From lattice QCD to experiment - general introduction

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Probing baryon weak decays - from experiment to lattice QCD   Warsaw, Poland   6-7 March 2023

Introduction

- Why do weak baryon decays?
- Baryon asymmetry of the universe requires (Sacharov)
  - C, P and CP violation
  - Out of equilibrium
  - Baryon number violation
- Generally testing the Standard Model
- C and P violation in baryons and mesons: late 1950s
- CP-violation in kaons: 1964 (indirect)
- CP-violation in kaons: direct 1999 (ε'/ε)
- CP-violation in bottom mesons: $\geq$ 2001 in lots of places
- CP-violation in charmed mesons: (LHCb 2019)
- all compatible with the Standard Model
- CP-violation in baryons is not observed yet
- $\Lambda_b \to p\pi^-\pi^+\pi^-$ to 3.3σ (LHCb 2016)

Introduction

- What is needed to see *CP*-violation?
  - Need a *CP*-violating amplitude
  - Need another amplitude for it to interfere with
  - *CP*-violation changes from particle to anti-particle
  - Strong phases do not
  - Interference leads to observable effects
- Baryons ony have direct *CP*-violation, no particle anti-particle mixing
- So to make theory predictions:
  - Need to know both amplitudes
  - Or look at observables where the "other" amplitude cancels
- Reminder of this talk:
  - A little bite more about *CP*-violation and the first steps
  - overview of how these amplitudes are calculated at (low) energies

CP-violation in the Standard Model

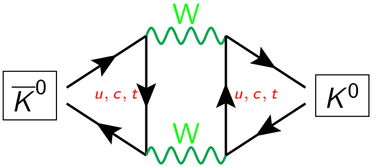
- Hidden in the W-couplings: $\frac{g_2}{\sqrt{2}} W^\mu \begin{pmatrix} \overline{u}_L & \overline{c}_L & \overline{t}_L \end{pmatrix} V \gamma_\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}$
- V is a general 3 by 3 unitary matrix.
- Redefine phases odf quark-fields: V_{11}, V_{12}, V_{13}, V_{21} and V_{31} real

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + s_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

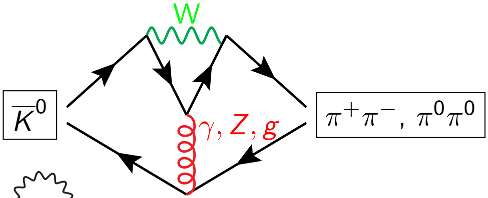
- $c_i = \cos\theta_1$, $s_i = \sin\theta_i$
- Unitary implies $\sum_i V^*_{ik} V_{il} = \delta_{il}$
- But need all three generations, otherwise can remove the phase
- CP-violation in SM is small because the angles are small

Need loops in the SM

- Box diagrams:



- Penguin diagrams:



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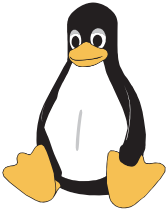
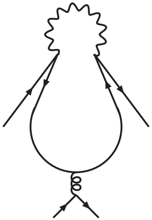
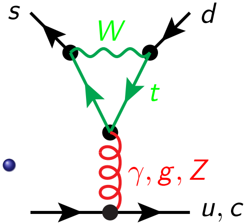




BSM etcetera

- 

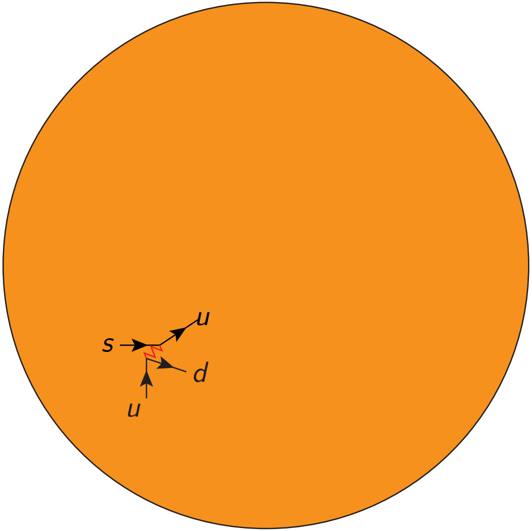
Heavy particles can contribute in loop

- Also heavy BSM particles can contribute: tree level and loops
- Competition with suppressed SM contributions allows very good limits
- Might need to know the SM very precisely to detect deviations
- $\boxed{\text{Need to be able to calculate amplitudes}}$

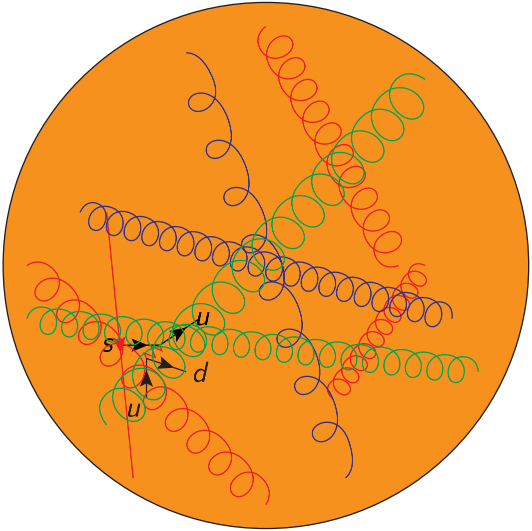
Methods of effective field theory

- Separate the problem at different scales
- High scale: $\geq M_W$
  - Reduce everything to quarks, leptons, gluons, photos
  - Integrate out everything else (W, Z,top,heavy BSM)
  - End up with a number of local operators of varying dimensions
  - Can be done using Feynman diagrams or other methods
  - Example: W exchange to $\bar{b}_L\gamma_\mu c_L \bar{u}_L\gamma^\mu d_L$
  - But many more options possible: full classsification: low-energy effective field theory
  - Matching BSM/SM to effective operators
- Intermediate scale: M_W to hadronic scale
  - Do renormalization group running down
  - Known typically to 2 and sometimes 3 loops
  - Running involves gluons and photons
- These two stages are under control and no problem in principle
- Can be very tedious though

The underlying problem: the low energy stuff

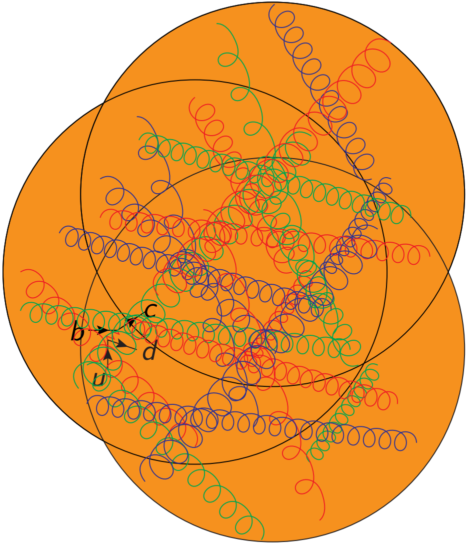


The underlying problem



The underlying problem

- quark-antiquark
- add gluons
- Two body decay



The underlying problem

- Flavour and Hadron Physics: need structure of hadrons
- Why is this so difficult?

- QED $\mathcal{L} = \overline{\psi}\gamma_\mu\left(\partial^\mu - ieA^\mu\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- QCD: $\overline{q}\gamma_\mu\left(\partial^\mu - i\frac{g}{2}G^\mu\right)q - \frac{1}{8}\mathrm{tr}\left(G_{\mu\nu}G^{\mu\nu}\right)$
- $G_\mu = G_\mu^a\lambda^a$ is a matrix

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- $G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig\left(G_\mu G_\nu - G_\nu G_\mu\right)$
- gluons interact with themselves
- $e(\mu)$ smaller for smaller μ, $g(\mu)$ larger for smaller μ
- QCD: low scales no perturbation theory possible

The low-energy step

- So simple QCD perturbation theory does not work
- What else?
- Lattice QCD
- QCD and other sum rules (light-cone sum rules)
- Using dispersion relations
- Schwinger-Dyson equations
- Effective field theor(y)(ies)
- Symmetries
- Models
- Large N_c
- Find observables where most of the difficulties cancel
- . . .

The low-energy step

- Usually a combination of several methods
- Rest of the talk: giving some indications of strenghts and weaknesses of the different methods (i.e. get everyone angry at me)
- In particular I will not discuss all the extra improvements that are used in practice
- Different observables call for different methods
- Cross checks are always valuable

Lattice QCD

- A recommended read:
  Lattice QCD: A Guide for people who want results, Christine Davies, hep-lat/0509046 [hep-lat]
- From first principles (in principle) (there are often extrapolations and other assumptions inherent in the analysis)
- Take the functional integral of QCD and integrate it numerically
- Discretize space and time and take a finite volume
- Go to Euclidean space or imaginary time
- The last two-points are needed to be able to do the functional integral numerically

Lattice QCD

- $\hbar c = 197.3$ MeV fm
- Lattice spacing a needs to be such that $1/a$ is scale significantly above 1 GeV: match to perturbative QCD
- The lattice size L must belarge enough so that (all) the hadrons "fit": need to be several fm.
- E.g. $1/a = 2$ GeV and $L = 4$ fm requires about 40 points in each direction
- Also explains why charm quark is difficult (but there are ways around it for some cases)
- The b-quark is typically treated as static (via HQET or NRQCD) but then needs $1/m_b$ corrections.
- Fast moving particles also difficult: wavelengths similar to lattice spacing

Lattice QCD: matrix elements of operators

- Calculate numerically vacuum expectation values: $\langle O_1(x_1)\ldots O_n(x_n)\rangle$
- From this (*LSZ* theorem for the theorists) get lots of physical quantities
- Hard work in lattice QCD summarized in a few words:
  - finding good O_i for the observable
  - Finding ways to get the vacuum expectation values calculated as accurately as possible

Lattice QCD: masses and decay constants

Two-point functions $\left\langle O_1(0,\vec{0})O_2(\tau,\vec{0})\right\rangle = \sum_i f_{1i}f_{2i}e^{-E_i\tau}$

- the sum is over all states i
- ALL: also multiparticle states, states with momentum,...
- $f_{1i} = \langle O_1|i\rangle$
- Can get energies and couplings of states
- Large τ dominated by E_0: get at the mass and coupling constants of the ground state.
- Example: O_i axial current: pion mass and decay constants
- Possible problems;
    - Need large τ: lattice length must be big enough (neglect around the end of the world)
    - Excited state contamination can happen
    - Typically: at small τ: $a \neq 0$ artefacts; at large τ: noise

Lattice QCD: form factors

- Form factors: need a matrix element like $\langle i|O_W|j\rangle = f_{Wij}$
- Get from
  $\langle O_1(0)O_W(\tau_W)O_2(\tau_2)\rangle = \sum_{ij} f_{1i}f_{2j}f_{Wij}e^{-E_i\tau_W}e^{-E_j(\tau_2-\tau_W)} + \sum_i f_{1i}f_{2i}e^{-E_i\tau_2}$
- So need to get the ground state in two legs
- The problems from previous page are amplified
- States should not move too fast so B-decays limited to maximal q^2

Lattice QCD: multi particle final states

- Problem when looking at scattering and multi (i.e. 2 or more) final states (note leptons don't count)
- Energy is not conserved so the lowest possible state with the same quantum numbers will dominate
- Scattering is in Minkowski: need to relate to Euclidean observables
- Problem for 2-body scattering: Lüscher: via volume dependence of energy levels
- Decays: Lellouch-Lüscher: same but with an insertion of the decay operator
- Three body: active research area

Lattice QCD and Baryons: added problems

- Three quark vs quark-antiquark: operators typically more complicated
- Baryons are bigger: need larger lattices
- Excited states closer by than for mesons
- NOISE MUCH LARGER

Explanation (long known):

- quantity x via $\langle x \rangle$; error is via $\langle x^2 \rangle - \langle x \rangle^2$
- baryons: $\left\langle N^\dagger(0) N(\tau) \right\rangle \sim e^{-m_N \tau}$
- Noise $\left\langle N^\dagger(0) N(0) \; N^\dagger(\tau) N(\tau) \right\rangle \sim e^{-E_0 \tau}$
- But $N^\dagger N$ couples to purely mesonic states and $E_0 < m_B$
- signal to noise decays as $e^{-(m_B - E_0)\tau}$: exponentially worse

Lattice QCD: status

- Masses and decay constants for lowest lying states: excellent
- Form factors: simple ones and near the largest q^2: very good
- Two body scattering for light mesons: starting to be good
- Weak decays if not via form factors: only starting
- Note: $\bar{B}$-B mixing is like a form factor

- I summarized a large community ($\geq$ 500 for a big conference) working for a long time ($\geq$ 40 years) here in a few slides
- I also stuck to lattice topics related to what we might want to do

Other full QCD only

Light cone quantization Brodsky, Pauli, Phys.Rept. 301 (1998) 299-486 [hep-ph/9705477]

- Only physical degrees of freedom (ie no ghosts . . .)
- Wave functions are expanded in Fock states: partons directly visible
- The perturbative vacuum is the physical vacuum
- In principle allows for a competing numerical nonperturbative method
- Was a very active field 1990s
- Main (unsolved) difficulty: dealing with the zero mode
  This is where all the trouble of spontaneous symmetry breaking and confinement hides in this approach

Note: this not quark models on the light cone

- Now often combined with holography (Brodsky, de Téramond,. . .)

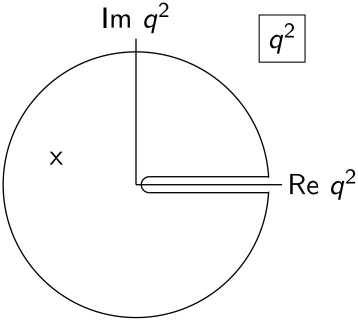
Other full QCD only

QCD, Finite Energy Sum Rules, . . .

- All rely on analyticity and Cauchy's theorem

$$\frac{1}{2\pi i}\oint_C dz\, f(z) = \sum_{\text{poles}} \text{residues}$$

- a typical curve



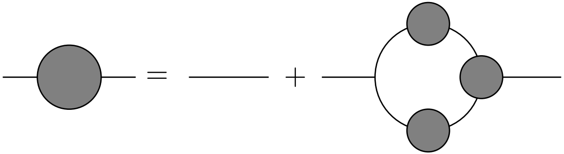
- Circle and residue points: perturbative QCD
- Axis: data and/or resonance saturation

Dispersion relations and unitarity

Dispersion relations and unitarity

- Again Cauchy's theorem
- But now choose $f(z)$ e.g. a decay or scattering amplitude
- s, t, u: more parameters
- Unitarity $1 = S^\dagger S = 1 + T^\dagger T + i(T - T^\dagger)$
- Due to the cuts: phases provide constraints
- Integral equations for the amplitudes
- Questions: subtraction constants, experimental input for phases, asymptotic behaviour

Schwinger-Dyson equations

- Idea (ϕ^3): 

- Full three-point function involves full four-point function
- Four involves five, . . .
- An infinite set of consistency equations
- Need to truncate: here the model aspects start
- Need for a starting ansatz to make life bearable (usually a full gluon propagator)
- Usually kept at the "quenched" approximation

Effective Field Theory (EFT)

- gap in the spectrum $\Longrightarrow$ separation of scales
- with the lower degrees of freedom, build the most general effective Lagrangian
- $\infty\#$ parameters
- Where did my predictivity go ?
- $\Longrightarrow$ Need some ordering principle: power counting
  Higher orders suppressed by powers of $1/\Lambda$
- Taylor series expansion does not work (convergence radius is zero when massless modes are present)
- Continuum of excitation states need to be taken into account
- Use field theory and loops

Examples of low-energy EFTs

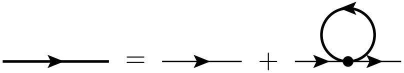
- Chiral perturbation theory
- Heavy quark effective theory
- Combinations of the above
- Main drawbacks:
  - many parameters, often unknown
  - Not always (often?) in range of validity
- Beware: Just because it says chiral perturbation theory or effective field theory doesn't mean it is (they are not protected trade marks)

Effective Lagrangians

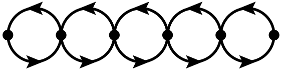
- Basic degrees of freedom: hadron fields
- Beware of product (mis)labelling
  - Chiral Perturbation Theory
  - Chiral Effective Theory
  - Are very popular names and “de vlag dekt niet altijd de lading” since they are not protected names (free flag doesn’t make free bottom)
- Note field redefinitions: same Lagrangian can look very different
- Hope: find a simple Lagrangian and then refine it
- A full classification attempt: Resonance chiral theory (RχT), also attempts to go to one-loop.
- This includes e.g. also hidden local symmetry implementations of the vectors
- many other partially successful attempts (e.g. by next speaker)
- In some ways holography belongs to this class as well
- Difficult to find a proper power counting (so not EFT)

Models with “Quarks”

- Nonrelativistic constituent quark models: understanding the spectrum (fill up octets and nonets)
- Chiral quark model: quarks plus pseudo-scalars, no confinement
- Nambu-Jona-Lasinio models: Quarks with a four quark interaction
    - Has spontaneous chiral symmetry breaking
    - Produces a constituent quark mass from a gap equation:



    - mesons from a bubble sum



    - Mesons but no confinement

Models with “Quarks”

- Add vector-like four quark interactions $\Longrightarrow$ vectors and axial-vertex
- Add a ‘t Hooft vertex to get η' better (or a variation on that vertex)
- make the vertex non-local
- Add Polyakov loop
- Many more variations possible
- Usually large N_c or tree level at the “meson” level
- Some attempts to go beyond that: many difficulties and not clear if it ever yielded something useful

Conclusions

- Why we want to study *CP*-violation
- Baryons give us more information beyond what we have (just mentioned really)
- Overview of lattice QCD
- Lightning overview of other methods
- Be aware of the advantages and drawbacks of what you use
- Do the best you can, sometimes using a model is all you can do
- Often best results by combining several methods