Probing baryon weak decays - from experiment to lattice QCD

Testing the standard model predictions with $B \to PP$ decays

7th March 2023

Adam Szabelski

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Motivation

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Nothing about baryons!!

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B decaying to a pair of psuedoscalars.

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Nothing about baryons!!

B decaying to a pair of psuedoscalars.

Although the results of this analysis lie beyond the topic of the workshop I hope you find some of the ideas inspiring.

Outline

1. Motivation and general idea
2. Diagrammatic approach
3. The weak phase ϕ in the SM
4. $SU(3)$ group decomposition
5. fits
6. Summary

Motivation and general idea

$B \to K\pi$ puzzle

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Newest LHCb results:
$B^+ \to K^+\pi^0$ - Phys. Rev. Lett. 126(9), 091802 (2021)
$B^0 \to K^+\pi^-$ - JHEP 03, 075 (2021)
World average:

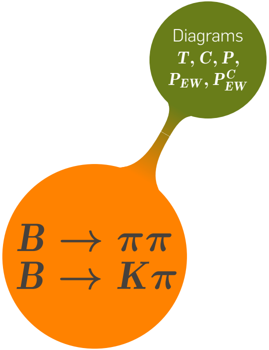
$$\Delta A_{CP}(K\pi) = A_{CP}(B^+ \to K^+\pi^0) - A_{CP}(B^0 \to K^+\pi^-) = 0.114 \pm 0.014$$

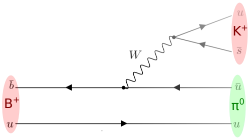
$$\Delta A_{CP}(K\pi) = 0 \quad \text{isospin symmetry}$$

$$\Delta A_{CP}(K\pi) = (1.8^{+4.1}_{-3.2})\% \quad \text{QCD factorisation}$$

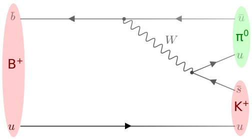
Over 8σ different from 0

Diagrammatic approach

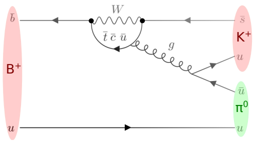

Diagrams
$T, C, P,$
P_{EW}, P_{EW}^C
$B \to \pi\pi$
$B \to K\pi$



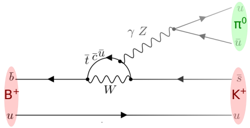
(a) tree colour-allowed



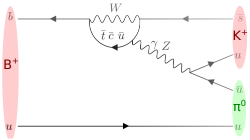
(b) tree colour-suppressed



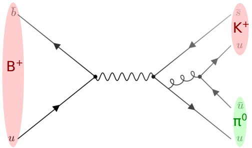
(c) QCD penguin



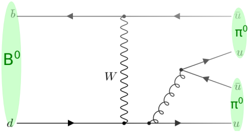
(d) EW penguin colour-allowed



(e) EW penguin colour-suppressed

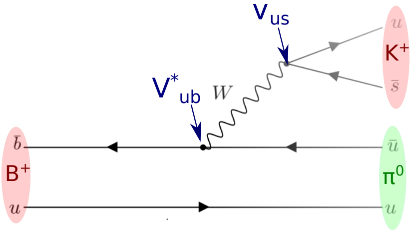


(f) annihilation

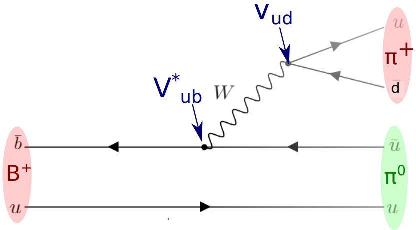


(g) W-exchange

CKM matrix elements



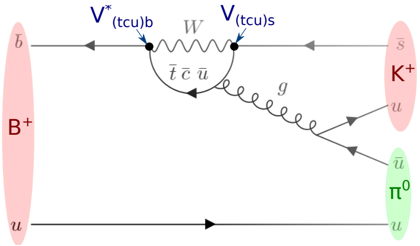
$$V^*_{ub} V_{us} = A\lambda^4(\rho + i\eta)$$



$$V^*_{ub} V_{ud} = A\lambda^3 \left(1 - \frac{\lambda^2}{2}\right)(\rho + i\eta)$$

$$\epsilon = \frac{\lambda^2}{1-\lambda^2} \quad \gamma = arg\left[-\frac{V^*_{ub} V_{ud}}{V^*_{cb} V_{cd}}\right] \quad \beta = arg\left[-\frac{V^*_{cb} V_{cd}}{V^*_{tb} V_{td}}\right]$$

Gluonic penguin



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$$P'_{QCD} = V^*_{tb}V_{ts}P'_t + V^*_{cb}V_{cs}P'_c + V^*_{ub}V_{us}P'_u$$

UT: $V^*_{tb}V_{ts} + V^*_{cb}V_{cs} + V^*_{ub}V_{us} = 0$

$$P'_{QCD} = -V^*_{cb}V^*_{cs}P'_{tc} - V^*_{ub}V_{us}P'_{tu} = A\lambda^3(P'_{tc} - e^{i\gamma}R_bP'_{tu})$$

$$R_b \equiv \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = 0.390 \pm 0.030$$

$B \to \pi\pi$ amplitudes

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$$\sqrt{2}\mathcal{A}(B^+ \to \pi^0\pi^+) = -\lambda^3 A R_b \left[e^{i\gamma}(\mathcal{T}+\mathcal{C}) + e^{-i\beta}(P_{EW}+P_{EW}^C) \right],$$

$$\mathcal{A}(B^0 \to \pi^-\pi^+) = -\lambda^3 A R_b \left[e^{i\gamma}(\mathcal{T} - \mathcal{P}_{tu} + \mathcal{E} - \mathcal{PA}_{tu}) + \frac{1}{R_b}(\mathcal{P}_{tc} + \mathcal{PA}_{tc}) \right],$$

$$\sqrt{2}\mathcal{A}(B^0 \to \pi^0\pi^0) = \lambda^3 A R_b \left[\frac{1}{R_b}(\mathcal{P}_{tc} + \mathcal{PA}_{tc}) - e^{i\gamma}(\mathcal{C} + \mathcal{P}_{tu} - \mathcal{E} + \mathcal{PA}_{tu}) + e^{-i\beta}(P_{EW}+P_{EW}^C) \right]$$

$$R_b \equiv \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = 0.390 \pm 0.030$$

$$\mathcal{P}_{tc} \equiv \mathcal{P}_t - \mathcal{P}_c$$

$$\mathcal{P}_{tu} \equiv \mathcal{P}_t - \mathcal{P}_u$$

$B \to \pi\pi$ amplitudes

$$\sqrt{2}\mathcal{A}(B^+ \to \pi^0\pi^+) = -\lambda^3 A R_b \left[e^{i\gamma}(\mathcal{T} + \mathcal{C}) + e^{-i\beta}(P_{EW} + P_{EW}^C) \right],$$

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$$\sqrt{2}\mathcal{A}(B^0 \to \pi^0\pi^0) = \lambda^3 A R_b \left[\frac{1}{R_b}(\mathcal{P}_{tc} + \mathcal{PA}_{tc}) - e^{i\gamma}(\mathcal{C} + \mathcal{P}_{tu} - \mathcal{E} + \mathcal{PA}_{tu}) + e^{-i\beta}(P_{EW} + P_{EW}^C) \right]$$

isospin symmetry example

$$\sqrt{2}\mathcal{A}(B^+ \to \pi^+\pi^0) = \mathcal{A}(B^0 \to \pi^+\pi^-) + \sqrt{2}\mathcal{A}(B^0 \to \pi^0\pi^0)$$

$$R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = 0.390 \pm 0.030$$

$$\mathcal{P}_{tc} \equiv \mathcal{P}_t - \mathcal{P}_c$$

$$\mathcal{P}_{tu} \equiv \mathcal{P}_t - \mathcal{P}_u$$

Parameterisation

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$$\tilde{T} = \lambda^3 A R_b(\mathcal{T} - \mathcal{P}_{tu} + \mathcal{E} - \mathcal{PA}_{tu}),$$
$$\tilde{C} = \lambda^3 A R_b(\mathcal{C} + \mathcal{P}_{tu} - \mathcal{E} + \mathcal{PA}_{tu}),$$
$$P = \lambda^3 A(\mathcal{P}_t - \mathcal{P}_c),$$

$$r_X = \frac{X}{P} \quad e.g.\ r_T = \frac{\tilde{T}}{P}$$
$$\sqrt{2}\mathcal{A}(B^+ \to \pi^0\pi^+) = -P\left[e^{i\gamma}(r_T + r_C) + e^{-i\beta}\tilde{q}(r_T + r_C)\right],$$
$$\mathcal{A}(B^0 \to \pi^-\pi^+) = P(1 - r_T e^{i\gamma})$$
$$\sqrt{2}\mathcal{A}(B^0 \to \pi^0\pi^0) = P(1 + r_C e^{i\gamma} + e^{-i\beta}\tilde{q}(r_T + r_C))$$

$$\tilde{q} \equiv \left|\frac{P_{EW} + P^C_{EW}}{T + C}\right|$$

$B \to K\pi$

$$A(B^+ \to \pi^+ K^0) = -P'\left[1 + r_{\rho_c} e^{i\gamma} - \frac{1}{3} a_C \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C)\right]$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = P'\left[1 + r_{\rho_c} e^{i\gamma} - \left\{ e^{i\gamma} - \left(1 - \frac{1}{3} a_C\right) \boldsymbol{q e^{i\omega} e^{i\phi}} \right\} (r'_T + r'_C)\right]$$

$$A(B^0_d \to \pi^- K^+) = P'\left[1 + r_{\rho_c} e^{i\gamma} + \frac{2}{3} a_C \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C) - r_T e^{i\gamma}\right]$$

$$\sqrt{2} A(B^0_d \to \pi^0 K^0) = -P'\left[1 + r_{\rho_c} e^{i\gamma} - \left(1 - \frac{2}{3} a_C\right) \boldsymbol{q e^{i\omega} e^{i\phi}} (r'_T + r'_C) + r'_C e^{i\gamma}\right]$$

$$P' \equiv \frac{\lambda^3 A}{\sqrt{\epsilon}} (\mathcal{P}'_t - \mathcal{P}'_c)$$

$$r_{\rho_c} \equiv \frac{\rho_c e^{i\theta_c}}{P'} \equiv \left(\frac{\lambda^2 R_b}{1 - \lambda^2}\right) \left[\frac{\mathcal{P}'_t - \tilde{\mathcal{P}}'_u - \mathcal{A}'}{\mathcal{P}'_t - \mathcal{P}'_c}\right] \qquad B^+ \to K^+ \bar{K}^0 : \quad \rho_c = 0.03 \pm 0.01, \quad \theta_c = (2.6 \pm 4.6)^\circ$$

$$a_C \equiv \frac{\hat{\mathcal{P}}'^C_{EW}}{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}$$

$$\boldsymbol{q e^{i\phi} e^{i\omega} \equiv -\left(\frac{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}{\hat{T}' + \hat{C}'}\right)}$$

In SM: $\boldsymbol{q e^{i\phi} e^{i\omega} \equiv \frac{-3}{2\lambda^2 R_b} \left[\frac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)}\right] R_q = (0.64 \pm 0.05) R_q}$

$R_q = 1 \pm 0.05$ - compensates for possible $SU(3)$ violation

$B \to K\pi$

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$$A(B^+ \to \pi^+ K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \frac{1}{3} a_C \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C)\right]$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} - \left\{ e^{i\gamma} - \left(1 - \frac{1}{3} a_C\right) \mathbf{q} e^{i\omega} e^{i\phi} \right\} (r'_T + r'_C)\right]$$

$$A(B_d^0 \to \pi^- K^+) = P' \left[1 + r_{\rho_c} e^{i\gamma} + \frac{2}{3} a_C \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C) - r_T e^{i\gamma}\right]$$

$$\sqrt{2} A(B_d^0 \to \pi^0 K^0) = -P' \left[1 + r_{\rho_c} e^{i\gamma} - \left(1 - \frac{2}{3} a_C\right) \mathbf{q} e^{i\omega} e^{i\phi} (r'_T + r'_C) + r'_C e^{i\gamma}\right]$$

$$P' \equiv \frac{\lambda^3 A}{\sqrt{\epsilon}} (\mathcal{P}'_t - \mathcal{P}'_c)$$

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$$a_C \equiv \frac{\hat{\mathcal{P}}'^C_{EW}}{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}$$

Isospin relation

$$\sqrt{2}\mathcal{A}(B^+ \to K^0 \pi^0) + \mathcal{A}(B^+ \to K^+ \pi^-) =$$
$$\sqrt{2}\mathcal{A}(B^+ \to K^+ \pi^0) + \mathcal{A}(B^0 \to K^0 \pi^+)$$

$$q e^{i\phi} e^{i\omega} \equiv -\left(\frac{\hat{\mathcal{P}}'_{EW} + \hat{\mathcal{P}}'^C_{EW}}{\hat{T}' + \hat{C}'}\right)$$

In SM: $q e^{i\phi} e^{i\omega} \equiv \dfrac{-3}{2\lambda^2 R_b} \left[\dfrac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)}\right] R_q = (0.64 \pm 0.05) R_q$

$R_q = 1 \pm 0.05$ - compensates for possible $SU(3)$ violation

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$B \to \pi\pi$
$B \to K\pi$

Diagrams
$T, C, P,$
P_{EW}, P_{EW}^{C}

$$qe^{i(\omega+\phi)} = -\frac{P_{EW}+P_{EW}^{C}}{T+C}$$

U-spin relation

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Connection between amplitudes describing $B \to \pi\pi$ and $B \to K\pi$ decays

$$B \to \pi\pi \text{ - } b \to d \iff B \to K\pi \text{ - } b \to s(')$$

$$R_{T+C} = \left|\frac{T' + C'}{T + C}\right| \equiv \left|\frac{(r'_T + r'_C)P'}{\epsilon(r_T + r_C)P}\right| = 1.21 \pm 0.015 \quad \text{QCDF}$$

$$R_{T+C} = 1.2 \pm 0.2 \quad \text{safe estimate}$$

$$\epsilon \equiv \frac{\lambda^2}{1-\lambda^2} = 0.0535 \pm 0.0002$$

$$arg(r'_T) - arg(r_T) = 0 \pm 20^\circ \quad arg(r'_C) - arg(r_C) = 0 \pm 20^\circ$$

same as arxiv:1806.08783 R. Fleischer et al.

$B \to \pi\pi$
$B \to K\pi$

Diagrams
$T, C, P,$
P_{EW}, P_{EW}^C

$qe^{i(\omega+\phi)} = -\frac{P_{EW}+P_{EW}^C}{T+C}$

P_{EW}, P_{EW}^C
vs
T, C

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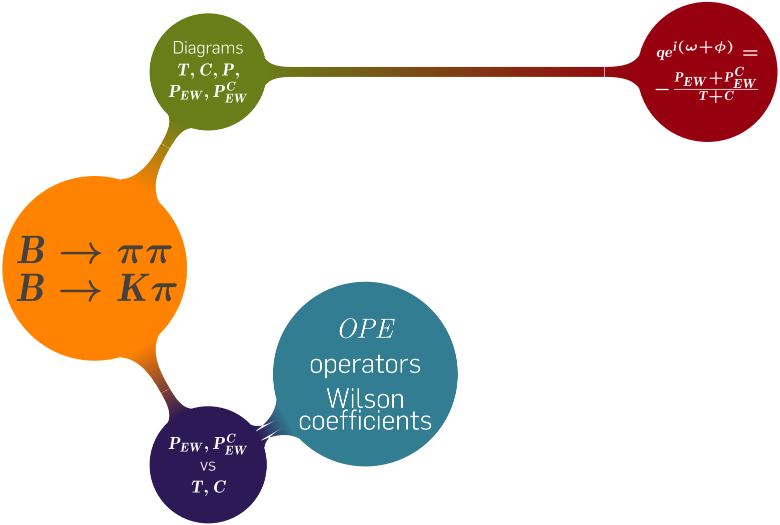
$B \to \pi\pi$
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P_{EW}, P^C_{EW}
$qe^{i(\omega+\phi)} = -\frac{P_{EW}+P^C_{EW}}{T+C}$
OPE
operators
Wilson coefficients
P_{EW}, P^C_{EW}
vs
T, C

The weak phase ϕ in the SM

Operator product expansion

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$B \to \pi K \;-\; b \to s,\ \Delta U = \Delta C = 0 \qquad (\bar{p}q)_{V\pm A} \equiv \bar{p}\gamma^{\mu}(1 \pm \gamma_5)q$

Current-current operators (tree operators):

$$Q_1^{\alpha} \equiv (\bar{b}_x \alpha_y)_{V-A}(\bar{\alpha}_y s_x)_{V-A},$$
$$Q_2^{\alpha} \equiv (\bar{b}\alpha)_{V-A}(\bar{\alpha}s)_{V-A};$$

Gluonic-penguin operators:

$$Q_3 \equiv (\bar{b}s)_{V-A}\sum_q (\bar{q}q)_{V-A},$$
$$Q_4 \equiv (\bar{b}_x s_y)_{V-A}\sum_q (\bar{q}_y q_x)_{V-A},$$
$$Q_5 \equiv (\bar{b}s)_{V-A}\sum_q (\bar{q}q)_{V+A},$$
$$Q_6 \equiv (\bar{b}_x s_y)_{V-A}\sum_q (\bar{q}_y q_x)_{V+A};$$

Electroweak-penguin operators:

$$Q_7 \equiv \frac{3}{2}(\bar{b}s)_{V-A}\sum_q (e_q\bar{q}q)_{V+A},$$
$$Q_8 \equiv \frac{3}{2}(\bar{b}_x s_y)_{V-A}\sum_q (e_q\bar{q}_y q_x)_{V+A},$$
$$Q_9 \equiv \frac{3}{2}(\bar{b}s)_{V-A}\sum_q (e_q\bar{q}q)_{V-A},$$
$$Q_{10} \equiv \frac{3}{2}(\bar{b}_x s_y)_{V-A}\sum_q (e_q\bar{q}_y q_x)_{V-A};$$

Effective Hamiltonian

[Matthias Neubert and Jonathan L. Rosner]

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$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=1,2} C_i(\lambda_u Q_i^u + \lambda_c Q_i^c) + \lambda_t \sum_{i=3}^{10} C_i Q_i\right] + \text{h.c.}$$

$$\lambda_\alpha = V_{\alpha b}^* V_{\alpha s} \quad \lambda_u + \lambda_c + \lambda_t = 0$$

$Q_{1,2}^u \sim \bar{b}s\bar{u}u$ - contribute to $\Delta I = 0$ and $\Delta I = 1$

$Q_{1,2}^c \sim \bar{b}s\bar{c}c$ - contribute only to $\Delta I = 0$

$Q_{3\ldots6} \sim \bar{b}s\sum\bar{q}q$ - contribute only to $\Delta I = 0$

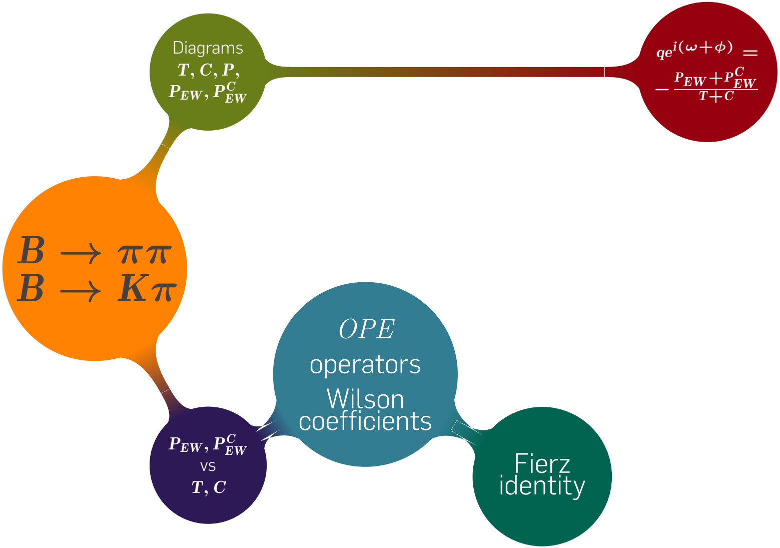
$Q_{7\ldots10} \sim \bar{b}s\sum e_q\bar{q}q$ - contribute to $\Delta I = 0$ and $\Delta I = 1$

$$H_{eff} = H_{\Delta I=0} + H_{\Delta I=1}$$

$$H_{\Delta I=0} = \frac{G_F}{\sqrt{2}}\left\{\sum_{i=1,2}\left[\frac{\lambda_u}{2} C_i(Q_i^u + Q_i^d) + \lambda_c Q_i^c\right] + \lambda_t \sum_{i=3}^{10} C_i Q_i - \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1}\right\} + h.c.$$

$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}}\left[\sum_{i=1,2}\frac{\lambda_u}{2} C_i(Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1}\right] + h.c.$$

$Q_{i=7\ldots10}^{\Delta I=1} \sim \frac{1}{2}\bar{b}s(\bar{u}u - \bar{d}d)$ - antisymmetric $d \leftrightarrow u \Rightarrow \Delta I = 1$



Diagrams
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$B \to \pi\pi$
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OPE
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P_{EW}, P_{EW}^C
vs
T, C
Fierz identity

Fierz identity $\Rightarrow \ \phi = 0$

when C_9 and C_{10} dominate the EW penguin contribution like in the SM, then $\phi = 0$.

$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1,2} \frac{\lambda_u}{2} C_i (Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right] + h.c.$$

$$Q_{i=7\ldots10}^{\Delta I=1} \sim \frac{1}{2} \bar{b}s(\bar{u}u - \bar{d}d)$$

$$C_7 = -0.002\alpha, \quad C_8 = 0.0054\alpha, \quad C_9 = -1.292\alpha, \quad C_{10} = 0.263\alpha$$

Fierz identity $\Rightarrow \ \phi = 0$

when C_9 and C_{10} dominate the EW penguin contribution like in the SM, then $\phi = 0$.

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$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1,2} \frac{\lambda_u}{2} C_i (Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right] + h.c.$$

$$Q^{\Delta I=1}_{i=7\ldots 10} \sim \frac{1}{2} \bar{b} s (\bar{u} u - \bar{d} d)$$

$$C_7 = -0.002\alpha, \quad C_8 = 0.0054\alpha, \quad C_9 = -1.292\alpha, \quad C_{10} = 0.263\alpha$$

$$Q_9 \equiv \frac{3}{2} (\bar{b}_x s_x)_{V-A} \sum_q (e_q \bar{q}_y q_y)_{V-A}, \quad Q_1^\alpha \equiv (\bar{b}_x \alpha_y)_{V-A} (\bar{\alpha}_y s_x)_{V-A}$$

$$Q_9^{\Delta I=1} = \frac{3}{2} (\bar{b}_x s_x)_{V-A} \frac{1}{2} (\bar{u}_y u_y - \bar{d}_y d_y)_{V-A} \overset{Fierz}{=}$$

$$-\frac{3}{2} \left\{ \frac{1}{2} \left[(\bar{b}_x u_y)_{V-A} (\bar{u}_y s_x)_{V-A} - (\bar{b}_x d_y)_{V-A} (\bar{d}_y s_x)_{V-A} \right] \right\} = -\frac{3}{2} \left[\frac{1}{2} (Q_1^u - Q_1^d) \right]$$

Fierz identity $\Rightarrow\ \phi = 0$

when C_9 and C_{10} dominate the EW penguin contribution like in the SM, then $\phi = 0$.

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$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1,2} \frac{\lambda_u}{2} C_i (Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right] + h.c.$$

$$Q_{i=7\ldots10}^{\Delta I=1} \sim \frac{1}{2} \bar{b}s(\bar{u}u - \bar{d}d)$$

$$C_7 = -0.002\alpha, \quad C_8 = 0.0054\alpha, \quad C_9 = -1.292\alpha, \quad C_{10} = 0.263\alpha$$

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$$Q_9^{\Delta I=1} = \frac{3}{2} (\bar{b}_x s_x)_{V-A} \frac{1}{2} (\bar{u}_y u_y - \bar{d}_y d_y)_{V-A} \stackrel{Fierz}{=}$$

$$-\frac{3}{2} \left\{ \frac{1}{2} \left[(\bar{b}_x u_y)_{V-A} (\bar{u}_y s_x)_{V-A} - (\bar{b}_x d_y)_{V-A} (\bar{d}_y s_x)_{V-A} \right] \right\} = -\frac{3}{2} \left[\frac{1}{2} (Q_1^u - Q_1^d) \right]$$

$$Q_{10} \equiv \frac{3}{2} (\bar{b}_x s_y)_{V-A} \sum_q (e_q \bar{q}_y q_x)_{V-A}, \quad Q_2^\alpha \equiv (\bar{b}_x \alpha_x)_{V-A} (\bar{\alpha}_y s_y)_{V-A}$$

$$Q_{10}^{\Delta I=1} = \frac{3}{2} (\bar{b}_x s_y)_{V-A} \frac{1}{2} (\bar{u}_y u_x - \bar{d}_y d_x)_{V-A} \stackrel{Fierz}{=}$$

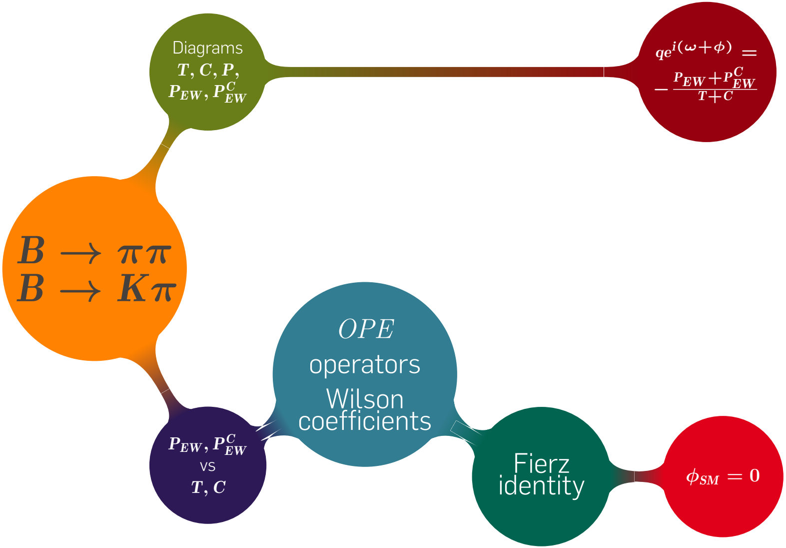
$$-\frac{3}{2} \left\{ \frac{1}{2} \left[(\bar{b}_x u_x)_{V-A} (\bar{u}_y s_y)_{V-A} - (\bar{b}_x d_x)_{V-A} (\bar{d}_y s_y)_{V-A} \right] \right\} = -\frac{3}{2} \left[\frac{1}{2} (Q_2^u - Q_2^d) \right]$$

$\phi_{SM} = 0$

NCBJ

$$Q_9^{\Delta I=1} \sim Q_1^u - Q_1^d \qquad Q_{10}^{\Delta I=1} \sim Q_2^u - Q_2^d$$

When the operators are linearly dependent the weak phase between them $\phi_{SM} = 0$
The weak interaction can break $SU(3)$ but this reasoning there is a stron constrain on the weak phase of $qe^{i(\phi+\omega)}$ in the SM.


$B \to \pi\pi$
$B \to K\pi$
Diagrams
$T, C, P,$
P_{EW}, P_{EW}^C
$q e^{i(\omega+\phi)} = -\frac{P_{EW}+P_{EW}^C}{T+C}$
P_{EW}, P_{EW}^C
vs
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operators
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coefficients
Fierz
identity
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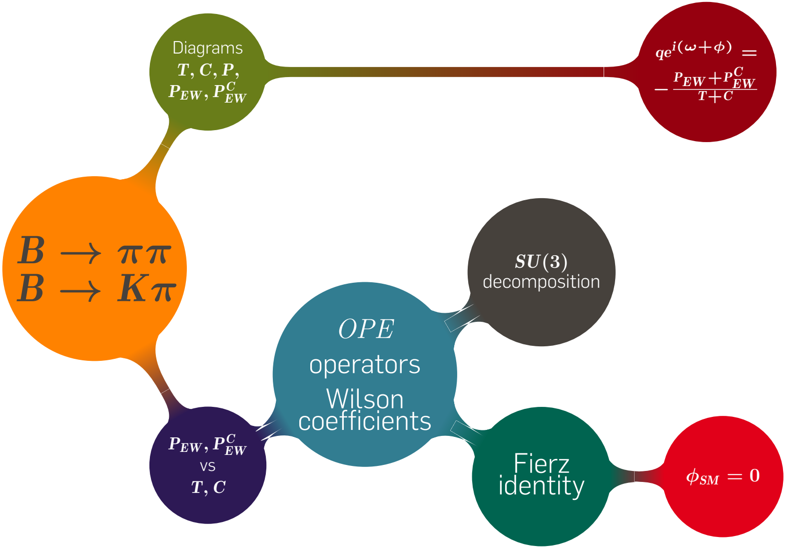
NCBJ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operators
Wilson coefficients
$SU(3)$
decomposition
Fierz identity
$\phi_{SM} = 0$

$SU(3)$ group decomposition

$SU(3)$ generators

NCBJ

[Michael Gronau, Dan Pirjol, Tung-Mow Yan]

Strong interaction conserves $SU(3)$

Operators: $\bar{q}_1\bar{q}_3 q_2 \simeq (\bar{b}q_1)(\bar{q}_2 q_3)$ form a SU(3) group that can be decomposed into:

$$\mathbf{\bar{3}} \otimes \mathbf{\bar{3}} \otimes \mathbf{3} = \mathbf{\bar{3}} \oplus \mathbf{\bar{3}} \oplus \mathbf{6} \oplus \mathbf{1\bar{5}}$$

$\Delta S = +1$ operators:

$$\mathbf{1\bar{5}}_{I=1} = -\frac{1}{2}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) + \frac{1}{2}(\bar{d}\bar{s}d + \bar{s}\bar{d}d),$$

$$\mathbf{1\bar{5}}_{I=0} = -\frac{1}{2\sqrt{2}}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) - \frac{1}{2\sqrt{2}}(\bar{d}\bar{s}d + \bar{s}\bar{d}d) + \frac{1}{\sqrt{2}}\bar{s}\bar{s}s,$$

$$\mathbf{6}_{I=1} = -\frac{1}{2}(\bar{u}\bar{s}u - \bar{s}\bar{u}u) + \frac{1}{2}(\bar{d}\bar{s}d - \bar{s}\bar{d}d),$$

$$\mathbf{\bar{3}}^{(a)}_{I=0} = -\frac{1}{2}(\bar{u}\bar{s}u - \bar{s}\bar{u}u) - \frac{1}{2}(\bar{d}\bar{s}d - \bar{s}\bar{d}d),$$

$$\mathbf{\bar{3}}^{(s)}_{I=0} = \frac{1}{2\sqrt{2}}(\bar{u}\bar{s}u + \bar{s}\bar{u}u) + \frac{1}{2\sqrt{2}}(\bar{d}\bar{s}d + \bar{s}\bar{d}d) + \frac{1}{\sqrt{2}}\bar{s}\bar{s}s.$$

Hamiltonian decomposition

NCBJ

$$\mathcal{H}_T = \frac{G_F}{\sqrt{2}}\Bigg(\lambda_u^{(s)}[\frac{1}{2}(c_1-c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0}-\mathbf{6}_{I=1})+\frac{1}{2}(c_1+c_2)(-\overline{\mathbf{15}}_{I=1}-\frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})$$

$$+\lambda_u^{(d)}[\frac{1}{2}(c_1-c_2)(\mathbf{6}_{I=\frac{1}{2}}-\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{1}{2}(c_1+c_2)(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=\frac{1}{2}}+\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\Bigg)$$

$$\mathcal{H}_{EWP} \simeq -\lambda_t^{(s)}\left(c_9 Q_9^{(s)}+c_{10}Q_{10}^{(s)}\right)-\lambda_t^{(d)}\left(c_9 Q_9^{(d)}+c_{10}Q_{10}^{(d)}\right)=$$

$$-\frac{\lambda_t^{(s)}}{2}\left(\frac{c_9-c_{10}}{2}(3\cdot\mathbf{6}_{I=1}+\bar{\mathbf{3}}^{(a)}_{I=0})+\frac{c_9+c_{10}}{2}(-3\cdot\overline{\mathbf{15}}_{I=1}-\frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})\right)$$

$$-\frac{\lambda_t^{(d)}}{2}\left(\frac{c_9-c_{10}}{2}(-3\cdot\mathbf{6}_{I=\frac{1}{2}}+\bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}})+\frac{c_9+c_{10}}{2}(-\sqrt{\frac{3}{2}}\cdot\overline{\mathbf{15}}_{I=\frac{1}{2}}-2\sqrt{3}\cdot\overline{\mathbf{15}}_{I=\frac{3}{2}}-\frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})\right)$$

$B \to PP$

initial B triplet:

$$B_u^+ = +\bar{b}u, \quad B_d^0 = +\bar{b}d, \quad B_s^0 = \bar{b}s.$$

Light meson nonet P (η_1 singlet $\oplus$ octet):

$$K^+ = +u\bar{s}, \qquad K^0 = +d\bar{s},$$

$$\pi^+ = +u\bar{d}, \qquad \pi^0 = -\frac{1}{\sqrt{2}}(+u\bar{u} - d\bar{d}), \qquad \pi^- = -d\bar{u},$$

$$\eta_8 = -\frac{1}{\sqrt{6}}(+u\bar{u} + d\bar{d} - 2s\bar{s}),$$

$$\bar{K}^0 = +s\bar{d}, \qquad K^- = -s\bar{u},$$

$$\eta_1 = -\frac{1}{\sqrt{3}}(+u\bar{u} + d\bar{d} + s\bar{s}).$$

Exclusion Principle allows only symmetric part of $P \otimes P$:

$$(\mathbf{8} \otimes \mathbf{8})_{\mathbf{S}} = \mathbf{1} \oplus \mathbf{8_S} \oplus \mathbf{27}$$

$B \to PP$

initial B triplet:

$$B_u^+ = +\bar{b}u, \quad B_d^0 = +\bar{b}d, \quad B_s^0 = \bar{b}s.$$

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$$\eta_1 = -\frac{1}{\sqrt{3}}(+u\bar{u} + d\bar{d} + s\bar{s}).$$

Exclusion Principle allows only symmetric part of $P \otimes P$:
$\mathbf{(8 \otimes 8)_S = 1 \oplus 8_S \oplus 27}$

Amplitude decomposition

NCBJ

Express the decay amplitudes with the $SU(3)$-transition amplitudes using Clebsch-Gordan coefficients $\boldsymbol{u}^{S=1} = \mathcal{O}\,\boldsymbol{v}^{S=1}$

Clebsch-Gordan coefficients

$$\boldsymbol{u}^{S=1} = \begin{pmatrix} \mathcal{A}(B^+ \to \pi^+ K^0) \\ \mathcal{A}(B^+ \to \pi^0 K^+) \\ \mathcal{A}(B^0 \to \pi^- K^+) \\ \mathcal{A}(B^0 \to \pi^0 K^0) \end{pmatrix} \qquad \boldsymbol{v}^{S=1} = \begin{pmatrix} \langle 1 || \bar{3}_{I=0} || 3 \rangle \\ \langle 8 || \bar{3}_{I=0} || 3 \rangle \\ \langle 8 || 6_{I=1} || 3 \rangle \\ \langle 8 || \overline{15}_{I=0} || 3 \rangle \\ \langle 8 || \overline{15}_{I=1} || 3 \rangle \\ \langle 27 || \overline{15}_{I=0} || 3 \rangle \\ \langle 27 || \overline{15}_{I=1} || 3 \rangle \end{pmatrix}$$

[Benjamin Grinstein and Richard F. Lebed]

$q^{i\omega}$ in $SU(3)$ decomposition

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(c_1 + c_2)\langle 27||\overline{15}_{I=1}||3\rangle$$

$q^{i\omega}$ in $SU(3)$ decomposition

NCBJ

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(c_1 + c_2)\langle 27||\bar{15}_{I=1}||3\rangle$$

$$\mathcal{P}^{EW}(B^+ \to \pi^+ K^0) + \sqrt{2}\mathcal{P}^{EW}(B^+ \to \pi^0 K^+) = \mathcal{P}'_{EW} + \mathcal{P}'^{C}_{EW} = -\lambda_t^{(s)} \sqrt{\frac{5}{2}}(c_9 + c_{10})\langle 27||\bar{15}_{I=1}||3\rangle$$

$q^{i\omega}$ in $SU(3)$ decomposition

NCBJ

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(c_1 + c_2)\langle 27||\bar{15}_{I=1}||3\rangle$$

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Combining the two:

$$\mathcal{P}_{EW} + \mathcal{P}^{C}_{EW} = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\frac{c_9 + c_{10}}{c_1 + c_2}(T + C)$$

$q^{i\omega}$ in $SU(3)$ decomposition

NCBJ

$$\mathcal{A}^{tree}(B^0 \to \pi^- K^+) + \sqrt{2}\mathcal{A}^{tree}(B^0 \to \pi^0 K^0) = T' + C' = -\lambda_u^{(s)} \frac{\sqrt{10}}{3}(c_1 + c_2)\langle 27 || \bar{15}_{I=1} || 3 \rangle$$

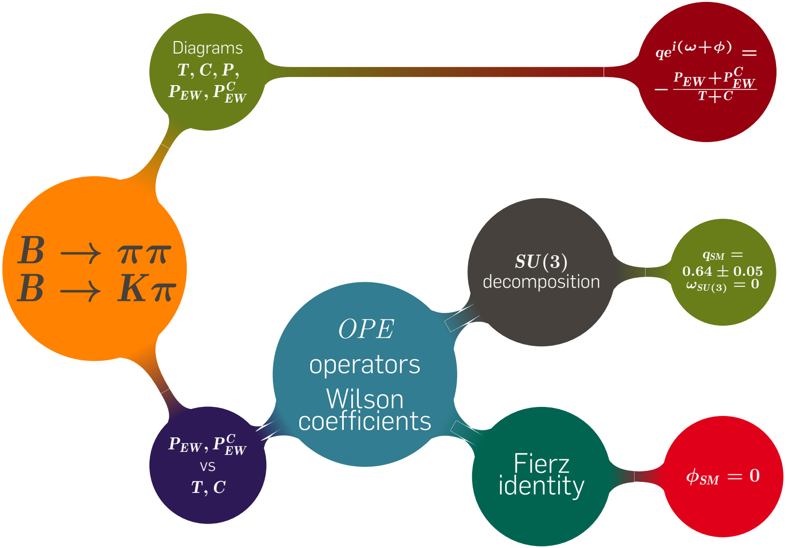
$$\mathcal{P}^{EW}(B^+ \to \pi^+ K^0) + \sqrt{2}\mathcal{P}^{EW}(B^+ \to \pi^0 K^+) = \mathcal{P'}_{EW} + \mathcal{P'}^{C}_{EW} = -\lambda_t^{(s)} \sqrt{\frac{5}{2}}(c_9 + c_{10})\langle 27 || \bar{15}_{I=1} || 3 \rangle$$

Combining the two:

$$\mathcal{P}_{EW} + \mathcal{P}_{EW}^{C} = \frac{3}{2} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} \frac{c_9 + c_{10}}{c_1 + c_2}(T + C)$$

In SM: $q e^{i\phi} e^{i\omega} \equiv \frac{-3}{2\lambda^2 R_b}\left[\frac{C_9(\mu) + C_{10}(\mu)}{C_1(\mu) + C_2(\mu)}\right] R_q = (0.64 \pm 0.05) R_q$

In $SU(3)$ limit: $\omega = 0$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$q_{SM} =$
0.64 ± 0.05
$\omega_{SU(3)} = 0$
Fierz
identity
$\phi_{SM} = 0$

P_{EW} and P_{EW}^C

NCBJ

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(c_9 - c_{10})\langle \mathbf{8}||\mathbf{6}||\mathbf{3}\rangle$$

$$b_4 = \lambda_t^{(s)} \frac{3}{2}(c_9 + c_{10})\langle \mathbf{8}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

$$b_5 = \lambda_t^{(s)} \frac{3}{2}(c_9 - c_{10})\langle \mathbf{27}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}} b_5 = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\frac{c_9 + c_{10}}{c_1 + c_2}(T + C) = -qe^{i\phi}(T' + C')$$

$$P'^{C}_{EW} = \frac{1}{2}\sqrt{\frac{3}{5}} b_3 + \frac{3}{2}\sqrt{\frac{3}{5}} b_4 - \frac{3}{2}\sqrt{\frac{2}{5}} b_5$$

$$= \frac{3}{4}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\left(-\frac{c_9 - c_{10}}{c_1 - c_2}(T' - C') - \frac{1}{5}\frac{c_9 + c_{10}}{c_1 + c_2}(T' + C')\right) - q^{i\phi}(T' + C')$$

$$P_{EW} + P^{C}_{EW} = -\frac{\sqrt{5}}{2} b_5 = \lambda^2 R_b \left|\frac{V_{td}}{V_{ub}}\right| qe^{i(\phi - \beta + \omega)}(T + C)$$

Exchange and annihilation diagrams neglected.

P_{EW} and P_{EW}^C

NCBJ

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(c_9 - c_{10})\langle \mathbf{8}||\mathbf{6}||\mathbf{3}\rangle$$

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New Physics

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}} b_5 = \frac{3}{2} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} \frac{c_9 + c_{10}}{c_1 + c_2}(T + C) = -q e^{i\phi}(T' + C')$$

$$P'^{C}_{EW} = \frac{1}{2}\sqrt{\frac{3}{5}} b_3 + \frac{3}{2}\sqrt{\frac{3}{5}} b_4 - \frac{3}{2}\sqrt{\frac{2}{5}} b_5$$

$$= \frac{3}{4} \frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} \left(-\frac{c_9 - c_{10}}{c_1 - c_2}(T' - C') - \frac{1}{5} \frac{c_9 + c_{10}}{c_1 + c_2}(T' + C') \right) - q^{i\phi}(T' + C')$$

$$P_{EW} + P_{EW}^C = -\frac{\sqrt{5}}{2} b_5 = \lambda^2 R_b \left| \frac{V_{td}}{V_{ub}} \right| q e^{i(\phi - \beta + \omega)}(T + C)$$

Exchange and annihilation diagrams neglected.

P_{EW} and P_{EW}^C

NCBJ

$$b_3 = \lambda_t^{(s)} \frac{3}{2}(c_9 - c_{10})\langle \mathbf{8}||\mathbf{6}||\mathbf{3}\rangle$$

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$$b_5 = \lambda_t^{(s)} \frac{3}{2}(c_9 - c_{10})\langle \mathbf{27}||\overline{\mathbf{15}}||\mathbf{3}\rangle$$

model-dependent

New Physics

$$P'_{EW} + P'^{C}_{EW} = -\sqrt{\frac{5}{2}}b_5 = \frac{3}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\frac{c_9 + c_{10}}{c_1 + c_2}(T + C) = -qe^{i\phi}(T' + C')$$

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$$= \frac{3}{4}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}}\left(-\frac{c_9 - c_{10}}{c_1 - c_2}(T' - C') - \frac{1}{5}\frac{c_9 + c_{10}}{c_1 + c_2}(T' + C')\right) - q^{i\phi}(T' + C')$$

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Exchange and annihilation diagrams neglected.

Two approaches

NCBJ

basic	$SU(3)$ decomposition

Two approaches

basic	$\boldsymbol{SU(3)}$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$

Two approaches

NCBJ

basic	$SU(3)$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$
$\phi_{SM} = 0$ Fierz identity	$\phi_{SM} = 0$ Fierz identity

Two approaches

NCBJ

basic	$\boldsymbol{SU(3)}$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$
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Two approaches

basic	$SU(3)$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$
$\phi_{SM} = 0$ Fierz identity	$\phi_{SM} = 0$ Fierz identity
$\omega = 0$ $SU(3)$ limit	$\omega = 0$ $SU(3)$ limit
$qe^{i\phi} = -\frac{P'_{EW}}{C'+T'}$ -> NP	$qe^{i\phi} = -\frac{P'_{EW}+P'^{C}_{EW}}{C'+T'}$ -> NP

Two approaches

NCBJ

basic	$SU(3)$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$
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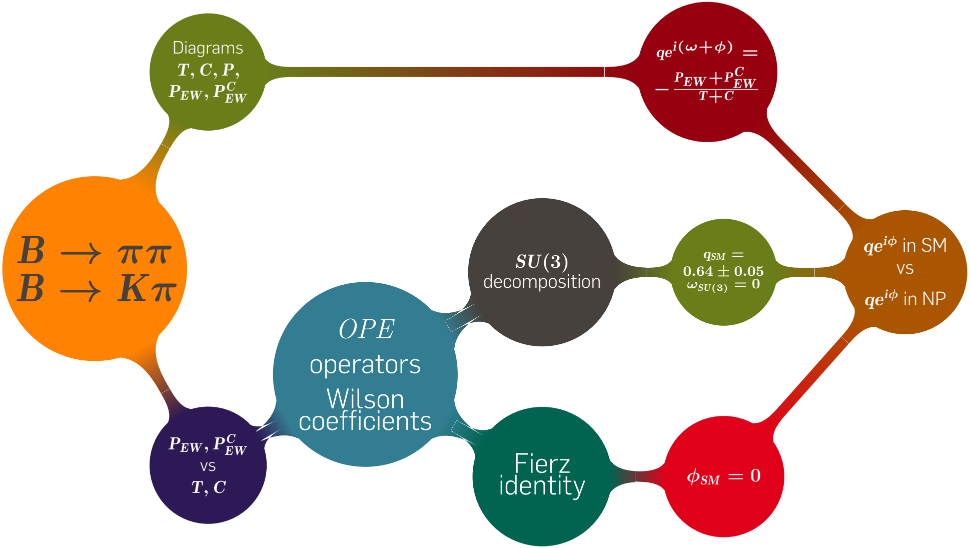
$qe^{i\phi} = -\frac{P'_{EW}}{C'+T'}$ -> NP	$qe^{i\phi} = -\frac{P'_{EW}+P'^{C}_{EW}}{C'+T'}$ -> NP
$P'^{C}_{EW} = 0$	P'^{C}_{EW} depends on C and T -> NP

Two approaches

NCBJ

basic	$\boldsymbol{SU(3)}$ decomposition
$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$	$q_{SM} = -\frac{3}{2}\frac{\lambda_t}{\lambda_u}\frac{c_9+c_{10}}{c_1+c_2}$
$\phi_{SM} = 0$ Fierz identity	$\phi_{SM} = 0$ Fierz identity
$\omega = 0$ $SU(3)$ limit	$\omega = 0$ $SU(3)$ limit
$qe^{i\phi} = -\frac{P'_{EW}}{C'+T'}$ -> NP	$qe^{i\phi} = -\frac{P'_{EW}+P'^{C}_{EW}}{C'+T'}$ -> NP
$P'^{C}_{EW} = 0$	P'^{C}_{EW} depends on C and T -> NP
$P_{EW} + P^{C}_{EW} = 0$	$P_{EW} + P^{C}_{EW} \sim C + T$ -> NP
	NP affects only $\langle \mathbf{27} \| \overline{\mathbf{15}} \| \mathbf{3} \rangle$ amplitude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operators
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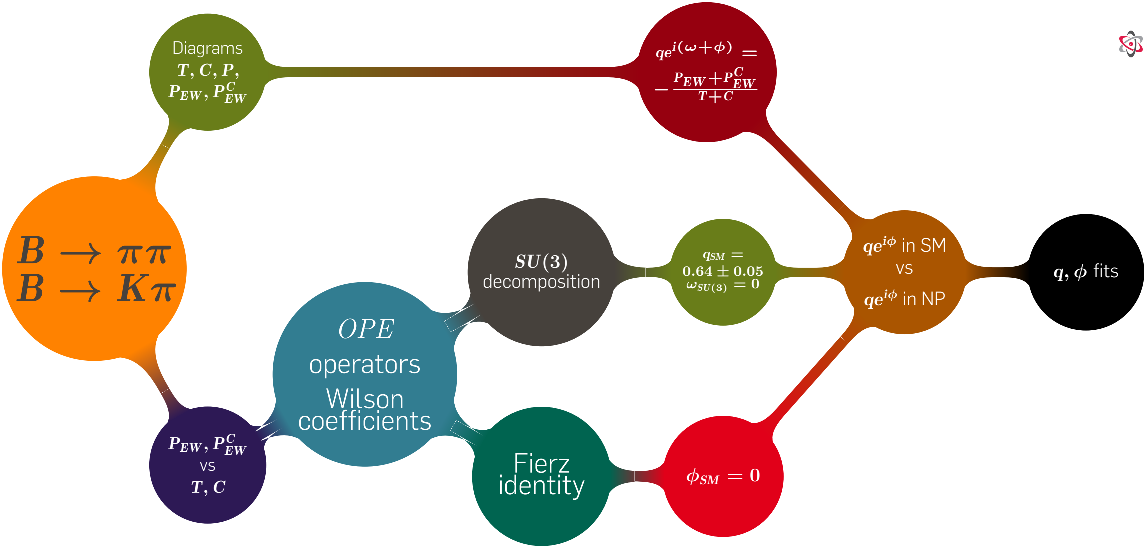
vs
T, C
Fierz identity
$\phi_{SM} = 0$

fits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Fitting procedure

Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\bar{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}$

and time-dependent S_{CP}: $A(t) = A_{CP}\cos\phi_d t + S_{CP}\sin\phi_d t$

Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

Fitting procedure

NCBJ

Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\bar{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}$

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Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

χ^2:

asymmetries $\chi^2 = \sum_i \frac{((A^{\text{exp}}_{CP})_i-(A^{\text{theory}}_{CP})_i)^2}{(\sigma_{\text{exp}})_i^2}$

ratios of BF $\chi^2 = \sum_{ij}(R_i^{\text{exp}} - R_i^{\text{theory}})\, Cov_{ij}^{-1}(R_j^{\text{exp}} - R_j^{\text{theory}})$

$R = \frac{\mathcal{B}_x}{\mathcal{B}_y}$

Fitting procedure

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Observables:

CP asymmetries direct $A_{CP} = \frac{\Gamma(\bar{i}\to\bar{f})-\Gamma(i\to f)}{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)} = \frac{|\bar{\mathcal{A}}|^2-|\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}$

and time-dependent S_{CP}: $A(t) = A_{CP}\cos\phi_d t + S_{CP}\sin\phi_d t$

Branching fraction $\mathcal{B} = \frac{\Gamma(\bar{i}\to\bar{f})+\Gamma(i\to f)}{2} = \frac{|\bar{\mathcal{A}}|^2+|\mathcal{A}|^2}{2}$

parameters (including $\boldsymbol{q}$ and $\boldsymbol{\phi}$)

χ^2:

asymmetries $\chi^2 = \sum_i \frac{((A_{CP}^{\text{exp}})_i-(A_{CP}^{\text{theory}})_i)^2}{(\sigma_{\text{exp}})_i^2}$

ratios of BF $\chi^2 = \sum_{ij}(R_i^{\text{exp}} - R_i^{\text{theory}})\, Cov_{ij}^{-1}(R_j^{\text{exp}} - R_j^{\text{theory}})$

$R = \frac{\mathcal{B}_x}{\mathcal{B}_y}$

Experimental data

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PDG world averages. Mainly BaBar, BELLE(2) and LHCb.

Observable	experimental value	source	corrected BF
$A^{\pi^+\pi^-}$	0.314 ± 0.030	PDG22	-
$A^{\pi^+\pi^0}$	0.01 ± 0.04	PDG22, Belle 2 CONF (2022)	-
$A^{\pi^0\pi^0}$	0.33 ± 0.22	PDG22	-
$S^{\pi^+\pi^-}$	-0.670 ± 0.030	PDG22	-
$BF(\pi^+\pi^-)$	$(5.16 \pm 0.19)10^{-6}$	PDG22, BELLE 2 CONF (2021)	$(1.80 \pm 0.64)10^4$
$BF(\pi^+\pi^0)$	$(5.6 \pm 0.4)10^{-6}$	PDG22, Belle 2 CONF (2022)	$(1.82 \pm 0.12)10^4$
$BF(\pi^0\pi^0)$	$(1.48 \pm 0.24)10^{-6}$	PDG22, BELLE 2 CONF (2021)	$(0.52 \pm 0.08)10^4$

$$R^{\pi^+\pi^-} \equiv= 2\frac{M_{B^+}}{M_{B_d^0}} \frac{\Phi(m_\pi/M_{B_d^0}, m_\pi/M_{B_d^0})}{\Phi(m_{\pi^0}/M_{B^+}, m_\pi/M_{B^+})} \left[\frac{BF(B^+ \to \pi^+\pi^0)}{BF(B_d^0 \to \pi^+\pi^-)}\right] \frac{\tau_{B_d^0}}{\tau_{B^+}}$$

$$R^{\pi^0\pi^0} \equiv= 2\frac{\Phi(m_\pi/M_{B_d^0}, m_\pi/M_{B_d^0})}{\Phi(m_{\pi^0}/M_{B_d^0}, m_{\pi^0}/M_{B_d^0})} \left[\frac{BF(B_d^0 \to \pi^0\pi^0)}{BF(B_d^0 \to \pi^+\pi^-)}\right]$$

$$\Phi(X, Y) = \sqrt{[1-(X+Y)^2][1-(X-Y)^2]}$$

Experimental data

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Observable	experimental value	source	corrected BF
$A^{\pi^+K^-}$	-0.0837 ± 0.0032	PDG22, BELLE 2 CONF (2021)	-
$A^{\pi^+K^0}$	-0.017 ± 0.016	PDG22, BELLE 2 CONF (2021)	-
$A^{\pi^0K^+}$	0.030 ± 0.013	PDG22, Belle 2 CONF (2022)	-
$A^{\pi^0K^0}$	-0.054 ± 0.121	PDG22, Belle 2 CONF (2022)	-
$S^{\pi^0K^0}$	0.58 ± 0.17	PDG22	-
$BF(\pi^+K^-)$	$(1.94 \pm 0.05)10^{-5}$	PDG22, BELLE 2 CONF (2021)	$(6.80 \pm 017)10^4$
$BF(\pi^+K^0)$	$(2.35 \pm 0.08)10^{-5}$	PDG22, BELLE 2 CONF (2021)	$(7.66 \pm 0.25)10^4$
$BF(\pi^0K^+)$	$(1.32 \pm 0.05)10^{-5}$	PDG22, Belle 2 CONF (2022)	(4.28 ± 0.15)
$BF(\pi^0K^0)$	$(10.0 \pm 0.5)10^{-6}$	PDG22, Belle 2 CONF (2022)	(3.51 ± 0.17)

$$R \equiv \left[\frac{BF^{corr}(B_d^0 \to \pi^- K^+)}{BF^{corr}(B^+ \to \pi^+ K^0)}\right]$$

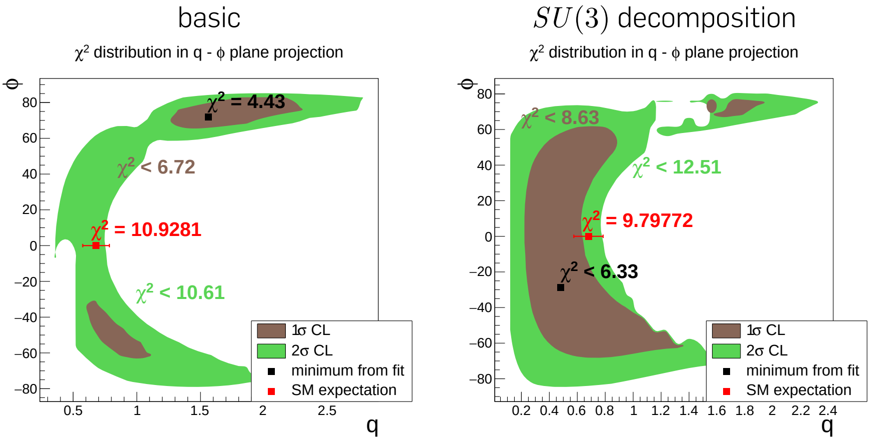
$$R_c \equiv \left[\frac{BF^{corr}(B^+ \to \pi^0 K^+)}{BF^{corr}(B^+ \to \pi^+ K^0)}\right]$$

$$R_n \equiv \left[\frac{BF^{corr}(B_d^0 \to \pi^- K^+)}{BF^{corr}(B_d^0 \to \pi^0 K^0)}\right] \qquad R^{\pi\pi-\pi K} \equiv \left[\frac{BF^{corr}(B^+ \to \pi^+\pi^0)}{BF^{corr}(B^+ \to \pi^0 K^+)}\right]$$

χ^2 scan

- For each fixed point in $q,\ \phi$ plane fit the rest of parameters - obtain χ^2 value for this point.
- find minimal χ^2 value.
- draw contours $(1\sigma,\ 2\sigma,\ ...)$ of the inverse χ^2 distribution.

fits


basic
χ2 distribution in q - ϕ plane projection
χ2 = 4.43
χ2 < 6.72
χ2 = 10.9281
χ2 < 10.61
1σ CL
2σ CL
minimum from fit
SM expectation
q
SU(3) decomposition
χ2 distribution in q - ϕ plane projection
χ2 < 8.63
χ2 < 12.51
χ2 = 9.79772
χ2 < 6.33
q

Solving the $B \to K\pi$ puzzle

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$$\Delta A_{CP}(K\pi) = A_{CP}(B^+ \to K^+\pi^0) - A_{CP}(^0\!\to K^+\pi^-) \simeq -2(\Im(r'_C))\sin\gamma + 2\Im(r'_T + r'_C)q\sin\phi$$

$$\Delta A_{CP}(K\pi) == 0.114 \pm 0.014$$

	SM scenario		NP scenario	
	basic	SU(3)decomposition	basic	SU(3)decomposition
ϕ	0	0	$(73 \pm 5)^\circ$	$(-31 \pm 45)^\circ$
q	0.61 ± 0.05	0.60 ± 0.06	1.7 ± 0.3	0.5 ± 0.3
γ	$(65.2 \pm 1.6)^\circ$	$(65.3 \pm 1.3)^\circ$	$(65.5 \pm 1.3)^\circ$	$(65.6 \pm 1.3)^\circ$
$\Im r'_C$	-0.062 ± 0.006	-0.064 ± 0.008	-0.006 ± 0.025	-0.054 ± 0.013
$\Im r'_T$	0.035 ± 0.010	0.035 ± 0.010	0.036 ± 0.010	0.0325 ± 0.012
$\mathbf{\Delta A_{CP}}$	$\mathbf{0.111 \pm 0.013}$	$\mathbf{0.111 \pm 0.013}$	$\mathbf{0.110 \pm 0.013}$	$\mathbf{0.111 \pm 0.013}$

Toy projection for the future

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The fit is sensitive to $\mathcal{B}$ and A_{CP}, and S_{CP} of $B \to \pi^0 K^0$.
BELLE 2 can bring results for all three. LHCb to $\mathcal{B}(B \to K\pi)$.

Check what happens when the result is shifted by 1σ and the accuracy increased.

Toy projection for the future

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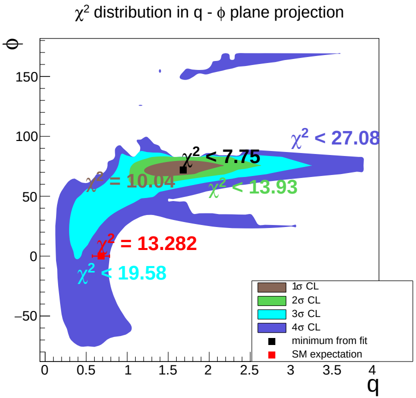
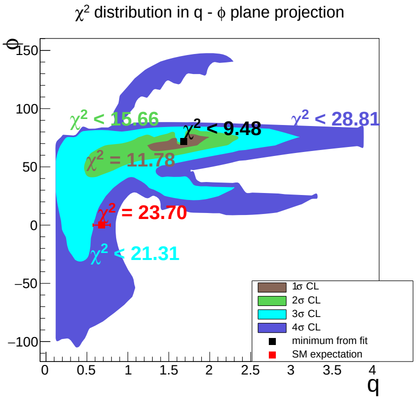
$$A_{CP}^{\text{new}}(\pi^0 K^0) = A_{CP}^{\text{old}}(\pi^0 K^0) + 1\sigma^{\text{old}}_{A_{CP}(\pi^0 K^0)}$$
$$S_{CP}^{\text{new}}(\pi^0 K^0) = S_{CP}^{\text{old}}(\pi^0 K^0) - 1\sigma^{\text{old}}_{S_{CP}(\pi^0 K^0)}$$

$$\sigma^{\text{new}}_{A_{CP}(\pi^0 K^0)} = 1/\sqrt{2}\sigma^{\text{old}}_{A_{CP}(\pi^0 K^0)}.$$
$$\sigma^{\text{new}}_{S_{CP}(\pi^0 K^0)} = 1/\sqrt{2}\sigma^{\text{old}}_{S_{CP}(\pi^0 K^0)}.$$

basic



$SU(3)$ decomposition



projection for the future

The fit is sensitive to $\mathcal{B}$ and A_{CP} of $B \to \pi^0 K^0$.

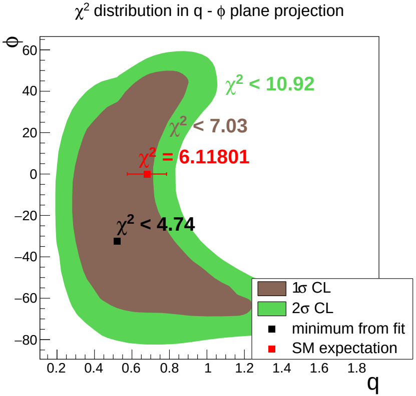
$$\mathcal{B}^{\text{new}}(\pi^0 K^0) = \mathcal{B}^{\text{old}}(\pi^0 K^0) - 1\sigma^{\text{old}}_{\mathcal{B}(\pi^0 K^0)}$$

$$A^{\text{new}}_{CP}(\pi^0 K^0) = A^{\text{old}}_{CP}(\pi^0 K^0) + 1\sigma^{\text{old}}_{A_{CP}(\pi^0 K^0)}$$

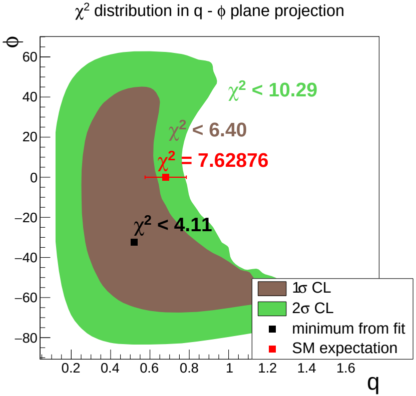
$$\sigma^{\text{new}}_{\mathcal{B}(\pi^0 K^0)} = 1/\sqrt{2}\sigma^{\text{old}}_{\mathcal{B}(\pi^0 K^0)}.$$

$$\sigma^{\text{new}}_{A_{CP}(\pi^0 K^0)} = 1/\sqrt{2}\sigma^{\text{old}}_{A_{CP}(\pi^0 K^0)}.$$

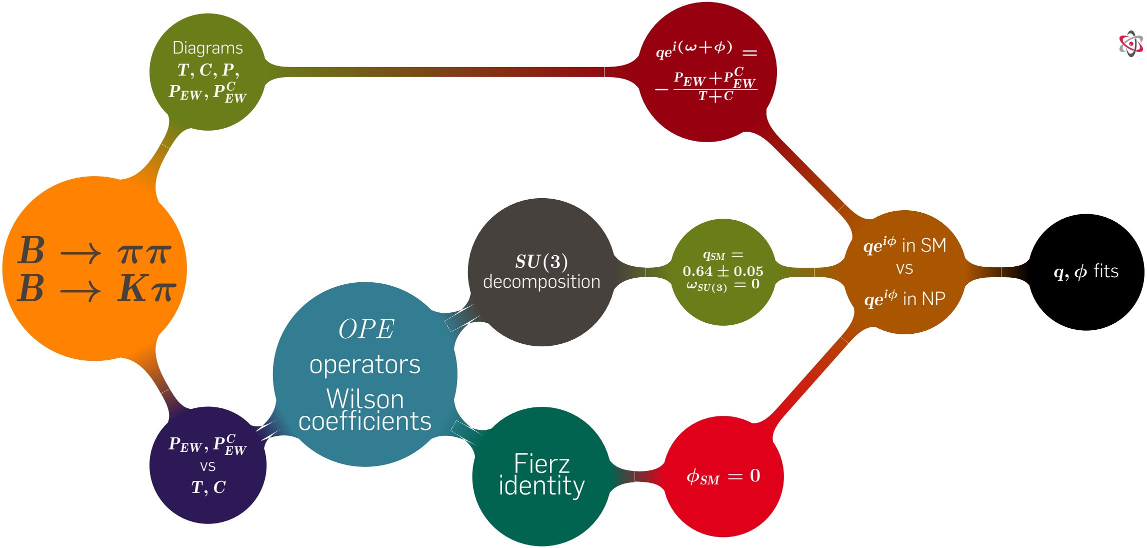
basic



$SU(3)$ decomposition



Summary


$B \to \pi\pi$
$B \to K\pi$
Diagrams
$T, C, P, P_{EW}, P^C_{EW}$
$qe^{i(\omega+\phi)} = -\frac{P_{EW}+P^C_{EW}}{T+C}$
P_{EW}, P^C_{EW} vs T, C
OPE
operators
Wilson coefficients
$SU(3)$ decomposition
$q_{SM} = 0.64 \pm 0.05$
$\omega_{SU(3)} = 0$
Fierz identity
$\phi_{SM} = 0$
$qe^{i\phi}$ in SM
vs
$qe^{i\phi}$ in NP
q, ϕ fits

Summary

- ✔ Diagrammatic approach is a helpful tool to study $B \to PP$ decays.
- ✔ The ad-hoc parameters $qe^{i\phi}$ are used to search for NP impact.
- ✔ The operator product expansion together with the $SU(3)$ decomposition brings relations between the EW penguin and tree amplitudes reducing the number of parameters in the problem.
- ✔ The joint fit to the $B \to \pi\pi$ and $B \to \pi K$ data does not exclude neither the SM nor the NP scenario.
- ✔ The fit is very sensitive to the measurement of $A_{CP}^{\pi^0 K^0}$ and $S_{CP}^{\pi^0 K^0}$.
- ✔ New results from BELLE 2 can shed more light to understand the situation.

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Thank you!

$SU(3)$

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$$\sqrt{2}\mathcal{A}(B \to \pi^+\pi^0) = -e^{i\gamma}(T' + C') \overset{SU(3)}{=} -\frac{V_{ud}}{V_{us}}\frac{f_\pi}{f_K}(C + T)$$

$\frac{f_\pi}{f_K} = 1.22 \pm 0.01$ - leading (i.e., factorisable) $SU(3)$ - breaking corrections
prime $'$ - labels $b \to d$ transitions.
Bose-Einstein statics brings $I(\pi^0\pi^+) = 2 \Rightarrow \Delta I = \frac{3}{2}$???
$\bar{Q}'_1 - \bar{Q}'_2$ contribute to $\Delta I = \frac{1}{2}$???
$\Rightarrow$ Only $\bar{Q}'_1 + \bar{Q}'_2$ contributes to $B^+ \to \pi^0\pi^+$
assuming $SU(3)$ only $\bar{Q}_1 + \bar{Q}_2$ contributes to $A_{3/2}$ in $B \to \pi K$

$$\frac{\langle \pi K(I = \frac{3}{2})|\, \bar{Q}_1 - \bar{Q}_2\, |B^+\rangle}{\langle \pi K(I = \frac{3}{2})|\, \bar{Q}_1 + \bar{Q}_2\, |B^+\rangle} \equiv -\delta_{SU(3)}\, e^{i\Delta\varphi}$$

general factorisation hypothesis - $\delta_{SU(3)} \approx 1 - 3\%$ and $\Delta\varphi$

Isospin decomposition

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$$H_{\Delta I=0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2} \left[\frac{\lambda_u}{2} C_i (Q_i^u + Q_i^d) + \lambda_c Q_i^c \right] + \lambda_t \sum_{i=3}^{10} C_i Q_i - \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right\} + h.c.$$

$$H_{\Delta I=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1,2} \frac{\lambda_u}{2} C_i (Q_i^u - Q_i^d) + \lambda_t \sum_{i=7}^{10} C_i Q_i^{\Delta I=1} \right] + h.c.$$

$$A_{3/2} = \sqrt{\frac{1}{3}} \left\langle \frac{3}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=1} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$A_{1/2} = \pm\sqrt{\frac{2}{3}} \left\langle \frac{1}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=1} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$B_{1/2} = \sqrt{\frac{2}{3}} \left\langle \frac{1}{2}, \pm\frac{1}{2} \middle| \mathcal{H}_{\Delta I=0} \middle| \frac{1}{2}, \pm\frac{1}{2} \right\rangle$$

$$\mathcal{A}_{-+} = \mathcal{A}(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2}$$

$$\mathcal{A}_{+0} = \mathcal{A}(B^+ \to \pi^+ K^0) = A_{3/2} + A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{00} = \sqrt{2}\mathcal{A}(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{0+} = \sqrt{2}\mathcal{A}(B^+ \to \pi^0 K^+) = 2A_{3/2} - A_{1/2} - B_{1/2}$$

$A_{3/2}$

$$\mathcal{A}_{-+} = \mathcal{A}(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2}$$

$$\mathcal{A}_{00} = \sqrt{2}\mathcal{A}(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2}$$

$$\mathcal{A}_{-+} = -Te^{i\gamma} + P_{tc} - P'_{uc}e^{i\gamma} - \frac{2}{3}P_{EW}^C$$

$$\mathcal{A}_{00} = -Ce^{i\gamma} - P_{tc} + P_{uc}e^{i\gamma} - P_{EW} - \frac{1}{3}P_{EW}^C$$

$$\mathcal{A}_{-+} + \mathcal{A}_{00} = 3A_{3/2} = -(C+T)e^{i\gamma} - P_{EW} - P_{EW}^C = -(C+T)(e^{i\gamma} - qe^{i\omega}e^{i\phi})$$

$$qe^{i(\phi+\omega)} \equiv -\frac{P_{EW} + P_{EW}^C}{C+T}$$

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$$A_{3/2} = \sqrt{\frac{1}{3}}\left\langle \frac{3}{2}, \pm\frac{1}{2}\right| \mathcal{H}_{\Delta I=1} \left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle$$

$$= \sqrt{\frac{1}{3}}\left\langle \frac{3}{2}, \pm\frac{1}{2}\right| \frac{1}{2}\left(1 - \frac{3}{2}\frac{\lambda_t(C_9 + C_{10})}{\lambda_u(C_1 + C_2)}\right)\lambda_u(C_1 + C_2)(\bar{Q}_1 + \bar{Q}_2)\left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle$$

$$A_{3/2} = -\frac{1}{3}(C + T)(e^{i\gamma} - qe^{i\omega}e^{i\phi})$$

If $\left\langle \frac{3}{2}, \pm\frac{1}{2}\right| \frac{\sqrt{3}}{2}\lambda_u(C_1 + C_2)(\bar{Q}_1 + \bar{Q}_2)\left|\frac{1}{2}, \pm\frac{1}{2}\right\rangle = (T + C)e^{i\gamma}$???

$$q = -\frac{3}{2\lambda^2 R_b}\frac{C_9 + C_{10}}{C_1 + C_2} = 0.64 \pm 0.05$$

$$\frac{\lambda_u}{\lambda_t} \approx -\lambda^2 R_b e^{i\gamma} \quad \lambda \approx 0.22 \text{ and } R_b = \frac{1}{\lambda}\frac{V_{ub}}{V_{cb}} \approx 0.41 \pm 0.07$$

$SU(3)$ breaking

[Benjamin Grinstein and Richard F. Lebec

NCBJ

Let us now consider SU(3)-breaking corrections to the lowest-order Hamiltonian. The simplest such breaking originates through insertions of the strange quark mass,

$$\mathcal{H}_s = m_s \bar{s} s, \tag{4.3}$$

which transforms as an $I = 0$, $Y = 0$ octet plus singlet in SU(3). Clearly neither piece changes the isospin of the Hamiltonian; this would be accomplished by insertions of the up or down masses, which are much smaller. Let us consider SU(3) breaking linear in m_s. In the case of $B \to PP$, the Hamiltonian contains pieces transforming under

$$(\bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}) \otimes (\mathbf{1} \oplus \mathbf{8}) = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}} \oplus \mathbf{24} \oplus \overline{\mathbf{42}}, \tag{4.4}$$