Towards CP violation in baryons from lattice QCD

Maxwell T. Hansen

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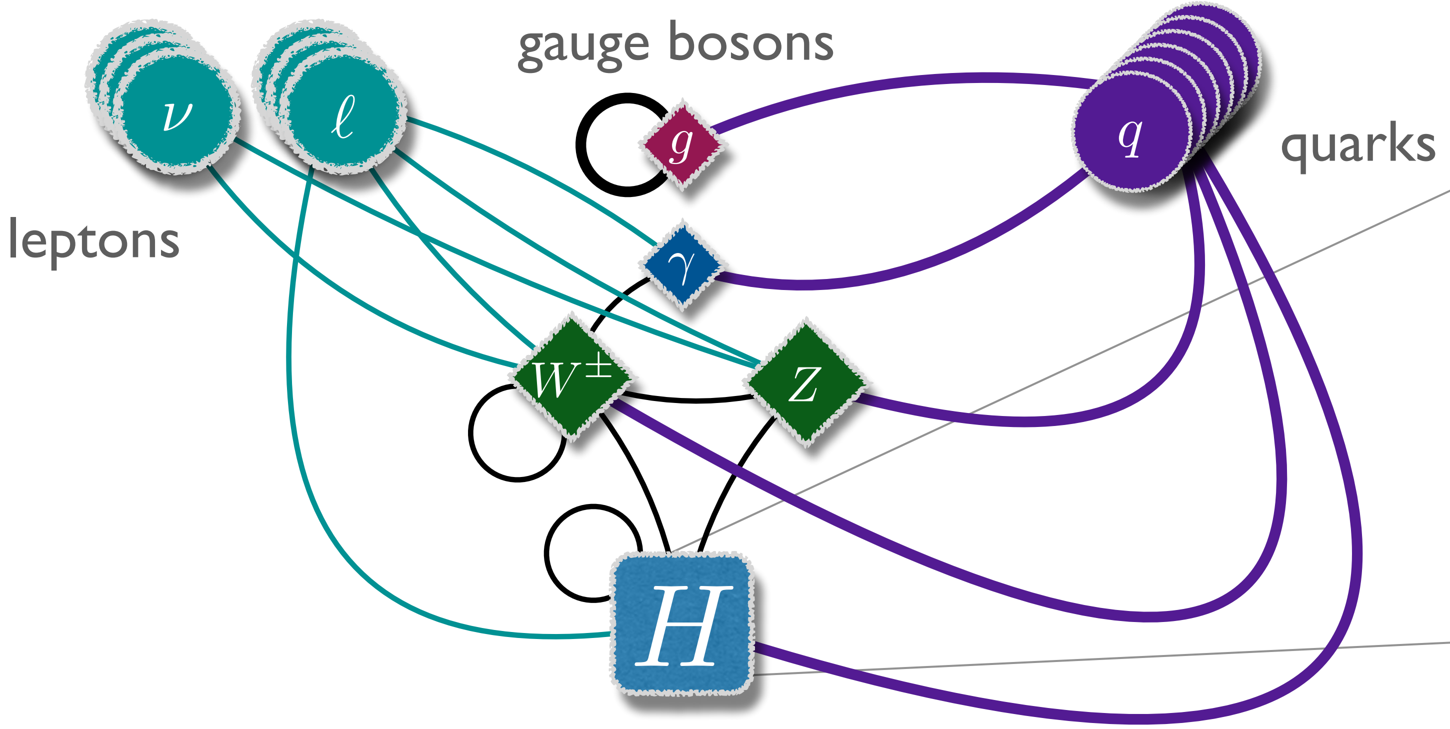
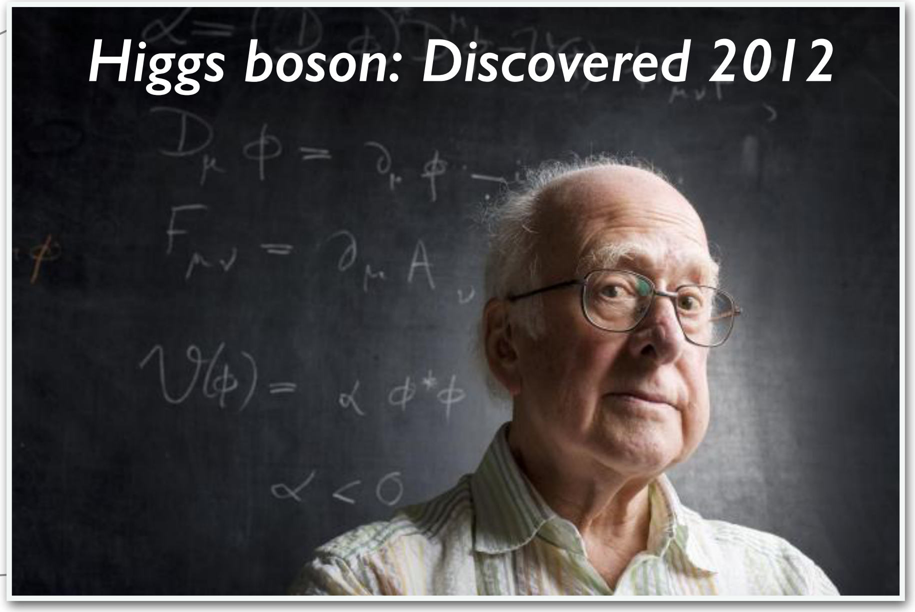
THE UNIVERSITY *of* EDINBURGH

The Standard Model

- A catalogue of particles and forces, highly constrained by *fundamental symmetries*




Higgs boson: Discovered 2012

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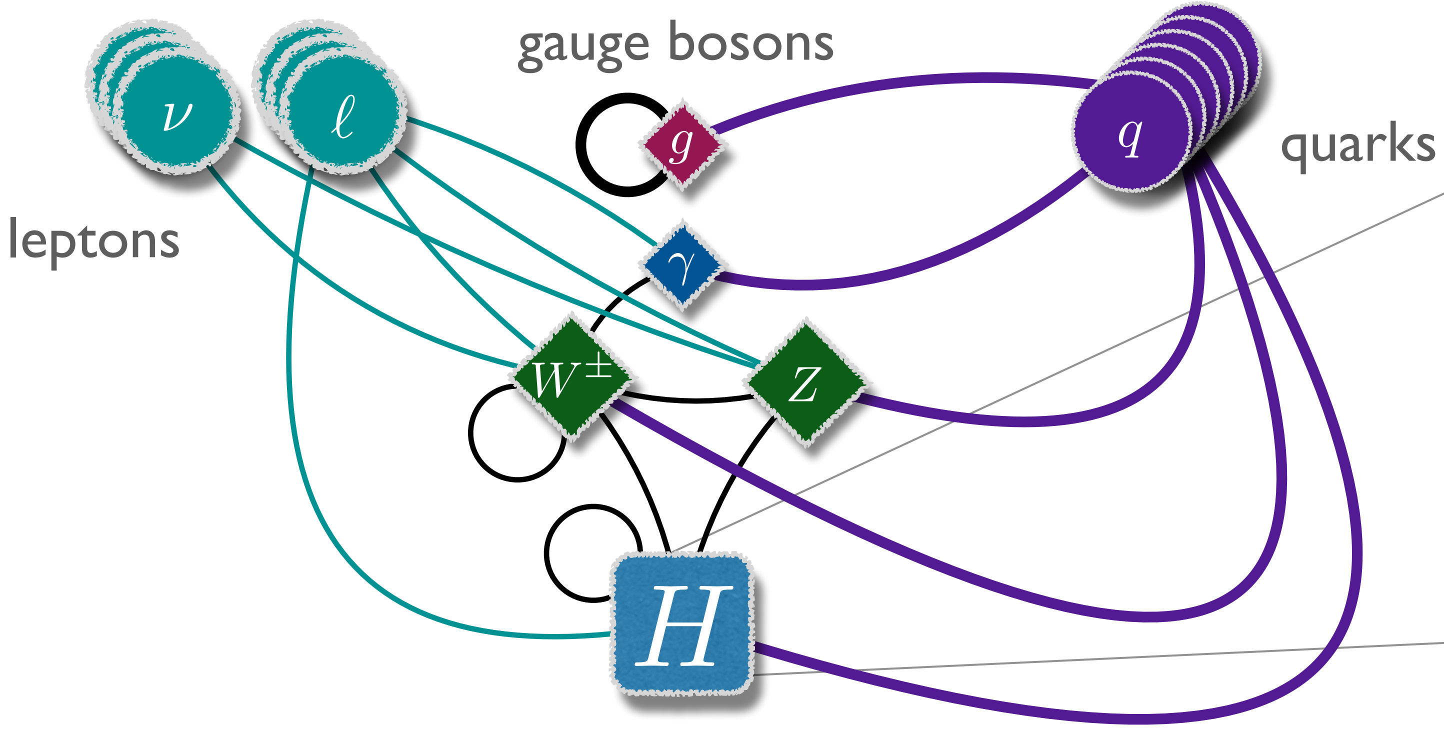
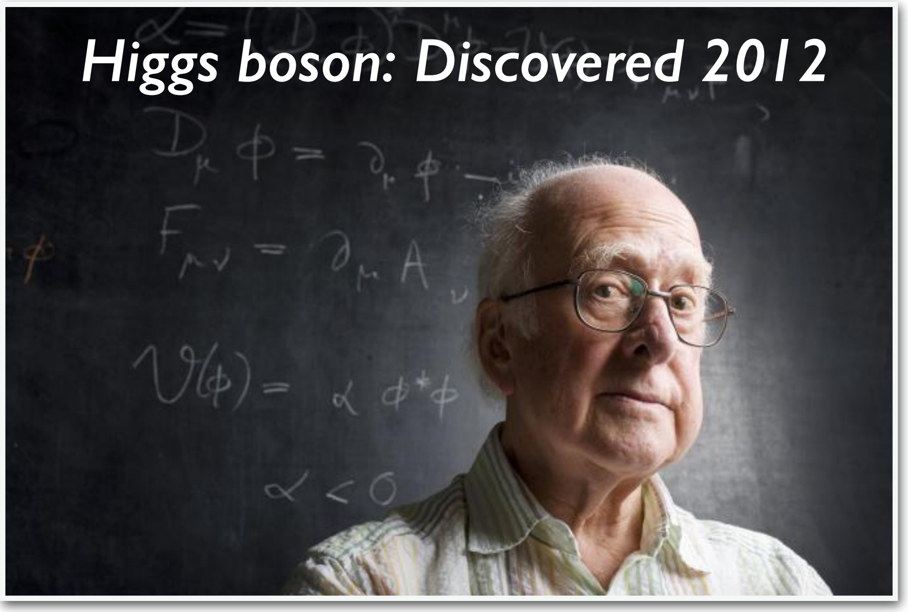
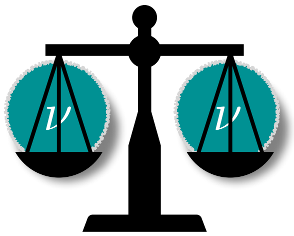


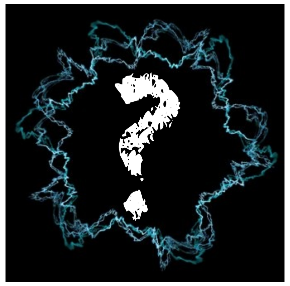

Higgs boson: Discovered 2012

☐ SM = a very ***effective theory***… but definitely not the whole story

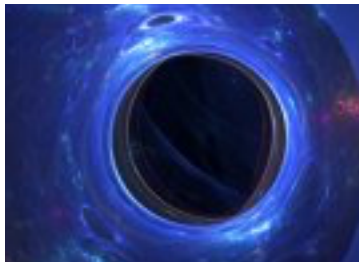
neutrino masses



gravity



dark matter



matter / anti-matter asymmetry

Why is our universe matter dominated?

Sakharov's conditions:

- Baryon number violation
- C and CP symmetry violation
- Interactions out of thermal equilibrium

Taking new physics seriously

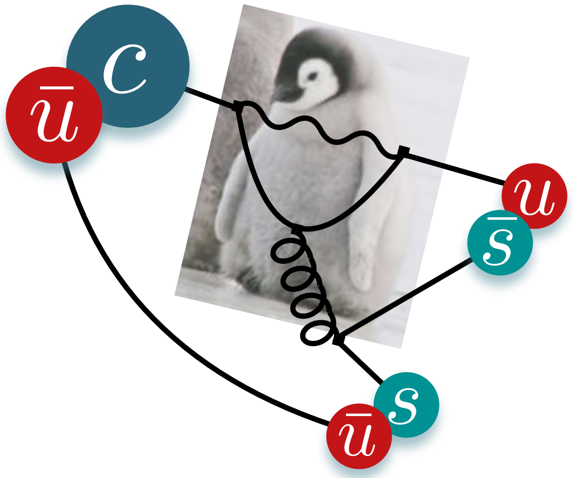
☐ SM is well known to have CPV

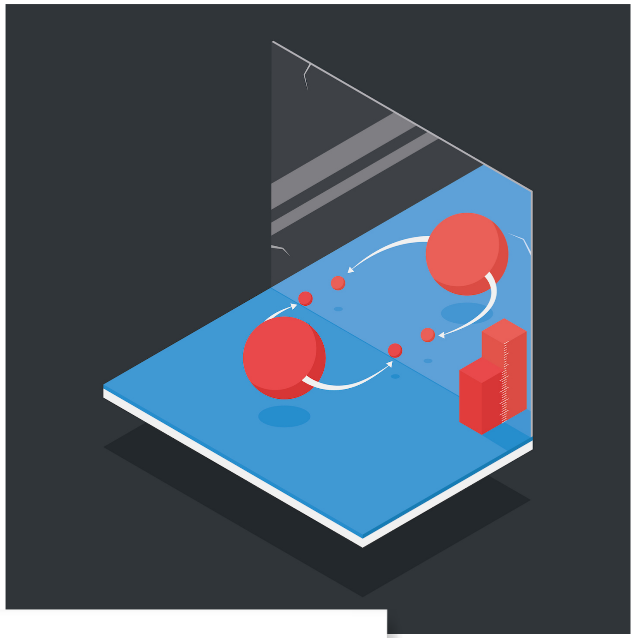
$$\mathrm{Im}[V_{\mathrm{CKM}}] \neq 0$$

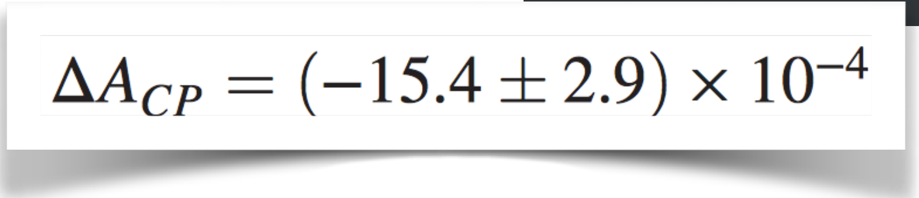
…but not enough for ***baryogenesis***!

☐ 2019: LHCb observed CP violation in *charm*

$$D \to \pi\pi, K\overline{K}$$







$$\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$$

- LHCb (PRL, 2019) •

Taking new physics seriously

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Is this new physics? →

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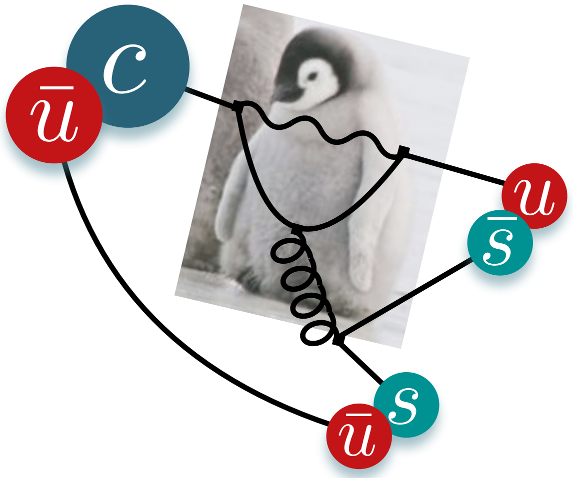

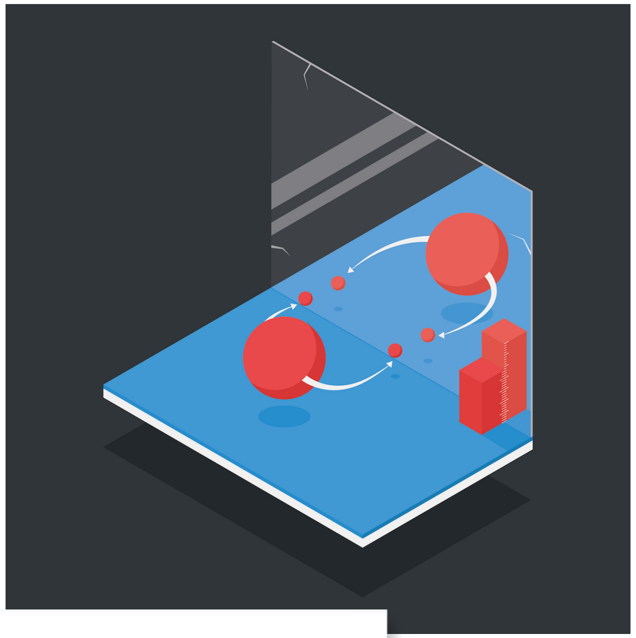

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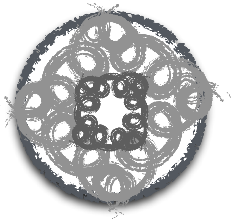
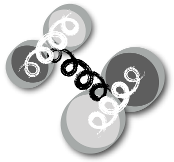
☐ No Standard Model prediction is available, due to complicated QCD dynamics

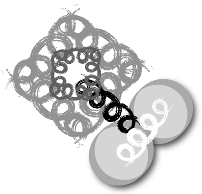
glueballs



tetraquarks



hybrids

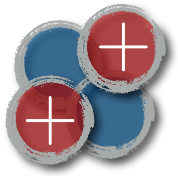


☐ What if CPV is seen in baryons? → same issue = *limited SM predictions*

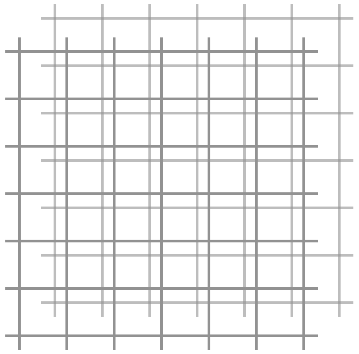
Lattice QCD: *Recipe for strong force predictions*

1. Lagrangian defining QCD
2. Formal / numerical machinery (lattice QCD)
3. A few experimental inputs (e.g. M_π, M_K, M_Ω)

$$\mathcal{L}_{\text{QCD}} = \sum_f \overline{\Psi}_f (i\not{D} - m_f)\Psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



Wide range of precision pre-/post-dictions



$m_q^{\overline{\text{MS}}}(\mu)$

m_p

$\alpha_{\overline{\text{MS}}}(\mu)$

m_n

f_π

g_A

f_K

$(g-2)_\mu^{\text{HVP}}$

f_D

f_B

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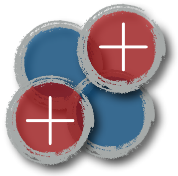

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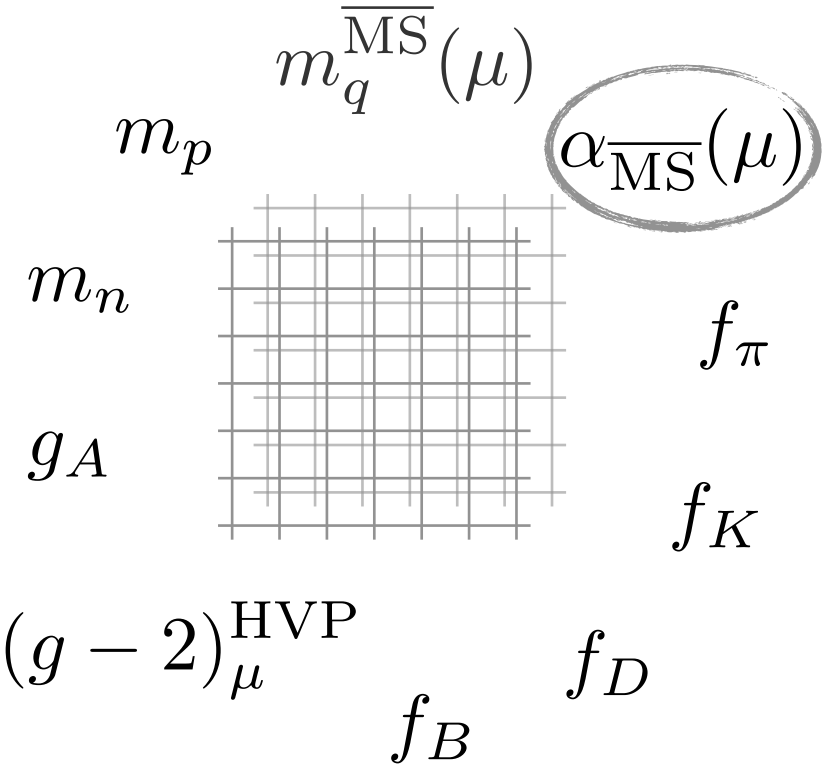


FLAG 2019 Flavour Lattice Averaging Group

$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1182(8)$ lattice average

$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1174(16)$ PDG 18 (non-lattice)

Overwhelming evidence for QCD ✓ Lattice QCD as a reliable tool ✓

More challenging observables? 😮

Baryonic amplitudes and LQCD

☐ Essential to classify by types of states and inserted operators

- 1. Single-hadron form factors

  $\boxed{\langle \mathrm{hadron} | \mathcal{O}(0) | \mathrm{hadron} \rangle}$

  $$n \to p\, e^-\, \overline{\nu}_e$$

- 2. Multi-hadron decays

  $\boxed{\langle \mathrm{multi\text{-}hadron\ state} | \mathcal{O}(0) | \mathrm{hadron} \rangle}$

  $$\Lambda_c^+ \to pK_S^0,\ \Lambda^0\pi^+,\ \Sigma^+\pi^0,\ \Sigma^0\pi^+ \qquad \Lambda_c^+ \to p\, K_S^0\, \pi^+$$

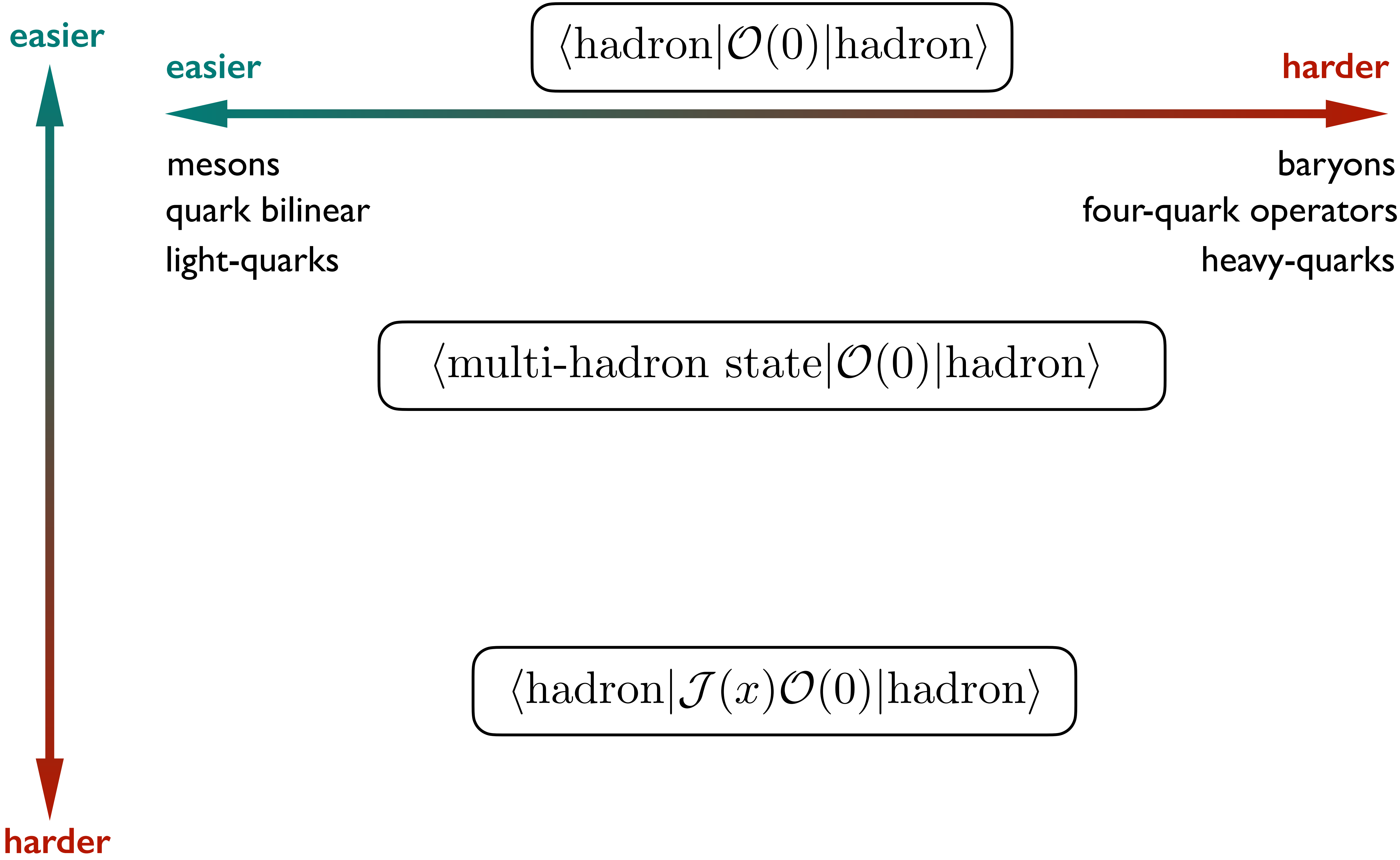
- 3. Intermediate multi-hadron states

  $\boxed{\langle \mathrm{hadron} | \mathcal{J}(x)\mathcal{O}(0) | \mathrm{hadron} \rangle}$

  $$\Sigma^+ \to p\ell^+\ell^-$$

☐ No published LQCD baryonic calculations for items 2. and 3.

Can LQCD calculate X for baryonic CPV?



easier

easier

$\langle \text{hadron} | \mathcal{O}(0) | \text{hadron} \rangle$

harder

mesons

quark bilinear

light-quarks

baryons

four-quark operators

heavy-quarks

$\langle \text{multi-hadron state} | \mathcal{O}(0) | \text{hadron} \rangle$

$\langle \text{hadron} | \mathcal{J}(x) \mathcal{O}(0) | \text{hadron} \rangle$

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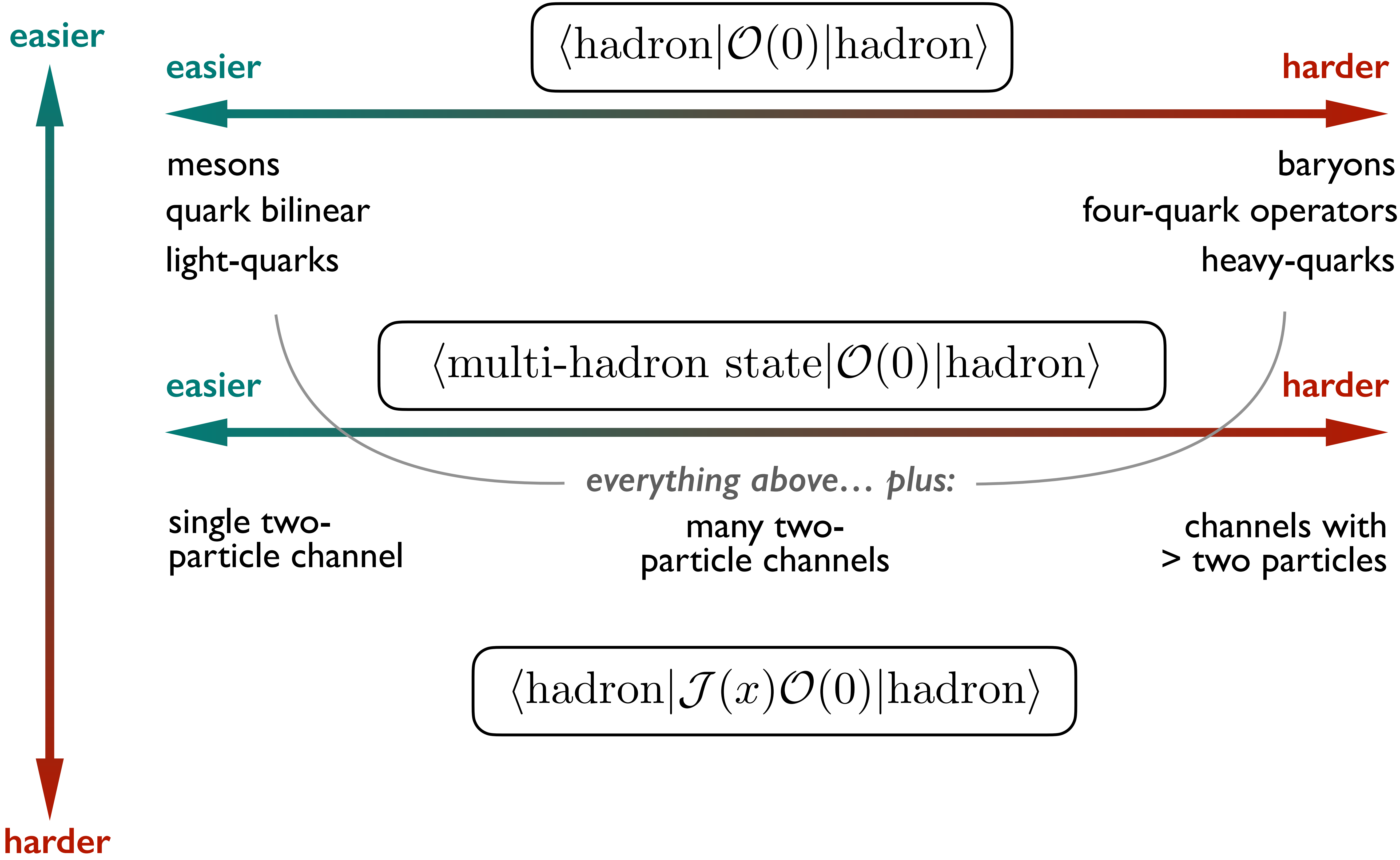

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easier

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everything above… plus:

single two-particle channel

many two-particle channels

channels with > two particles

$$\langle \text{hadron} | \mathcal{J}(x) \mathcal{O}(0) | \text{hadron} \rangle$$

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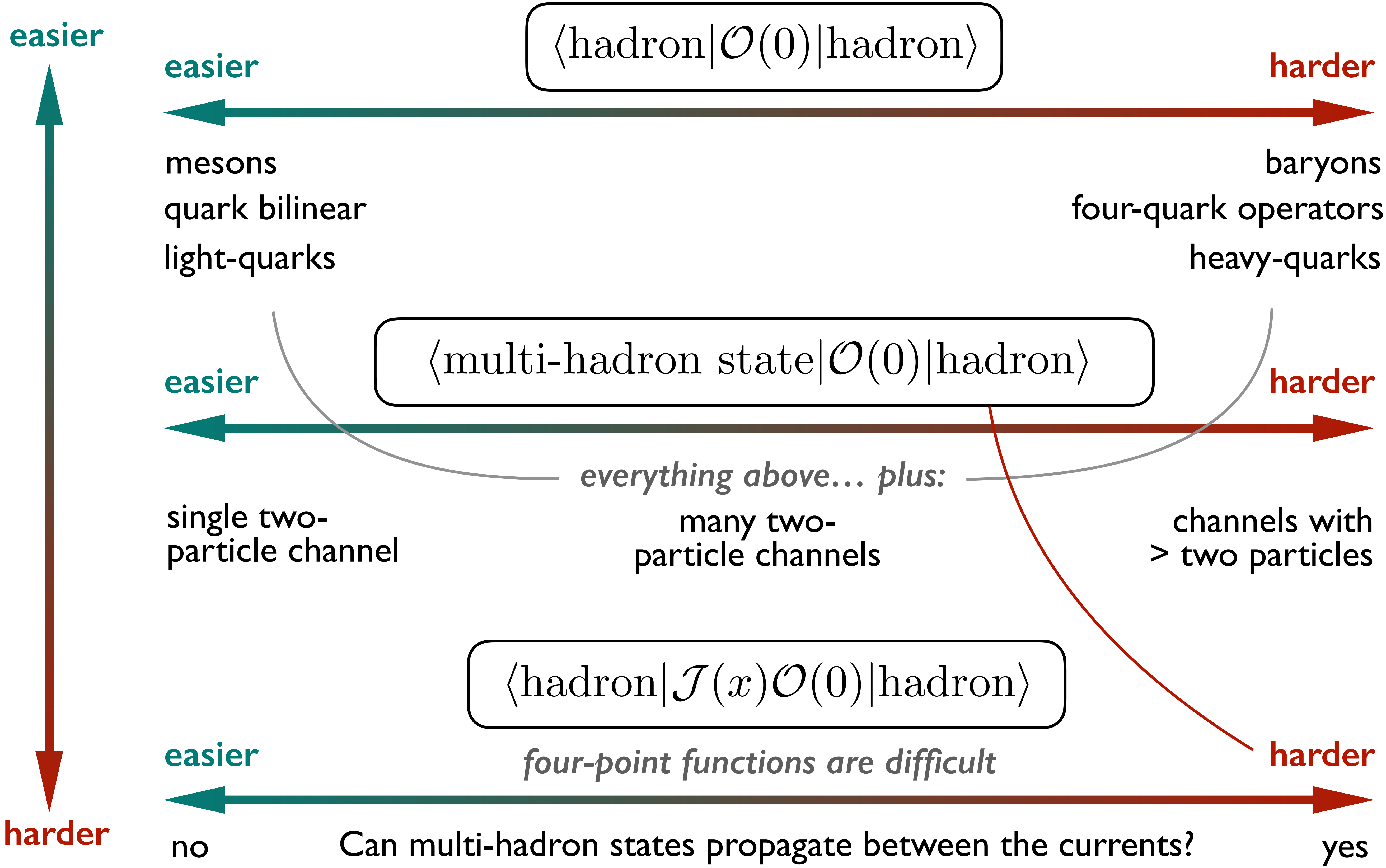

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everything above… plus:

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$\langle \text{hadron} | \mathcal{J}(x)\mathcal{O}(0) | \text{hadron} \rangle$

easier

four-point functions are difficult

harder

no

Can multi-hadron states propagate between the currents?

yes

Resonances

☐ If multi-hadron states play a role… ***resonances*** could be relevant

☐ Meson decays

What is the role of the $\sigma/f_0(500)$?

CP violation in strange $K \to \pi\pi$

$f_0(1710)$ could enhance ΔA_{CP}

• Soni (2017) •

CP violation in charm $D \to \pi\pi, K\overline{K}$

Resonant B decays $B \to K^* \ell\ell \to K\pi\,\ell\ell$

Resonances

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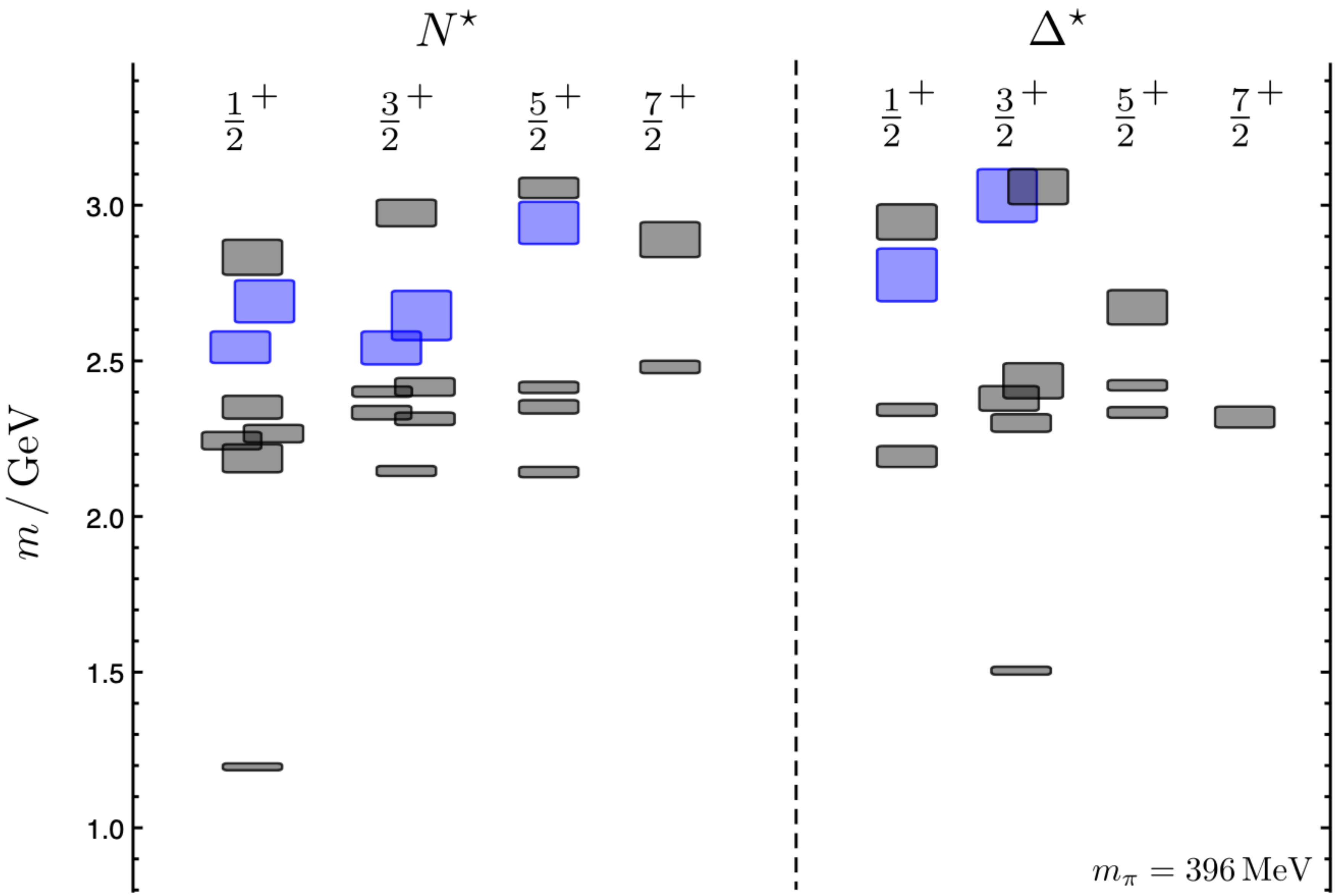
- ☐ Of course, resonances = also important for baryonic processes
  - ○ Any reliable approach should consider whether such effects are relevant
  - ○ Can be both a challenge and an opportunity (also for lattice QCD)

Baryonic resonances



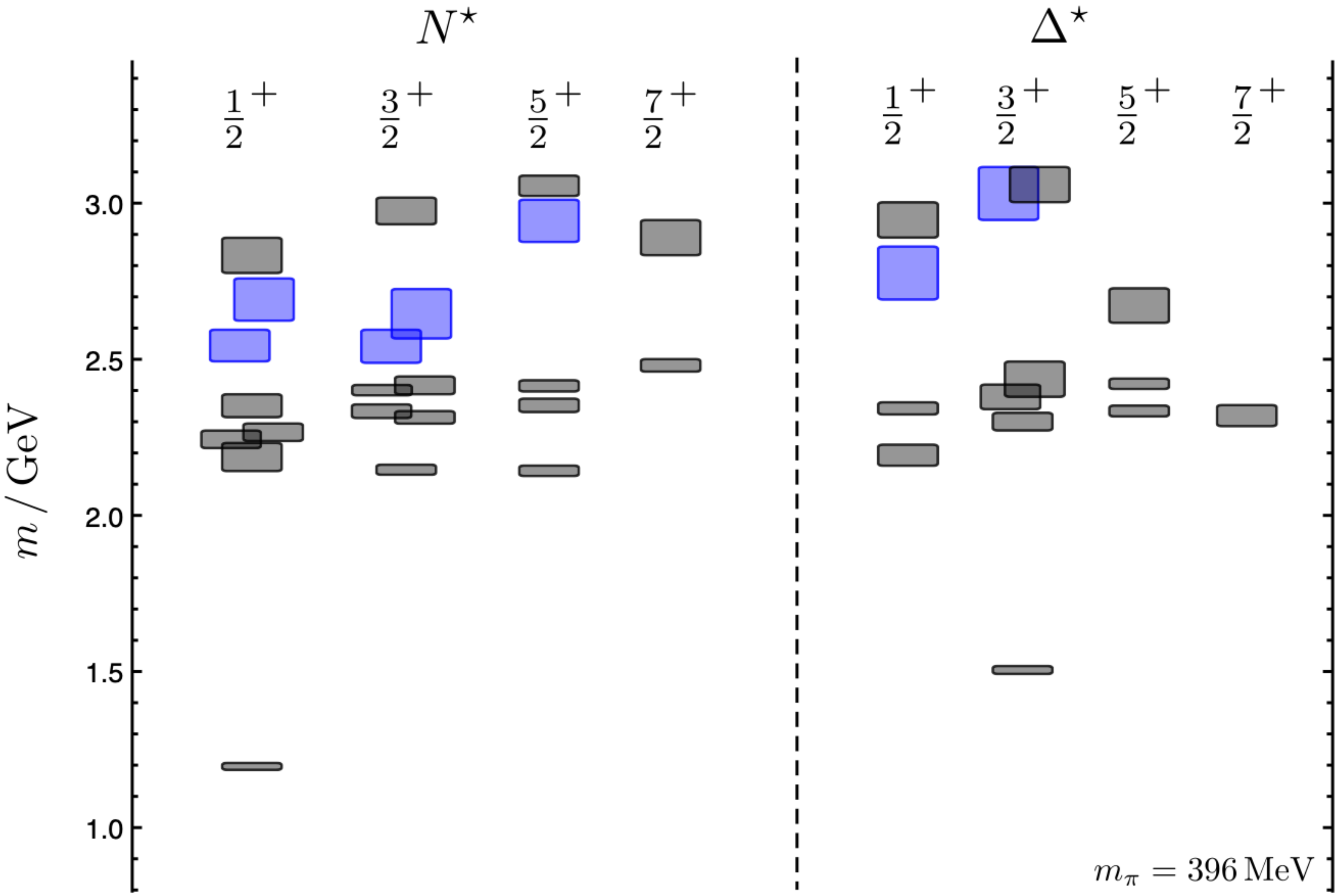
heavier than physical M_π

Height of rectangle = statistical uncertainty

States created using local operators

- Dudek and Edwards, *Hybrid Baryons in QCD*, PRD, 2012

Baryonic resonances



heavier than physical M_π

Height of rectangle = statistical uncertainty

States created using local operators

Not quite right to say "*States cannot decay*" (more later)

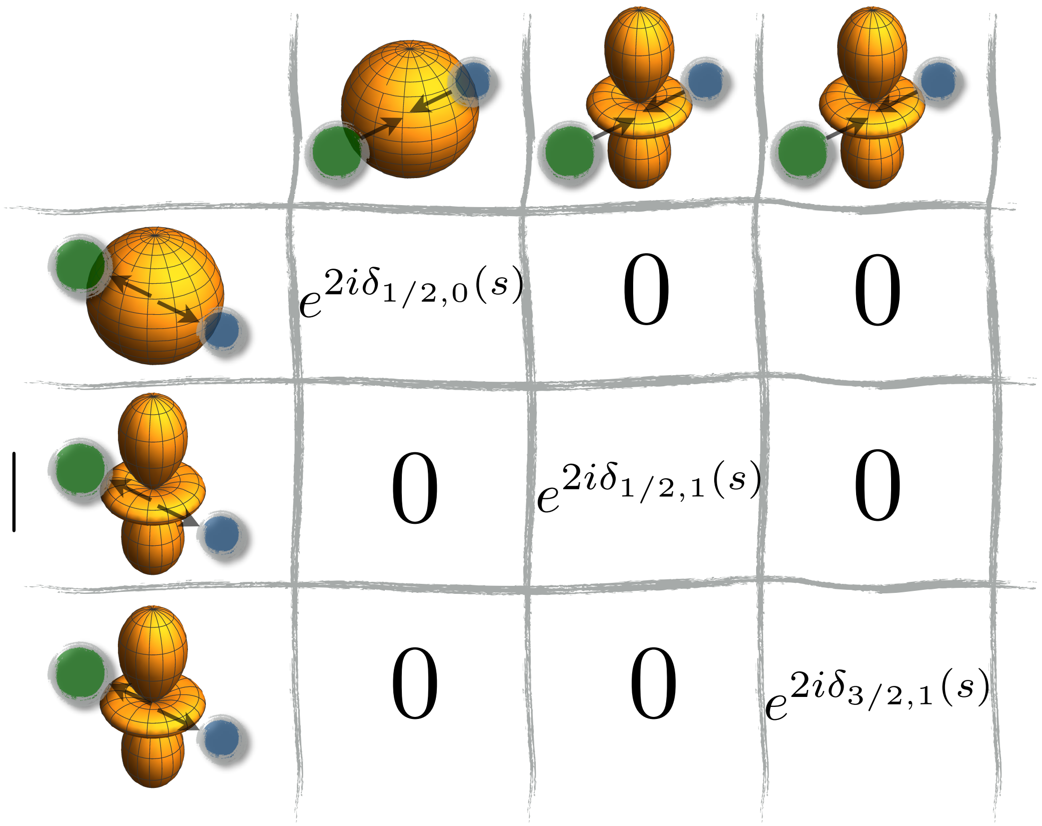
Remarkable progress… but not the complete picture!

- Dudek and Edwards, *Hybrid Baryons in QCD*, PRD, 2012 •

QCD Fock space

- ☐ At low-energies QCD = hadronic degrees of freedom $\pi \sim \bar{u}d,\ K \sim \bar{s}u,\ p \sim uud$
- ☐ Overlaps of multi-hadron ***asymptotic states*** → S matrix

$$|N\pi, \text{in}\rangle$$



$$S(s) \equiv \langle N\pi, \text{out}| \quad \begin{pmatrix} e^{2i\delta_{1/2,0}(s)} & 0 & 0 \\ 0 & e^{2i\delta_{1/2,1}(s)} & 0 \\ 0 & 0 & e^{2i\delta_{3/2,1}(s)} \end{pmatrix}$$

depends on $s = E_{\text{cm}}^2$
and angular variables

diagonal in total angular momentum

$$\mathcal{M}_{J,\ell}(s) \propto e^{2i\delta_{J,\ell}(s)} - 1$$

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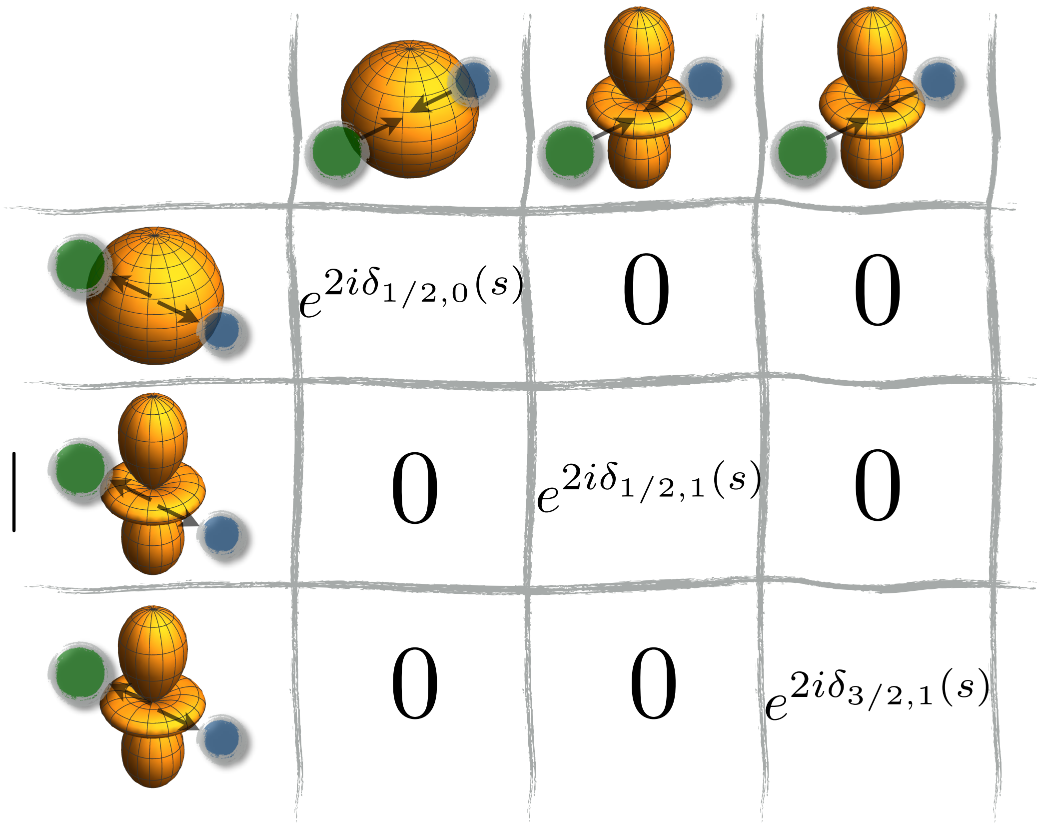

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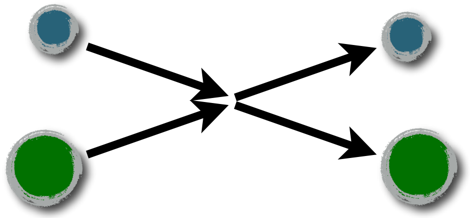
- ☐ Poles on the second Riemann sheet give resonances

Full QCD demands this description… lattice QCD cannot escape it

Unitarity and analyticity

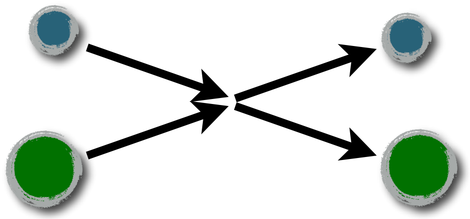


- ☐ For $s < (M_N + 2M_\pi)^2$, the optical theorem tells us…

$$\rho_{N\pi}(s)|\mathcal{M}_{J,\ell}(s)|^2 = \operatorname{Im}\mathcal{M}_{J,\ell}(s)$$

where $\rho_{N\pi}(s) = \sqrt{p^2(s, M_\pi, M_N)/s}$ is the phase space… continuum of $N\pi$ states

Unitarity and analyticity



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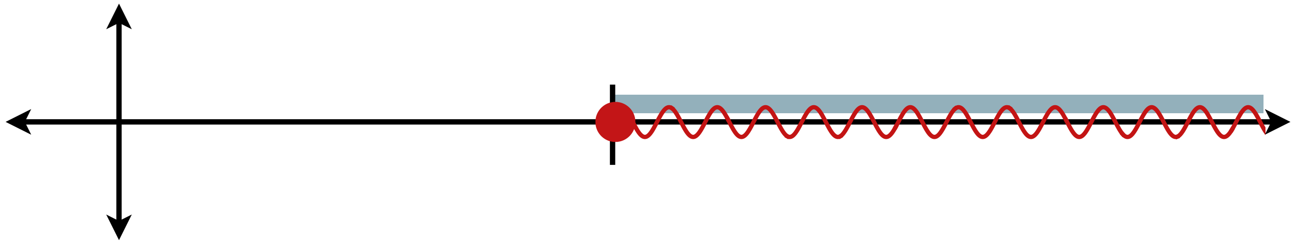
☐ Unique solution is…

$$\mathcal{M}_{J,\ell}(s) = \frac{1}{\mathcal{K}_{J,\ell}(s)^{-1} - i\rho_{N\pi}(s)}$$

K matrix (short distance)

phase-space cut (long distance)



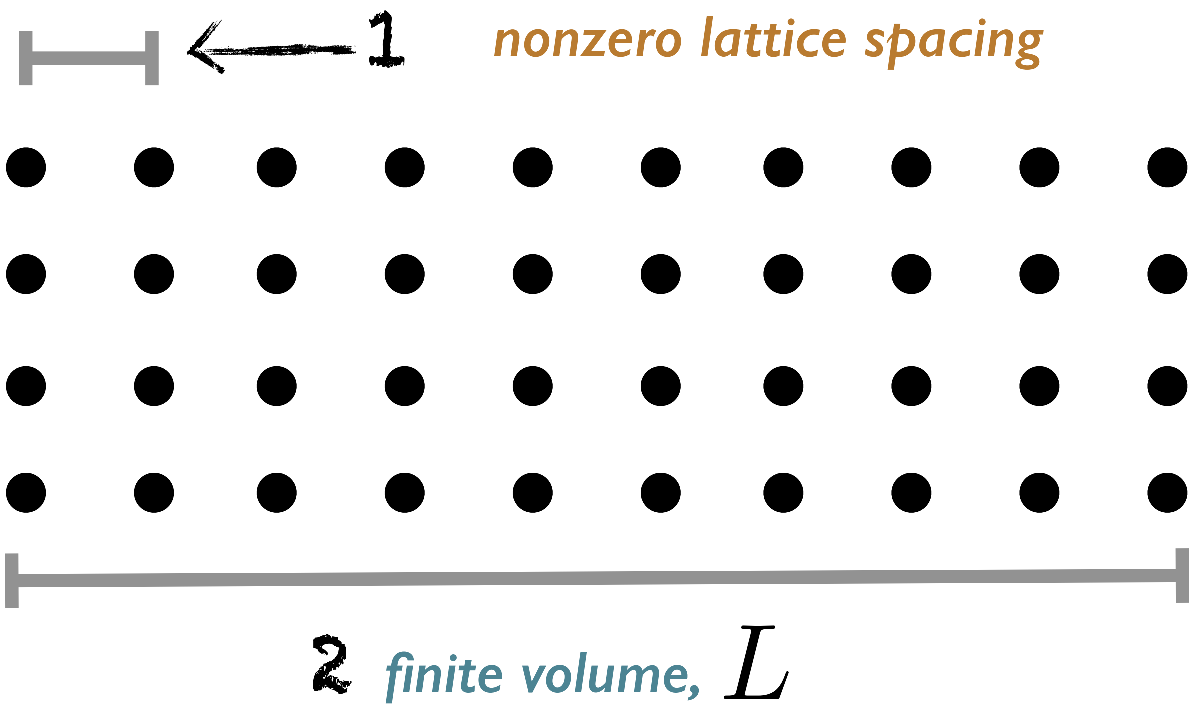
Amplitude has a branch cut ✓

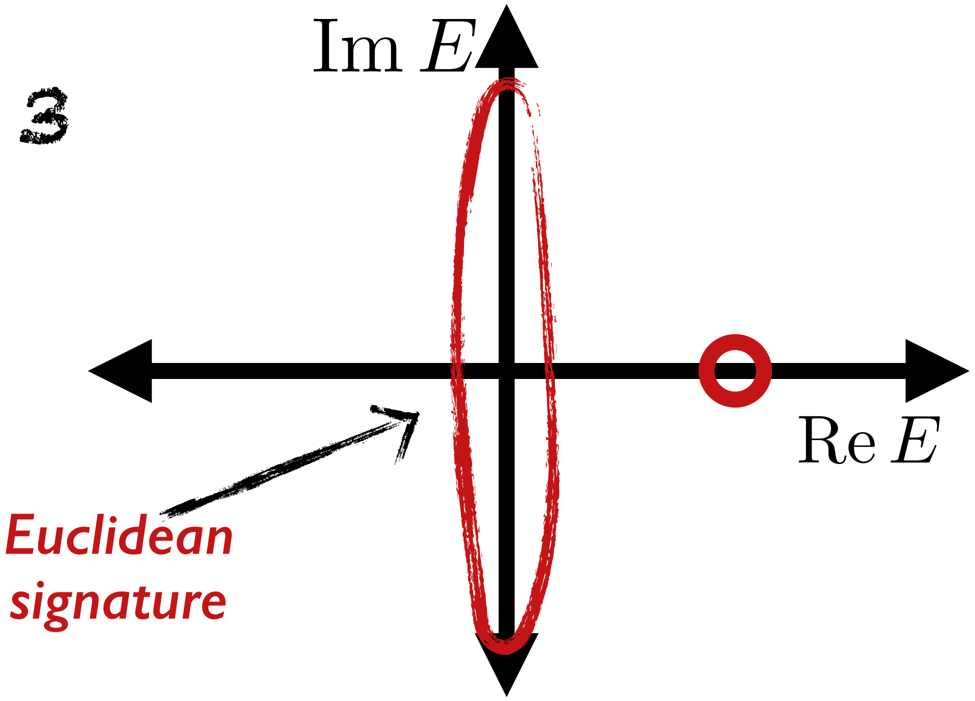
K-matrix is useful for parametrizing ✓

Lattice QCD

$$\text{observable?} = \int d^N\phi \, e^{-S} \left[\begin{array}{c} \textbf{interpolator} \\ \textbf{for observable} \end{array} \right]$$

To proceed we have to make *three modifications*



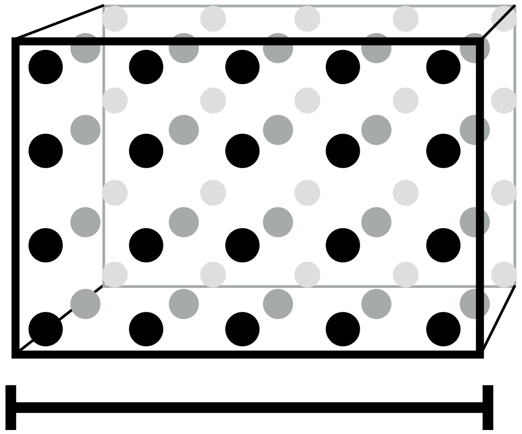


Also… $M_{\pi,\text{lattice}} > M_{\pi,\text{our universe}}$

(but physical masses → increasingly common)

Difficulties for multi-hadron observables

- The ***Euclidean signature***…
  - ***Prevents*** usual on-shell approach (want $p_4^2 = -E(p)^2$, but have only $p_4^2 > 0$)



- The ***finite volume***…
  - *Discretizes* the spectrum
  - *Eliminates* the branch cuts and extra sheets
  - *Hides* the resonance poles

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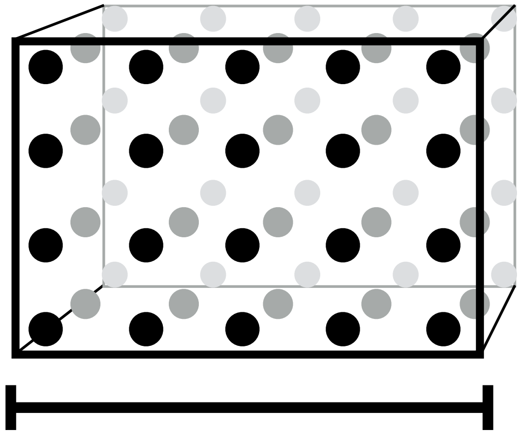

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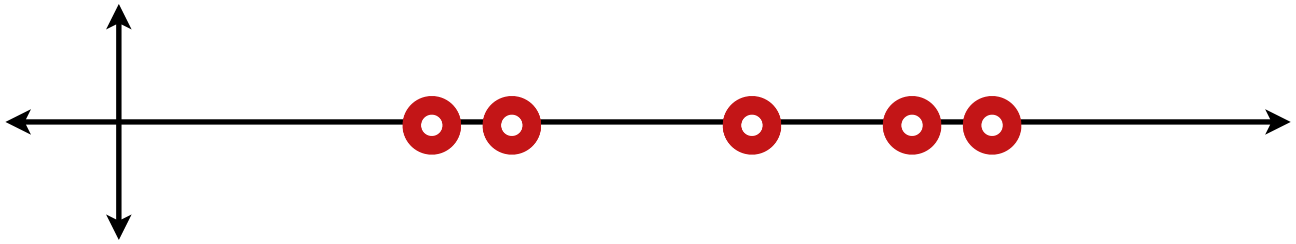
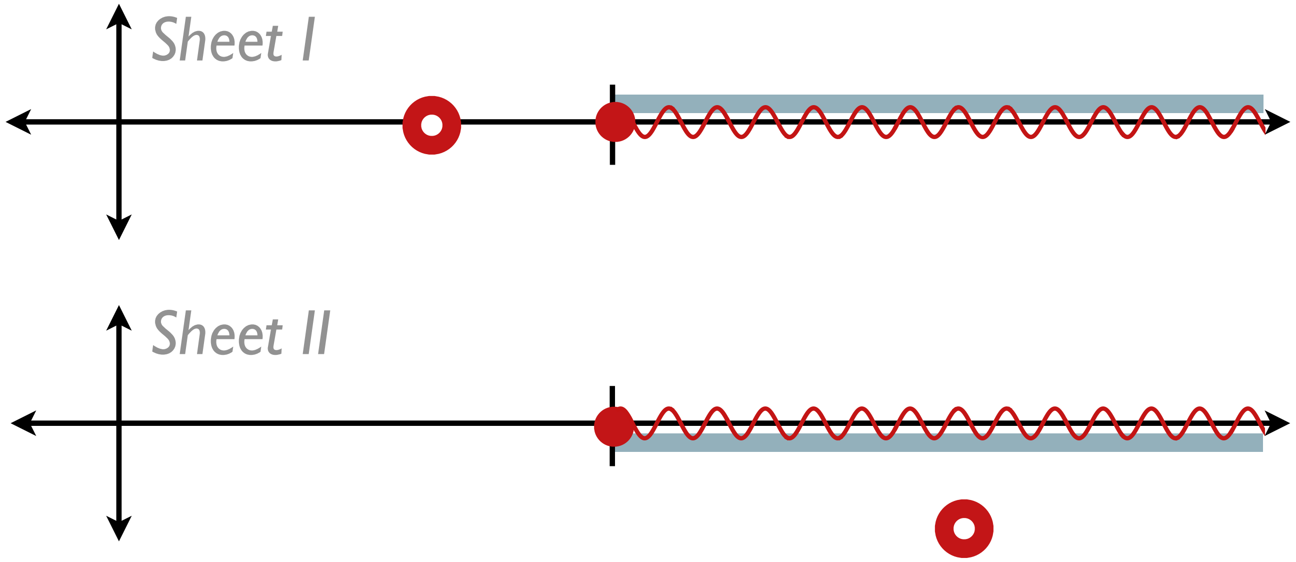
Finite-volume analytic structure



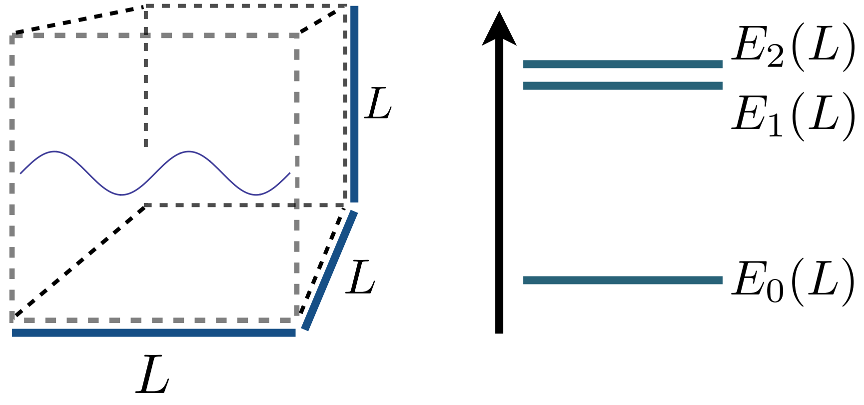
Note: cannot count finite-volume energies to count resonance poles!

Infinite-volume analytic structure



The finite-volume as a tool

☐ Finite-volume set-up



☐ **cubic**, spatial volume (extent *L*)

☐ **periodic**

☐ *L* is large enough to neglect $e^{-M_\pi L}$

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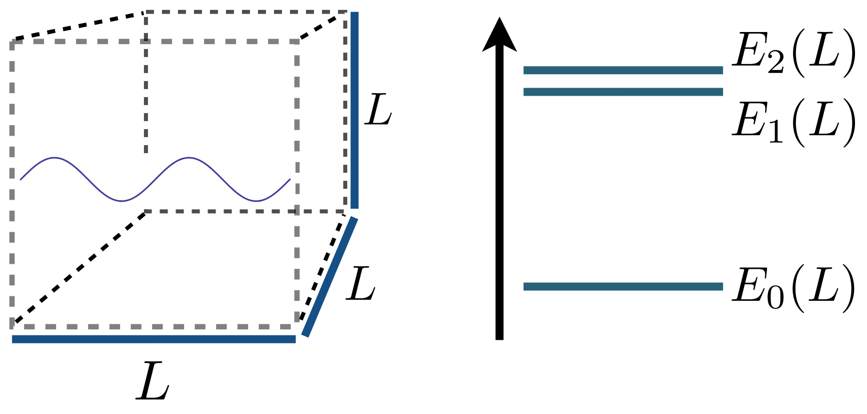
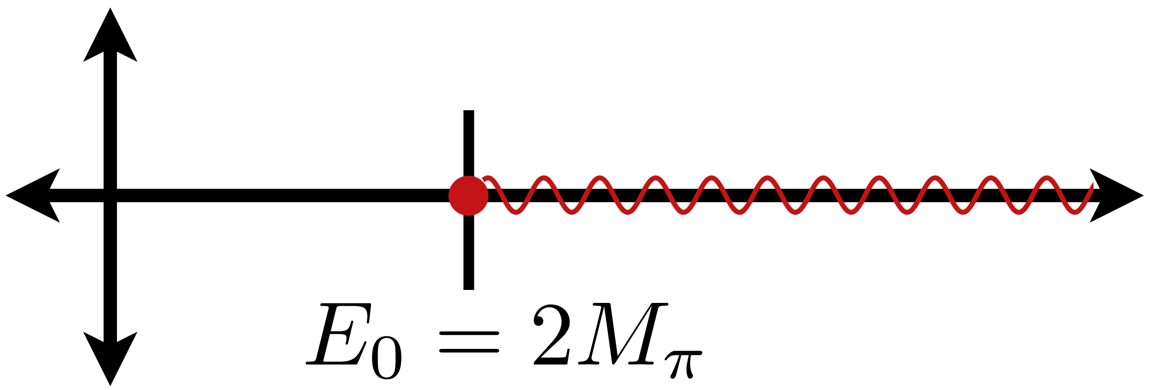


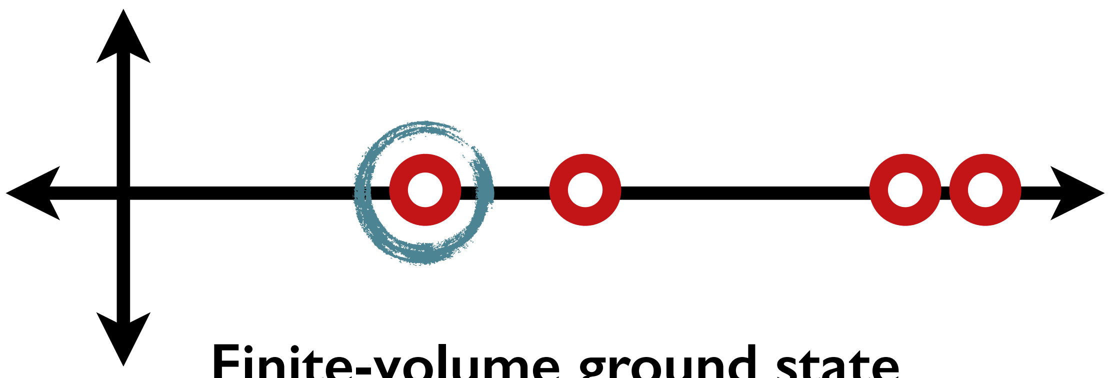
- ☐ **cubic**, spatial volume (extent *L*)
- ☐ **periodic**
- ☐ *L* is large enough to neglect $e^{-M_\pi L}$

- ☐ Scattering leaves an ***imprint*** on finite-volume quantities



$$E_0 = 2M_\pi$$

Infinite-volume threshold



Finite-volume ground state

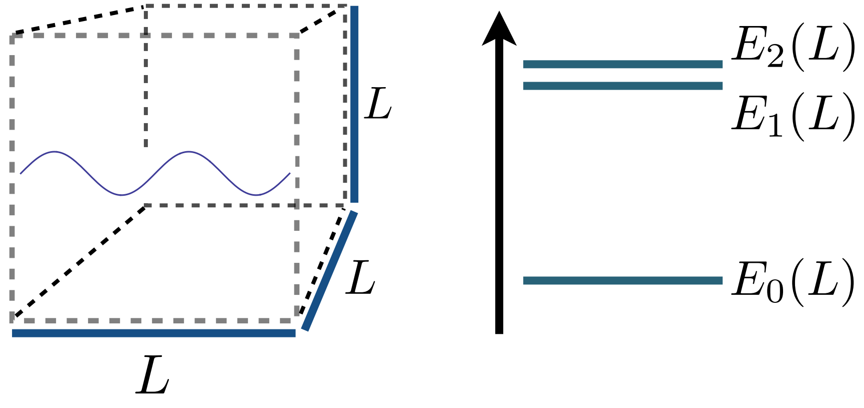
$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi a$$

$$E_0(L) = 2M_\pi + \frac{4\pi a}{M_\pi L^3} + \mathcal{O}(1/L^4)$$

• Huang, Yang (1958) •

The finite-volume as a tool

☐ Finite-volume set-up



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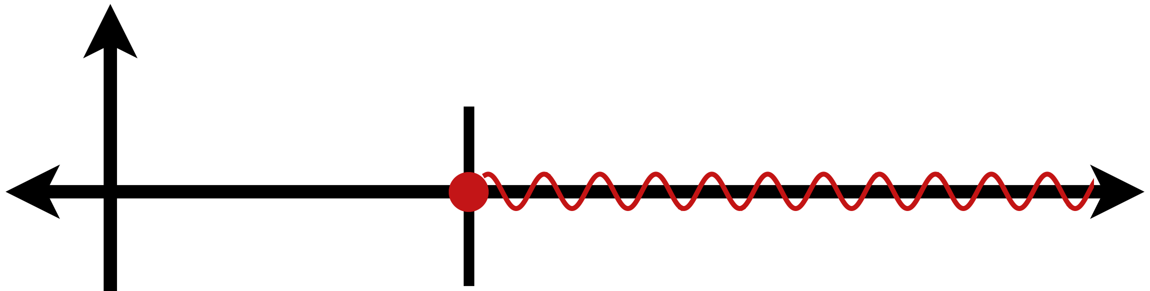
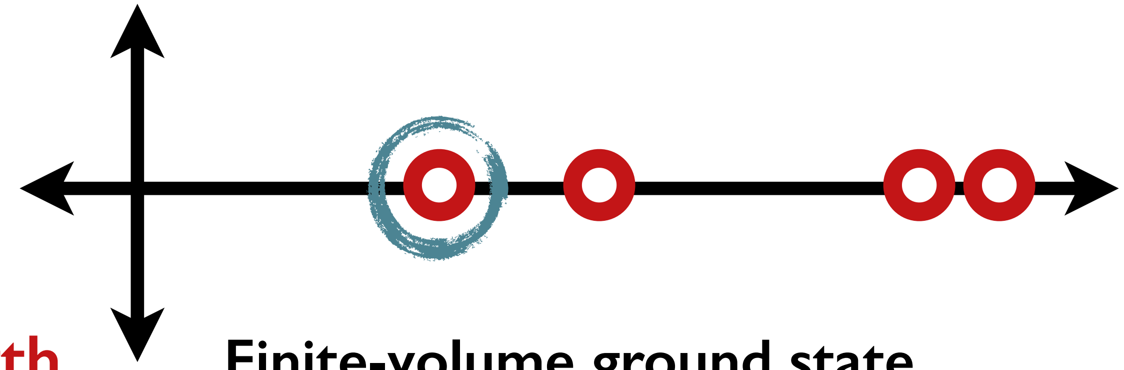


$E_0 = 2M_\pi$

Infinite-volume threshold



Finite-volume ground state

scattering length

$$\mathcal{M}_{\ell=0}(2M_\pi) = -32\pi M_\pi \boxed{a}$$

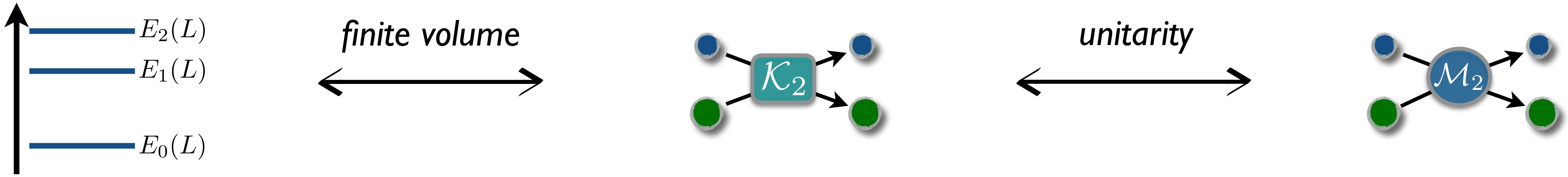
$$E_0(L) = 2M_\pi + \frac{4\pi \boxed{a}}{M_\pi L^3} + \mathcal{O}(1/L^4)$$

• Huang, Yang (1958) •

General method

$$\det[\mathcal{K}^{-1}(s) + F(P,L)] = 0$$

$F(P,L) \equiv$ Matrix of known geometric functions



Holds only for two-particle energies $s < (M_N + 2M_\pi)^2$ Neglects $e^{-M_\pi L}$

Generalized to *non-degenerate masses, multiple channels, spinning particles*

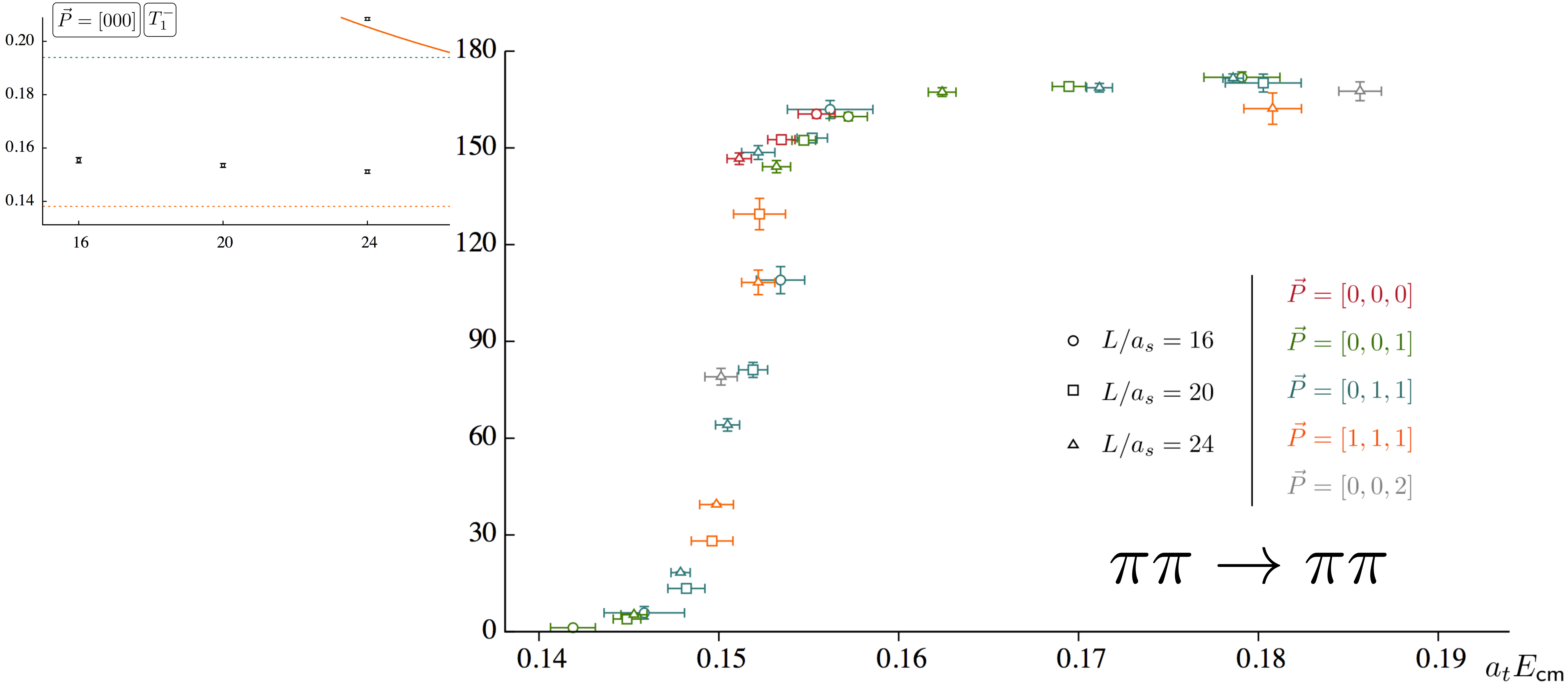
Encodes angular momentum mixing

Huang, Yang (1958) • Lüscher (1986, 1991) • Rummukainen, Gottlieb (1995)
Kim, Sachrajda, Sharpe (2005) • Christ, Kim, Yamazaki (2005) • He, Feng, Liu (2005)
Leskovec, Prelovsek (2012) • Bernard *et. al.* (2012) • MTH, Sharpe (2012) • Briceño, Davoudi (2012)
Li, Liu (2013) • Briceño (2014)

Using the result

- ☐ Single-channel case *(pions in a p-wave)*

$$\mathcal{K}(s_n)^{-1} = \rho \cot \delta(s_n) = -F(E_n, \vec{P}, L)$$



• Dudek, Edwards, Thomas in *Phys.Rev*. D87 (2013) 034505 •

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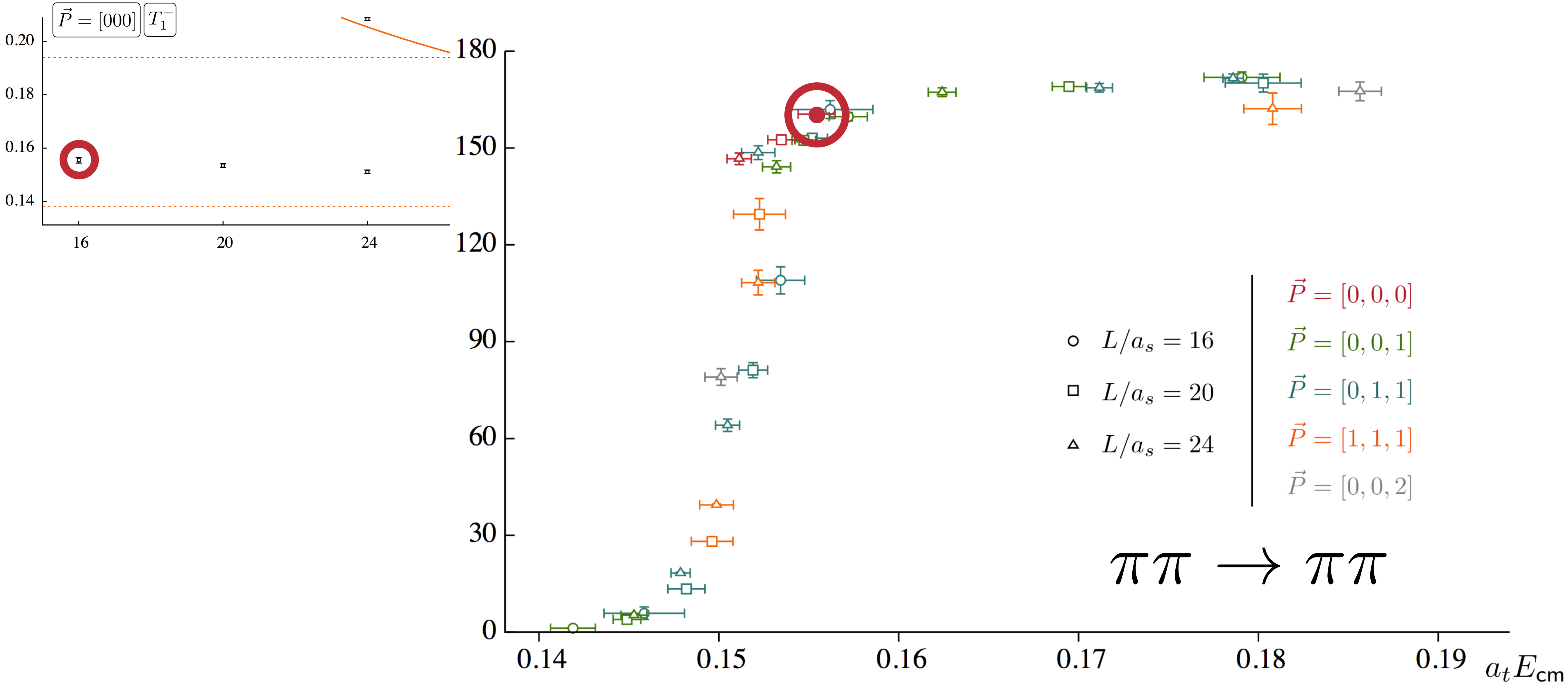

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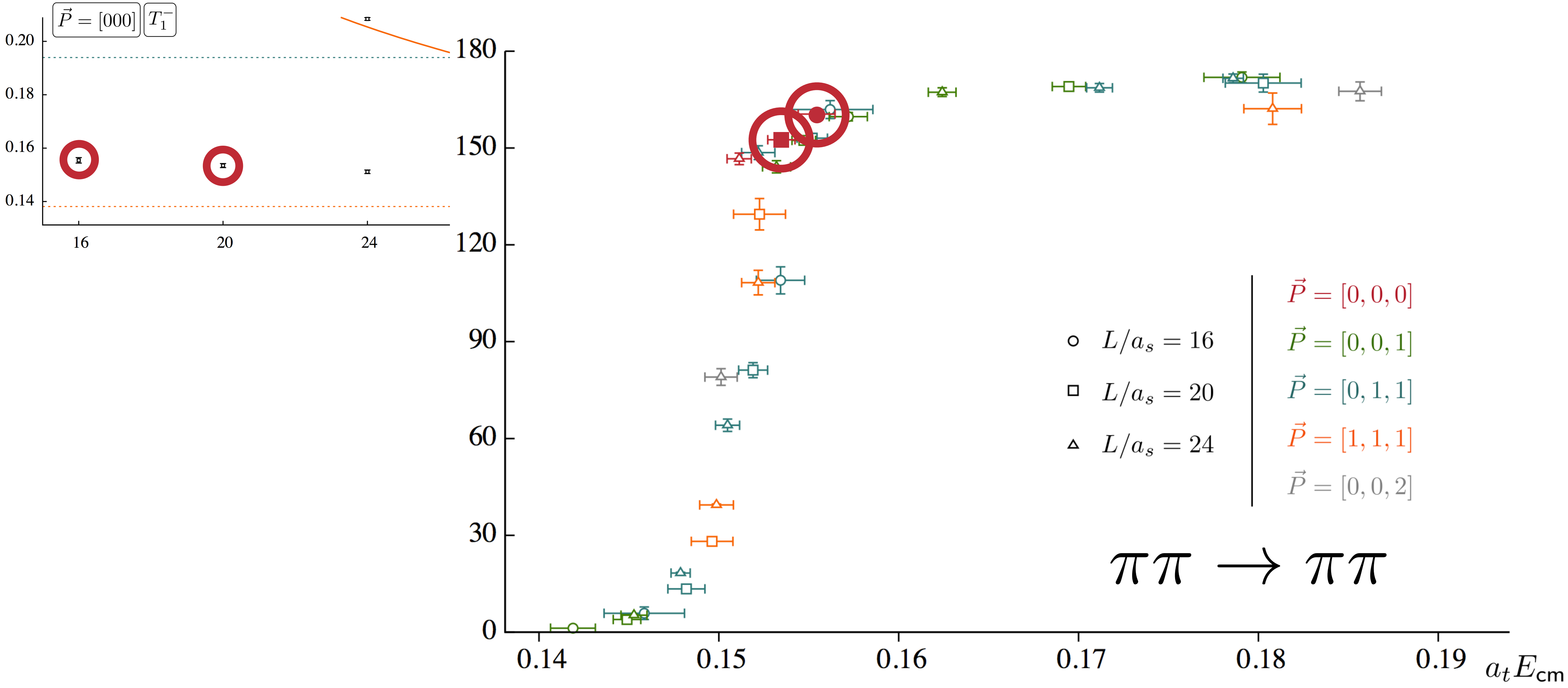

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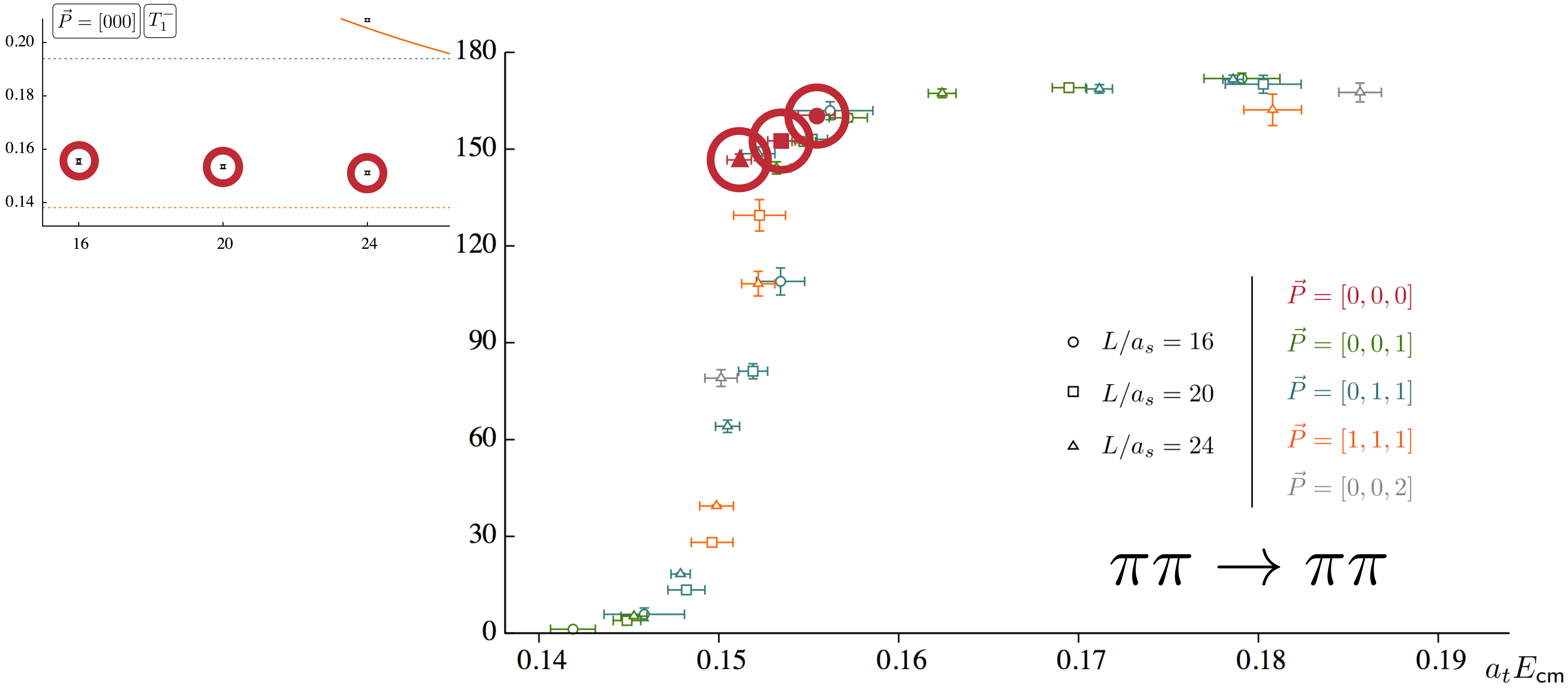

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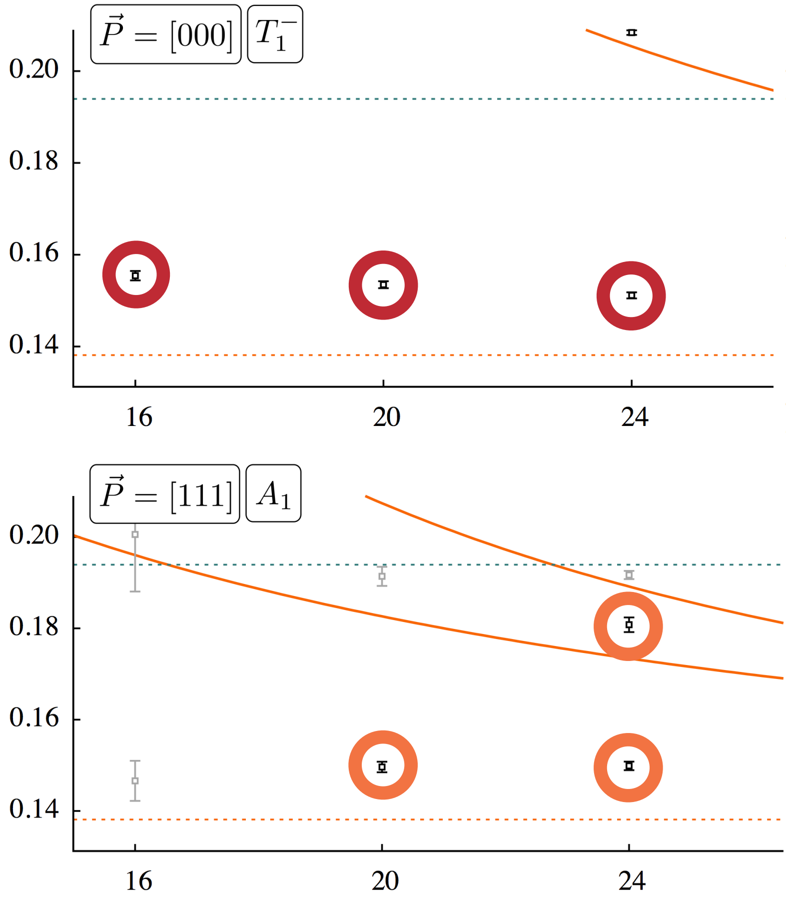
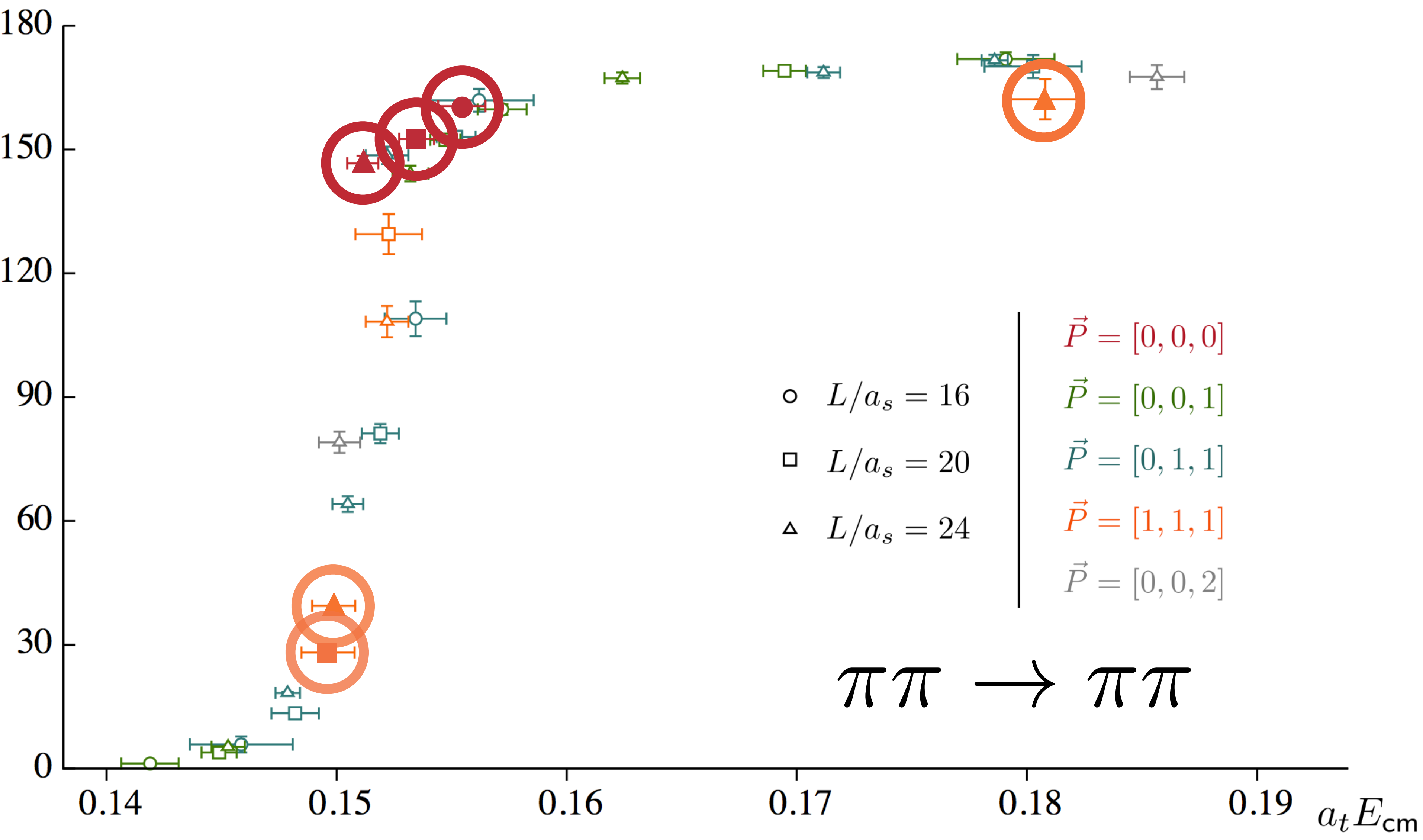


vary box-size + total momenta to ensure results are robust



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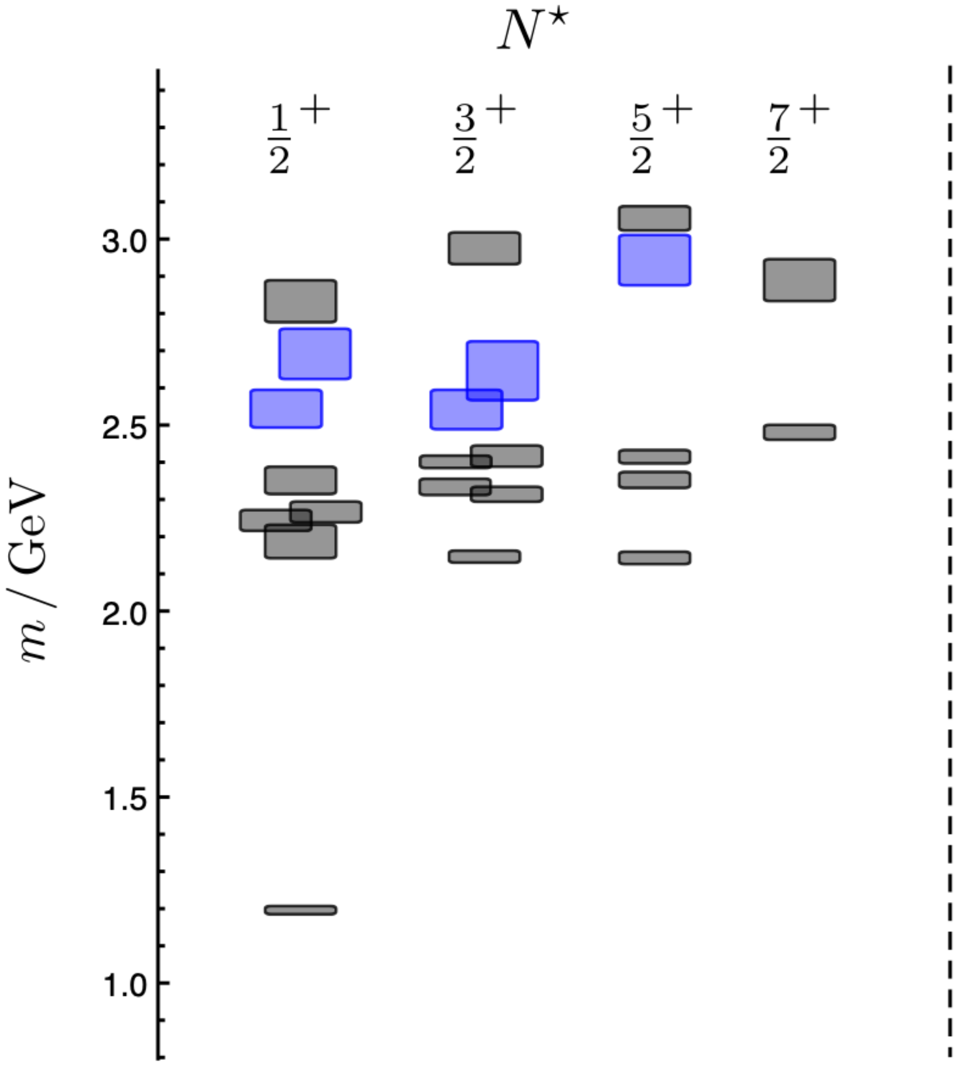
Two types of spectroscopy

Explore the spectrum of compact QCD excited states

(via quark-model inspired local operators)



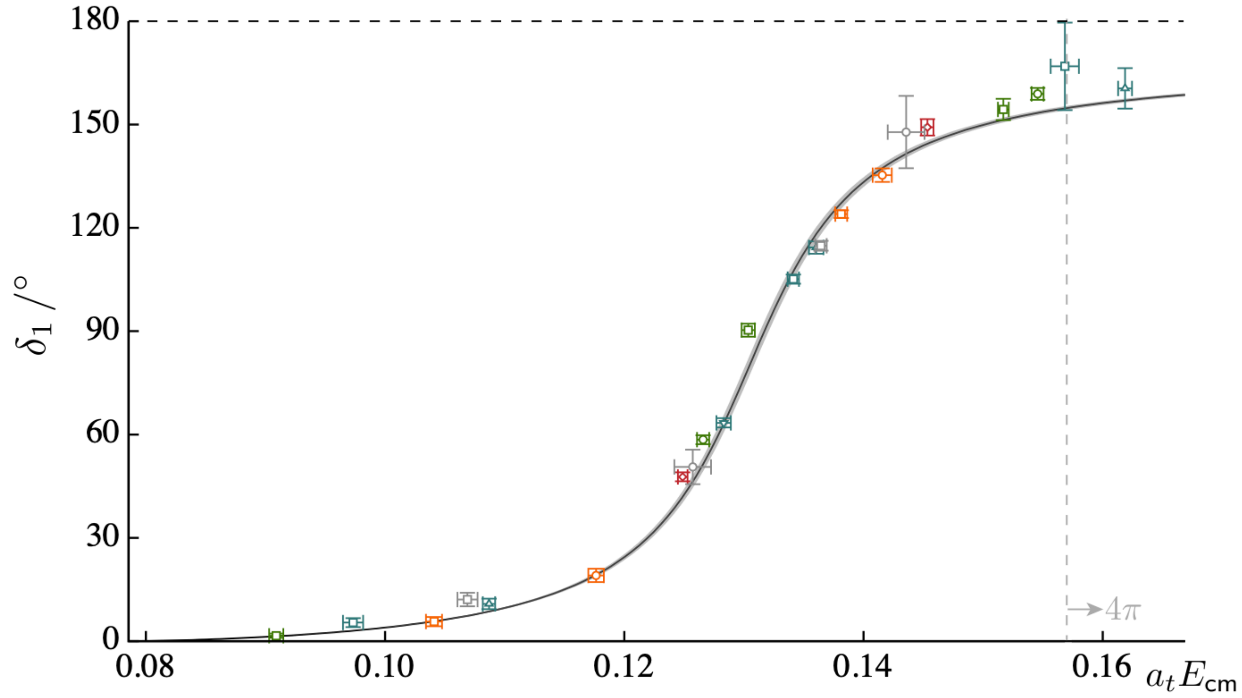
Dudek, Edwards (2012)

Extract the full finite-volume energy spectrum

local operators

+ many multi-hadron operators

$$\pi\pi \to \rho \to \pi\pi$$



Wilson, Briceño, Dudek, Edwards, Thomas (2015)

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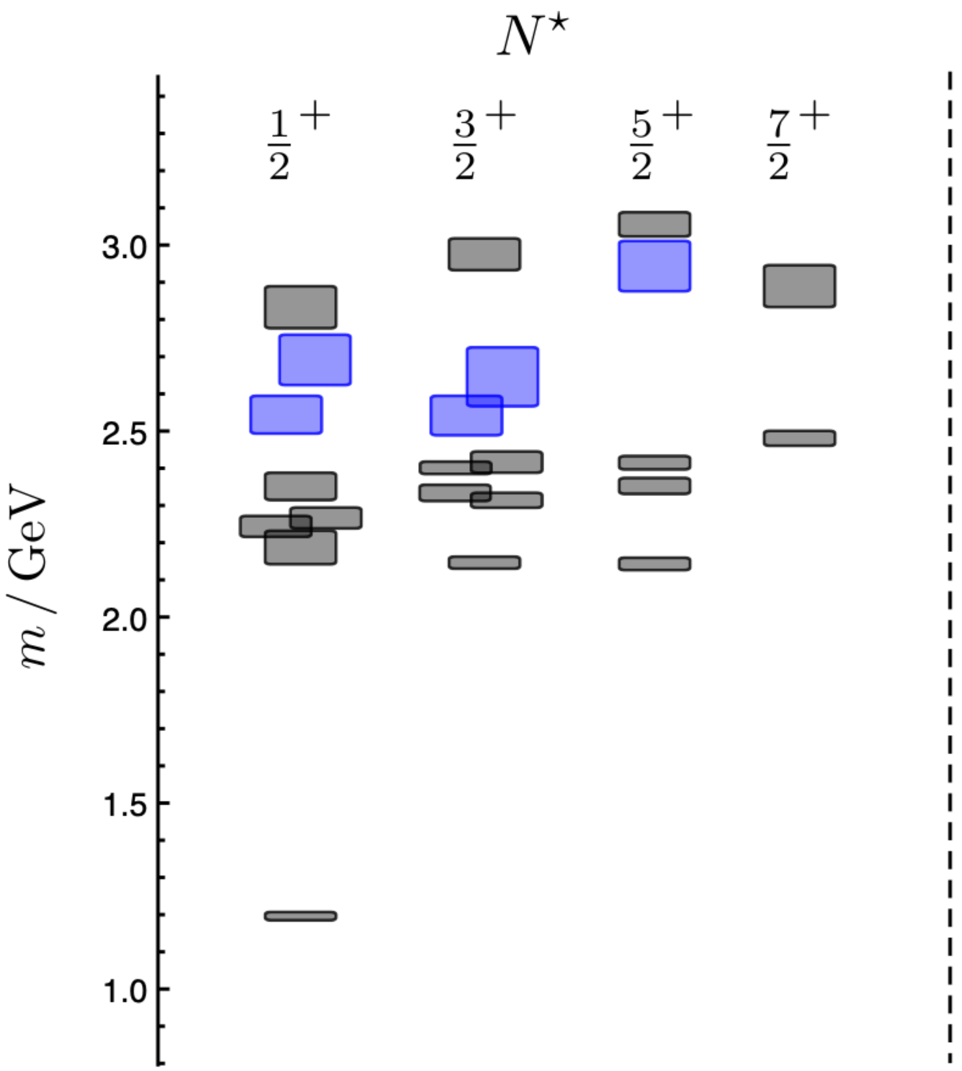
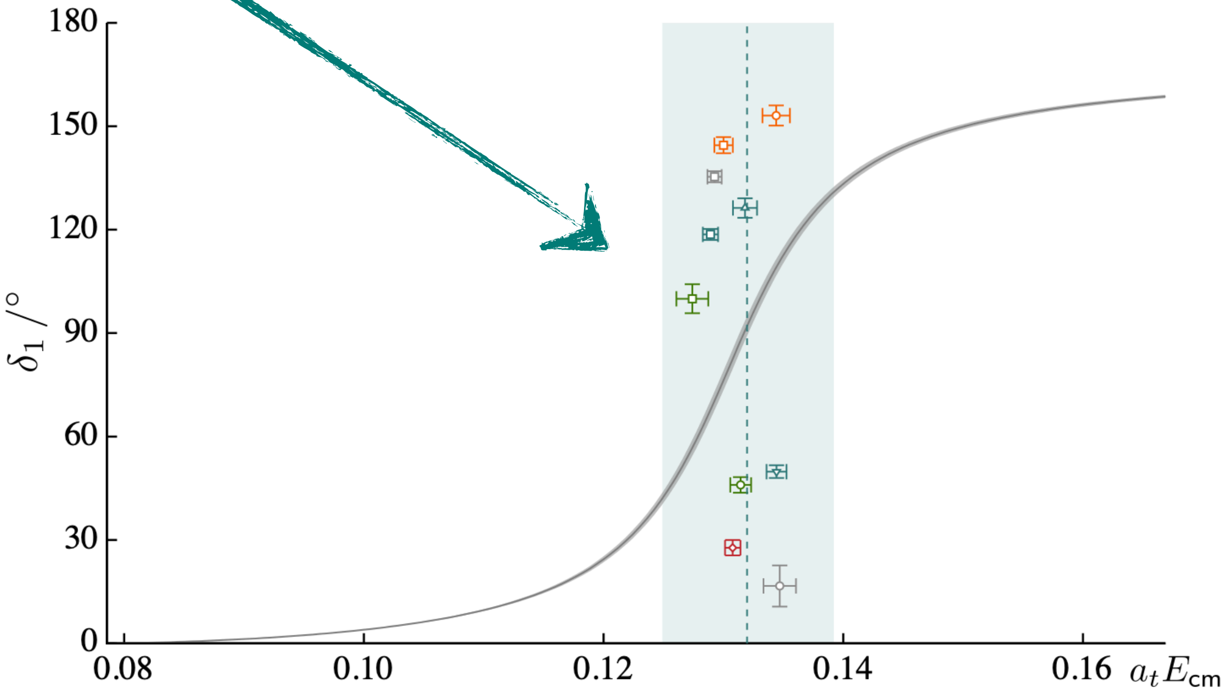


local operator spectrum =

not suitable for phase shift extraction

$$\pi\pi \to \rho \to \pi\pi$$



Note: cannot count finite-volume energies to count resonance poles!

Dudek, Edwards (2012)

Wilson, Briceño, Dudek, Edwards, Thomas (2015)

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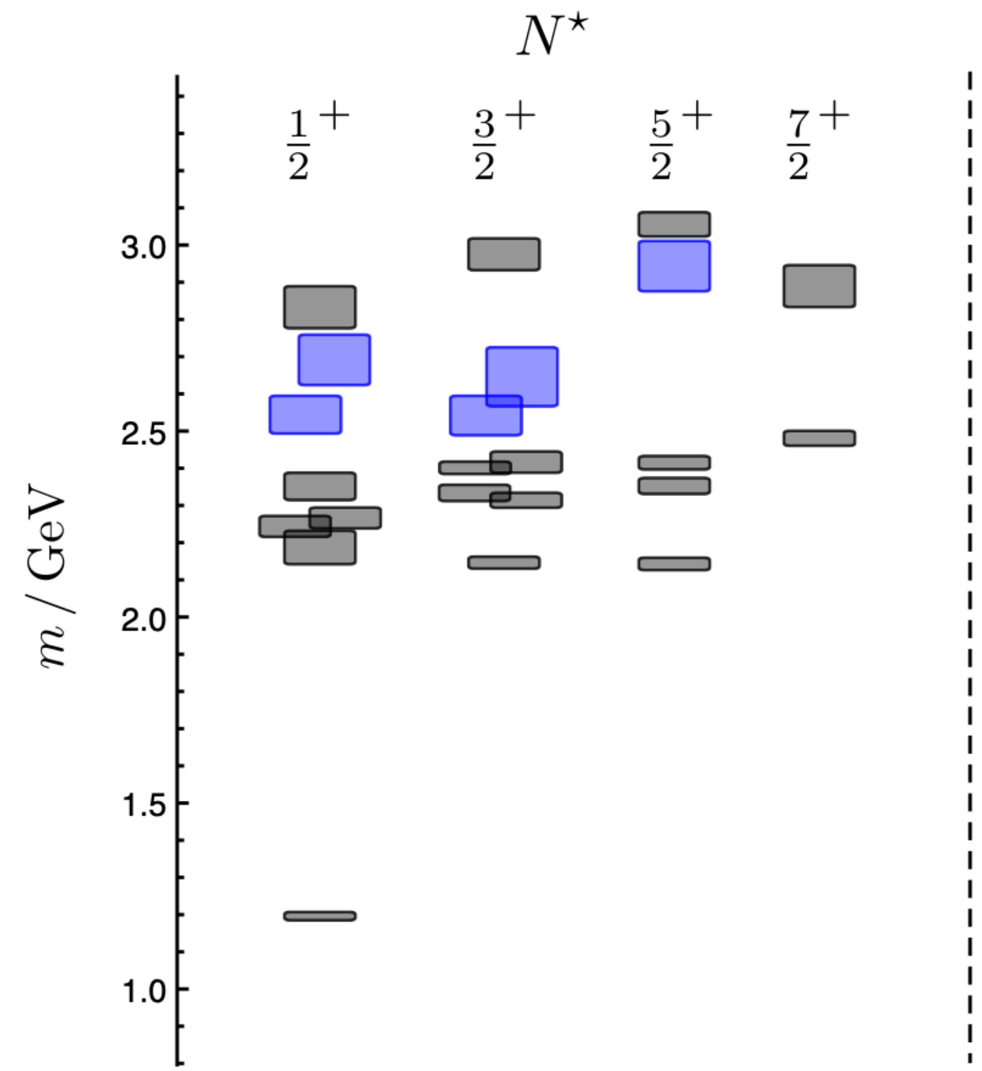

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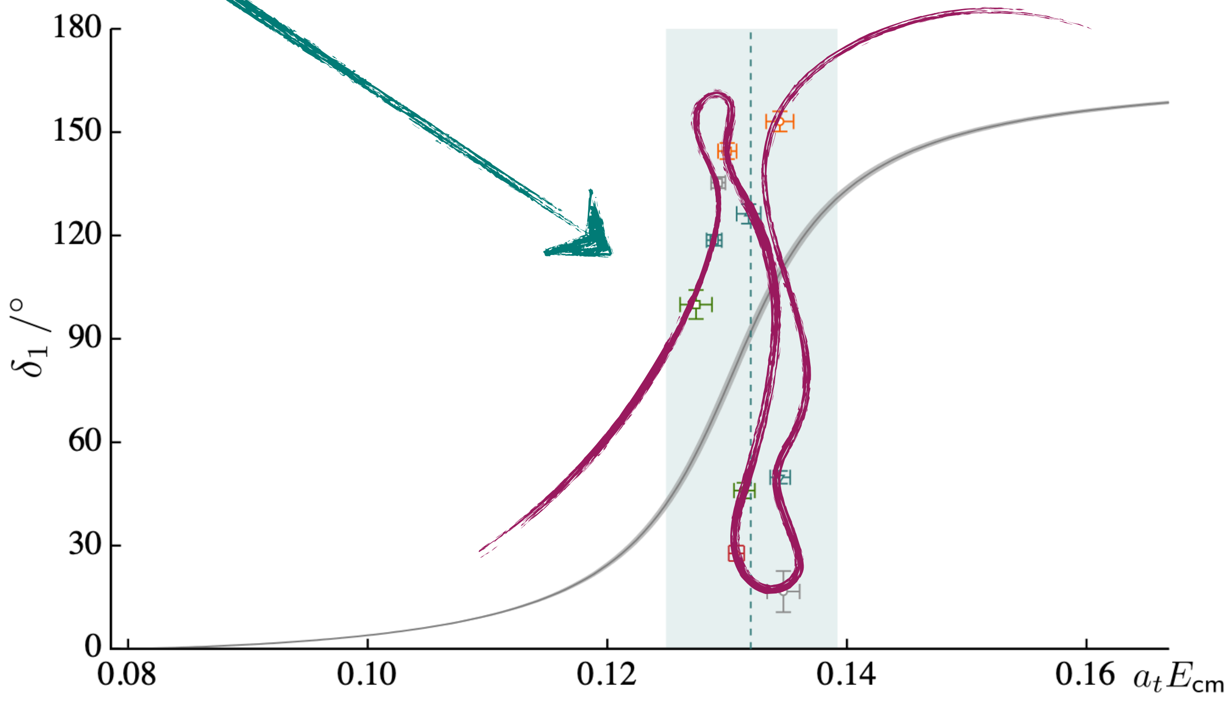
😂



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Wilson, Briceño, Dudek, Edwards, Thomas (2015)

$$\Delta \to N\pi$$

- ☐ Andersen et al. 2018
- ☐ Andersen et al. 2019
- ☐ Silvi et al. 2021
- ☐ Pittler et al. 2021
- ☐ Bulava et al. 2022

$$N^* \to N\pi$$

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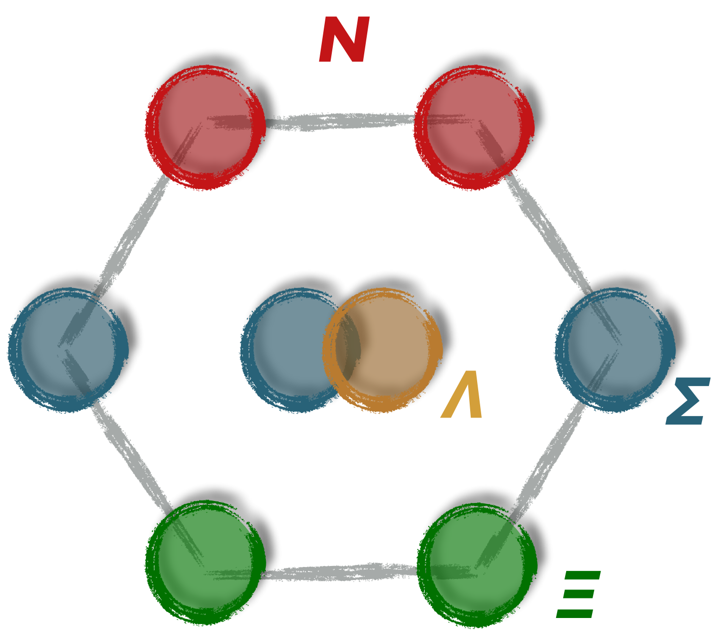
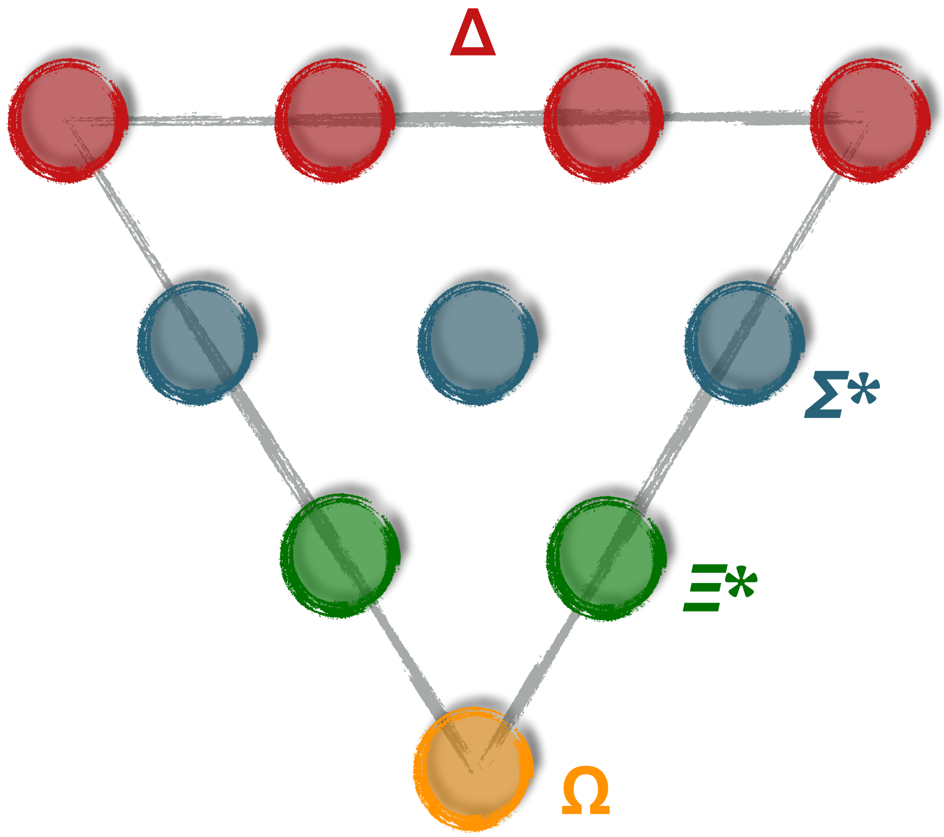
$$\Lambda \to \overline{K}N$$

- ☐ Hall et al. 2015



(focusing here on studies with scattering states)

Baryons are difficult!



See also…

- ☐ Detmold and Nicholson 2015
- ☐ Wu et al. 2018
- ☐ Xing & Liu, LATT2022 (in prep)

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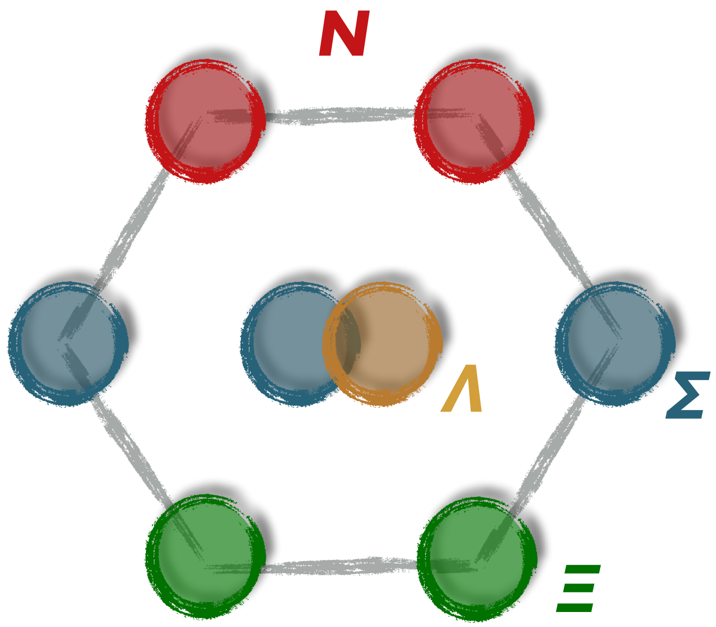

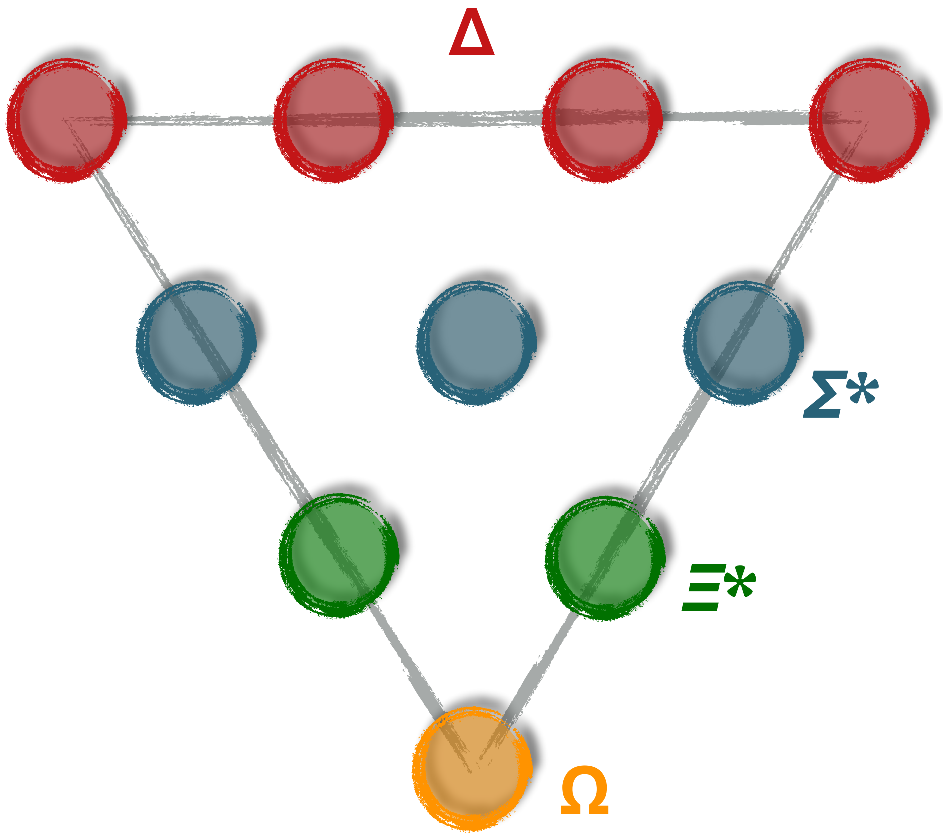

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$N\pi$ elastic scattering ($M_\pi = 200$ MeV)

arXiv > hep-lat > arXiv:2208.03867

High Energy Physics – Lattice

[Submitted on 8 Aug 2022]

Elastic nucleon–pion scattering at $m_\pi \approx 200$ MeV from lattice QCD

John Bulava, Andrew D. Hanlon, Ben Hörz, Colin Morningstar, Amy Nicholson, Fernando Romero-López, Sarah Skinner, Pavlos Vranas, André Walker-Loud

☐ Studied scattering-lengths and the Δ channel

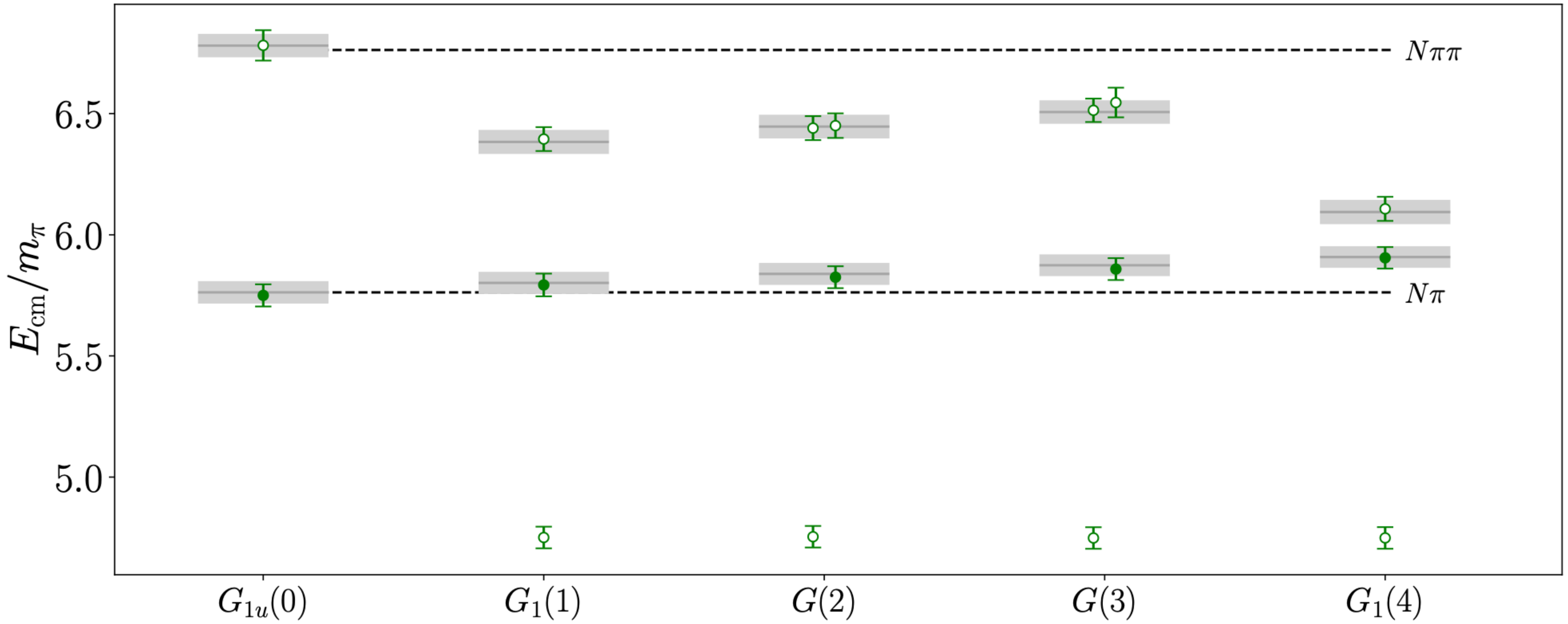
$$I = 1/2: \quad J^P = 1/2^- \ (S)$$
$$I = 3/2: \quad J^P = 1/2^- \ (S), \ \ 3/2^+ \ (P) \qquad [1/2^+ \ (P), \ \ 3/2^- \ (D), \ \ 5/2^- \ (D)]$$

☐ Advanced operators + fits to extract finite-volume energies

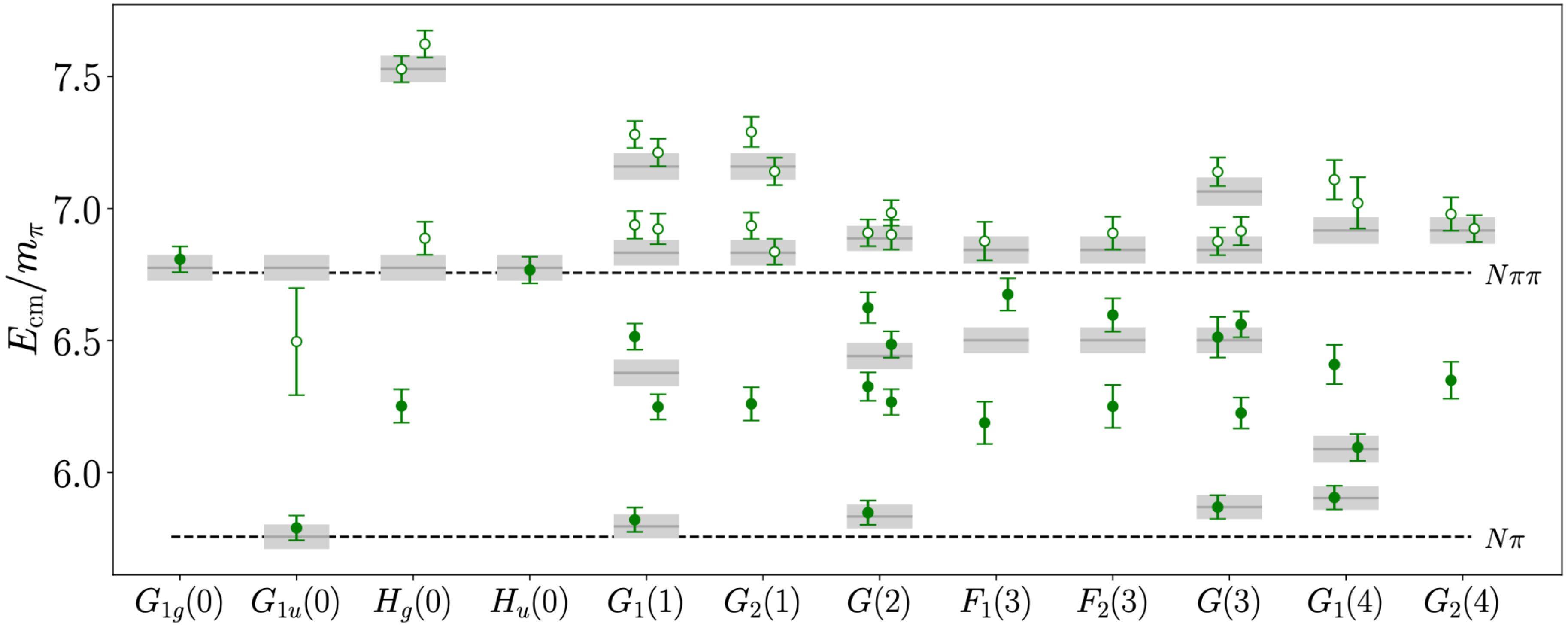
• Bulava et al. (2022) 2208.03867 •

$N\pi$ finite-volume energies ($M_\pi = 200$ MeV)

$I = 1/2$



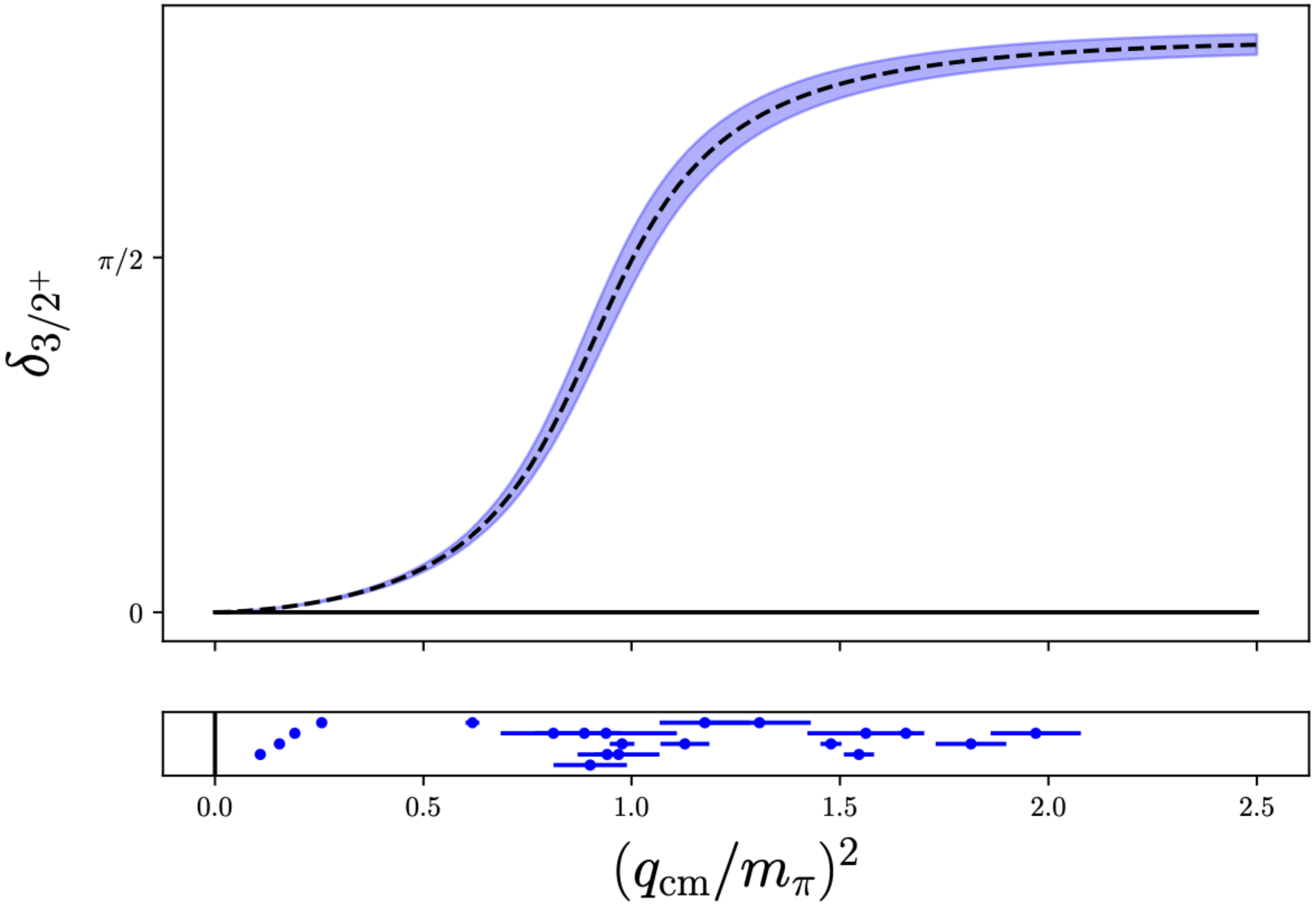
$I = 3/2$



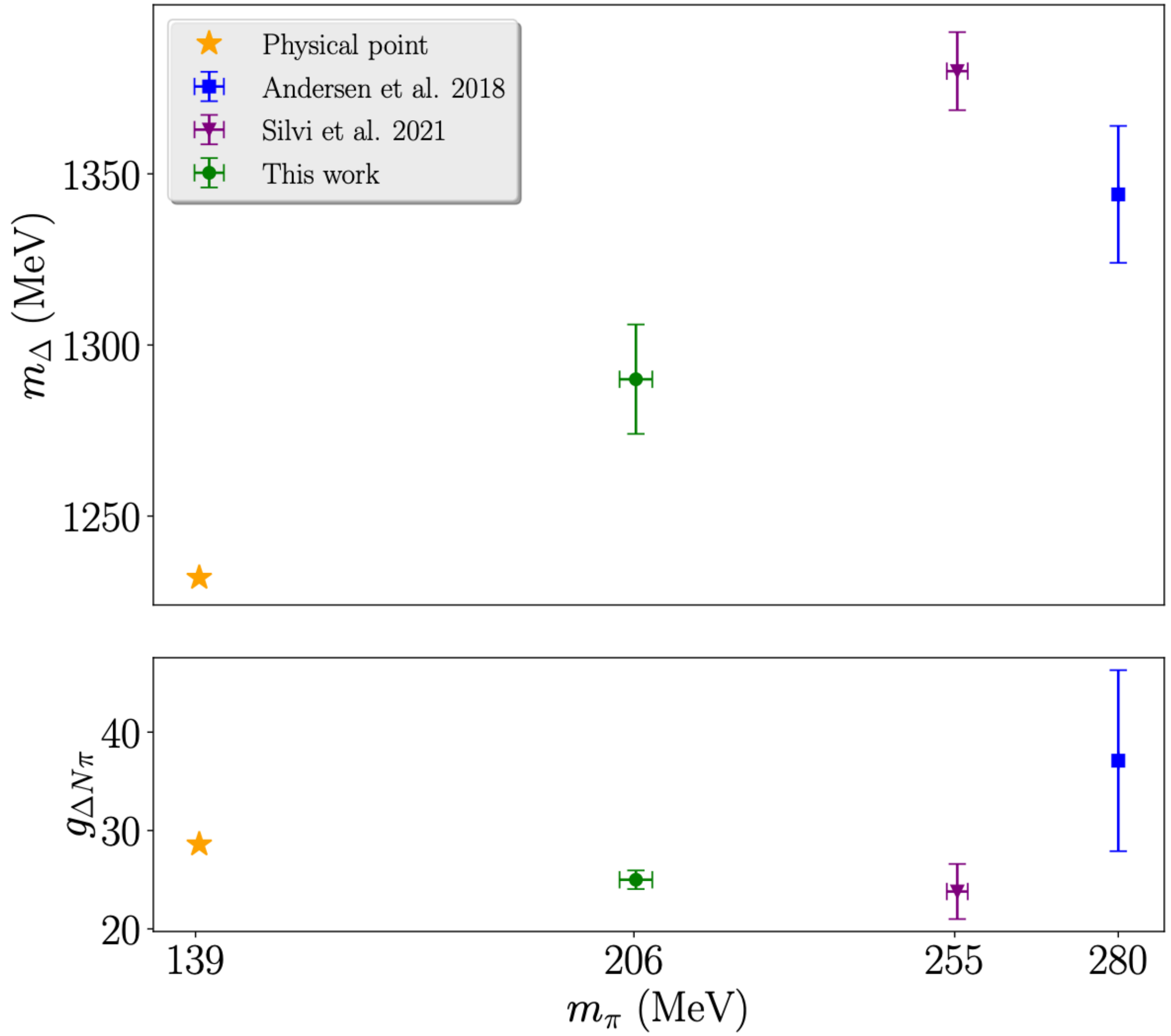
- Bulava et al. (2022) 2208.03867 •

$N\pi \rightarrow \Delta \rightarrow N\pi$

Scattering phase shift



Δ summary plot



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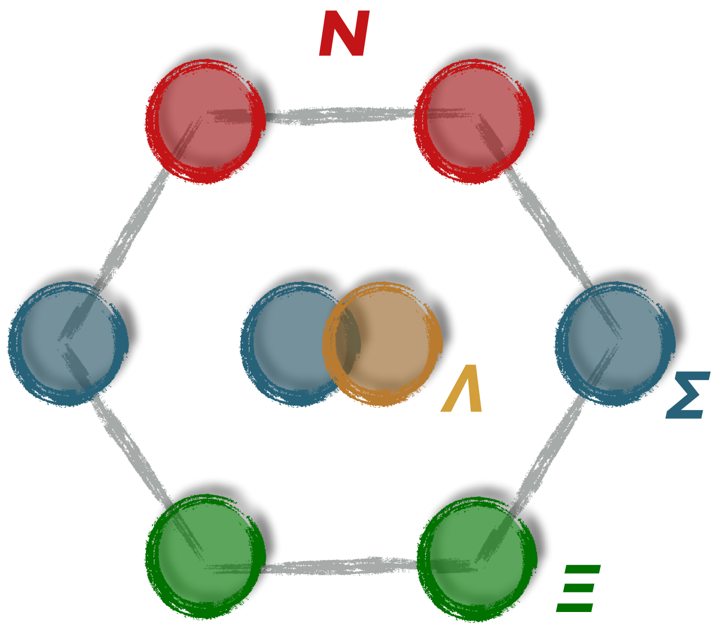
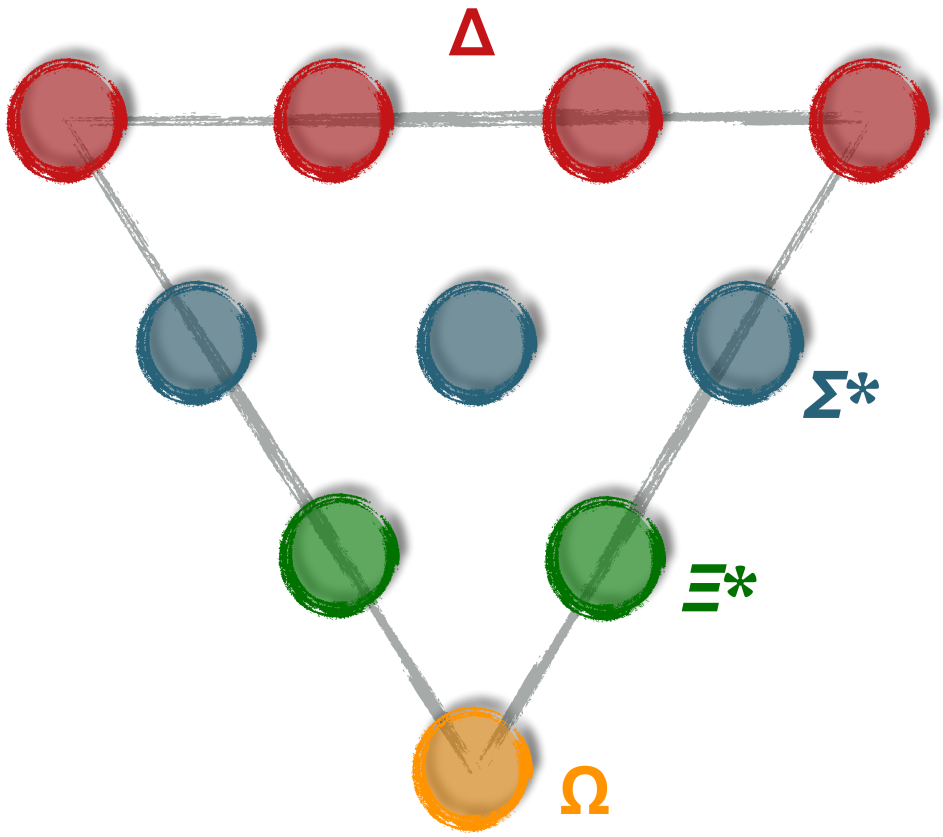


(focusing here on studies with scattering states)

Baryons are difficult!



See also…

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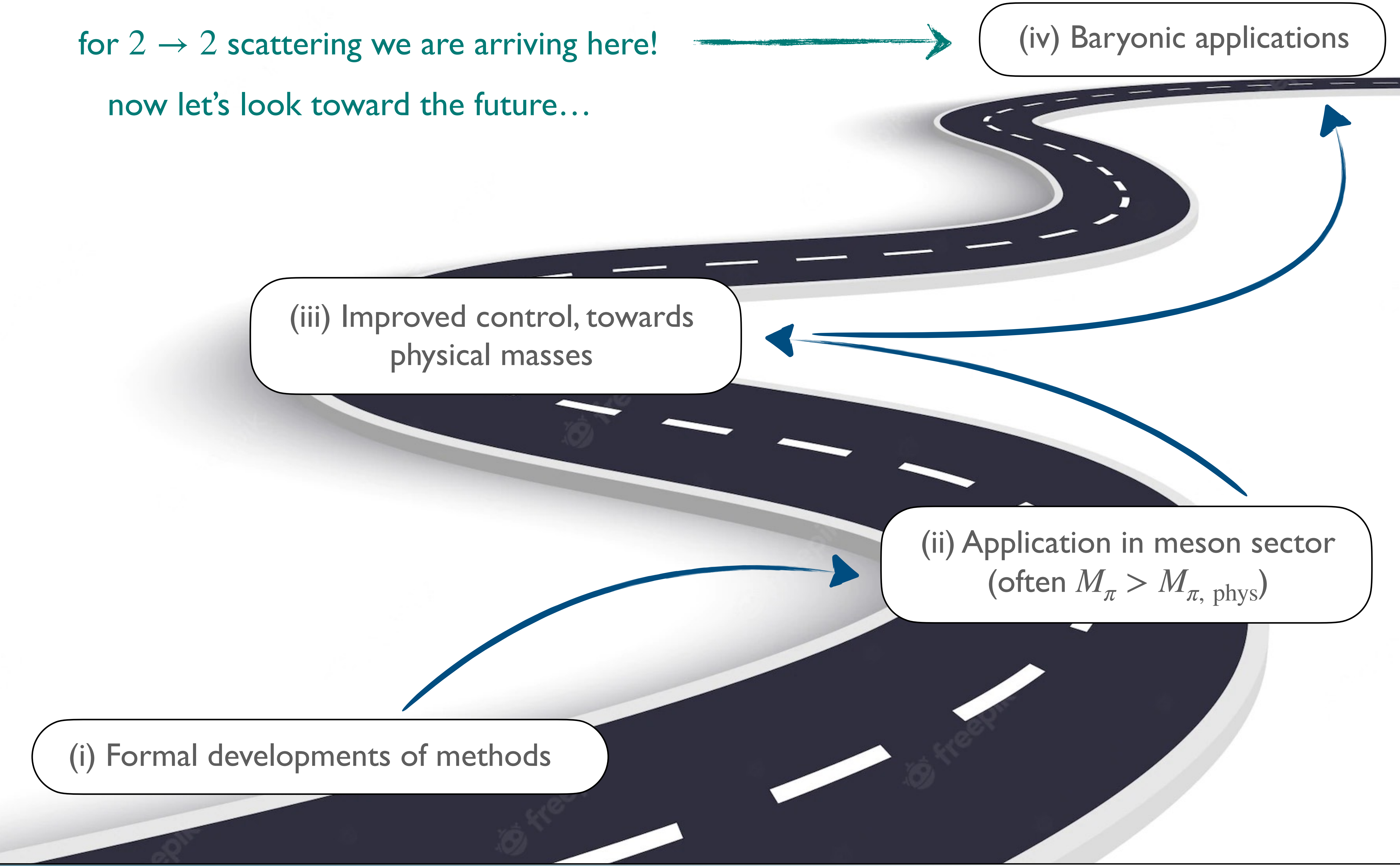
☐ Xing & Liu, LATT2022 (in prep)

Journey of a lattice calculation

for $2 \to 2$ scattering we are arriving here!

now let's look toward the future…



(iv) Baryonic applications

(iii) Improved control, towards physical masses

(ii) Application in meson sector (often $M_\pi > M_{\pi,\,\mathrm{phys}}$)

(i) Formal developments of methods

Can LQCD calculate X for baryonic CPV?

easier

$\langle \text{hadron} | \mathcal{O}(0) | \text{hadron} \rangle$

easier **harder**

mesons

quark bilinear

light-quarks

baryons

four-quark operators

heavy-quarks

$\langle \text{multi-hadron} | \text{multi-hadron} \rangle$ ✓

$\langle \text{multi-hadron state} | \mathcal{O}(0) | \text{hadron} \rangle$

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everything above… plus:

single two-particle channel

many two-particle channels

channels with > two particles

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easier **harder**

four-point functions are difficult

no

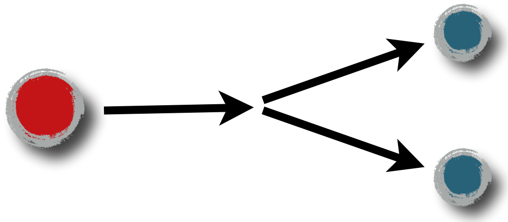
Can multi-hadron states propagate between the currents?

yes

harder

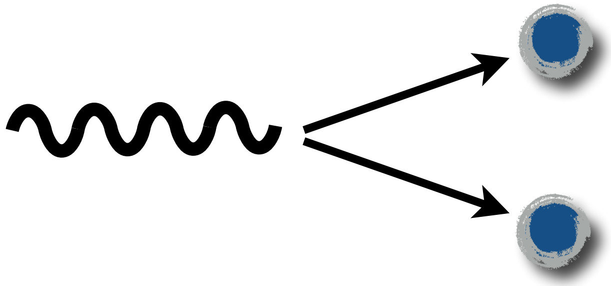
Formal progress: Transition amplitudes

☐ Weak decay $\langle \pi\pi, \mathrm{out}|\mathcal{H}|K\rangle \equiv$



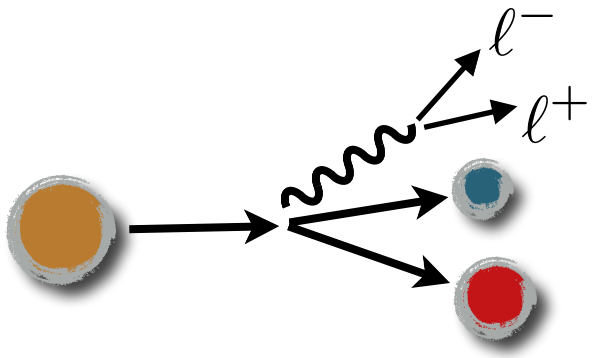
Lellouch, Lüscher *(2001)* • Kim, Sachrajda, Sharpe *(2005)* • Christ, Kim, Yamazaki *(2005)* • MTH, Sharpe *(2012)*

☐ Time-like form factors $\langle \pi\pi, \ \mathrm{out}|\mathcal{J}_\mu|0\rangle \equiv$

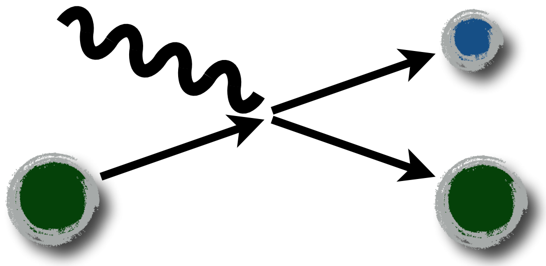


Meyer *(2011)*

☐ Resonance form factors $\langle K\pi, \mathrm{out}|\mathcal{J}_{\alpha\beta}|B\rangle \equiv$



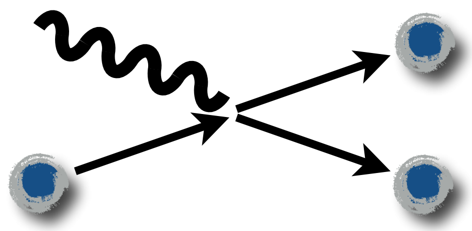
☐ Particles with spin $\langle N\pi, \mathrm{out}|\mathcal{J}_\mu|N\rangle \equiv$



Agadjanov *et al. (2014)* • Briceño, MTH, Walker-Loud *(2015)* • Briceño, MTH *(2016)*

Pion photo-production

$\langle \pi\pi, \text{out}|\mathcal{J}_\mu|\pi\rangle \equiv$



☐ Formal relation

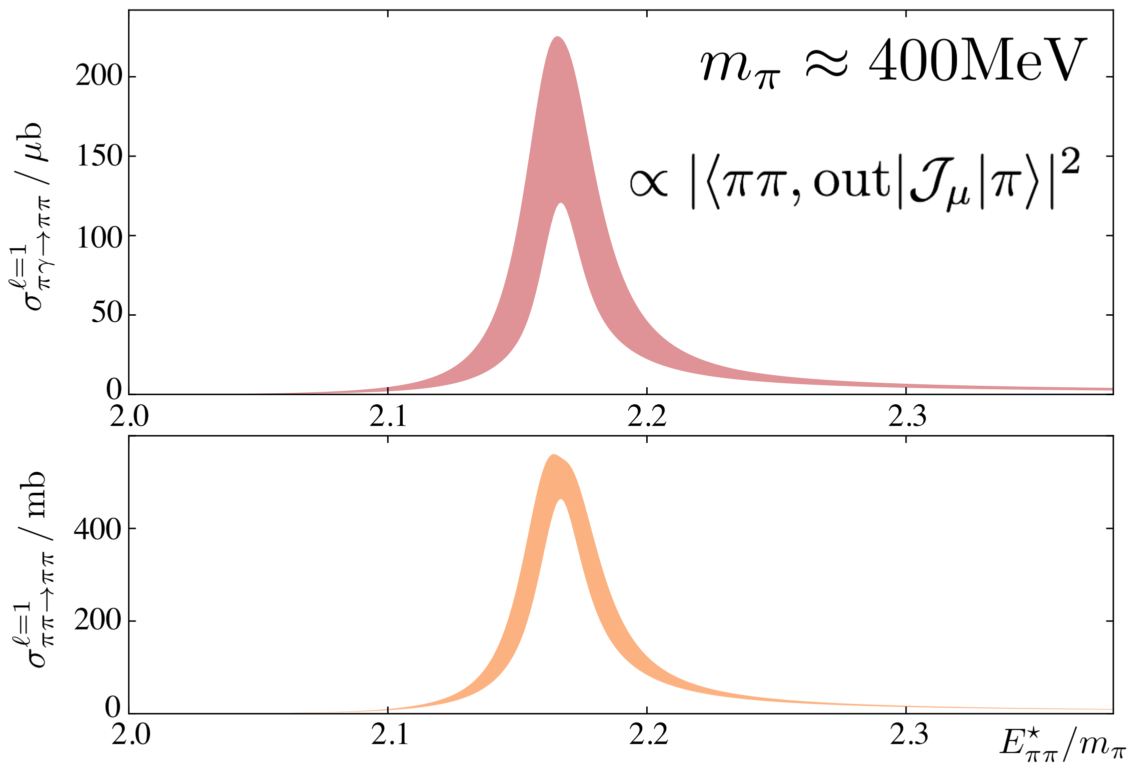
get this from the lattice

experimental observable

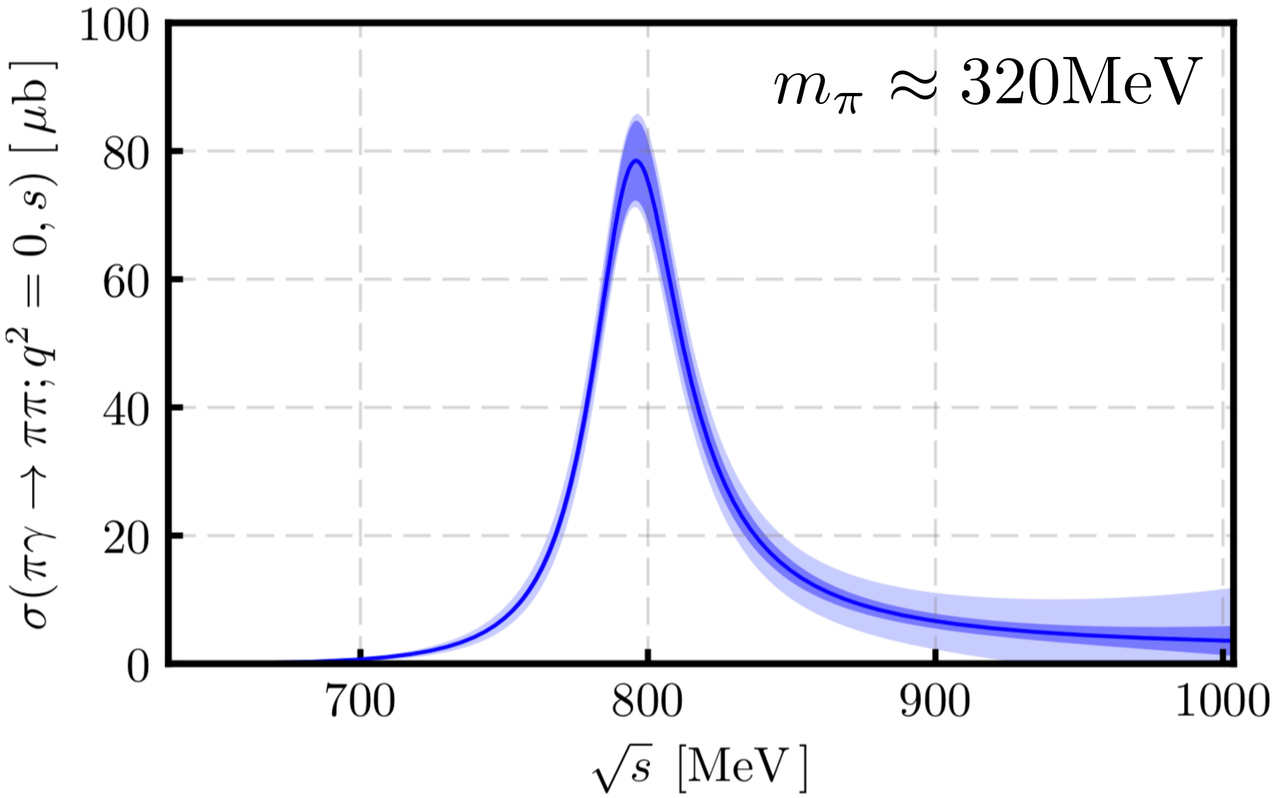
$$|\langle n, L|\mathcal{J}_\mu|\pi\rangle|^2 = \langle \pi|\mathcal{J}_\mu|\pi\pi, \text{in}\rangle \mathcal{R}(E_n, L) \langle \pi\pi, \text{out}|\mathcal{J}_\mu|\pi\rangle$$

Briceño, MTH, Walker-Loud (2015)

☐ Numerical implementation



Briceño et. al., Phys. Rev. D93, 114508 (2016)



Alexandrou et. al., Phys. Rev. D98, 074502 (2018)

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Formal & numerical progress: Long-distance matrix elements

☐ Formal method for extracting these processes is understood

☐ Key complication is multi-hadron intermediate states

$$\Sigma^+ \xrightarrow[H_W]{} N\pi \to p\gamma^*$$

Christ, Feng, Martinelli, Sachrajda *(2015)* • Christ *et al. (2016)*
• Briceño, Davoudi, MTH, Schindler, Baroni *(2019)* • Erben, Gülpers, MTH, Hodgson, Portelli *(2022)*

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- ○ Issue of growing exponentials *(Christ et al.)*

$$\langle p | j_\mu(0) \mathcal{H}_W(-|\tau|) | \Sigma^+ \rangle_L = \sum_n c_n(L) e^{-(E_n(L) - M_\Sigma)|\tau|} \quad \underset{\int_{-T}^{0} d\tau}{\Longrightarrow} \quad \sum_n c_n \frac{1 - e^{-(E_n - M_\Sigma)T}}{M_\Sigma - E_n}$$

Christ, Feng, Martinelli, Sachrajda *(2015)* • Christ *et al. (2016)*
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- Issue of power-like finite-volume effects (after discarding exponential)

$$F_L = \sum_n \frac{c_n}{M_\Sigma - E_n}$$

☐ See lattice proceedings (and forthcoming thesis) of Raoul Hodgson

Christ, Feng, Martinelli, Sachrajda *(2015)* • Christ *et al. (2016)*
• Briceño, Davoudi, MTH, Schindler, Baroni *(2019)* • Erben, Gülpers, MTH, Hodgson, Portelli *(2022)*

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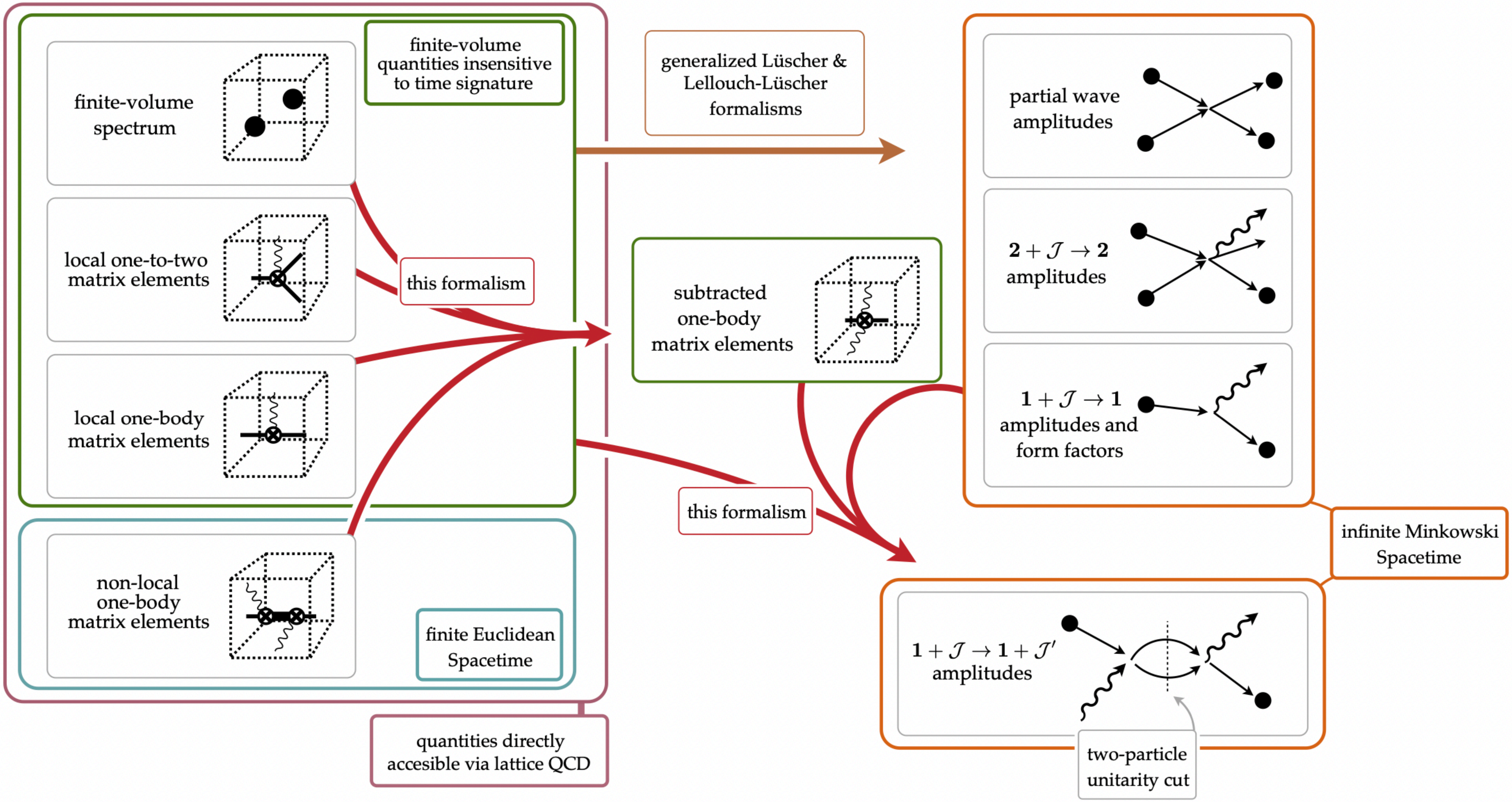

finite-volume spectrum

local one-to-two matrix elements

local one-body matrix elements

finite-volume quantities insensitive to time signature

non-local one-body matrix elements

finite Euclidean Spacetime

quantities directly accesible via lattice QCD

this formalism

generalized Lüscher & Lellouch-Lüscher formalisms

subtracted one-body matrix elements

this formalism

partial wave amplitudes

$\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes

$\mathbf{1} + \mathcal{J} \to \mathbf{1}$ amplitudes and form factors

infinite Minkowski Spacetime

$\mathbf{1} + \mathcal{J} \to \mathbf{1} + \mathcal{J}'$ amplitudes

two-particle unitarity cut

Christ, Feng, Martinelli, Sachrajda *(2015)* • Christ *et al. (2016)*
• Briceño, Davoudi, MTH, Schindler, Baroni *(2019)* • Erben, Gülpers, MTH, Hodgson, Portelli *(2022)*

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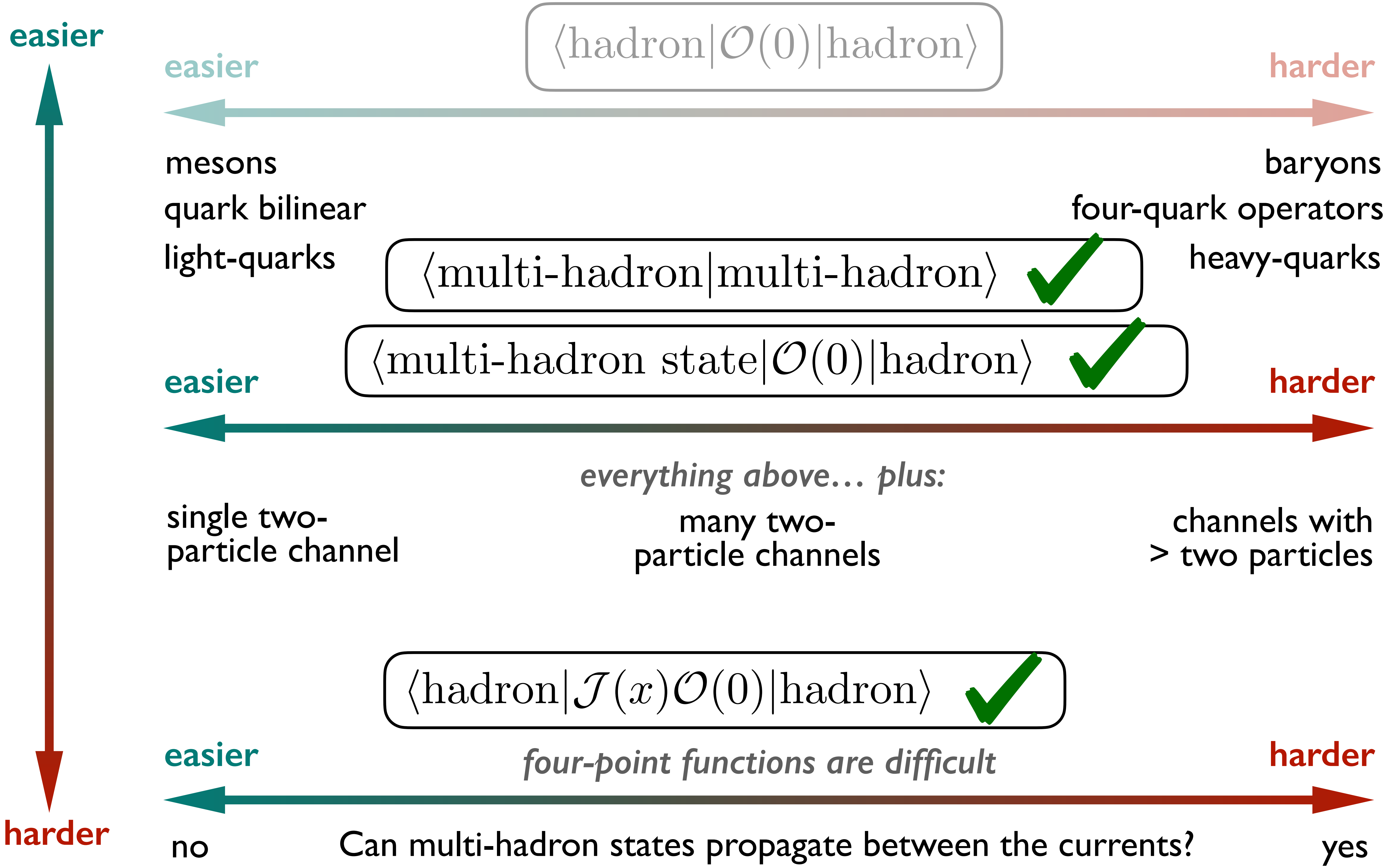

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Towards $N\pi\pi$

- Multiple three-particle finite-volume formalisms developed (so far only spin zero)

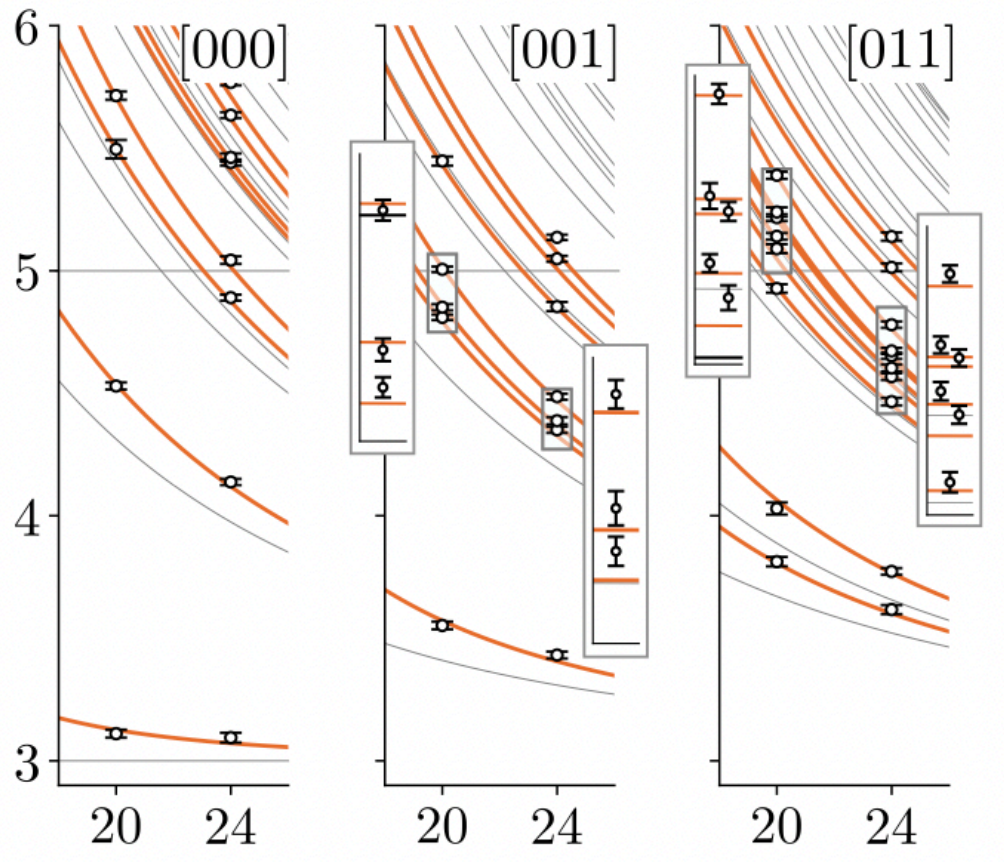
MTH, Sharpe (2014-2016) *See also Döring, Mai, Hammer, Pang, Rusetsky*

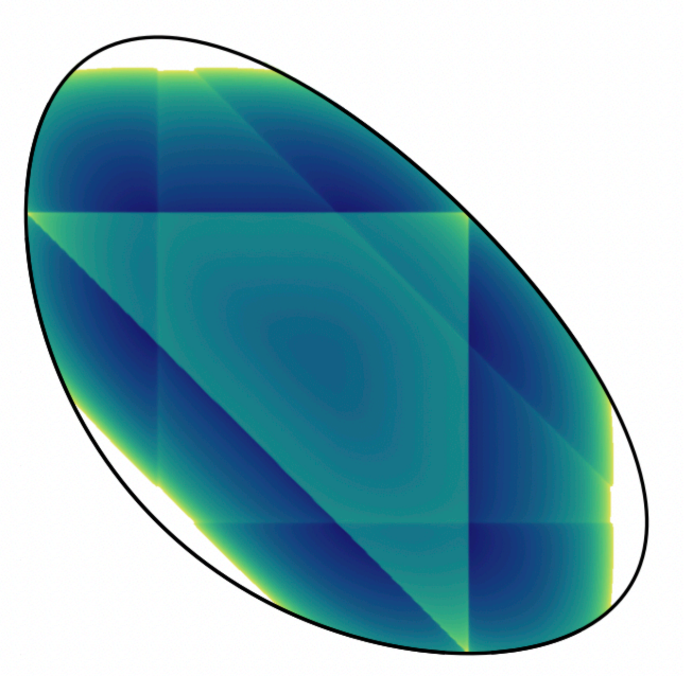
Towards $N\pi\pi$

☐ Multiple three-particle finite-volume formalisms developed (so far only spin zero)

MTH, Sharpe (2014-2016) *See also Döring, Mai, Hammer, Pang, Rusetsky*

☐ First lattice calculations appearing… e.g. $\pi^+\pi^+\pi^+ \to \pi^+\pi^+\pi^+$





○ Extract reliable spectrum

○ Use formalism to fit scheme-dependent K-matrix

○ Solve integral equations to reach physical amplitude

MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001

☐ See Blanton et al. 2022 and 2023 for pion and kaon results

☐ See Sadasivan et al. 2022 for application to $a_1(1260)$

Not discussed here…

- Signal-to-noise problem and solutions
    - For a single nucleon **(signal/noise) ~** $\exp[-\,(M_N - 3/2 M_\pi)\,\tau_{\text{Euclidean}}]$
    - One method to solve this is multi-level = "only update gauge field in regions”

- HAL-QCD potential method
    - Extract effective potential from lattice calculation
    - Requires derivative expansion

See… Murakami et al.,
Lattice QCD studies on decuplet baryons as meson-baryon bound states in the HAL QCD method

- Spectral-function reconstruction methods
    - Approach that does not use finite-volume as a tool
    - Attempts to solve regulated inverse Laplace transform
    - Delivers observables “smeared” over a range of energies

Points for discussion

- With lattice QCD experts…
  - What are the biggest limitations & next steps for **single-baryon observables**?
  - Will multi-level algorithms play an important role?
  - How far are multi-meson observables from full-error-budget physical point?
    - Will we get there?… When?
  - How far are multi-baryon observables from full-error-budget physical point?
- With other theorists…
  - What are the single-baryon observables where lattice QCD can be most impactful?
  - What are the simplest multi-hadron observables for lattice QCD?
  - Can theory provide scattering data to be used in lattice finite-volume formulae?
    - Also away from physical pion mass?
  - How light to pions have to be in lattice QCD calculations to be useful?
- With experimentalists…
  - Which processes are on the horizon?
  - Can lattice + experiment identify more lattice-friendly observables?

Points for discussion

- ☐ With lattice QCD experts…
  - ○ What are the biggest limitations & next steps for **single-baryon observables**?
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- ☐ With experimentalists…
  - ○ Which processes are on the horizon?
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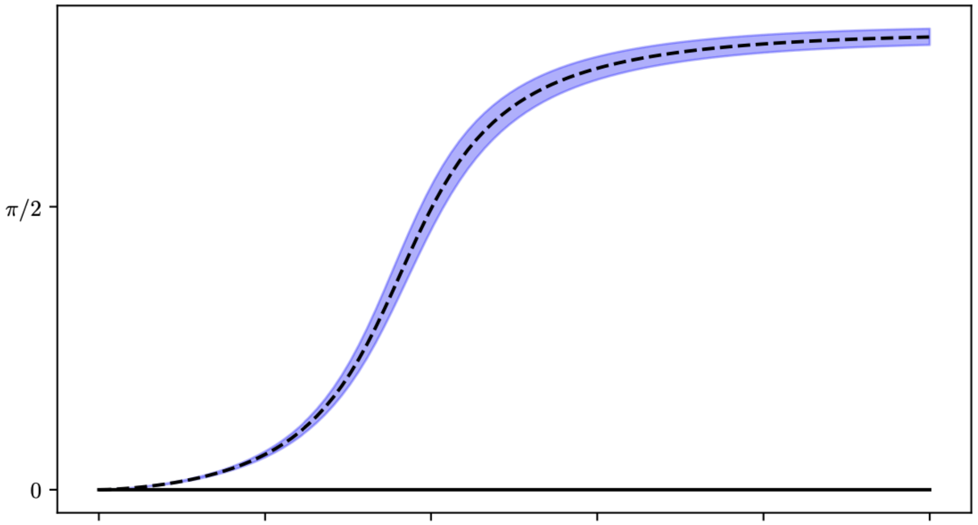
Summary and outlook

☐ If amplitude has multi-hadron contributions

→ LQCD requires complicated formalism (*volume* + *Euclidean*)



☐ Robust baryonic scattering studies now appearing

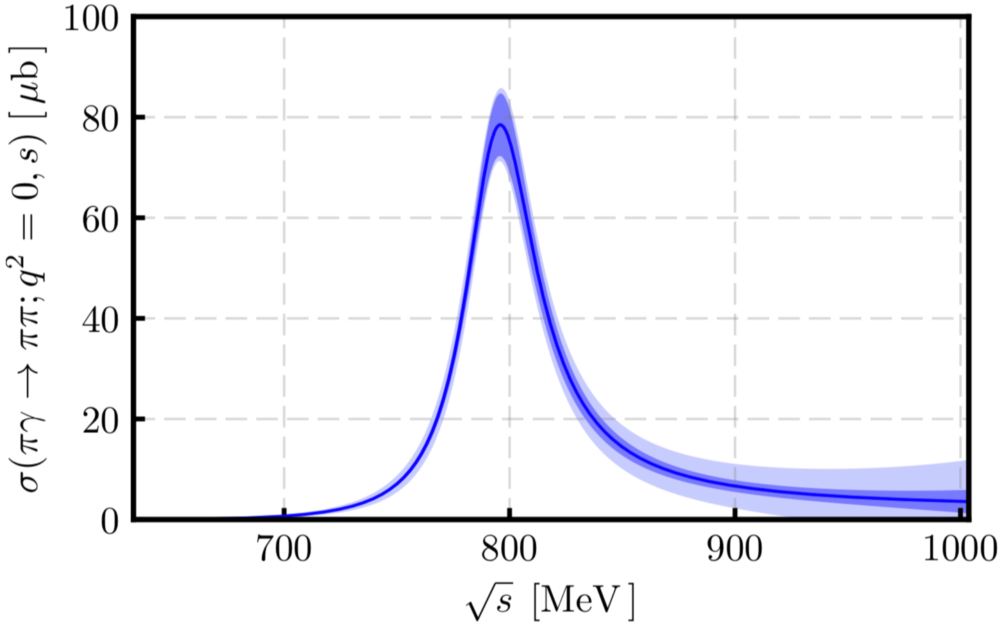
complete finite-volume spectrum, field theoretically mapped to amplitudes

☐ Formal developments ahead of numerical calculations

Supported by UKRI FLF

UK Research and Innovation

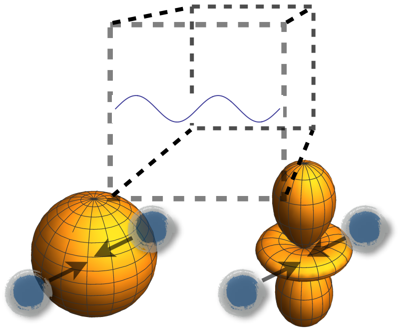
Thanks for listening!… questions?



Coupled channels

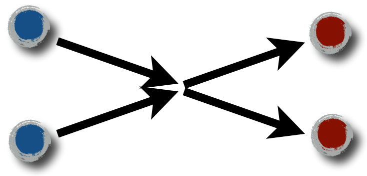
☐ The cubic volume mixes different partial waves…

e.g. $K\pi \to K\pi$, $\vec{P} \neq 0$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_s^{-1} & 0 \\ 0 & \mathcal{K}_p^{-1} \end{pmatrix} + \begin{pmatrix} F_{ss} & F_{sp} \\ F_{ps} & F_{pp} \end{pmatrix}\right] = 0$

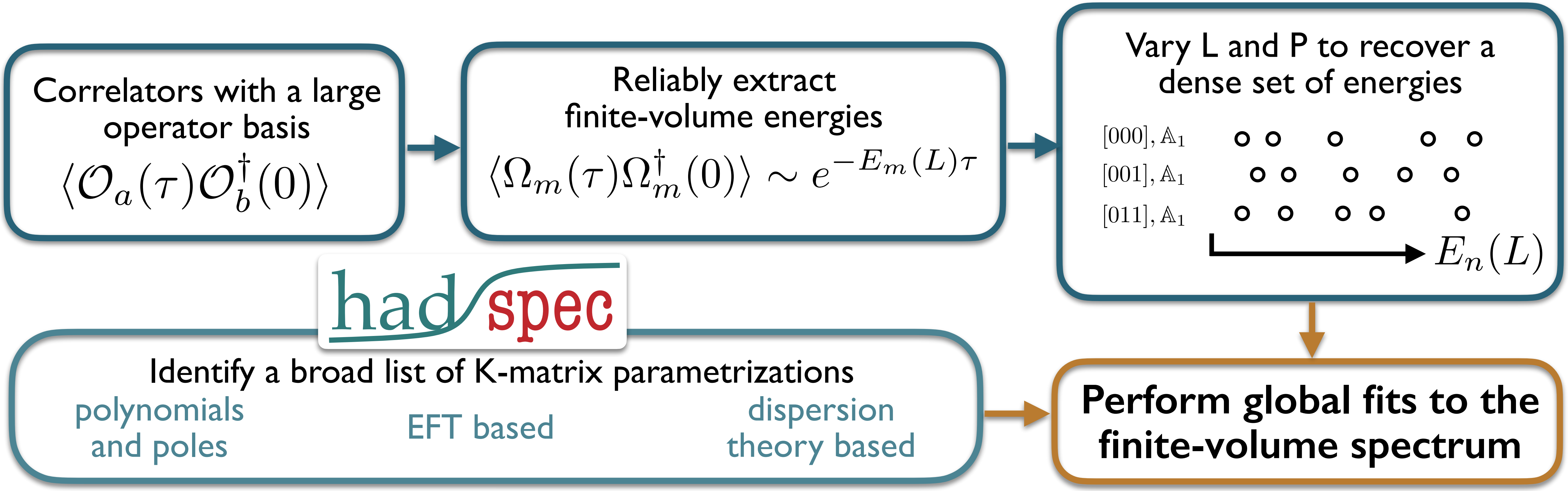


…as well as different flavor channels…

e.g. $a = \pi\pi$, $b = K\overline{K}$ $\longrightarrow$ $\det\left[\begin{pmatrix} \mathcal{K}_{a\to a} & \mathcal{K}_{a\to b} \\ \mathcal{K}_{b\to a} & \mathcal{K}_{b\to b} \end{pmatrix}^{-1} + \begin{pmatrix} F_a & 0 \\ 0 & F_b \end{pmatrix}\right] = 0$



☐ Workflow…



Correlators with a large operator basis

$\langle \mathcal{O}_a(\tau)\mathcal{O}_b^\dagger(0)\rangle$

Reliably extract finite-volume energies

$\langle \Omega_m(\tau)\Omega_m^\dagger(0)\rangle \sim e^{-E_m(L)\tau}$

Vary L and P to recover a dense set of energies

Identify a broad list of K-matrix parametrizations

polynomials and poles

EFT based

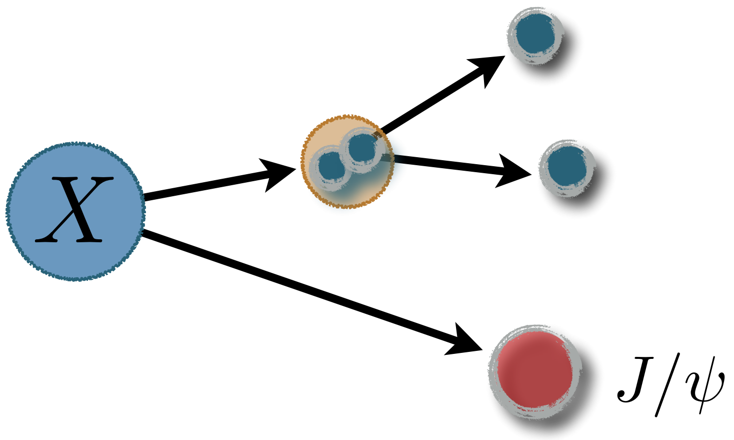
dispersion theory based

Perform global fits to the finite-volume spectrum

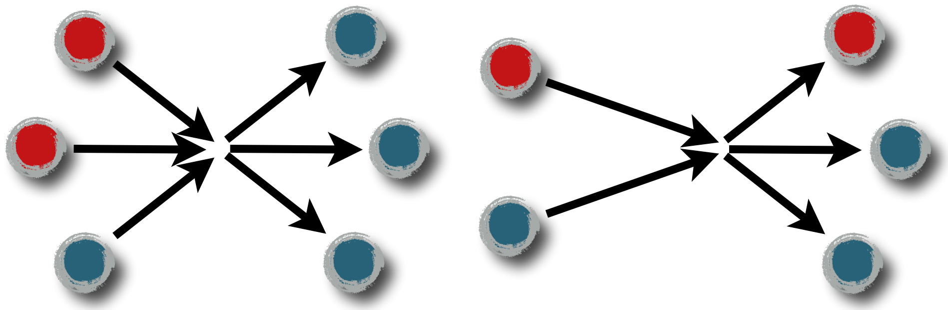
3-particle amplitudes

2-to-2 only samples J^P 0^+ 1^- $2^+ \cdots$

many interesting resonances have significant 3-body decays



Goal: ***finite-volume + unitarity formalism*** for generic two- and three-particle systems



Applications…

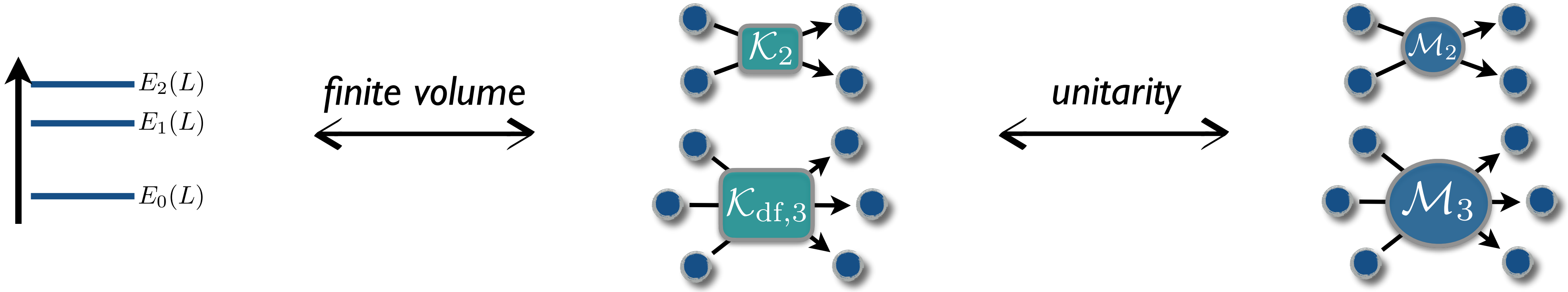
exotic resonance pole positions, couplings, quantum numbers

$\omega(782),\, a_1(1420) \to \pi\pi\pi$ $X(3872) \to J/\psi\pi\pi$ $X(3915)[Y(3940)] \to J/\psi\pi\pi$

form factors and transitions

and much more!… *(3-body forces, weak transitions, gluons content)*

Status…



☑ Identical spin-zero, no 2-to-3, no K2 poles • MTH, Sharpe (2014, 2015) •

☑ as above… but including 2-to-3 • Briceño, MTH, Sharpe (2017) •

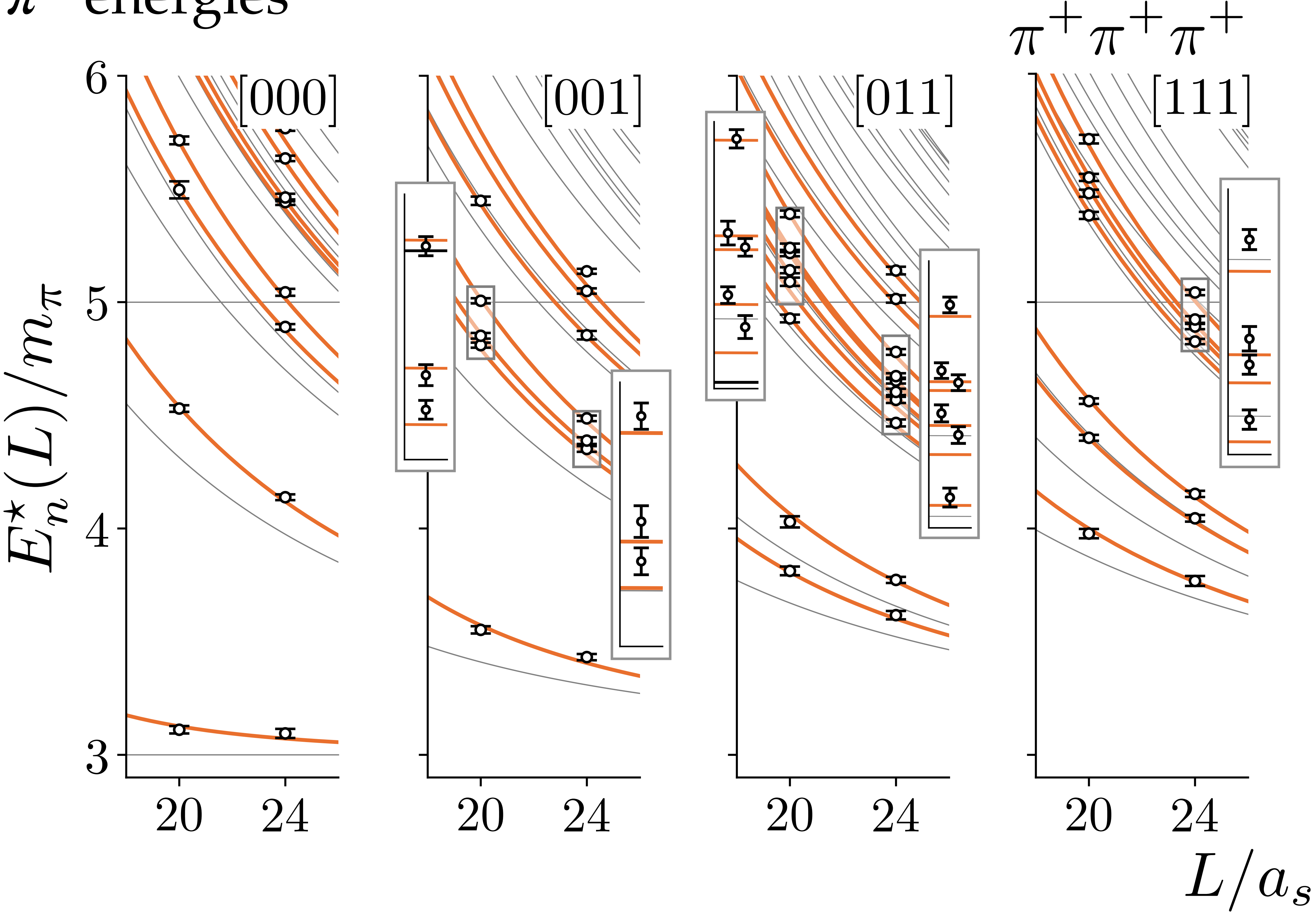
☑ as above… but including K2 poles • Briceño, MTH, Sharpe (2018) •

☑ Non-identical, non-degenerate spin-zero $\pi\pi\pi \to \rho\pi \to \omega \to \rho\pi \to \pi\pi\pi$

• MTH, Romero-López, Sharpe (2020) • Blanton, Sharpe (2020, 2021)

☐ Multiple three-particle channels… Spin!

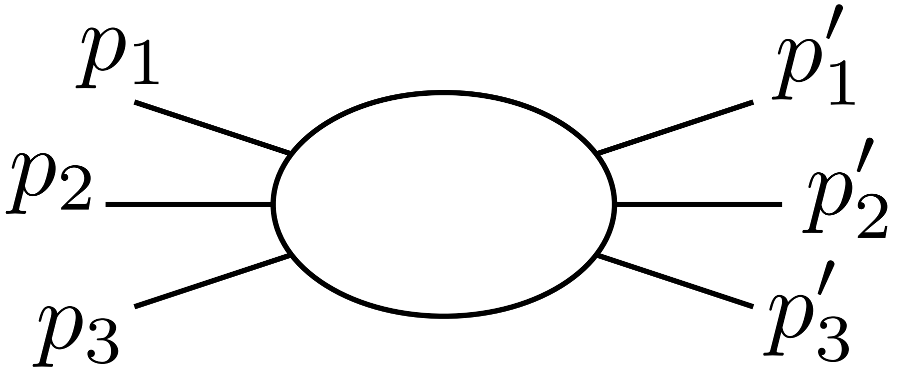
$\pi^+\pi^+\pi^+$ energies

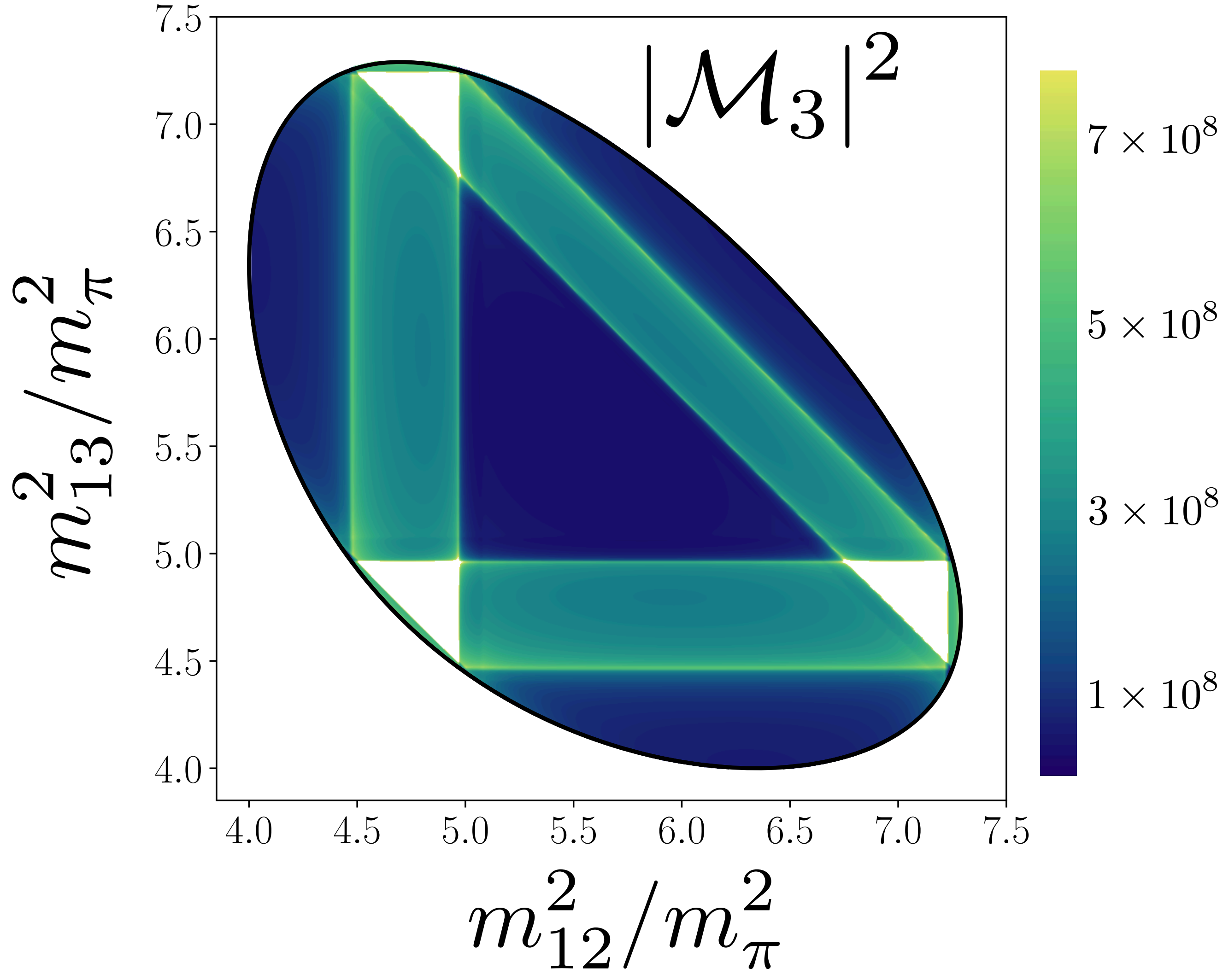


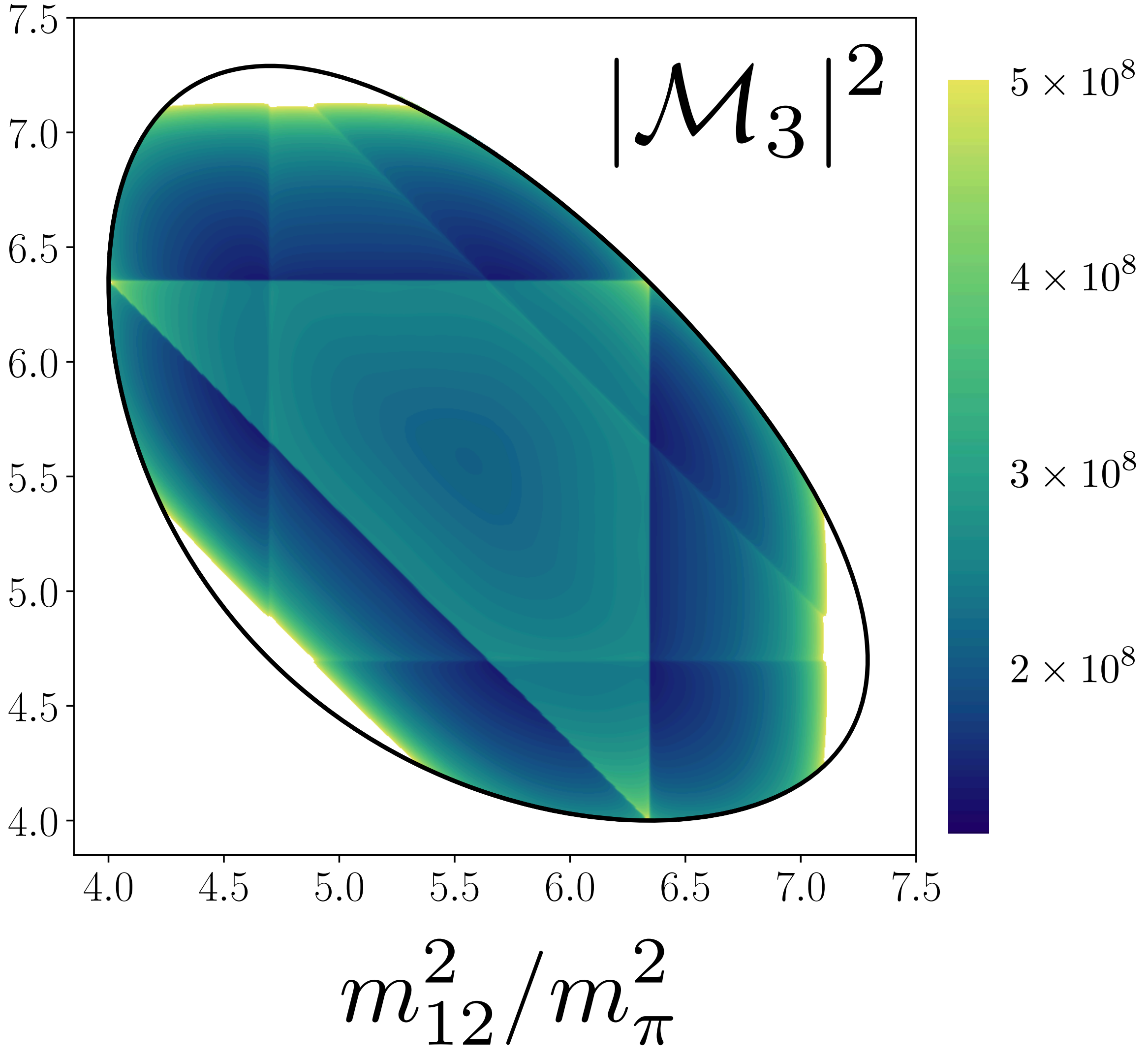
MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001,

see also work by… Culver, Döring, Hanlon, Hörz, Mai, Morningstar, Romero-Lopez, Sharpe + ETMC

$$\mathcal{M}_3 = \sum_{i,j\in\{1,2,3\}} \mathcal{M}_3^{\mathsf{un}}(p_i', p_j)$$







MTH, Briceño, Edwards, Thomas, Wilson, *Phys.Rev.Lett.* 126 (2021) 012001

$$\Delta \to N\pi$$

- ☐ Andersen et al. 2018
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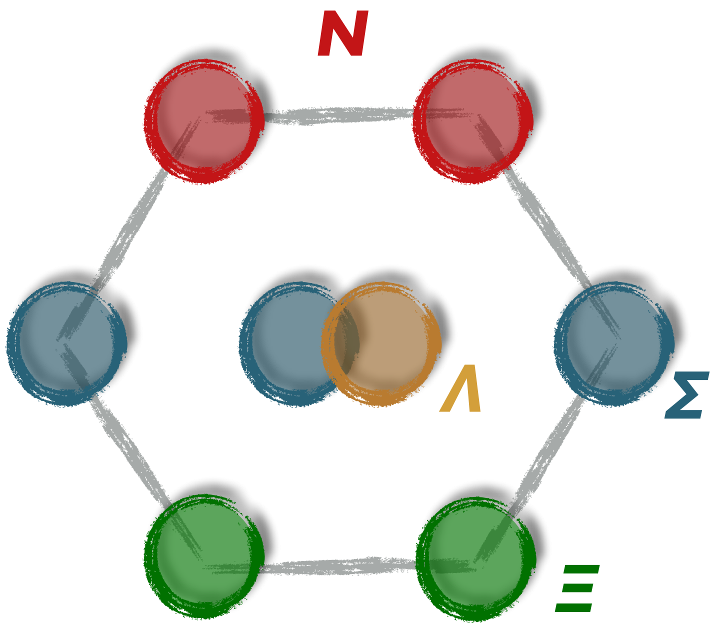
- ☐ Silvi et al. 2021
- ☐ Pittler et al. 2021
- ☐ Bulava et al. 2022

$$N^* \to N\pi$$

- ☐ Lang et al. 2017
- ☐ Wu et al. 2017
- ☐ Kiratidis et al. 2017

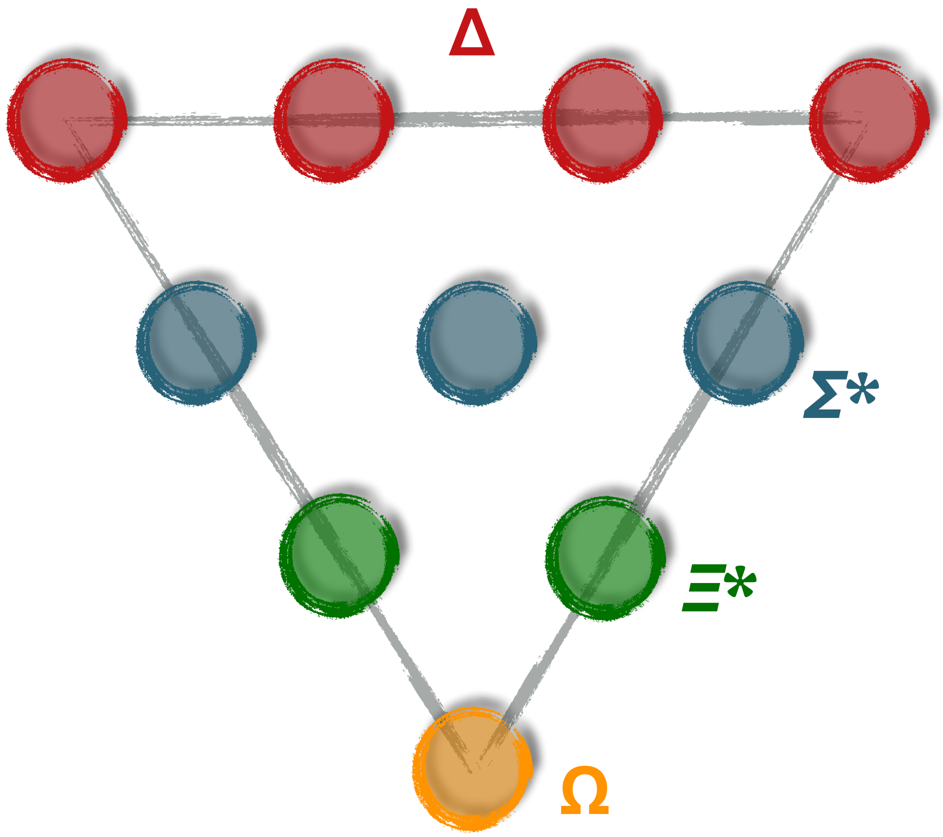
$$\Lambda \to \overline{K}N$$

- ☐ Hall et al. 2015



(focusing here on studies with scattering states)

Baryons are difficult!



See also…

- ☐ Detmold and Nicholson 2015
- ☐ Wu et al. 2018
- ☐ Xing & Liu, LATT2022 (in prep)

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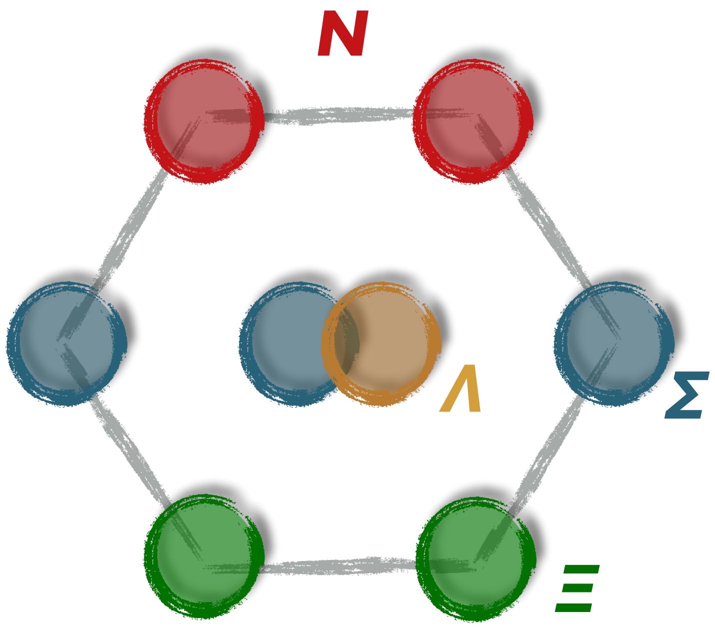

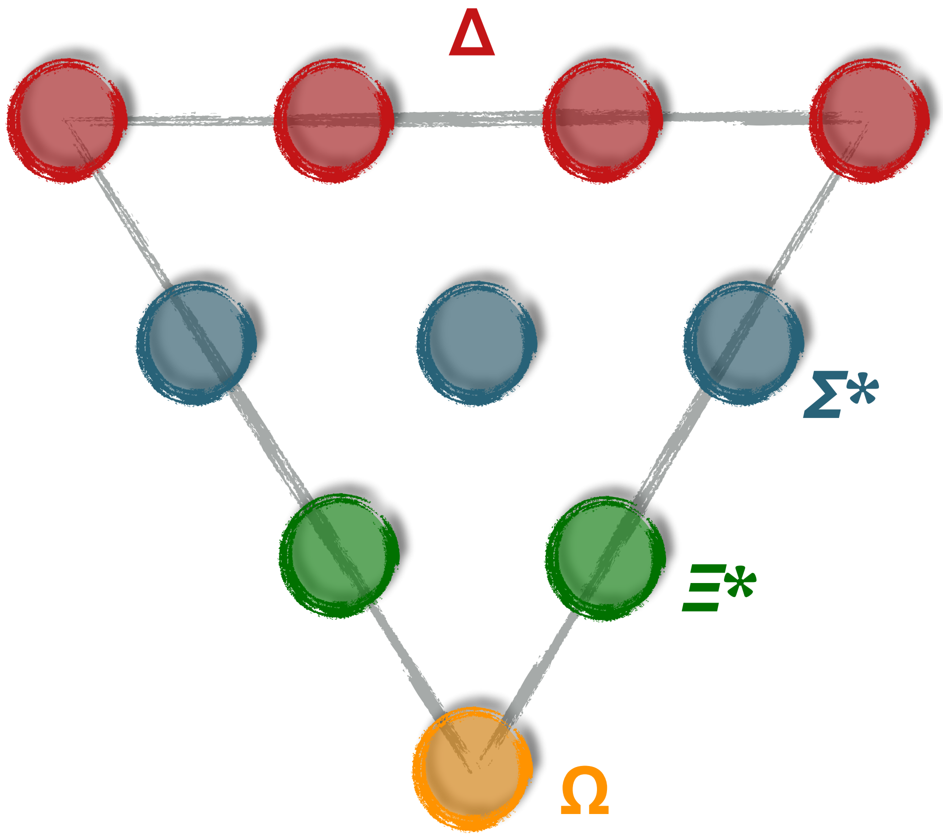

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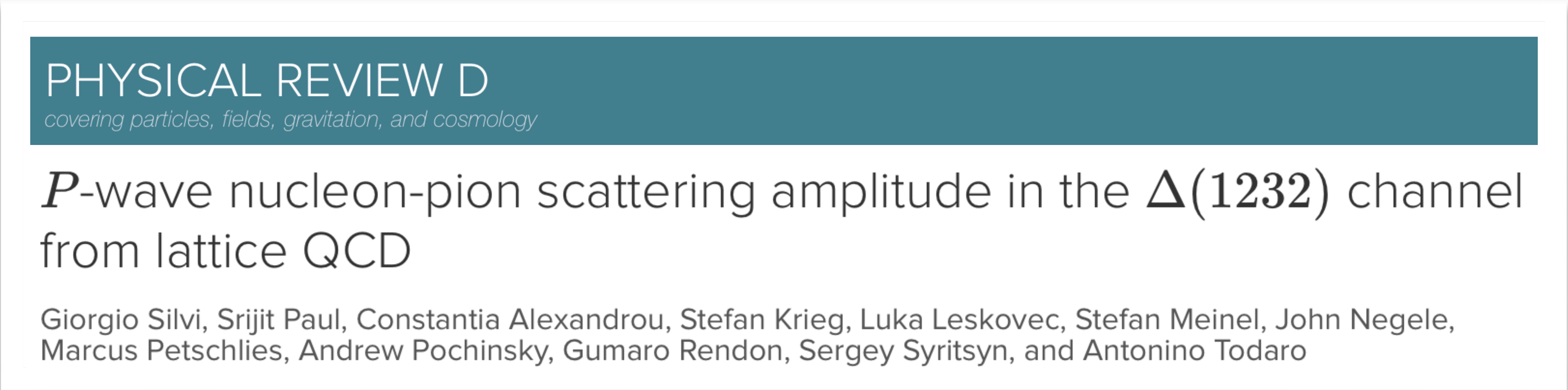
$N\pi$ elastic scattering ($M_\pi = 255\ \mathrm{MeV}$)



PHYSICAL REVIEW D

covering particles, fields, gravitation, and cosmology

P-wave nucleon-pion scattering amplitude in the $\Delta(1232)$ channel from lattice QCD

Giorgio Silvi, Srijit Paul, Constantia Alexandrou, Stefan Krieg, Luka Leskovec, Stefan Meinel, John Negele, Marcus Petschlies, Andrew Pochinsky, Gumaro Rendon, Sergey Syritsyn, and Antonino Todaro

☐ $M_\pi = 255\ \mathrm{MeV}$

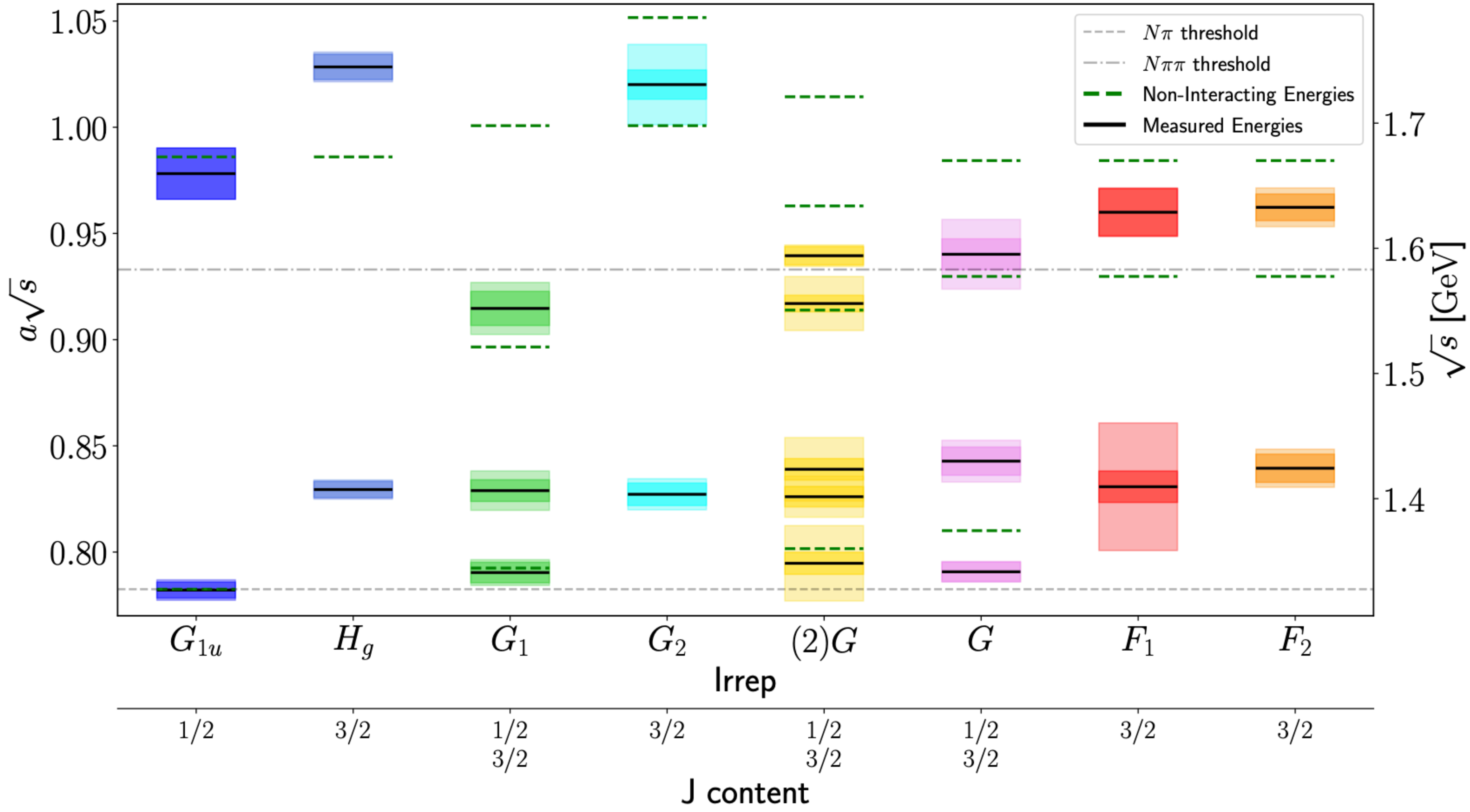
☐ Studied scattering-lengths and the Δ channel

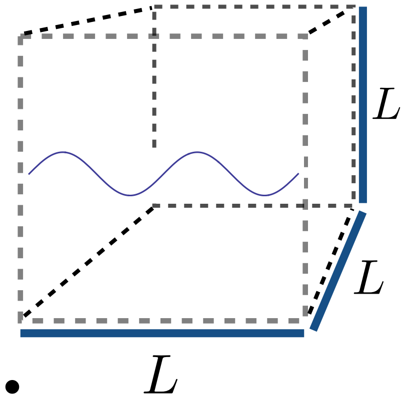
$I = 3/2 : \quad J^P = 1/2^-\ (S),\ \ 3/2^+\ (P) \qquad [1/2^+\ (P),\ \ 3/2^-\ (D),\ \ 5/2^-\ (D)]$

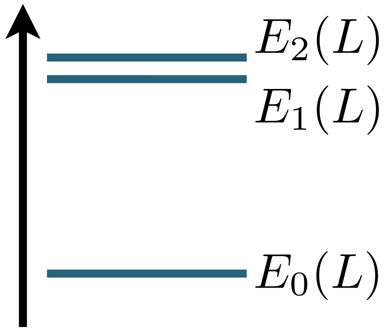
☐ Local Δ-like operators + $N(\boldsymbol{p}_1)\pi(\boldsymbol{p}_2)$ operators

• Silvi et al., PRD 2021, 2101.00689 •

$N\pi$ finite-volume energies $(M_\pi = 255\ \text{MeV})$

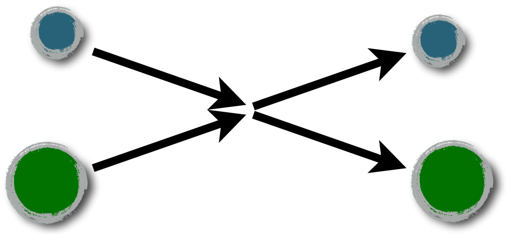


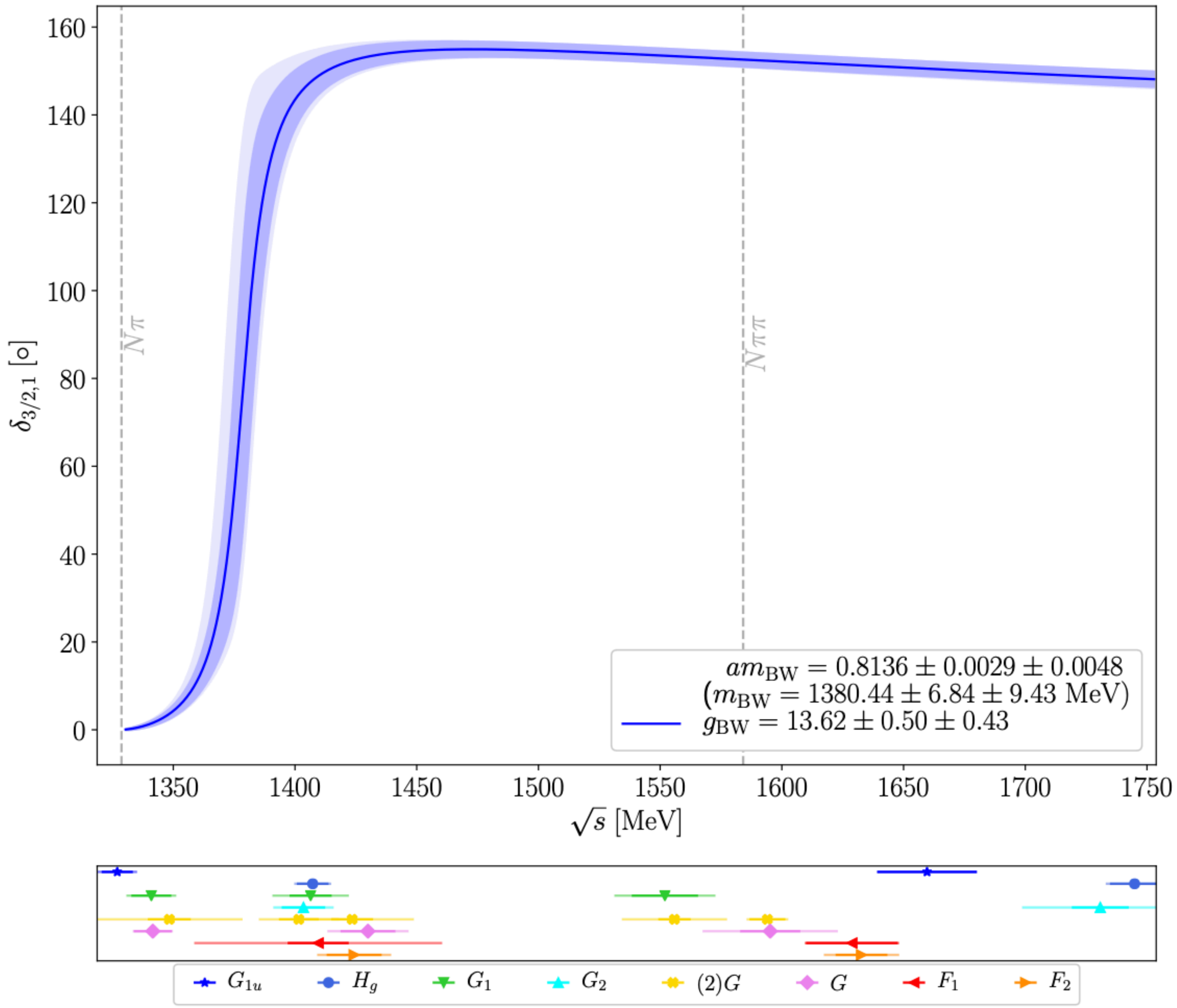




- Silvi et al., PRD 2021, 2101.00689
-

$N\pi \rightarrow \Delta \rightarrow N\pi \quad (M_\pi = 255\ \mathrm{MeV})$





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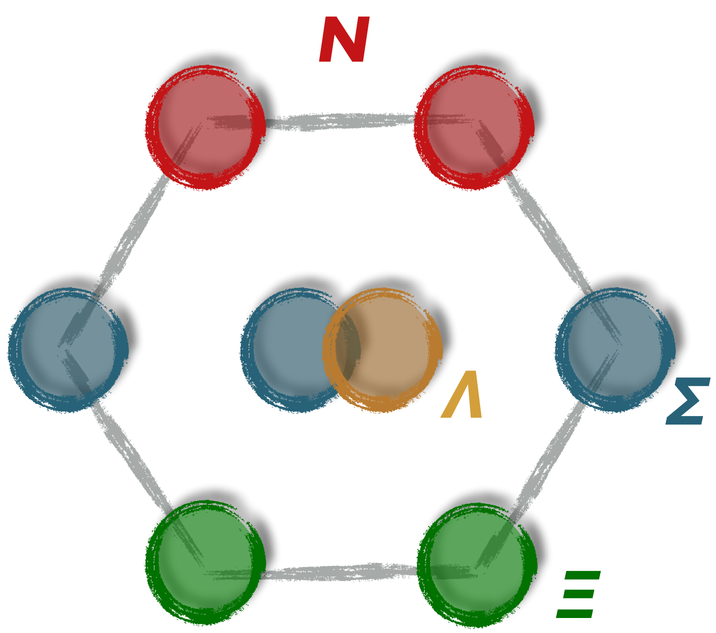
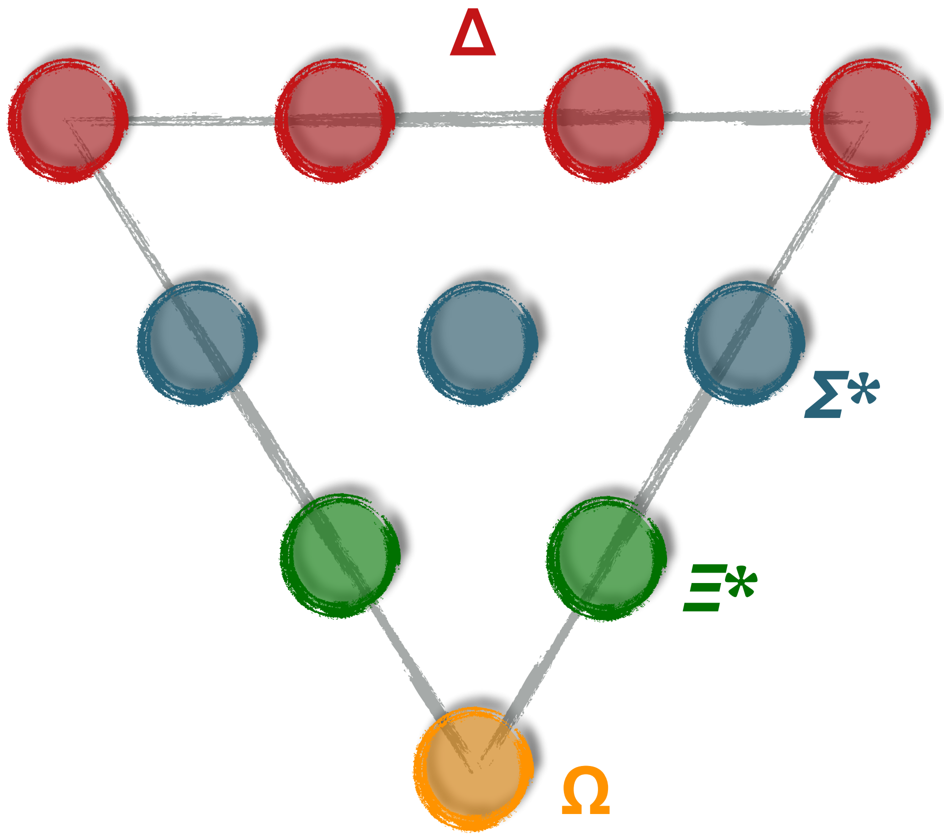




(focusing here on studies with scattering states)

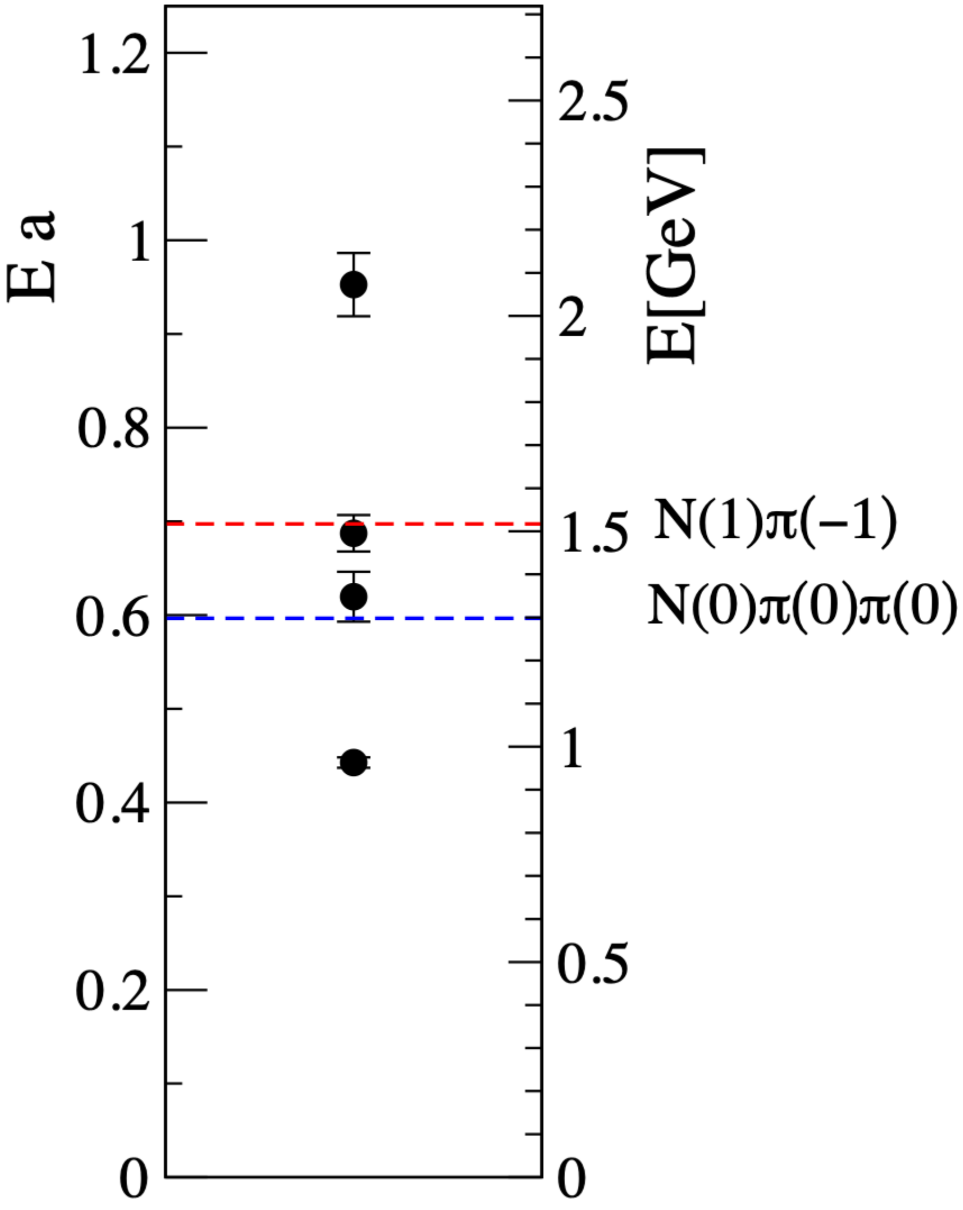
Baryons are difficult!

See also…

- ☐ Detmold and Nicholson 2015
- ☐ Wu et al. 2018
- ☐ Xing & Liu, LATT2022 (in prep)

Roper resonance

- ☐ Results with impressive operators from Lang et al., $M_\pi = 156\ \text{MeV}$
- ☐ Interpretation presented by D. Leinweber this morning



• Lang, Leskovec, Padmanath, Prelovsek (2017) •

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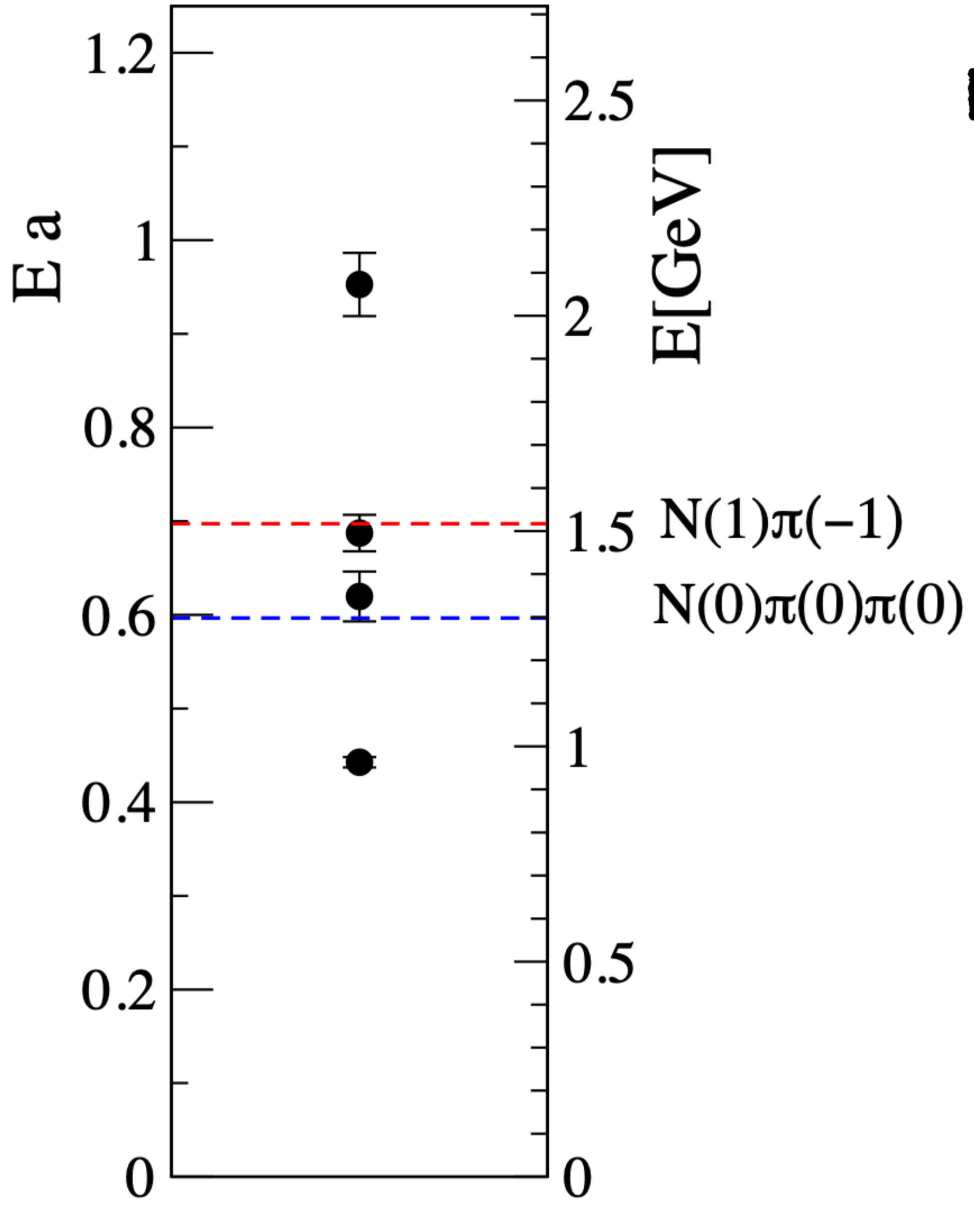
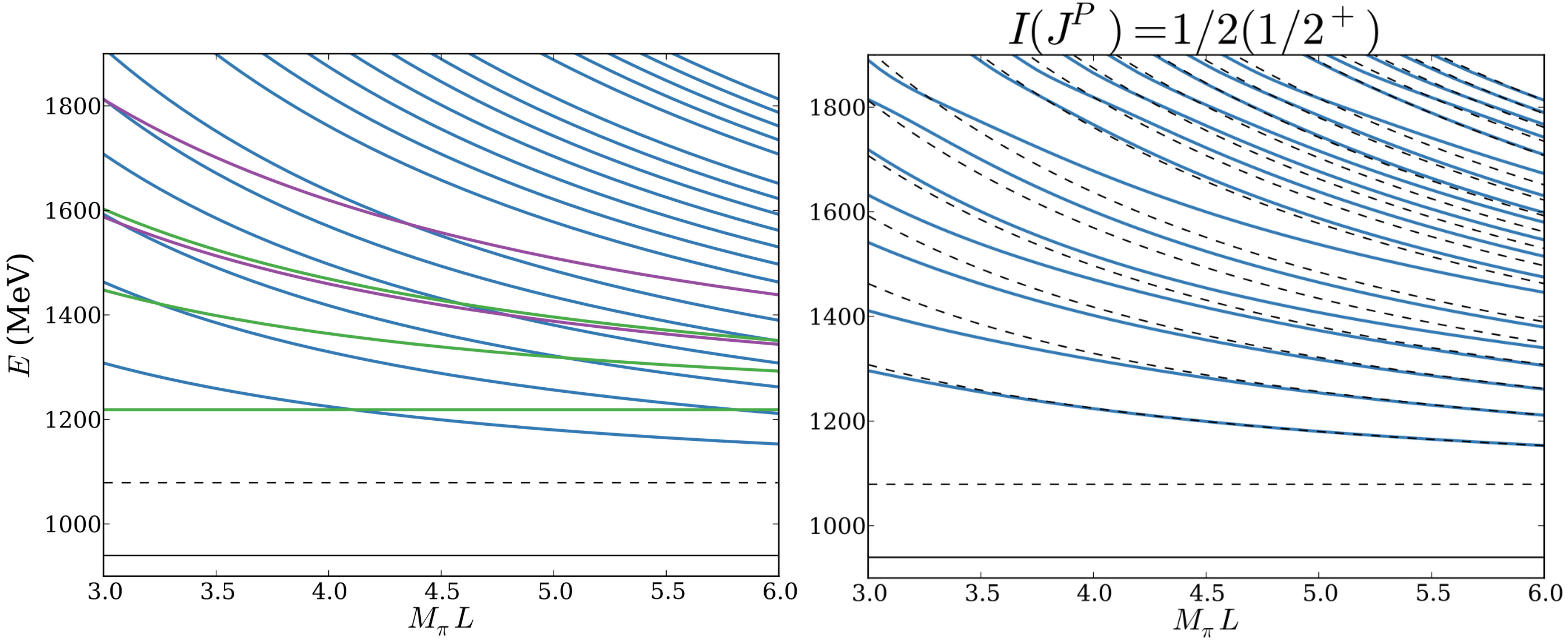


- My personal take, need more data…
  - Lattice "rule-of-thumb" is to take $M_\pi L \sim 4$ or larger
  - Here $M_\pi L \sim 2.2$
  - First three states are statistically consistent with non-interacting nucleons and pions
  - Careful with operator overlaps… discretisation dependent, no guarantee of continuum limit

• Lang, Leskovec, Padmanath, Prelovsek (2017) •

Roper resonance

- ☐ Naive spectra from MTH and Meyer 2016
  - ○ Non-interacting
  - ○ GWU WI08 + Lüscher



- ☐ But, right panel ignores three-body effects!… I would say work is needed

- MTH and Meyer (2016) • see also Döring et al. (2013) • Daniel Severt LATT2022 •

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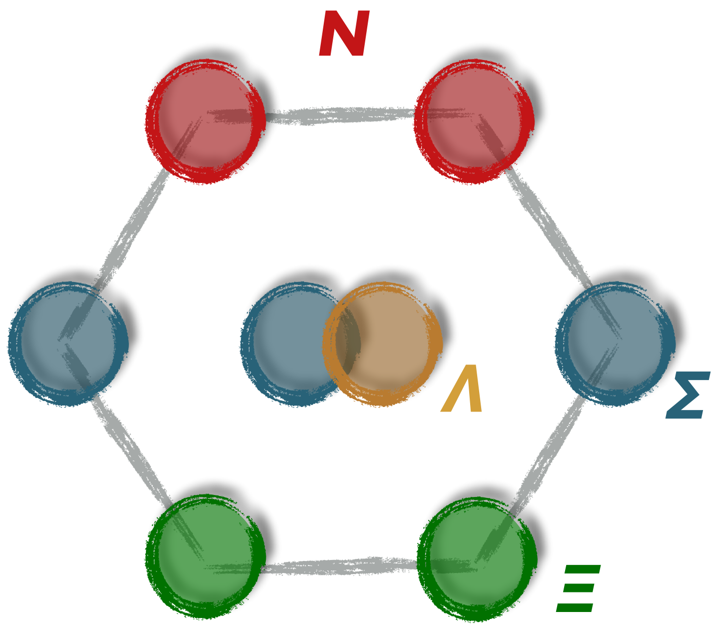
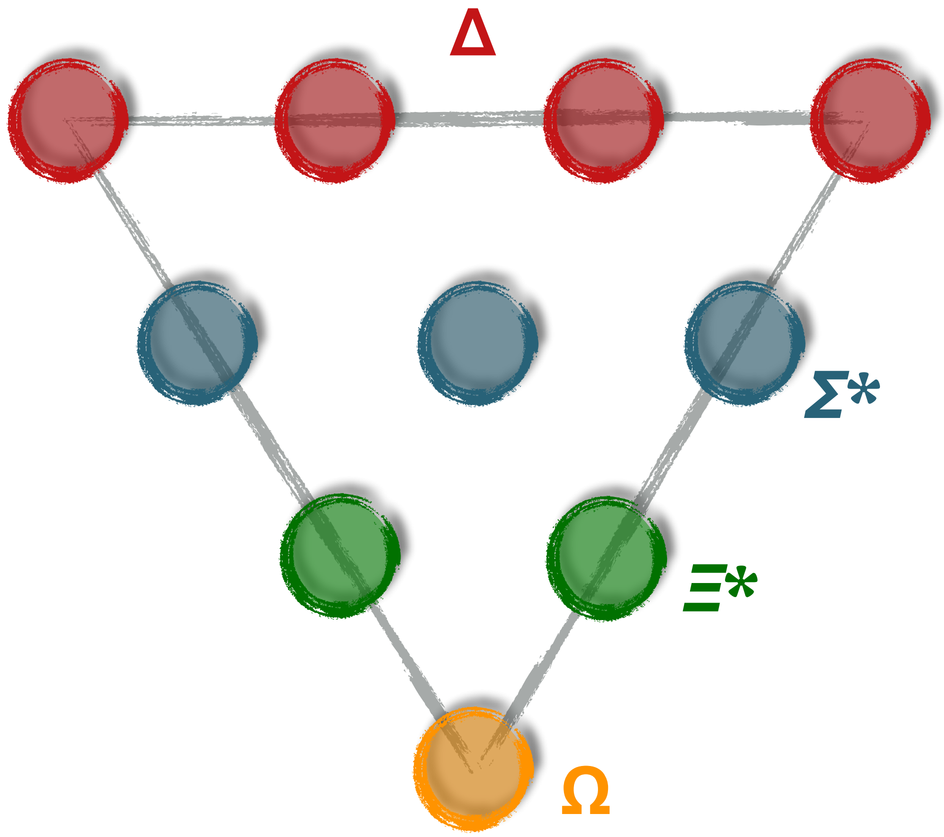


(focusing here on studies with scattering states)

Baryons are difficult…

Stay tuned!



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