Semileptonic form factors: Lattice perspectives for charmed baryons

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Probing baryon weak decays – from experiment to lattice QCD, Warszawa March 7, 2023

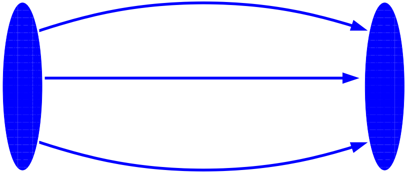
Why baryons?

- Is theory under control? Consistency with meson decays?
- More channels → precision increases.
- Mesons: except for $B_c \to J/\psi$, only pseudoscalar final states are stable against strong decay. Decays like $D \to K^* \to K\pi$, $B \to D^* \to D\pi$ are a challenge for theory.
- All positive parity spin $1/2$ light baryons as well as the Ω do not undergo strong decay. Ξ^* is quite narrow. Also spin $3/2$ initial states are possible.
- Currents that do not exist at tree-level in the SM can in principle be probed with "stable" initial and final hadronic states, e.g., $\Omega_c^* \to \Omega$ or Ξ ($J = 3/2 \to 3/2$ or $1/2$), $\Omega_c \to \Omega$ or $\Xi_c \to \Xi^*$ ($1/2 \to 3/2$).
- Experimental challenge: polarization.

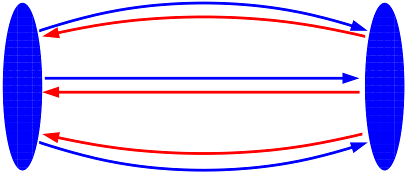
Problem with baryons: noise/signal (c.f. talk by J Bijnens)

`HW Hamber, E Marinari, G Parisi, C Rebbi, NPB225 (83) 475 (Appendix B)`

`GP Lepage, http://inspirehep.net/record/287173`



$$C_N(t) \sim \quad \sim \exp\left(-m_N t\right)$$



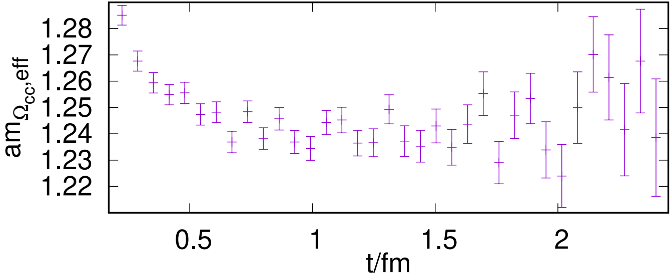
$$[\Delta C_N(t)]^2 \sim \quad \sim \exp\left(-3m_\pi t\right)$$

$$\frac{\Delta C_N(t)}{C_N(t)} \sim \exp\left[\left(m_N - \frac{3}{2}m_\pi\right) t\right]$$

Noise is less of a problem for charmed baryons

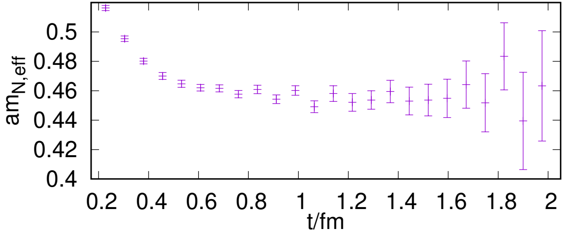
- $m_N - \frac{3}{2}M_\pi \approx 740\,\text{MeV}$.
- $m_{\Lambda_c} - M_D - \frac{1}{2}M_\pi \approx 350\,\text{MeV}$, $m_{\Lambda_c} - \frac{1}{2}M_{\eta_c} - M_\pi \approx 660\,\text{MeV}$.
- $m_N - 3m_\ell \approx m_N$, $m_{\Lambda_c} - 2m_\ell - m_c < m_N$: a smaller fraction of the mass is “dynamical”. Most is due to the heavy spectator quark and does not contribute to the “noise”.
- $q^2 < q^2_{\max} = (m_i - m_f)^2$ where q^2 near $q^2_{\max}$ is easiest to achieve. $m_{\Lambda_c} - m_\Lambda \approx 1170\,\text{MeV} < M_D - M_\pi \approx 1730\,\text{MeV}$ “easier” (although there is no problem with D decay either).
- Spectrum of excitations of the initial state is very similar as for heavy-light mesons. However, $(E_n - m)/m$ is smaller for final state baryons than for mesons and, unlike in the pseudoscalar meson case, t cannot be made very large.
- Note that $m_c \ll 1/a$ at present-day lattice spacings. For m_b HQET is needed.

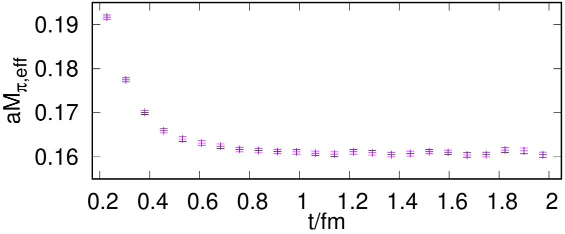
Effective masses



$$m_{\mathrm{H,eff}}(t) = a^{-1} \ln\left[\frac{C_H(t)}{C_H(t+a)}\right] \to m_H$$

Error of $m_{\Omega_{cc}}$ increases much slower than that of m_N.

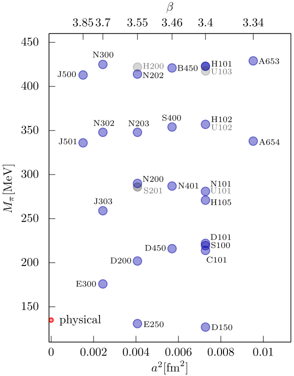




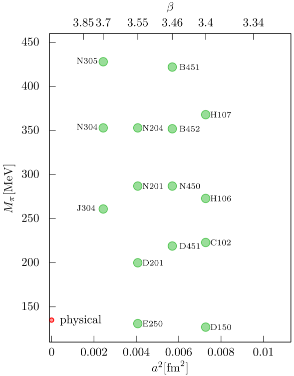
CLS ensembles: M_π vs a^2

$N_f = 2+1$ flavours of non-perturbatively $\mathcal{O}(a)$ improved Wilson fermions on tree level Symanzik improved glue.
High statistics: mostly > 6000 MDUs. Always open-boundary conditions in time for $a < 0.06\,\mathrm{fm}$.
Aim to control all main sources of systematics (a, m_q and V).

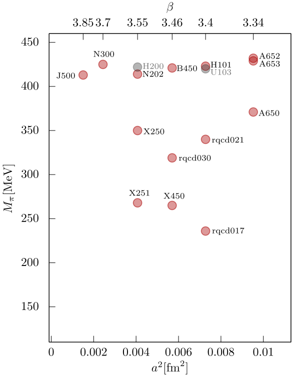
Six lattice spacings: $a = (0.1 \searrow 0.039)\,\mathrm{fm}$, $LM_\pi \gtrsim 4$ + smaller volumes, $M_\pi = (420 \searrow 130)\,\mathrm{MeV}$.



$2m_\ell + m_s = \mathrm{const.}$

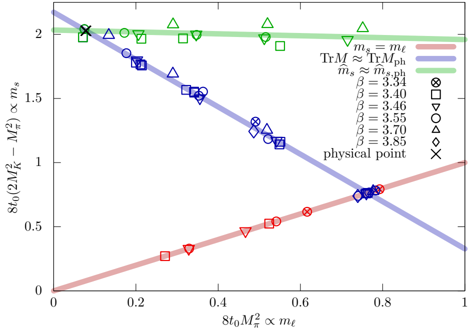


$m_s = \mathrm{const.}$



$m_\ell = m_s$

CLS ensembles: m_ℓ-m_s plane

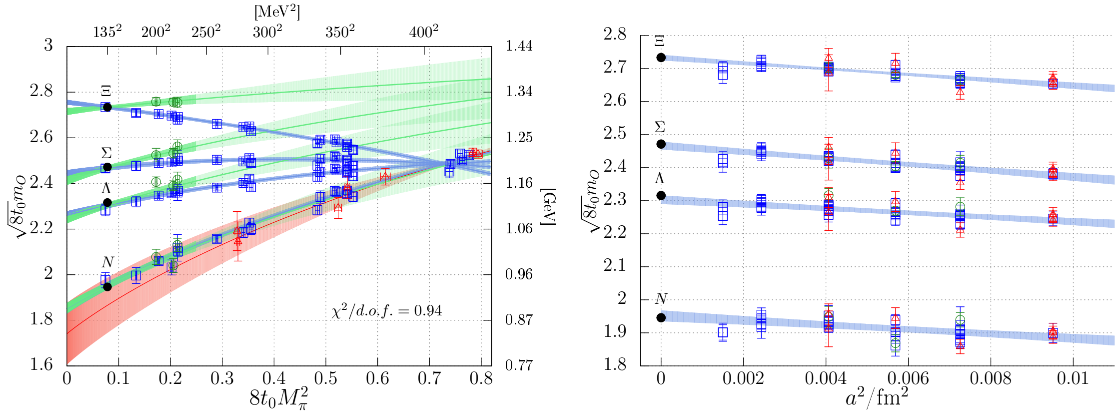


Three trajectories: can correct for mis-tuning. Good control over the quark mass dependence.

$2m_\ell + m_s = \mathrm{const.}$: investigate SU(3) flavour breaking (flavour averaged quantities roughly constant).

Approach to the physical point involves $M_K \uparrow$ as $M_\pi \downarrow$.

The light octet baryon spectrum

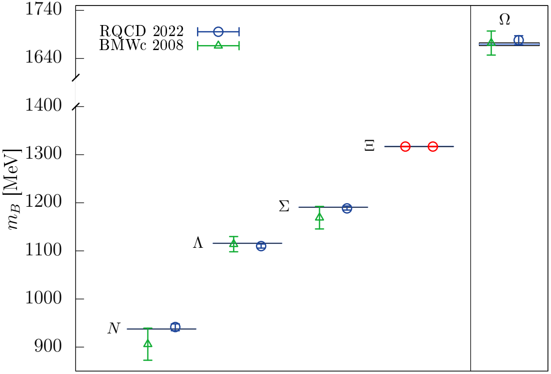


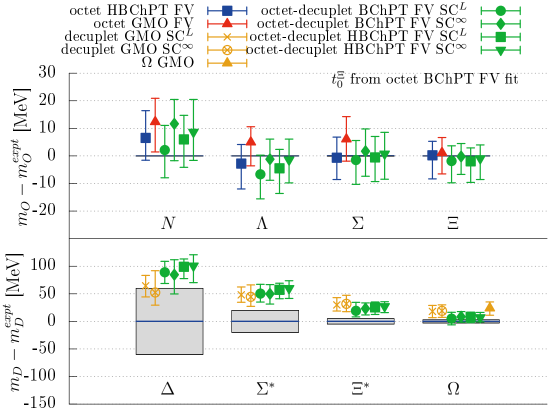
Above is a fit to NNLO EOMS BChPT, with the cuts $\overline{M}^2 < (440\,\mathrm{MeV})^2$ and $LM_\pi > 4$ on the average meson mass and lattice size.

Varying data cuts and fit ansatz, we obtain the lattice scale t_0 from m_Ξ. [RQCD, 2211.03744]

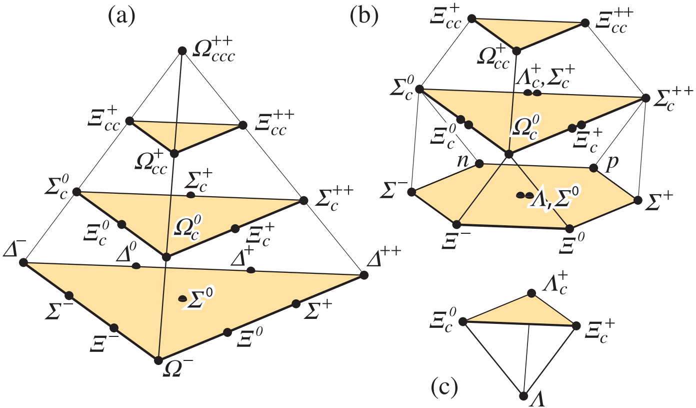
Progress since 2008

Comparison with [BMW, S. Dürr et al, 0906.3599] and the precision achieved.



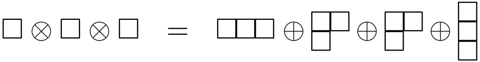


- SU(4) representations (→ more detail in S Grote's talk)

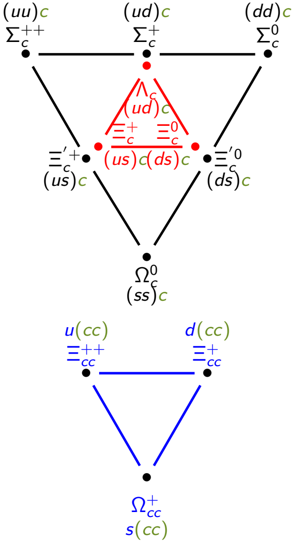


- Flavour symmetry is not respected but
- simplest way to see which baryons should exist.
- SU(4):

$$4 \otimes 4 \otimes 4 \quad = \quad 20 \oplus 20 \oplus 20 \oplus \overline{4}$$



Gell-Mann Okubo mass formulae for charmed baryons



Sextet

$$m_{\Sigma_c^{(*)}} = m_6^{(*)} - \tfrac{2}{3}A^{(*)}\delta m + \mathrm{O}(\delta m^2)$$

$$m_{\Xi_c^{\prime|*}} = m_6^{(*)} + \tfrac{1}{3}A^{(*)}\delta m + \mathrm{O}(\delta m^2)$$

$$m_{\Omega_c^{(*)}} = m_6^{(*)} + \tfrac{4}{3}A^{(*)}\delta m + \mathrm{O}(\delta m^2)$$

Anti-triplet

$$m_{\Lambda_c} = m_{\bar{3}} + \tfrac{2}{3}B\delta m + \mathrm{O}(\delta m^2)$$

$$m_{\Xi_c} = m_{\bar{3}} - \tfrac{1}{3}B\delta m + \mathrm{O}(\delta m^2)$$

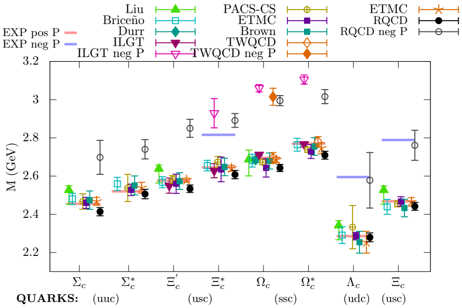
Triplet

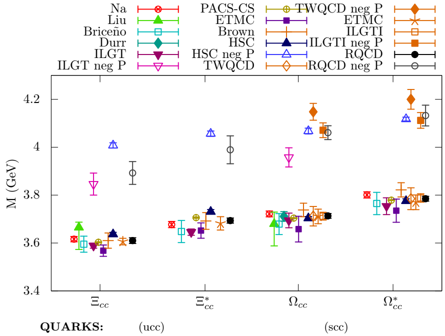
$$m_{\Xi_{cc}^{(*)}} = m_3^{(*)} + \tfrac{1}{3}C^{(*)}\delta m + \mathrm{O}(\delta m^2)$$

$$m_{\Omega_{cc}^{(*)}} = m_3^{(*)} - \tfrac{2}{3}C^{(*)}\delta m + \mathrm{O}(\delta m^2)$$

$$\delta m = m_s - m_{ud}$$

Spectrum of charmed baryons 2018



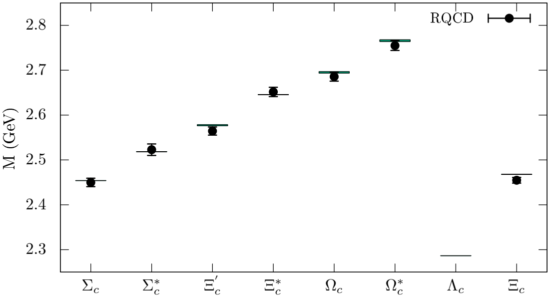


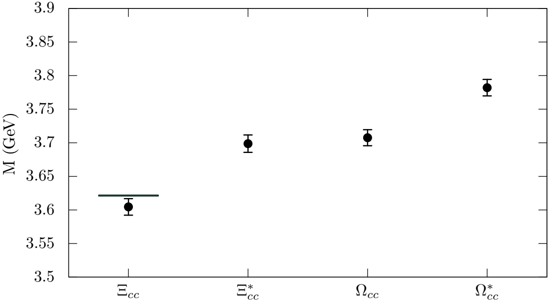
- RQCD: Some $c\ell\ell$ states below experiment (no continuum limit in 2015).
- Negative parity: $J^P = \frac{1}{2}^-, \frac{3}{2}^-$: [ILGTI,1211.6277], [HSC,1502.01845], [RQCD,1503.08440], [TWQCD,1701.02581], [ILGTI,1807.00174].
- Continuum, physical quark mass extrapolation: [Briceno,1207.3536], [ETMC,1406.4310], [Brown,1409.0497], [ILGTI,1807.00174].

Charmed baryon spectrum: RQCD on CLS ensembles

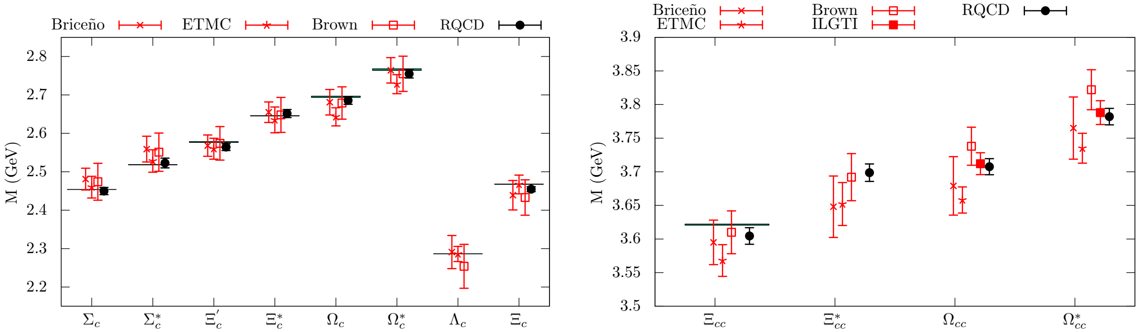
Continuum, infinite volume, physical quark mass limit (very preliminary).

Λ_c used to set the charm quark mass. We will switch to D_s for the final results.





Comparison with other continuum limit results



- [Briceno,1207.3536], [ETMC,1406.4310], [Brown,1409.0497], [ILGTI,1807.00174].
- RQCD: (very) preliminary.
- Also ∃ preliminary ETMC results (talk by C Alexandrou).

Semileptonic decay form factors of charmed baryons

Only a few lattice results exist so far.

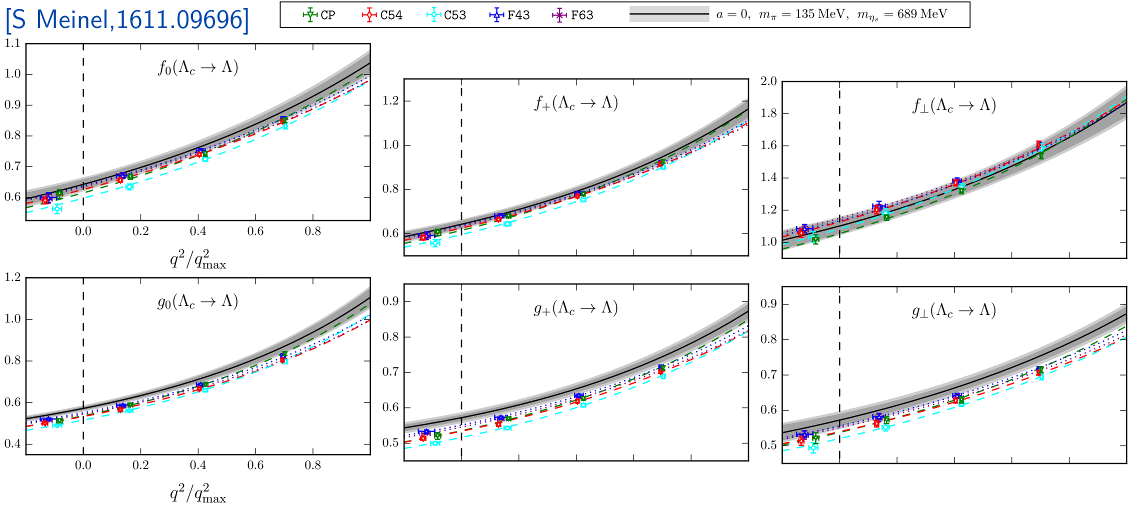
- $\Lambda_c \to \Lambda \ell \nu_\ell$: [S Meinel, 1611.09696]. Five ensembles at $a = 0.085\,\mathrm{fm}$ and $0.112\,\mathrm{fm}$. One ensemble close to the physical point, four at $M_\pi \gtrsim 300\,\mathrm{MeV}$.
- $\Lambda_c \to n \ell \nu_\ell$: [S Meinel, 1712.05783]. Different set of six ensembles at $a = 0.085\,\mathrm{fm}$ and $0.112\,\mathrm{fm}$ with $350\,\mathrm{MeV} \gtrsim M_\pi \gtrsim 230\,\mathrm{MeV}$.
- $\Xi_c \to \Xi \ell \nu_\ell$: [Qi-An Zhang et al, 2103.07064]. Two ensembles at $a = 0.080\,\mathrm{fm}$ and $0.108\,\mathrm{fm}$ with $M_\pi \approx 300\,\mathrm{MeV}$.
- $\Lambda_c \to \Lambda^*(1520) \ell \nu_\ell$ ($J^P = \frac{1}{2}^+$, $\Gamma \approx 15\,\mathrm{MeV}$, decays to $N\overline{K}, \Sigma\pi, \Lambda\pi\pi$): [S Meinel & G Rendon, 2107.13084; 2107.13140]. Three ensembles at $a = 0.083\,\mathrm{fm}$ and $0.111\,\mathrm{fm}$ and $M_\pi \approx 300, 340$ and $430\,\mathrm{MeV}$.

Λ_b decays have also been investigated!

Six form factors ($f_0(q^2), f_+(q^2), f_\perp(q^2)$ and $g_0(q^2), g_+(q^2), g_\perp(q^2)$) are needed to parameterize $\langle \Lambda(p') | \bar{d} \gamma_\mu c | \Lambda_c(p) \rangle$ and $\langle \Lambda(p') | \bar{d} \gamma_\mu \gamma_5 c | \Lambda_c(p) \rangle$ ($q = p' - p$).

Form-factors for $\Lambda_c \to \Lambda$

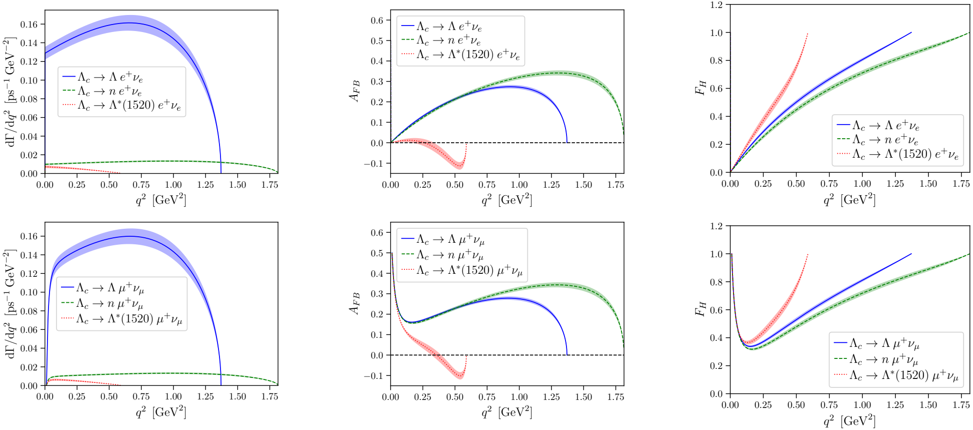
[S Meinel,1611.09696]



SM decay rate predictions

Differential decay rate, forward-backward asymmetry and flat term. [S Meinel,2107.13084].

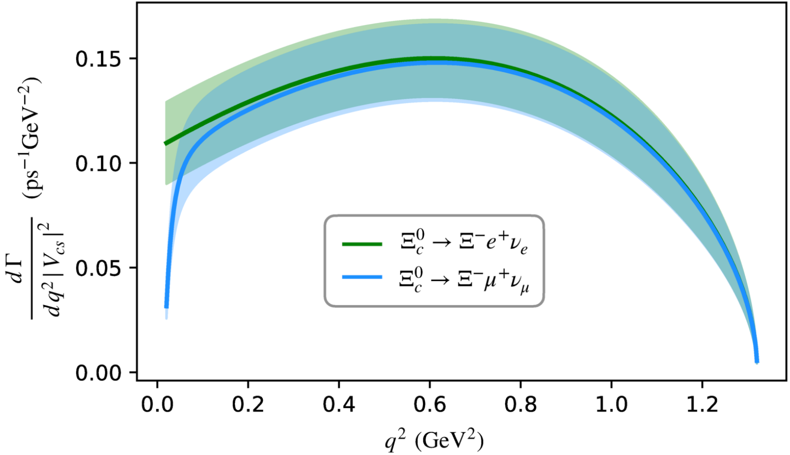
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\mathrm{d}\cos\theta} = A + B\cos\theta + C\cos\theta^2, \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = 2A + \frac{2}{3}C, \quad A_{FB} = \frac{3B}{6A+2C}, \quad F_H = \frac{6A+6C}{6A+2C}.$$



Ξ_c decay

[Qi-An Zhang et al,2103.07064]

Total branching fractions obtained within errors $\sim 20\%$, similar to Alice and Belle.

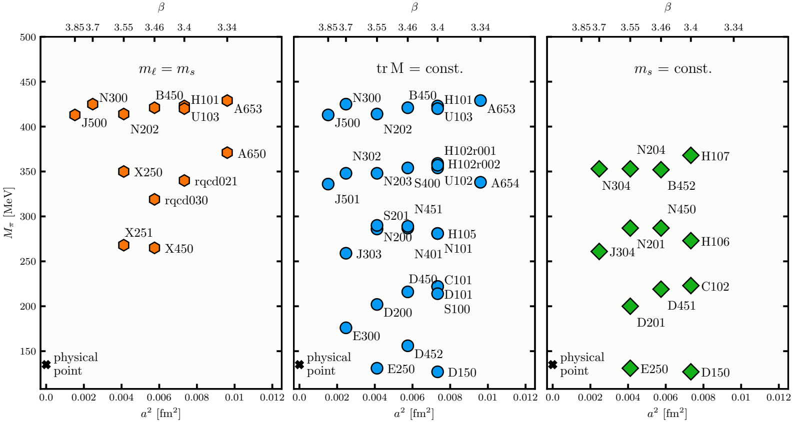


Light octet baryon charges

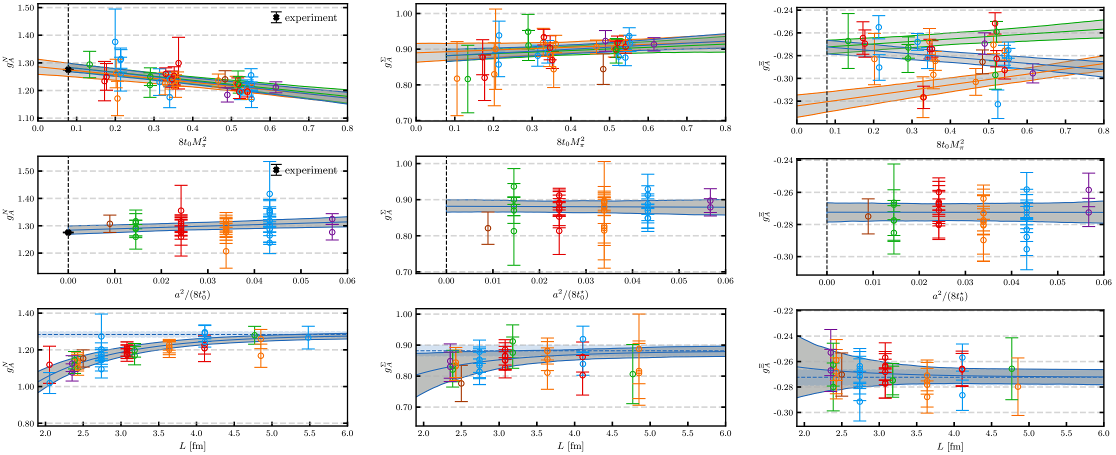
[RQCD: S Weishäupl, GB, S Collins et al, in preparation]

We computed all octet baryon vector charges (u-d).

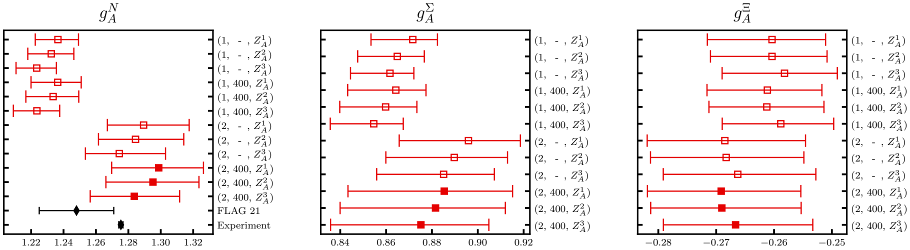
Measurements for all (transition) form factors are on disk but need to be analysed.

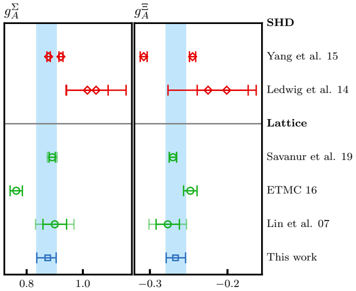


The axial charge

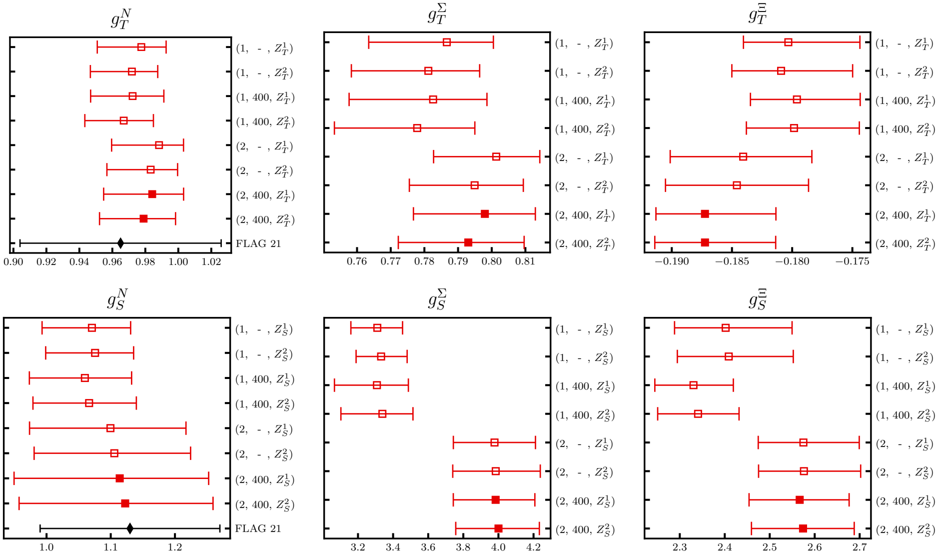


The axial charge II





The tensor and the scalar charges ($\overline{\mathrm{MS}}$ at $\mu = 2\,\mathrm{GeV}$ for $N_f = 3$)



Violation of flavour symmetry

For $m_\ell = m_s$, SU(3) symmetry gives ($g_J^\Lambda = 0$ due to isospin symmetry)

$$g_J^p = F_J + D_J, \quad g_J^{\Xi^+} = F_J - D_J, \quad g_J^{\Sigma^+} = 2F_J.$$

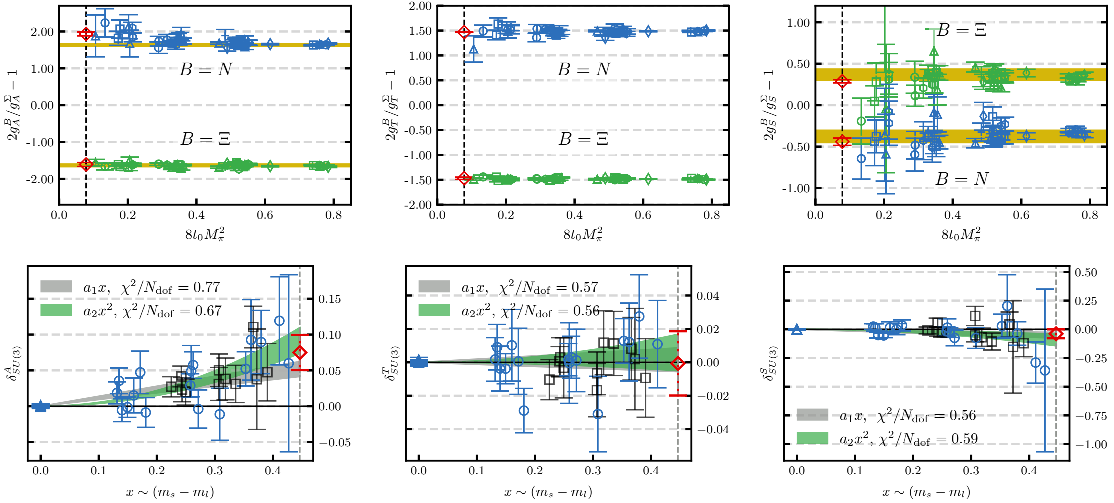
This means that

$$2g_J^N/g_A^\Sigma - 1 = D_J/F_J = -\left(2g_J^\Xi/g_A^\Sigma - 1\right).$$

However, SU(3) symmetry is broken. The violation can be parametrized through

$$\delta_J = \frac{g_J^\Xi + g_J^N - g_J^\Sigma}{g_J^\Xi + g_J^N + g_J^\Sigma} \neq \frac{2F_J - 2F_J + D_J - D_J}{4F_J}.$$

Flavour symmetry breaking



SU(3) symmetry breaking is only significant for g_A and can be attributed to the nucleon.

Summary

- Hadronic decays difficult (not covered). In principle possible for hyperons (similar to $K \to \pi\pi$) but a major effort.
- Data for light baryon to light baryon matrix elements with arbitrary current exist but need to be analysed.
- Semileptonic decay matrix elements for charmed and bottom baryons have been computed. Should be studied more systematically wrt continuum limit, quark mass and volume dependence.
- In the absence of direct form factor calculations $\exists$ moments of octet baryon light cone distribution amplitudes [RQCD: GB et al,1903.12590]. Particularly interesting for decays of b baryons but have also been used for $\Lambda_c \to \Lambda\gamma$ [Yu-Yi Shi et al,2212.01111].