Hadron structure from lattice QCD

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Extended Twisted Mass Collaboration

STIMULATE
European Joint Doctorates

Probing baryon weak decays
Warsaw, 6 March 2023

Outline

- ✳Introduction
    - State-of-the-art lattice QCD simulations
- ✳ 3D structure of the nucleon
    - ➡Electromagnetic form factors and strangeness of the nucleon
    - ➡Axial form factors
    - ➡Direct computation of parton distributions (talk by K. Cichy)
- ✳Conclusions

Quantum ChromoDynamics (QCD)

- QCD-Gauge theory of the strong interaction

- Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^{\mu} D_{\mu} - m_f \right) \psi_f \qquad D_{\mu} = \partial_{\mu} - ig\frac{\lambda^{a}}{2} A^{a}{}_{\mu}$$

Harald Fritzsch

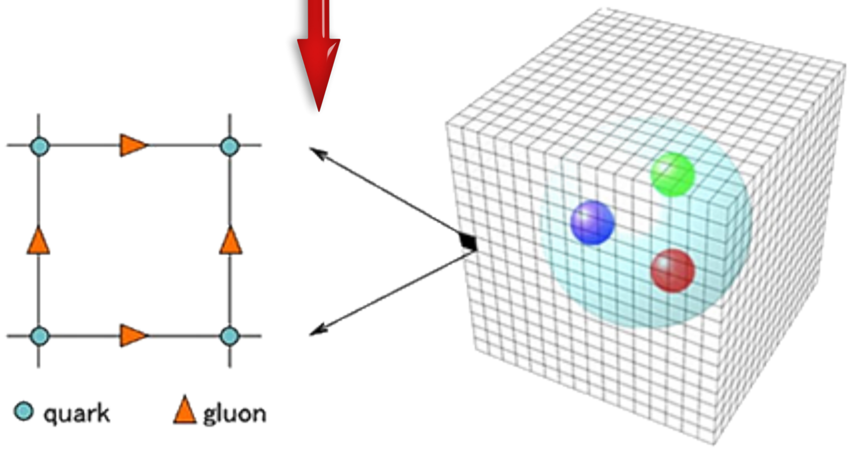
Murray Gell-Mann

Heinrich Leutwyler

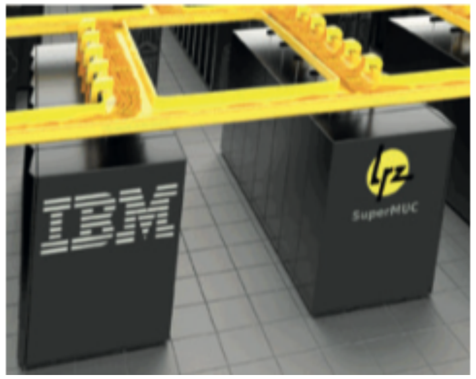
Phys. Lett. 47B (1973) 365

Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}[U]\,\mathcal{O}(D_f^{-1}[U],U)\left(\prod_{f=u,d,s,c}\mathrm{Det}(D_f[U])\right)e^{-S_{\mathrm{QCD}}[U]}$$



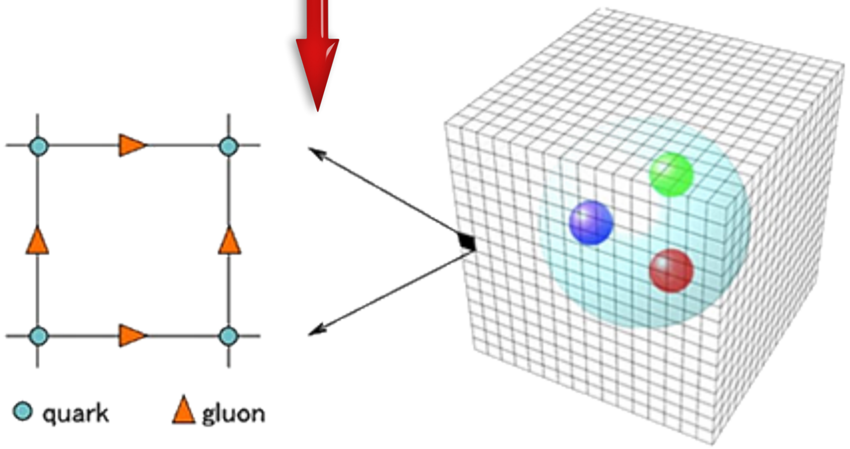
Simulation of gauge ensembles {U}



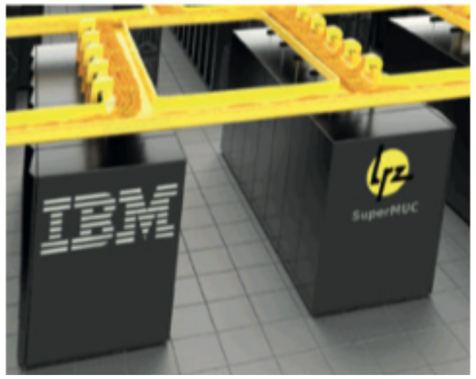
Quark & gluon propagators

Lattice QCD

$$\langle \mathcal{O}\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}[U]\; O(D_f^{-1}[U],U)\left(\prod_{f=u,d,s,c}\mathrm{Det}(D_f[U])\right)e^{-S_{\rm QCD}[U]}$$

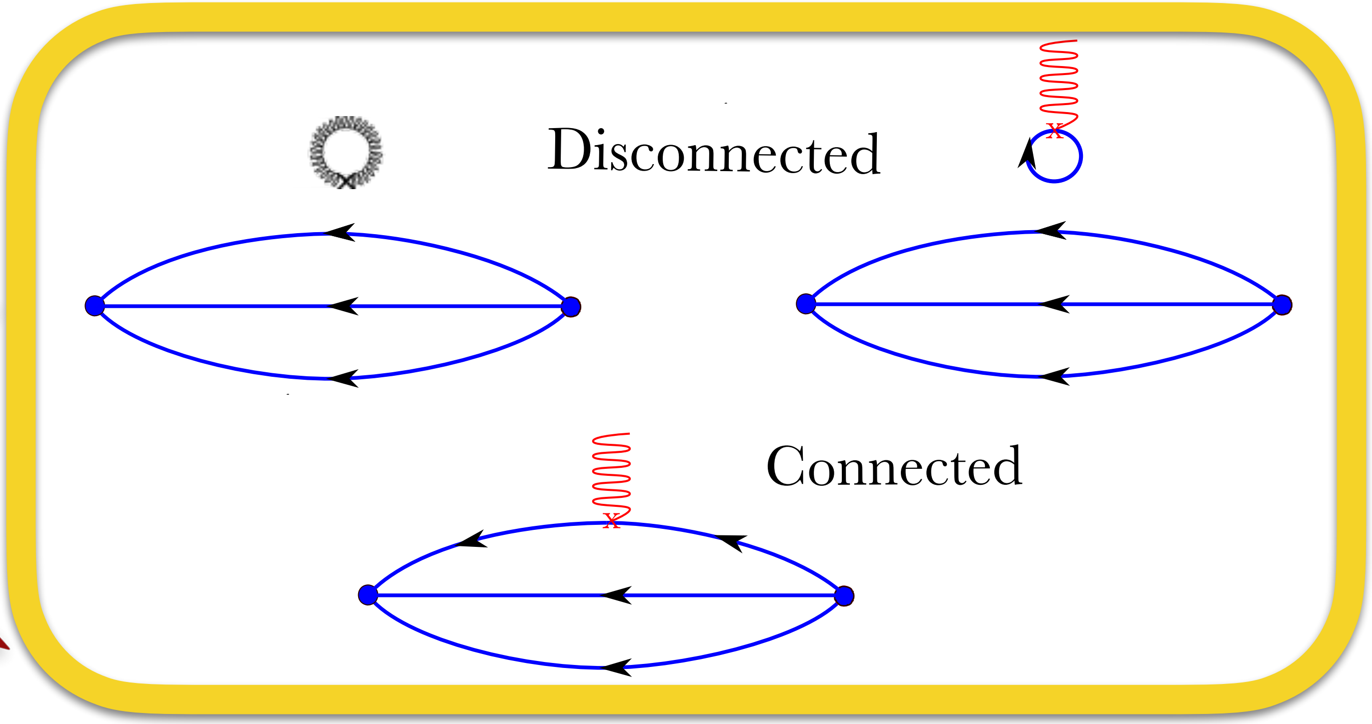


Simulation of gauge ensembles {U}



Quark & gluon propagators

contractions

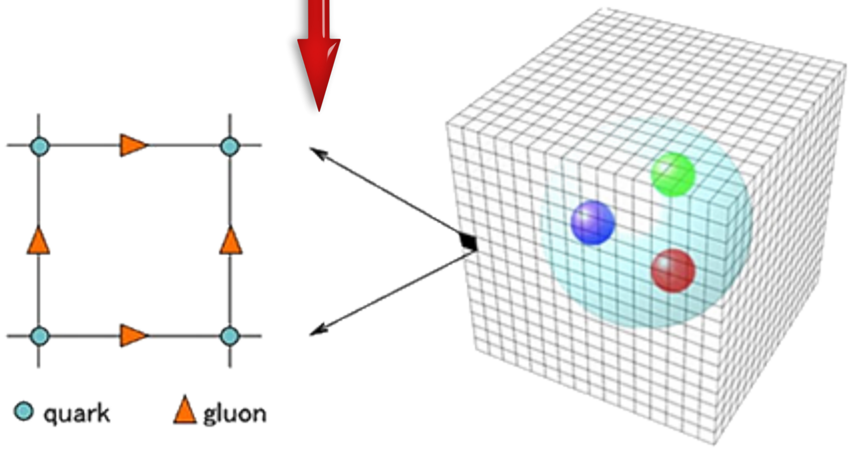


Disconnected

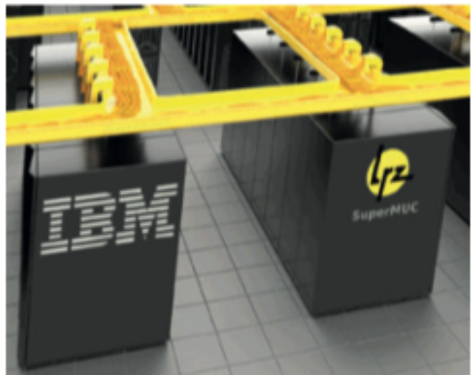
Connected

Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$

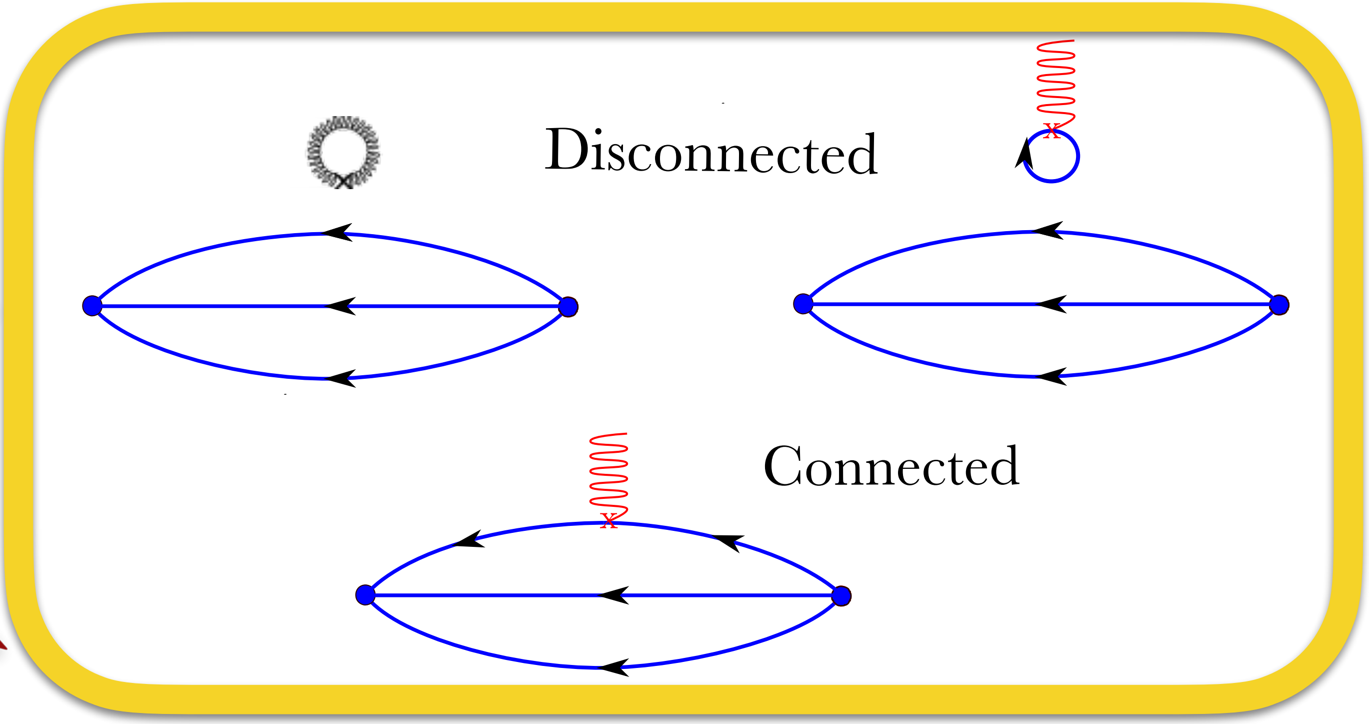


Simulation of gauge ensembles {U}



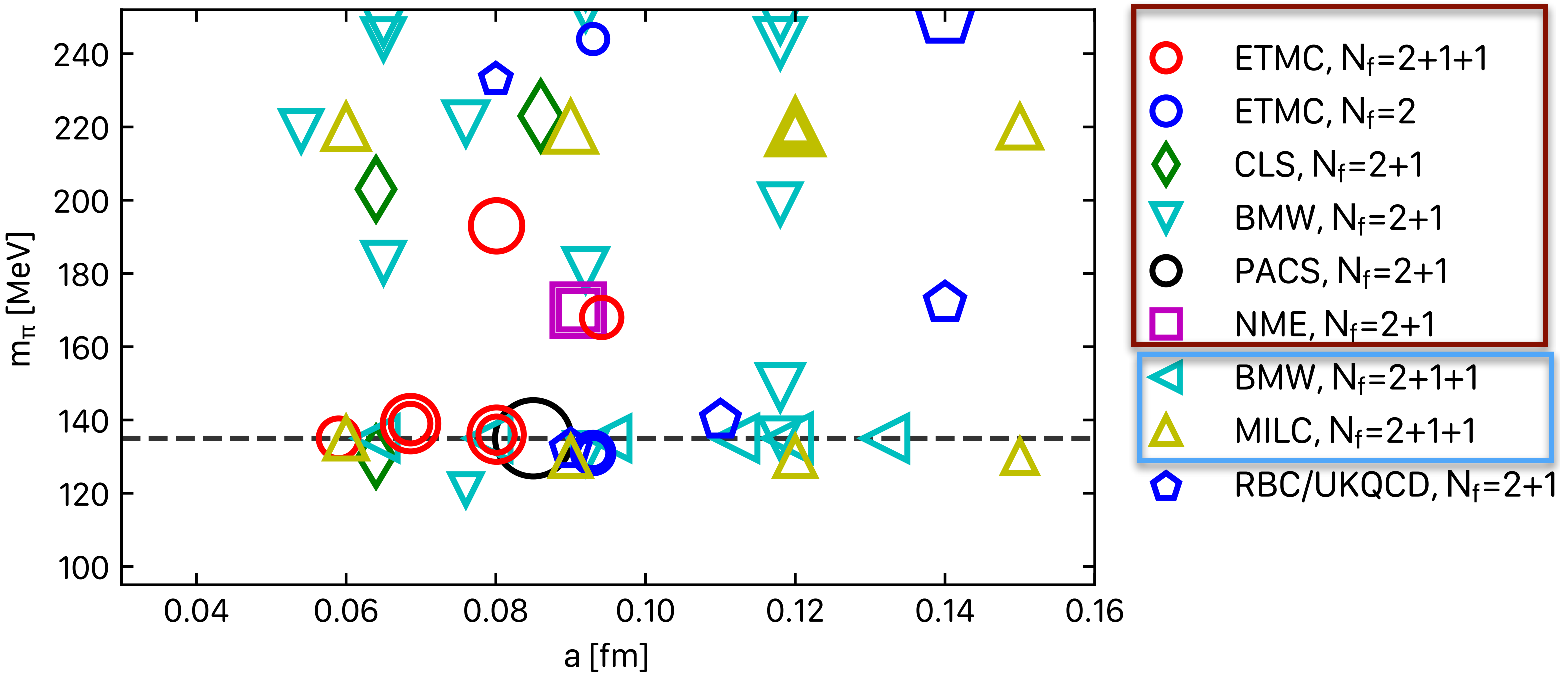
Quark & gluon propagators

contractions



Data Analysis

Status of current simulations

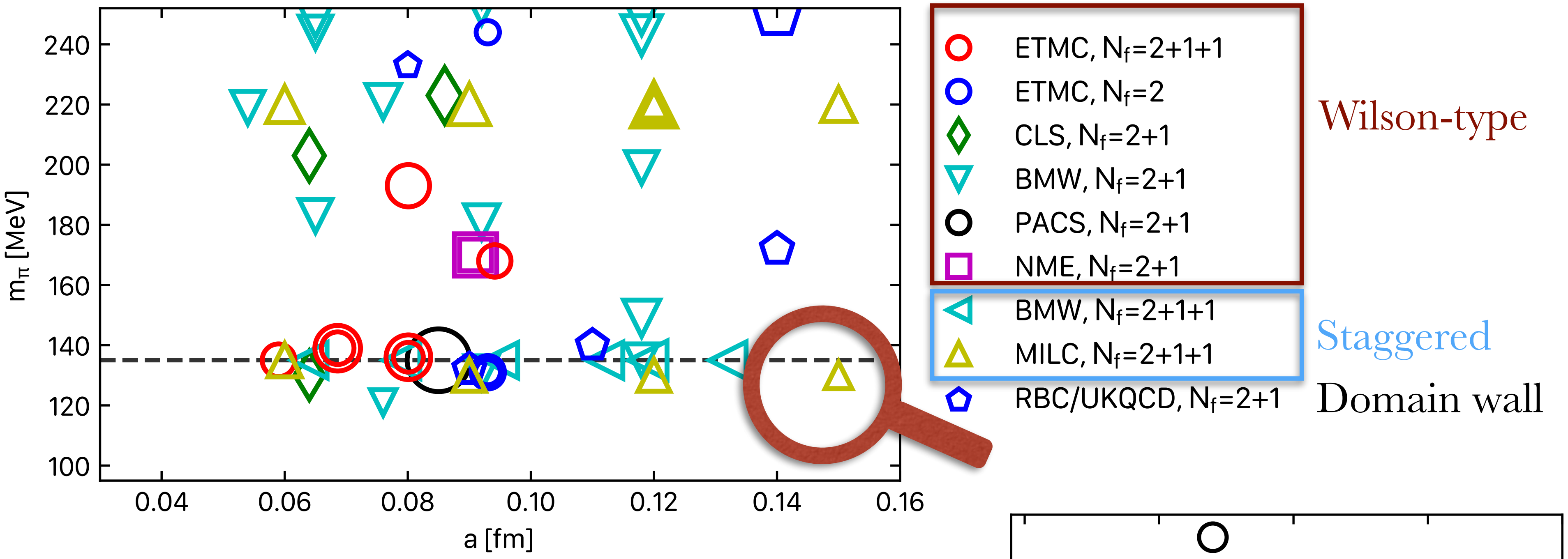


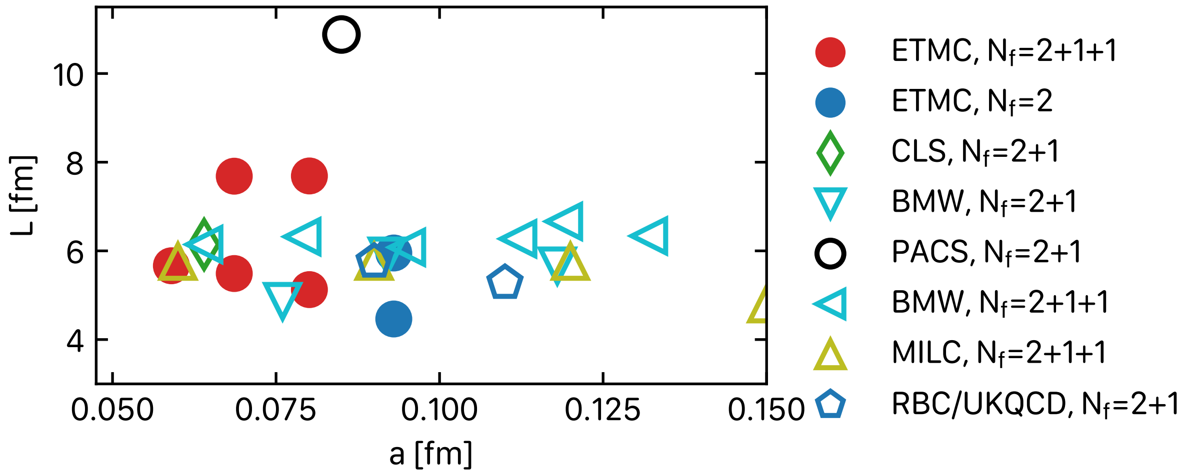
Wilson-type

Staggered

Domain wall

Status of current simulations



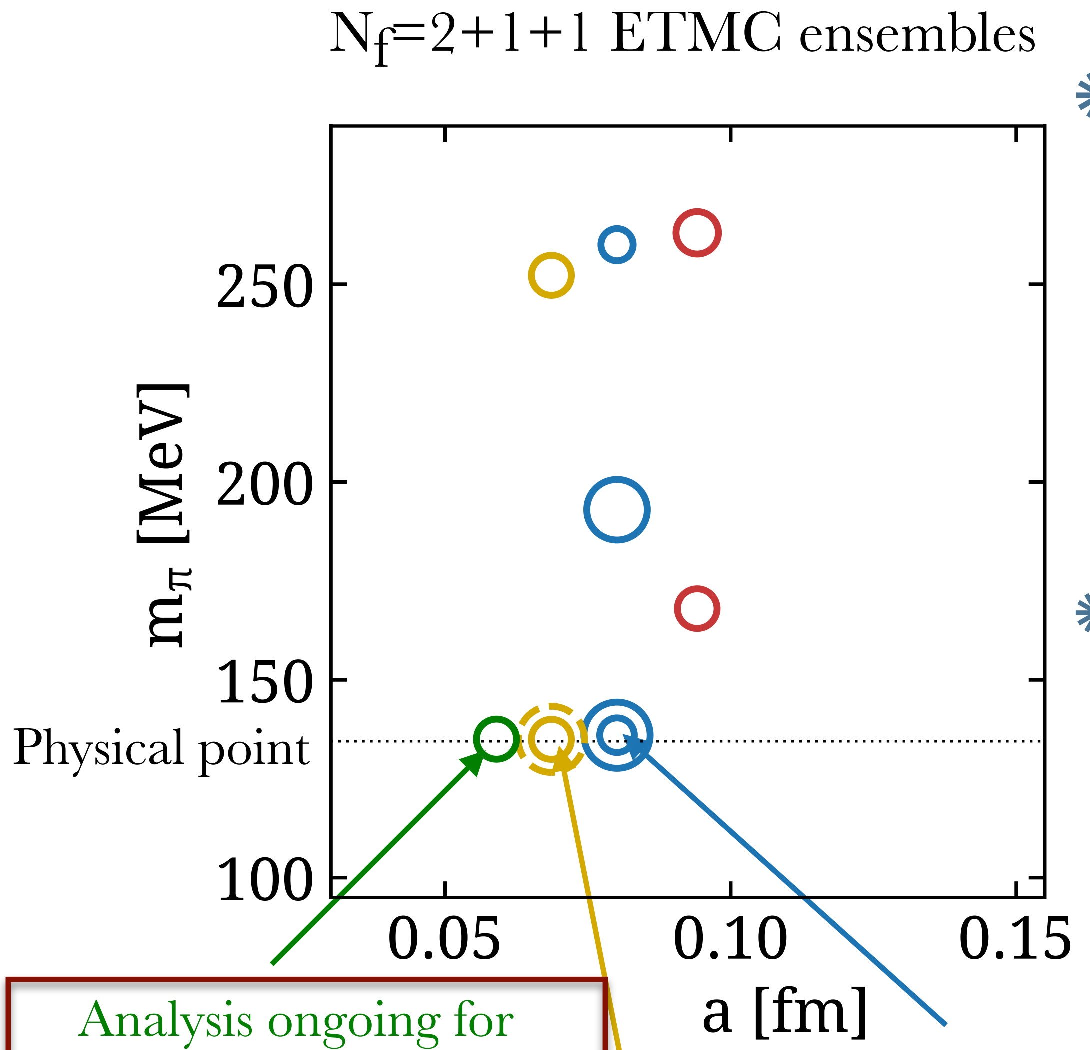


- A number of collaborations has generated gauge ensembles with physical values of the u/d, s and c quarks

- Algorithmic improvements needed to simulate for a<0.05 fm due to critical slow down in Markov chain Monte Carlo (long autocorrelations) —> new sampling algorithms, e.g. flow-based algorithms

G. Kanwar *et al.*, Phys. Rev. Lett. 125 (2020) 121601, arXiv:2003.06413; D. Boyda *et al.*, Phys.Rev.D 103 (2021) 074504, arXiv: 2008.05456; M. S. Albergo *et al.*, Phys.Rev.D 104 (2021) 114507, arXiv:2106.05934, J. Finkenrath arXiv:2201.02216

Gauge ensembles generated by ETMC



Analysis ongoing for 96^3x192, a~0.06 fm -D96

- Analysis completed for 64^3x128 a=0.08 fm -B64
- Analysis ongoing for 96^3x192 a=0.08 fm

- Analysis ongoing for 80^3x160, a=0.07 fm -C80
- Simulation ongoing for 112^3x224, a=0.07 fm

- 5 ensembles at physical pion mass
  - 3 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
  - Two volumes at a=0.08 fm and 0.07 fm of Lm_π~3.6 (5.1 fm) and Lm_π ~5.4 (7.7 fm)

- Algorithmic improvements needed to go to a<0.05 fm due to critical slow down in HMC (long autocorrelations) —> new approaches e.g. Machine learning approaches using equivariant flows

G. Kanwar, et al., Phys. Rev. Lett. 125 (2020) 121601, arXiv:2003.06413; D. Boyda, et al., Phys.Rev.D 103 (2021) 074504, arXiv: 2008.05456; M. S. Albergo *et al.*, *Phys.Rev.D* 104 (2021) 114507, arXiv:2106.05934

C. A. *et al.* (ETMC), "Simulating twisted mass fermions at physical light, strange and charm quark masses" Phys. Rev. D98 (2018) 054518

Systematics & Challenges

- **Simulations directly at the physical point** ✓

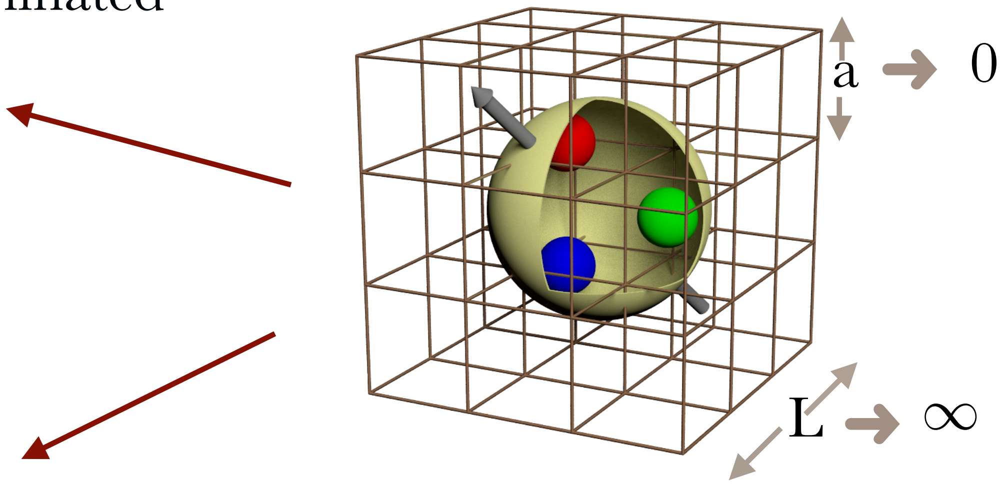
  Systematic effects from chiral extrapolation are eliminated

- **Discretisation effects:** Continuum limit

  —> need simulations for at least 3 lattice spacings

- **Finite volume effects:** Infinite volume limit

  —> need simulations for at least 3 volumes



- **Renormalisation** ✓

  Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts

- **Ground-state identification**

  Cross-check (one-, two- and three-state fits, summation)

  Two-particle state contribution complicate the identification of the ground state

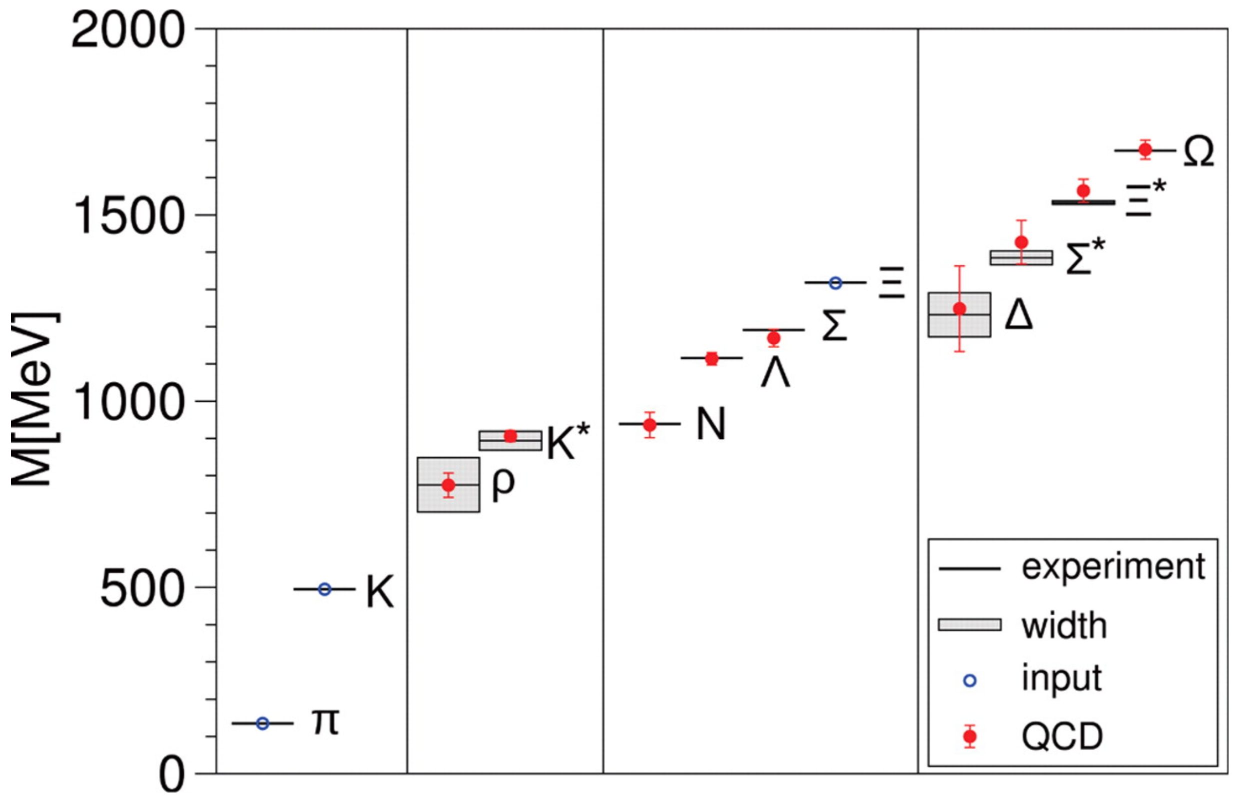
- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects**

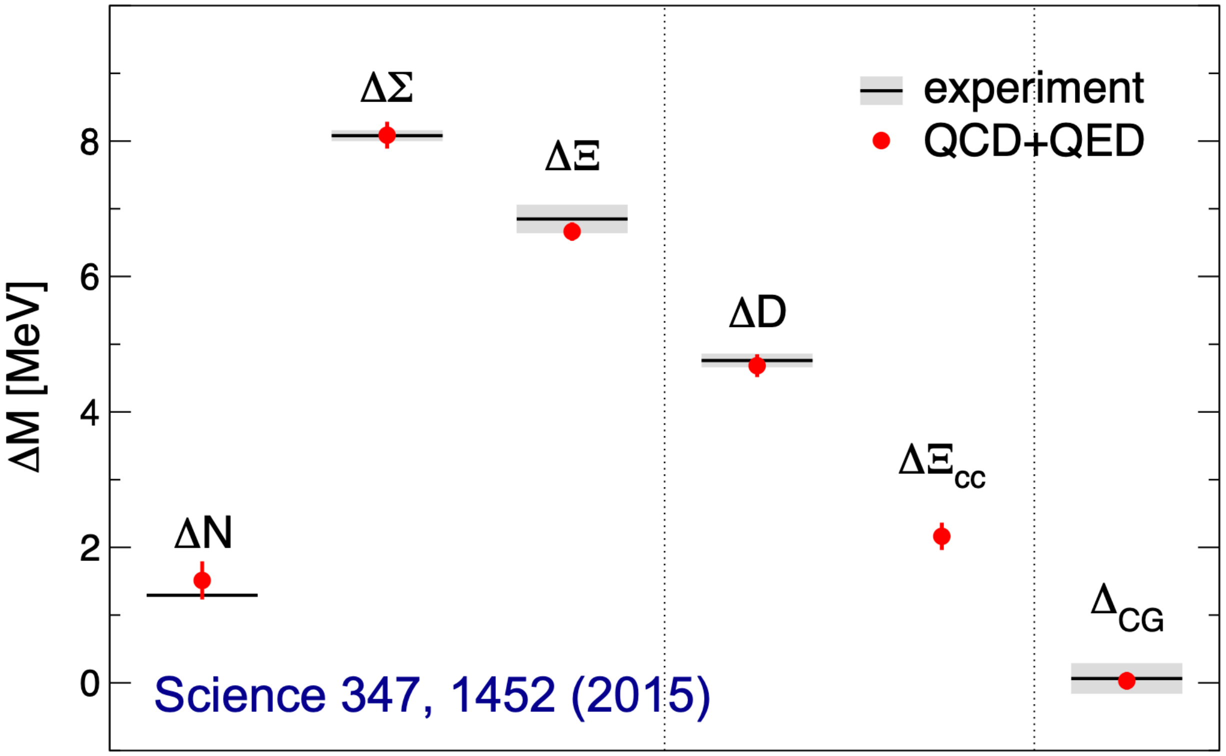
Low-lying hadron spectrum

- BMW collaboration determined the low-lying hadron masses, as well as the mass splittings

S. Durr *et al.*, Science 322 (2008) 1224

Sz. Borsanyi *et al.*, Science 347 (2015) 1452





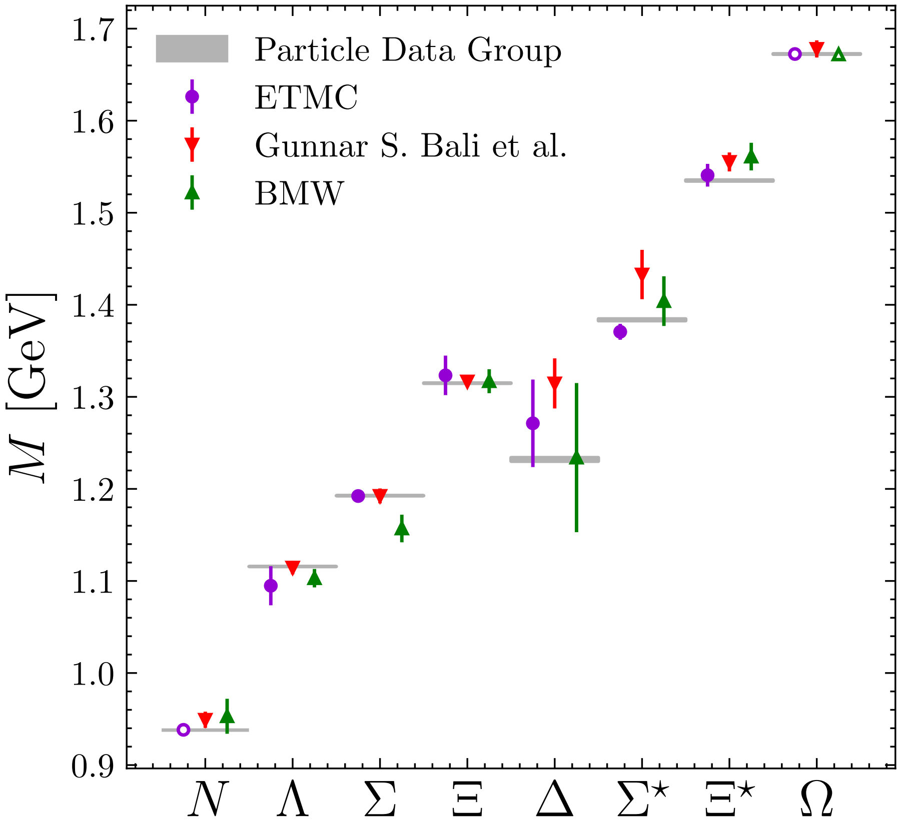
	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{\mathrm{CG}} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

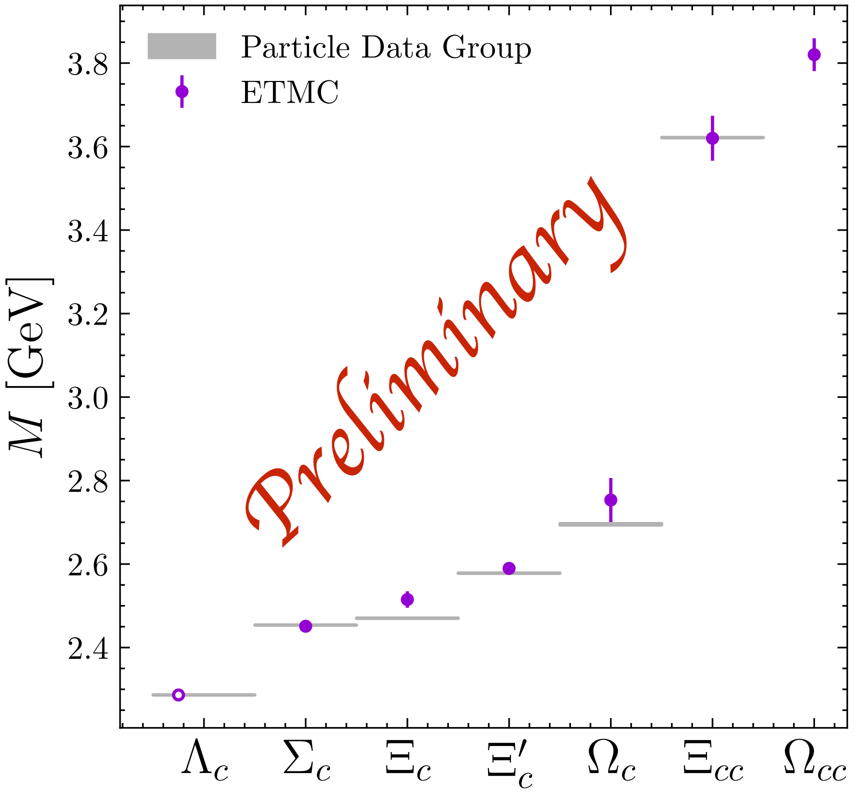
Lattice QCD reproduces the low-lying hadron masses and mass splittings

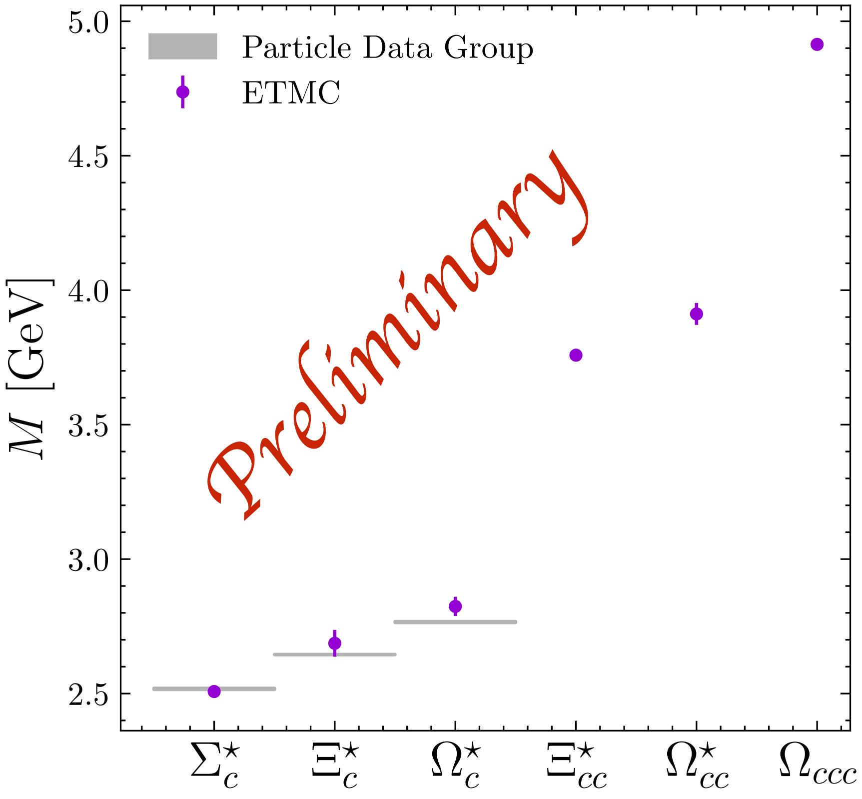
Recent results on the low-lying masses

- ETMC: only using physical point ensembles —> no uncontrolled systematics due to chiral extrapolation

- RQCD: extrapolated to the physical pion mass







Baryonic semi-leptonic decays

Write transition matrix elements $H \to H' l \bar{\nu}_l$ of in term of hadronic J^h and leptonic J^l components

$$\mathcal{M}_{H\to H' l\bar{\nu}} = \frac{G_F}{\sqrt{2}}\mathcal{C}\boxed{\langle H'|J^h_\mu|H\rangle}\langle l|J^{\mu,l}|\bar{\nu}_l\rangle$$

Relevant Cabibbo angle

V-A structure and for u, d and s quarks and each current we have ΔS=0 and ΔS=1 components

$$\langle H'|J^h_\mu|H\rangle = \langle H'|V_\mu - A_\mu|H\rangle$$

The matrix elements in Euclidean space are written as

$$\langle H'(\vec{p}',s')|V_\mu|H\rangle(\vec{p},s) = \bar{u}_{H'}(\vec{p}',s')\left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}q_\nu\frac{f_2(q^2)}{M_H+M_{H'}} - q_\mu\frac{f_3(q^2)}{M_H+M'_H}\right]u_H(\vec{p},s)$$

$$\langle H'(\vec{p}',s')|A_\mu|H\rangle(\vec{p},s) = \bar{u}_{H'}(\vec{p}',s')\left[\gamma_\mu\gamma_5 g_1(q^2) + i\sigma_{\mu\nu}q_\nu\gamma_5\frac{g_2(q^2)}{M_H+M_{H'}} - q_\mu\gamma_5\frac{g_3(q^2)}{M_H+M'_H}\right]u_H(\vec{p},s)$$

- At q^2=0 we get the vector and axial coupling constants: $f_1(0) = g_V \qquad g_1(0) = g_A$
- For diagonal matrix elements the form factors $f_3(q^2)$ and $g_2(q^2)$ vanish as they do in the SU(3) limit

$$\boxed{n \to p e^- \bar{\nu}};\ \Sigma^- \to n e^- \bar{\nu};\ \Xi^- \to \Lambda e^- \bar{\nu};\ \Lambda \to p e^- \bar{\nu};\ \cdots$$

Very few lattice studies on the rest

Most studied in lattice QCD

$$\Xi^0 \to \Sigma^+ e^- \bar{\nu};\ \Sigma^- \to n e^- \bar{\nu}$$

S. Sasaki, 1209.6115;
M. Goeckeler *et al.* (QCDSF) 1101.2806;
A. N. Cooke *et al.*, 1311.4916 and 1212.2564; P.E. Shanahan *et al.*, 1508.06923.

Nucleon matrix elements

$$\langle N(\vec{p}', s')|V_\mu|N(\vec{p}, s)\rangle = \bar{u}_N(\vec{p}', s')\left[\gamma_\mu \underset{\nwarrow\,\text{Dirac}}{F_1(q^2)} + i\frac{\sigma_{\mu\nu}q_\nu}{2M_N}\underset{\nwarrow\,\text{Pauli}}{F_2(q^2)}\right]u_N(\vec{p}, s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{2M_N}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\langle N(\vec{p}', s')|A_\mu|N(\vec{p}, s)\rangle = \bar{u}_N(\vec{p}', s')\left[\gamma_\mu\gamma_5 G_A(q^2) - \frac{q_\mu\gamma_5}{2M_H}G_P(q^2)\right]u_N(\vec{p}, s)$$

Our work:

- **Nucleon electromagnetic and axial form factors**
- **Δ transition form factors:**
  - Previous work not using state-of-the-art ensembles and Delta stable
  - Formalism for unstable Delta developed, work is ongoing
- Delta electromagnetic and axial form factors - only done assuming stable Delta and older ensembles

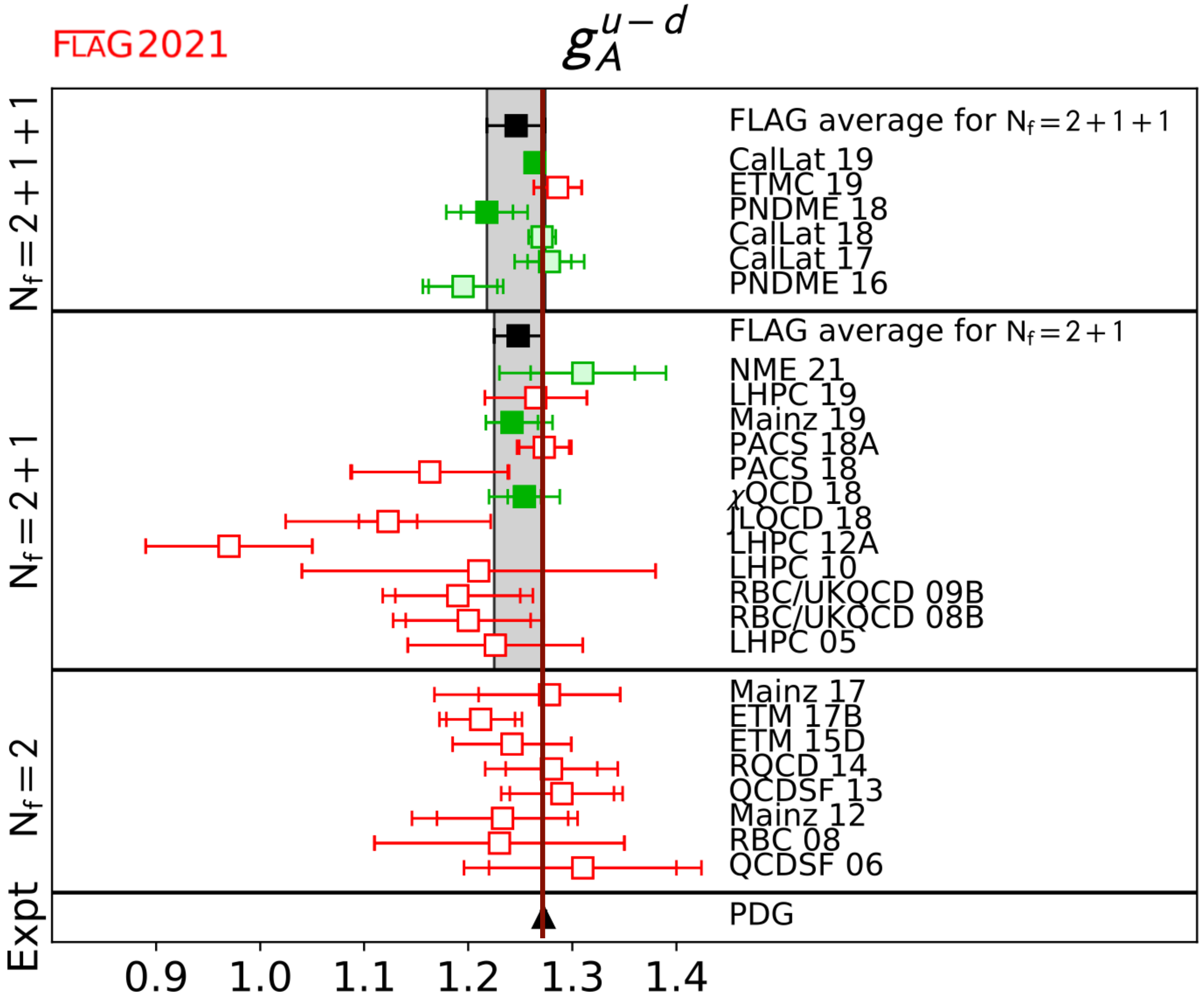
Charges of baryons

- Readily accessible in lattice QCD
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)

Nucleon isovector charges

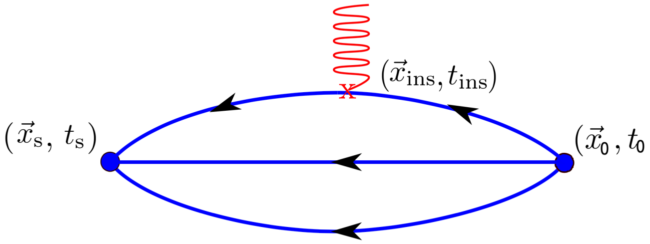
$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2723 \pm 0.0023$ ☜ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

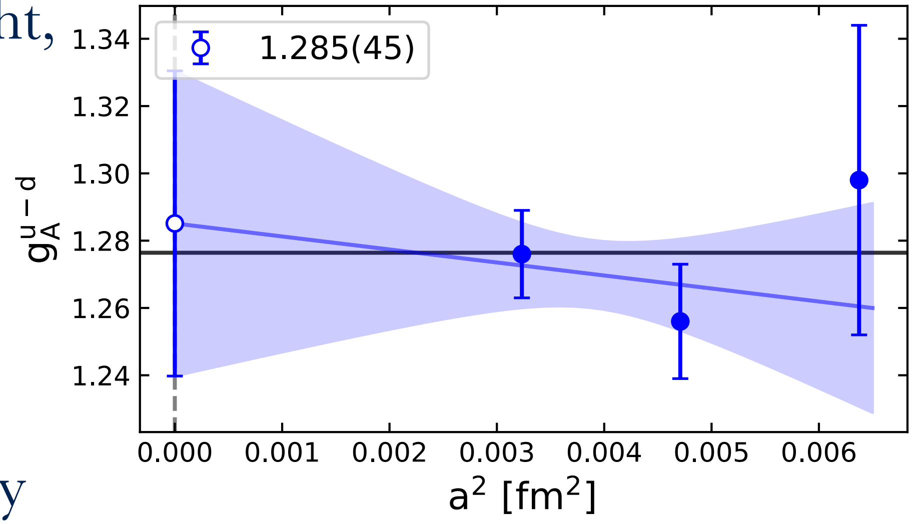


(1) Lattice QCD results on g_A consistent with experimental value

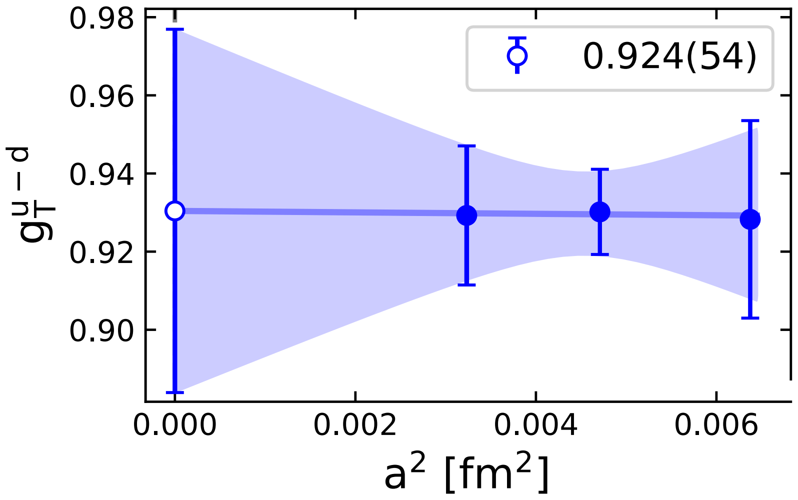
Nucleon isovector (u-d) axial and tensor charges



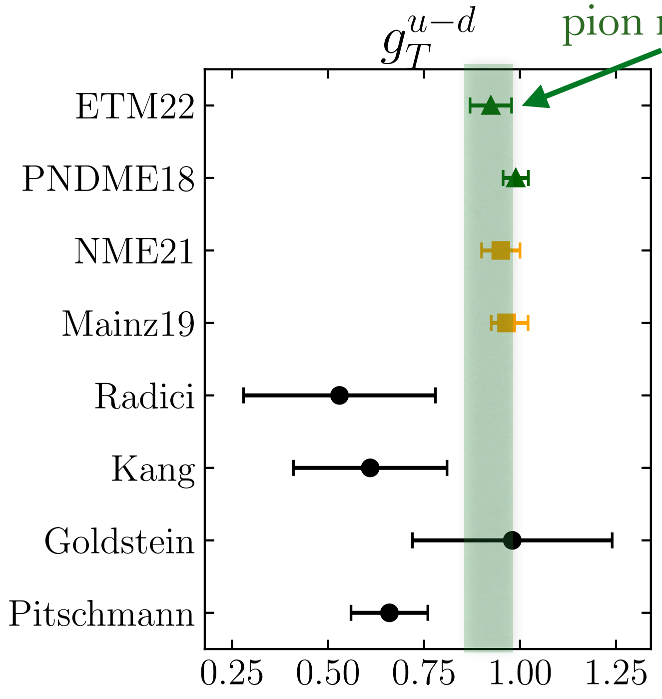
- Only connected contributions
- Use three gauge ensembles generated using physical values of the light, strange and charm quarks:
  - B64-ensemble: 64^3 x 128, a~0.08 fm
  - C80-ensemble: 80^3x160, a~0.07 fm
  - D96-ensemble:96^3x192, a~0.06 fm



- Obtain the tensor charge for the first time in the continuum using only physical point ensembles



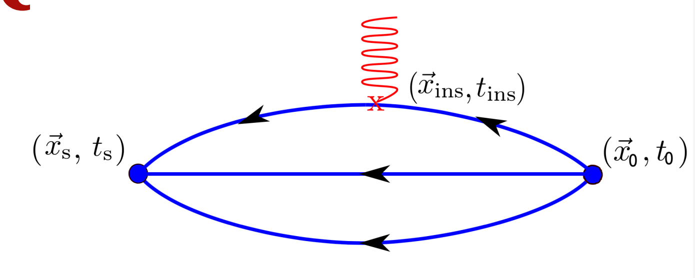
Continuum limit directly at physical pion mass

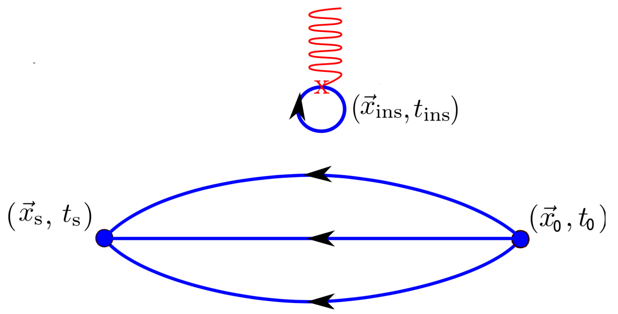


- Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

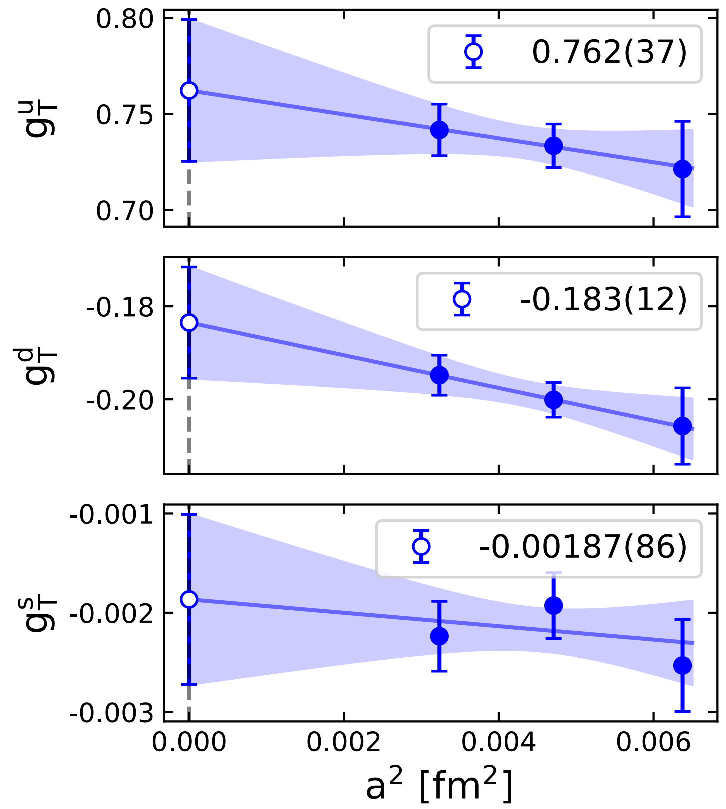
Phys.Rev.D 106 (2022) 3, 034014, arXiv:2205.00999

Flavor diagonal tensor charge

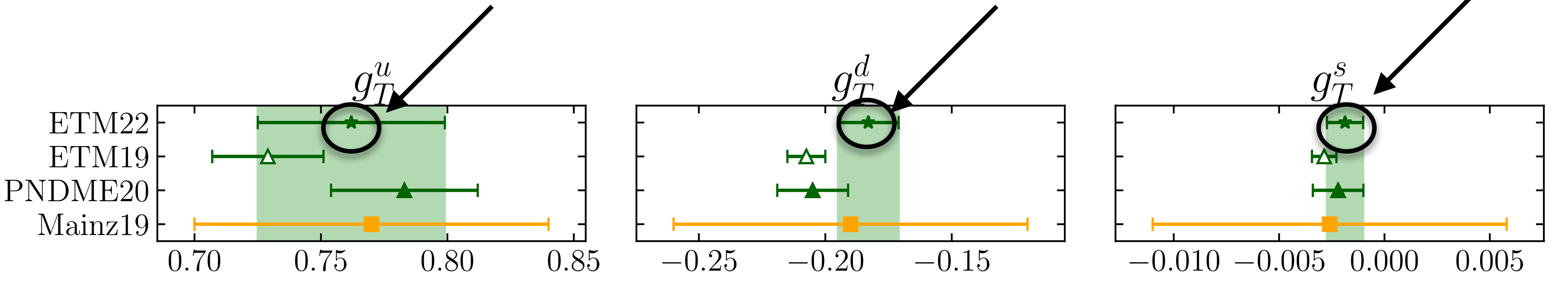




- Evaluate both connected and disconnected contributions
- Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology



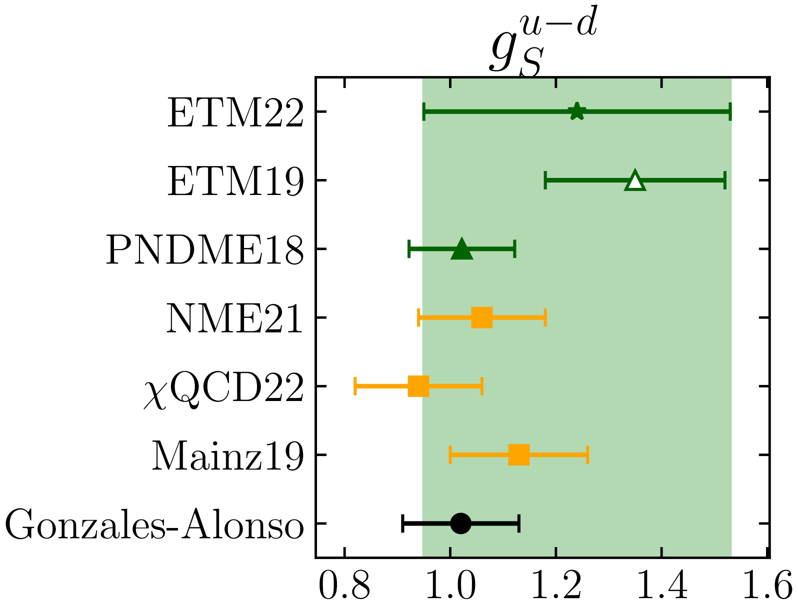
Only calculation in the continuum limit directly at the physical point

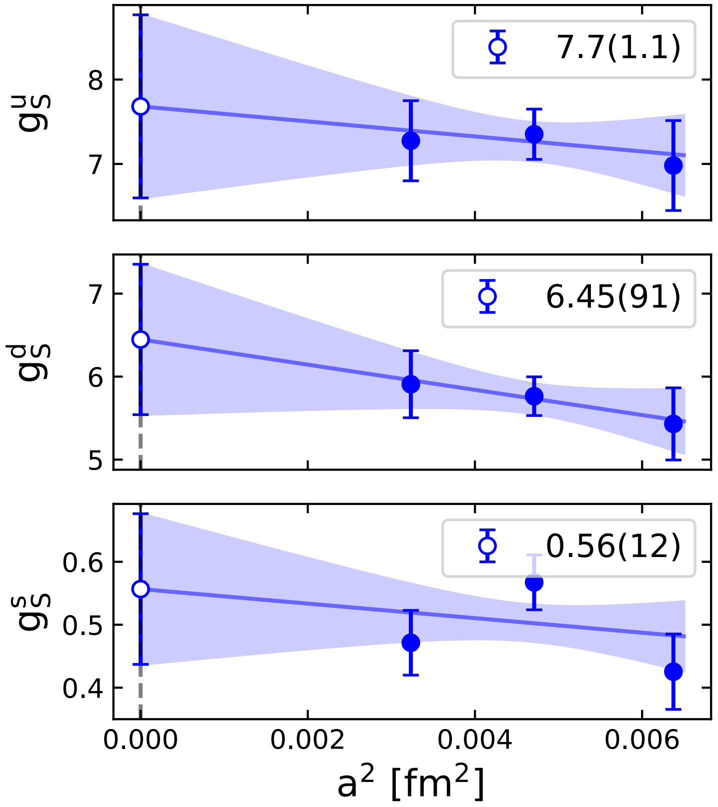


(2) Precision era of lattice QCD for charges including flavor diagonal

Nucleon scalar charge

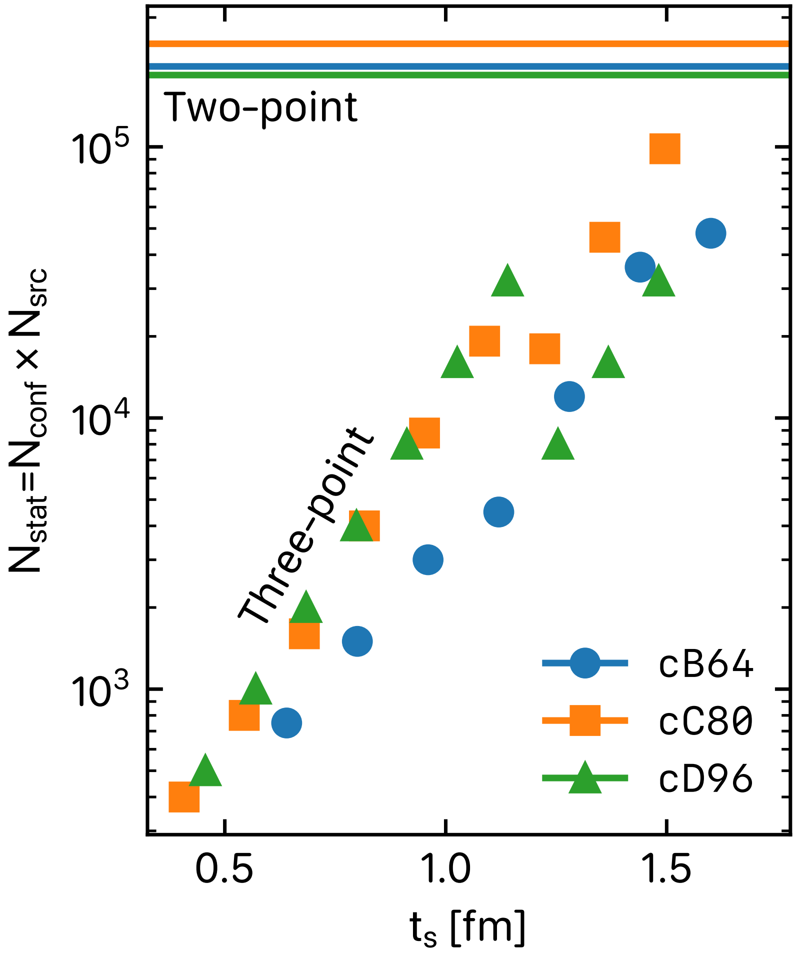
- Perform a similar analysis for the scalar charge - important input for direct dark matter searches



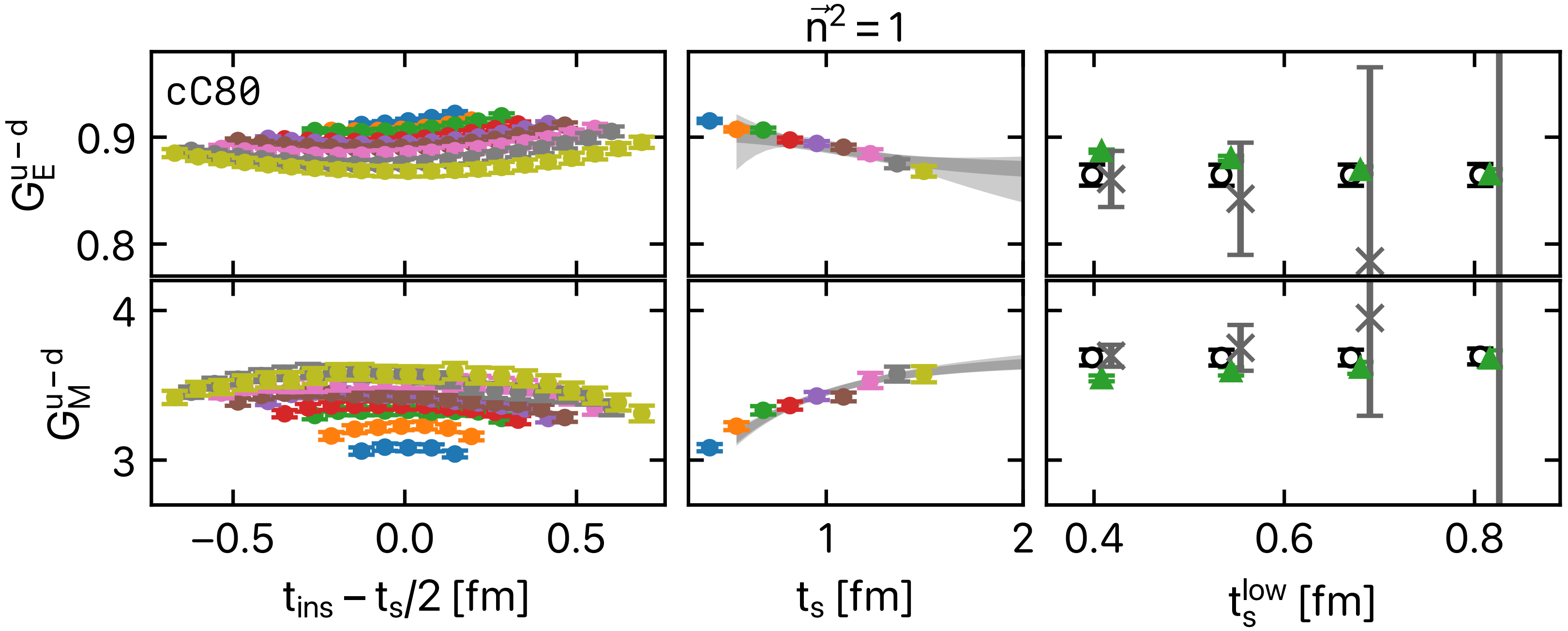


- Scalar charge is also directly related to the nucleon σ-terms or quark content $\sigma_q = m_q \langle N|\bar{q}q|N\rangle$

Electromagnetic form factors



- Aim for *constant statistical errors* over all values of t_s of a given ensemble to enable excited state analysis
- Large two-point functions needed for disconnected



▲ Summation

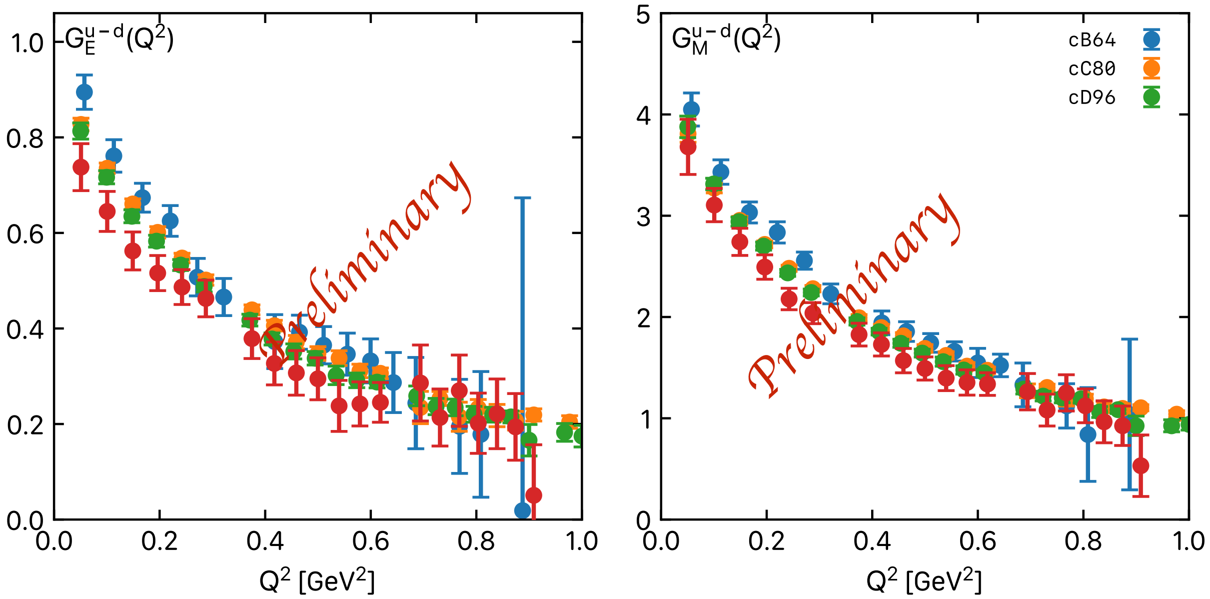
○ Two-state,
$E_1(0)=\varepsilon_1(0)$
$E_1(\vec{q})=\varepsilon_1(\vec{q})$

× Two-state,
$E_1(\vec{q})\neq\varepsilon_1(\vec{q})$
$E_1(0)\neq\varepsilon_1(0)$

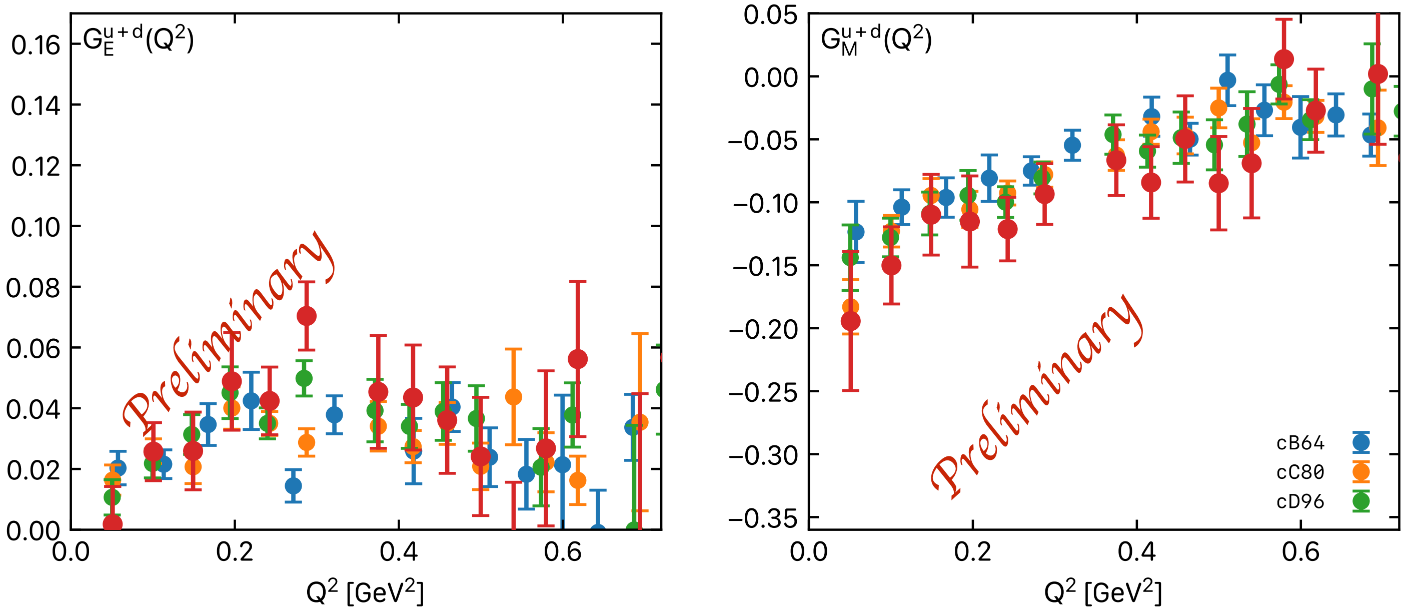
$$G_\Gamma(\vec{q};t_s,t_{ins})=\sum_{i=0}^{1}\sum_{j=0}^{1}c_{ij}e^{-E_i(0)(t_s-t_{ins})}e^{-E_j(\vec{q})t_{ins}}$$

$$G(\vec{q};t_s)=a_0(\vec{q})e^{-\varepsilon_0(\vec{q})t_s}+a_1(\vec{q})e^{-\varepsilon_1(\vec{q})t_s}$$

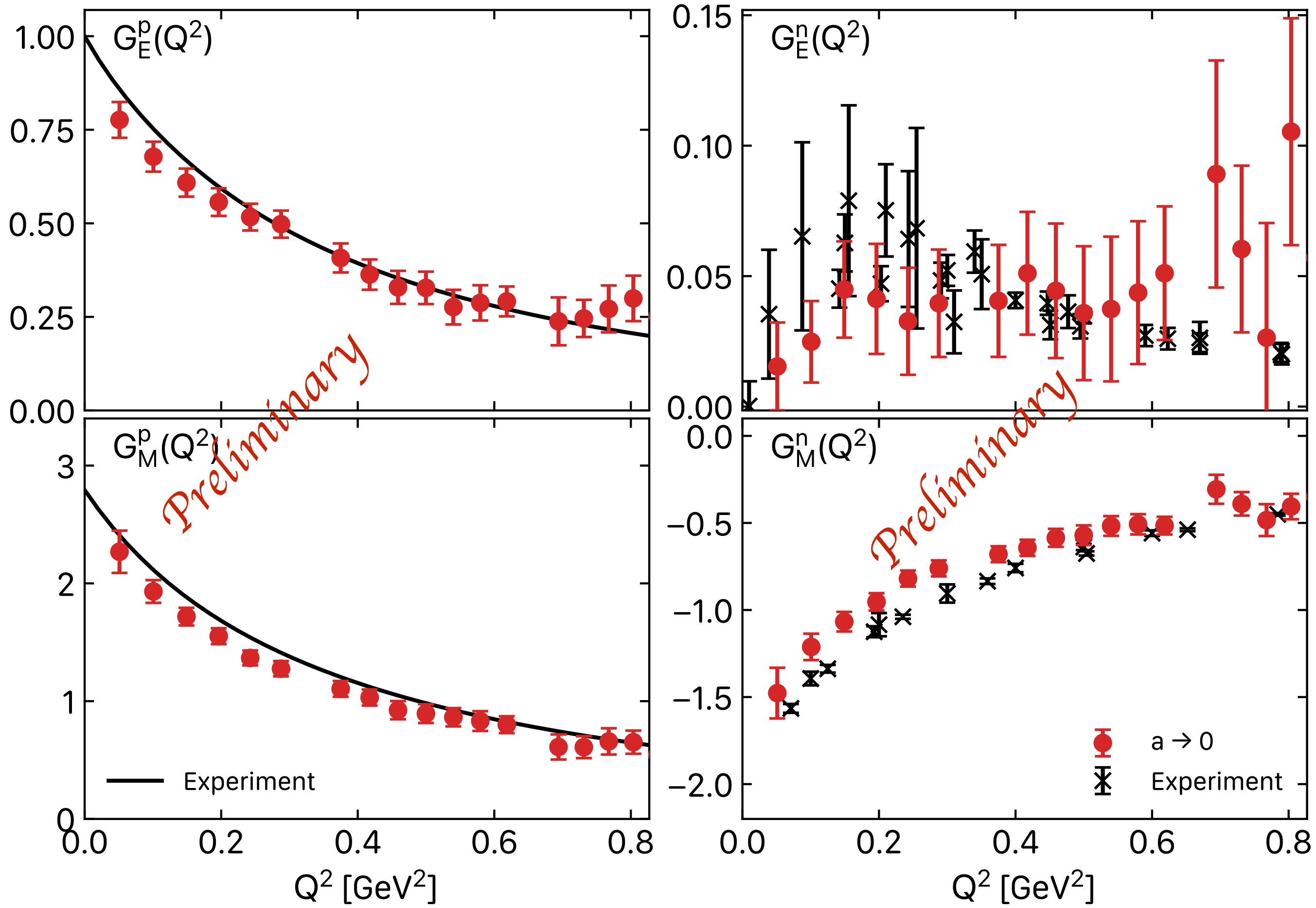
Continuum limit



Disconnected (u+d)



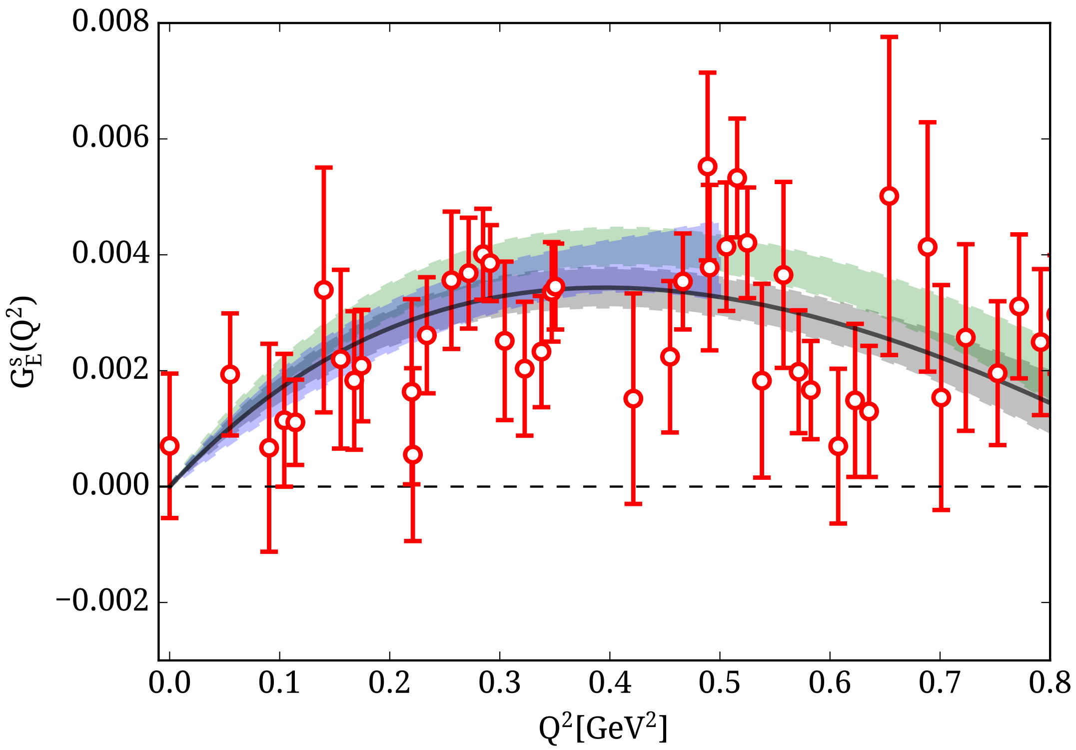
Proton and Neutron EM form factors compared to experiment



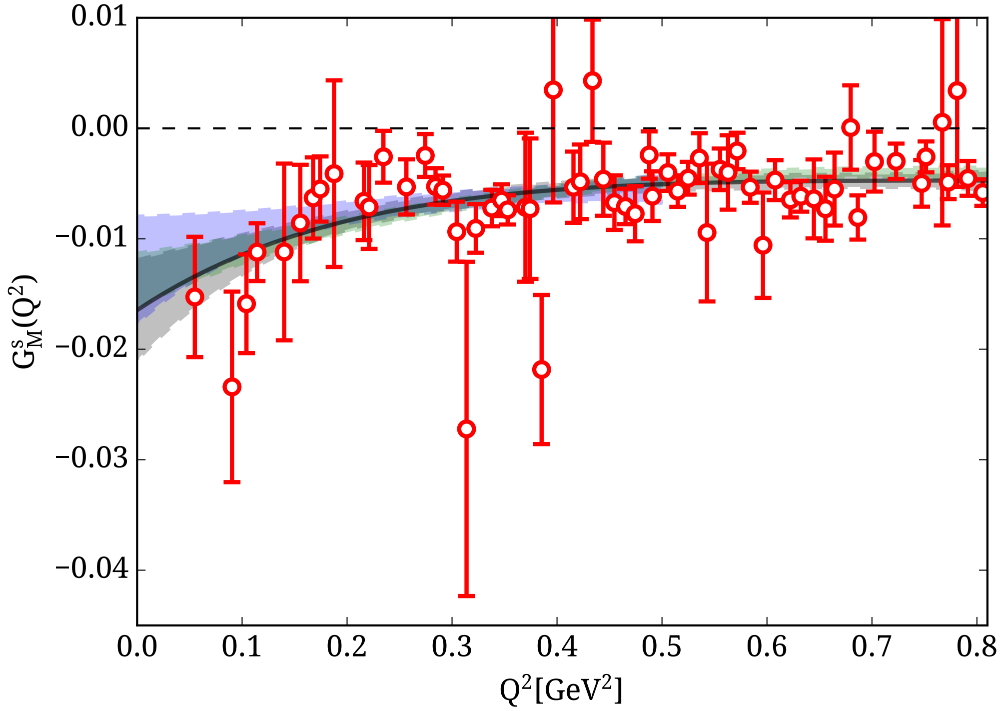
Strangeness of the nucleon

- Sea quark effects can be accurately determined for EM form factors —> provide precise input to experiments

B64-ensemble: 64^3 x 128, a~0.08 fm

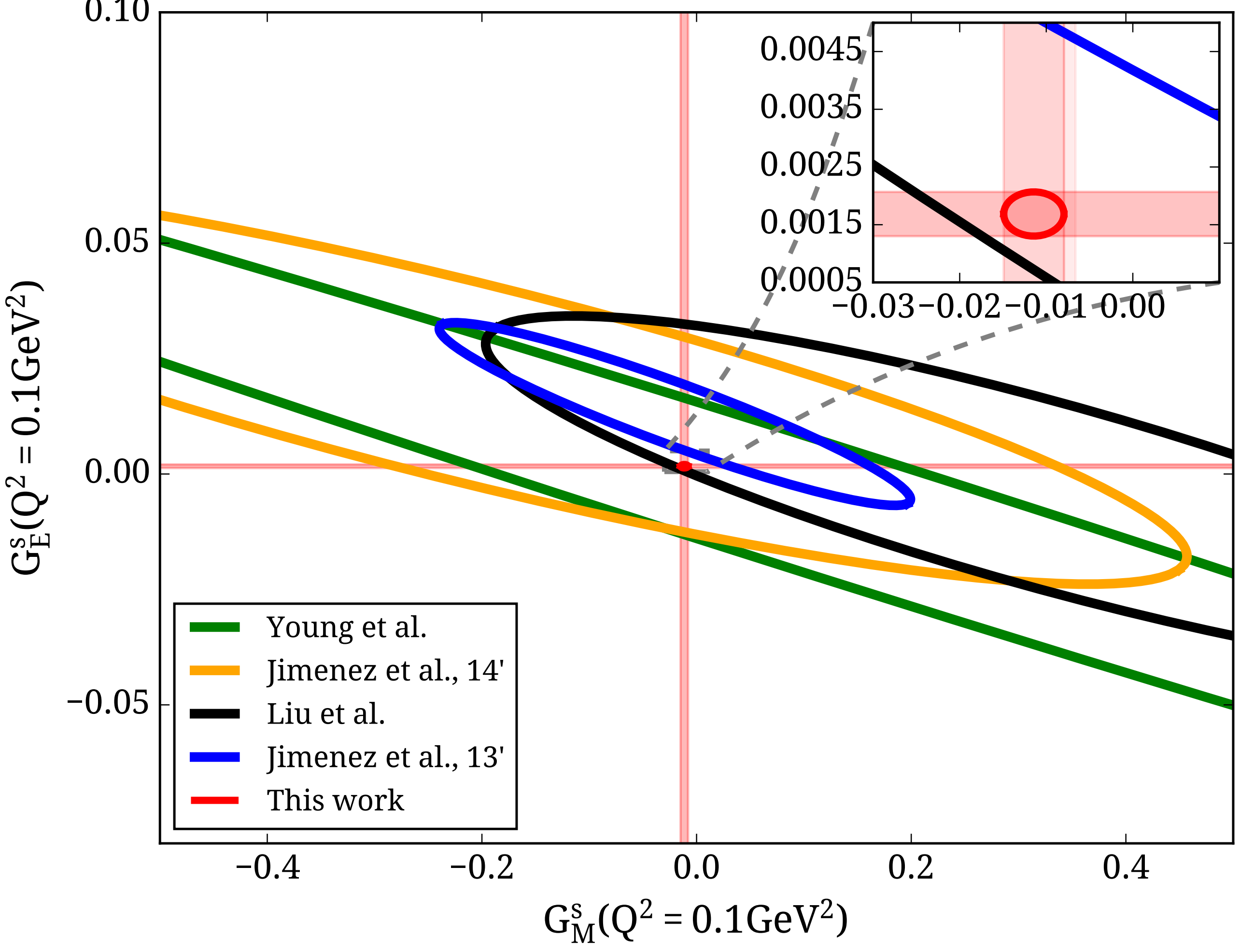


Negative magnetic moment



Significant input to experimental searches

e.g Q-Weak, SolID, etc



C. A. *et al.*, Phys. Rev. D 101 (2020) 031501, arXiv:1909.10744

Axial and induced pseudoscalar form factors

Extract from lattice QCD →

$$\boxed{\langle N(p',s')|A_\mu|N(p,s)\rangle} = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N} G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\boxed{\langle N(p',s')|P_5|N(p,s)\rangle} = G_5(Q^2)\bar{u}_N(p',s')\gamma_5 u_N(p,s)$$

$q^2=-Q^2$

- Check the PCAC : $\partial^\mu A_\mu = 2m_q P, \quad m_q = m_u = m_d$

$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)$$

Goldberger-Treiman relation

- Relate to pion field: $G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_q}\frac{G_{\pi NN}(Q^2)}{m_\pi^2 + Q^2}$

$$\longrightarrow \quad G_P(Q^2) = \frac{4m_N^2}{Q^2 + m_\pi^2} G_A(Q^2) \qquad m_N G_A(Q^2) = F_\pi G_{\pi NN}(Q^2)$$

- At the pion pole we get the pion nucleon coupling: $g_{\pi NN} \equiv G_{\pi NN}(Q^2 = -m_\pi^2)$

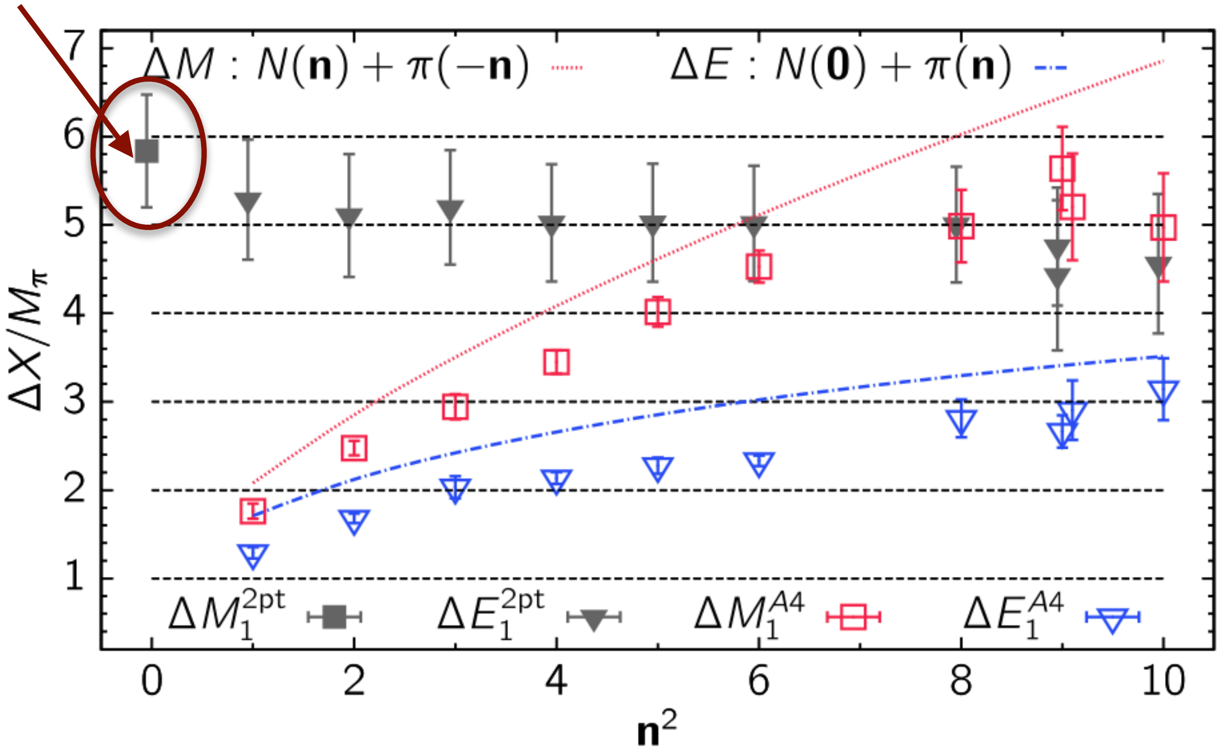
$$\lim_{Q^2 \to -m_\pi^2} (Q^2 + m_\pi^2) G_P(Q^2) = 4 m_N F_\pi g_{\pi NN}$$

and $g_{\pi NN} = m_N G_A(-m_\pi^2)/F_\pi \xrightarrow{m_\pi \to 0} \frac{m_N}{F_\pi} g_A$

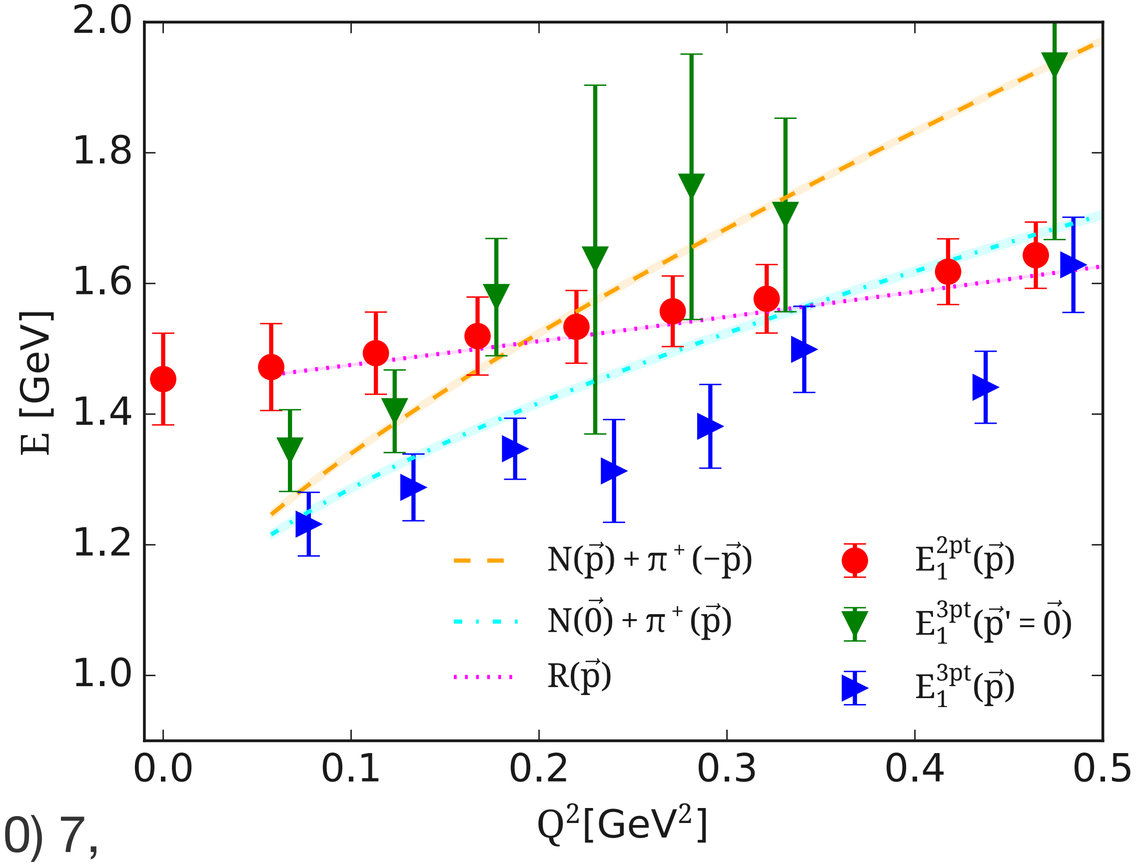
Analysis of excited states

We allow the first excited state to be different in the two- and three-point functions, O. Baer, Phys. Rev. D 99, 054506 (2019).

✳Extract first excited state from the zero component of the axial-vector current



ΔM₁~800 MeV

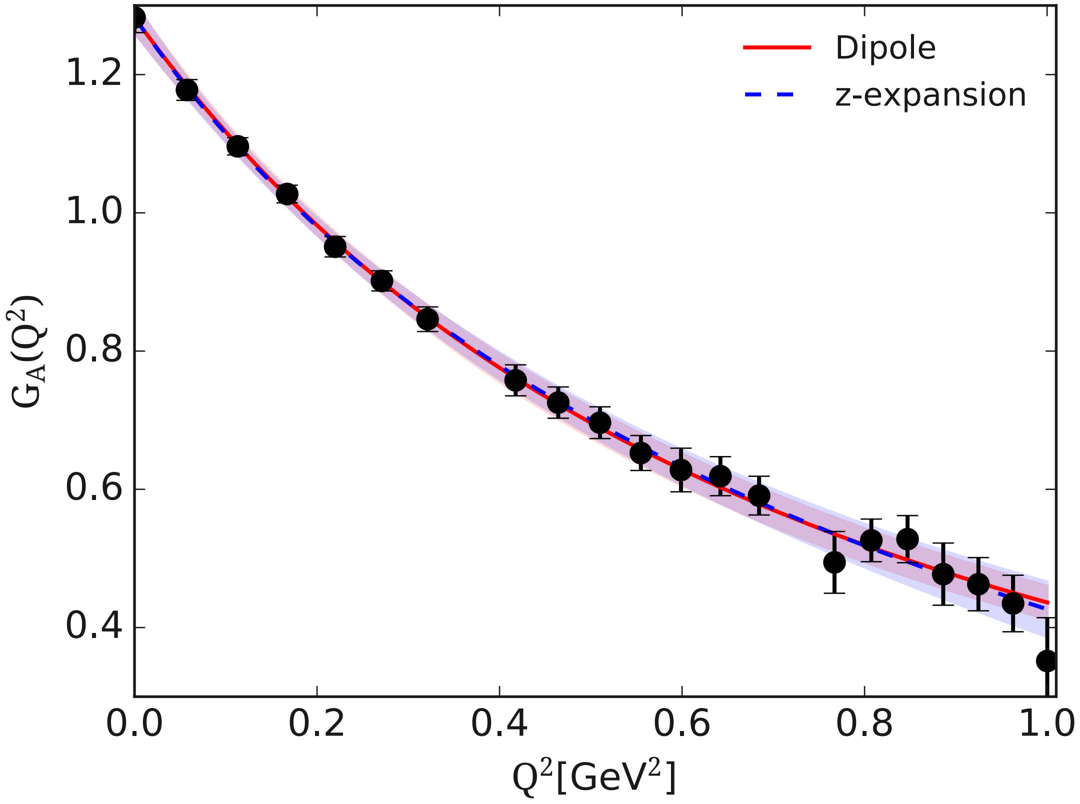


PNDME: Yong-Chull Jang et al., Phys. Rev. Lett. 124 (2020) 7, 072002, 1905.06470

Axial form factor using the B64 ensemble

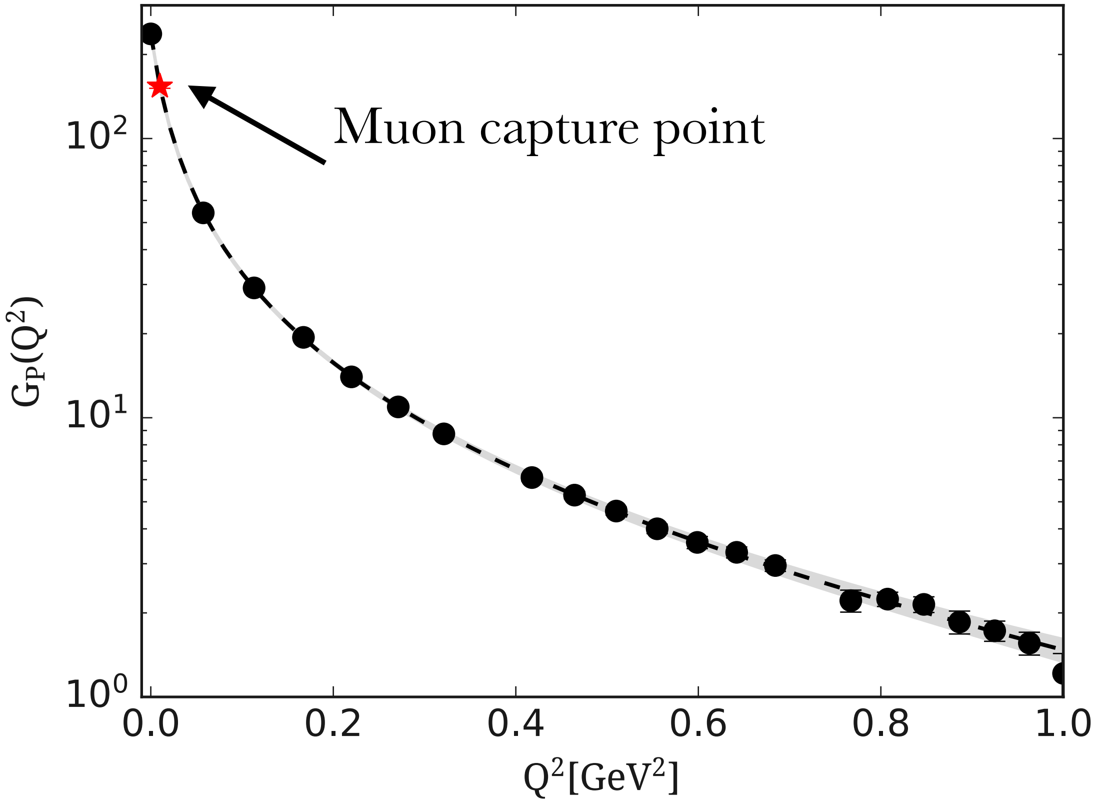
- Fit the Q^2 dependence using a dipole and z-expasion

$$G_A(Q^2) = \frac{g_A}{(1+Q^2/m_A^2)^2} \quad \text{or} \quad G(Q^2) = \sum_{k=0}^{k_{\max}} a_k z^k(Q^2) \text{ and } z(Q^2) = \frac{\sqrt{t_{\rm cut}+Q^2}-\sqrt{t_{\rm cut}}}{\sqrt{t_{\rm cut}+Q^2}+\sqrt{t_{\rm cut}}}$$

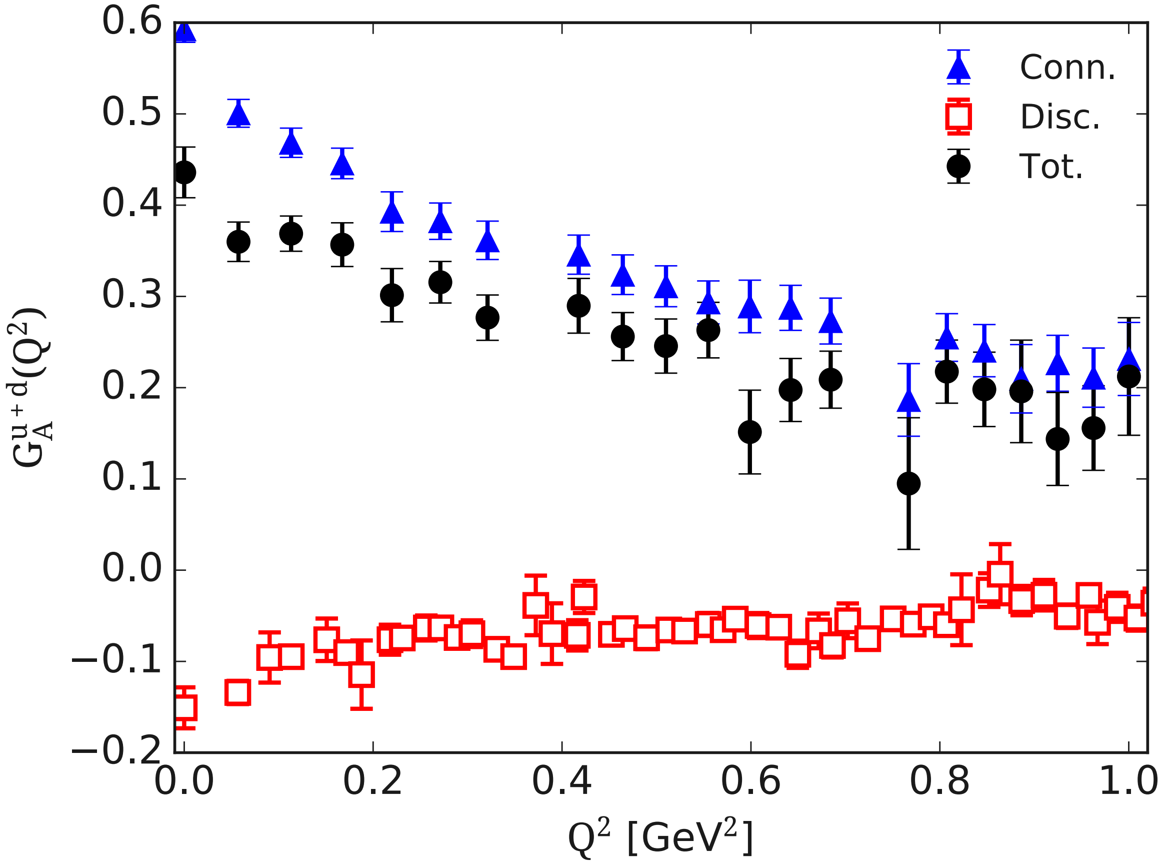


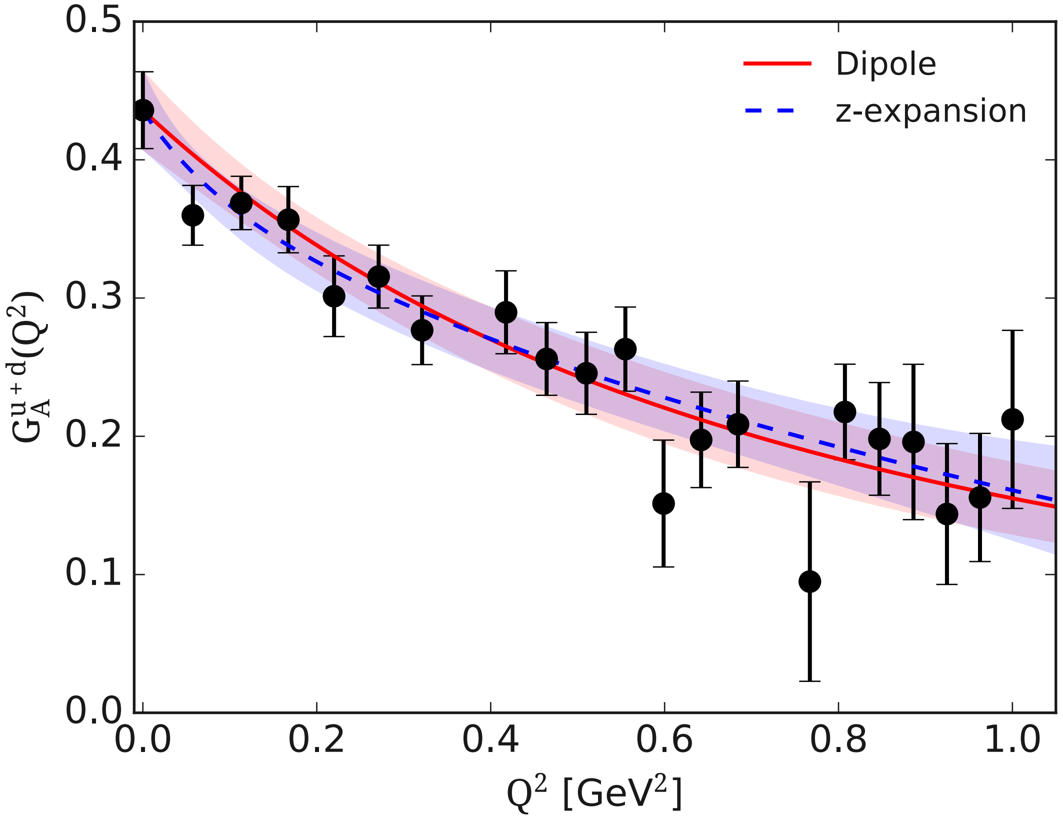
Using the z-expansion we find:
m_A= 1.169(72)(27) GeV
$\sqrt{\langle r_A \rangle^2}$ =0.585(36)(14) fm

Use PCAC to get G_P from G_A



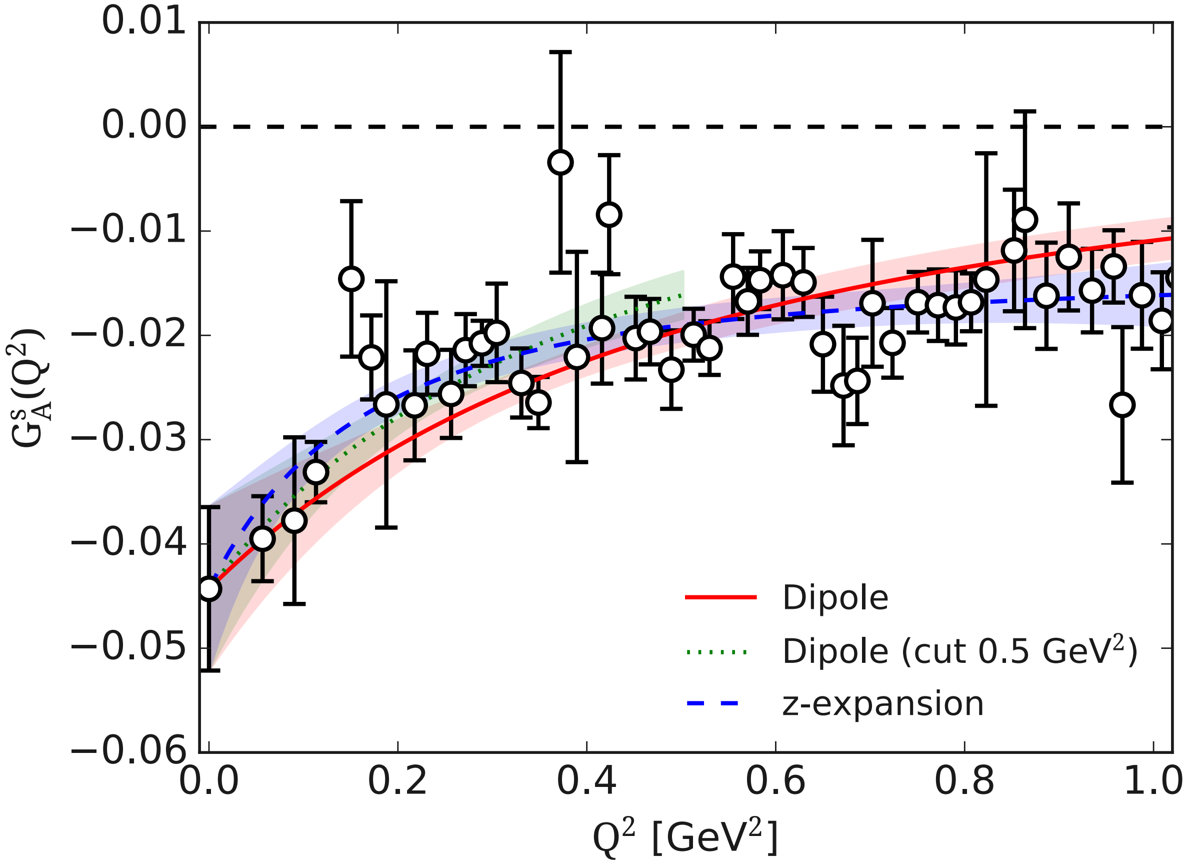
Quark flavour decomposition of G_A



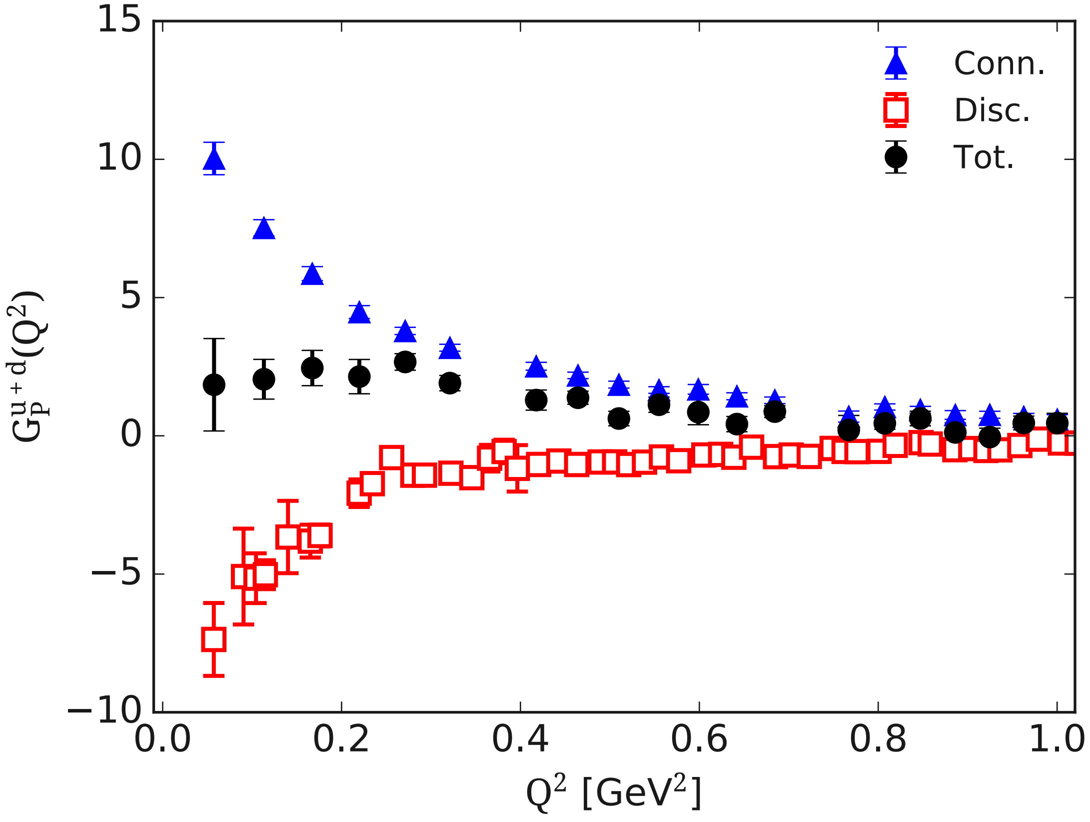


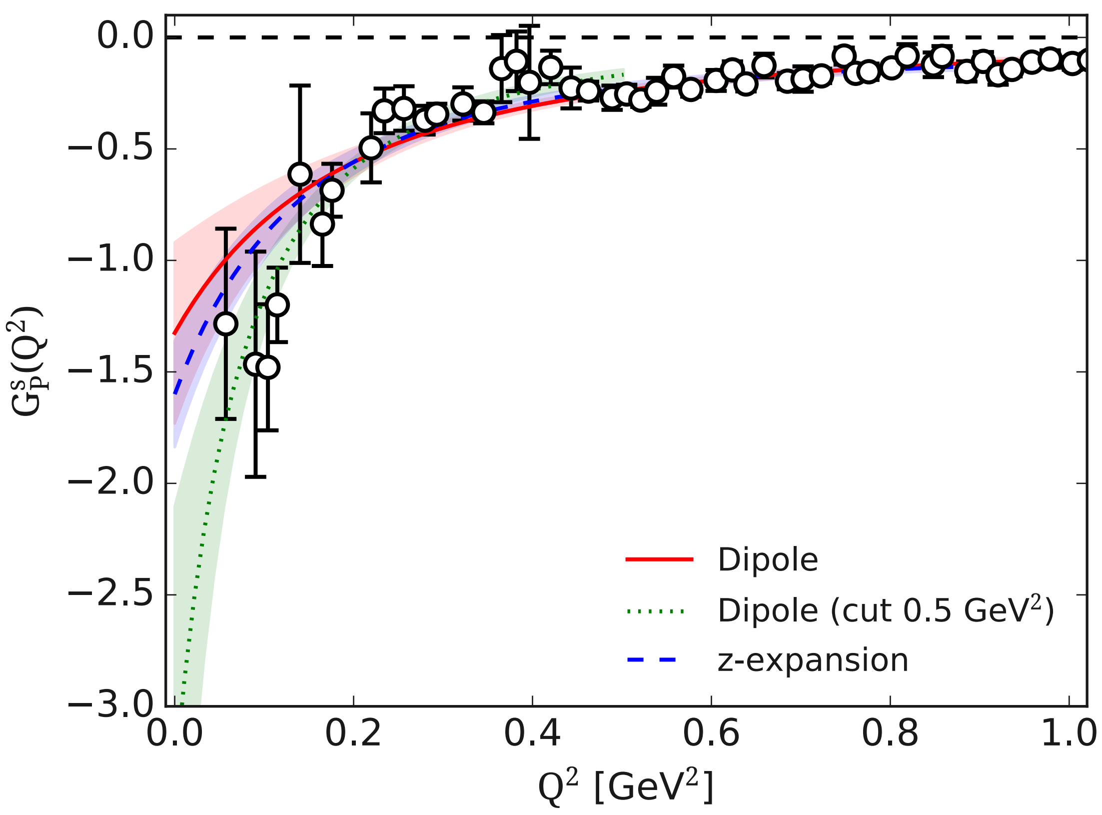
Contribution of disconnected diagrams

- Significant contribution of disconnected to u+d combination
- Negative disconnected contribution: subtracts from connected
- Good signal for strange contribution: clearly non-zero and negative



Quark flavour decomposition of G_P

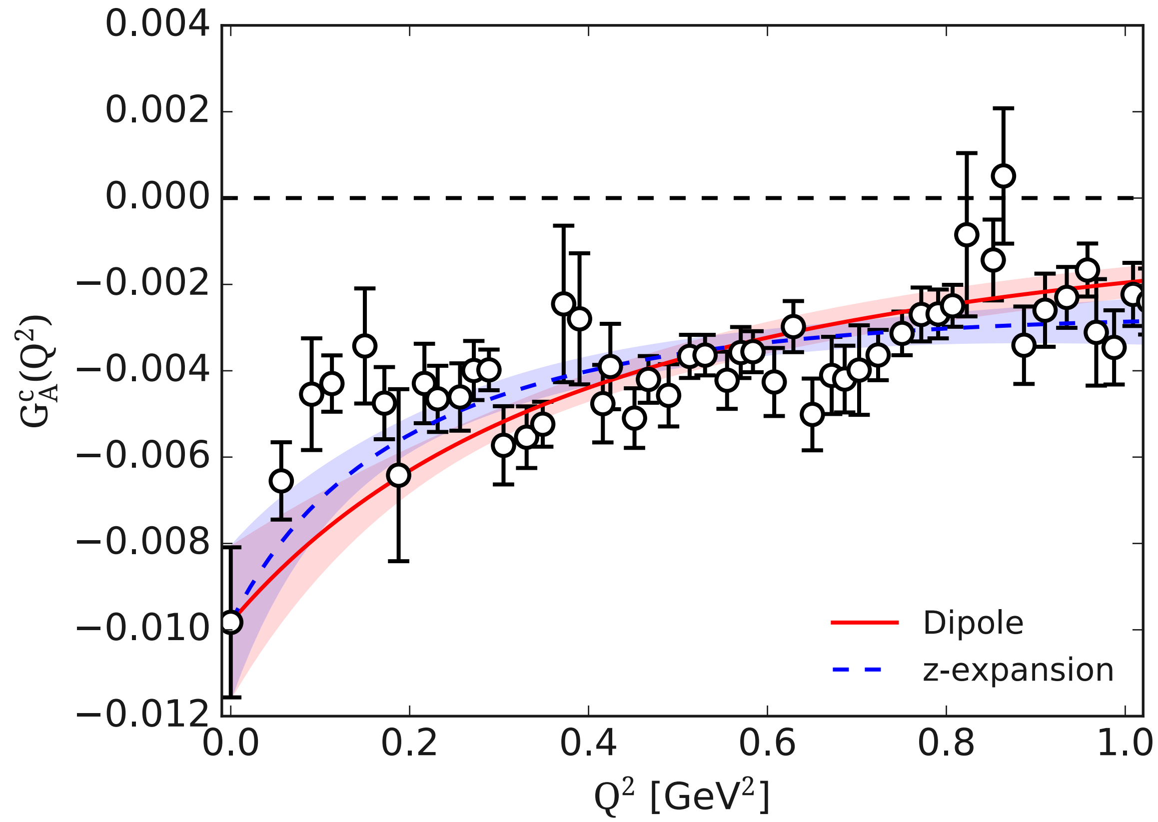


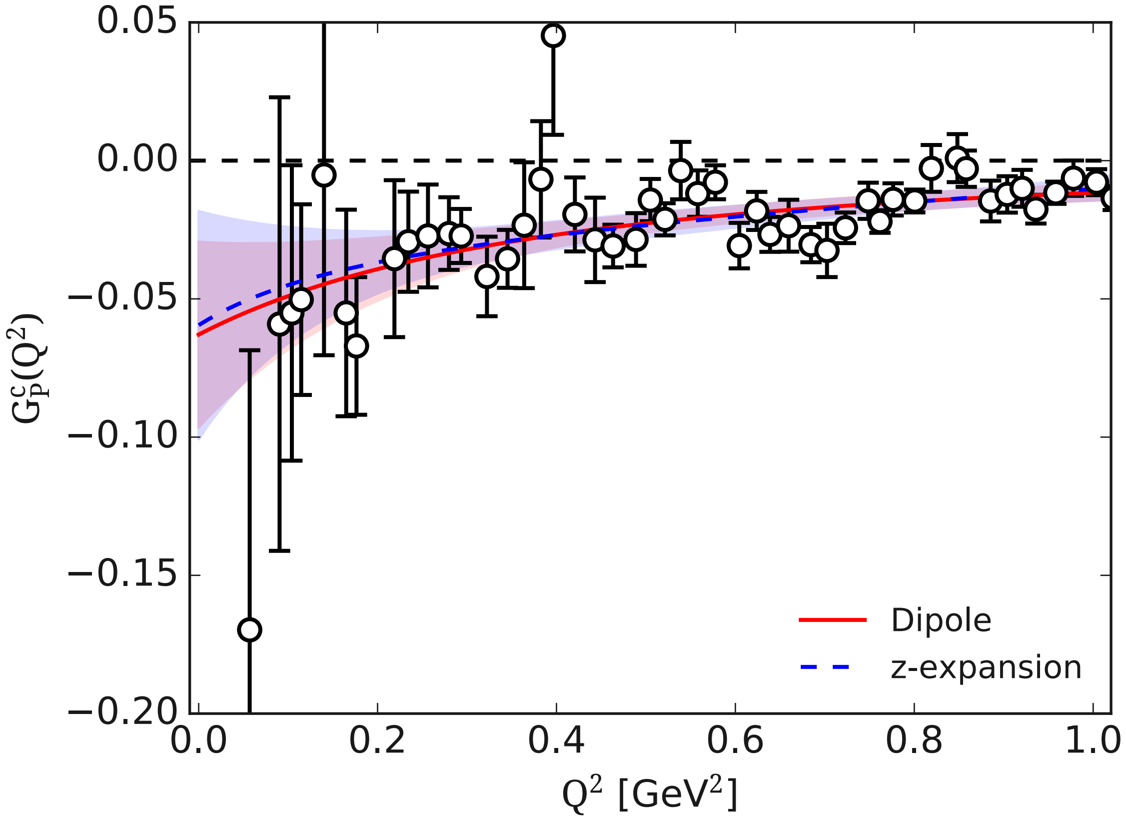


Contribution of disconnected diagrams

- Significant contribution of disconnected to u+d combination
- Negative large disconnected contribution: subtracts from connected
- Large negative strange quark contribution

Charm quark contributions

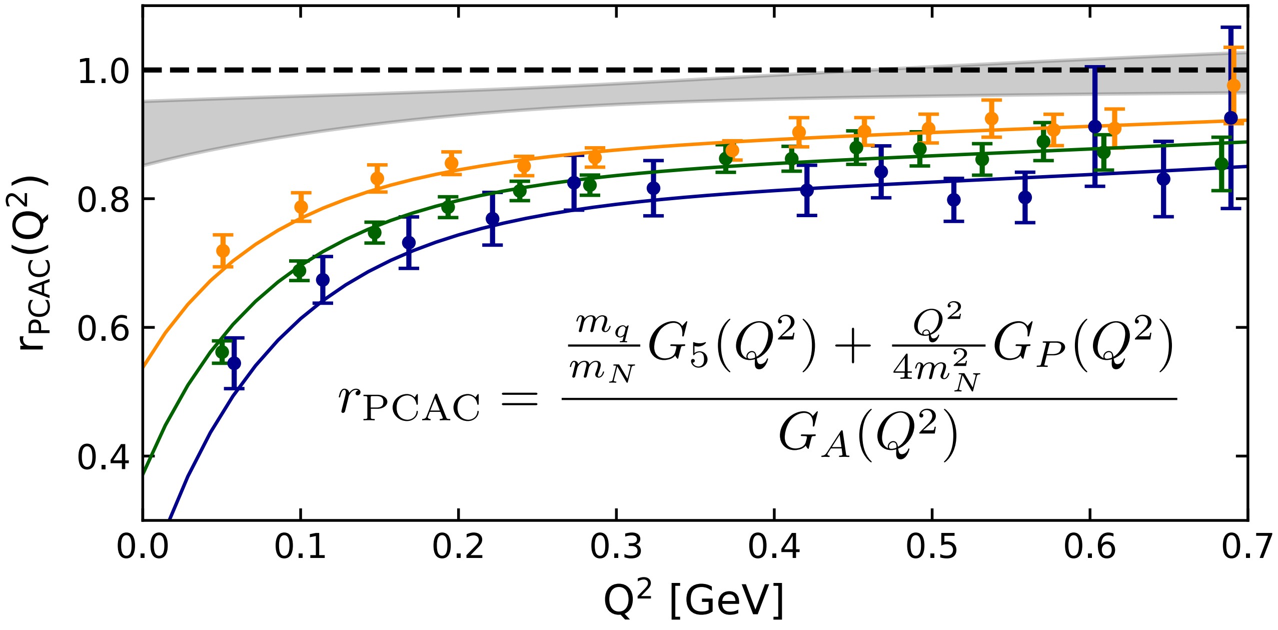




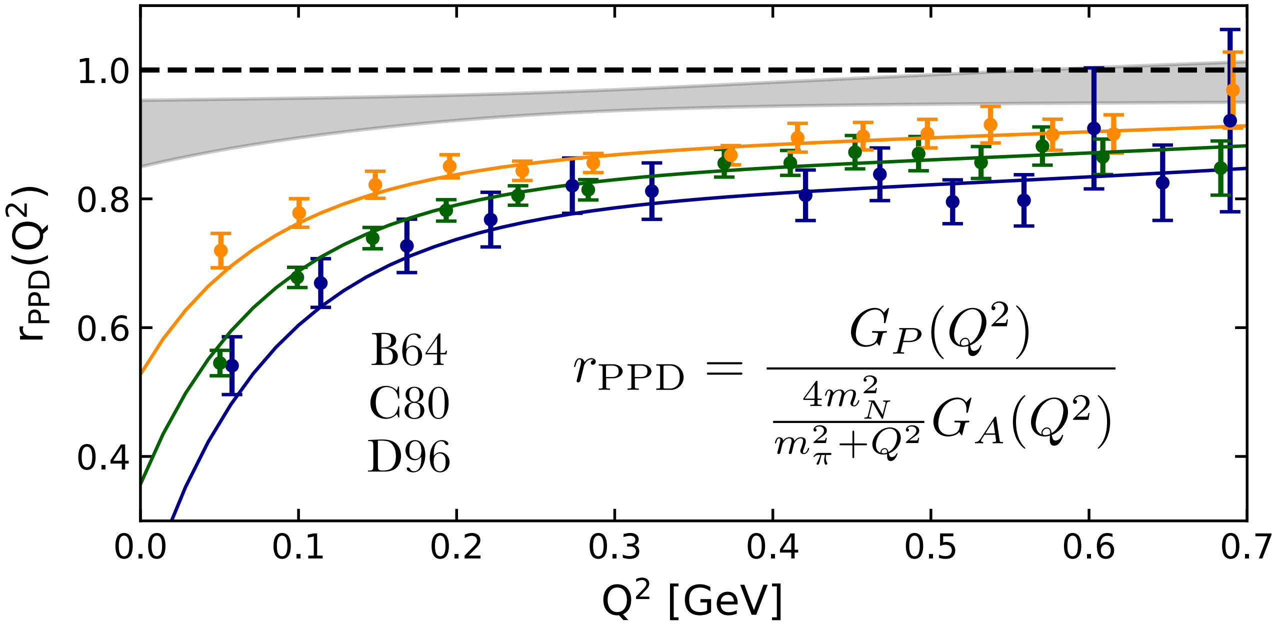
- Clearly non-zero negative contributions for both axial form factors

(3) Sea quarks effects clearly seen on the form factors

PCAC and pion pole dominance (PPD) at the continuum limit

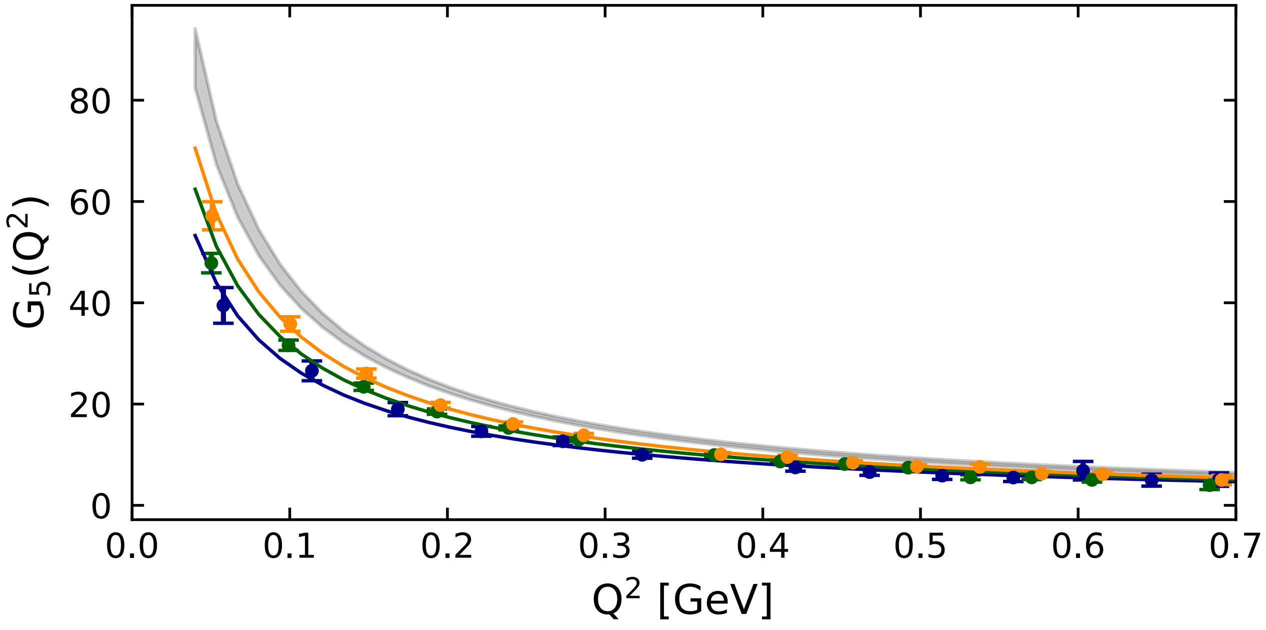


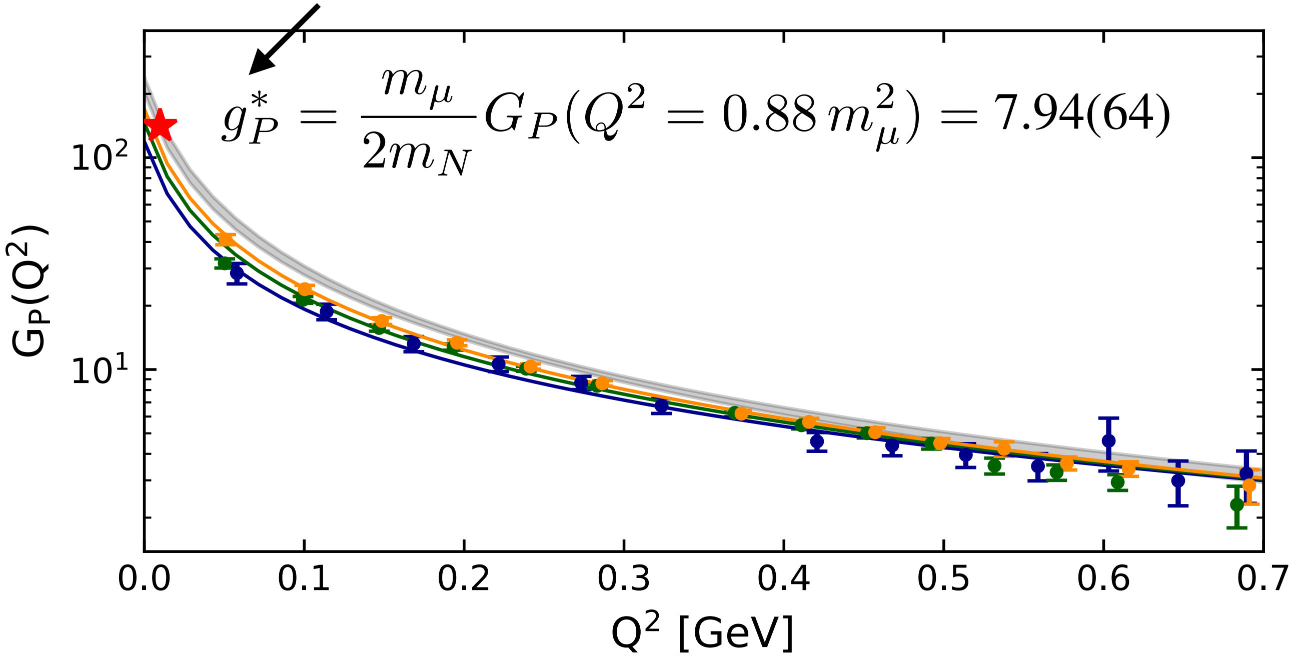
$$r_{\rm PCAC} = \frac{\frac{m_q}{m_N} G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$



$$r_{\rm PPD} = \frac{G_P(Q^2)}{\frac{4m_N^2}{m_\pi^2+Q^2} G_A(Q^2)}$$

In agreement with other lattice studies
e.g. RQCD and PNDME





$$g_P^* = \frac{m_\mu}{2m_N} G_P(Q^2 = 0.88\, m_\mu^2) = 7.94(64)$$

(4) Both PCAC and PPD are recovered in the continuum limit

Conclusions

(1) Lattice QCD results converge to the experimental values of e.g. nucleon axial charge, form factors, etc

(2) A number of accurate results with controlled systematics on less known quantities provide valuable input for searches of new physics, e.g nucleon scalar and tensor charges including flavor diagonal, strangeness, …

(3) Lattice QCD reveals clearly strange and charm effects in the nucleon Direct computation of PDFs is a very active field

(4)Continuum limit is important for recovering important relations

(5) Same approaches can be used for other baryons and for non-diagonal matrix elements

Extended Twisted Mass Collaboration

Backup slides

Computational resources