Partonic distributions from Lattice QCD

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

NATIONAL SCIENCE CENTRE POLAND

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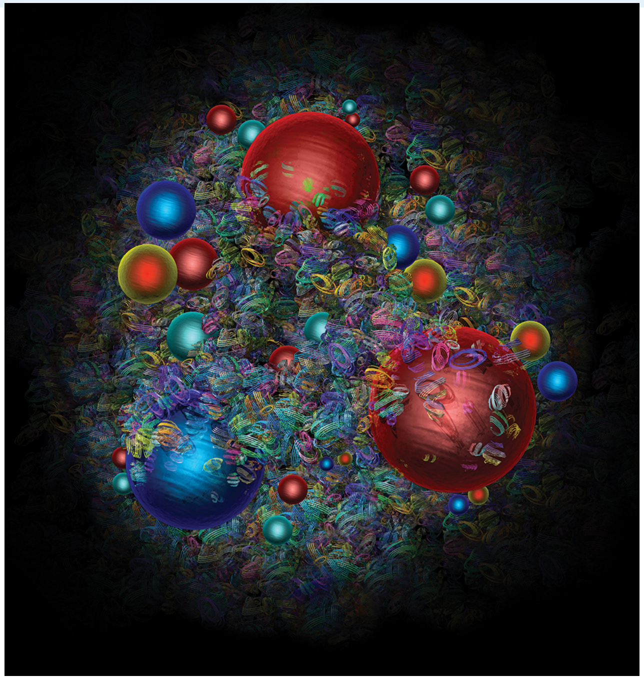
Outline:

Introduction
PDFs from lattice:
– how to access
– status
GPDs
Prospects/conclusion

Many thanks to my Collaborators for work presented here:

C. Alexandrou, M. Bhat, S. Bhattacharya, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, J. Miller, S. Mukherjee, A. Scapellato, F. Steffens, Y. Zhao



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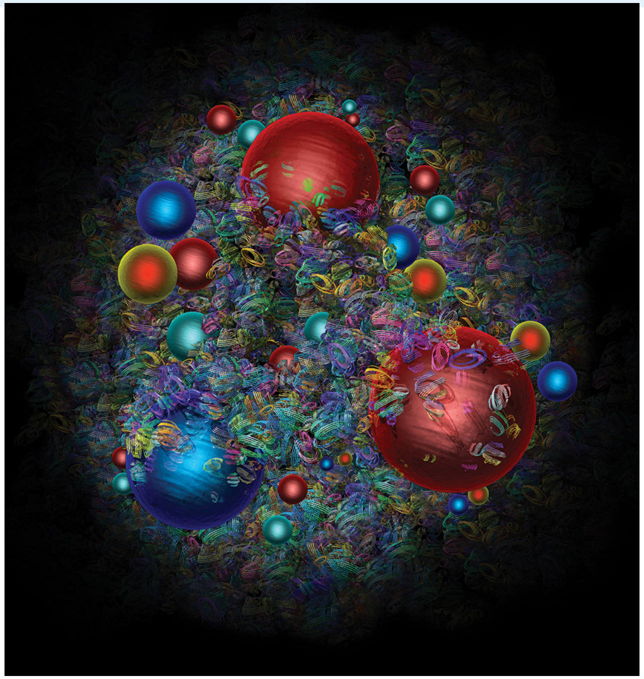

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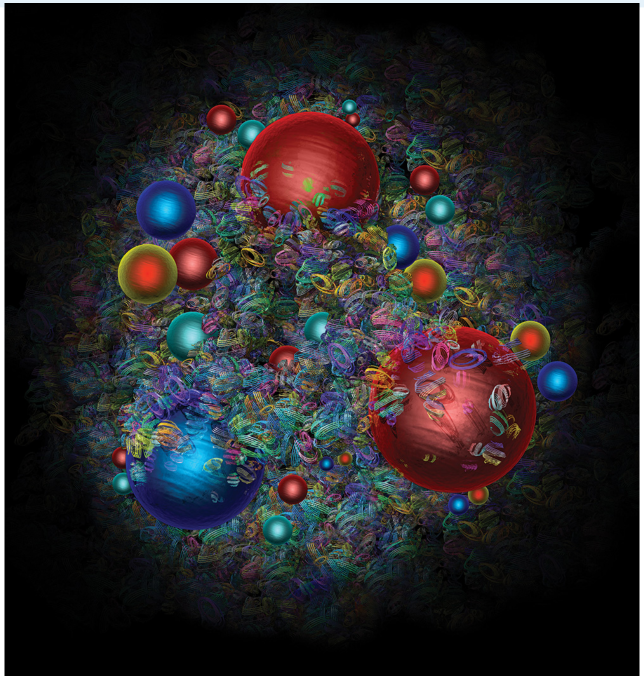
- For this, we need to probe the 3D structure.
- Thus, we need to access new kinds of functions: GPDs, TMDs.
- Also higher-twist is of growing importance for the full picture.
- Both theoretical and experimental input needed.



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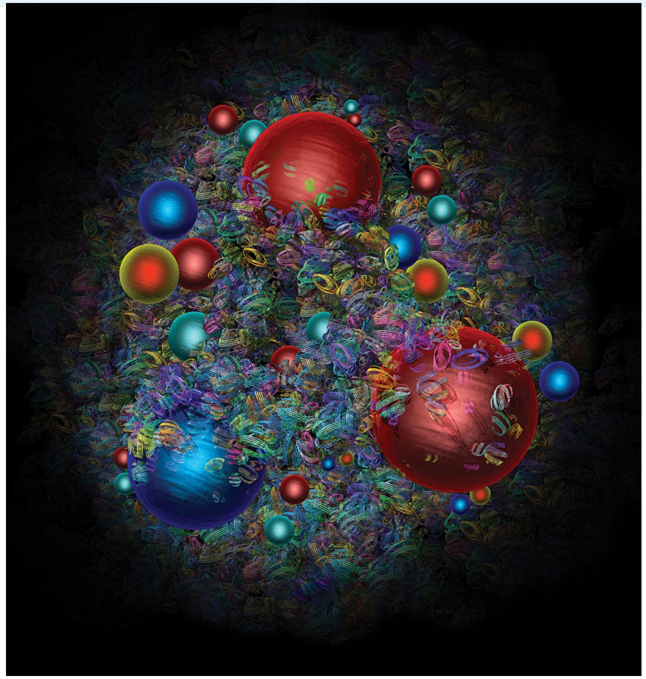
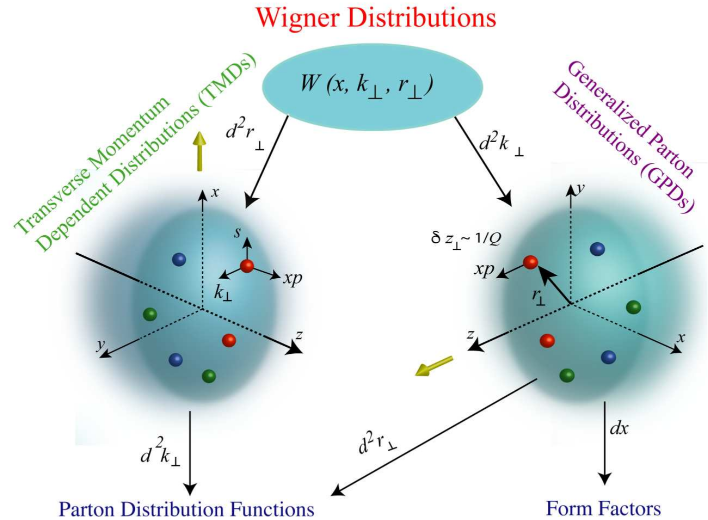


Lattice can provide *qualitative* and eventually *quantitative* knowledge of different functions and their moments:

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function



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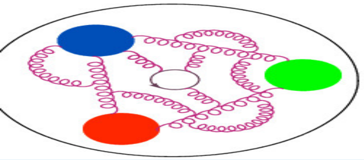

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     - $\star$ quarks $\rightarrow$ sites
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     typical lattice parameters:
     $L/a = [32, 96]$, $a \in [0.04, 0.15]$ fm, $m_\pi \in [135, 500]$ MeV
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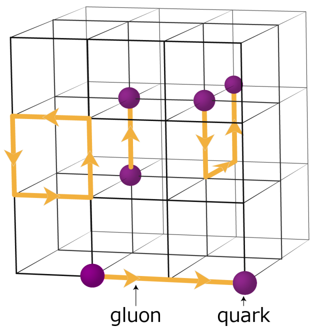

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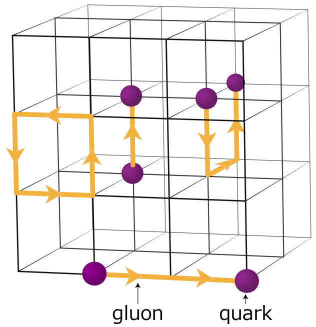

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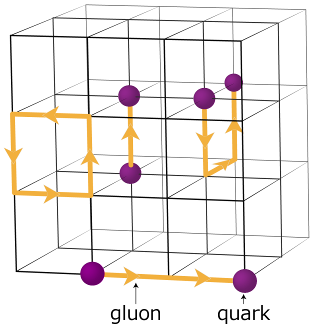

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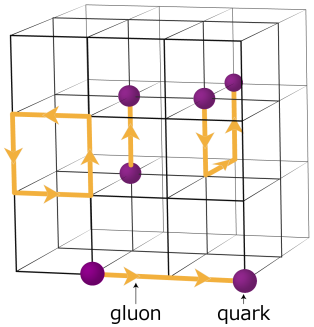

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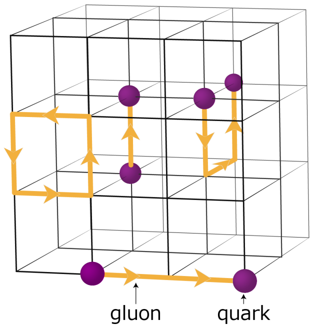

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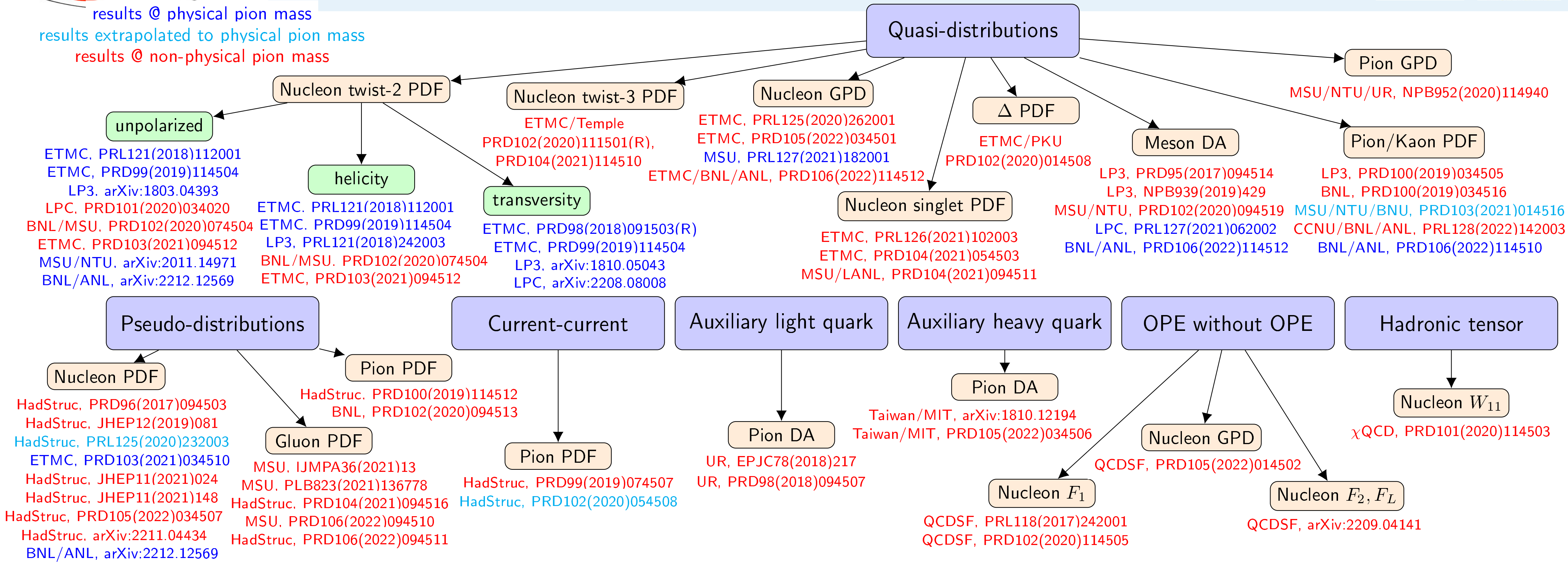
  3. Optimized computation setup.
  4. A lot of computing time!
  5. Ingenious analysis techniques, with inputs from perturbation theory.

Lattice PDFs/GPDs: dynamical progress



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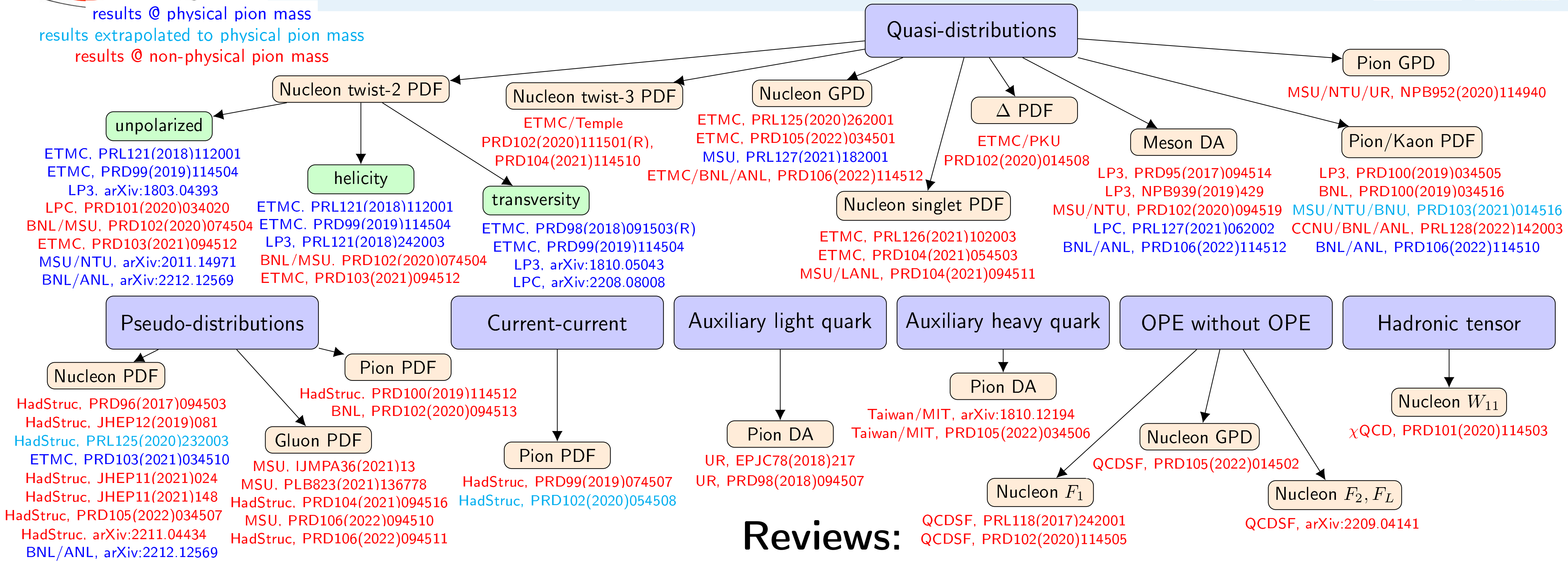

Reviews:

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

Quasi-PDFs

Quasi-distribution approach:

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002

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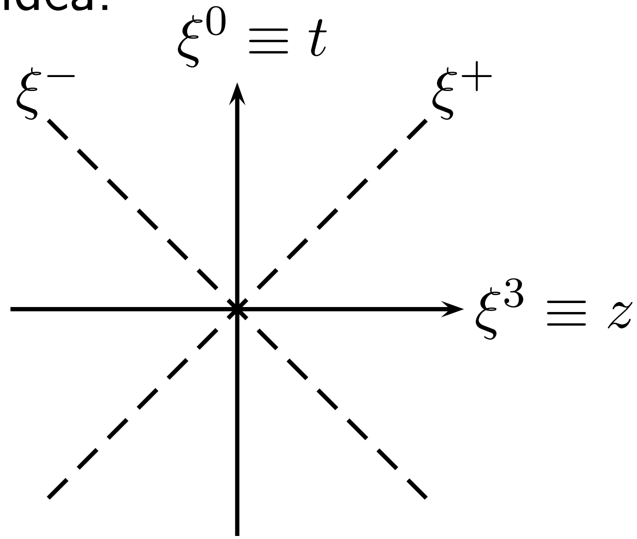
Main idea:



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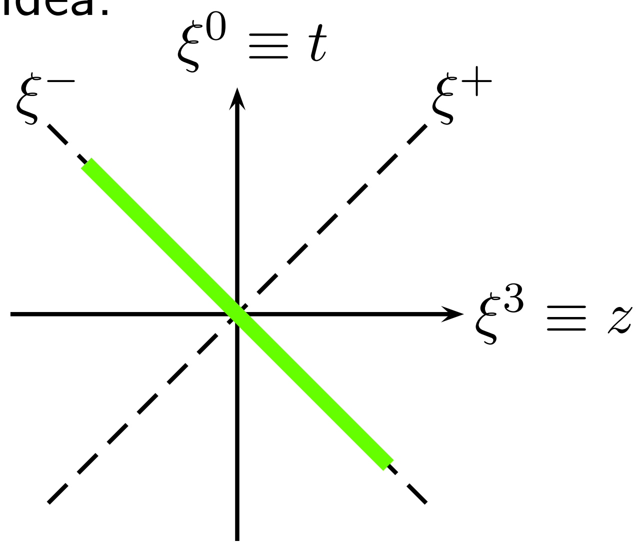

Correlation along the ξ^--direction:

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle$$

$|N\rangle$ – nucleon at rest in the light-cone frame

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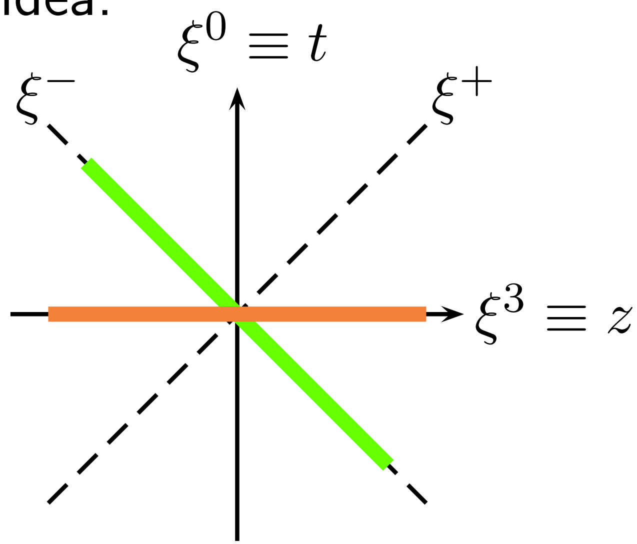

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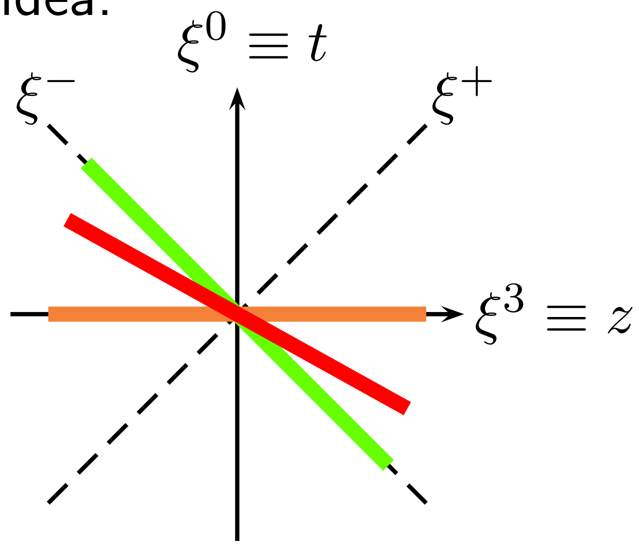

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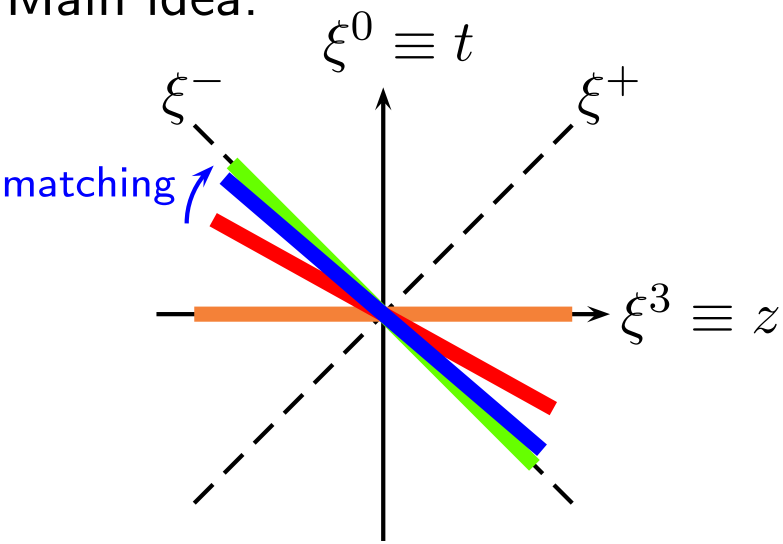

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Matching (Large Momentum Effective Theory (LaMET)

X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, Sci.China Phys.Mech.Astron. 57 (2014) 1407

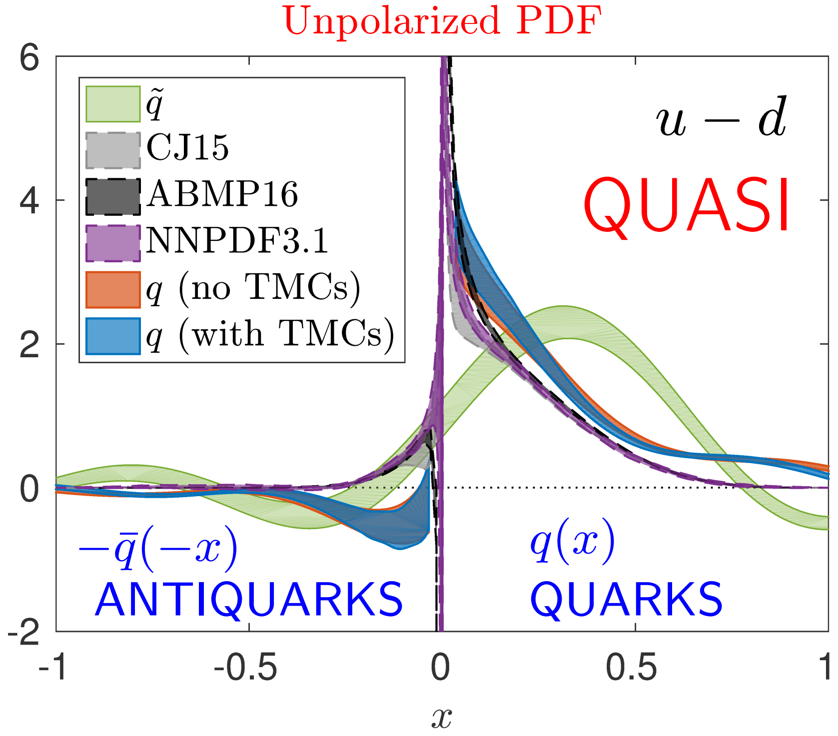
$\rightarrow$ brings quasi-distribution to the light-cone distribution, up to power-suppressed effects:

$$\underbrace{\tilde{q}(x,\mu,P_3)}_{\text{quasi-PDF}} = \int_{-1}^{1} \frac{dy}{|y|} \underbrace{C\left(\frac{x}{y}, \frac{\mu}{P_3}\right)}_{\text{pert.kernel}} \underbrace{q(y,\mu)}_{\text{PDF}} + \underbrace{\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)}_{\text{higher-twist effects}}$$

Current state-of-the-art: unpolarized PDFs

ETMC, Phys. Rev. Lett. 121 (2018) 112001
ETMC, Phys. Rev. D 99 (2019) 114504

$Q^2 = 4$ GeV^2



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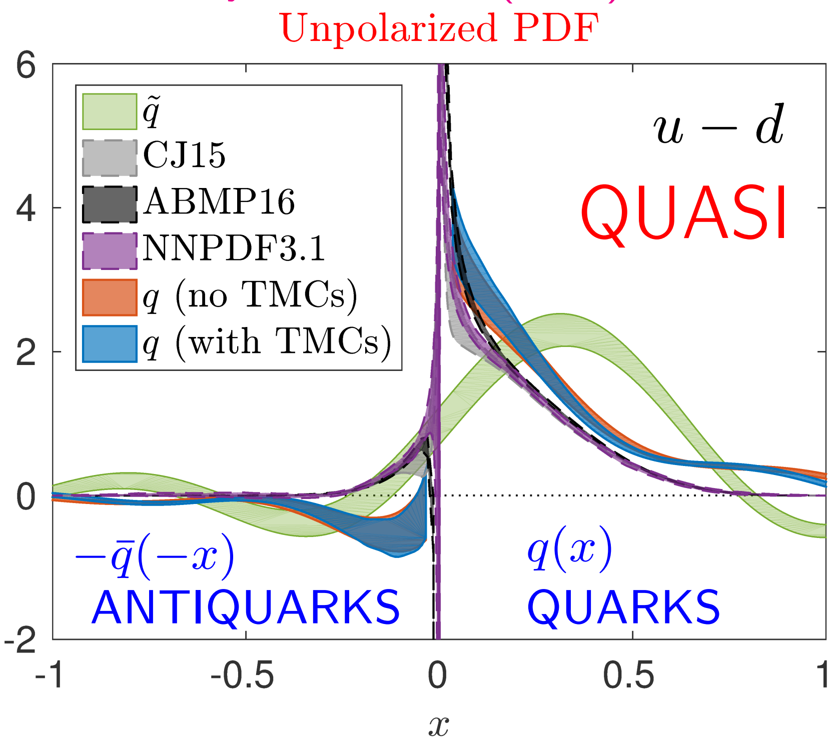

Qualitative agreement with pheno

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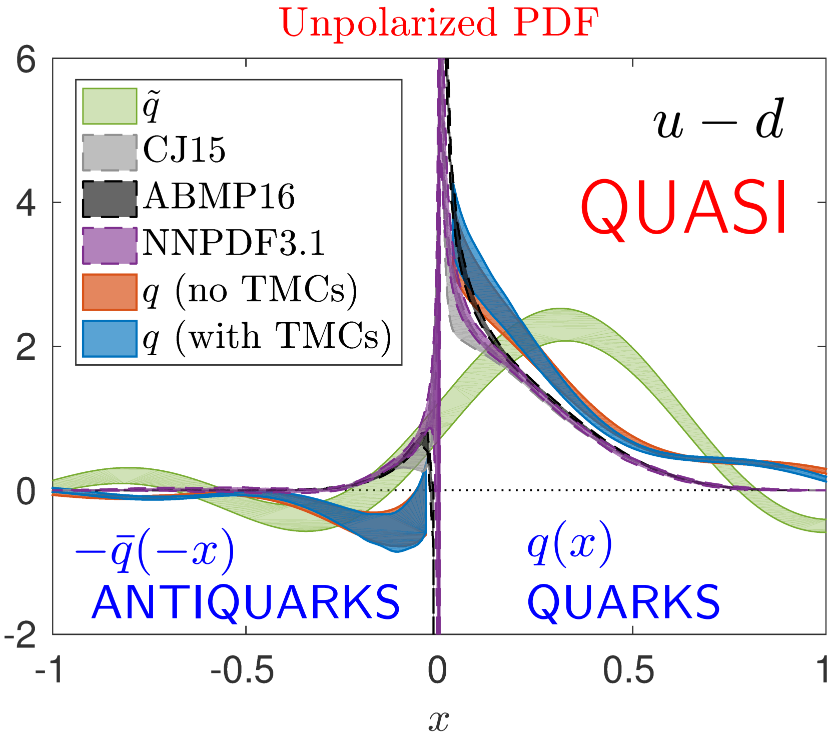

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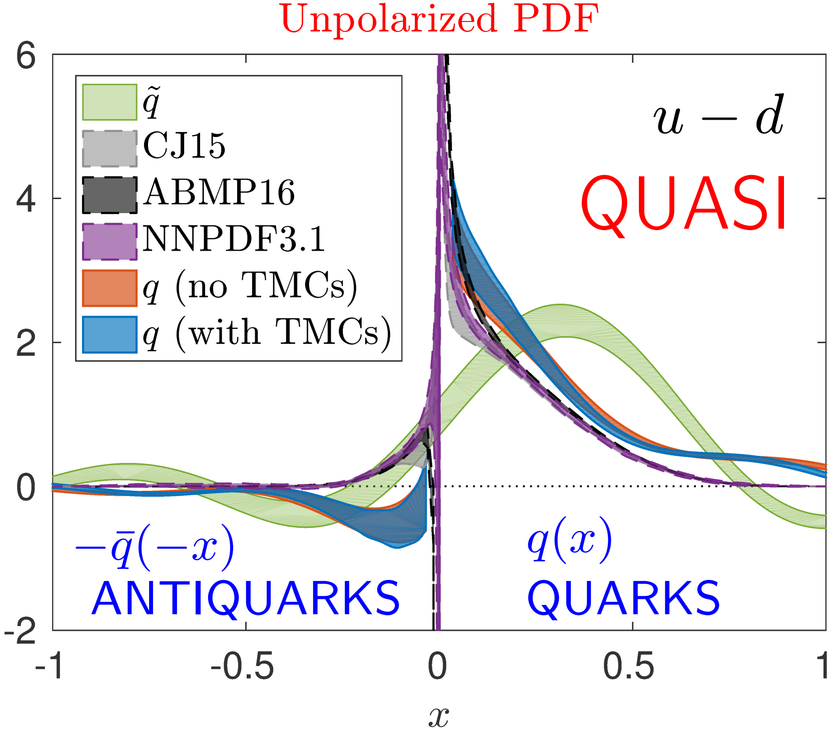

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cut-off effects
truncation (matching)
higher-twist effects
reconstruction of x-dep.
finite volume effects

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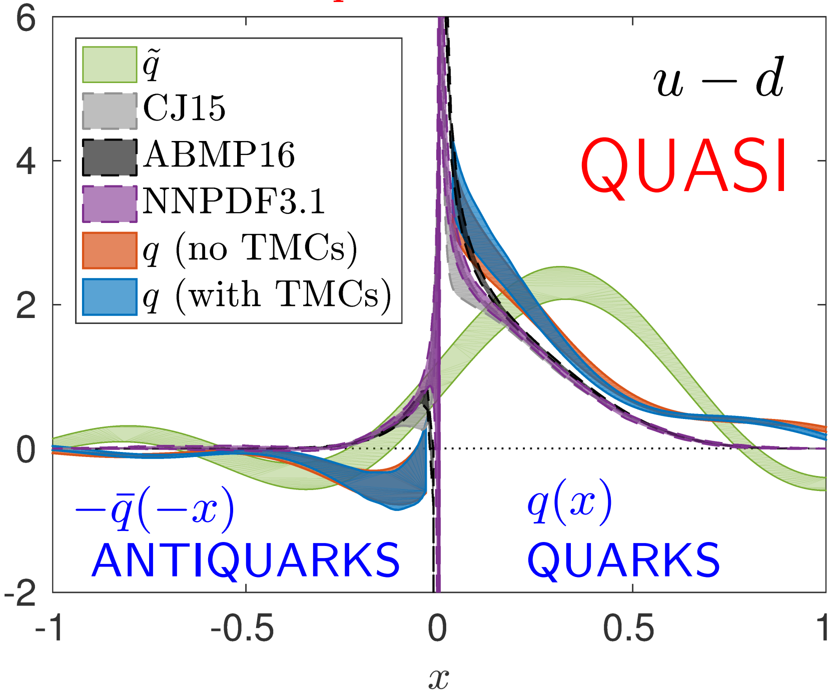
PSEUDO

Unpolarized PDF



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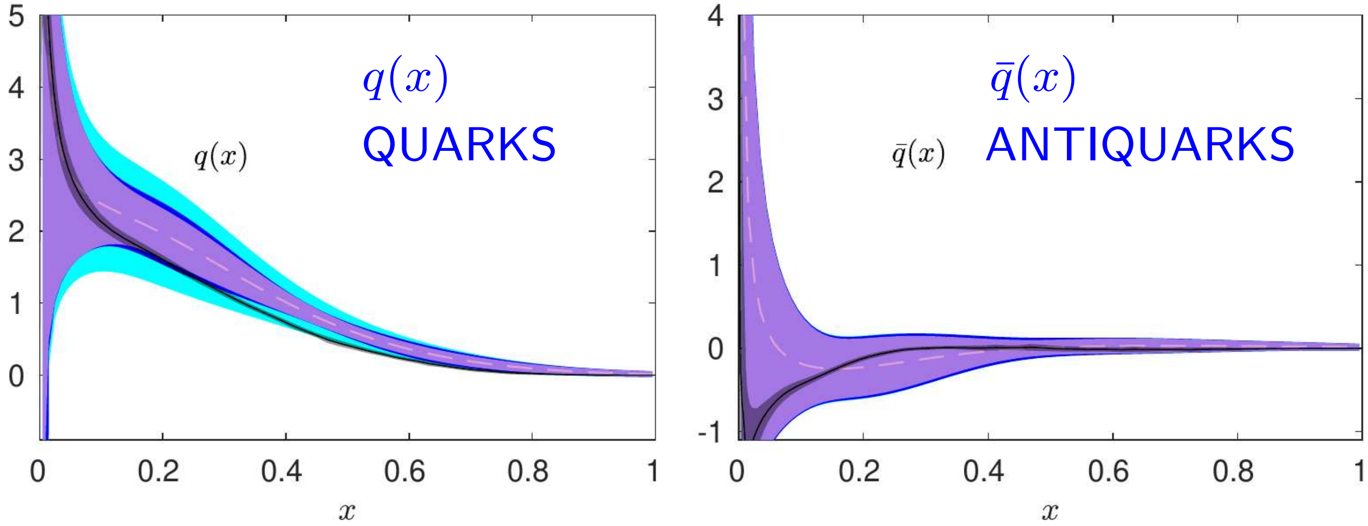

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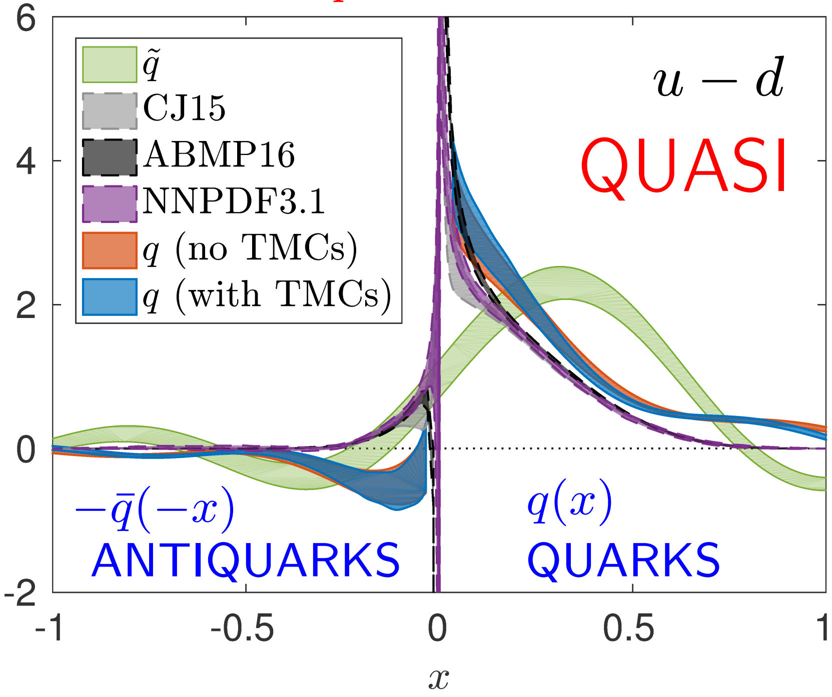

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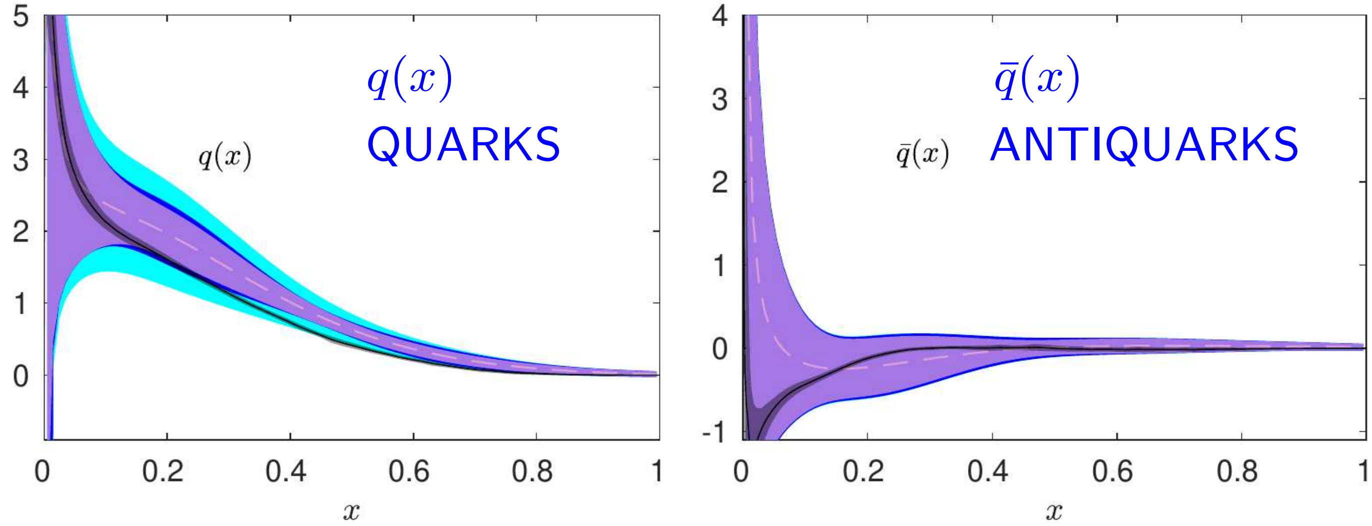

Different approach starting from the same MEs

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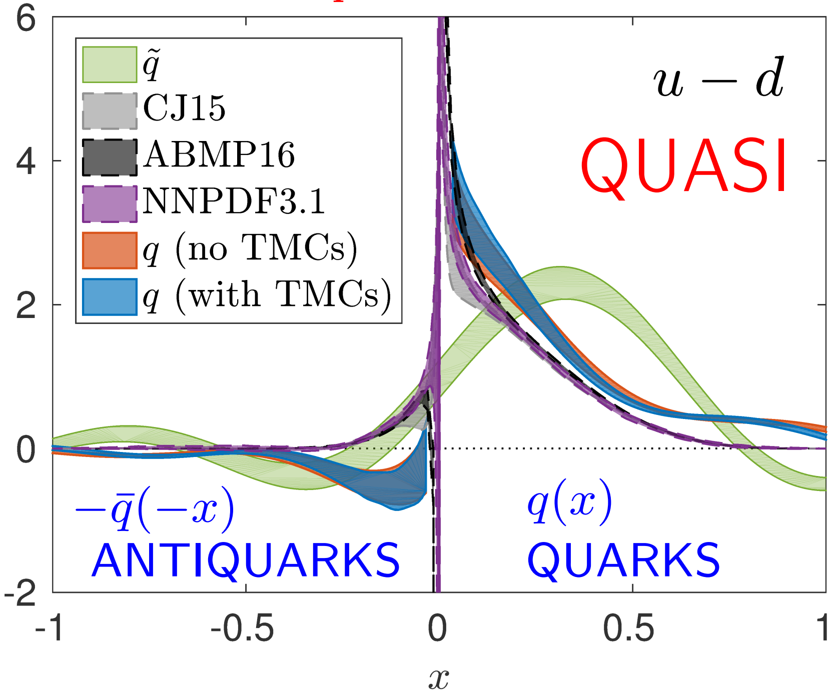

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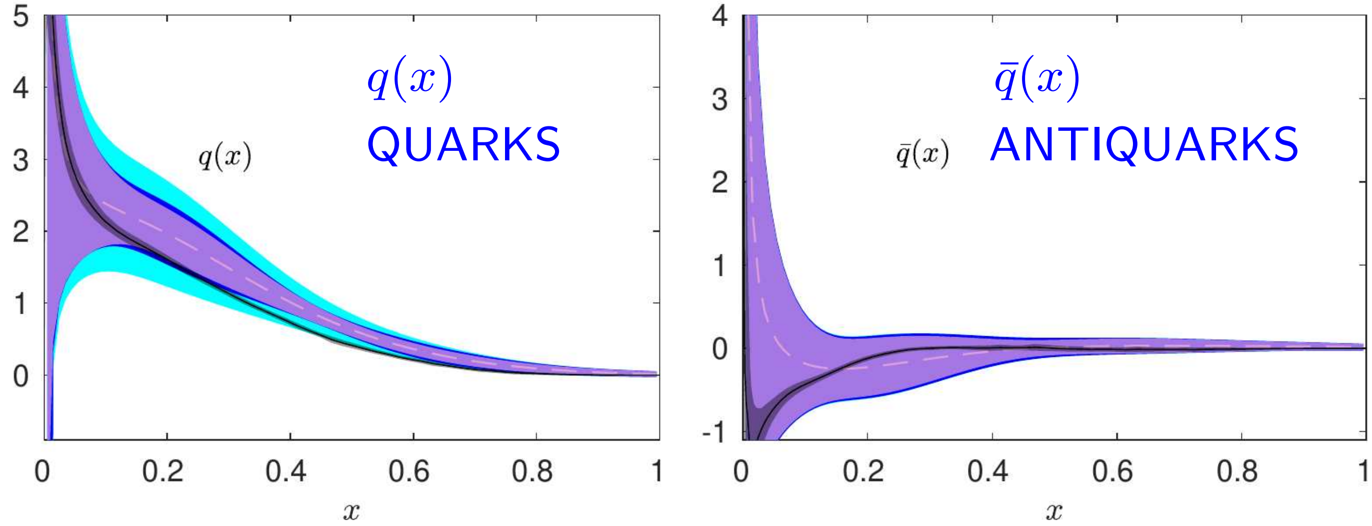

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Also: reconstruction using a pheno-inspired ansatz

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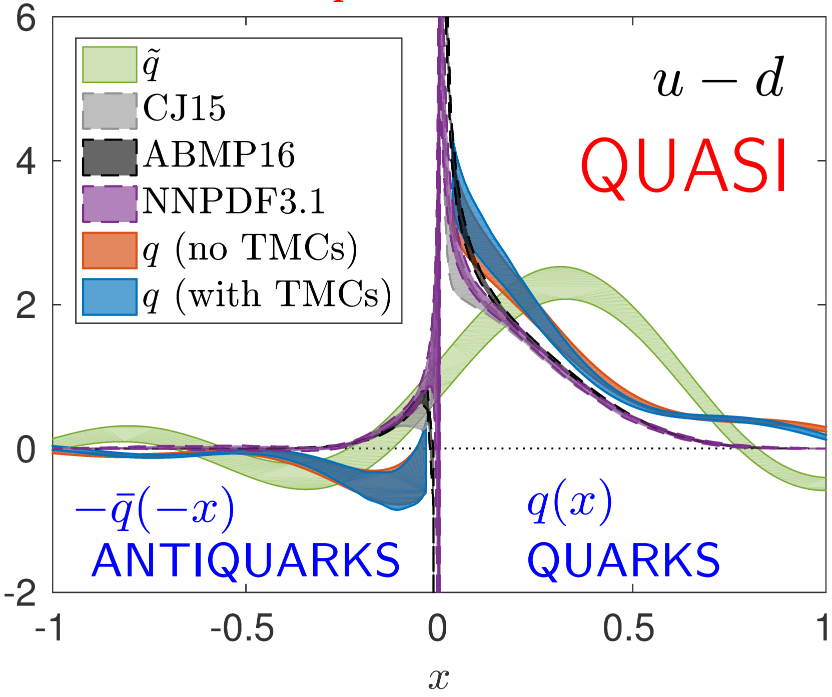

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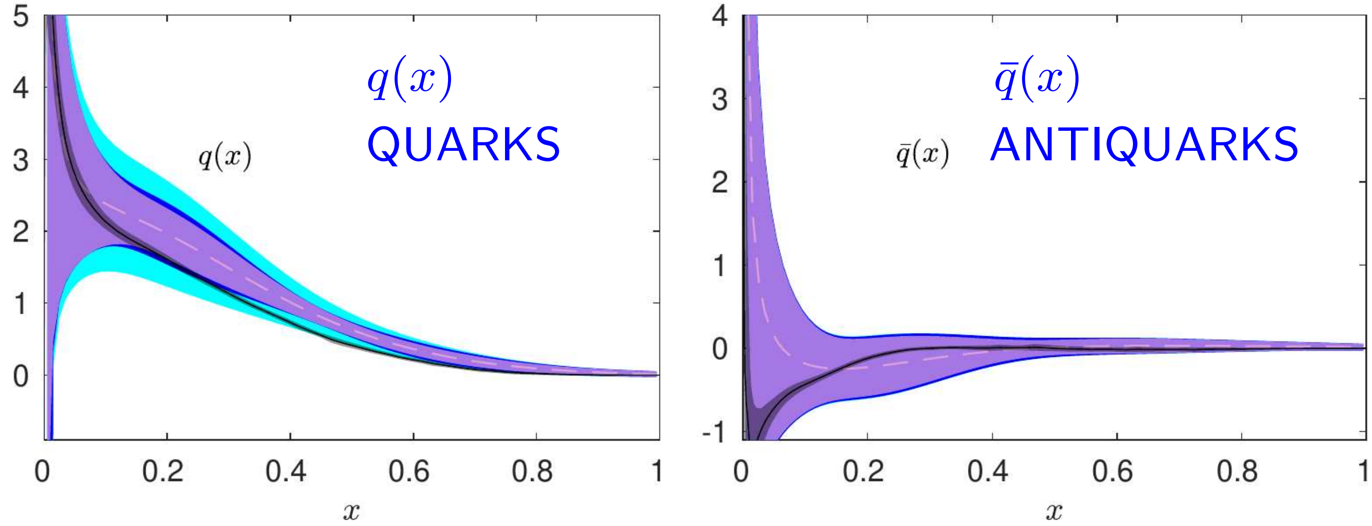

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Different approach starting from the same MEs
Also: reconstruction using a pheno-inspired ansatz
And: added plausible estimates of systematics

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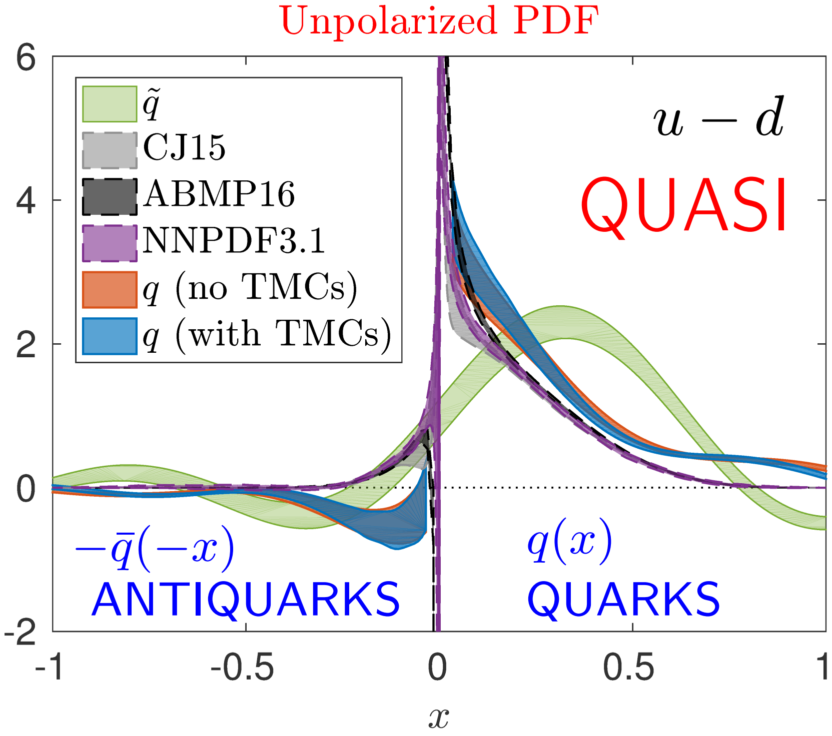

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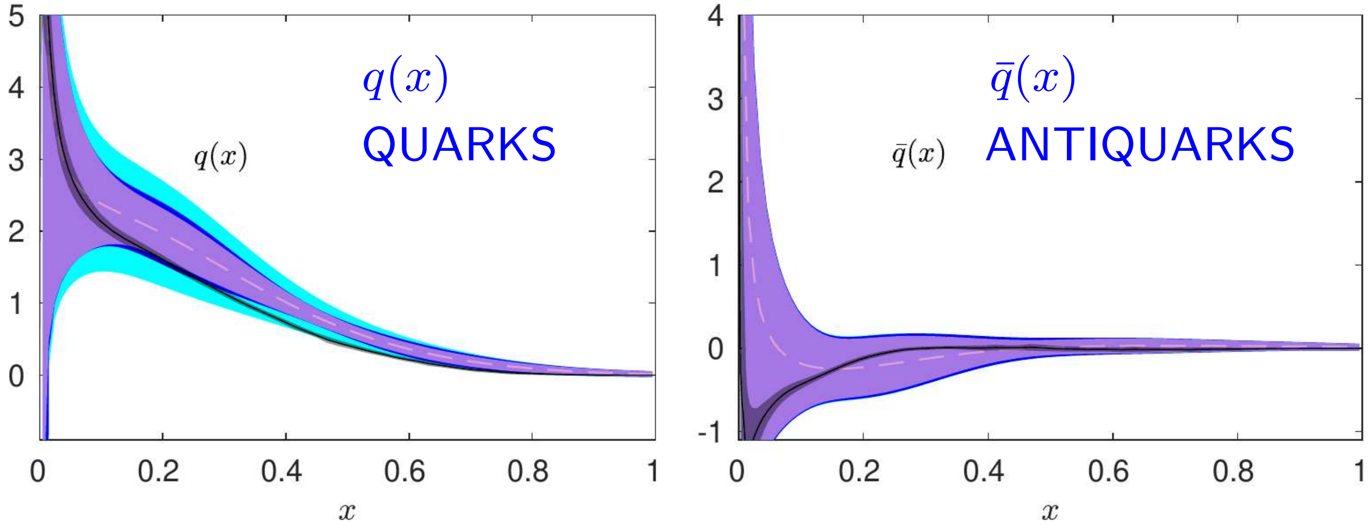


Qualitative agreement with pheno
Systematics to be investigated

cut-off effects
truncation (matching)
higher-twist effects
reconstruction of x-dep.
finite volume effects

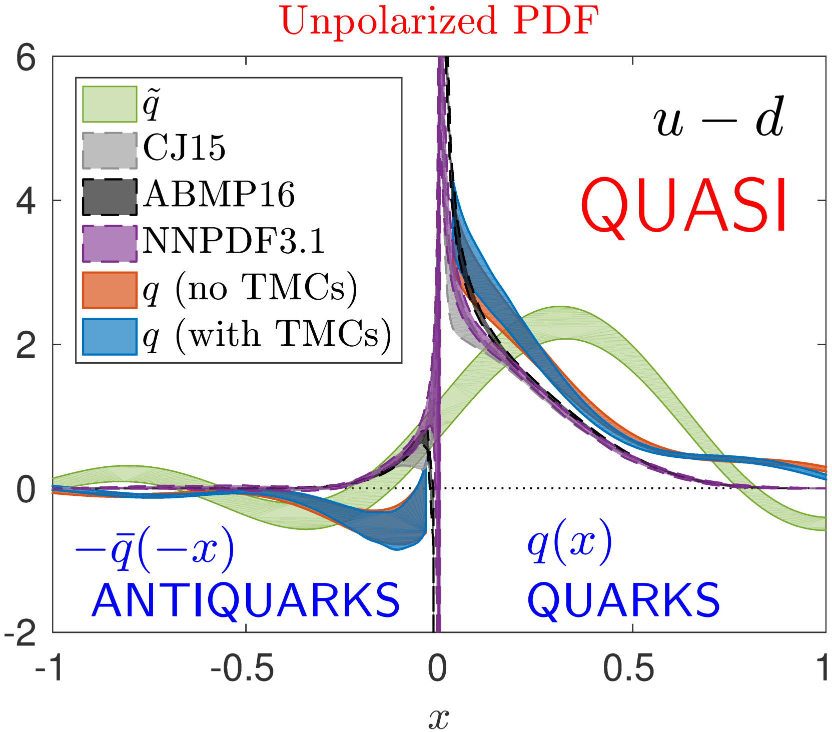
Different approach starting from the same MEs
Also: reconstruction using a pheno-inspired ansatz
And: added plausible estimates of systematics
purple – statistical error

Current state-of-the-art: unpolarized PDFs

ETMC, Phys. Rev. Lett. 121 (2018) 112001
ETMC, Phys. Rev. D 99 (2019) 114504

$Q^2 = 4$ GeV^2

PSEUDO



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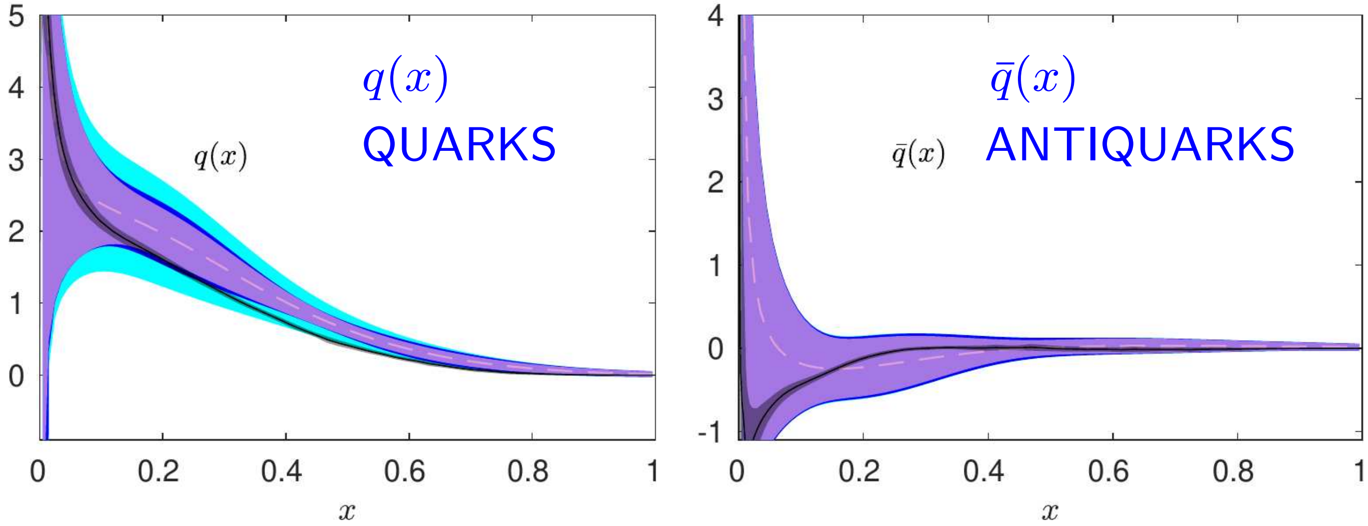

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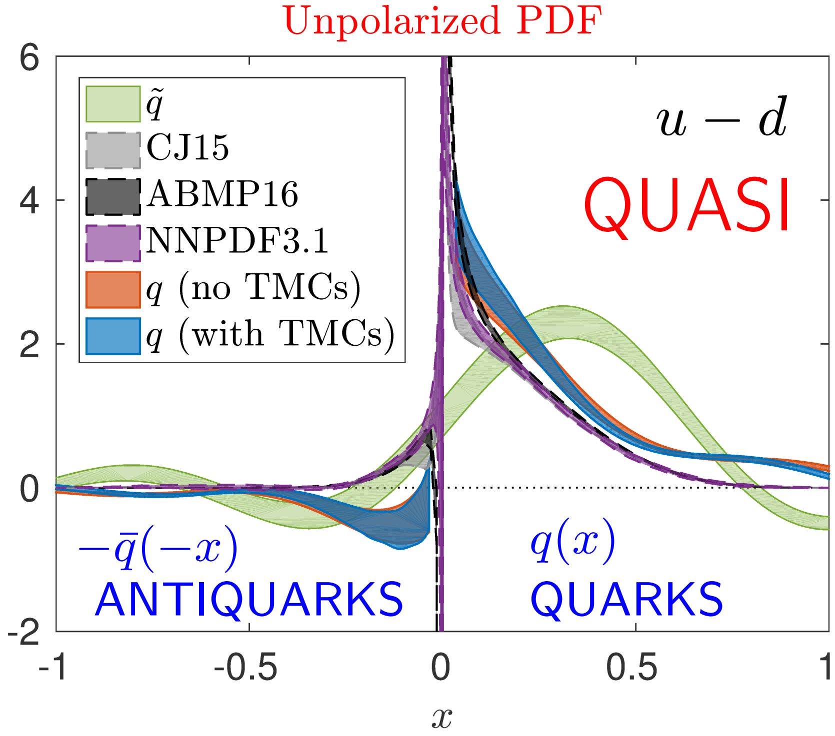

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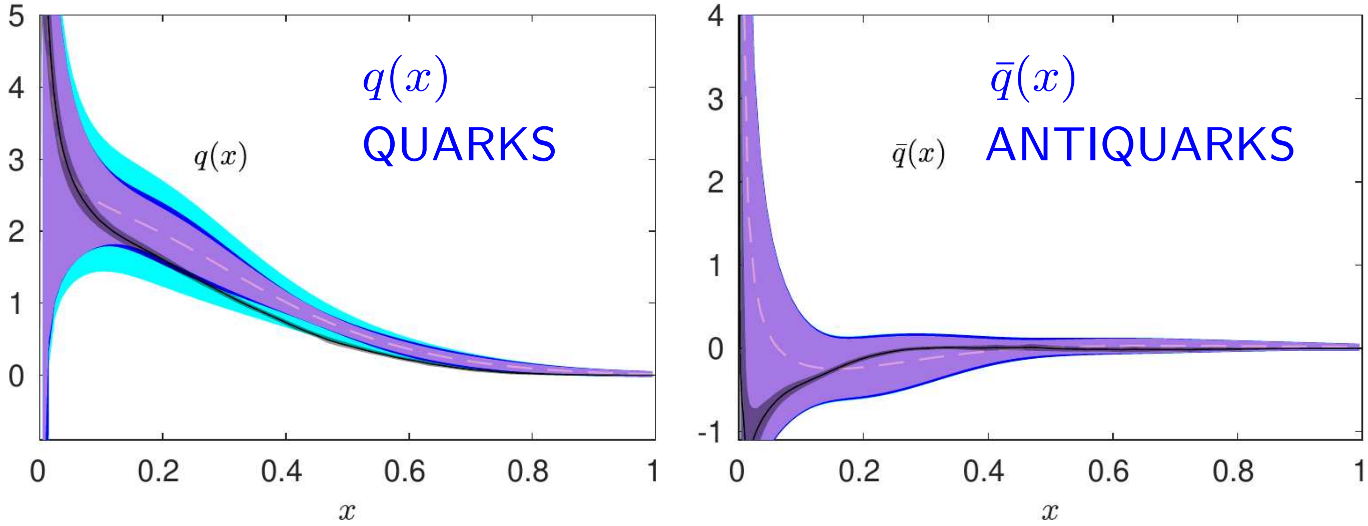

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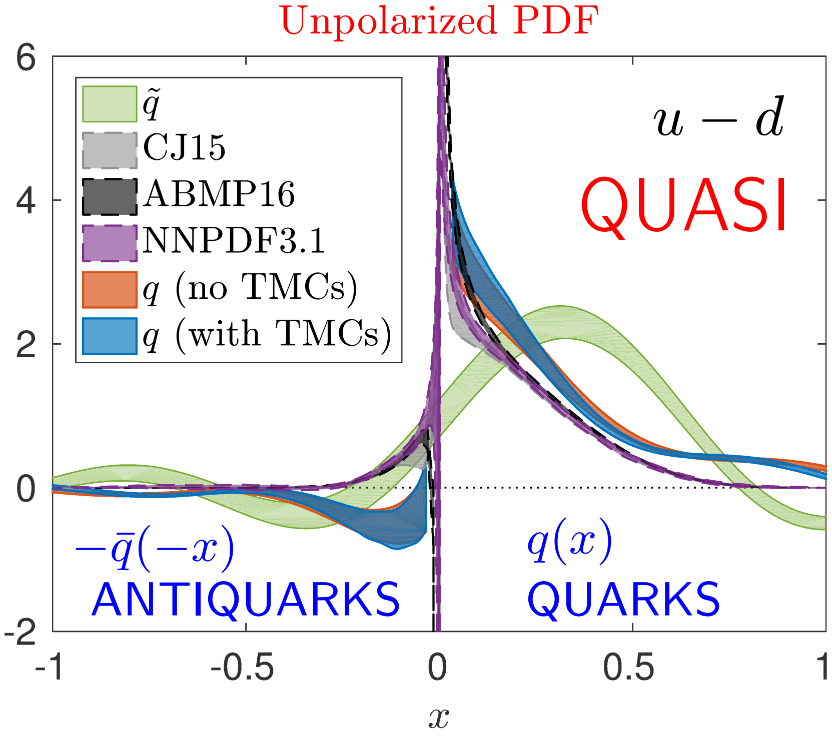

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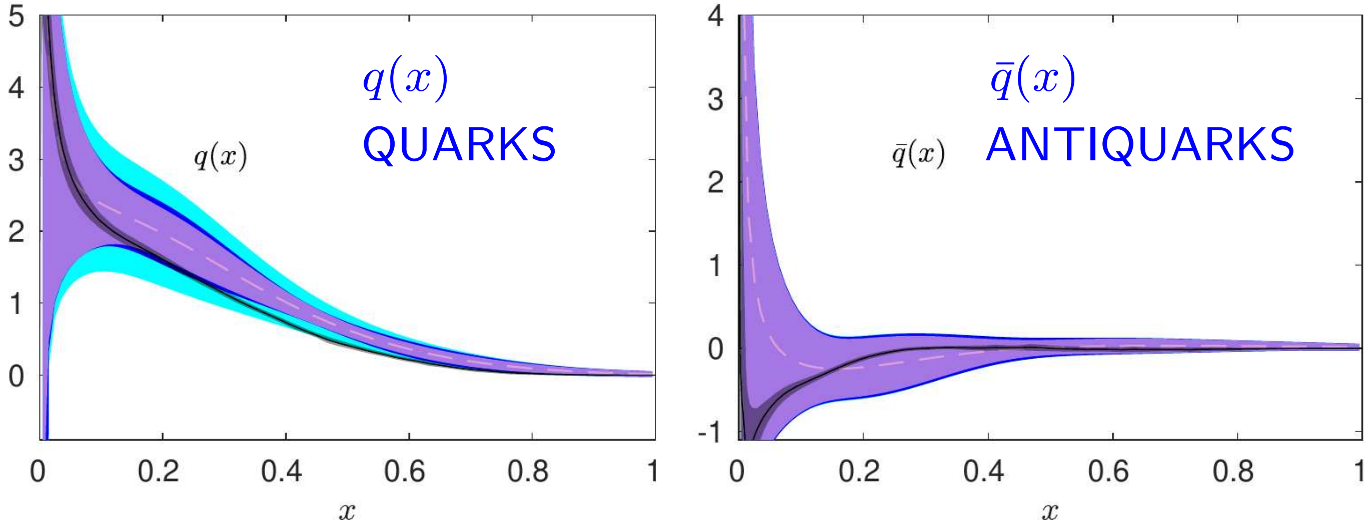

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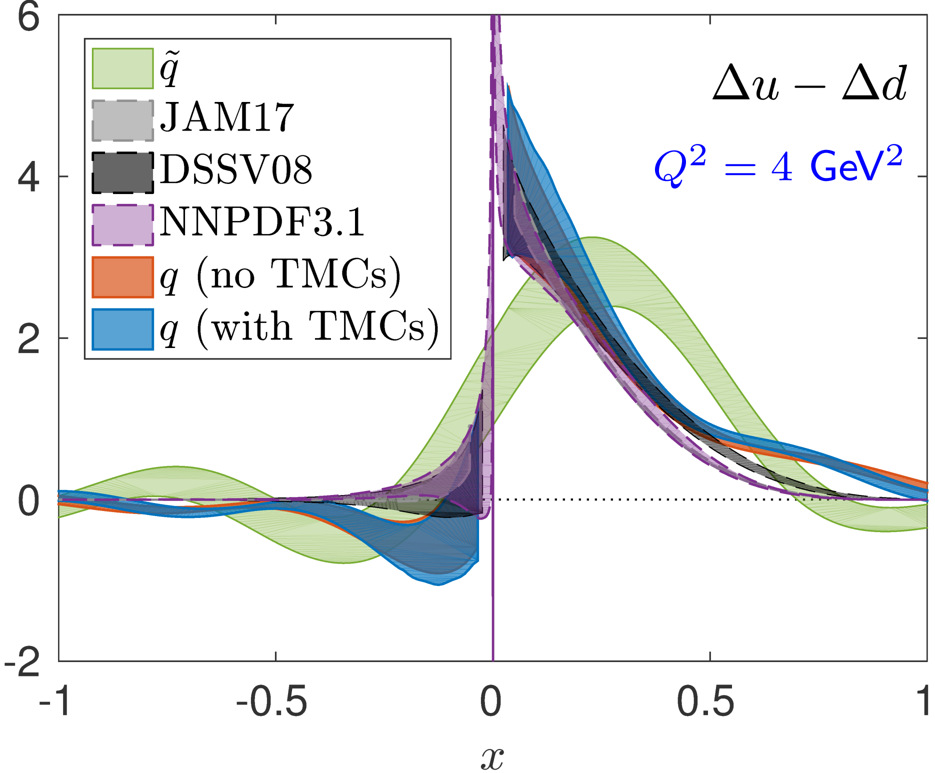
Quantitative agreement with phenomenology within stat. + plausible syst. error!

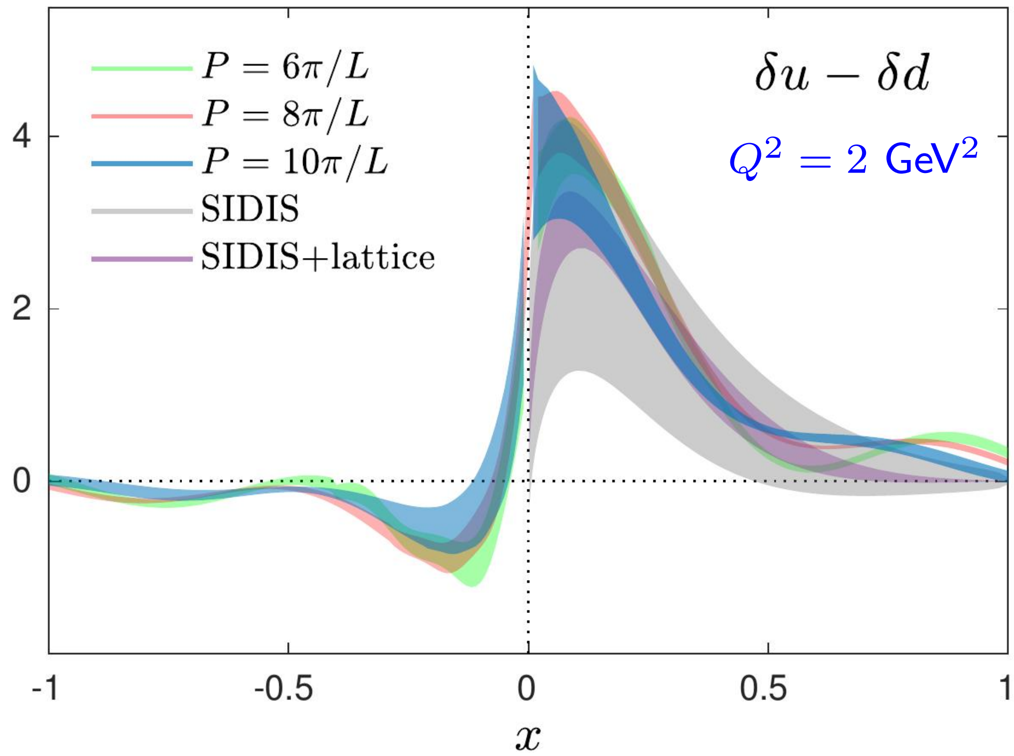
Polarized PDFs

Helicity PDF



ETMC, PRL121(2018)112001

Transversity PDF



ETMC, PRD98(2018)091503(R)

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.
LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3 PDFs

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Twist-3:

- no density interpretation,
- contain important information about qgq correlations,
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Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e

  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005

  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025

  BC-type sum rules S. Bhattacharya, A. Metz, 2105.07282

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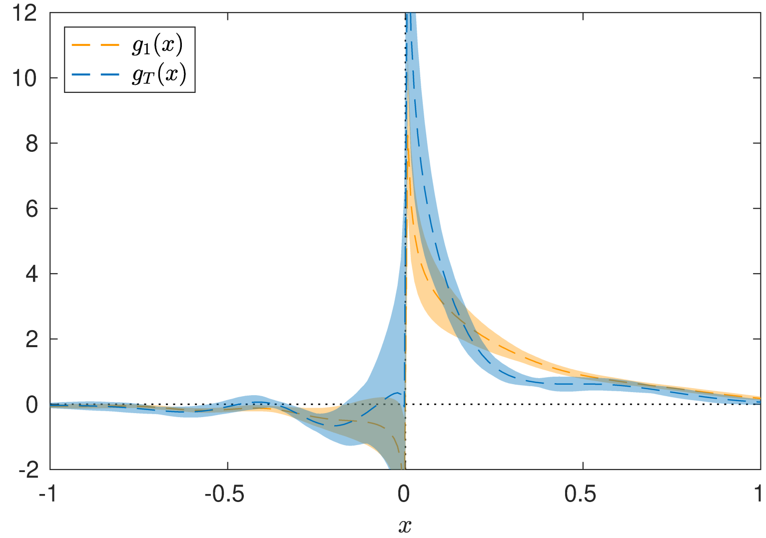

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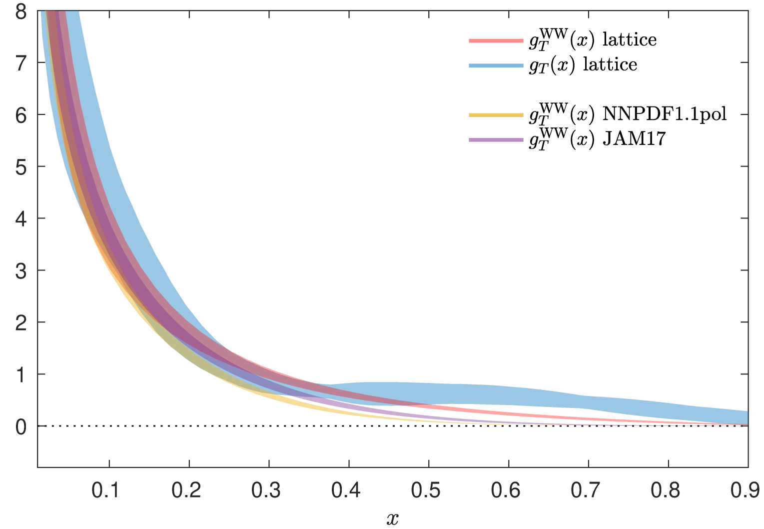

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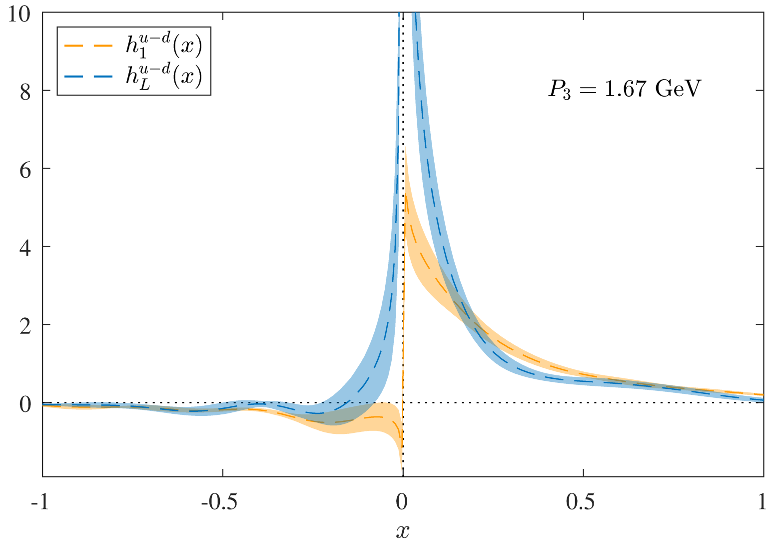

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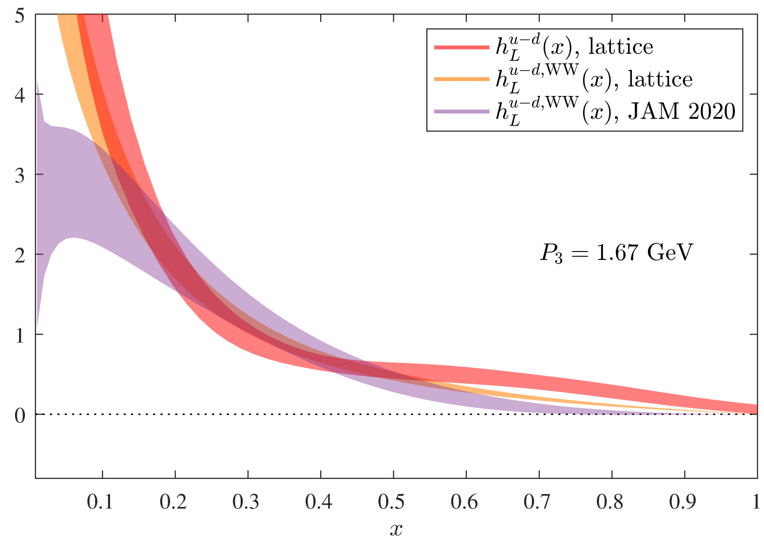

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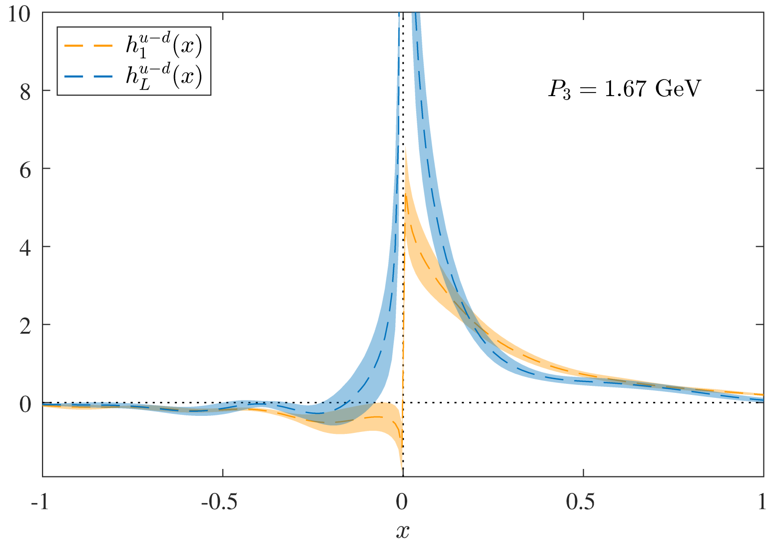

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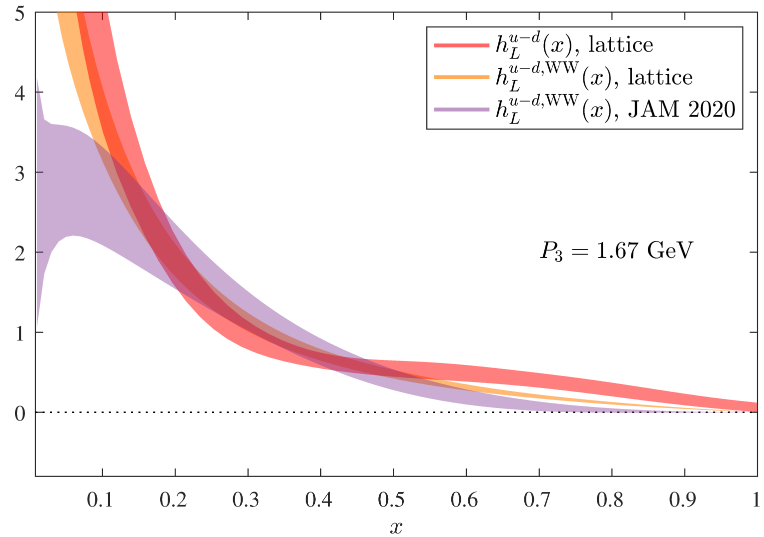
- first exploration of twist-3 GPDs
  S. Bhattacharya et al., 2112.05538



Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon

$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$

$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer

lattice computation of bare ME

↓

renormalization of bare ME
intermediate RI scheme

↓

reconstruction of x-dependence
z-space$\rightarrow$$x$-space
Backus-Gilbert

↓

matching to light cone
RI$\rightarrow$ $\overline{\mathrm{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

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most costly part of the procedure!

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Breit frame: separate calculations for each $\vec{Q}$

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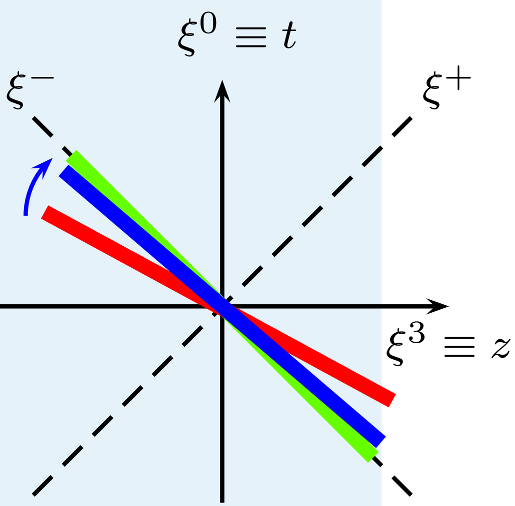

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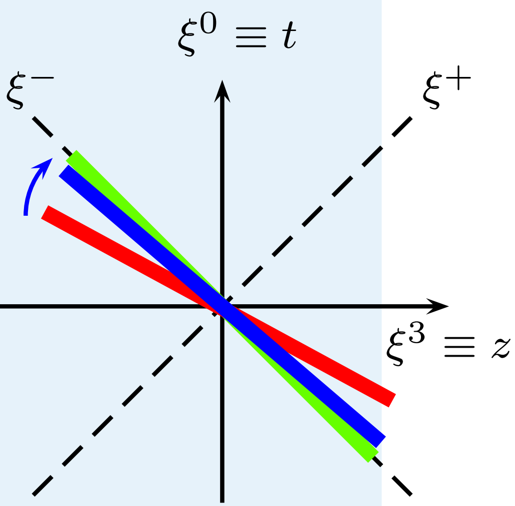
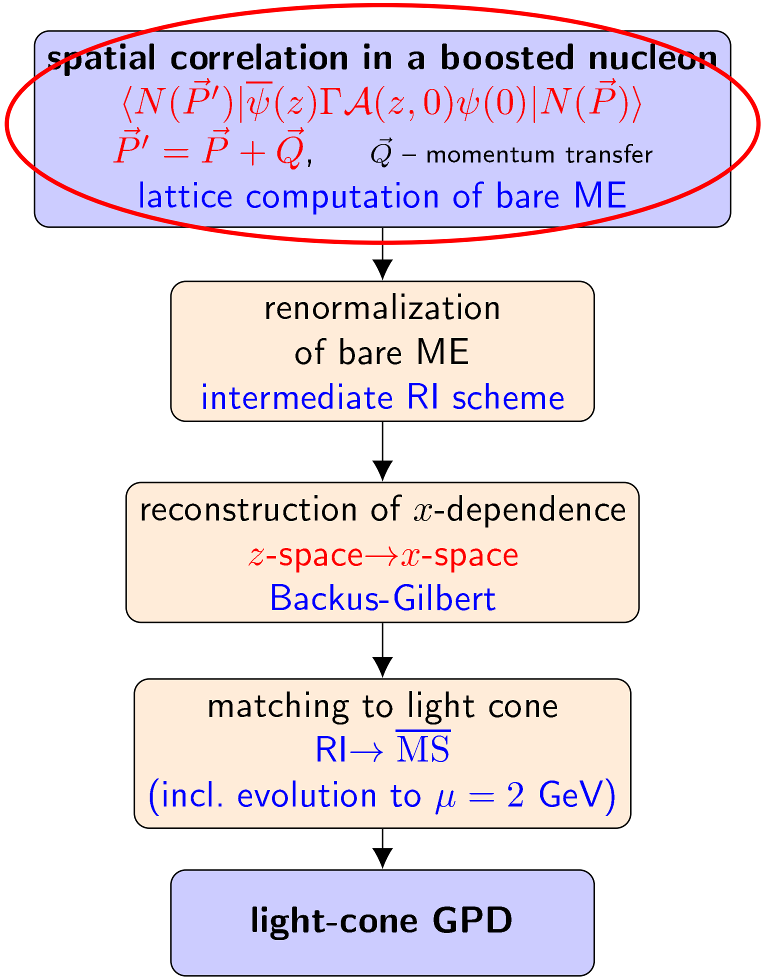
light-cone GPD

the final desired object!



Bare matrix elements

Lattice matrix elements need to be computed with 2 different projections (unpolarized/polarized).
Below for the unpolarized Dirac insertion (for unpolarized GPDs)



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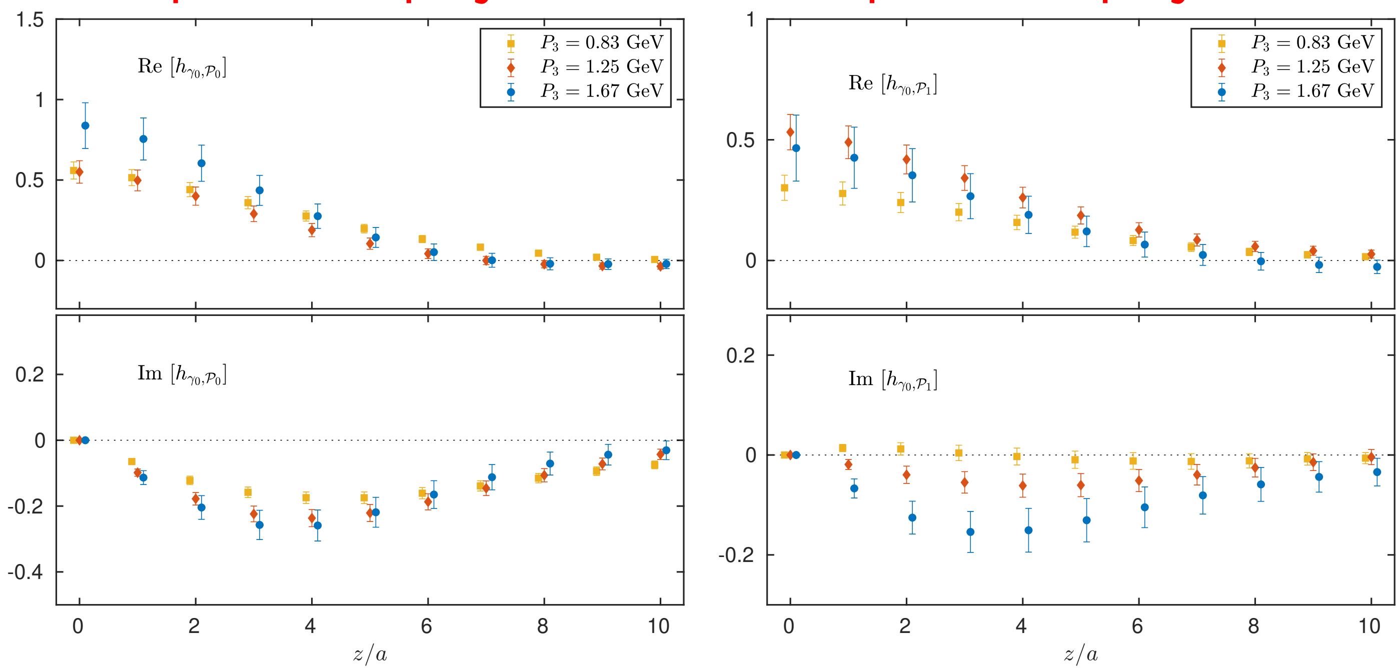
light-cone GPD

unpolarized projector | polarized projector



Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Disentangled renormalized matrix elements

Removal of divergences and disentangling of H- and E-GPDs.
Unpolarized Dirac insertion (for unpolarized GPDs)

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matching to light cone
RI$\rightarrow$ $\overline{\text{MS}}$
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light-cone GPD

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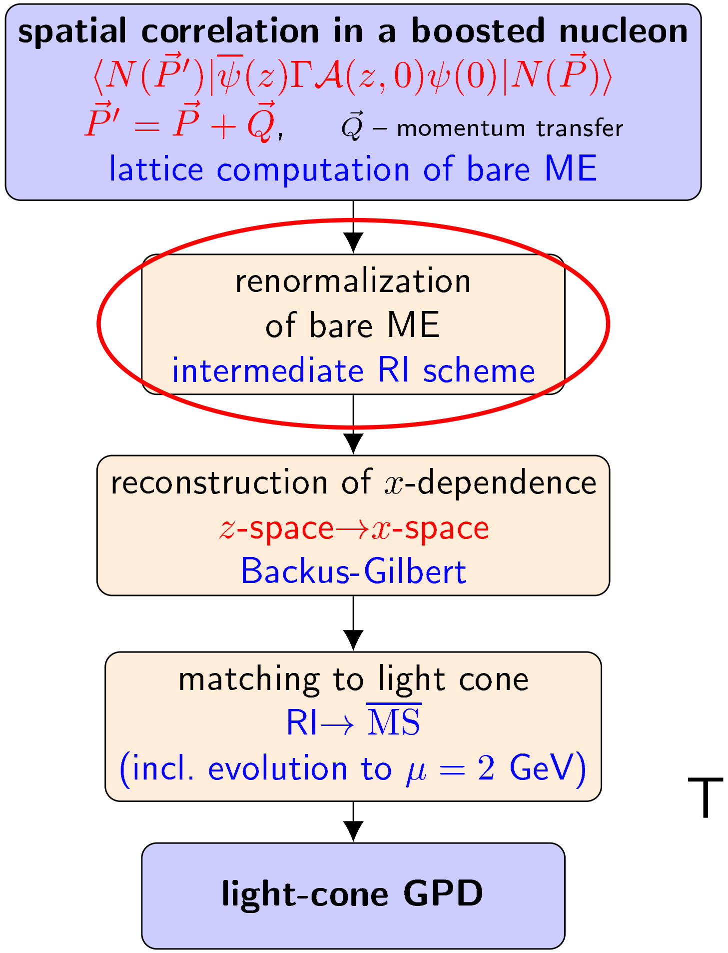
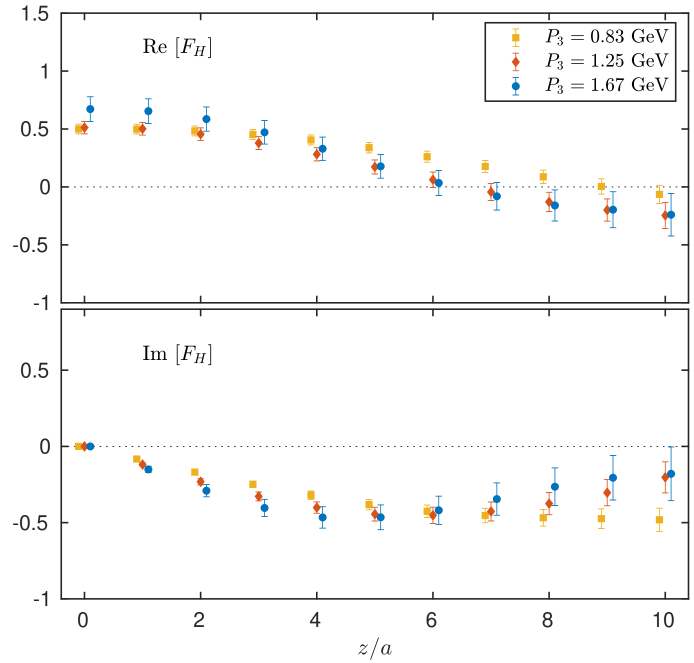
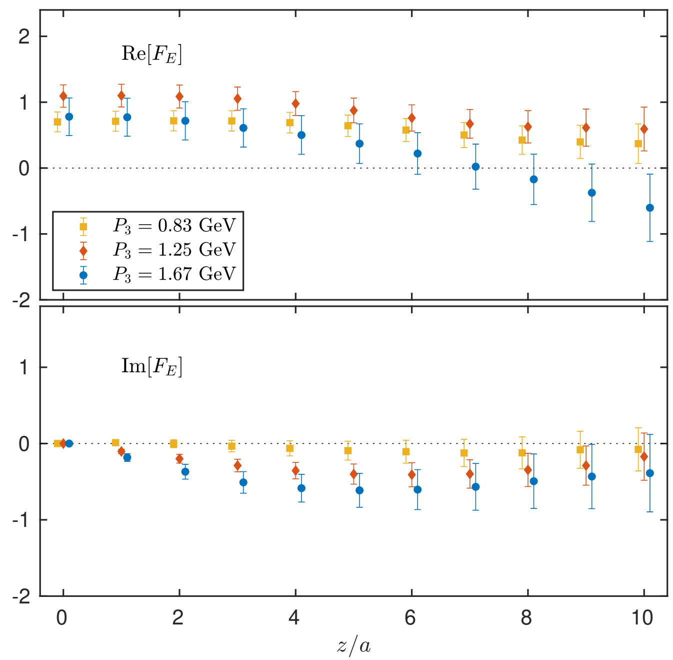
Unpolarized Dirac insertion (for unpolarized GPDs)



ME of H-function



ME of E-function



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ETMC, Phys. Rev. Lett. 125 (2020) 262001

Light-cone distributions

Reconstruction of x-dependence and matching to light cone.
Unpolarized Dirac insertion (for unpolarized GPDs)

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↓

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intermediate RI scheme

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reconstruction of x-dependence
z-space→x-space
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light-cone GPD

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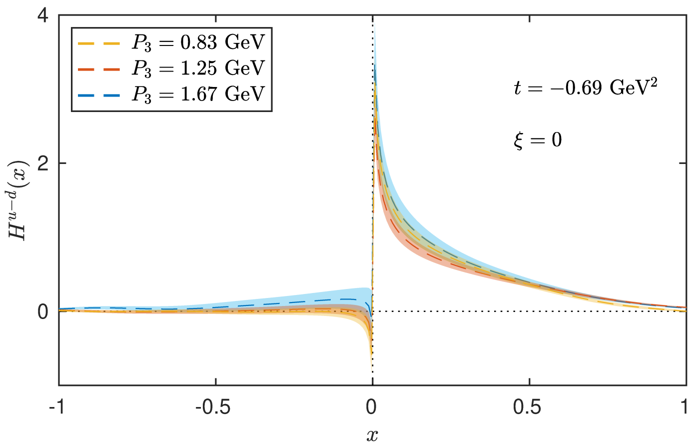
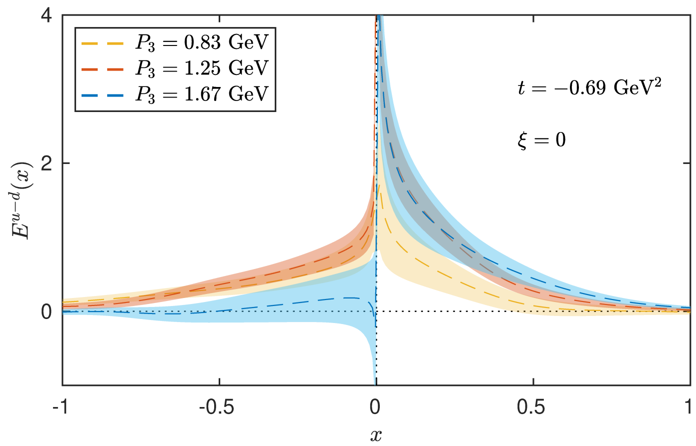
light-cone GPD

H-GPD



E-GPD



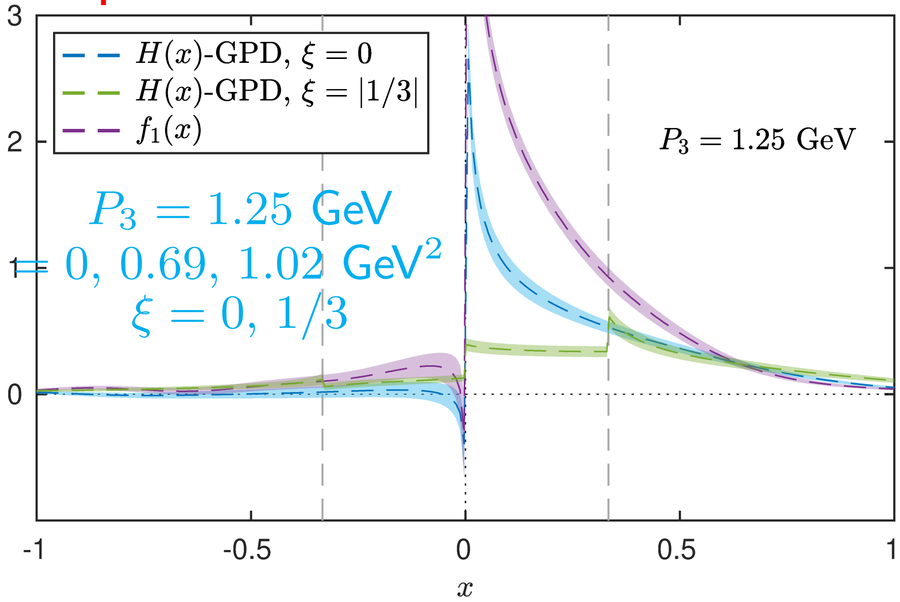
Three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV
Momentum transfer: $-t = 0.69$ GeV^2
Zero skewness: $\xi = 0$

ETMC, Phys. Rev. Lett. 125 (2020) 262001

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

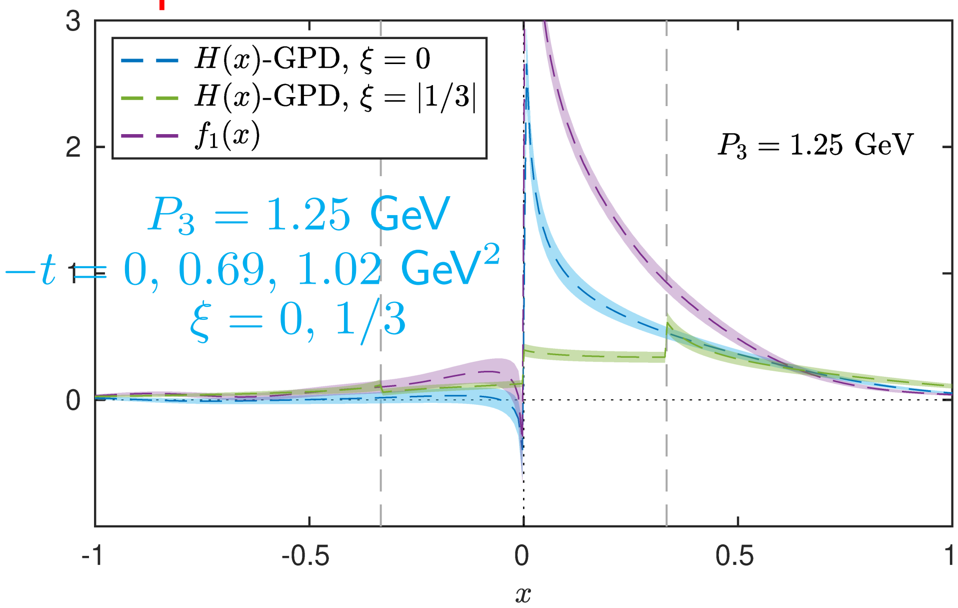


$P_3 = 1.25$ GeV
$-t = 0,\ 0.69,\ 1.02$ GeV^2
$\xi = 0,\ 1/3$

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001



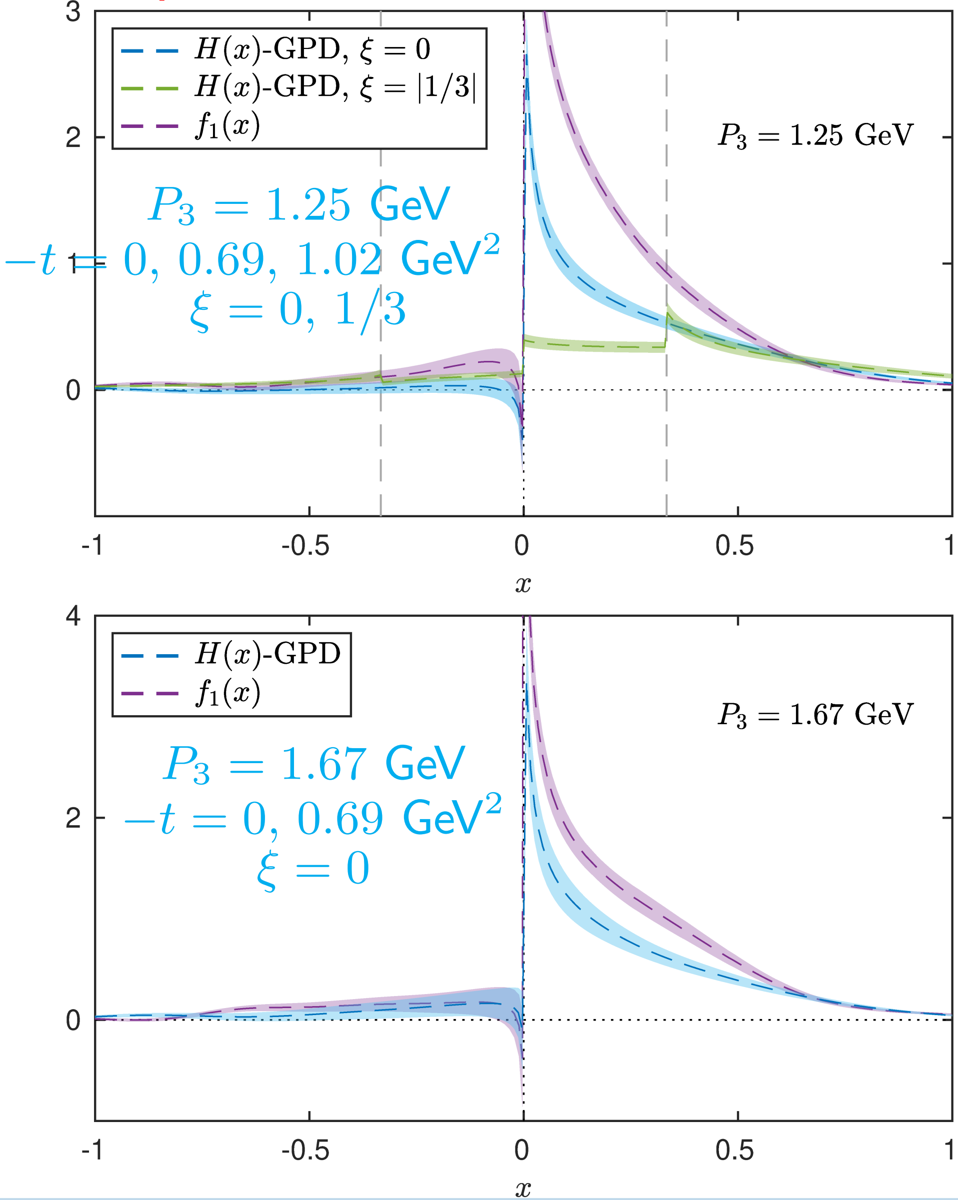
Important insights from models:

S. Bhattacharya, C. Cocuzza, A. Metz
Phys. Lett. B788 (2019) 453
Phys. Rev. D102 (2020) 054201

Comparison of PDFs and H-GPDs

unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

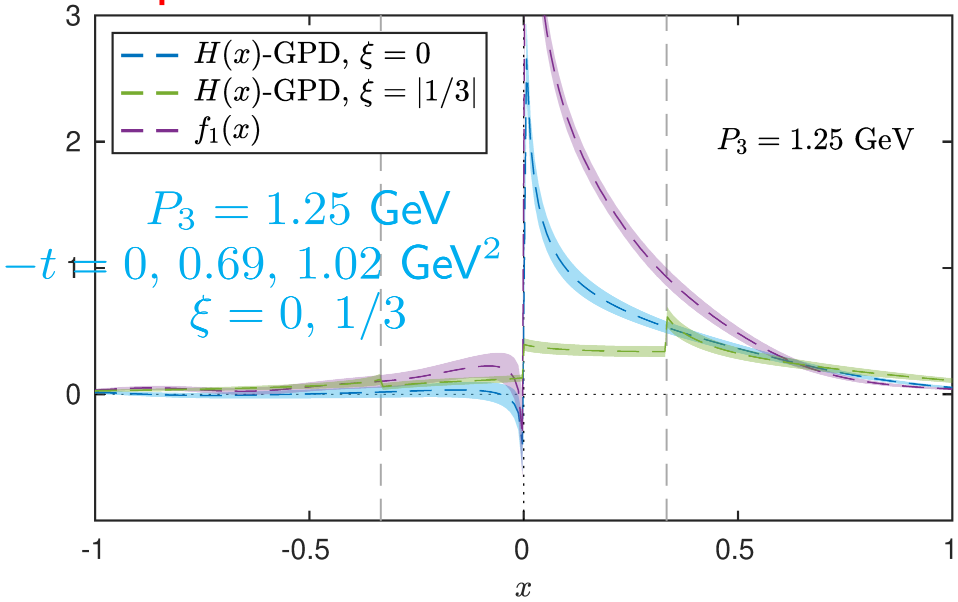


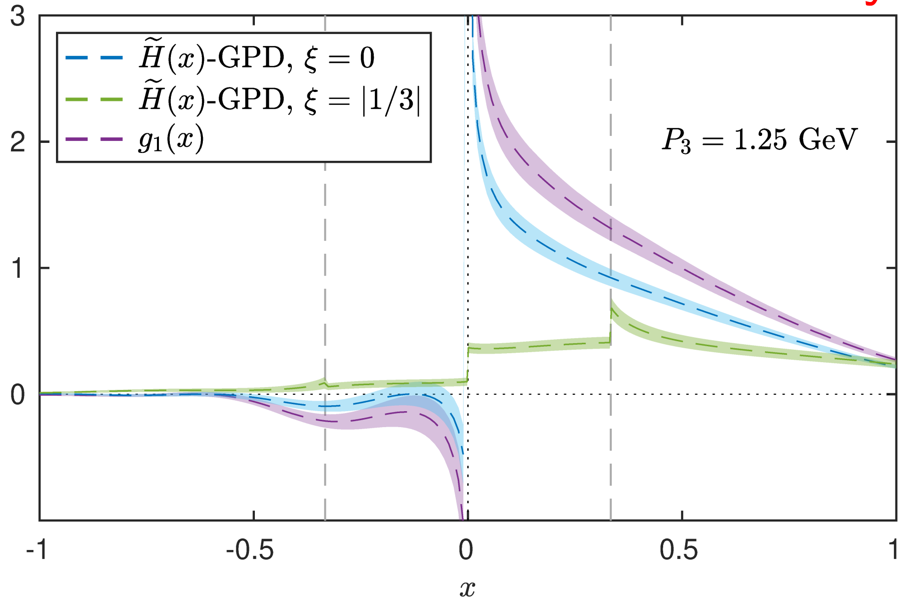
Comparison of PDFs and H-GPDs

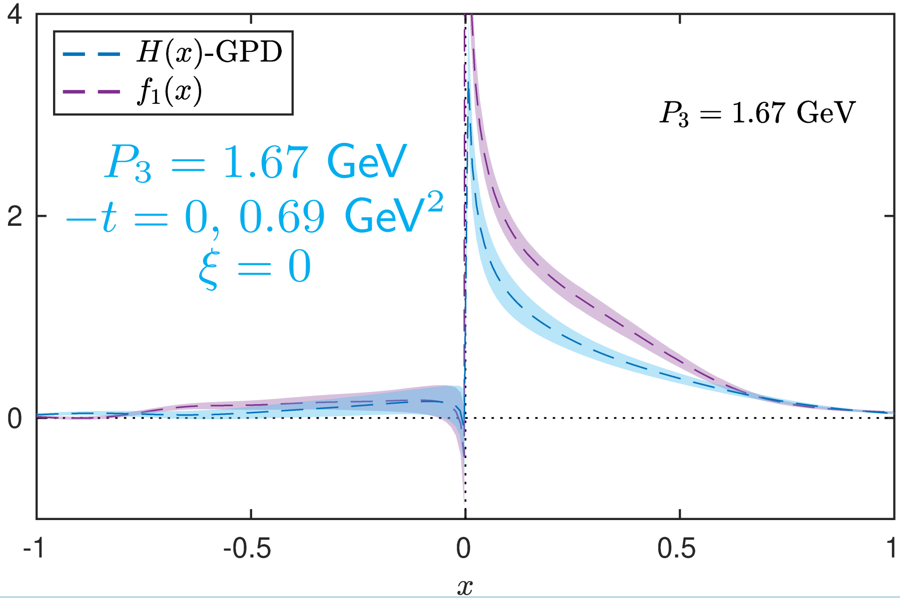
unpolarized

ETMC, Phys. Rev. Lett. 125 (2020) 262001

helicity

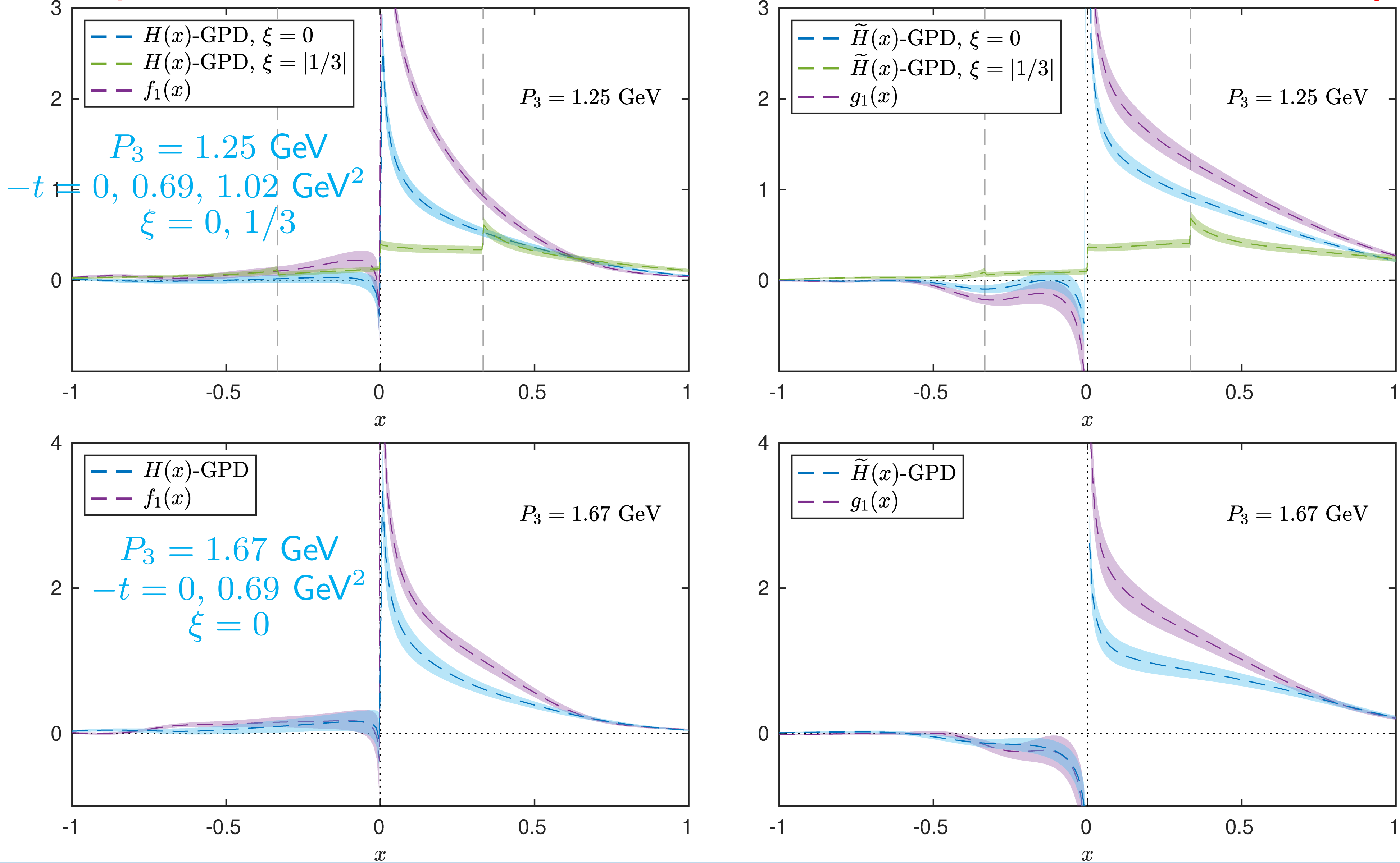






Comparison of PDFs and H-GPDs

unpolarized ETMC, Phys. Rev. Lett. 125 (2020) 262001 helicity

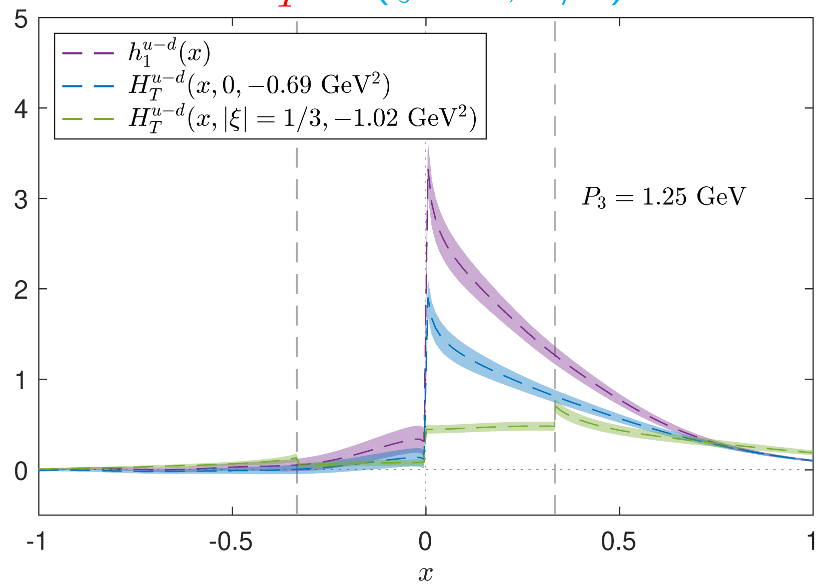


Transversity GPDs

Transversity GPDs:
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

ETMC, Phys. Rev. D105 (2022) 034501

H_T^{u-d} $(\xi = 0,\ 1/3)$



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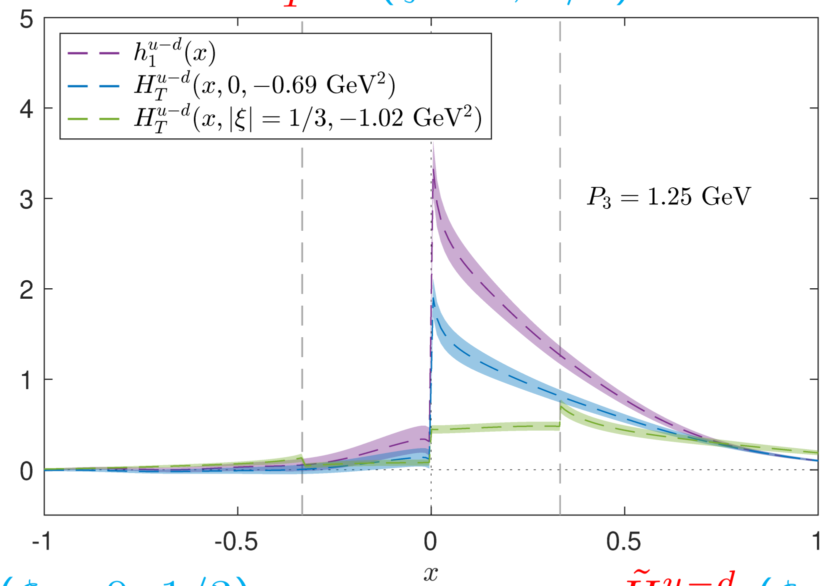
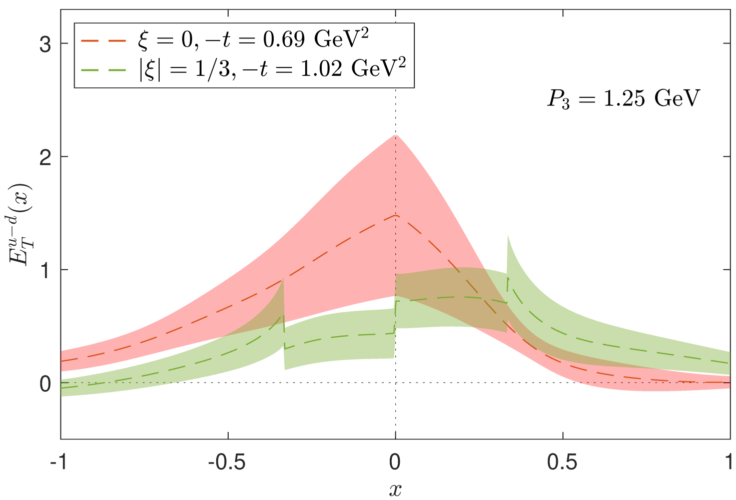
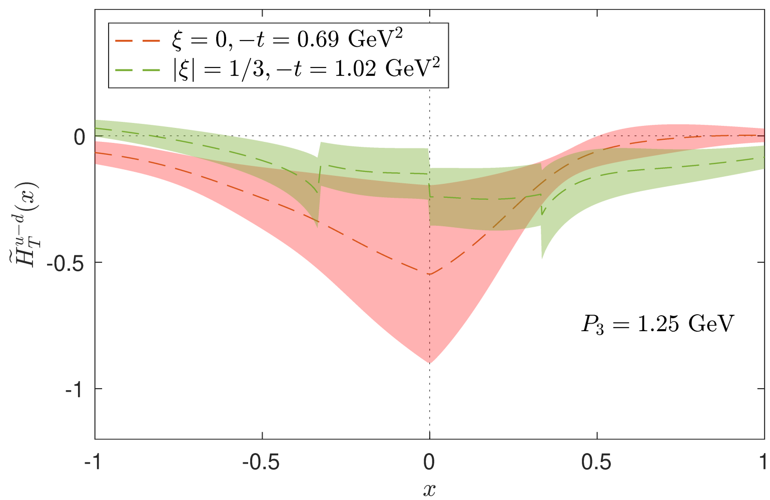


E_T^{u-d} $(\xi = 0,\ 1/3)$



$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$

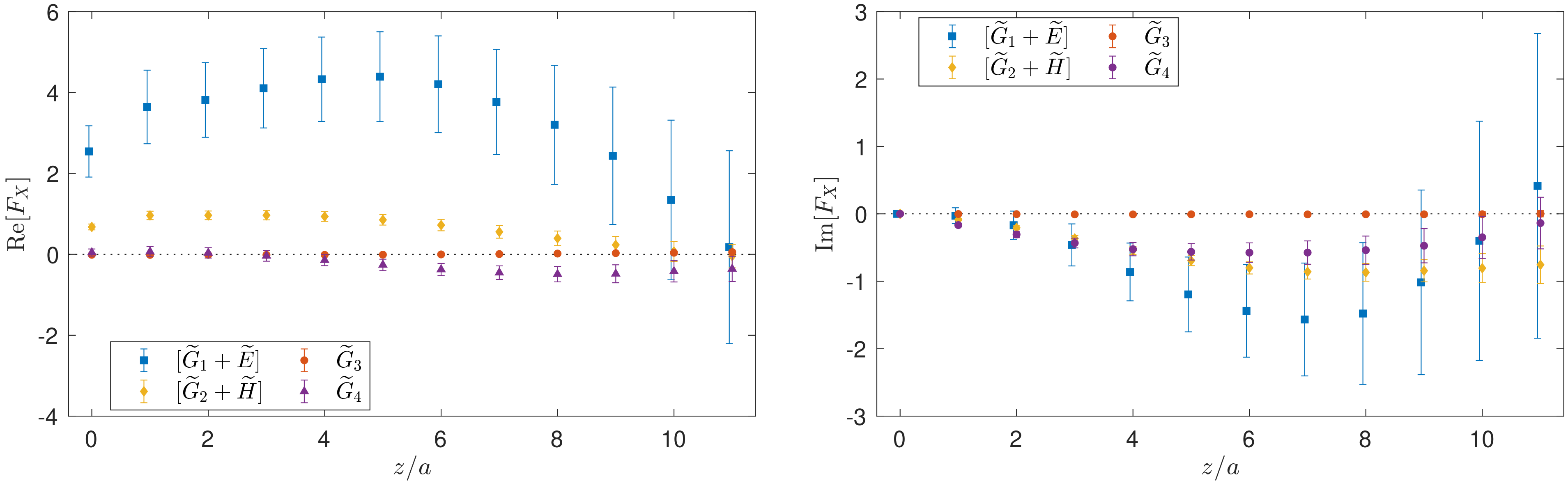


Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., 2112.05538

Twist-3 axial GPDs: $\widetilde{G}_1,\ \widetilde{G}_2,\ \widetilde{G}_3,\ \widetilde{G}_4$

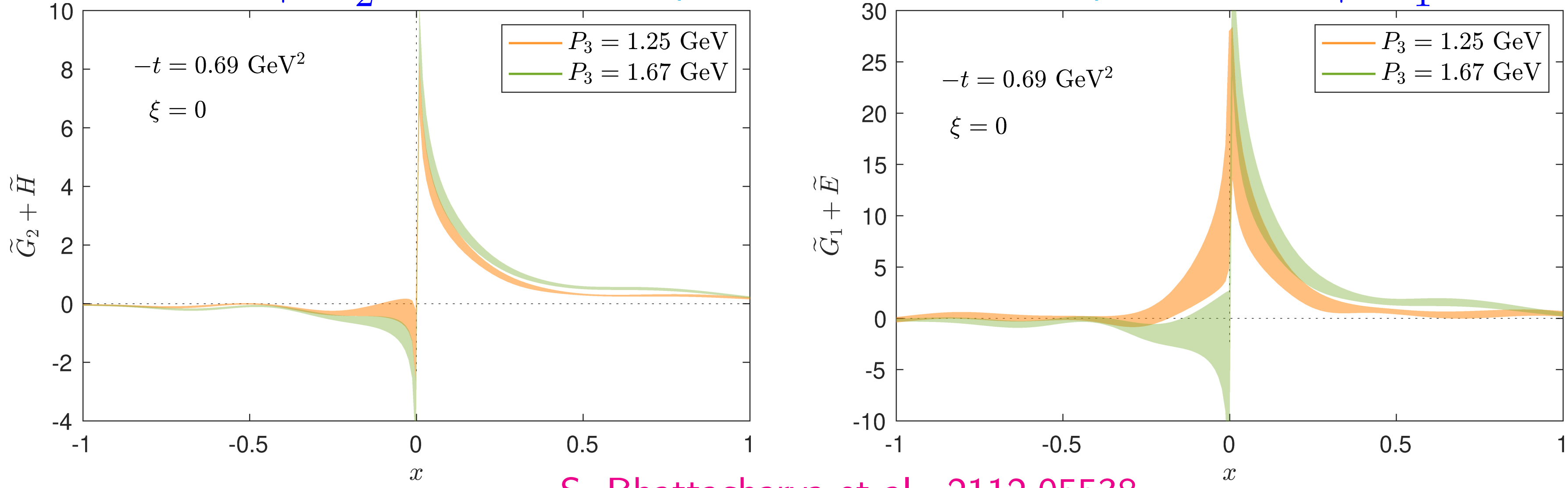
$$h_{\gamma^j\gamma_5} = \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma_5}{2m}\rangle\rangle[F_{\widetilde{E}} + F_{\widetilde{G}_1}] + \langle\langle g_\perp^{j\rho}\gamma_\rho\gamma_5\rangle\rangle[F_{\widetilde{H}} + F_{\widetilde{G}_2}] + \langle\langle \frac{g_\perp^{j\rho}\Delta_\rho\gamma^+\gamma_5}{P^+}\rangle\rangle F_{\widetilde{G}_3} + \langle\langle \frac{i\epsilon_\perp^{j\rho}\Delta_\rho\gamma^+}{P^+}\rangle\rangle F_{\widetilde{G}_4}\,.$$

Disentangled renormalized ME:



First exploration of twist-3 GPDs

$\widetilde{H} + \widetilde{G}_2$ $(\xi = 0, -t = 0.69\ \text{GeV}^2)$ $\widetilde{E} + \widetilde{G}_1$



S. Bhattacharya et al., 2112.05538

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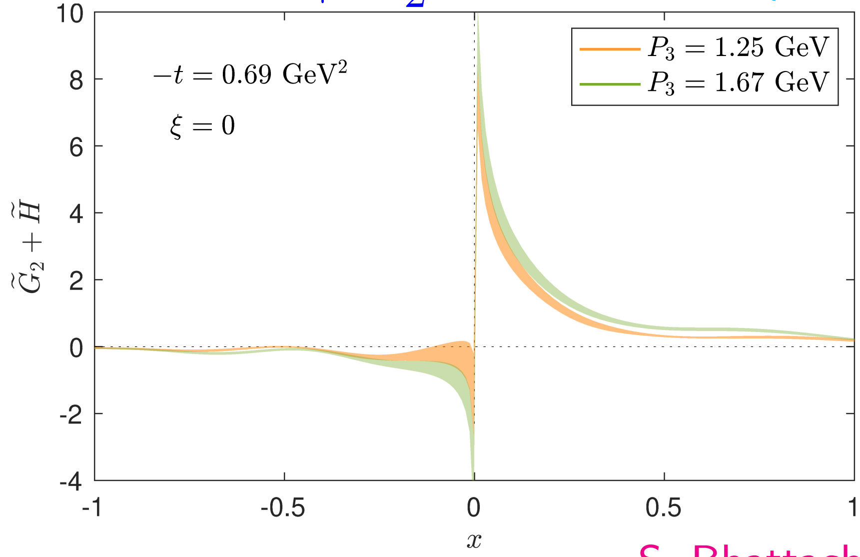
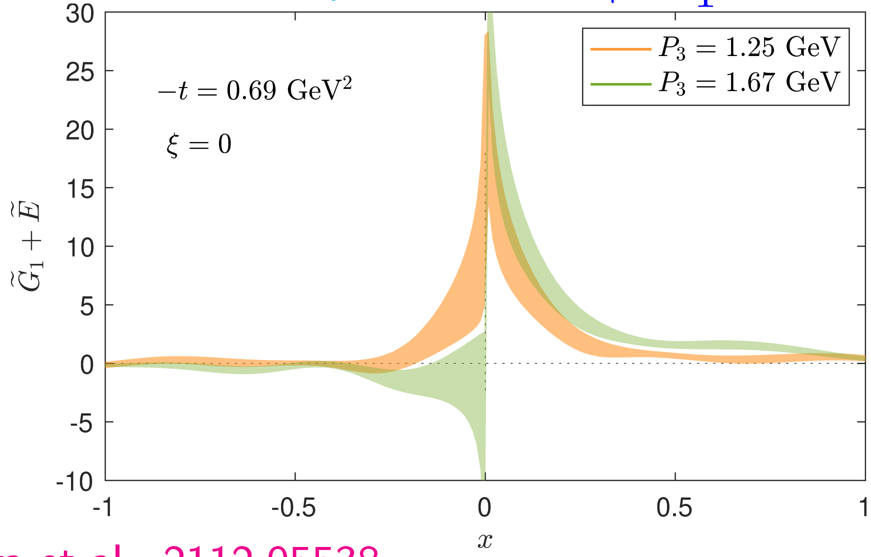
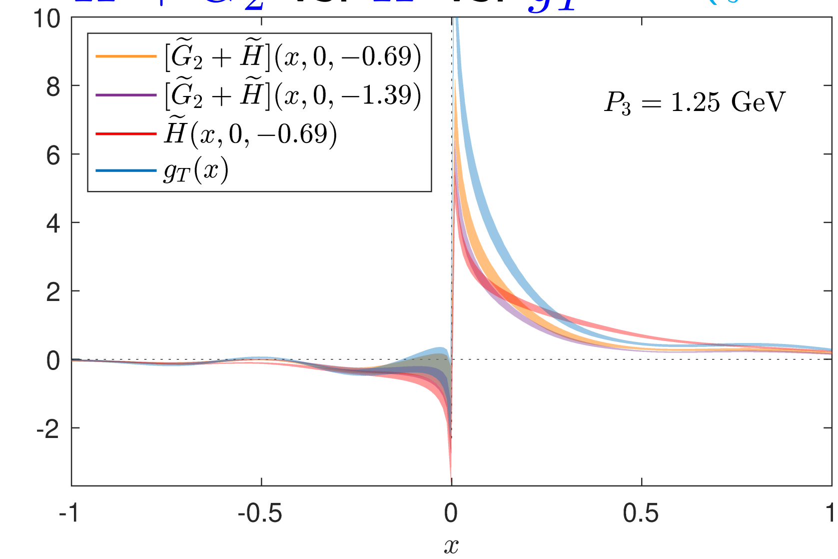
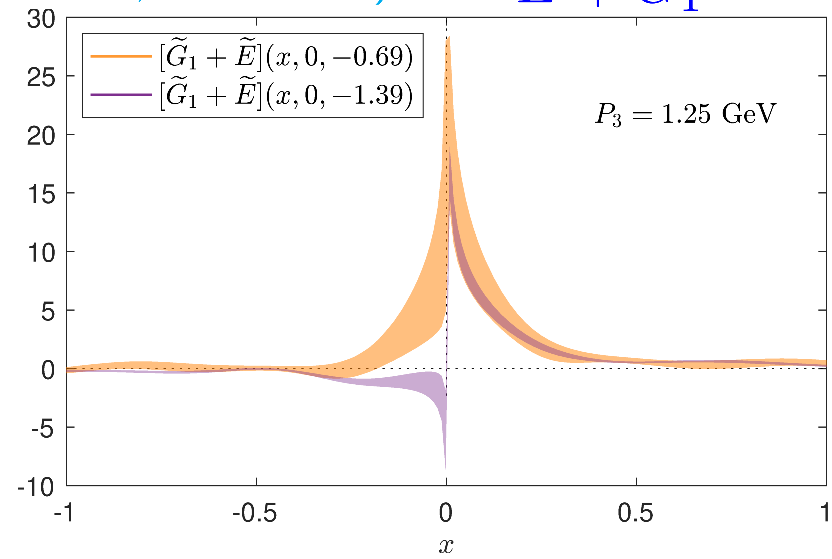




S. Bhattacharya et al., 2112.05538

$\widetilde{H} + \widetilde{G}_2$ vs. $\widetilde{H}$ vs. g_T $(\xi = 0, -t = 0.69, 1.39$ GeV$^2)$ $\widetilde{E} + \widetilde{G}_1$





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Main theoretical tool: S. Bhattacharya et al., PRD106(2022)114512
Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta) = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{m}A_1 + m z^{\mu} A_2 + \frac{\Delta^{\mu}}{m}A_3 + i m \sigma^{\mu z} A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{m}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{m}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{m}A_8\right]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example

S. Bhattacharya et al., PRD106(2022)114512

The relation between lattice-calculated matrix elements and the Lorentz-invariant amplitudes A_i is different in the symmetric and the non-symmetric frame.

For example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0) = C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1 + \frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5 + \frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

asymmetric frame:

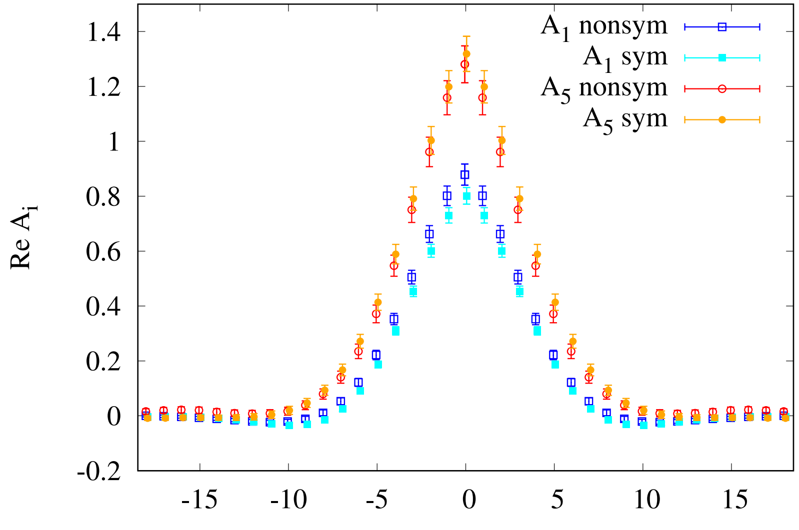
$$\Pi_0^a(\Gamma_0) = C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1 - \frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3 + \frac{(E_i-E_f)P_3 z}{4m}A_4$$
$$+ \frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5 + \frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6 + \frac{E_f P_3(E_f-E_i)^2 z}{2m^3}A_8\Bigg).$$

Thus,

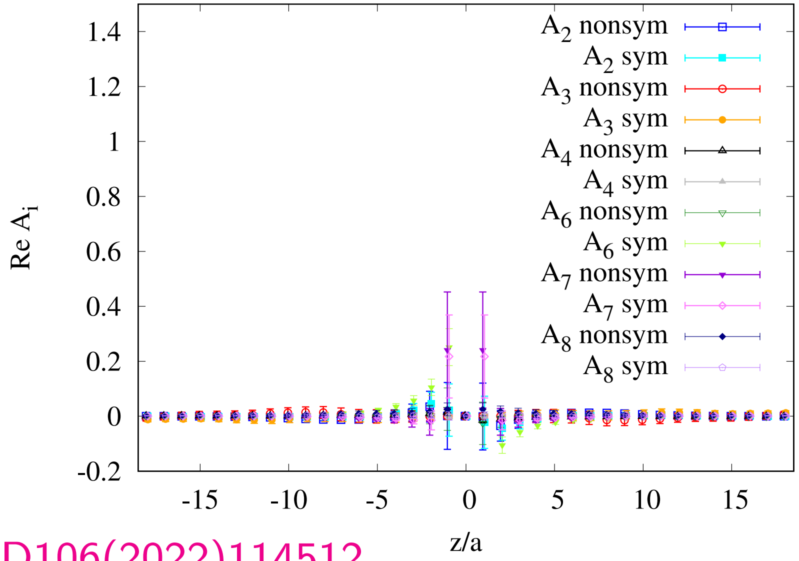
- matrix elements $\Pi_\mu(\Gamma_\nu)$ are frame-dependent,
- but the amplitudes A_i are frame-invariant.

Comparison of amplitudes between frames

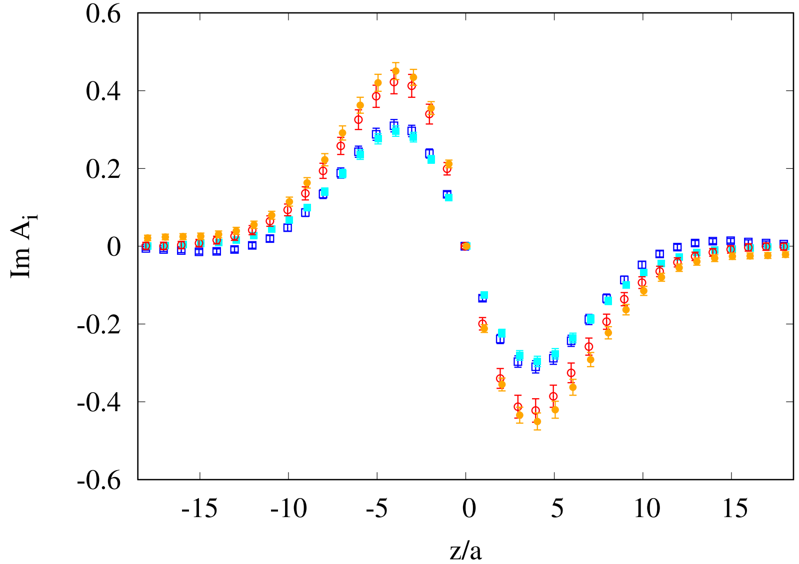
A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (subleading ones)



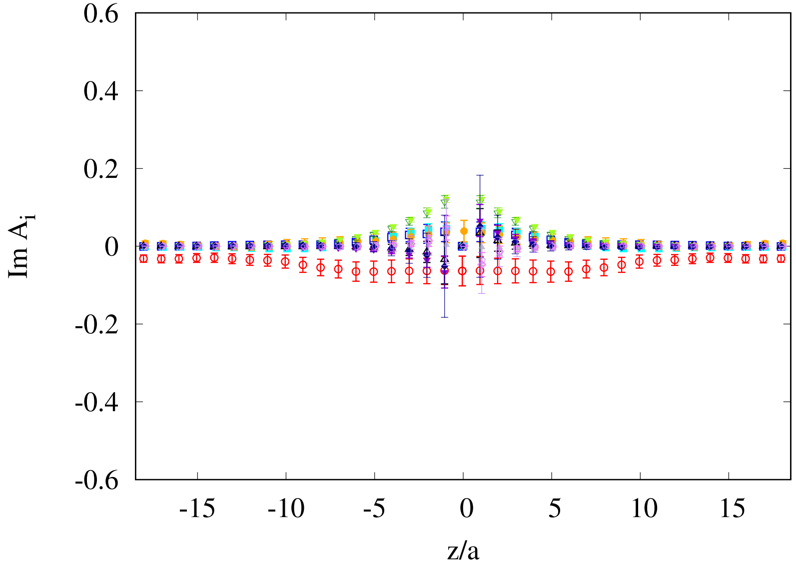
Re



S. Bhattacharya et al., PRD106(2022)114512



Im



H and E GPDs – possible definitions

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,,$$

$$F_{E^{(0)}} = -A_1 - \frac{m^2 z}{P_3} A_4 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6 \,.$$

ASYMMETRIC frame:

$$F_{H^{(0)}} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8 \,,$$

$$F_{E^{(0)}} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8 \,.$$

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3} A_6\,,$$

$$F_{E(0)} = -A_1 - \frac{m^2 z}{P_3} A_4 + 2A_5 + \frac{z\left(4E^2-\Delta_1^2-\Delta_2^2\right)}{2P_3} A_6\,.$$

ASYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z\Delta_0}{2P_0P_3} A_4 + \frac{z(\Delta_0^2+\Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3+\Delta_0\Delta_\perp^2)}{2P_0P_3} A_8\,,$$

$$F_{E(0)} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0+2P_0)}{2P_0P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2+2P_0\Delta_0+4P_0^2+\Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2+2\Delta_0P_0+4P_0^2+\Delta_\perp^2\right)}{2P_0P_3} A_8\,.$$

One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

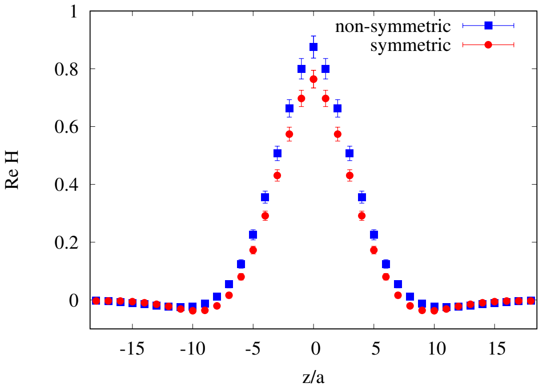
$$F_H = A_1\,,$$

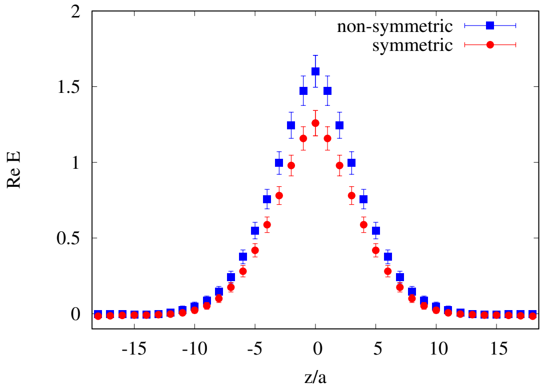
$$F_E = -A_1 + 2A_5 + 2zP_3A_6\,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

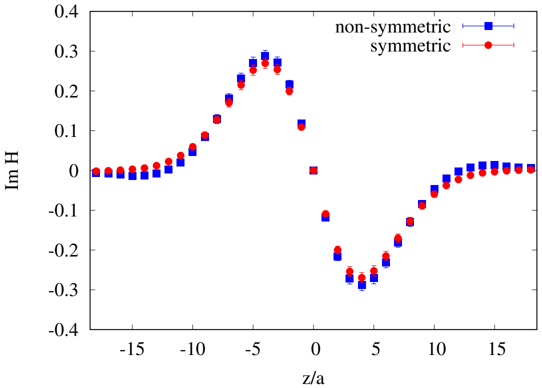
STANDARD DEFINITION

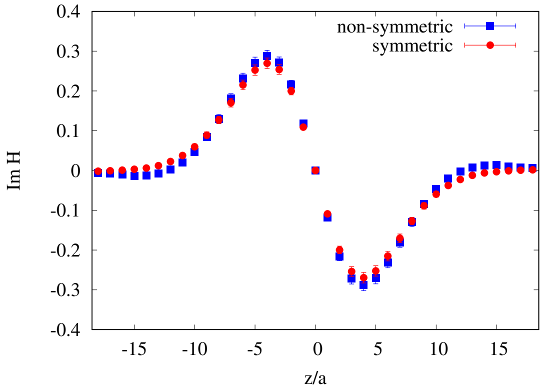
H-GPD E-GPD





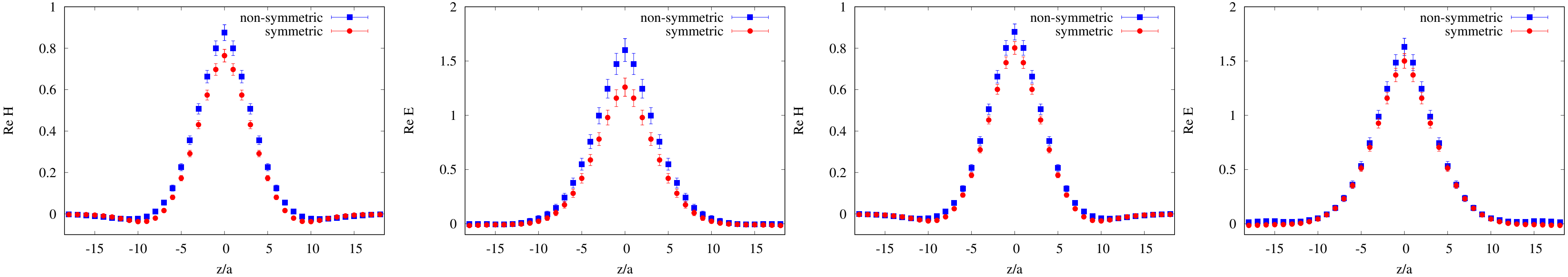
S. Bhattacharya et al., PRD106(2022)114512





STANDARD DEFINITION | LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD



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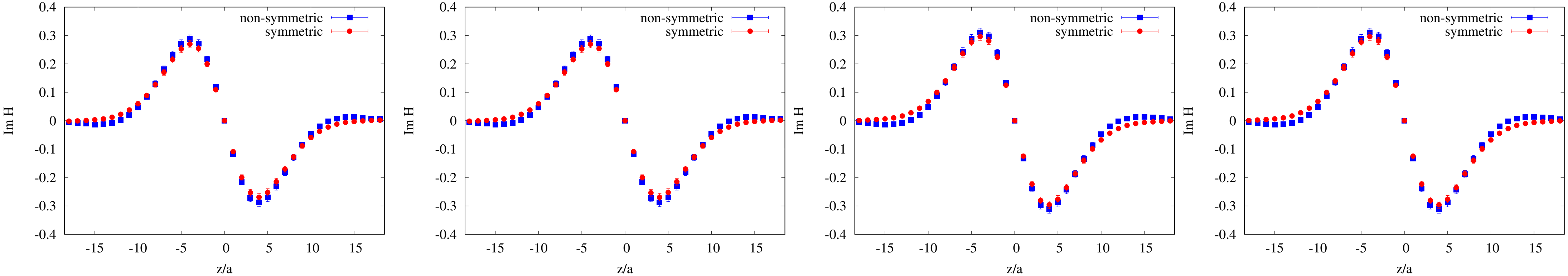
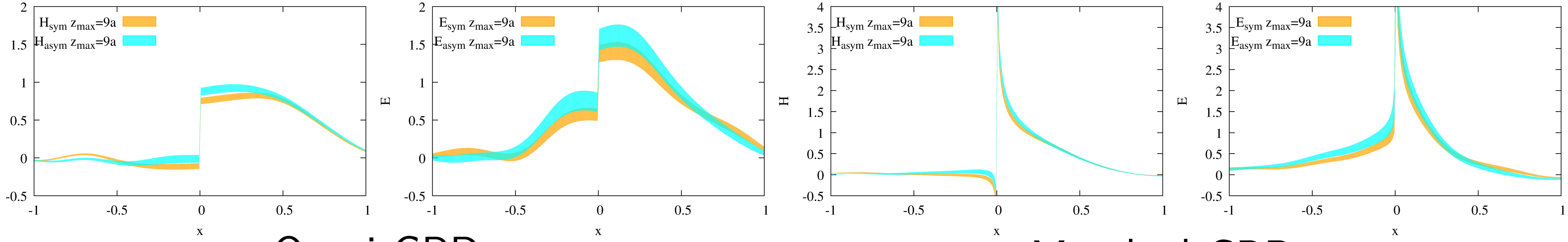


Quasi- and matched H and E GPDs

STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

Quasi- and matched H and E GPDs

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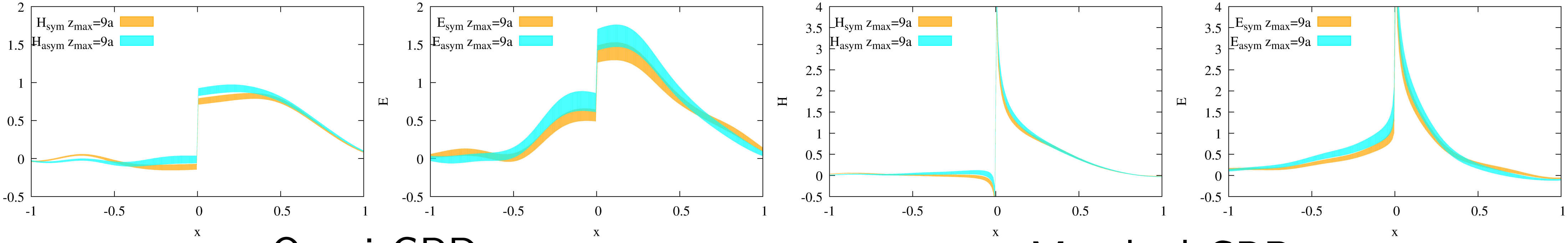
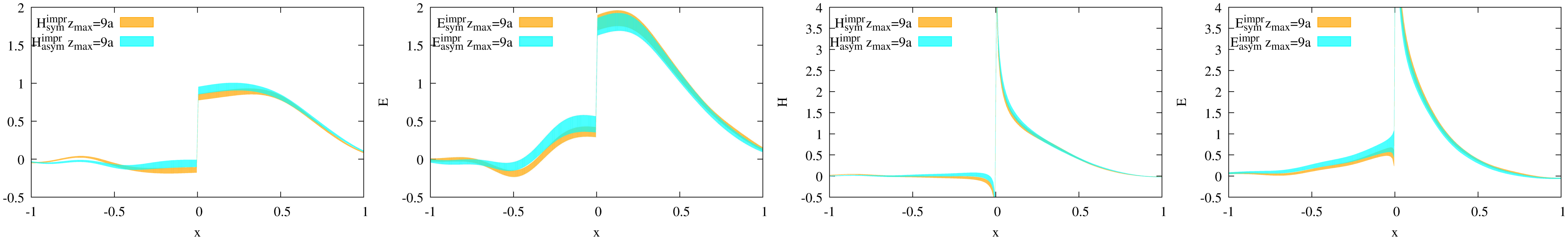


Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



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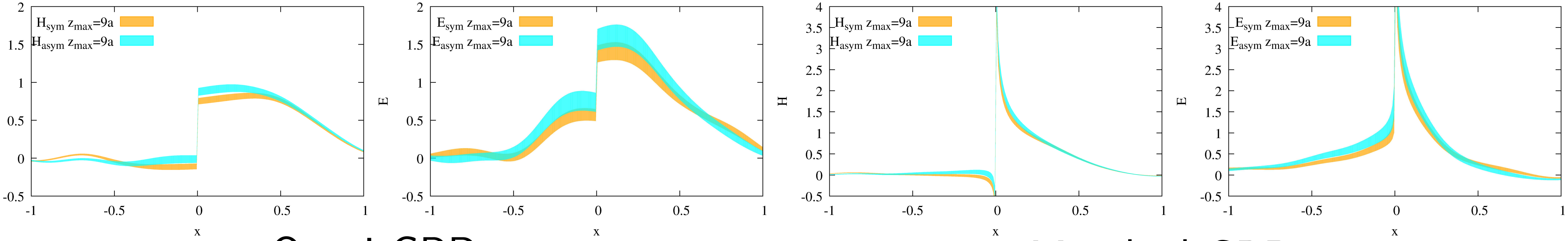

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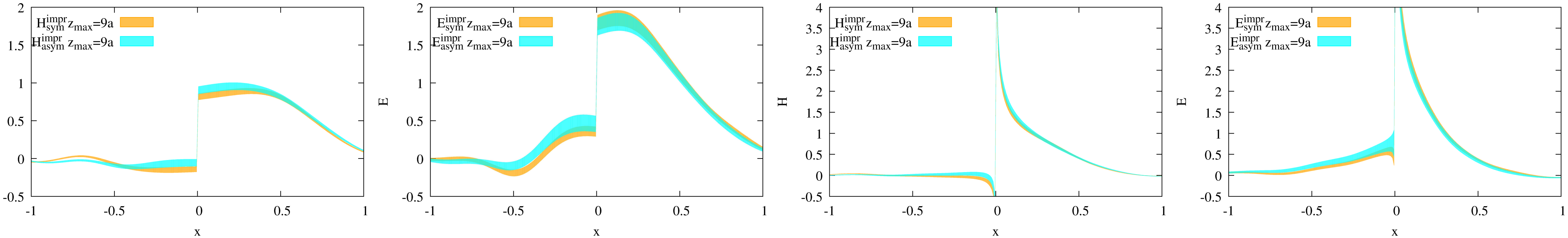
H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



Main conclusions:

– GPDs can be computed in non-symmetric frames, reducing the computational cost

– GPDs can be made frame-independent (Lorentz-invariant definition) – potentially better convergence

Overall, it gives much better perspectives for lattice GPDs!

Work in progress

A lot of extensions currently in progress:

- helicity GPDs,
- transversity GPDs,
- twist-3 GPDs,
- other hadrons: pion and kaon GPDs.

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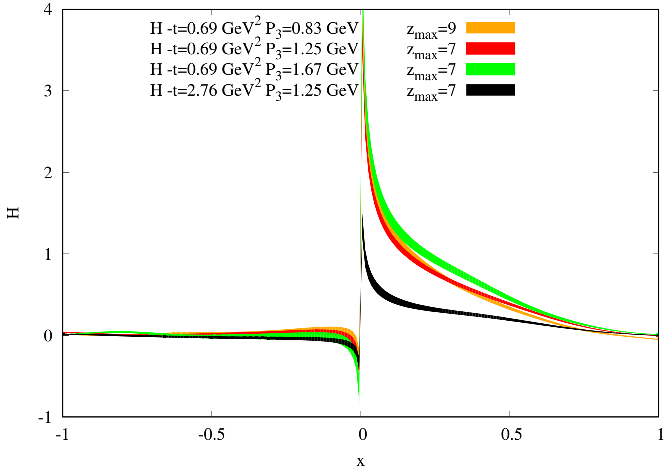
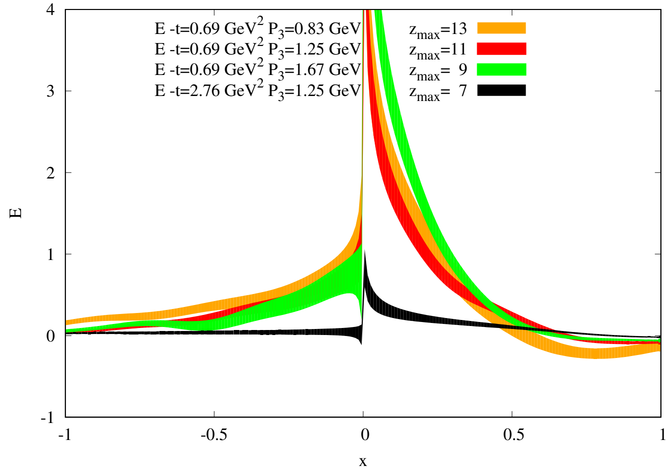
- transversity GPDs,
- twist-3 GPDs,
- other hadrons: pion and kaon GPDs.

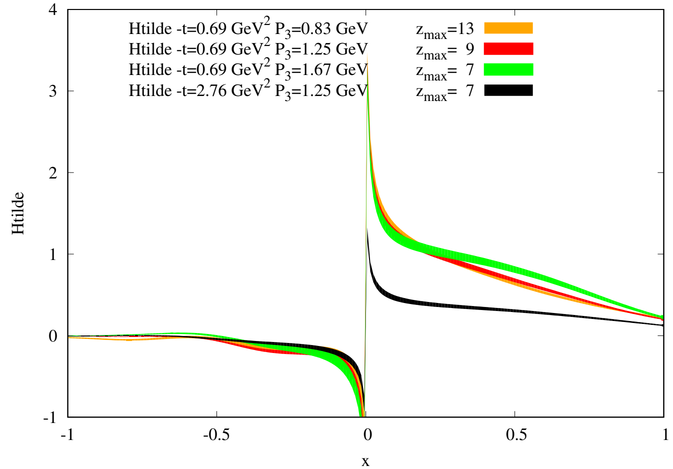
Also, intensive calculations extending the kinematic coverage:

- investigation of convergence towards the light cone: $P_3 = 0.83,\ 1.25,\ 1.67$ GeV for $-t = 0.69$ GeV^2 ($Q = (2,0,0)$),
- additional momentum transfers:
  - ⋆ symmetric: $-t = 0.69,\ 1.38,\ 2.76$ GeV^2 ($Q = (2,0,0),\ (2,2,0),\ (4,0,0)$),
  - ⋆ asymmetric: $-t = 0.17,\ 0.34,\ 0.64,\ 0.81,\ 1.24,\ 1.38,\ 1.52,\ 2.29$ GeV^2 ($Q = (1,0,0),\ (1,1,0),\ (2,0,0),\ (2,1,0),\ (2,2,0),\ (3,0,0),\ (3,1,0),\ (4,0,0)$),

STANDARD DEFINITION







UNPOLARIZED

H-GPD

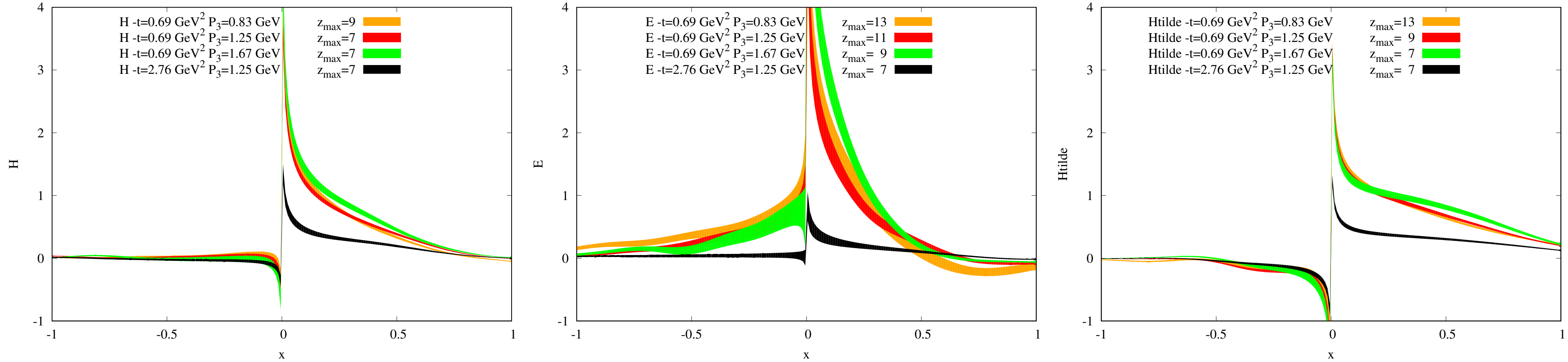
E-GPD

HELICITY

$\widetilde{H}$-GPD

Convergence of different definitions

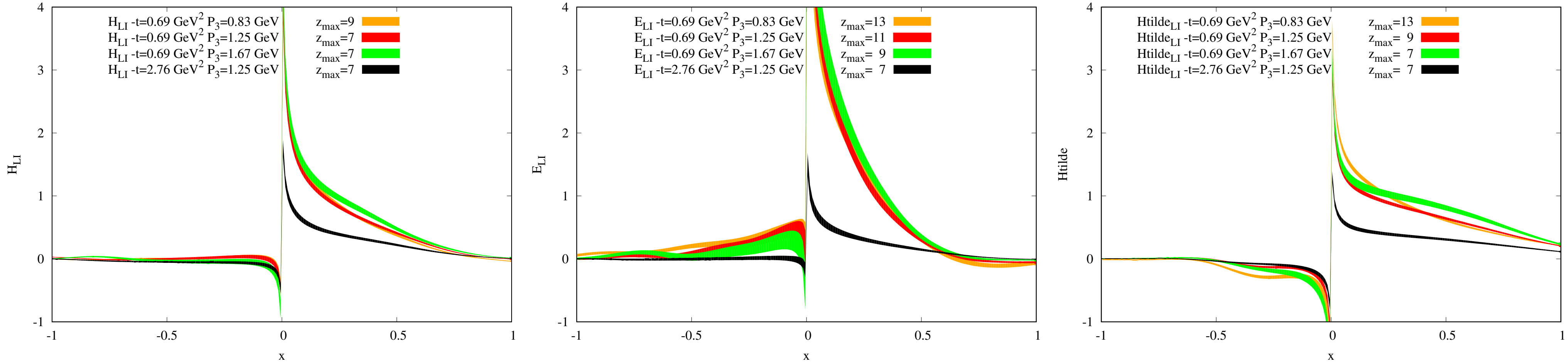
STANDARD DEFINITION



UNPOLARIZED HELICITY

H-GPD E-GPD $\widetilde{H}$-GPD

LORENTZ-INVARIANT DEFINITION



Conclusions and prospects

- **Last years: huge progress in lattice calculations of partonic distributions!**
- Recent breakthrough for GPDs:
  - ⋆ computationally more efficient calculations in non-symmetric frames,
  - ⋆ alternative definitions of GPDs may provide faster convergence to the light-cone.
- **Overall very encouraging results!**
- Still several challenges to overcome (control of systematics).
- Expect slow, but consistent progress and complementary role to pheno.

Conclusions and prospects

- **Last years: huge progress in lattice calculations of partonic distributions!**
- Recent breakthrough for GPDs:
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  - ⋆ alternative definitions of GPDs may provide faster convergence to the light-cone.
- **Overall very encouraging results!**
- Still several challenges to overcome (control of systematics).
- Expect slow, but consistent progress and complementary role to pheno.

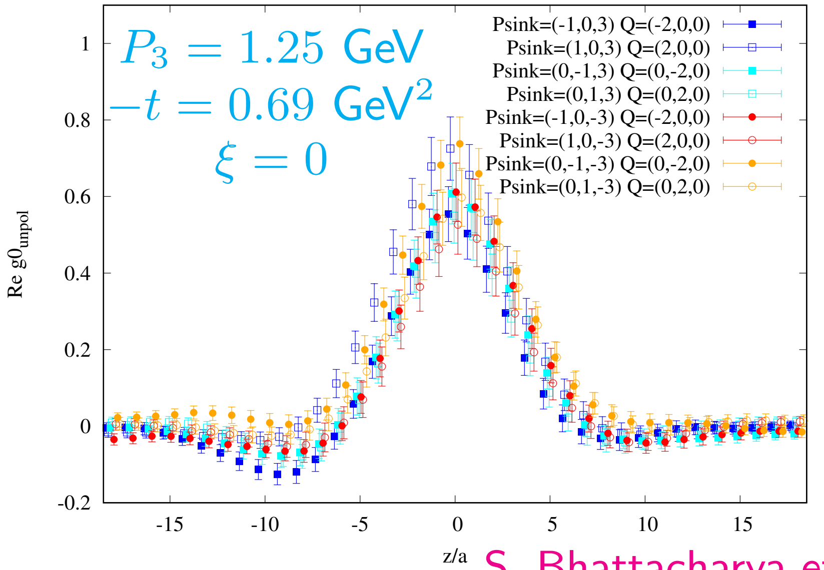
Thank you for your attention!

Backup slides

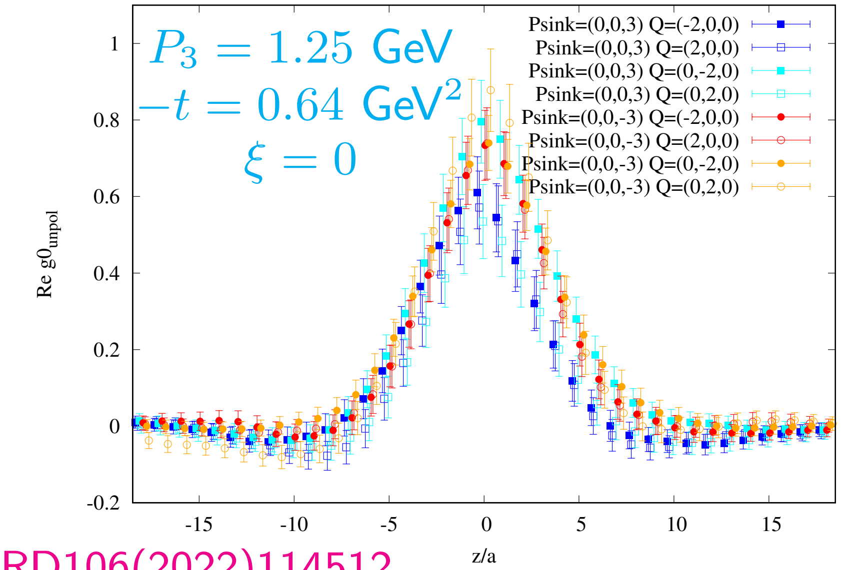
Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame

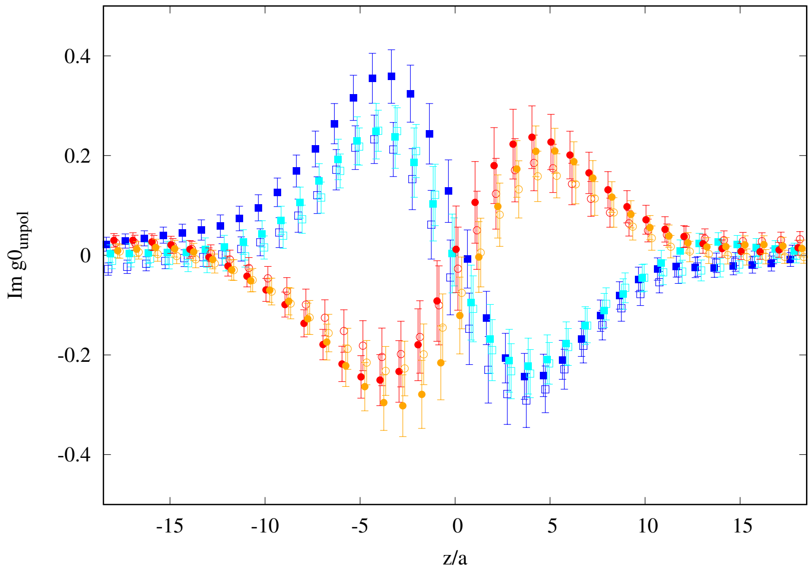
non-symmetric frame



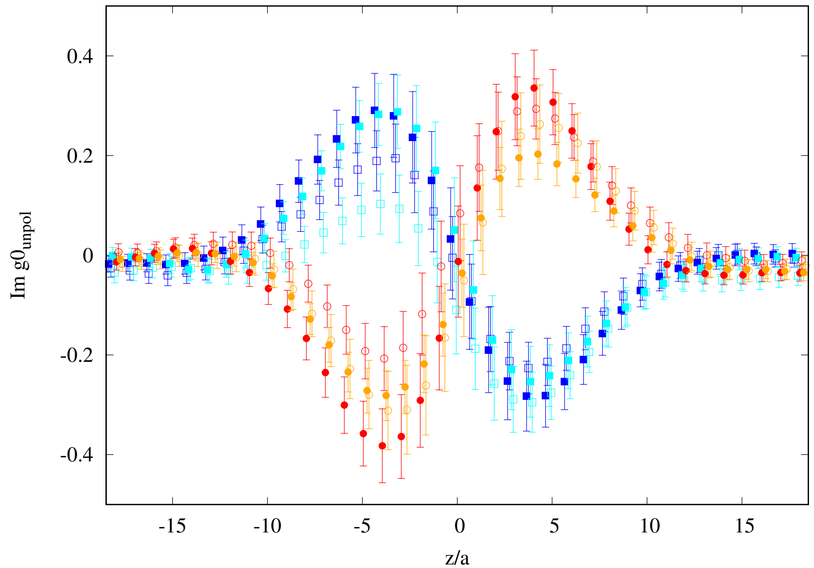
Re



S. Bhattacharya et al., PRD106(2022)114512

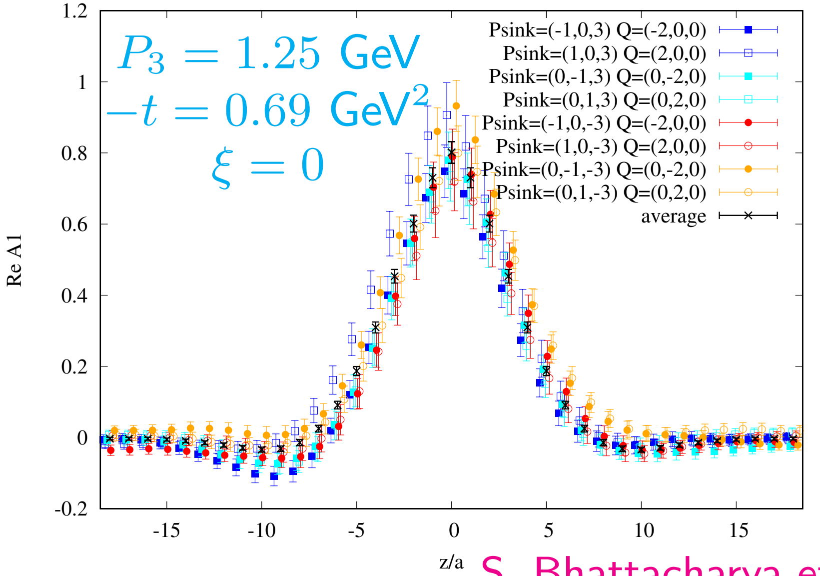


Im



Example amplitude A_1

symmetric frame | non-symmetric frame



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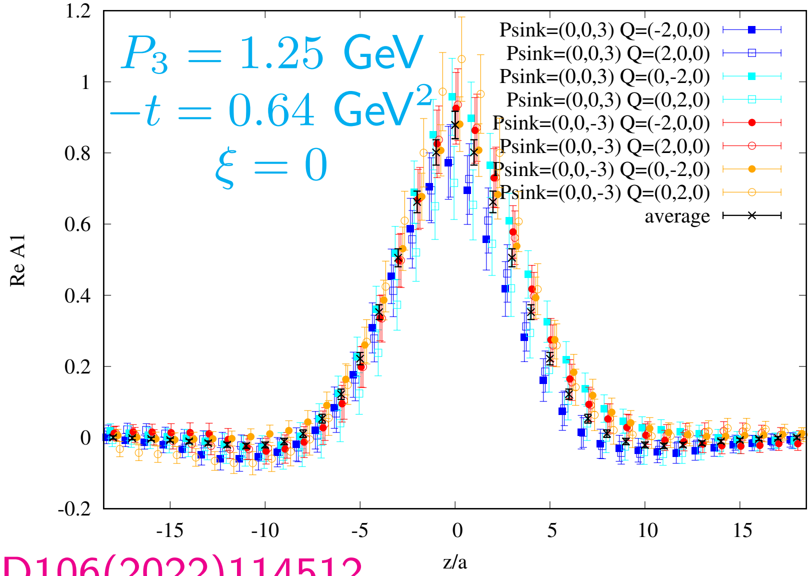
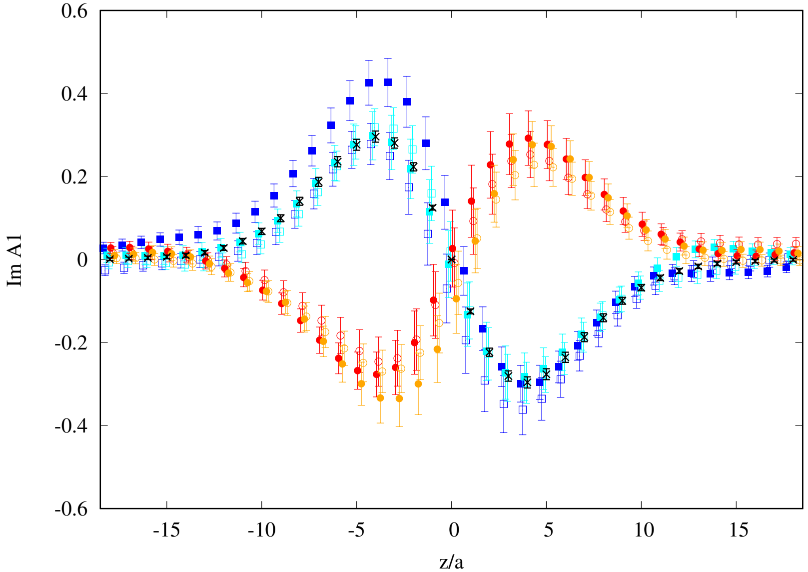


S. Bhattacharya et al., PRD106(2022)114512



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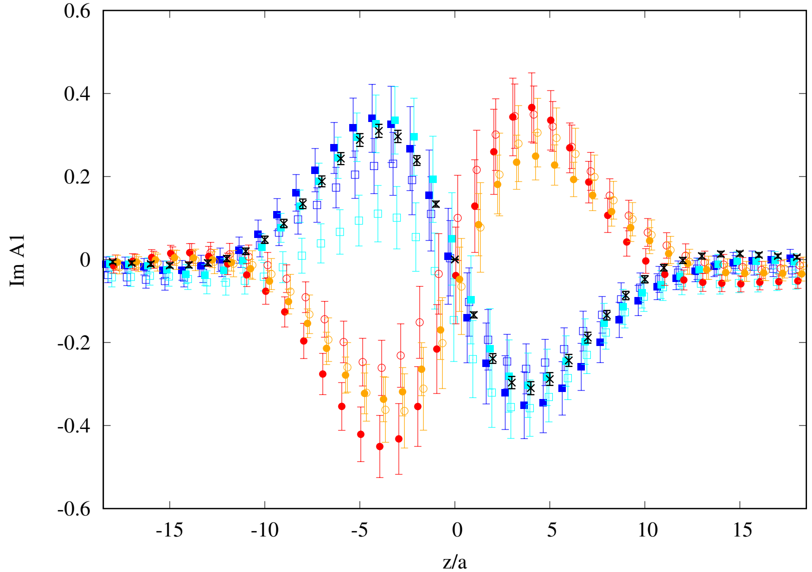
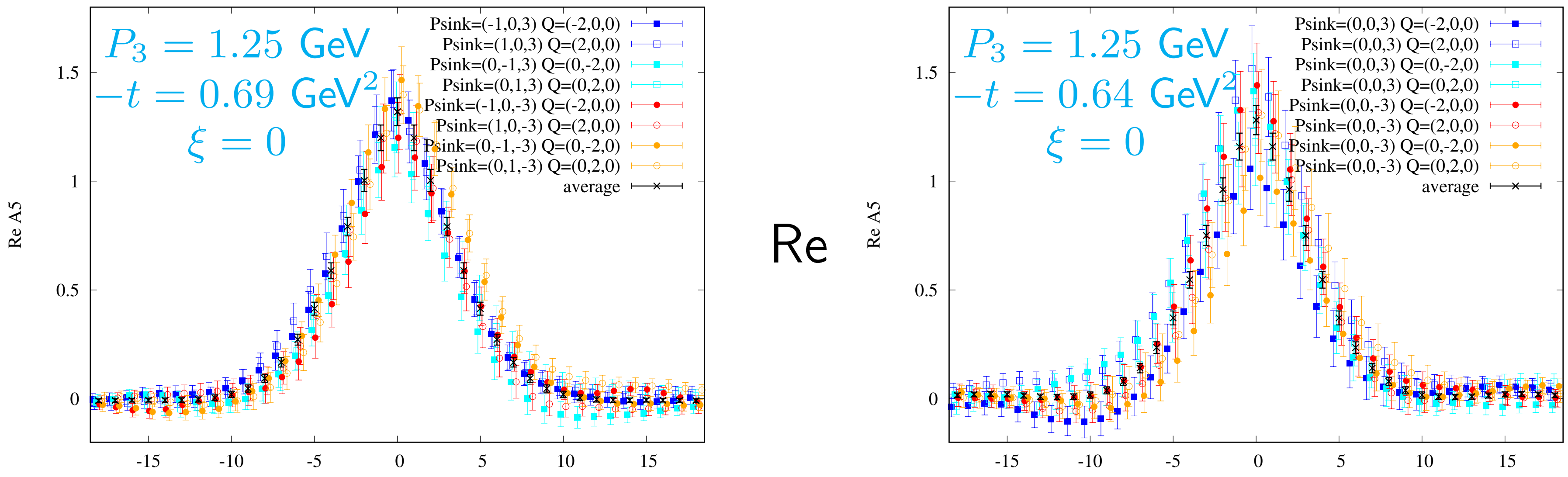
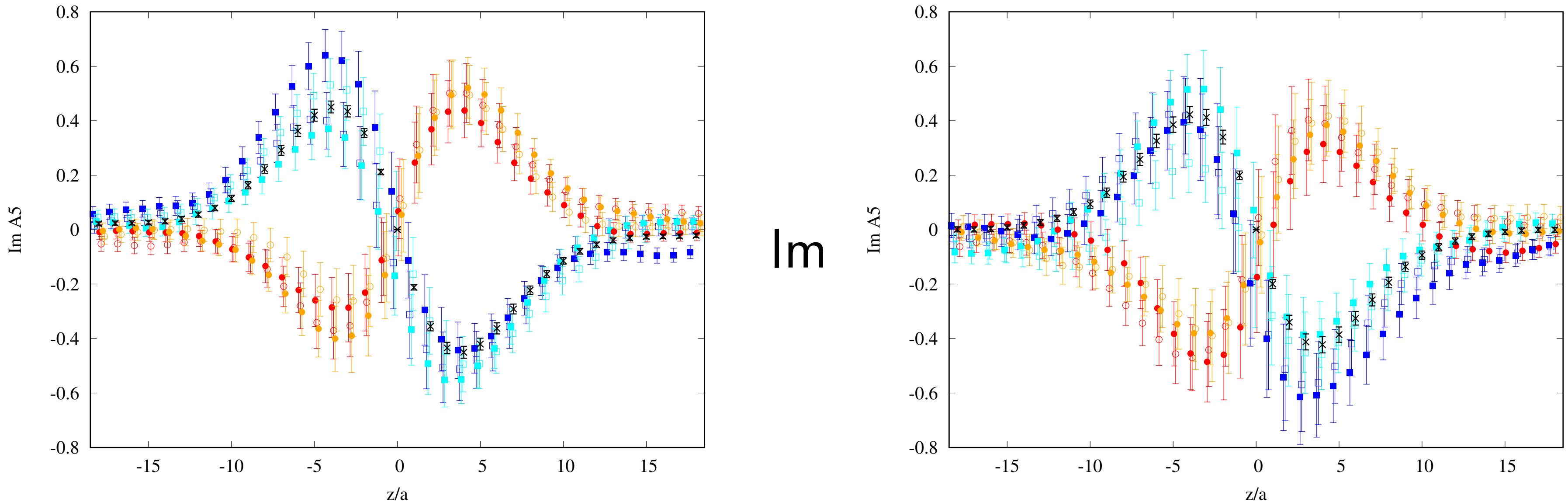


Example amplitude A_5

symmetric frame | **non-symmetric frame**

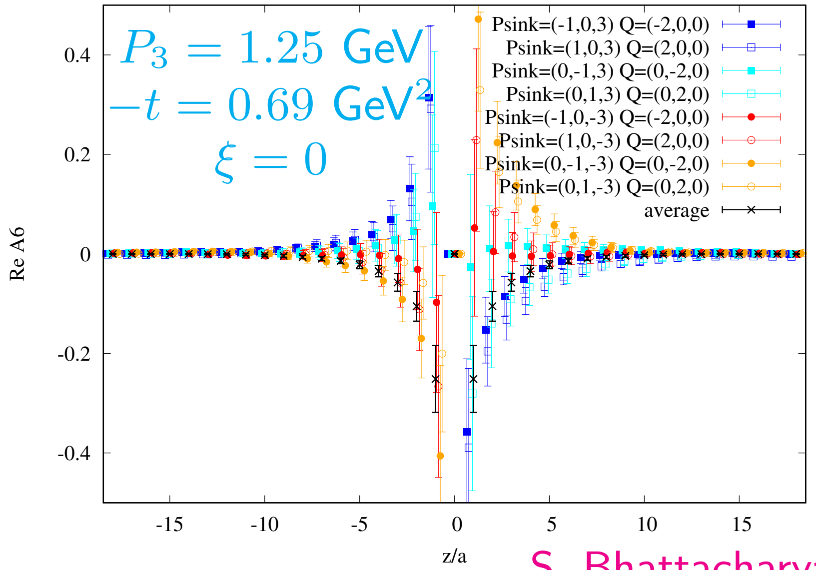


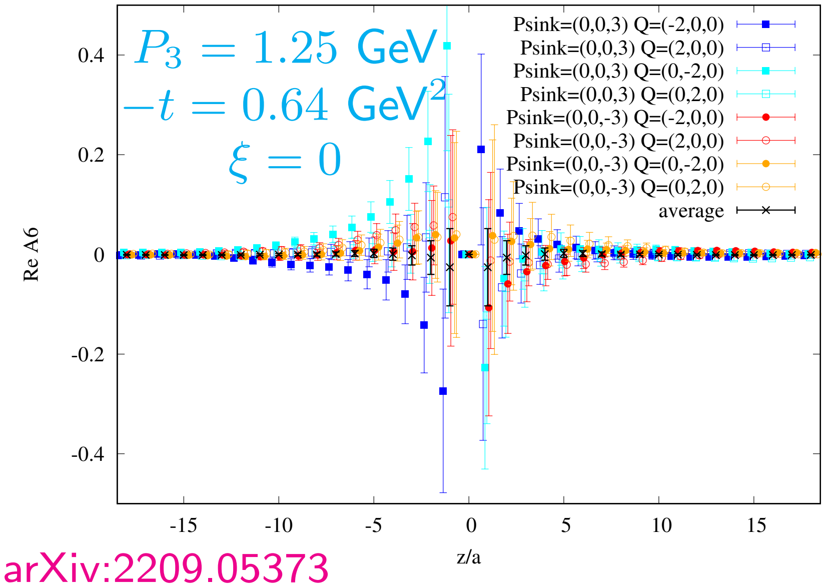
S. Bhattacharya et al., arXiv:2209.05373



Example amplitude A_6

symmetric frame | non-symmetric frame





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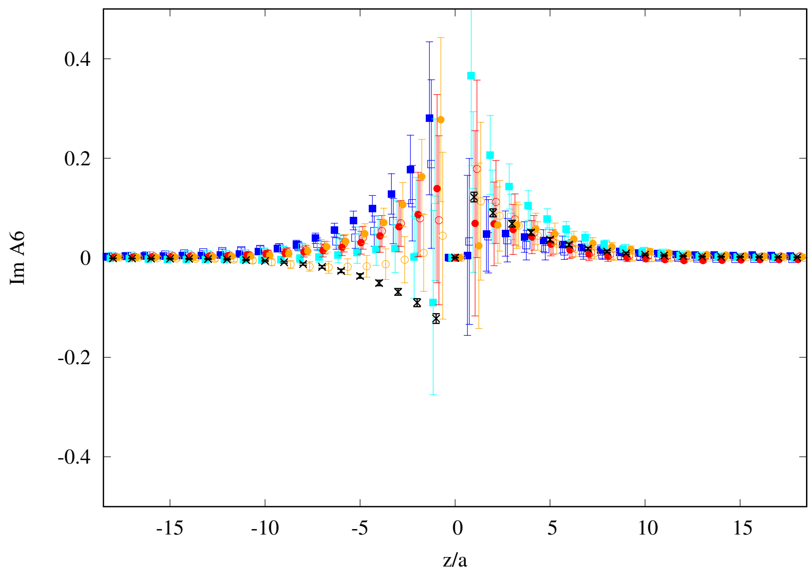
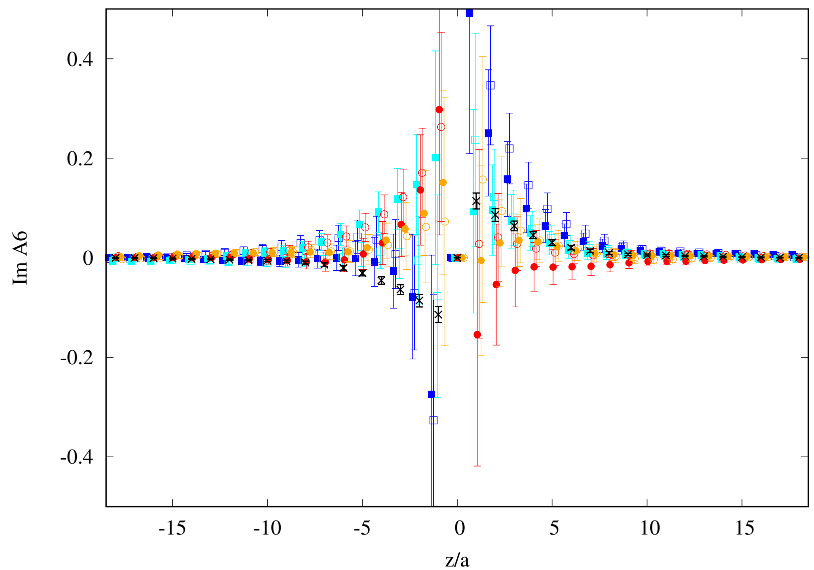
S. Bhattacharya et al., arXiv:2209.05373

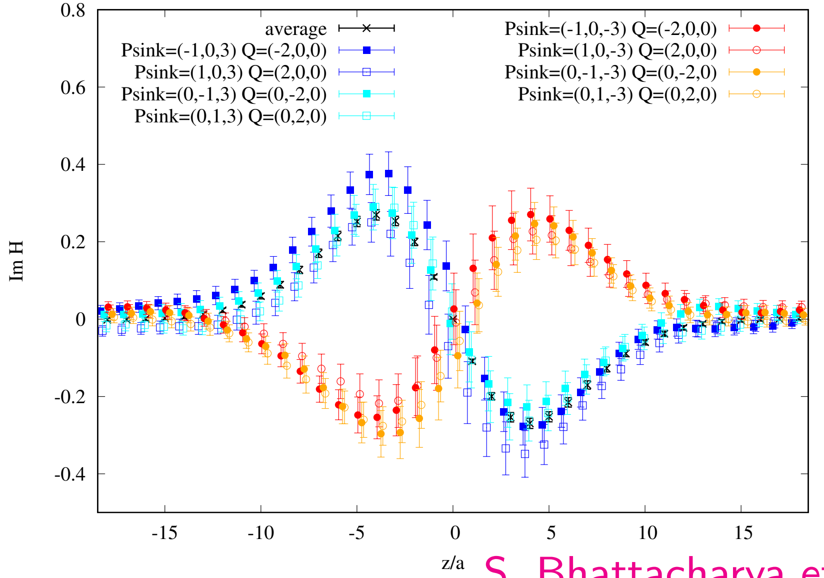


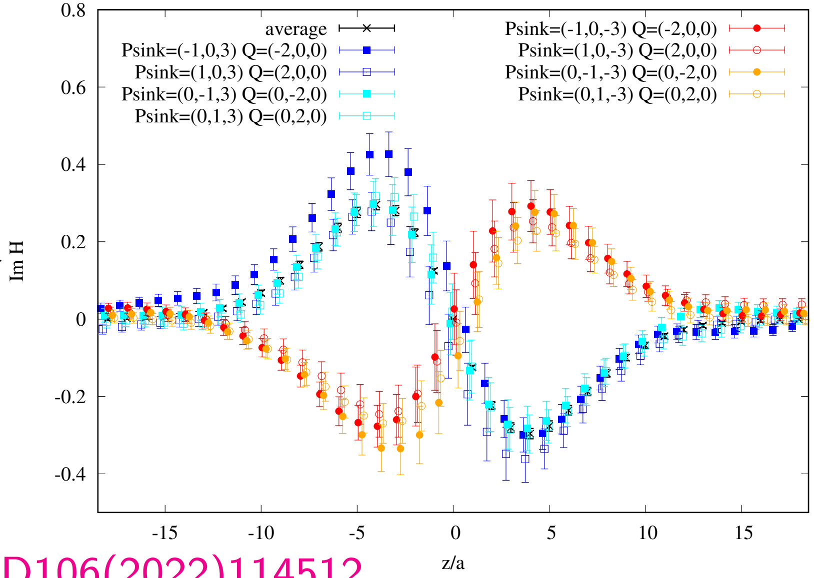


Im

H and E GPDs – signal improvement

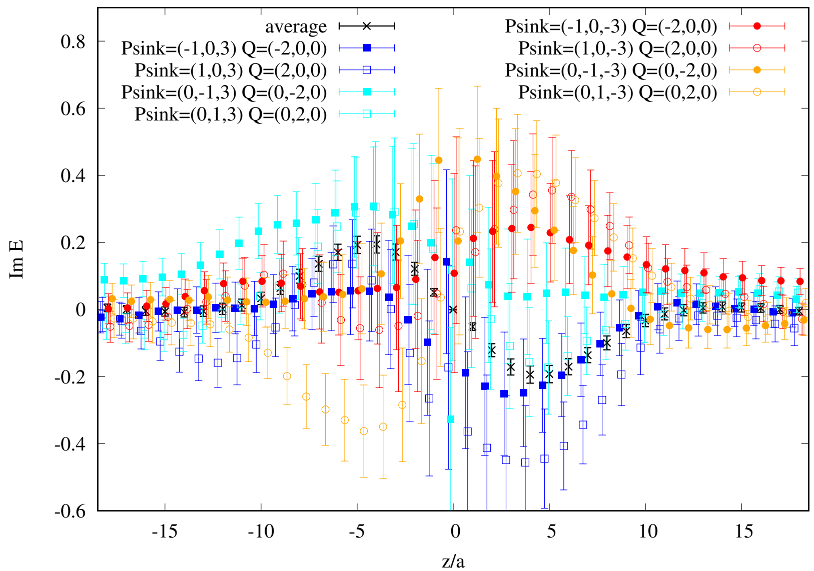
standard | Lorentz-invariant

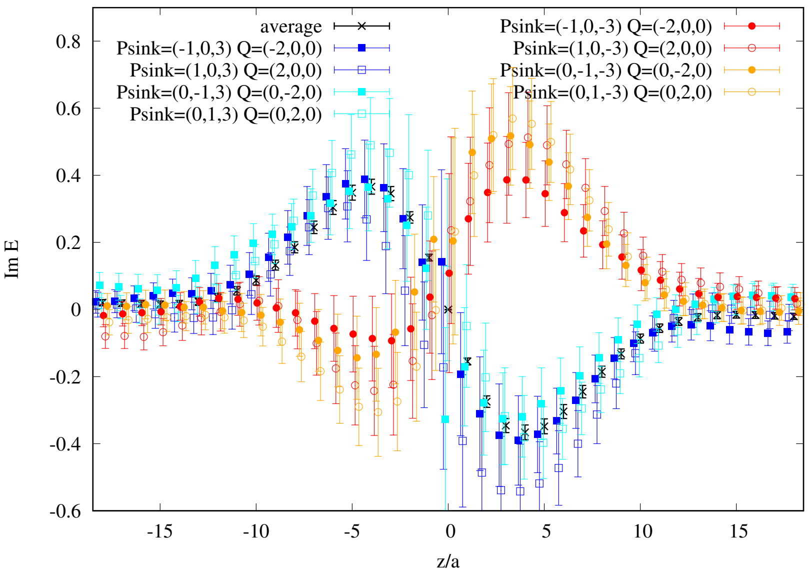




Im H

S. Bhattacharya et al., PRD106(2022)114512





Im E

Lorentz-covariant parametrization of matrix elements (axial vector case):

$$F^{[\gamma^\mu\gamma_5]} = \bar{u}(p',\lambda')\Big[\frac{i\epsilon^{\mu P z\Delta}}{m}A_1+\gamma^\mu\gamma_5 A_2+\gamma_5\Big(\frac{P^\mu}{m}A_3+mz^\mu A_4+\frac{\Delta^\mu}{m}A_5\Big)+m\not{z}\gamma_5\Big(\frac{P^\mu}{m}A_6+mz^\mu A_7+\frac{\Delta^\mu}{m}A_8\Big)\Big]u(p,\lambda)$$

Two definitions of $\widetilde{H}$:

standard: $F_{\tilde{H}} = A_2 + zP_3A_6 - m^2z^2A_7\,,$

Lorentz-invariant: $F_{\tilde{H}} = A_2 + zP_3A_6\,.$

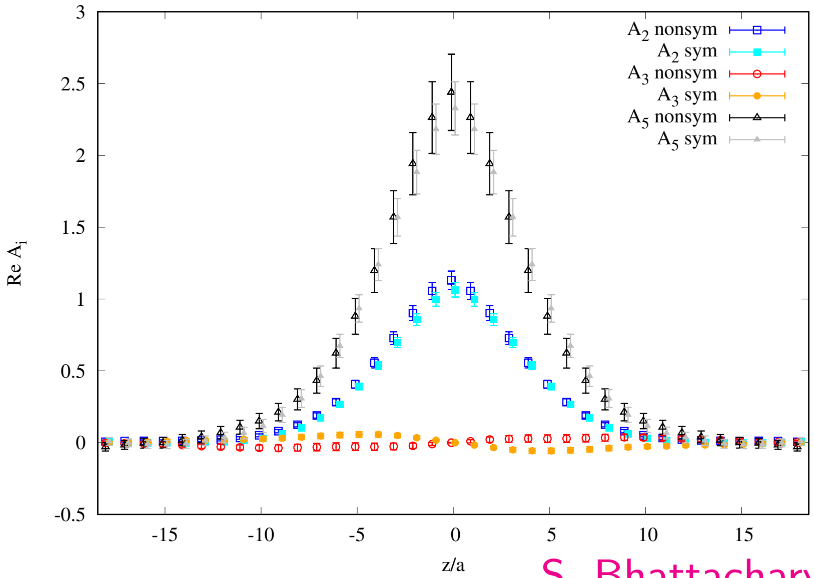
$\widetilde{E}$ seems impossible to extract at $\xi = 0$:

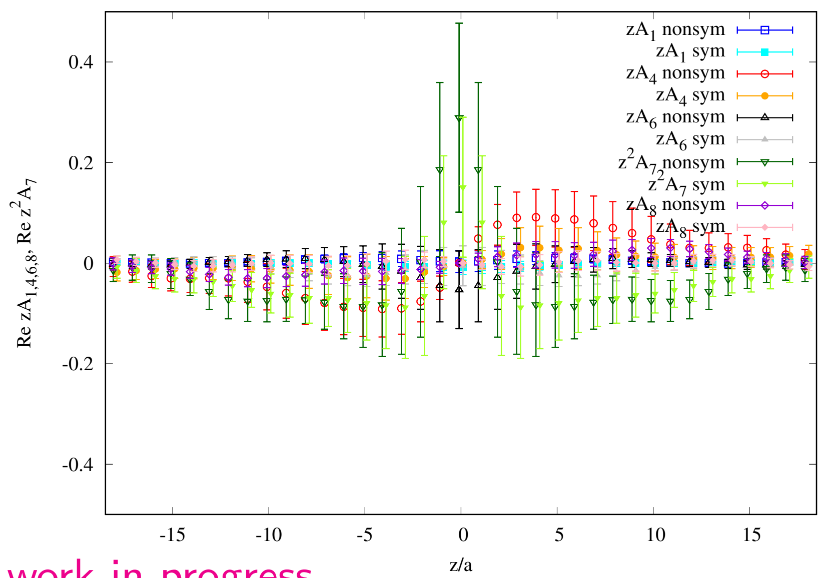
$$F_{\tilde{E}} = 2\,\frac{P\cdot z}{\Delta\cdot z}A_3 + 2A_5\,.$$

Helicity GPDs – work in progress

$A_2,\ A_3,\ A_5$ / $zA_1,\ zA_4,\ zA_6,\ z^2A_7,\ zA_8$

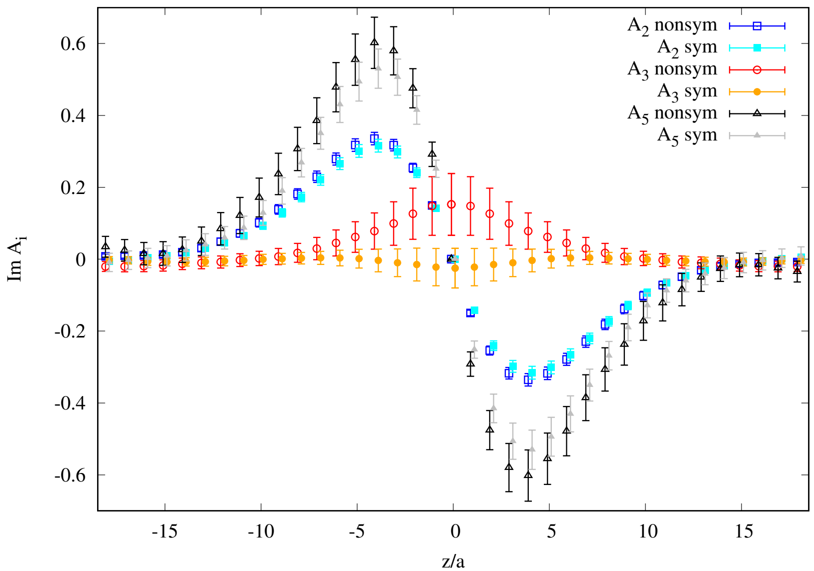
Re

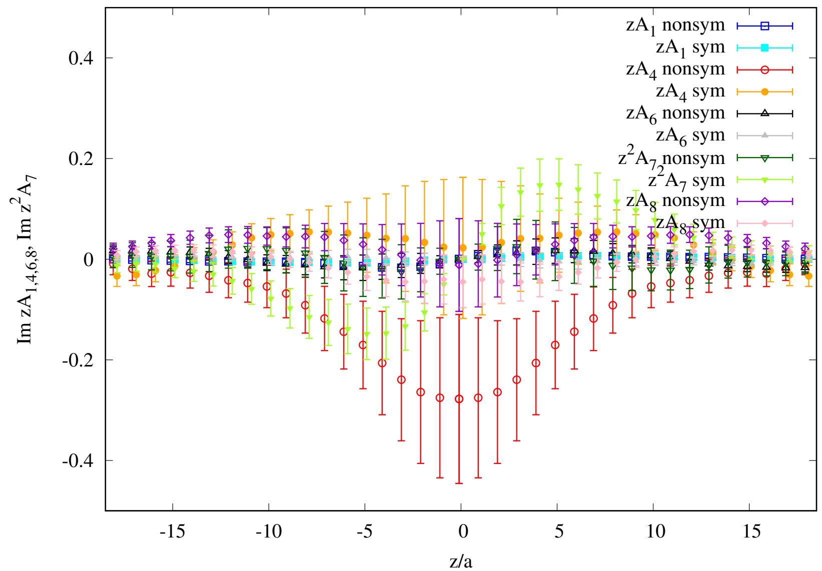




S. Bhattacharya et al., work in progress

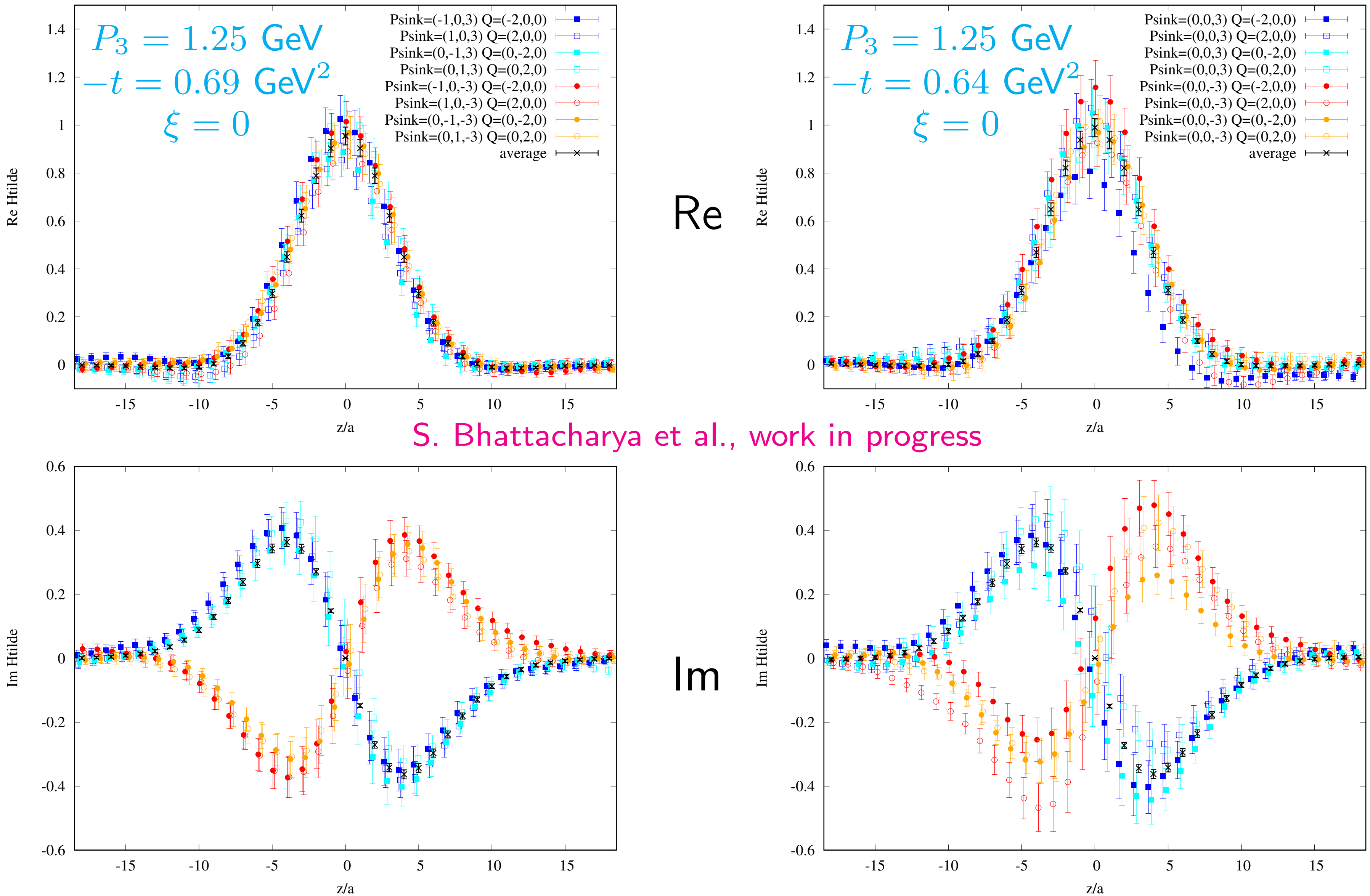
Im





Helicity GPD $\widetilde{H}$ (std. def.) – work in progress

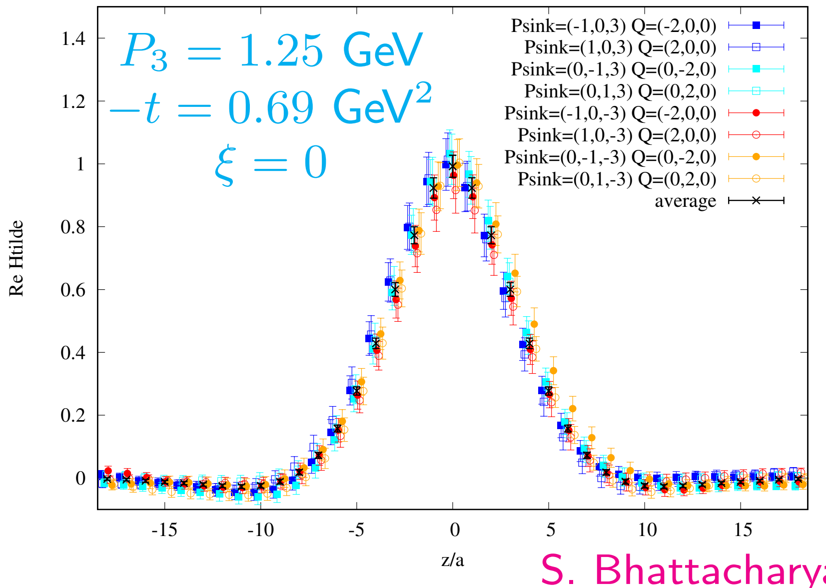
symmetric frame — non-symmetric frame

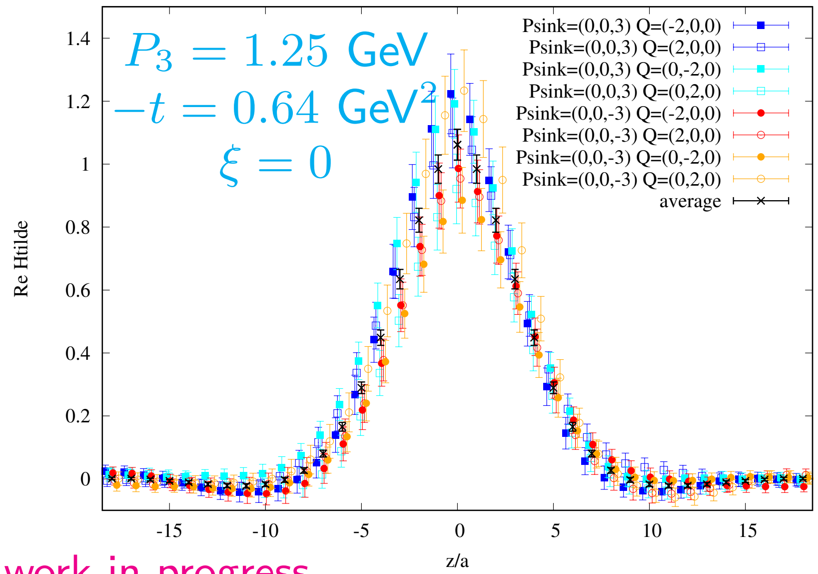


Helicity GPD $\widetilde{H}$ (LI def.) – work in progress

symmetric frame

non-symmetric frame





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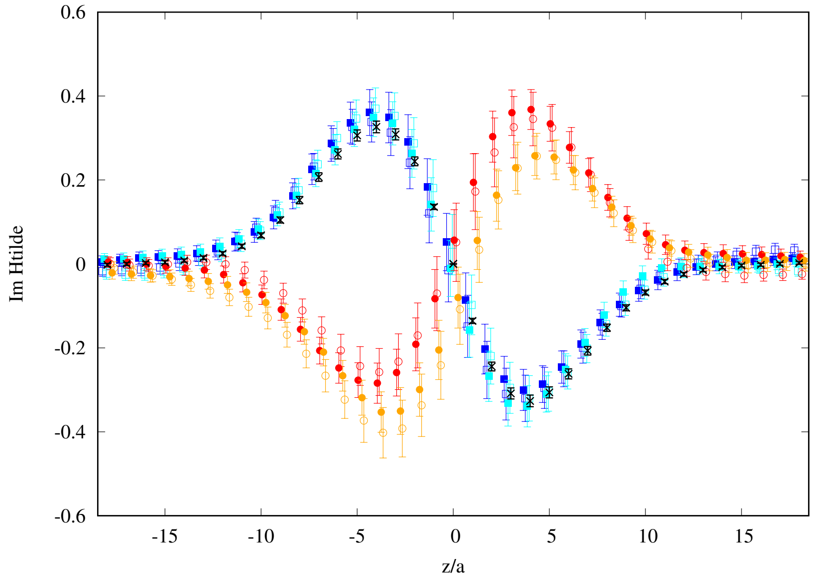
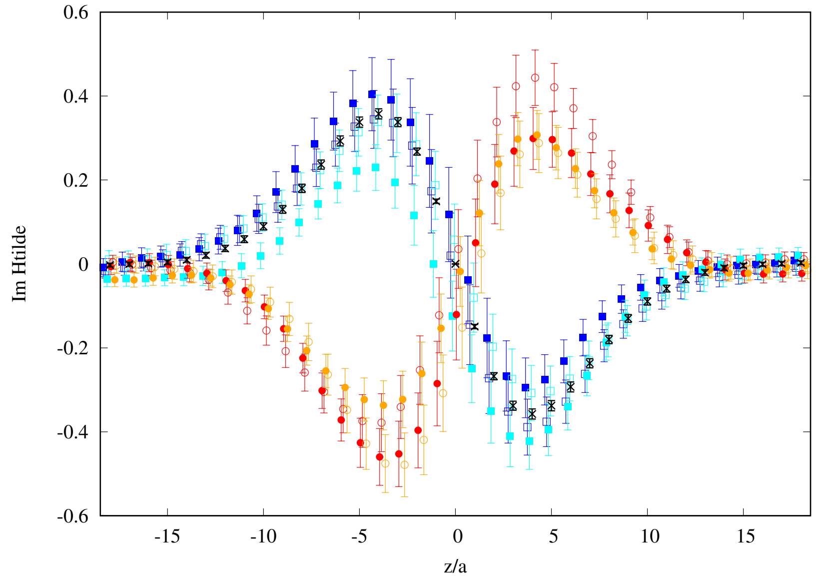
S. Bhattacharya et al., work in progress





Im

Transversity GPDs: ETMC, Phys. Rev. D105 (2022) 034501
4 GPDs: H_T, E_T, $\tilde{H}_T$, $\tilde{E}_T$

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{Q}$, $\vec{Q}$ – momentum transfer
lattice computation of bare ME

↓

renormalization
of bare ME
intermediate RI scheme

↓

reconstruction of x-dependence
z-space→x-space
Backus-Gilbert

↓

matching to light cone
RI→ $\overline{\text{MS}}$
(incl. evolution to $\mu = 2$ GeV)

↓

light-cone GPD

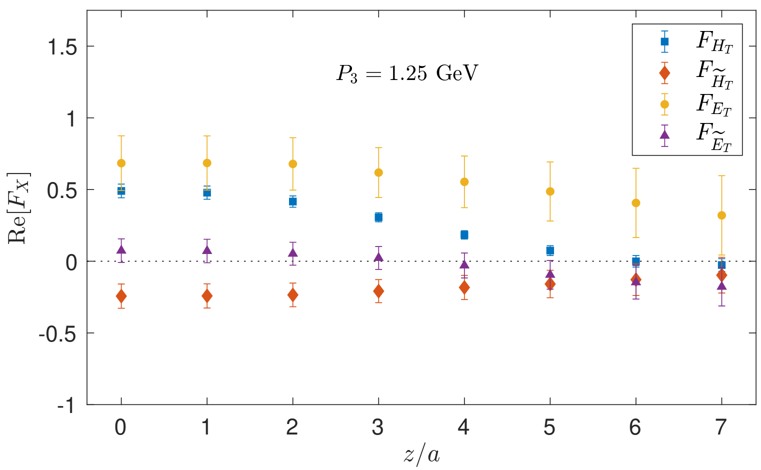
Three nucleon boosts ($\xi = 0$): $P_3 = 0.83, 1.25, 1.67$ GeV
Nucleon boost ($\xi \neq 0$): $P_3 = 1.25$ GeV

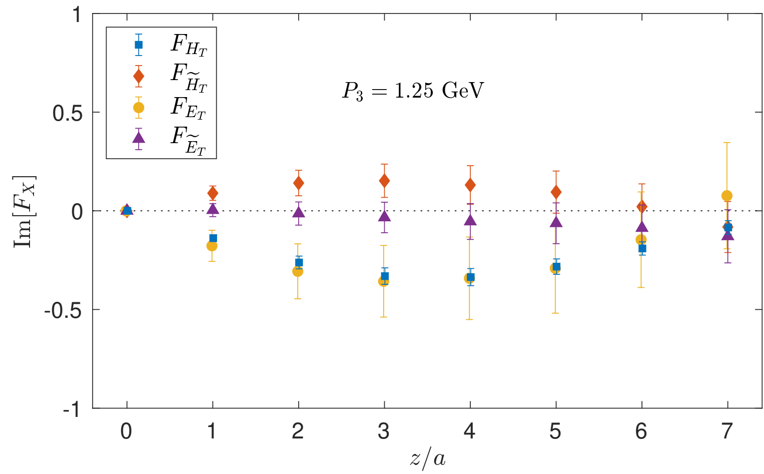
Momentum transfer ($\xi = 0$): $-t = 0.69$ GeV^2
Momentum transfer ($\xi \neq 0$): $-t = 1.02$ GeV^2

Renormalized ME

Real part | Imaginary part

$\xi = 1/3$





Transversity GPDs

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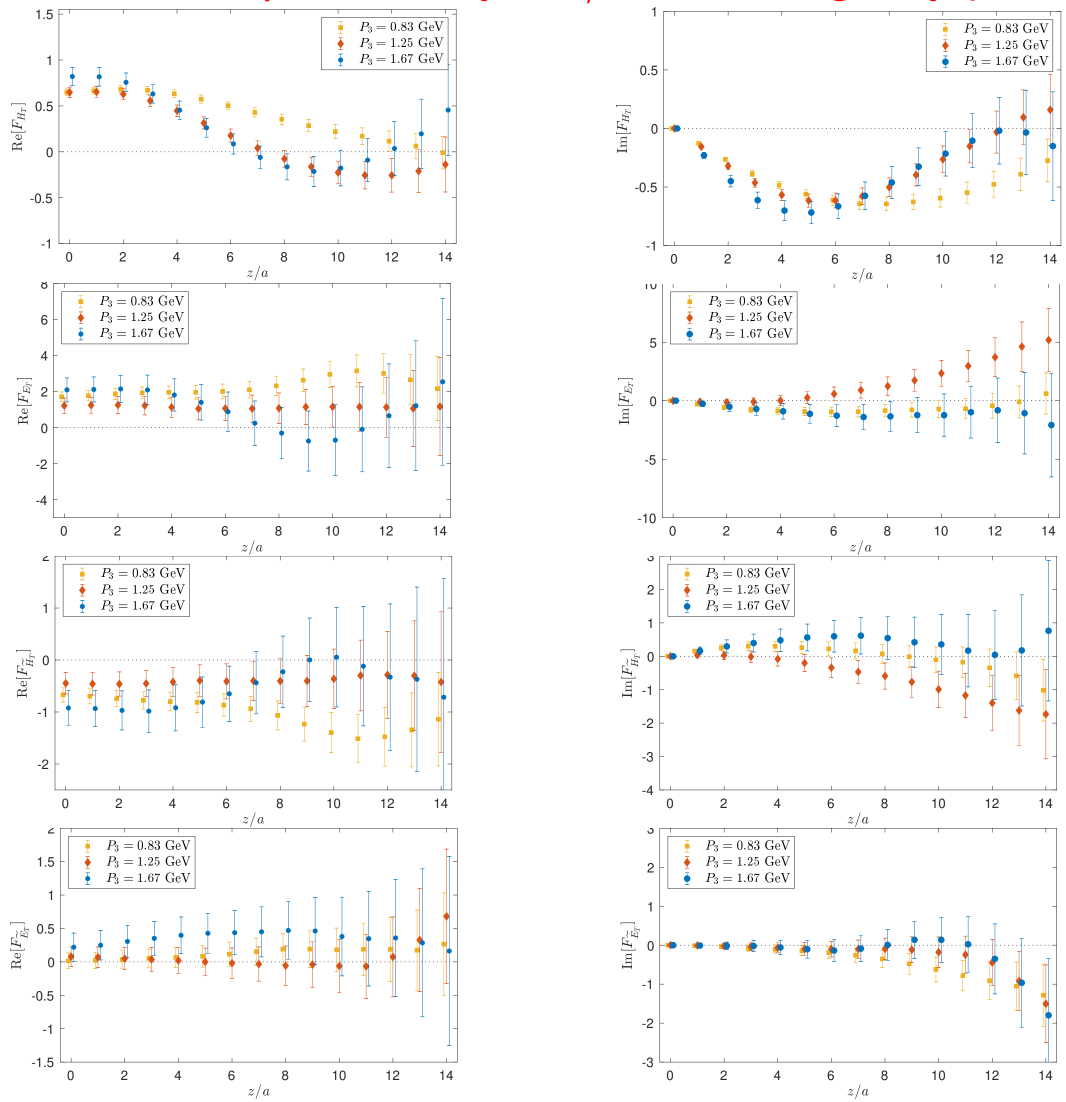
ETMC, Phys. Rev. D105 (2022) 034501

Real part $\xi = 1/3$ Imaginary part



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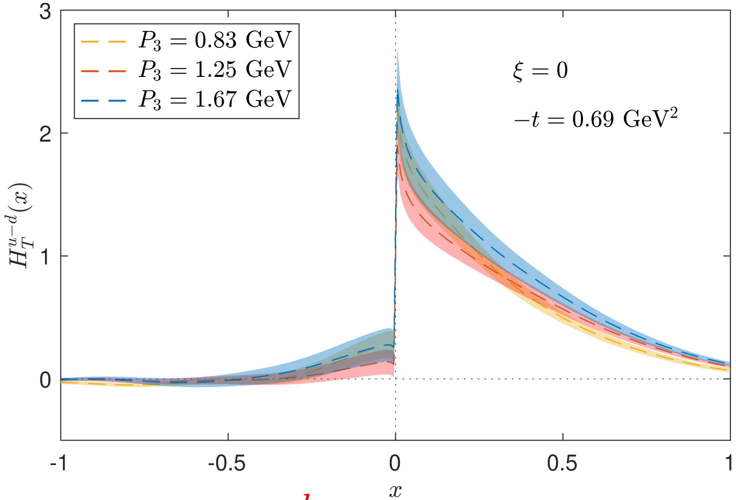
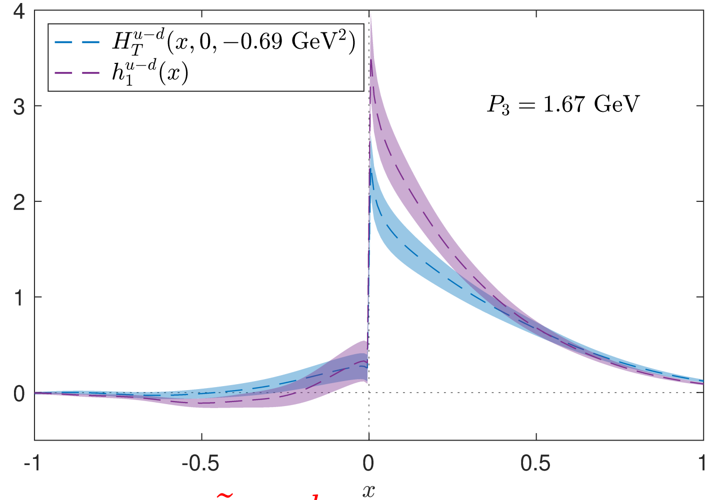
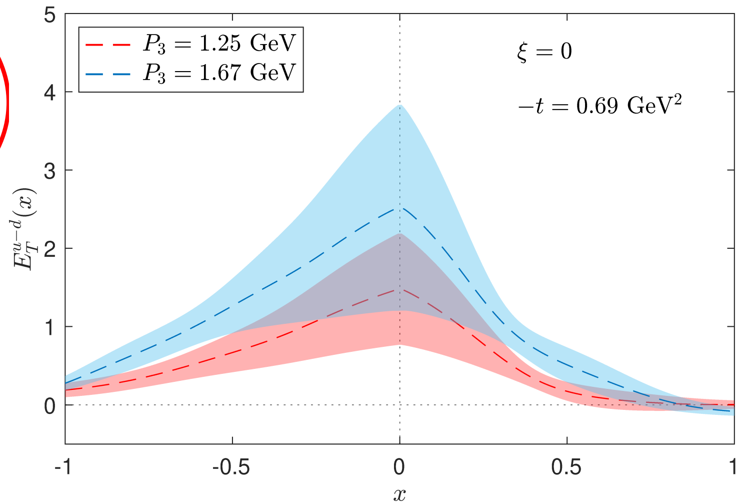
light-cone GPD

H_T^{u-d} ($\xi = 0$)

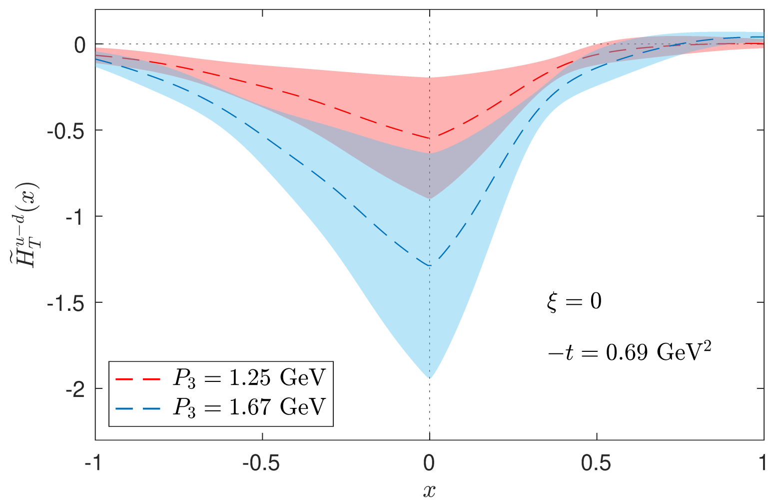




E_T^{u-d} ($\xi = 0$)



$\tilde{H}_T^{u-d}$ ($\xi = 0$)



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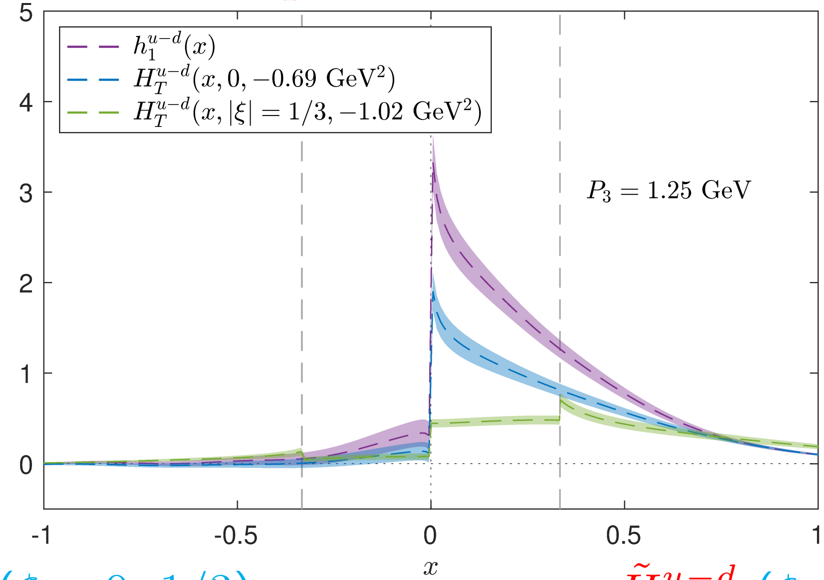
↓

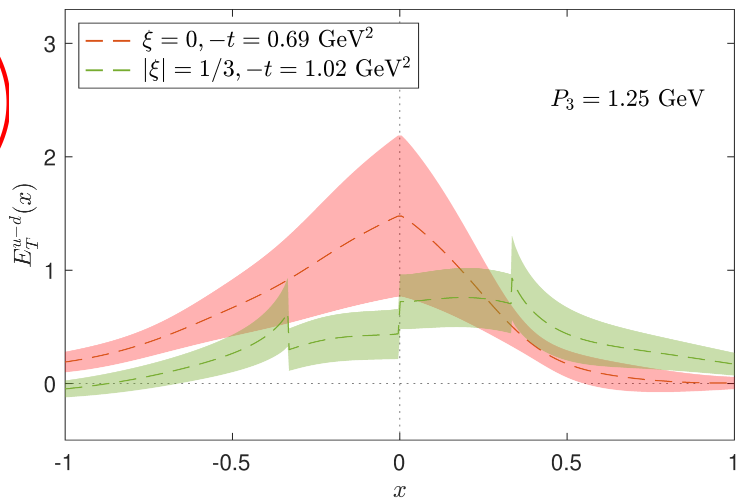
light-cone GPD

ETMC, Phys. Rev. D105 (2022) 034501

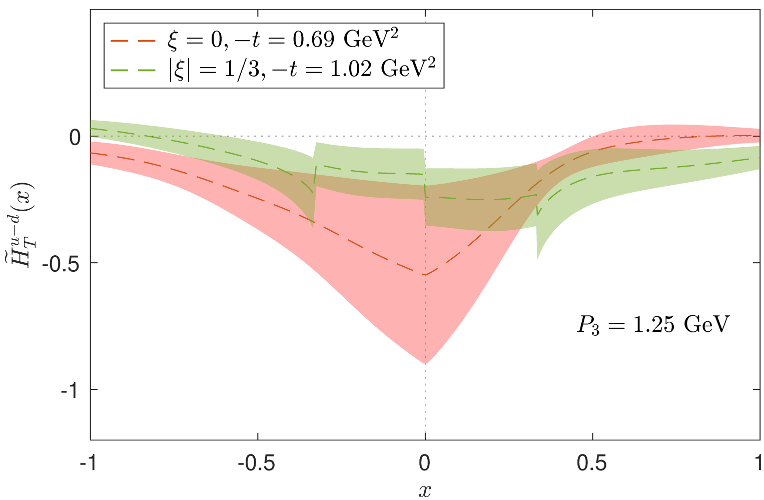
H_T^{u-d} $(\xi = 0,\ 1/3)$



E_T^{u-d} $(\xi = 0,\ 1/3)$



$\tilde{H}_T^{u-d}$ $(\xi = 0,\ 1/3)$



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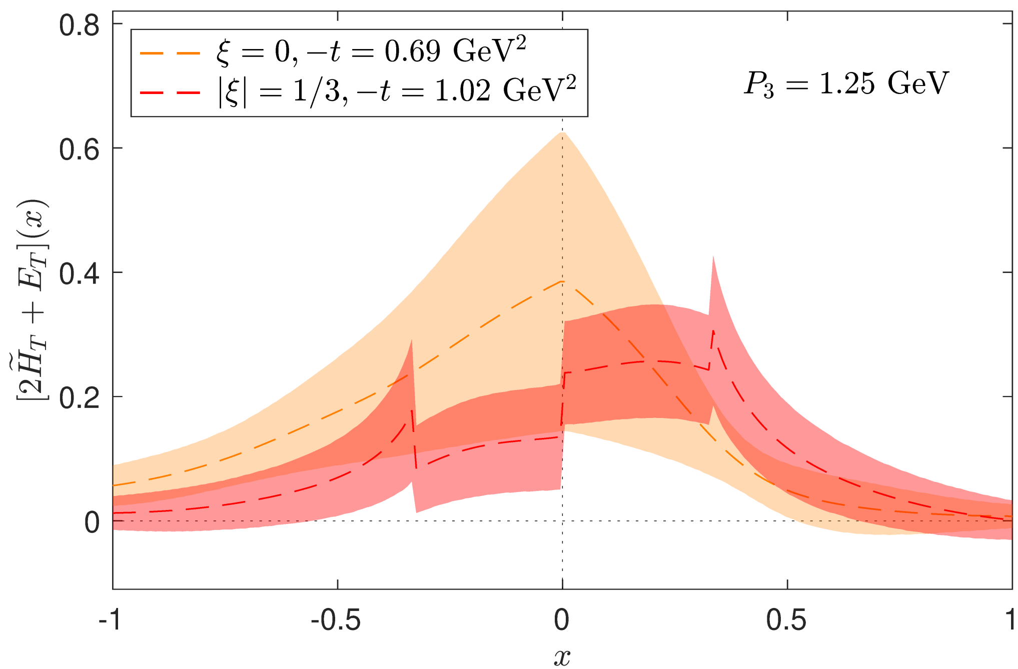
light-cone GPD

More fundamental quantity: $E_T + 2\tilde{H}_T$

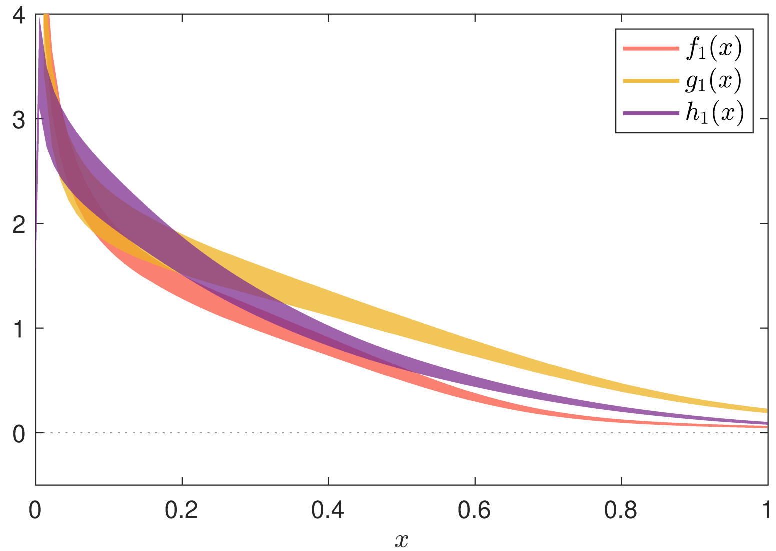
– related to the transverse spin structure of the proton
– physically interpreted as lateral deformation in the distribution of transversely polarized quarks in an unpolarized proton
– lowest Mellin moment in the forward limit: transverse spin-flavor dipole moment in an unpolarized target (k_T)
– second moment related to the transverse-spin quark angular momentum in an unpolarized proton

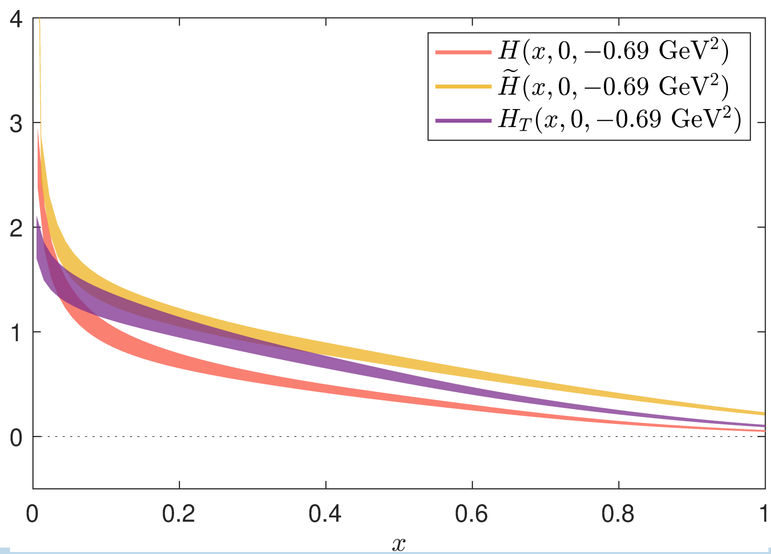


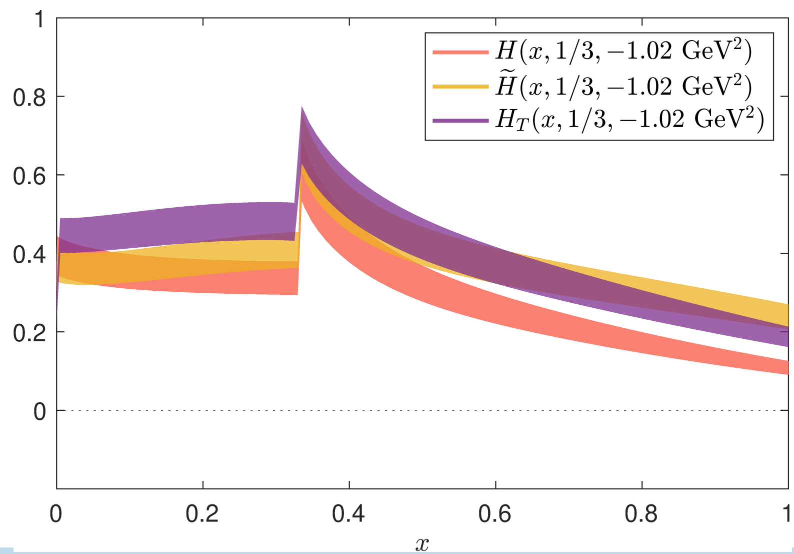
Comparison of different types of PDFs/GPDs

ETMC, Phys. Rev. Lett. 125 (2020) 262001

ETMC, Phys. Rev. D105 (2022) 034501







Moments of transversity GPDs

$n = 0$ Mellin moments:

$$\begin{aligned}
\int_{-1}^{1} dx\, H_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,, \\
\int_{-1}^{1} dx\, E_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,, \\
\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) &= \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,,
\end{aligned} \tag{1}$$

- lowest moments of GPDs skewness-independent,
- lowest moments of quasi-GPDs boost-independent.

$n = 1$ Mellin moments (related to GFF of one-derivative tensor operator):

$$\begin{aligned}
\int_{-1}^{1} dx\, x\, H_T(x,\xi,t) &= A_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, E_T(x,\xi,t) &= B_{T20}(t)\,, \\
\int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) &= \widetilde{A}_{T20}(t)\,, \qquad (3) \\
\int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) &= 2\xi \widetilde{B}_{T21}(t)\,, \qquad (2)
\end{aligned}$$

- skewness-dependence only in for $\widetilde{E}_T$ (only ξ-odd GPD).

Moments of transversity GPDs

Moments of	$H_T(x, \xi=0, t=-0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi=1/3, t=-1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
H_{Tq}	0.65(4)	0.64(6)	0.81(10)	0.49(5)
H_T	0.69(4)	0.67(6)	0.84(10)	0.45(4)
xH_T	0.20(2)	0.21(2)	0.24(3)	0.15(2)
$A_{T10}\ (z=0)$	0.65(4)	0.65(6)	0.82(10)	0.49(5)

Mellin moments P_3-independent, preserved by matching, suppressed with increasing $-t$.

Moments of	$E_T(x, \xi=0, t=-0.69\,\mathrm{GeV}^2)$			$H_T(x, \xi=1/3, t=-1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
E_{Tq}		1.20(42)	2.05(65)	0.67(19)
E_T		1.15(43)	2.10(67)	0.73(19)
xE_T		0.06(4)	0.13(5)	0.11(11)
$B_{T10}\ (z=0)$	1.71(28)	1.22(43)	2.10(67)	0.68(19)

Moments of	$\widetilde{H}_T(x, \xi=0, t=-0.69\,\mathrm{GeV}^2)$			$\widetilde{H}_T(x, \xi=1/3, t=-1.02\,\mathrm{GeV}^2)$
	$P_3 = 0.83$ GeV	$P_3 = 1.25$ GeV	$P_3 = 1.67$ GeV	$P_3 = 1.25$ GeV
$\widetilde{H}_{Tq}$		-0.44(20)	-0.90(32)	-0.26(9)
$\widetilde{H}_T$		-0.42(21)	-0.92(33)	-0.27(9)
$x\widetilde{H}_T$		-0.17(8)	-0.30(10)	-0.05(5)
$\widetilde{A}_{T10}\ (z=0)$	-0.67(14)	-0.45(21)	-0.92(33)	-0.24(8)

Similar conclusions (but very large errors).