Semileptonic decays of spin-entangled baryon-antibaryon pairs

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Probing baryon weak decays -
from experiment to lattice QCD
Warsaw, Poland
6-7 March 2023

Narodowe Centrum Badań Jądrowych
National Centre for Nuclear Research
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中国科学院高能物理研究所
Institute of High Energy Physics
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Study of semileptonic hyperon (SLH) decays

- Presentation is based on recent manuscript: [2302.07665]
- **Motivation (theory)**:
  - Development of formalism for SLH decays that allow to study the spin correlations and polarization
    → similar way as developed for hadronic hyperon decays [PRD99(2019)056008]
    → have not been done before
  - Test of CP symmetry in SLH decays
- **Motivation (experiment)**:
  - Analysis of process $e^+e^- \to J/\psi, \psi(2S) \to (B_1 \to SL)(\bar{B}_1 \to H)$ + c.c.
    → extraction of decay parameters using provided modular method
    →→ some of them has been measured > 30 y.a.
  - Measurement of V_{ij} matrix elements in SLH decays

Production process

[PRD99(2019)056008]

- Two spin-1/2 particle state: $$\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}}\sigma_{\mu}^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_1}$$

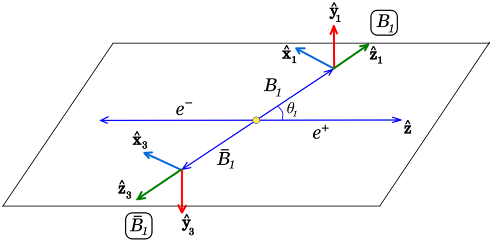
B_1 transverse polarization

$$C_{\mu\bar{\nu}} \propto \begin{pmatrix} 1+\alpha_\psi\cos^2\theta & 0 & \beta_\psi\sin\theta\cos\theta & 0 \\ 0 & \sin^2\theta & 0 & \gamma_\psi\sin\theta\cos\theta \\ -\beta_\psi^2\sin\theta\cos\theta & 0 & \alpha_\psi\sin^2\theta & 0 \\ 0 & -\gamma_\psi\sin\theta\cos\theta & 0 & -\alpha_\psi-\cos^2\theta \end{pmatrix}$$

$\bar{B}_1$ transverse polarization

spin correlations

- $\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)$ and $\gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$
- Main parameters of $C_{\mu\bar{\nu}}$: θ; $\alpha_\psi \in [-1,+1]$, $\Delta\Phi \in [-\pi,+\pi]$



Semileptonic/Hadronic Hyperon decays

- $B_1 \to B_2 + W^-_{\text{off-shell}}(\to l^- \bar{\nu}_l)$ — $\bar{B}_1 \to \bar{B}_3 + \pi^+$
- $\mathcal{B}_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + \{0, \pm 1\}\}$ — $a_{\mu\nu}$ for $\{\frac{1}{2} \to \frac{1}{2} + 0\}$

$\sigma_\mu^{B_1} \to \frac{(q^2 - m_l^2)}{\pi^2} \sum_{\nu=0}^3 \mathcal{B}_{\mu\nu} \sigma_\nu^{B_2}$ — $\sigma_\mu^{\bar{B}_1} \to \sum_{\nu=0}^3 a_{\mu\nu} \sigma_\nu^{\bar{B}_3}$

- Helicity amplitudes:

  $H_{\frac{1}{2}0}, H_{-\frac{1}{2}0}, H_{\frac{1}{2}1}, H_{-\frac{1}{2}-1}$ — $B_{\frac{1}{2}}, B_{-\frac{1}{2}}$

- Main parameters:

  $\Omega_2 = \{\phi_2, \theta_2, 0\}$, $\Omega_l = \{\phi_l, \theta_l, 0\}$ — $\Omega_3 = \{\bar{\phi}_3, \bar{\theta}_3, 0\}$

  $q^2 \in (m_l^2, (M_1 - M_2)^2)$, $g_{\text{av}}^D(q^2), g_{\text{w}}^D(q^2)$ — $\bar{\alpha}_D, \bar{\phi}_D$

where $g_{\text{av}}^D(q^2) = F_1^A(q^2)/F_1^V(0)$

and $g_{\text{w}}^D(q^2) = F_2^V(q^2)/F_1^V(0)$

Semileptonic Hyperon decays (1)

- Initial baryon B_1 with spin-density matrix $\rho_1^{\kappa\kappa'}$ transforms to final baryon B_2 with spin-density matrix $\rho_2^{\lambda_2\lambda_2'}$
$$\rho_2^{\lambda_2\lambda_2'} = T^{\kappa\kappa',\lambda_2\lambda_2'}\rho_1^{\kappa\kappa'}$$
- Transition tensor:
$$T^{\kappa\kappa',\lambda_2\lambda_2'} = \frac{1}{4\pi}\sum_{\underline{\lambda}_W,\underline{\lambda}'_W} T^{\kappa\kappa',\lambda_2\lambda_2'}_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_2)L_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_l)$$
  - Hadronic tensor
$$T^{\kappa\kappa',\lambda_2\lambda_2'}_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_2) = H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}'_W}\mathcal{D}^{1/2*}_{\kappa,\lambda_2-\lambda_W}(\Omega_2)\mathcal{D}^{1/2}_{\kappa',\lambda_2'-\lambda_W'}(\Omega_2)$$
  - Lepton tensor with $\varepsilon = m_l^2/(2q^2)$
$$L_{\underline{\lambda}_W,\underline{\lambda}'_W}(q^2,\Omega_l) = \frac{8(q^2-m_l^2)}{4\pi}\left[\ell^{\mathrm{nf}}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l) + \varepsilon\ell^{\mathrm{f}}_{\underline{\lambda}_W,\underline{\lambda}'_W}(\Omega_l)\right]$$
    - nonflip$(\underline{\lambda}_W = \mp 1)$: $|h^l_{\lambda_l=\mp\frac{1}{2},\lambda_\nu=\pm\frac{1}{2}}|^2 = 8\delta(\lambda_l+\lambda_\nu)(q^2-m_l^2)$
    - flip$(\underline{\lambda}_W = 0, t)$: $|h^l_{\lambda_l=\pm\frac{1}{2},\lambda_\nu=\pm\frac{1}{2}}|^2 = 8\delta(\lambda_l-\lambda_\nu)\varepsilon(q^2-m_l^2)$

Semileptonic Hyperon decays (2)

$$\sigma_\mu^{B_1} \longrightarrow \frac{3(q^2-m_l^2)}{4\pi^3}\sum_{\nu=0}^{3}\mathcal{B}_{\mu\nu}\sigma_\nu^{B_2}$$

- $\mathcal{B}_{\mu\nu}$ can be obtained by inserting Pauli matrices for mother and daughter baryons in $T^{\kappa\kappa',\lambda_2\lambda_2'}$ tensor expression

$$\mathcal{B}_{\mu\nu} = \frac{2\pi^3}{3(q^2-m_l^2)}\sum_{\lambda_2,\lambda_2'=-1/2}^{1/2}\sum_{\kappa,\kappa'=-1/2}^{1/2} T^{\kappa\kappa',\lambda_2\lambda_2'}\sigma_\mu^{\kappa,\kappa'}\sigma_\nu^{\lambda_2',\lambda_2}$$

$$\Downarrow$$

$$\mathcal{B}_{\mu\nu} = \sum_{\kappa=0}^{3}\mathcal{R}_{\mu\kappa}(\Omega_2)b_{\kappa\nu}(q^2,\Omega_l)$$

- $\mathcal{R}_{\mu\kappa}$ - 4×4 rotation matrix
- $b_{\kappa\nu}$ coefficients correspond to $B_1 \to B_2$ transition where axes orientation of the r.s. are aligned $\Omega_2 = \{0,0,0\}$

$$b_{\kappa\nu} = \frac{\pi}{6(q^2-m_l^2)}\sum_{\underline{\lambda}_W,\underline{\lambda}_W'}\sum_{\lambda_2,\lambda_2'} H_{\lambda_2\underline{\lambda}_W}H^*_{\lambda_2'\underline{\lambda}_W'}\sigma_\kappa^{\lambda_2-\underline{\lambda}_W,\lambda_2'-\underline{\lambda}_W'}\sigma_\nu^{\lambda_2',\lambda_2}L_{\underline{\lambda}_W,\underline{\lambda}_W'}(q^2,\Omega_l)$$

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- Replace $b_{\kappa\nu}$ by other decay matrices
  $\rightarrow$ can describe $B_1 \to B_2 + \pi$ or $B_1 \to B_2 + \gamma$

Rotation matrix $\mathcal{R}_{\mu\kappa}$

- 4D rotation matrix $\mathcal{R}_{\mu\kappa}(\Omega)$ with $\Omega \equiv \{\phi, \theta, \chi\}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta\cos\chi\cos\phi - \sin\chi\sin\phi & -\cos\theta\sin\chi\cos\phi - \cos\chi\sin\phi & \sin\theta\cos\phi \\ 0 & \cos\theta\cos\chi\sin\phi + \sin\chi\cos\phi & \cos\chi\cos\phi - \cos\theta\sin\chi\sin\phi & \sin\theta\sin\phi \\ 0 & -\sin\theta\cos\chi & \sin\theta\sin\chi & \cos\theta \end{pmatrix}$$

- $\mathcal{R}_{\mu\kappa}(\Omega_2) = \mathcal{R}_{\mu\kappa}(\phi_2, \theta_2, 0)$

Decay matrices $b^i_{\kappa\nu}$

$$B_1 \to B_2 + \pi$$

$$b^D_{\kappa\nu} \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_D \\ 0 & \gamma_D & -\beta_D & 0 \\ 0 & \beta_D & \gamma_D & 0 \\ \alpha_D & 0 & 0 & 1 \end{pmatrix}$$

$$B_1 \to B_2 + \gamma$$

$$b^\gamma_{\kappa\nu} \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_\gamma \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_\gamma & 0 & 0 & -1 \end{pmatrix}$$

Decay matrices $b^i_{\kappa\nu}$

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$B_1 \to B_2 + W^-_{\text{off-shell}}(\to l^- \bar{\nu}_l)$: $b^{\text{SLW}}_{\kappa\nu} = b^{\text{nf}}_{\kappa\nu} + \varepsilon b^{\text{f}}_{\kappa\nu}$

$$b^{\text{nf}}_{\kappa\nu} = \begin{pmatrix} b^{\text{nf}}_{00} & -\Re(\mathcal{I}^{\text{nf}}_{01}) & \Im(\mathcal{I}^{\text{nf}}_{10}) & b^{\text{nf}}_{03} \\ -\Re(\mathcal{I}^{\text{nf}}_{10}) & \Re(\mathcal{E}^{\text{nf}}_{00} + \mathcal{E}^{\text{nf}}_{11}) & -\Im(\mathcal{E}^{\text{nf}}_{00} + \mathcal{E}^{\text{nf}}_{11}) & \Re(\mathcal{I}^{\text{nf}}_{13}) \\ \Im(\mathcal{I}^{\text{nf}}_{10}) & \Im(\mathcal{E}^{\text{nf}}_{00} - \mathcal{E}^{\text{nf}}_{11}) & \Re(\mathcal{E}^{\text{nf}}_{00} + \mathcal{E}^{\text{nf}}_{11}) & -\Im(\mathcal{I}^{\text{nf}}_{13}) \\ b^{\text{nf}}_{30} & -\Re(\mathcal{I}^{\text{nf}}_{31}) & \Im(\mathcal{I}^{\text{nf}}_{31}) & b^{\text{nf}}_{33} \end{pmatrix}$$

$$b^{\text{f}}_{\kappa\nu} = \begin{pmatrix} b^{\text{f}}_{00} & -\Re(\mathcal{I}^{\text{f}}_{01}) & \Im(\mathcal{I}^{\text{f}}_{10}) & b^{\text{f}}_{03} \\ -\Re(\mathcal{I}^{\text{f}}_{10}) & \Re(\mathcal{E}^{\text{f}}_{00} - \mathcal{E}^{\text{f}}_{11}) & -\Im(\mathcal{E}^{\text{f}}_{00} - \mathcal{E}^{\text{f}}_{11}) & \Re(\mathcal{I}^{\text{f}}_{13}) \\ \Im(\mathcal{I}^{\text{f}}_{10}) & \Im(\mathcal{E}^{\text{f}}_{00} + \mathcal{E}^{\text{f}}_{11}) & \Re(\mathcal{E}^{\text{f}}_{00} + \mathcal{E}^{\text{f}}_{11}) & -\Im(\mathcal{I}^{\text{f}}_{13}) \\ b^{\text{f}}_{30} & -\Re(\mathcal{I}^{\text{f}}_{31}) & \Im(\mathcal{I}^{\text{f}}_{31}) & b^{\text{f}}_{33} \end{pmatrix}$$

Polarization $\vec{P}$ of baryon B_2

- Represent first row of $b_{\kappa 0}$ matrix

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \frac{1}{b_{00}^{\rm nf}+\varepsilon b_{00}^{\rm f}} \begin{bmatrix} -\cos\phi_l & \sin\phi_l & 0 \\ \sin\phi_l & \cos\phi_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Re(\mathcal{I}_{01}) \\ \Im(\mathcal{I}_{01}) \\ b_{03}^{\rm nf}+\varepsilon b_{03}^{\rm f} \end{bmatrix}$$

where

$$b_{00/03}^{\rm nf} = \frac{1}{4}(1\mp\cos\theta_l)^2|H_{\frac{1}{2}1}|^2 +- \frac{1}{4}(1\pm\cos\theta_l)^2|H_{-\frac{1}{2}-1}|^2 +- \frac{1}{2}\sin^2\theta_l(|H_{-\frac{1}{2}0}|^2 +- |H_{\frac{1}{2}0}|^2),$$

$$b_{00/03}^{\rm f} = |H_{\frac{1}{2}t}|^2 +- |H_{-\frac{1}{2}t}|^2 + \frac{1}{2}\sin^2\theta_l(|H_{\frac{1}{2}1}|^2 +- |H_{-\frac{1}{2}-1}|^2) +- \cos^2\theta_l(|H_{\frac{1}{2}0}|^2 +- |H_{-\frac{1}{2}0}|^2)$$
$$- \cos\theta_l\Re(H^*_{\frac{1}{2}0}H_{\frac{1}{2}t} +- H^*_{-\frac{1}{2}0}H_{-\frac{1}{2}t}),$$

$$\mathcal{I}_{01}^{\rm nf} = \pm\frac{1}{2\sqrt{2}}\sin\theta_l\left[(1\pm\cos\theta_l)H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0} + (1\mp\cos\theta_l)H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1}\right],$$

$$\mathcal{I}_{01}^{\rm f} = \frac{1}{\sqrt{2}}\sin\theta_l\left[(H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}t} - H^*_{-\frac{1}{2}t}H_{\frac{1}{2}1}) + \cos\theta_l(H^*_{-\frac{1}{2}0}H_{\frac{1}{2}1} - H^*_{-\frac{1}{2}-1}H_{\frac{1}{2}0})\right].$$

Semileptonic baryon decay

- Momenta and masses: $B_1(p_1, M_1) \to B_2(p_2, M_2) + l^-(p_l, m_l) + \bar{\nu}_l(p_{\bar{\nu}_l}, 0)$
- FF for the weak current-induced baryonic $1/2^+ \to 1/2^+$ transitions [EPJ C59 (2009) 27]:

$$\langle B_2(p_2)|J_\mu^{V+A}|B_1(p_1)\rangle = \bar{u}(p_2)\left[\gamma_\mu(F_1^V(q^2) + F_1^A(q^2)\gamma_5) + \frac{i\sigma_{\mu\nu}q^\nu}{M_1}(F_2^V(q^2) + F_2^A(q^2)\gamma_5) + \frac{q^\mu}{M_1}(F_3^V(q^2) + F_3^A(q^2)\gamma_5)\right]u(p_1)$$

where $\quad q_\mu = (p_1 - p_2)_\mu$

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- For $B_1 \to B_2 e^- \bar{\nu}_e$ at $\mathcal{O}(\frac{m_e^2}{2q^2}) \to 0 \Longrightarrow F_3^{V,A}(q^2) \to 0$
- $H_{\lambda_2 \lambda_W} = (H^V_{\lambda_2 \lambda_W} + H^A_{\lambda_2 \lambda_W})$ with $(\lambda_2 = \pm 1/2; \lambda_W = t, 0, \pm 1)$: $H^{V,A}_{\lambda_2 \lambda_W} \equiv H^{V,A}_{\lambda_2 \lambda_W}(F^{V,A}_{1,2}(q^2))$

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vector helicity amplitudes

$$H^V_{\frac{1}{2}1} = \sqrt{2Q_-}\left(-F_1^V - \frac{M_+}{M_1}F_2^V\right),$$
$$H^V_{\frac{1}{2}0} = \sqrt{\frac{Q_-}{q^2}}\left(M_+F_1^V + \frac{q^2}{M_1}F_2^V\right),$$
$$H^V_{\frac{1}{2}t} = \sqrt{\frac{Q_+}{q^2}}\left(M_-F_1^V + \frac{q^2}{M_1}F_3^V\right),$$

axial-vector helicity amplitudes

$$H^A_{\frac{1}{2}1} = \sqrt{2Q_+}\left(F_1^A - \frac{M_-}{M_1}F_2^A\right),$$
$$H^A_{\frac{1}{2}0} = \sqrt{\frac{Q_+}{q^2}}\left(-M_-F_1^A + \frac{q^2}{M_1}F_2^A\right),$$
$$H^A_{\frac{1}{2}t} = \sqrt{\frac{Q_-}{q^2}}\left(-M_+F_1^A + \frac{q^2}{M_1}F_3^A\right)$$

where $Q_\pm = (M_1 \pm M_2)^2 - q^2 \equiv M_\pm^2 - q^2$, $\quad H^{V,A}_{-\lambda_2, -\lambda_W} = \pm H^{V,A}_{\lambda_2, \lambda_W}$

Form factors

- Neglecting possible CP-odd weak phase, $\mathrm{FF}(l^-, \bar{\nu}_l) = sign\ \mathrm{FF}(l^+, \nu_l)$
- In limit of exact SU(3) symmetry, F_2^A and $F_3^V \to 0$

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- FF parametrization for **hyperons** [PLB478(2000)417][EPJC81(2021)226]:

$$F_i^{V,A}(q^2) = \frac{F_i^{V,A}(0)}{1-\frac{q^2}{M^2_{V,A}}}\frac{1}{1-\alpha_{\mathrm{BK}}\frac{q^2}{M^2_{V,A}}} \Longrightarrow F_i^{V,A}(q^2) = F_i^{V,A}(0)\left[1+r_i^{V,A}q^2+...\right]$$

with $r^{V,A} = 2/m^2_{V,A}$ [AnnRevNuclPartSci34(1984)351] [AnnRevNuclPartSci53(2003)39]

- $\Delta S = 0$: $m_V = 0.84$ GeV [RivNuovoCim2(1972)241], $m_A = 1.08$ GeV [BNL-24848]
- $|\Delta S| = 1$: $m_V = m_{K^*(892)} = 0.89$ GeV, $m_A = m_{K^*(1270)} = 1.27$ GeV

Decay	$\mathcal{B}(\times 10^{-4})$	$g^D_{av}(0)$	$g^D_w(0)$	$M_1 - M_2$ [MeV]	Ref.
$\Lambda \to pe^-\bar{\nu}_e$	8.32(14)	0.718(15)	1.066	177	[1, 2]
$\Sigma^+ \to \Lambda e^+\nu_e$ [a]	0.20(05)	0.01(10)	2.4(17)	74	[1]
$\Xi^- \to \Lambda e^-\bar{\nu}_e$	5.63(31)	0.25(5)	0.085	206	[2, 3]

[a] Since $F_1^{\Sigma} = 0$, g_{av} and g_w are defined as F_1^V/F_1^A and F_2^V/F_1^A, respectively

[1] PTEP2022 083C01(2022) [2] AnnRevNuclPartSci53(2003)39 [3] ZPhysC21(1983)1

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- FF parametrization for **charm baryons**:
  [EPJC76(2016)628] [PRD93(2016)034008] [PRD80(2009)074011][PRC72(2005)035201]

  and many others

Form factors for charm baryons (1)

- Light-front approach [Chin.Phys.C42(2018)093101]:

$$F_i(q^2) = F_i(0) / \left(1 \mp \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2}\right)^2\right)$$

  where m_{fit}, δ fitted from numerical results

- Pole-dominance model:
  SU(4)-symmetry limit [PRD40(1989)2944], MIT bag model [PRD40(1989)2955]:

$$F_i^{V,A}(q^2) = F_i^{V,A}(0) \left[1 + r_i^{V,A} q^2\right] \quad \text{with} \quad r^{V,A} = n/m_{V,A}^2$$

  - $|\Delta C| = 1, \Delta S = 0$: $m_V = m_{D^*} = 2.01$ GeV, $m_A = m_{D^{*0}} = 2.42$ GeV
  - $|\Delta C| = |\Delta S| = 1$: $m_V = m_{D_s^*} = 2.11$ GeV, $m_A = m_{D_{s1}} = 2.54$ GeV

Form factors for charm baryons (2)

- Relativistic quark model based on quasi-potential approach with QCD-motivated potential:

$$F_i(q^2) = \frac{1}{1 - q^2/(M_{\text{pole}}^{F_i})^2} \sum_{n=0}^{n_{\max}} a_n^{F_i} [z(q^2)]^n$$

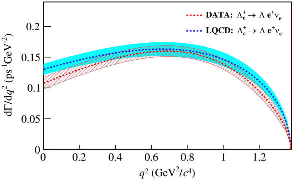
where $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$ with $t_0 = (M_1 - M_2)^2$

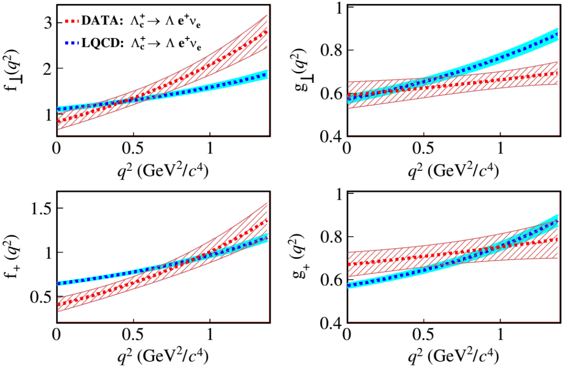
Decay	$\sqrt{t_+}$	$m(F^V_{1,2})$ [GeV]	$m(F^V_3)$ [GeV]	$m(F^A_{1,2})$ [GeV]	$m(F^A_3)$ [GeV]	$M_1 - M_2$ [GeV]	Ref.
$\Lambda_c^+ \to \Lambda l^+ \nu_l$	$m_D + m_K$	2.11	2.32	2.46	1.97	1.17	[1]
$\Xi_c \to \Xi l \nu_l$	$m_{D_s} + m_K$	2.11	2.54	2.54	1.97	1.15	[2]
$\Xi_c \to \Lambda l \nu_l$	$m_D + m_\pi$	2.01	2.42	2.42	1.87	1.35	[2]

[1] [PRL118(2017)082001] [2] [EPJC79(2019)695] [3] ZPhysC21(1983)1

$\Lambda_c^+ \to \Lambda e^+ \bar{\nu}_e$ FFs

- First measurement by BESIII [PRD129(2022)231803]
- Comparison with LQCD calculation [PRL118(2017)082001]
  - Different kinematic behaviour for FF(q^2)
  - Agreement for decay rate
- $\{F_1^V, F_2^V, F_1^A, F_2^A\} \longrightarrow \{f_+, f_\perp, g_+, g_\perp\}$





Conclusion and Outline

- Presented general formalism [2302.07665] can be applied in the BESIII analyses to fit data and to generate MC samples
- Neglecting hadronic CP-violating effects, CP-symmetry tests can be performed using FFs
- Measurement of FFs and $\mathcal{BR}$ will allow to measure CKM matrix elements V_{ij} within one data analysis
- Provided modular description is very flexible:
  - Non-leptonic, semileptonic and radiative decays of baryons with spin 1/2
  - One- and the-step decays
- **Next step**: extension of method for charm baryons
  - Semileptonic decays
  - Three-body decays with mesons (spin $\{0, \pm 1\}$) in the final state

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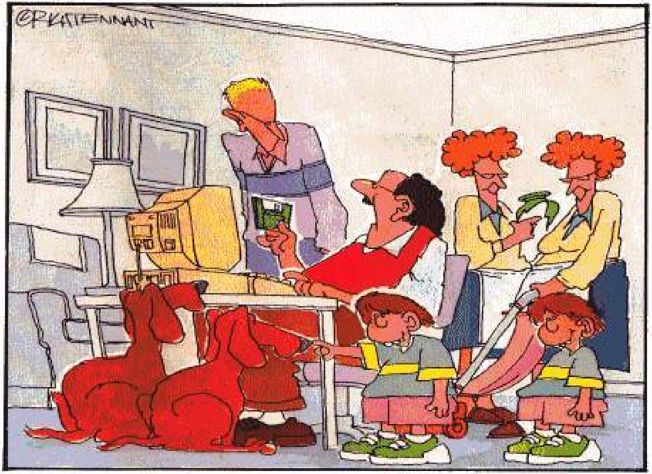
Thank you for your attention!

Backups



" I ALWAYS BACK UP EVERYTHING."

Joint angular distribution (1)

- $e^+e^- \to J/\psi, \psi(2S) \to B_1 \to B_2 W^-_{\text{off-shell}}(\to l^- \bar{\nu}_l)$

$$\mathrm{Tr}\rho_{B_2} \propto \sum_{\mu=0}^{3} C_{\mu 0} \mathcal{B}^{B_1 B_2}_{\mu 0} = \sum_{\mu=0}^{3} C_{\mu 0} \sum_{\kappa=0}^{3} \mathcal{R}_{\mu\kappa}(\Omega_2) b^{B_1 B_2}_{\kappa 0}(q^2, \Omega_l)$$

  - $C_{\mu 0} \equiv (1, P_x, P_y, P_z)$
  - $\mathcal{B}^{B_1 B_2}_{\mu 0} \equiv \mathcal{R}_{\mu\kappa}(\theta_2, \phi_2) b_{\kappa 0}(\theta_l, \phi_l, q^2; g^{B_1}_{\text{av}}, g^{B_1}_{\text{w}})$

- $e^+e^- \to J/\psi, \psi(2S) \to (B_1 \to B_2 W^-_{\text{off-shell}}(\to l^- \bar{\nu}_l))(\bar{B}_1 \to \bar{B}_3 \pi^+)$

$$\mathrm{Tr}\rho_{B_2 \bar{B}_3} \propto \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} \mathcal{B}^{B_1 B_2}_{\mu 0} a^{\bar{B}_1 \bar{B}_3}_{\bar{\nu} 0}$$

  - $C_{\mu\bar{\nu}} \equiv C_{\mu\bar{\nu}}(\theta_1; \alpha_\psi, \Delta\Phi)$
  - $\mathcal{B}^{B_1 B_2}_{\mu 0} \equiv \mathcal{R}_{\mu\kappa}(\theta_2, \phi_2) b_{\kappa 0}(\theta_l, \phi_l, q^2; g^{B_1}_{\text{av}}, g^{B_1}_{\text{w}})$
  - $a^{\bar{B}_1 \bar{B}_3}_{\bar{\nu} 0} \equiv a_{\bar{\nu} 0}(\bar{\theta}_3, \bar{\phi}_3; \bar{\alpha}_{B_1})$

Joint angular distribution (2)

- $e^+e^- \to J/\psi, \psi(2S) \to (\Xi^- \to \Lambda(\to pe^-\bar{\nu}_e)\pi^-)(\bar{\Xi}^+ \to \bar{\Lambda}(\to \bar{p}\pi^+)\pi^+)$

$$\mathrm{Tr}\rho_{p\bar{p}} \propto \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}}^{\Xi\bar{\Xi}} \sum_{\mu'=0}^{3} a_{\mu\mu'}^{\Xi\Lambda} \mathcal{B}_{\mu'0}^{\Lambda p} \sum_{\bar{\nu}'=0}^{3} a_{\bar{\nu}\bar{\nu}'}^{\bar{\Xi}\bar{\Lambda}} a_{\bar{\nu}'0}^{\bar{\Lambda}\bar{p}}$$

  - $\mathcal{B}_{\mu'0}^{\Lambda p} \equiv \mathcal{R}_{\mu'\kappa}(\theta_p, \phi_p) b_{\kappa 0}(\theta_e, \phi_e, q^2; g_{\mathrm{av}}^{\Lambda}, g_{\mathrm{w}}^{\Lambda})$

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  - $\mathcal{B}_{\mu\mu'}^{\Xi\Lambda} \equiv \mathcal{R}_{\mu\kappa}(\theta_\Lambda, \phi_\Lambda) b_{\kappa\mu'}(\theta_e, \phi_e, q^2; g_{\mathrm{av}}^{\Xi}, g_{\mathrm{w}}^{\Xi})$