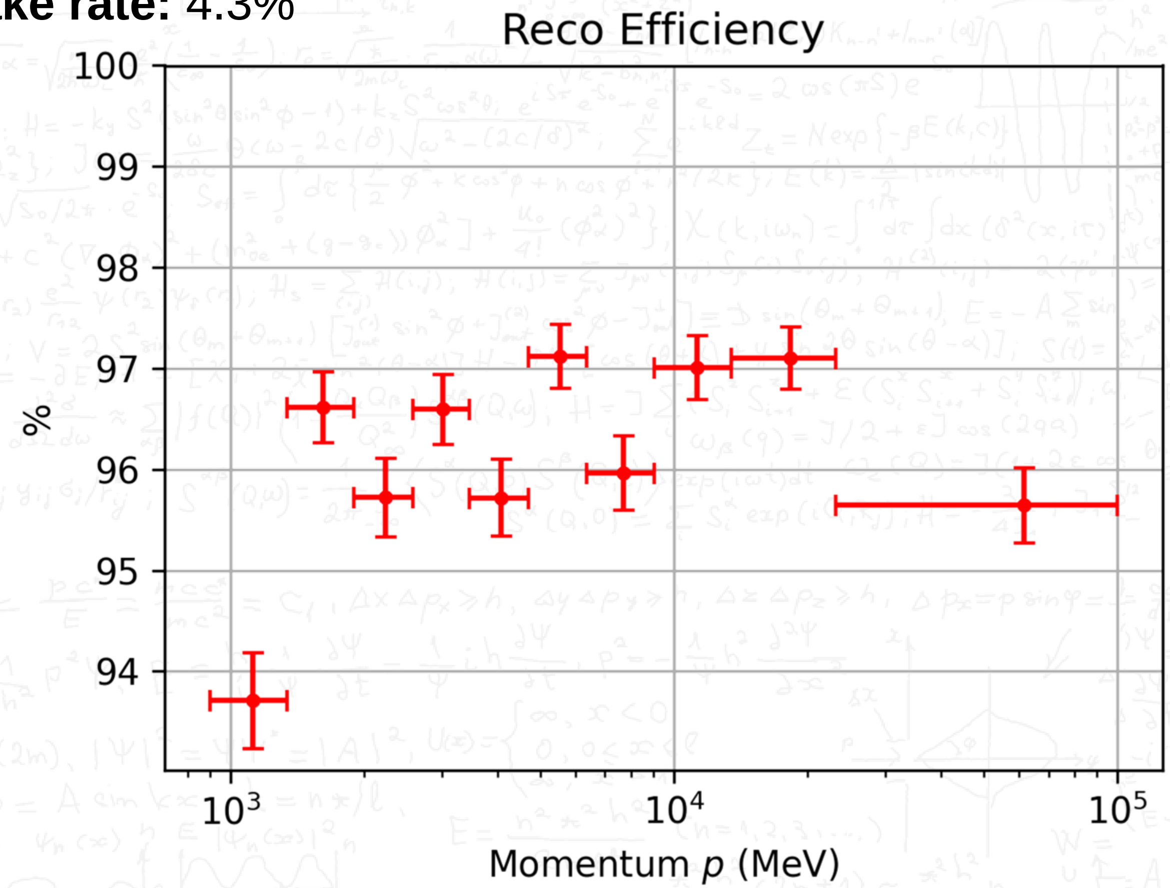
◉ A **paper** draft on these results will be ready soon… stay tuned!

Tracking performances

- Very good performance
- Close to state of the art

Integrated fake rate: 4.3%





A quantum algorithm for tracking

From Ising to an Linear Algebra

- Spins take discrete values: **on** or **off**

Spins take discrete values:
Relaxation of the problem
Hamiltonian is a differentia
Solve this huge $N \times N$ syste
Discretize back the solutio

$$S_i \in \{0,1\}$$

- **Relaxation** of the problem promoting $S_i \in \mathbb{R}$
- Hamiltonian is a differentiable

$$\nabla \mathcal{H} = 0 \Rightarrow A\mathbf{S} = \mathbf{b}$$

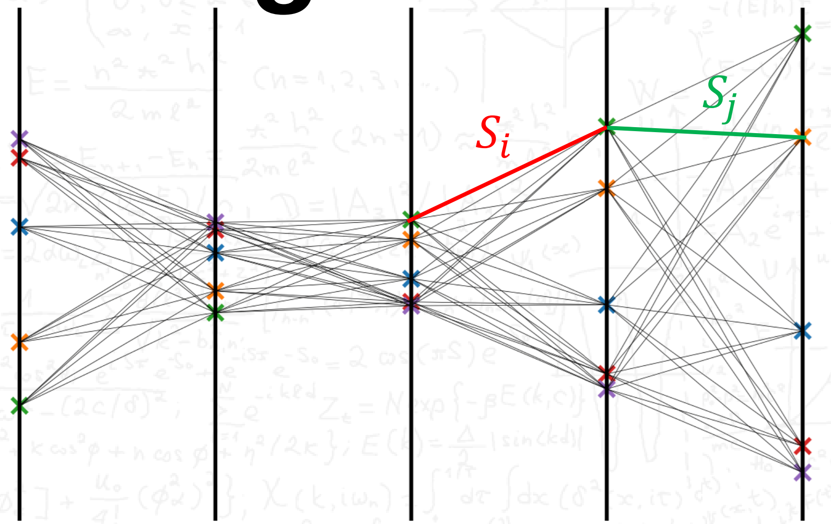
- Solve this **huge** $N \times N$ system

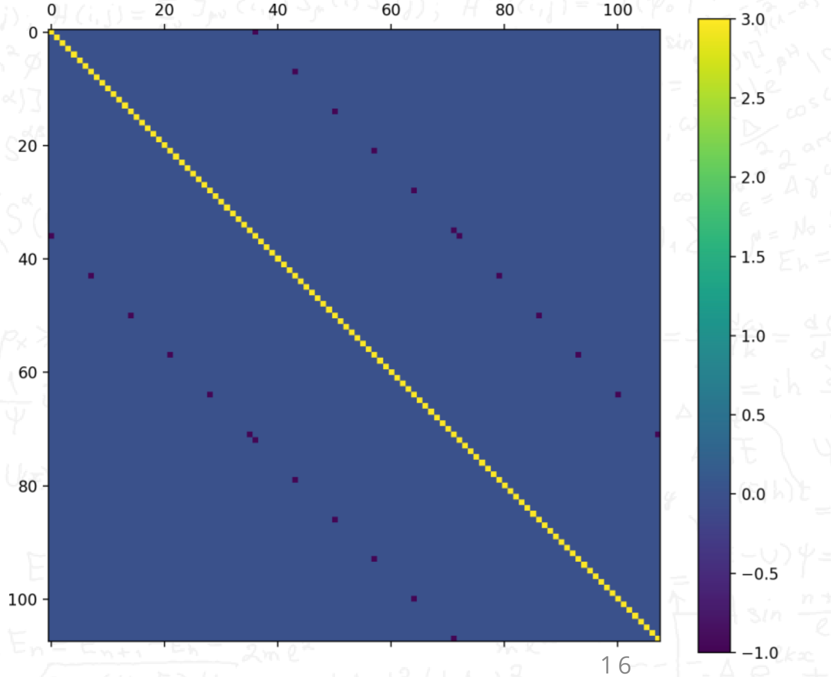
$$N = \#Layers \times \#Particles^2$$

- **Discretize** back the solution

$$S_i \in \{0,1\}$$

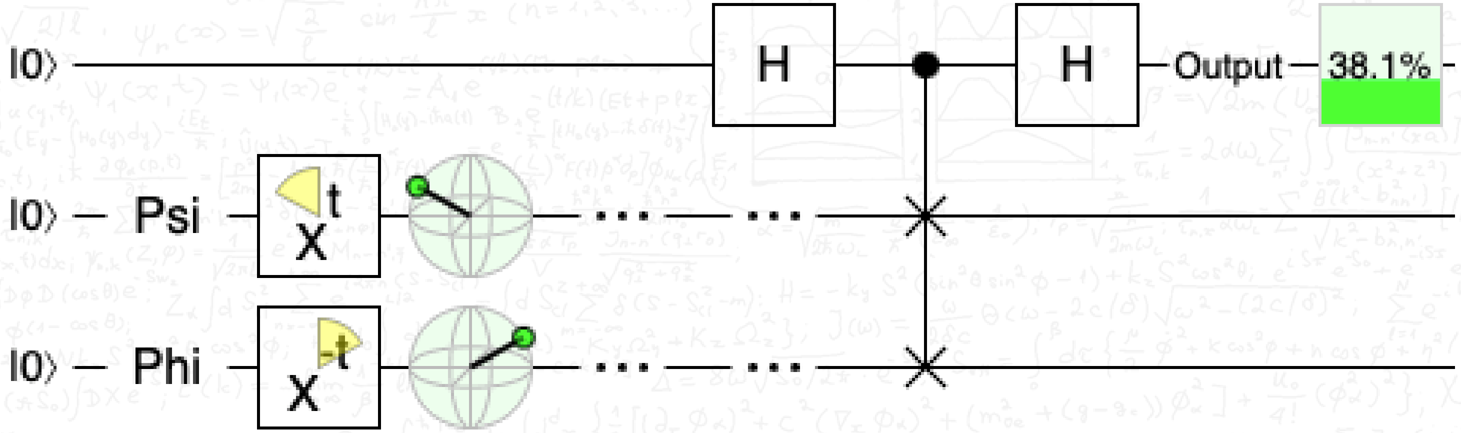
$$S_i \in \mathbb{R}$$





Backup

QC example - swap test



Pros

✓ Complexity **scales exponentially** better for some problems

✓ **Active field of research** (theory and hardware)

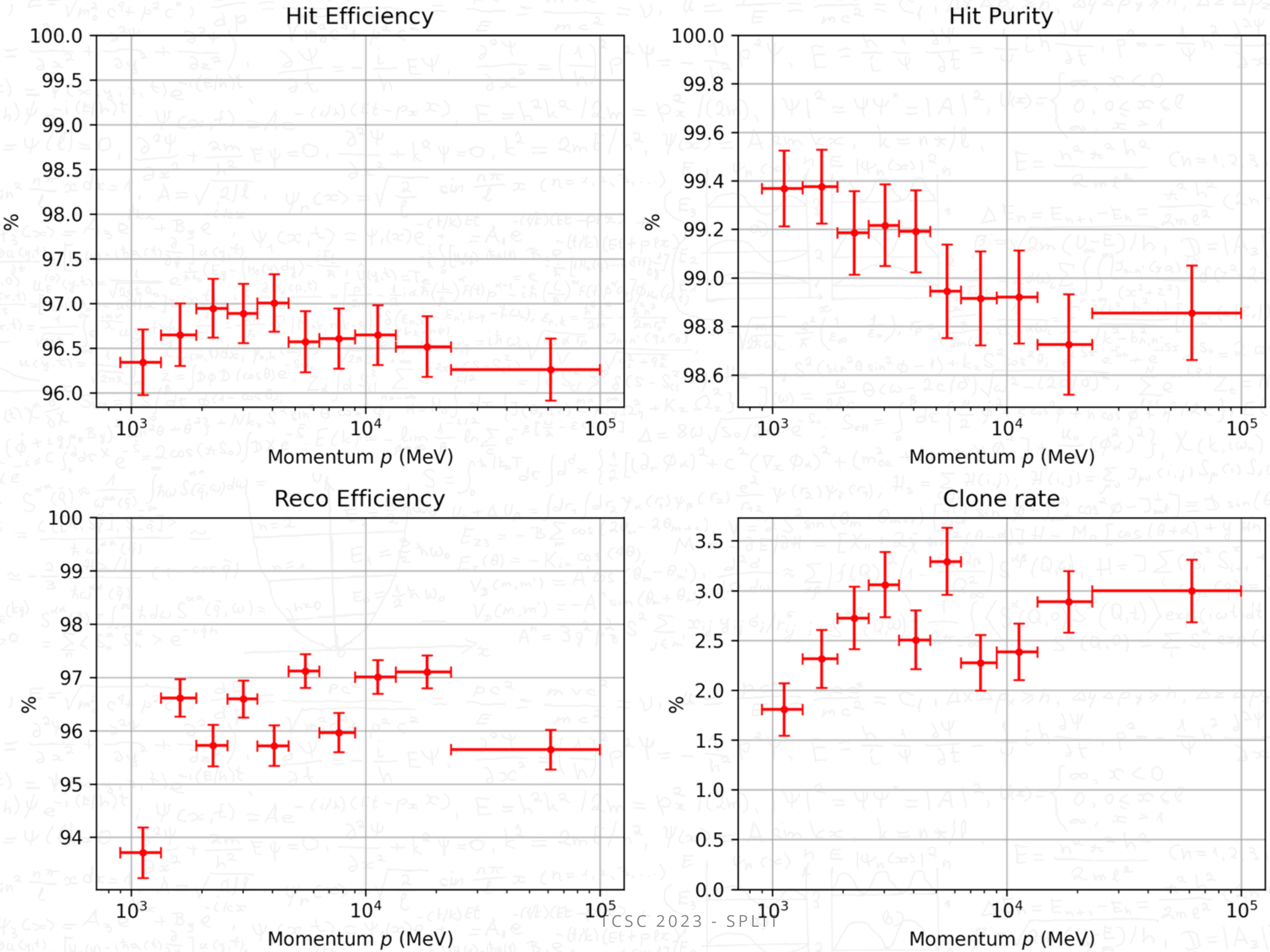
✓ Potentially powerful for **optimization** and **simulation**

Cons

✗ A concrete **useful** application is yet **to be found**

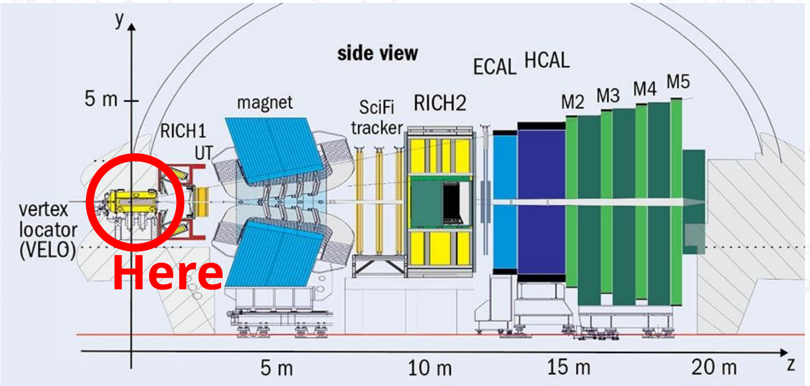
✗ Developing **algorithms** is extremely **hard**
https://quantumalgorithmzoo.org/

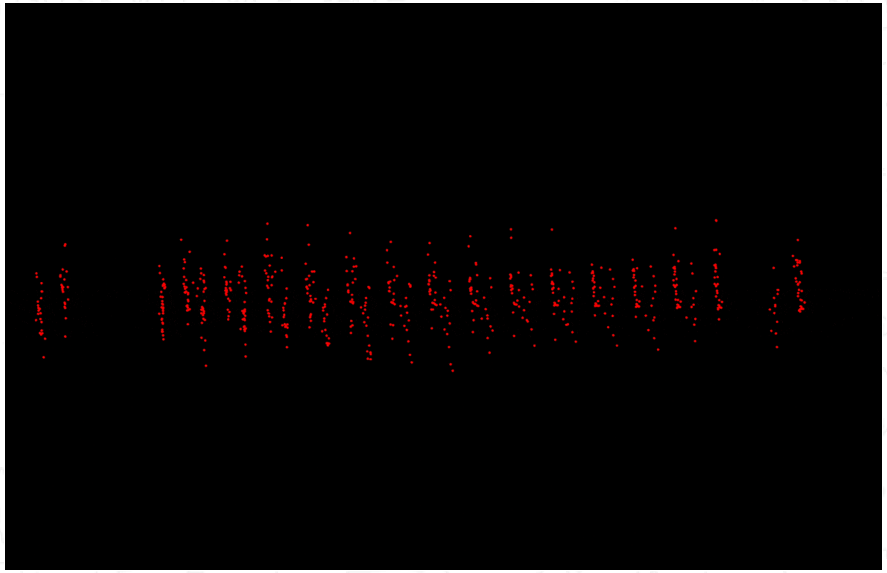
✗ Current **quantum hardware** is not mature enough:
Limited access to **noisy** qubits ~100


Hit Efficiency
Hit Purity
Reco Efficiency
Clone rate
%
Momentum p (MeV)

Connecting dots in the LHCb VELO

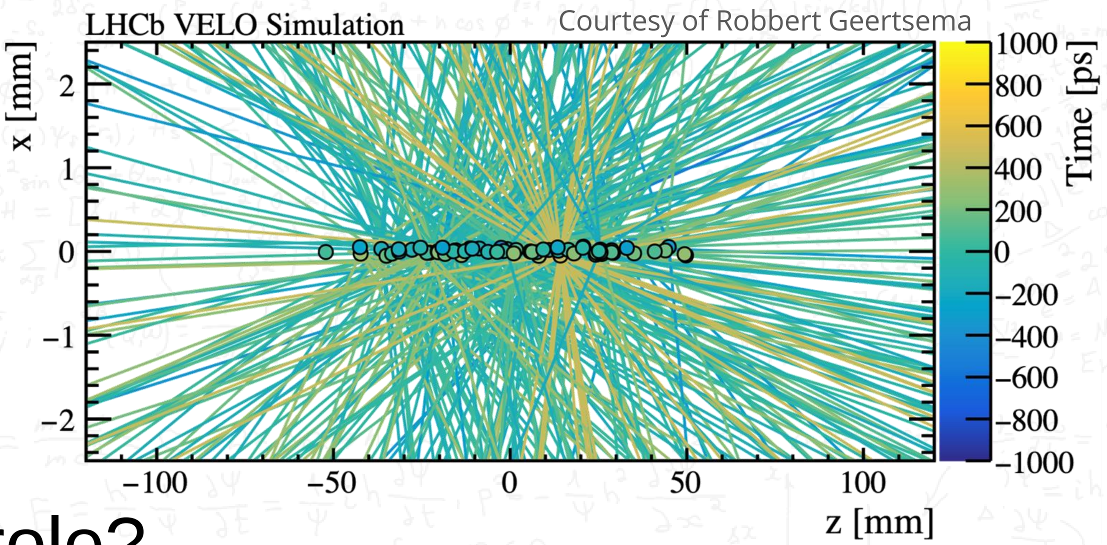
- Charged particles produced in the proton-proton collisions leave **hits** crossing the layers of the LHCb Vertex Locator.
- The reconstruction of the **tracks** of the particles is a crucial step of the LHCb pipeline.
- The **most computationally expensive** algorithm at trigger level.
- In the VELO the effect of the magnetic field is negligible: **straight tracks**.





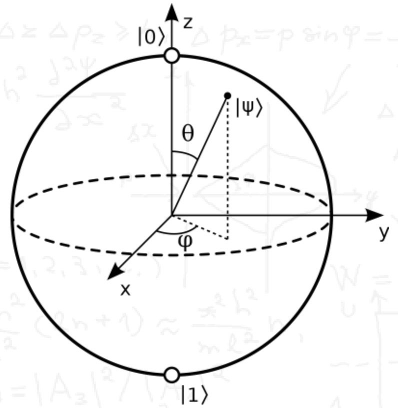
Tracking challenges in HL-LHC

- Current number of interactions per bunch crossing: **~1.6**
- The LHCb detector is going to face a substantial **boost in luminosity** that will increase the complexity of tracking
  - **10x** more **vertices** multiplicity
  - **10x** more **particles** multiplicity
- New **algorithms** will be required to deal with the new operating conditions

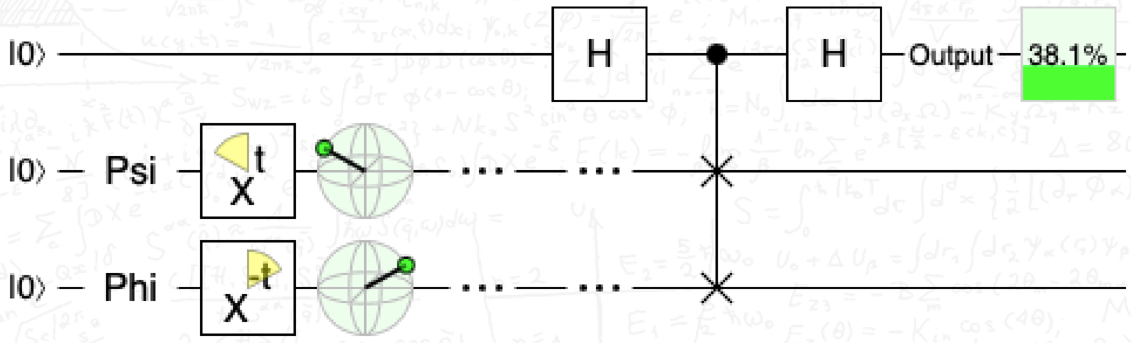


- Can **quantum computing** play a role?

Quantum Computing



- Using a **quantum system** as a fancy calculator
- **Example**: computing the overlap between two quantum states



SWAP test algorithm

1. Prepare state $|\psi\rangle$ and $|\phi\rangle$
2. Apply an H gate to the ancilla
3. Apply the CSWAP gate
4. Reapply an H gate to the ancilla
5. Measure the probability distribution on the ancilla

$$P(0) = \frac{1}{2}\left(1 - |\langle\psi|\phi\rangle|^2\right)$$

✓ Complexity **scales exponentially** better than classical for some problems

✓ Extremelly **active field of research** in the last few years (theory and hardware)

✓ Potentially a very powerful tool for **optimization** and **simulation** tasks

✗ A concrete **useful** application is yet **to be found**

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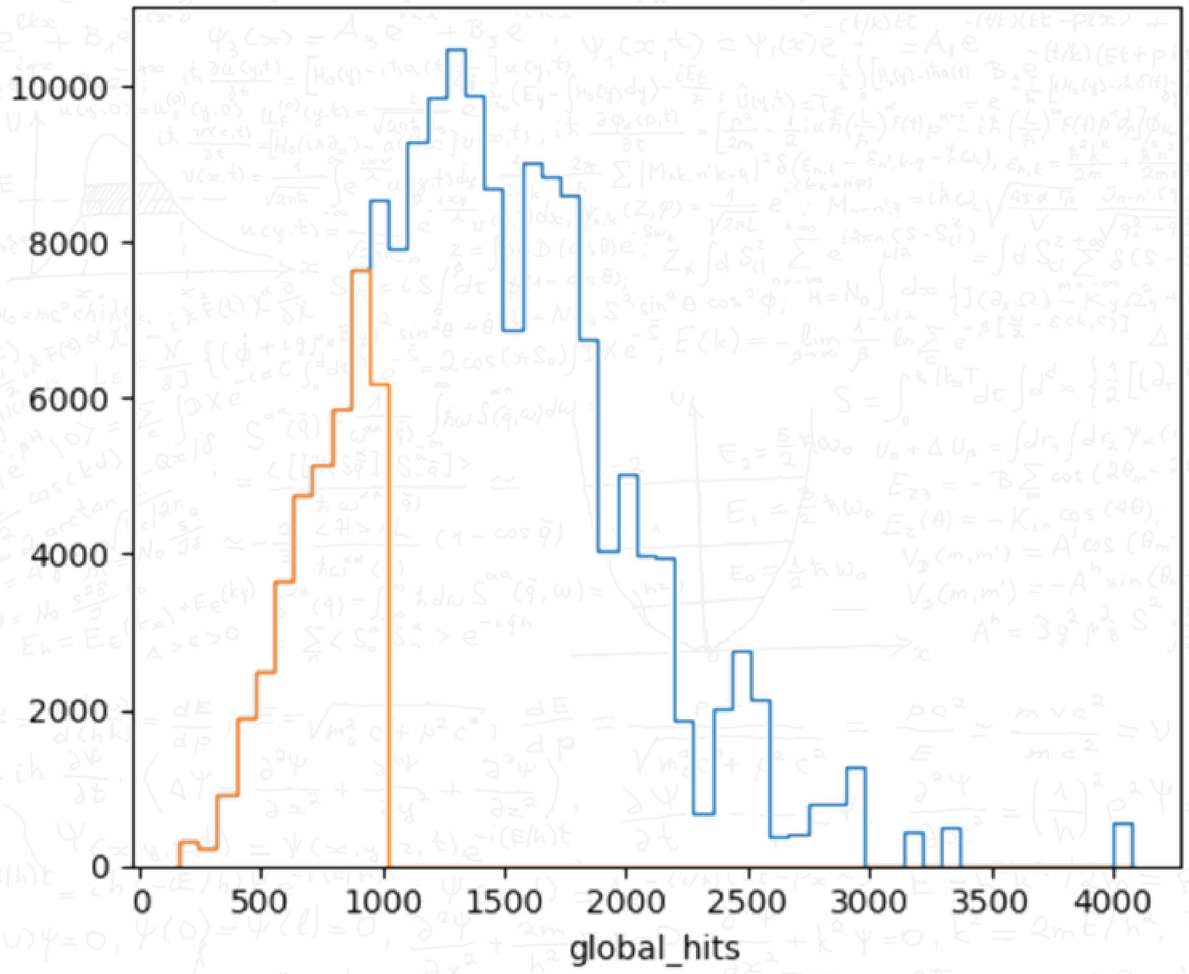
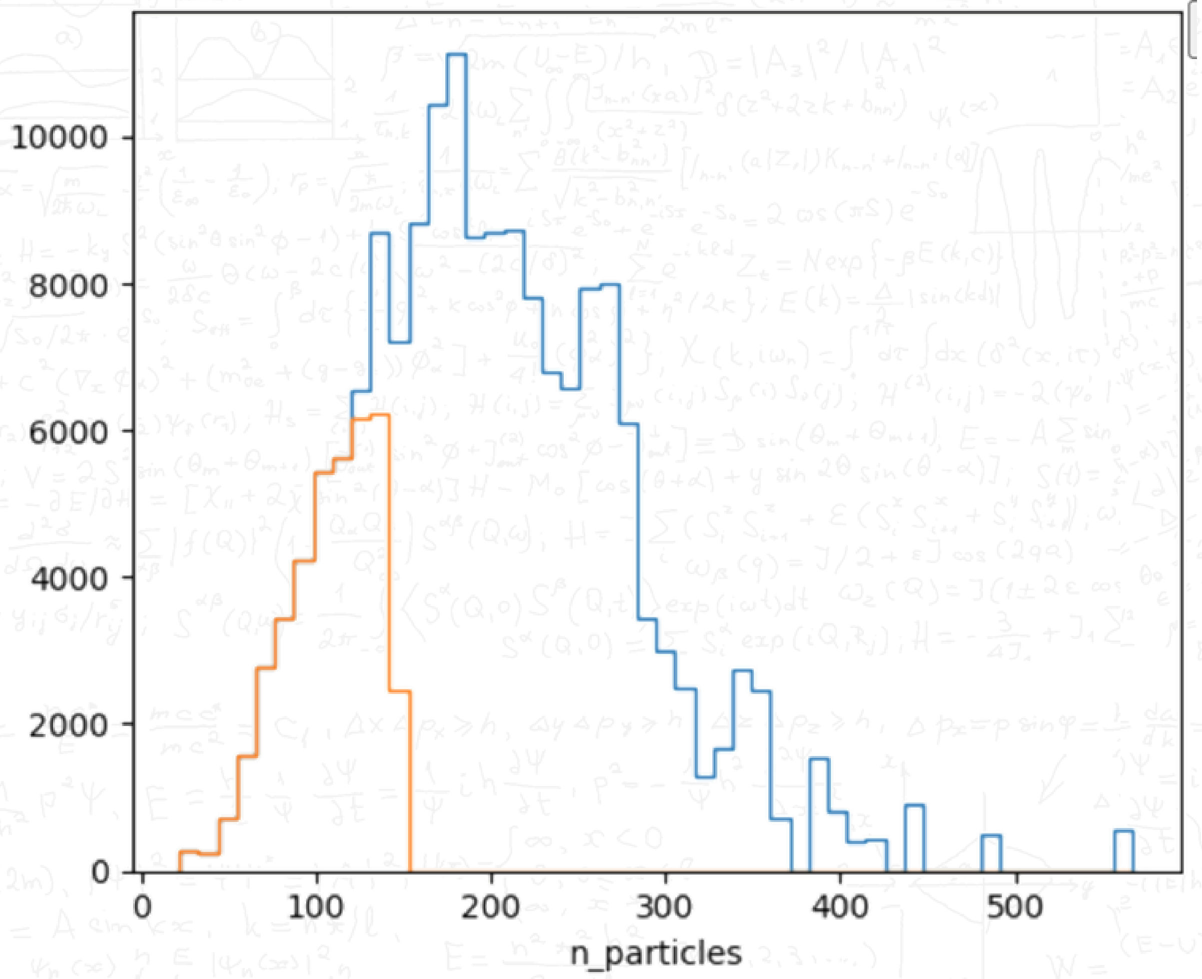
N. hits and particles





Tracking challenges in HL-LHC

- Number of collisions per crossing: **~7**
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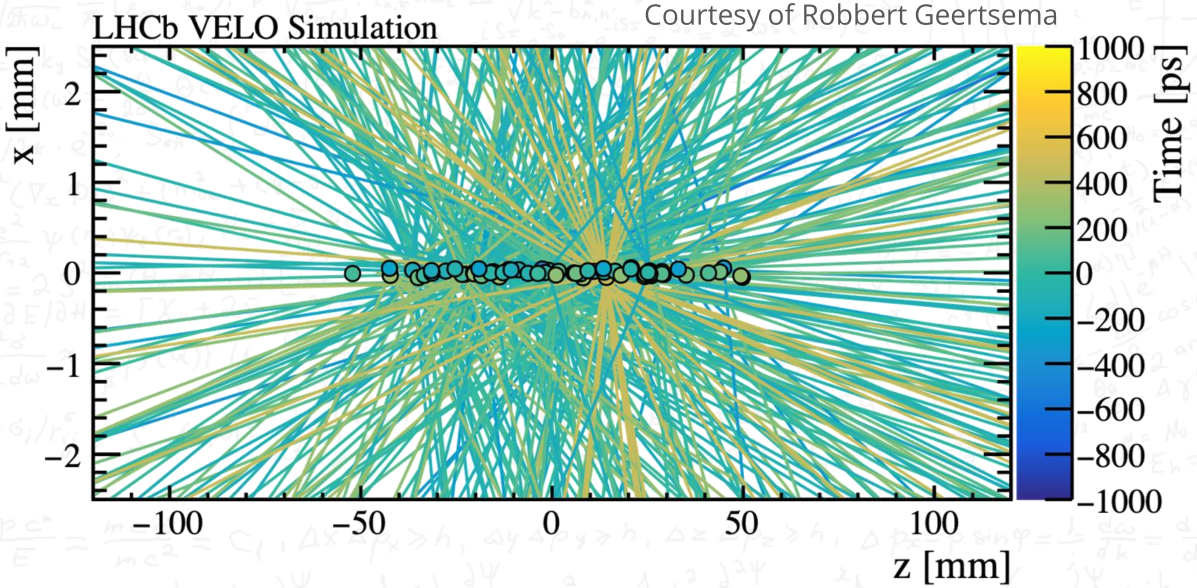
  - **10x** more **collisions**
  - **10x** more **particles**
- New **algorithms** will be required to deal with the new operating conditions



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