QCD equation of state at finite density with a critical point from an alternative expansion scheme

Micheal KAHANGIRWE

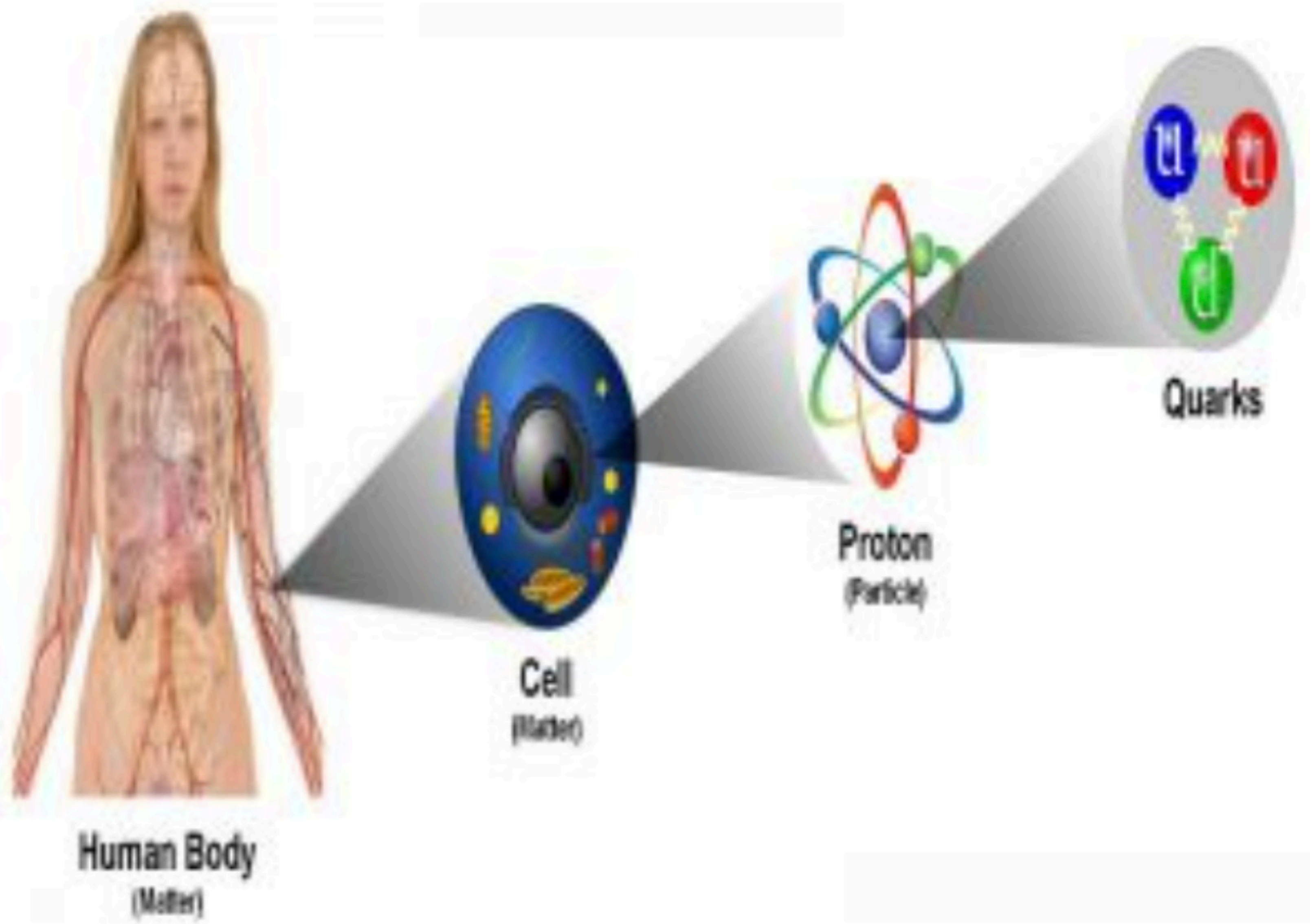
UNIVERSITY of HOUSTON

Collaborators: Claudia Ratti, Elena Bratkovskaya, Joerg Aichelin, Misha Stephanov, Pierre Moreau, Olga Soloveva, Steffen A. Bass

NuSTEAM Nuclear Science in Texas to Enhance and Advance Minorities

muses

NP3M



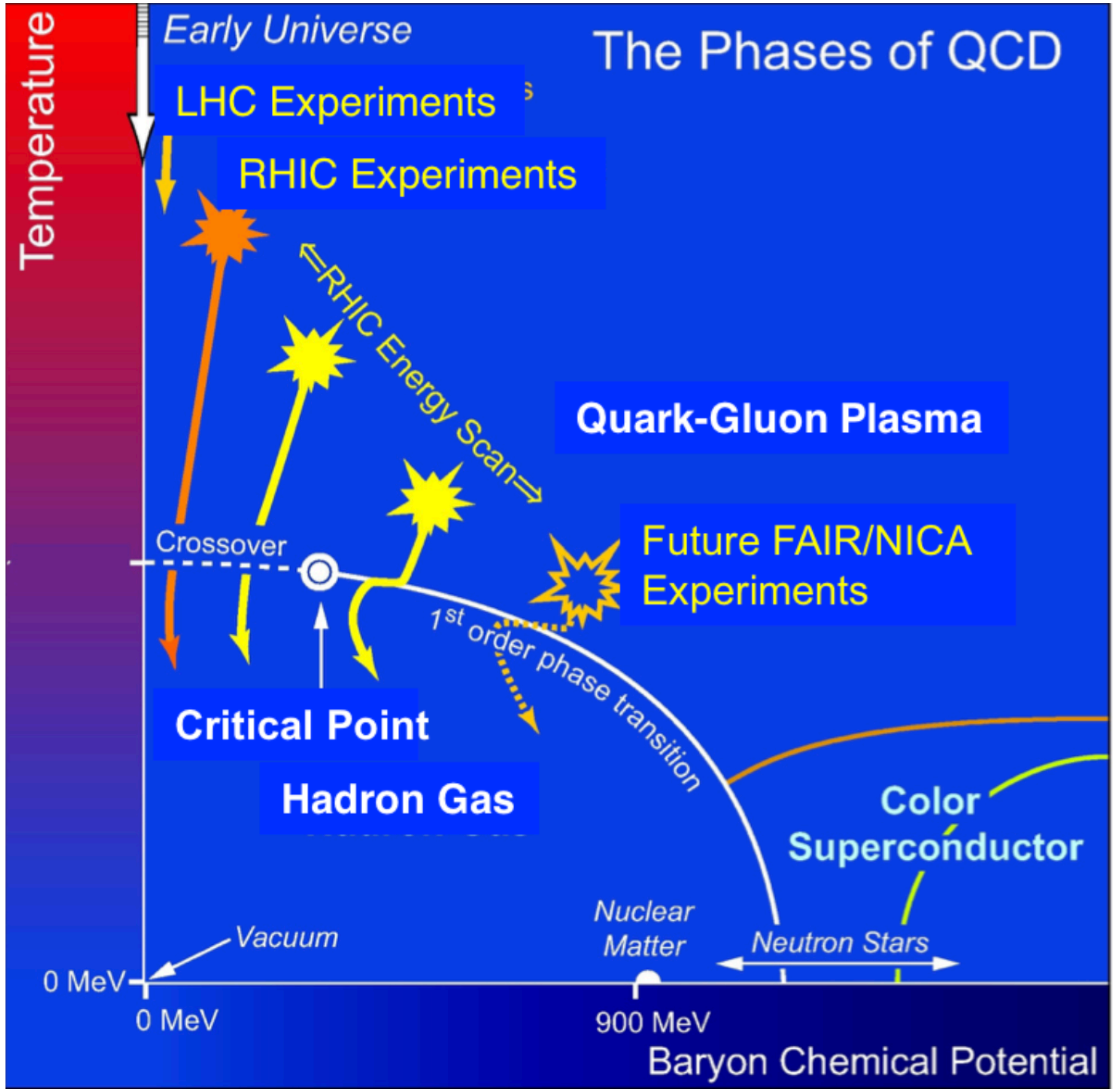
Physics Research Day

02/18/2023

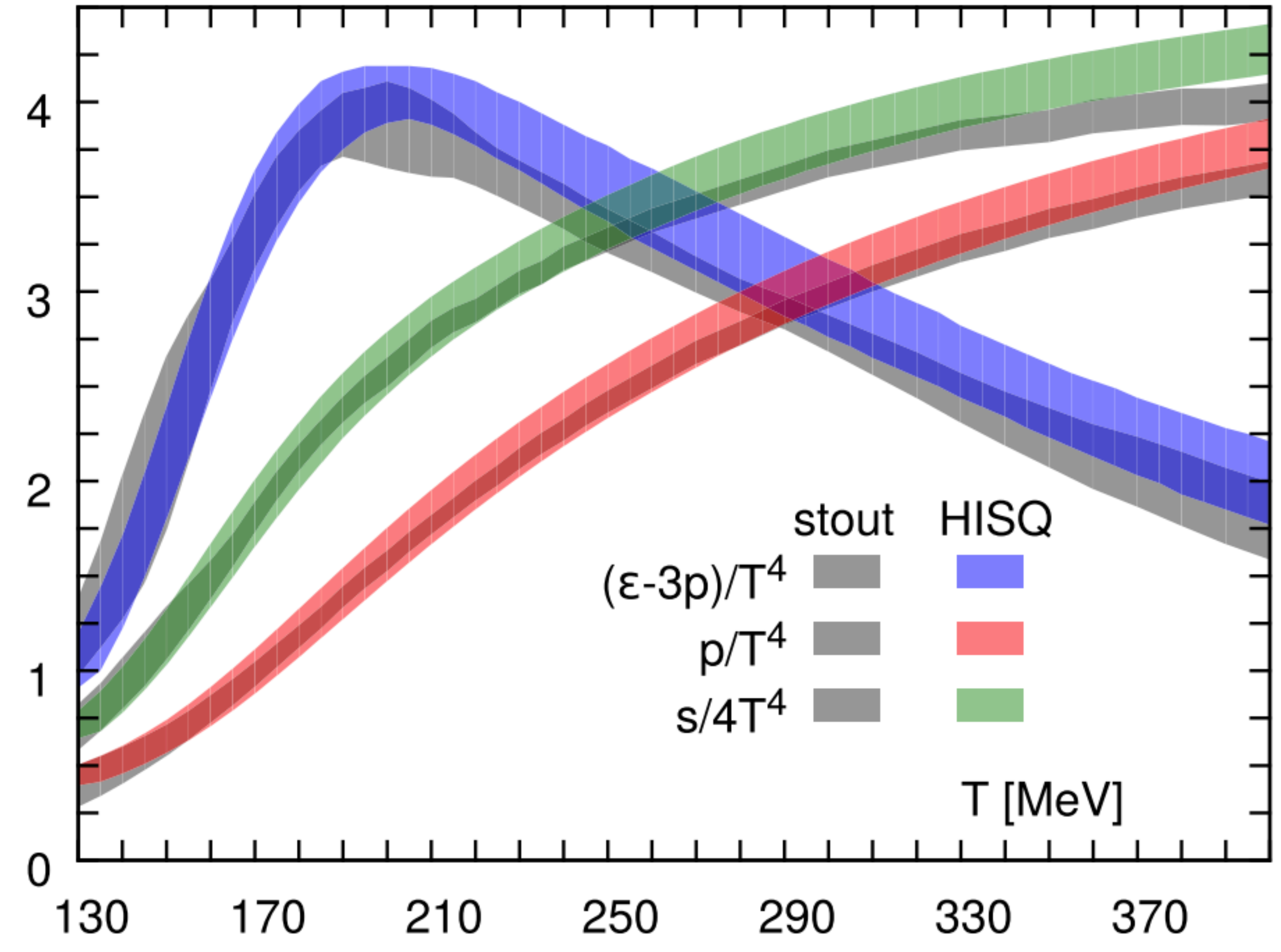
Introduction

Strongly interacting matter (QCD)

QCD Phase Diagram



Lattice EoS $\mu_B = 0$ MeV



[Wuppertal-Budapest (stout) and HotQCD (HISQ) collaborations]

- $\mu_B \neq 0$ **MeV** **Fermi Sign Problem!**

Way out!

- **Simulating Imaginary $\mu_B \to$ Analytical Continuation**
- **Expansion around $\mu_B = 0$**

Part 1: Lattice EoS: Taylor Expansion

Lattice EoS at Finite Density

Taylor Expansion around $\mu_B = 0$

Most Straightforward

$$\frac{P(T,\mu_B)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n(T,\mu_B = 0) \left(\frac{\mu_B}{T}\right)^n$$

$$c_n(T) = \frac{\chi_n^B(T,\mu_B = 0)}{n!} = \frac{1}{n!} \left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)\Big|_{\mu_B=0}$$

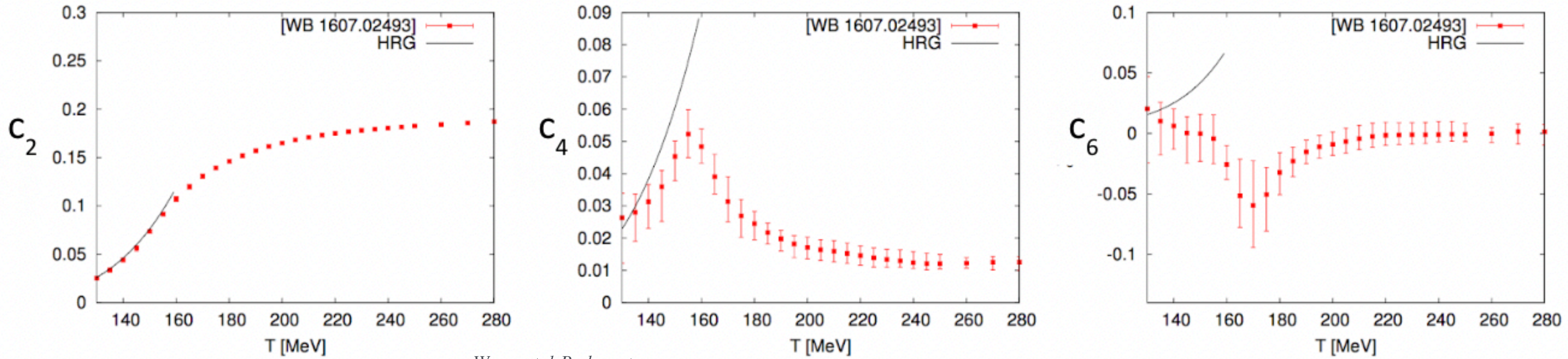
Limitations

- **Currently limited to $\frac{\mu_B}{T} \leq 2.5$ despite great Computational Power**
- **Adding one more Higher-Order term does not help in convergence**
- **Taylor expansion is carried out at T= constant and doesn't cope well with μ_B-dependent transition temperature**

[Borsányi, S. et al. PRL (2021)]

- **Re-summation with Pade' Approximation can reach $\frac{\mu_B}{T} = 3.0$**

D. Bollweg et al., arXiv:2212.09043v1 [hep-lat] (2022)



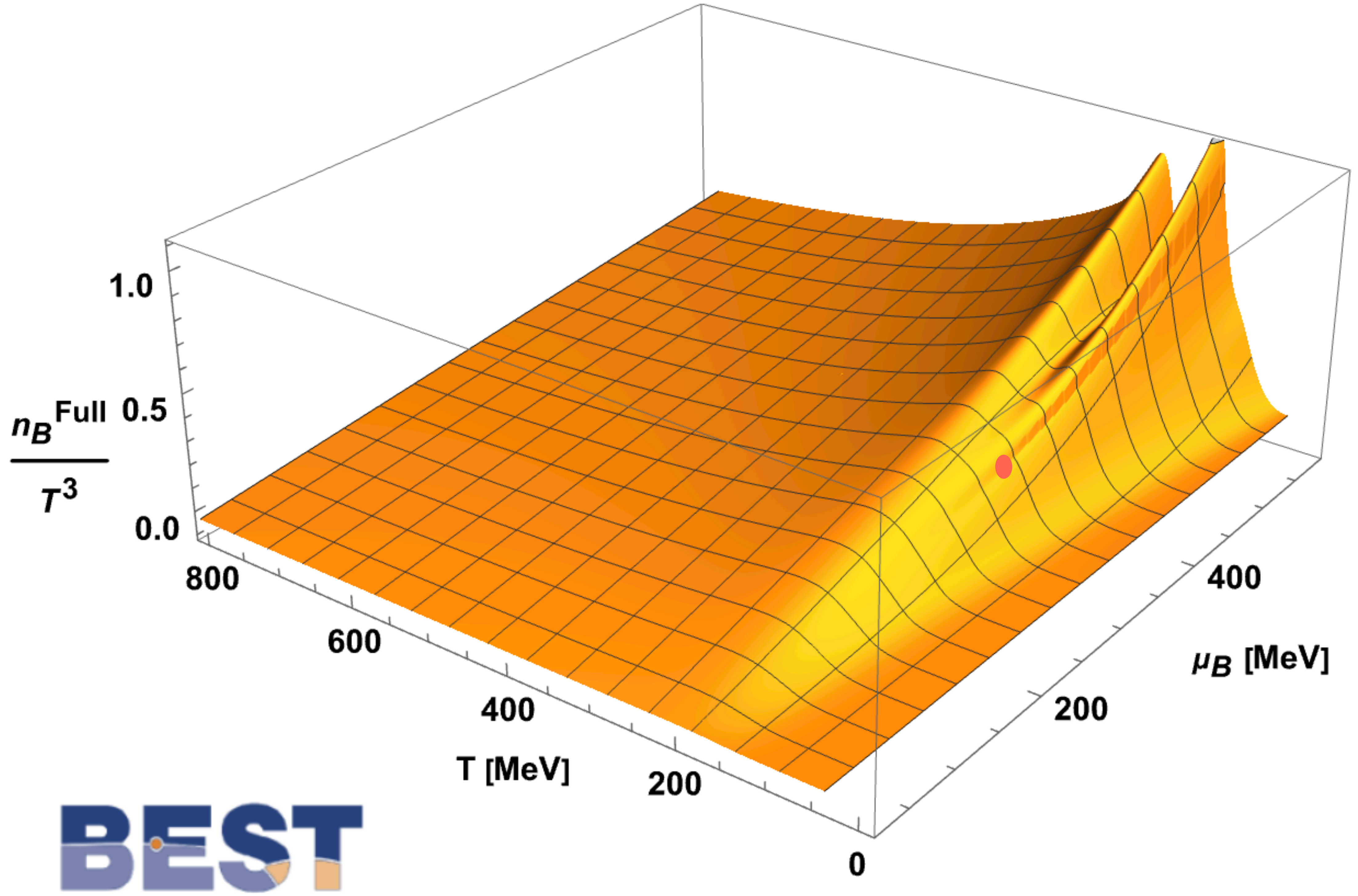
Wuppertal-Budapest

Taylor EoS with Critical point

$$P(T,\mu_B) = T^4 \sum_{n=0}^{2} \frac{1}{(2n)!} \chi_{2n}^{Non-Ising}(T) \left(\frac{\mu_B}{T}\right)^{2n} + T_C^4 P_{symm}^{Ising}(T,\mu_B)$$

Baryon density $$\frac{n_B(T,\mu_B)}{T^3} = \frac{\partial(P/T^4)}{\partial(\mu_B/T)}$$

$$\chi_n^{Lat}(T) = \chi_n^{Non-Ising}(T) + \frac{T_C^4}{T^4}\chi_n^{Ising}(T)$$



Critical Point at $\mu_{BC} = 350\ [MeV]$

- **Thermodynamic Observables at** $\mu_B \geq 450\ MeV$ **show unphysical behavior**

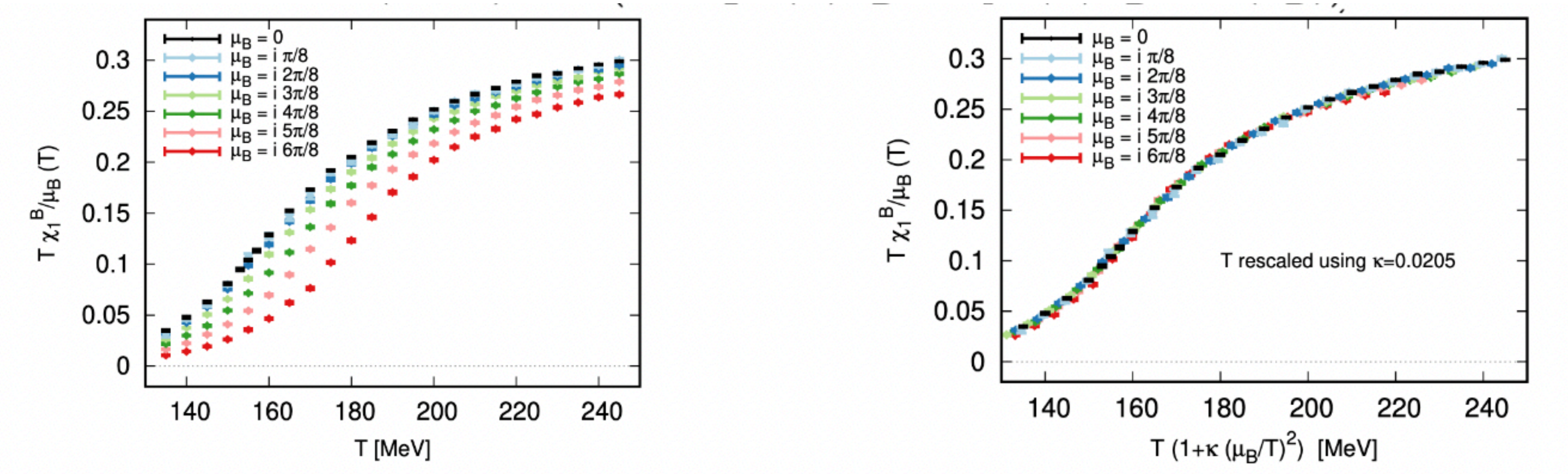
[Parotto, Paolo, et al. Physical Review C 101.3 (2020): 034901.]

BEST COLLABORATION

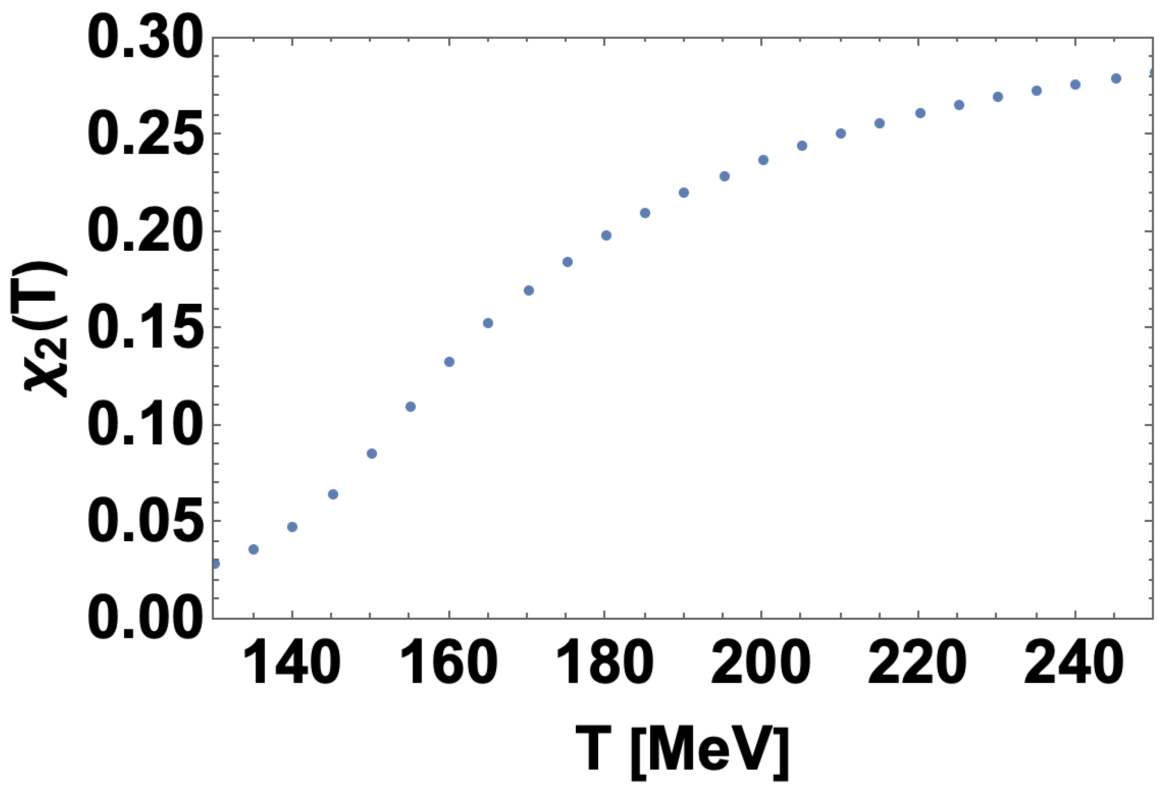
Part 2: Lattice EoS: Alternative Expansion Scheme

Alternative Expansion Scheme: EoS

Simulating at Imaginary μ_B



$\sim$



[Borsányi, S. et al. PRL (2021)]

$$T\frac{\chi_1^B(T,\mu_B)}{\mu_B} = \chi_2^B(T',0)$$

$$\chi_n^B(T,\mu_B) = \frac{1}{n!}\left(\frac{\partial}{\partial(\mu_B/T)}\right)^n (P/T^4)$$

$$T'[T,\mu_B] = T\left[1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right]$$

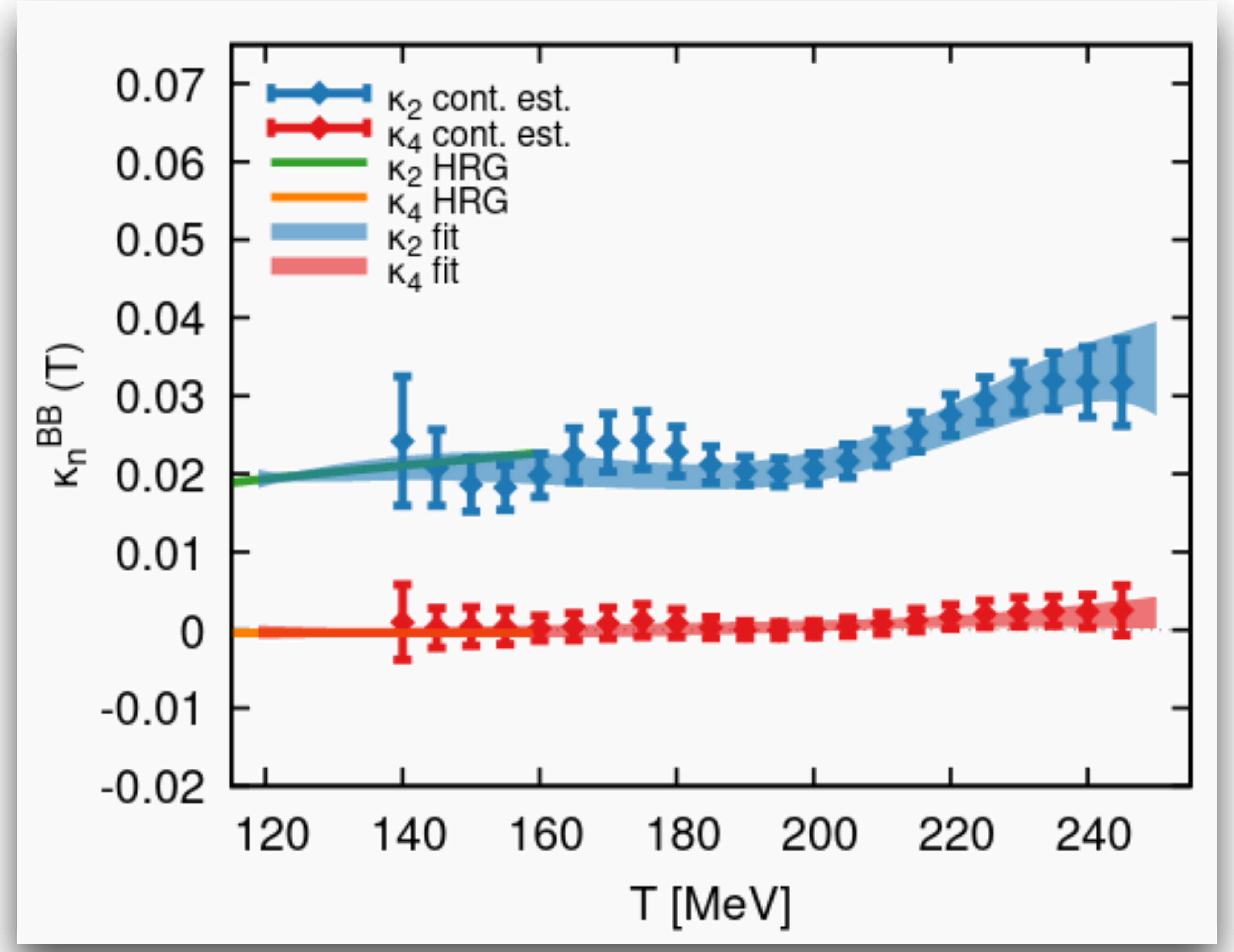
- **μ_B dependence is captured in T-rescaling.**
- **Trusted up to** $\frac{\mu_B}{T} = 3.5$

[Borsányi, S. et al. PRL (2021)]

Alternative expansion scheme

Comparing Taylor expansion and Alternative expansion

- $$\kappa_2^{BB}(T) = \frac{1}{6T}\frac{\chi_4^B(T)}{\left(\frac{\partial \chi_2^B(T)}{\partial T}\right)}$$
- $$\kappa_4^{BB}(T) = \frac{1}{360\chi_2^B(T)^3}\left(3{\chi'}_2^{B2}\chi_6^B(T) - 5\chi_2^B(T)''\chi_4^B(T)^2\right)$$



[Borsányi, S. et al. PRL (2021)]

Pros

- $\kappa_2(T)$ **is fairly constant over a large T-Range**
- **There is a separation of scale between** $\kappa_2(T)$ **and** $\kappa_4(T)$
- $\kappa_4(T)$ **is almost zero** $\rightarrow$ **faster convergence**
- **A good agreement with HRG results at Low Temperature**

Part 3: Putting Critical Point into Alternative Expansion: EoS

EoS with a Critical point

Strategy

Scale 3D-Ising model EoS → Map Ising to QCD → Merging Ising with Lattice → Estimate the critical Contribution

Scale 3D-Ising Model EoS

Close to the critical point, we define a parametrization for Magnetization M, Magnetic field h, and reduced temperature

QCD Critical point is in the 3D-Ising model Universality class

$$M = M_0 R^{\beta} \theta$$

$$h = h_0 R^{\beta\delta}(\theta + a\theta^3 + b\theta^5)$$

$$r = R(1 - \theta^2)$$

$$(R \geq 0, \ |\theta| \leq \theta_0)$$

$$(R, \theta) \longmapsto (r, h)$$

$$\delta \sim 4.8$$

$$\beta \sim 0.326$$

$$r = \frac{T - T_C}{T_C}$$

$h \rightarrow$ **External magnetic field**

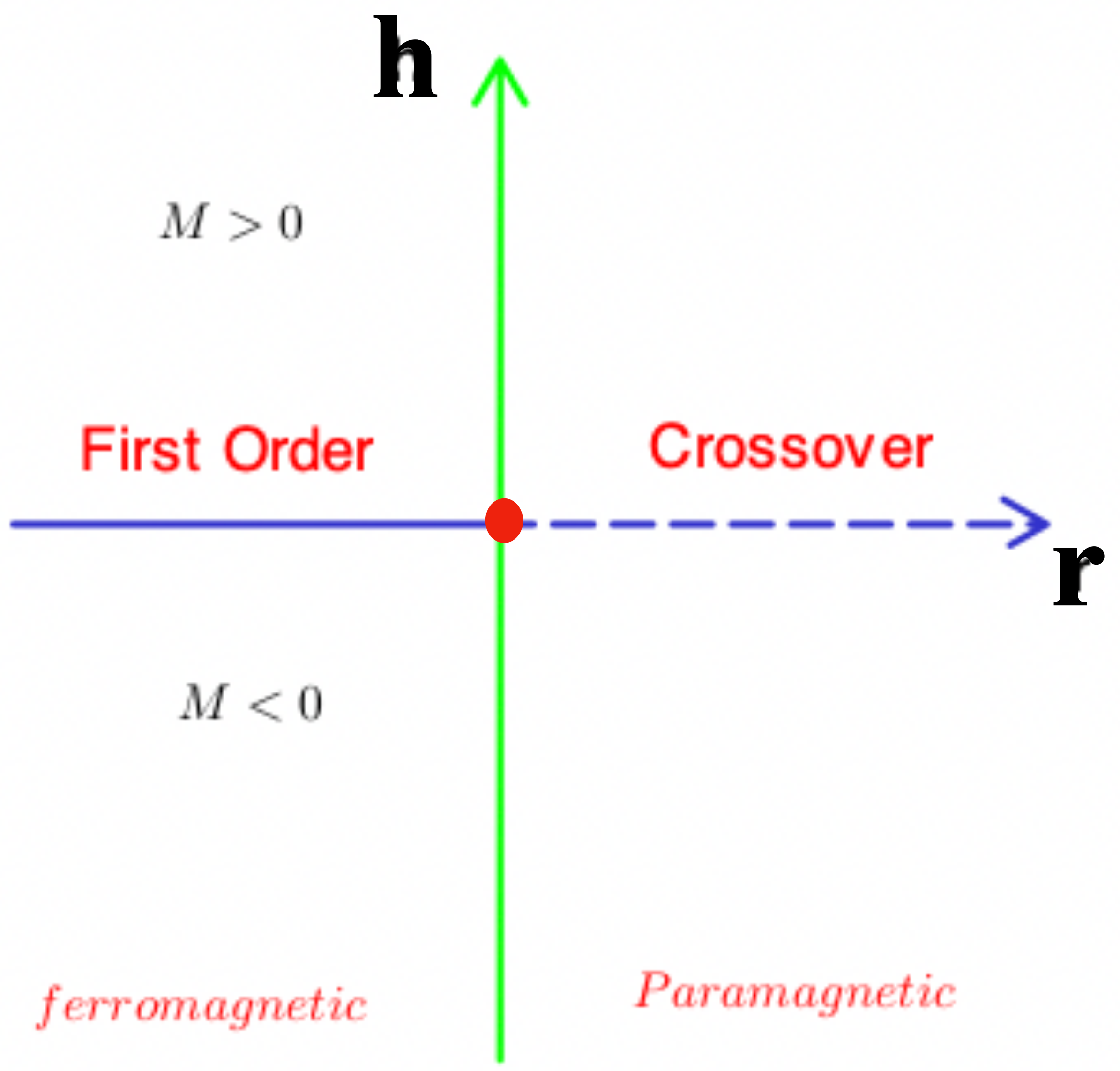
[Parotto et al PhysRevC.101.034901(2020)]

[Nonaka et al Physical Review C, 71(4), 044904.(2005)]

[Guida et al Nuclear Physics B, 489(3), 626-652.(1997)]

EoS with a Critical point

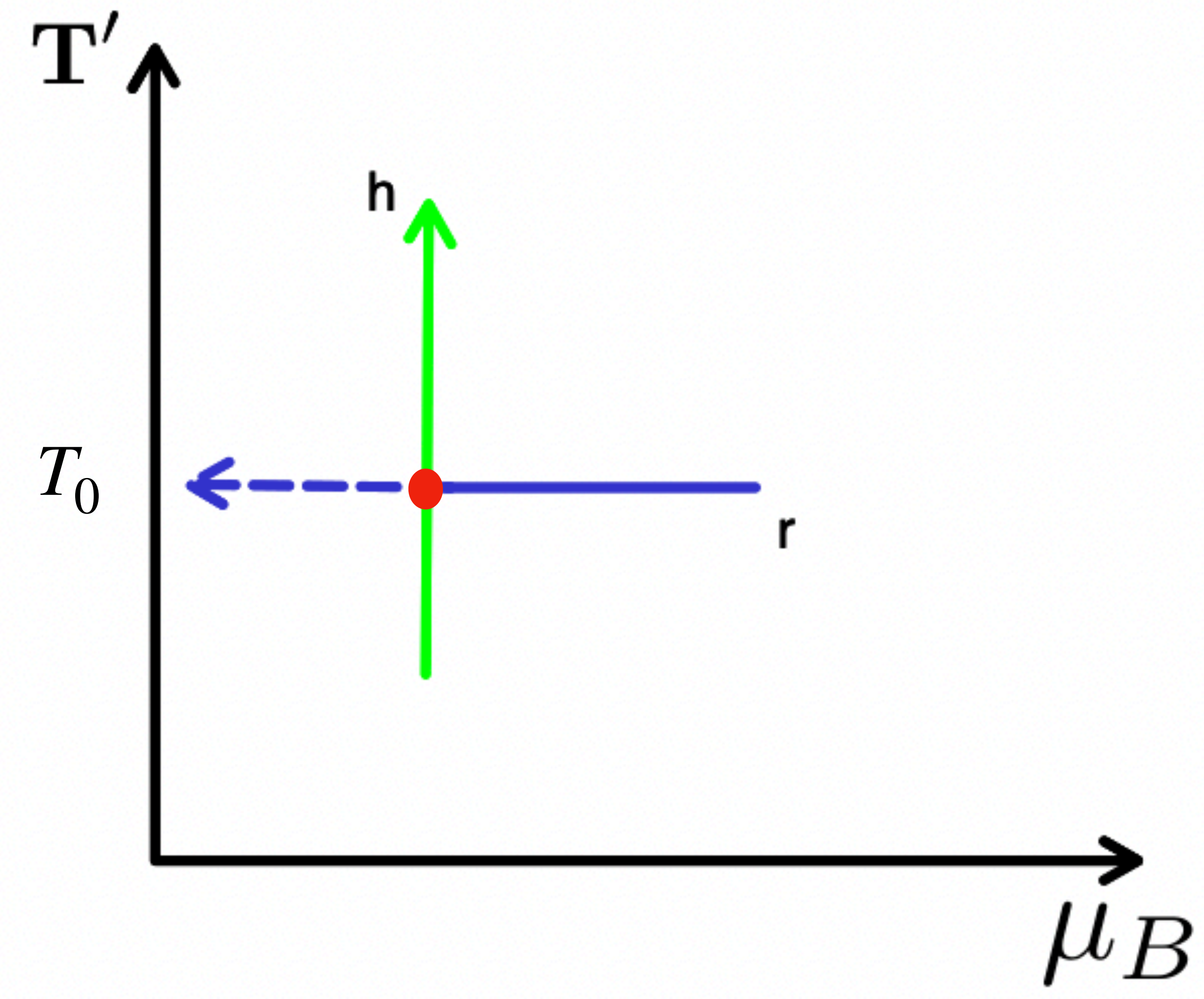
3D Ising to QCD Mapping



3D-Ising Co-ordinates

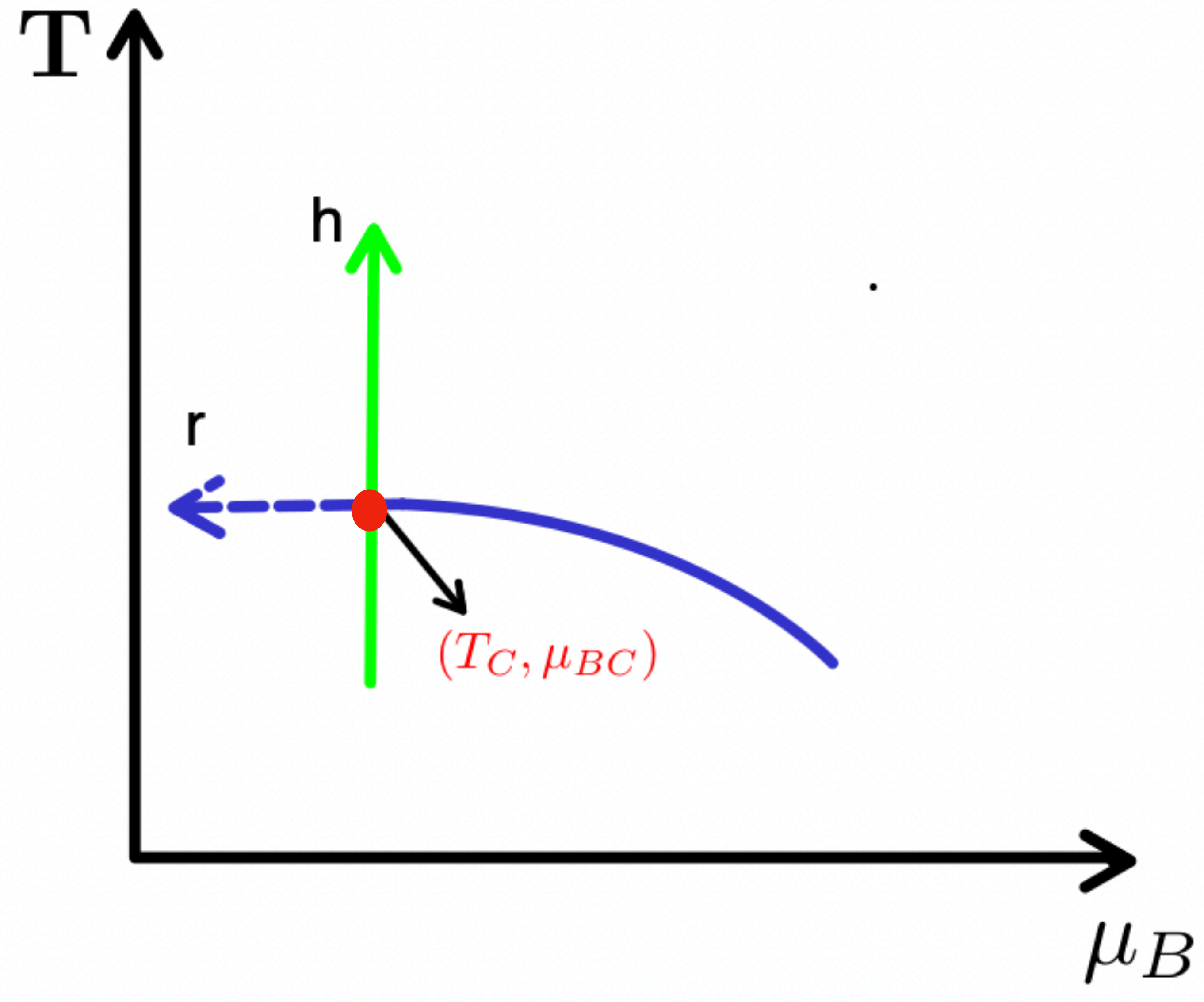
$$\frac{T'-T_0}{T_0} = w\; h \sin\, \alpha_{12}$$

$$\frac{\mu_B-\mu_{BC}}{T_0} = w\,(-r\rho - h\cos\, \alpha_{12})$$



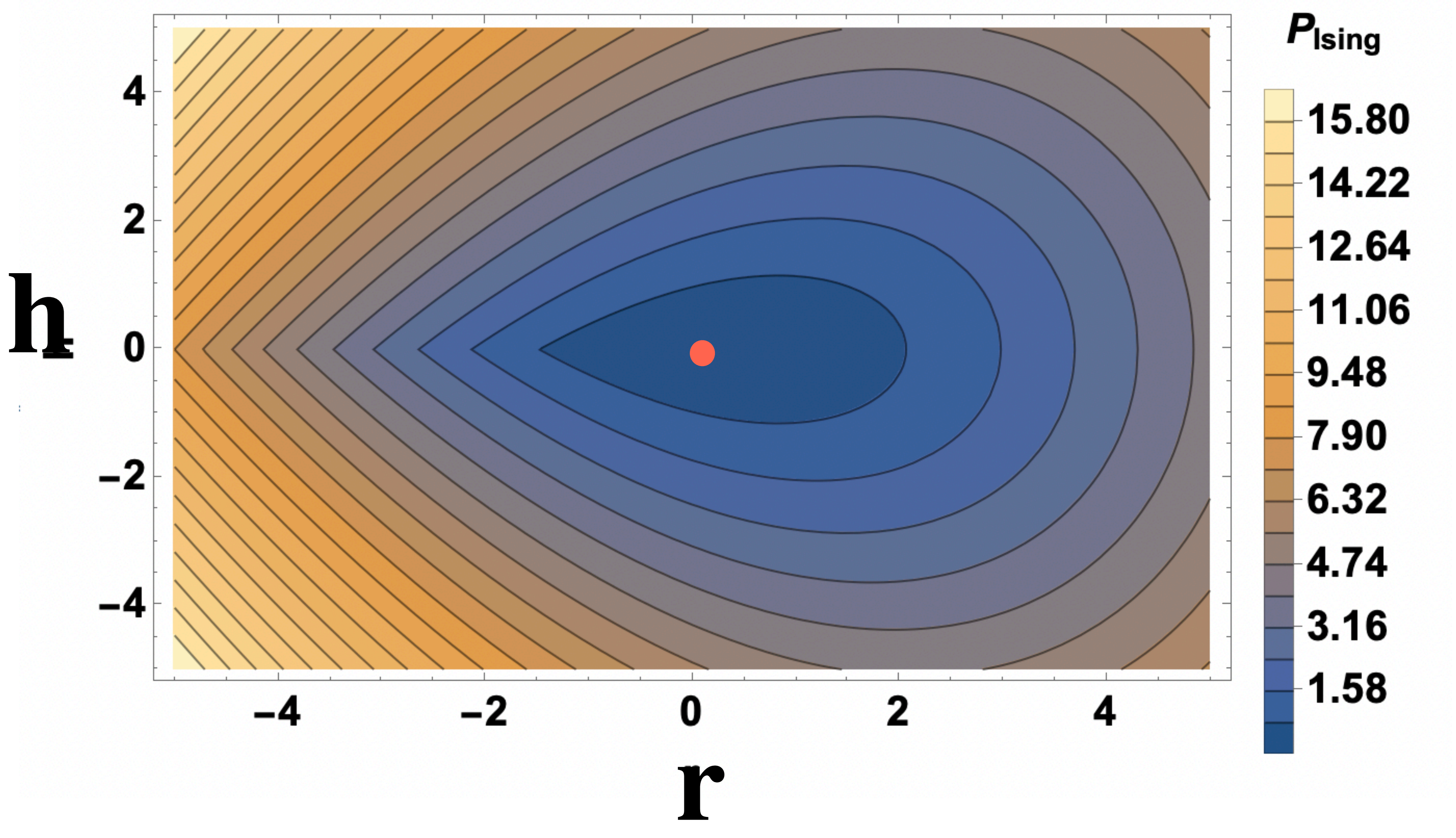
(T′, μ_B)-Co-ordinates

$$T' = T\left[1+\left(\frac{\mu_B}{T}\right)^2 \kappa_2^{BB}(T) + \mathcal{O}\left(\frac{\mu_B}{T}\right)^4\right]$$

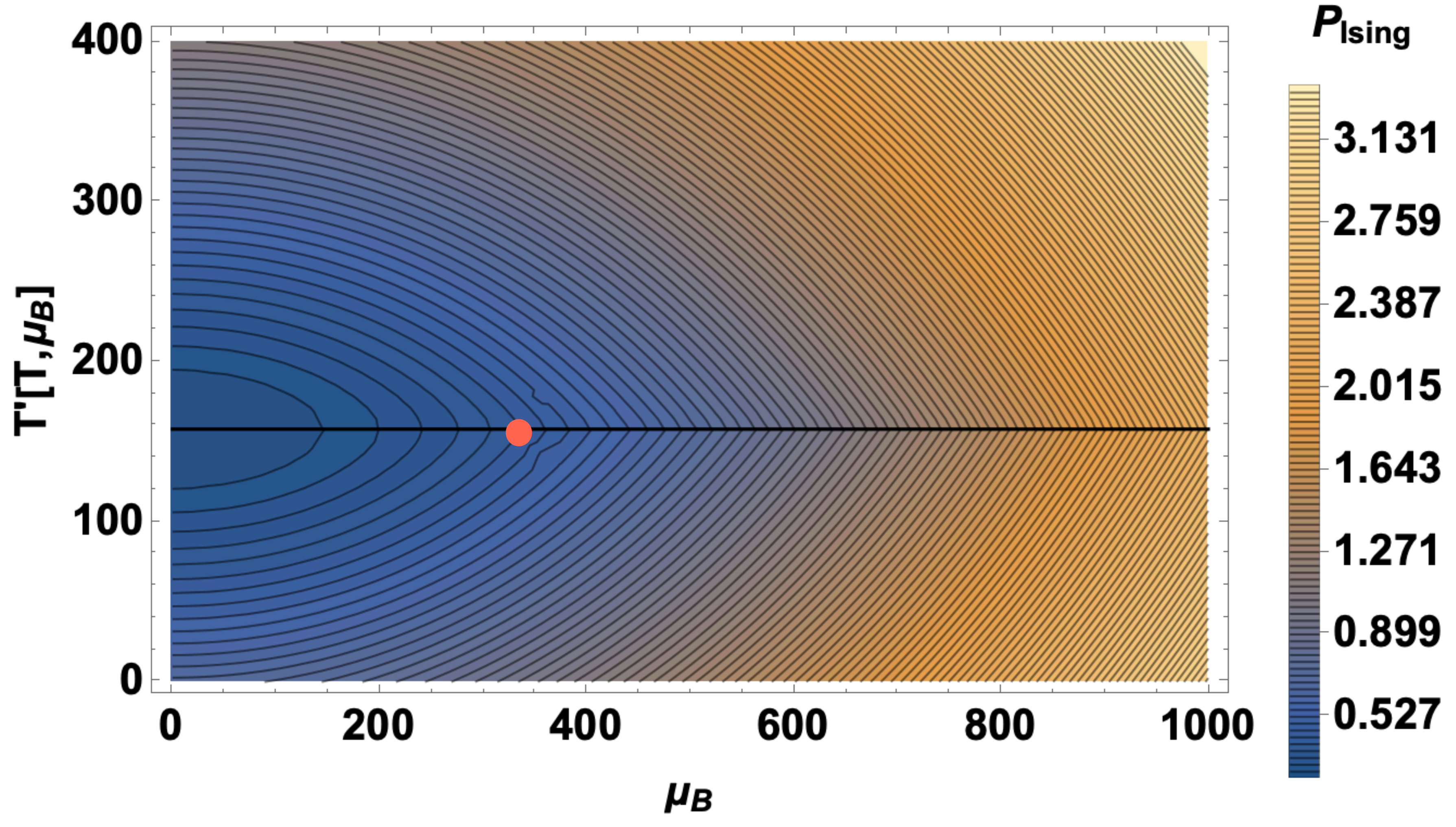


QCD - Co-ordinates

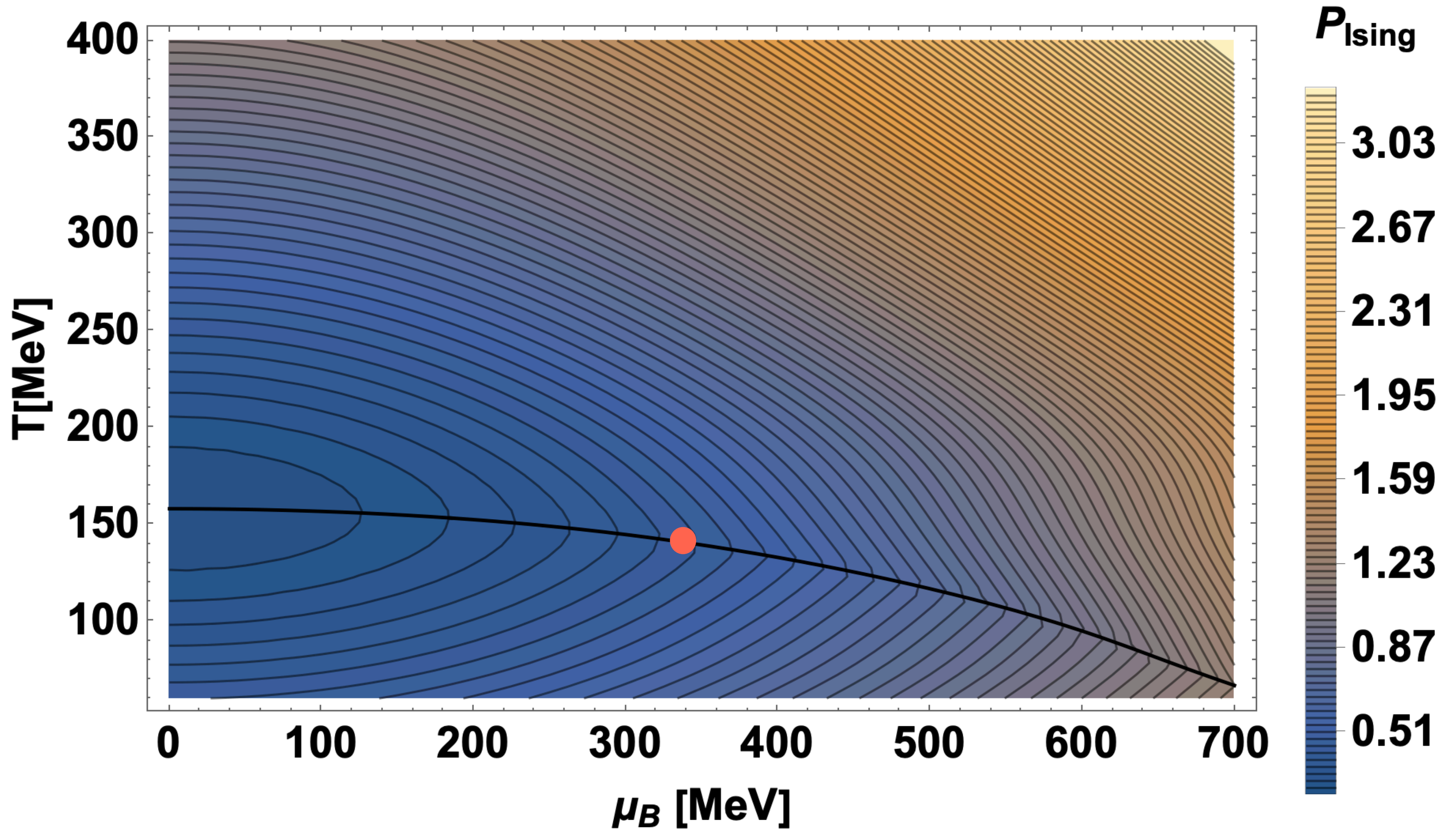
Ising Pressure



Ising Coordinates



(μ_B, T') **Coordinates**



QCD Coordinates

Parameters

$$w = 1,\ \rho = 2,\ \mu_{BC} = 350\ MeV,\ T_0 = 158\ [MeV]$$

$$T_C(\mu_B) = T_0\left[1 - \kappa_2(T_0)\left(\frac{\mu_B}{T_0}\right)^2\right]$$

Merging Ising with Lattice

Full Baryon Density

$$\frac{n_B^{full}(T,\mu_B)}{T^3} = \left(\frac{\mu_B}{T}\right)\chi^B_{2,Lattice}(T'_{full}(T,\mu_B),0)$$

$$T'_{full}(T,\mu_B) = \underbrace{T'_{Lattice}(T,\mu_B)}_{\text{lowest order in } \left(\frac{\mu_B}{T}\right)} + \underbrace{T'_{crit}(T,\mu_B) - Taylor[T'_{crit}(T,\mu_B)]}_{\text{higher orders in } \left(\frac{\mu_B}{T}\right)}$$

Lattice Term

Ising Term

Introducing a Critical Point

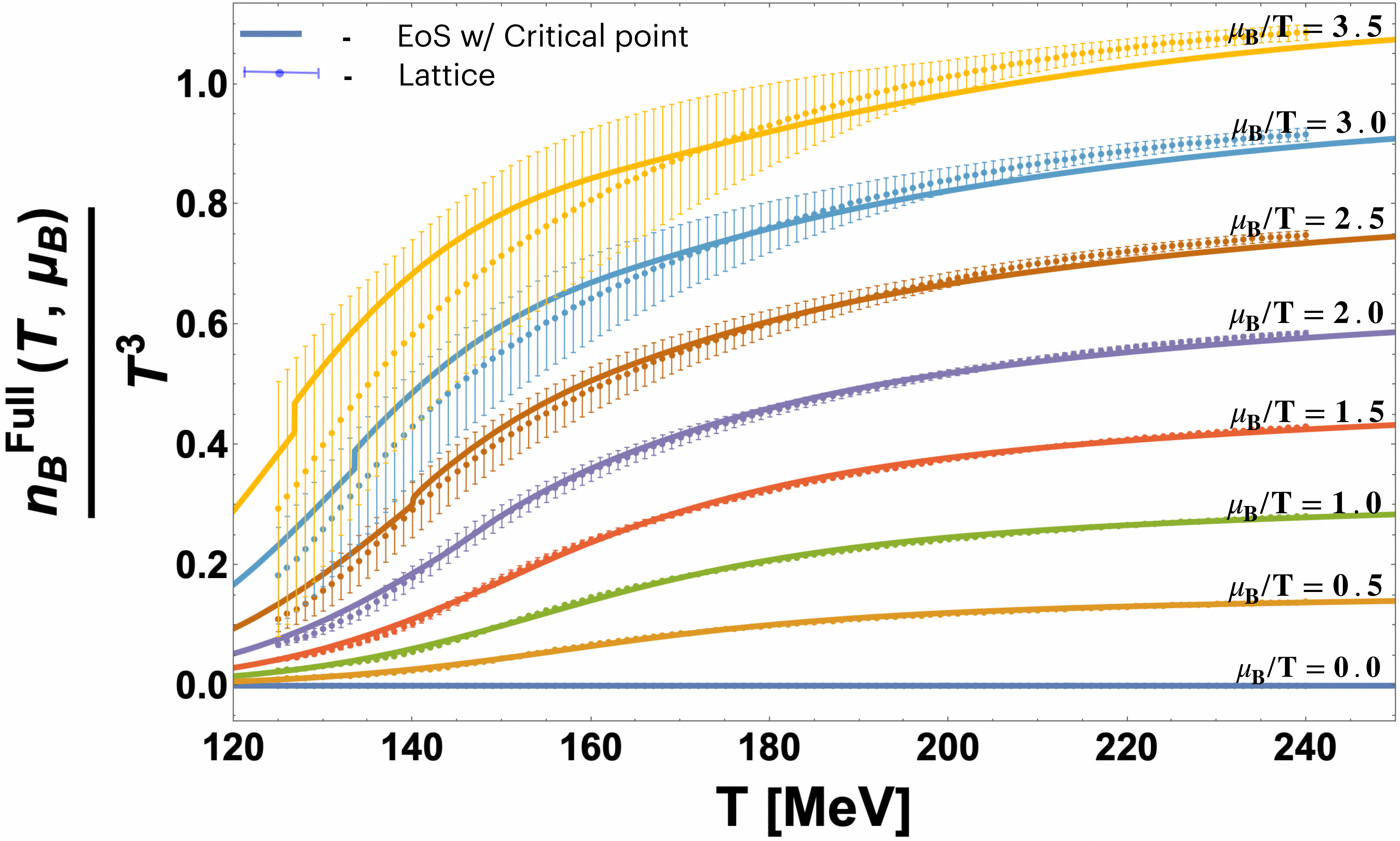
$$T'_{crit}(T,\mu_B) \approx T_0 + \left(\left.\frac{\partial\chi^B_{2,lattice}(T,0)}{\partial T}\right|_{T=T_0}\right)^{-1}\frac{n_B^{crit}(T,\mu_B)}{T^3(\mu_B/T)} + \ldots$$

$$\chi_1^{crit}(T,\mu_B) = \frac{n_B^{crit}(T,\mu_B)}{T^3} = \frac{\partial(P^{crit}(T,\mu_B)/T^4)}{\partial(\mu_B/T)}$$

Baryon density results

Full Baryon Density at a constant $\frac{\mu_B}{T}$ compared with Lattice

$\mu_B = 350$ [MeV], $\alpha_{12} = 90$, $\rho = 2$, w $= 2$



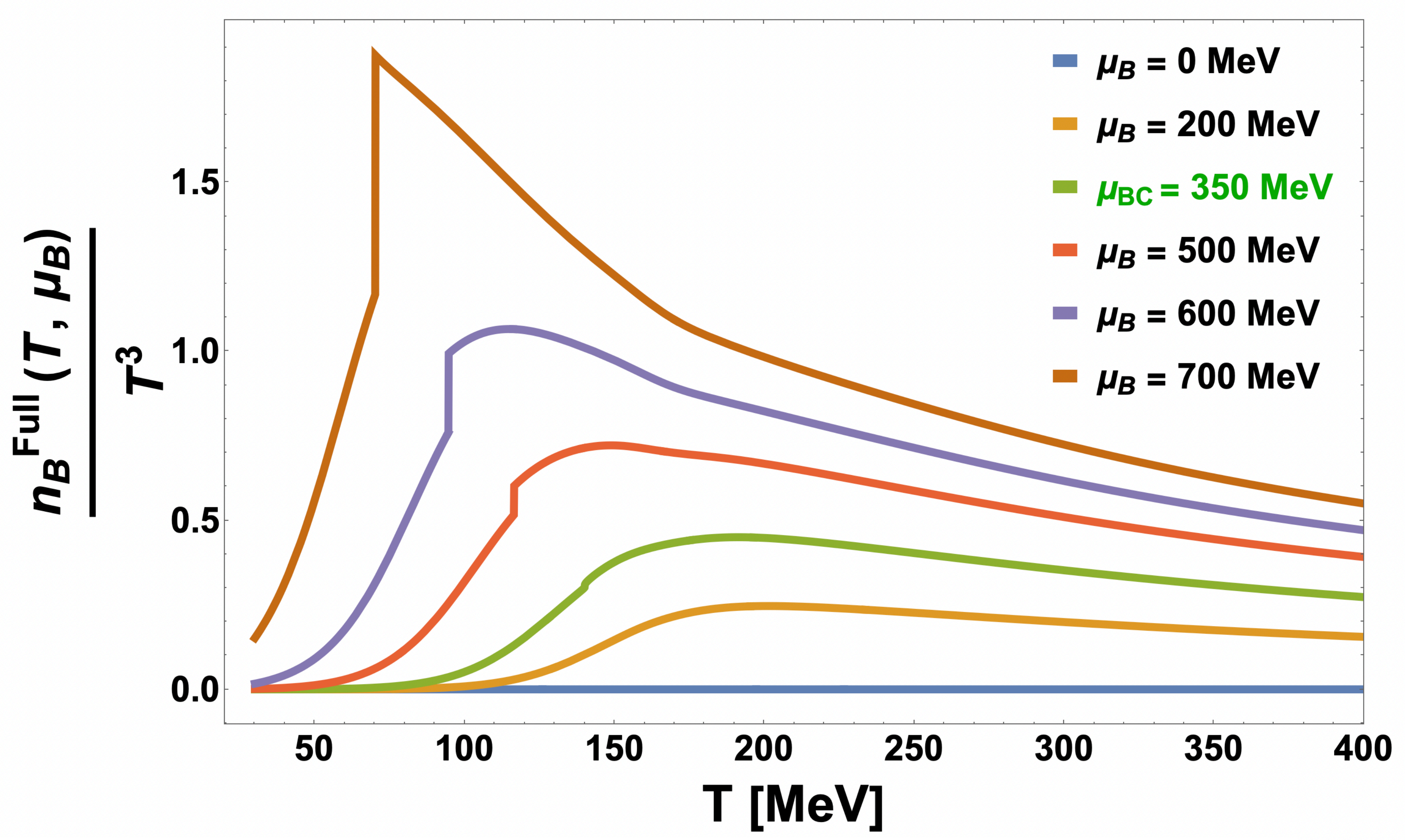
[Borsányi, S. et al. PRL (2021)]

Baryon density results

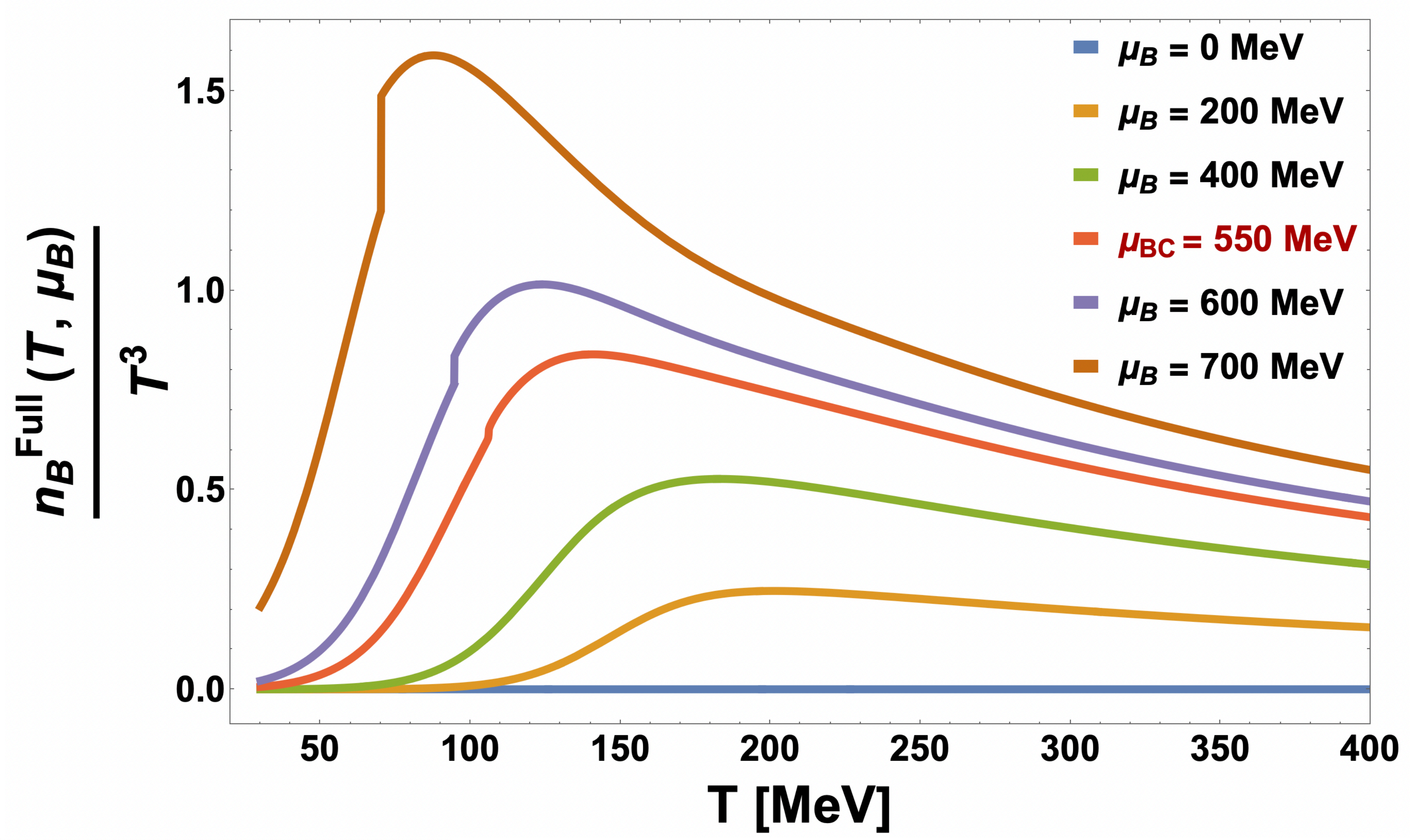
Baryon Density at a constant μ_B for different μ_{BC}

$\alpha_{12} = 90, \quad \rho = 2, \quad w = 2$

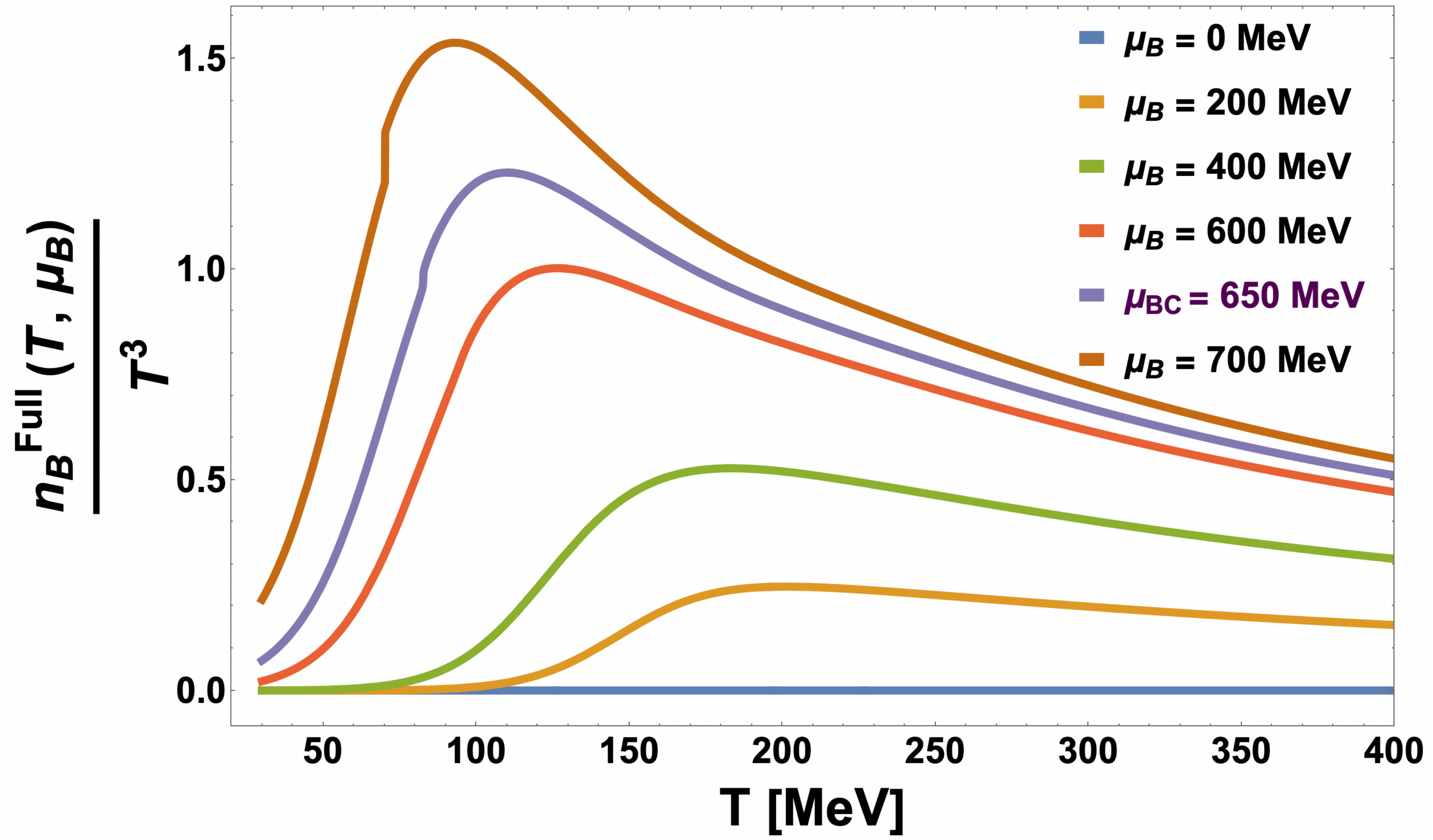
$\mu_{BC} = 350 \quad MeV$



$\mu_{BC} = 550 \quad MeV$



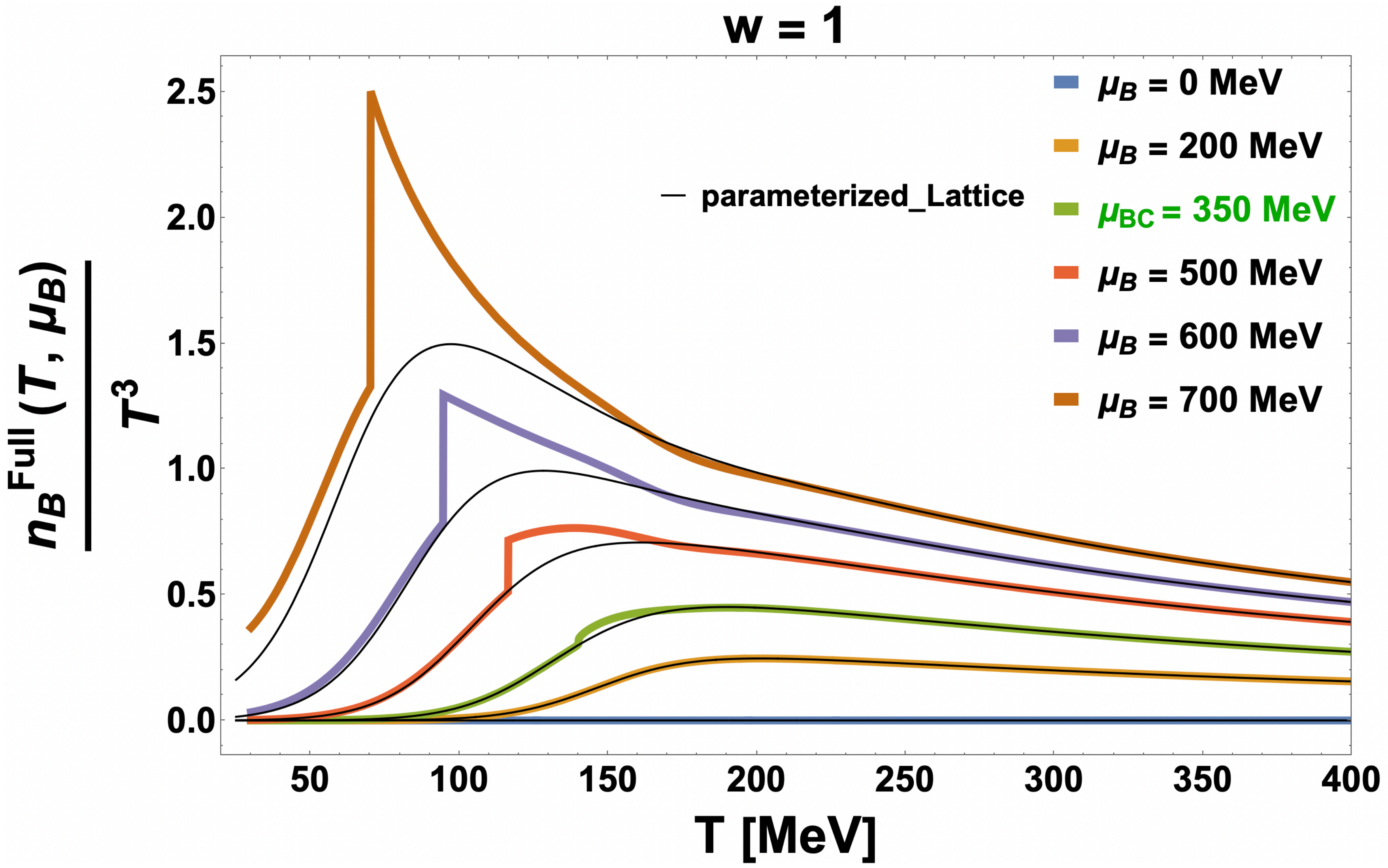
$\mu_{BC} = 650 \quad MeV$

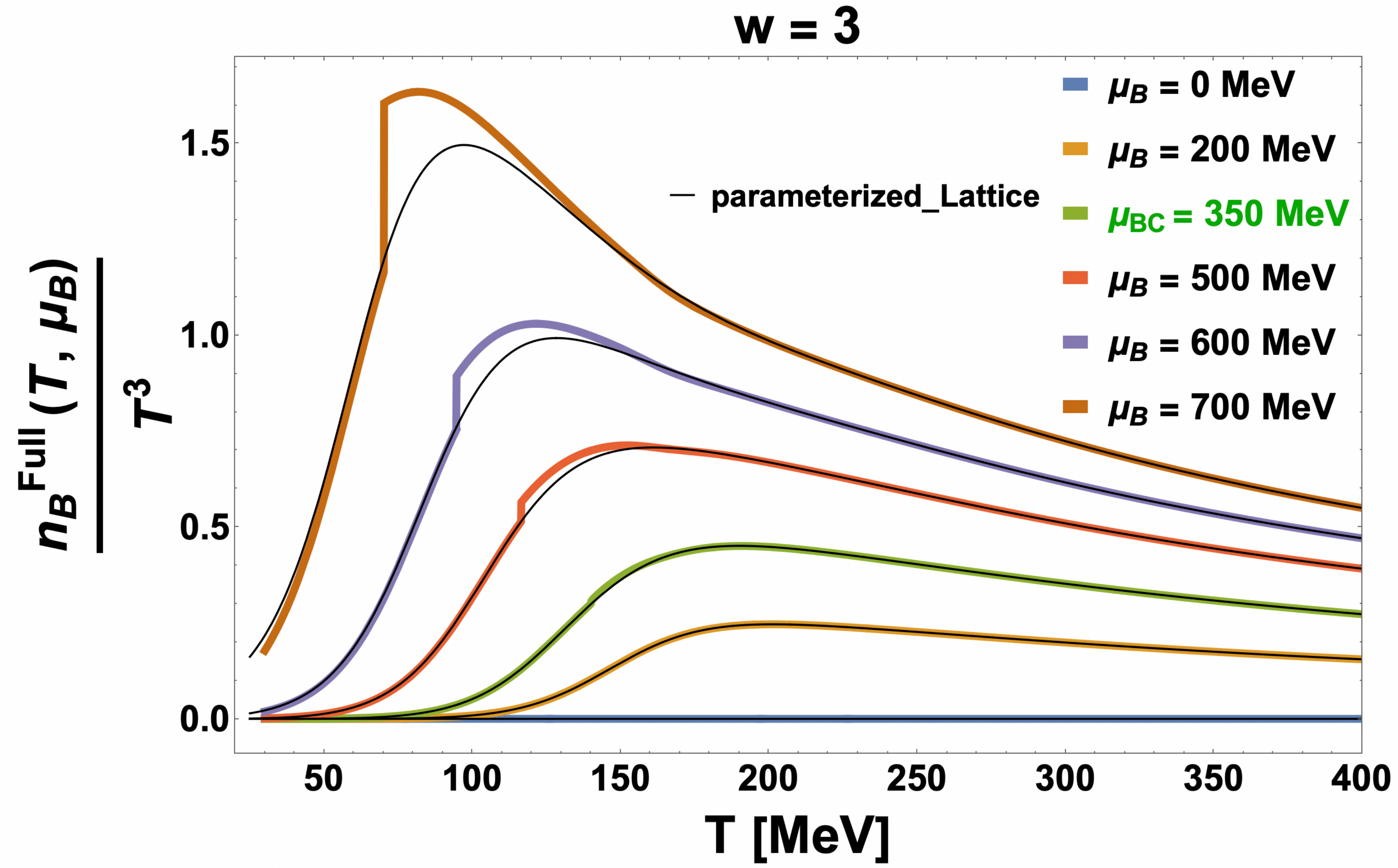


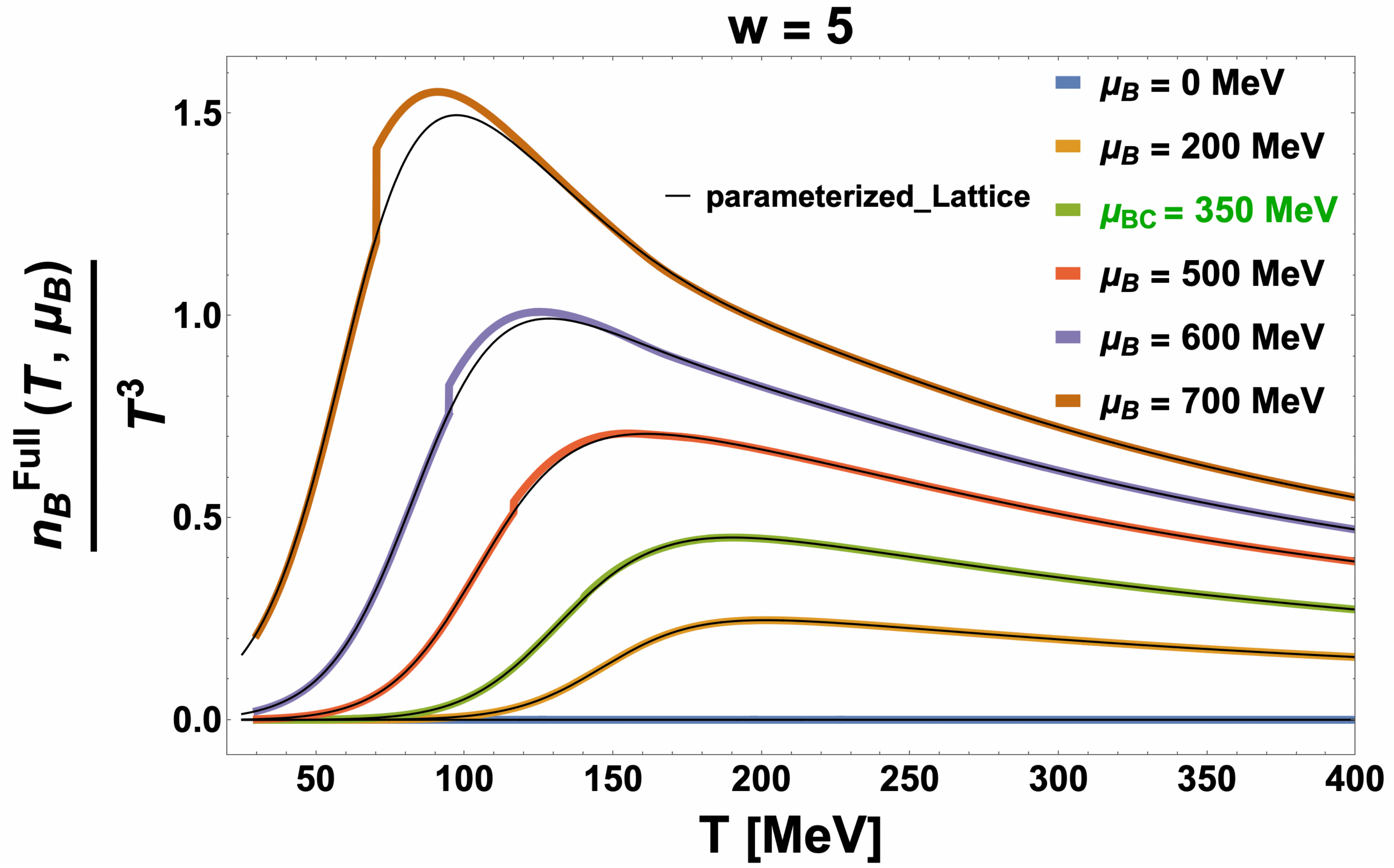
Estimating the Critical contribution

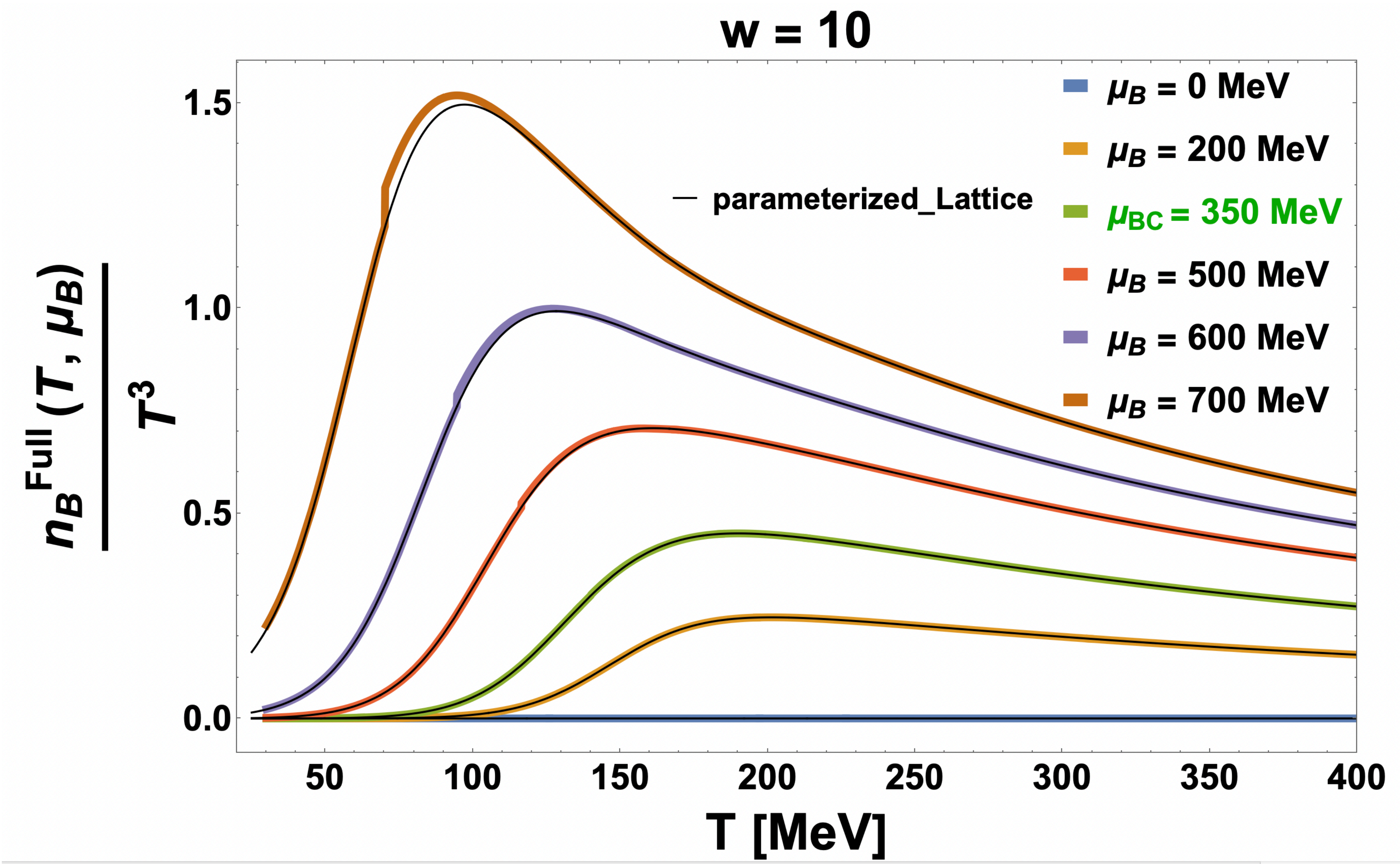
Baryon Density at a constant μ_B for 1 to w=10

$\mu_{BC} = 350\ [\text{MeV}], \quad \alpha_{12} = 90, \quad \rho = 2$



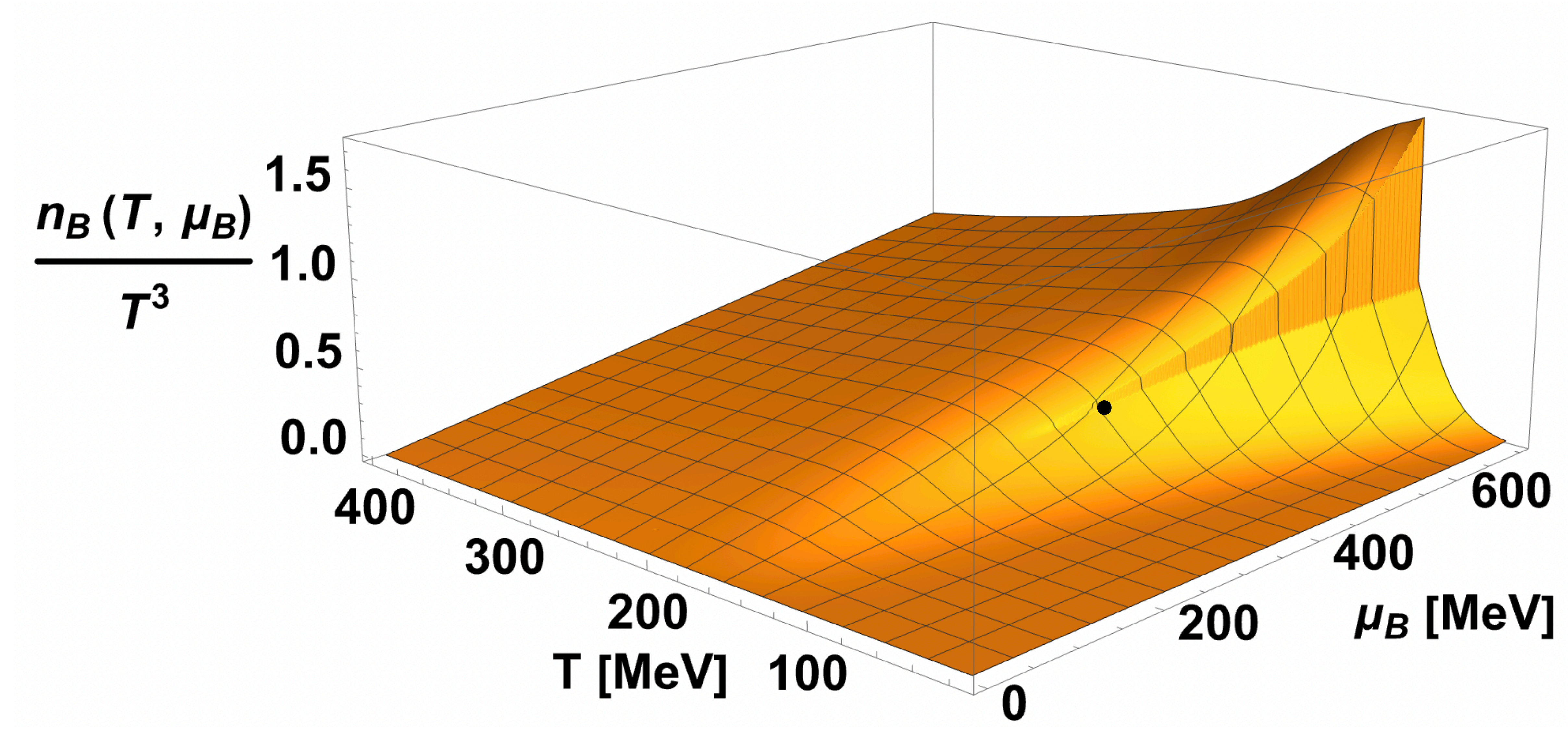




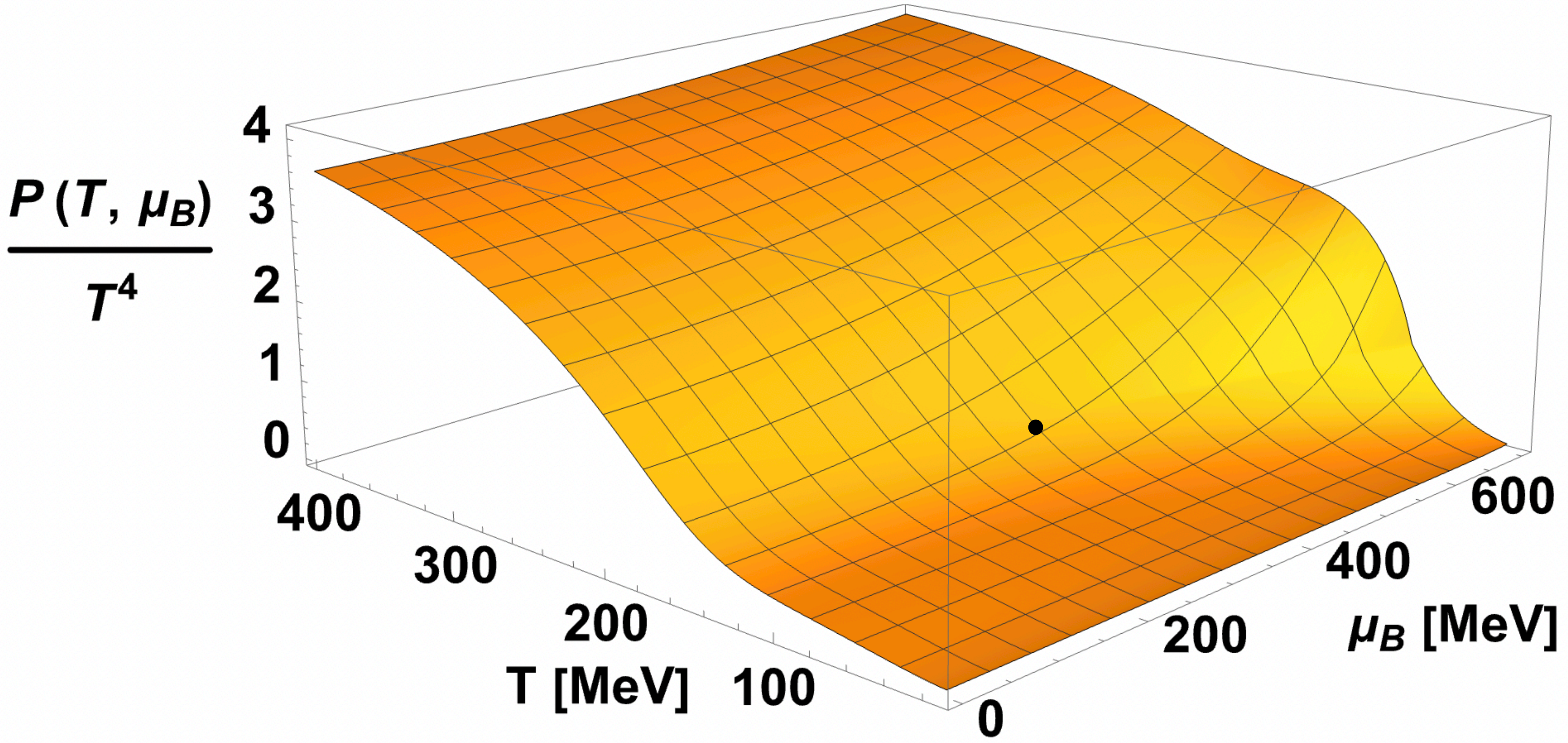


Thermodynamic observables

Baryon Density



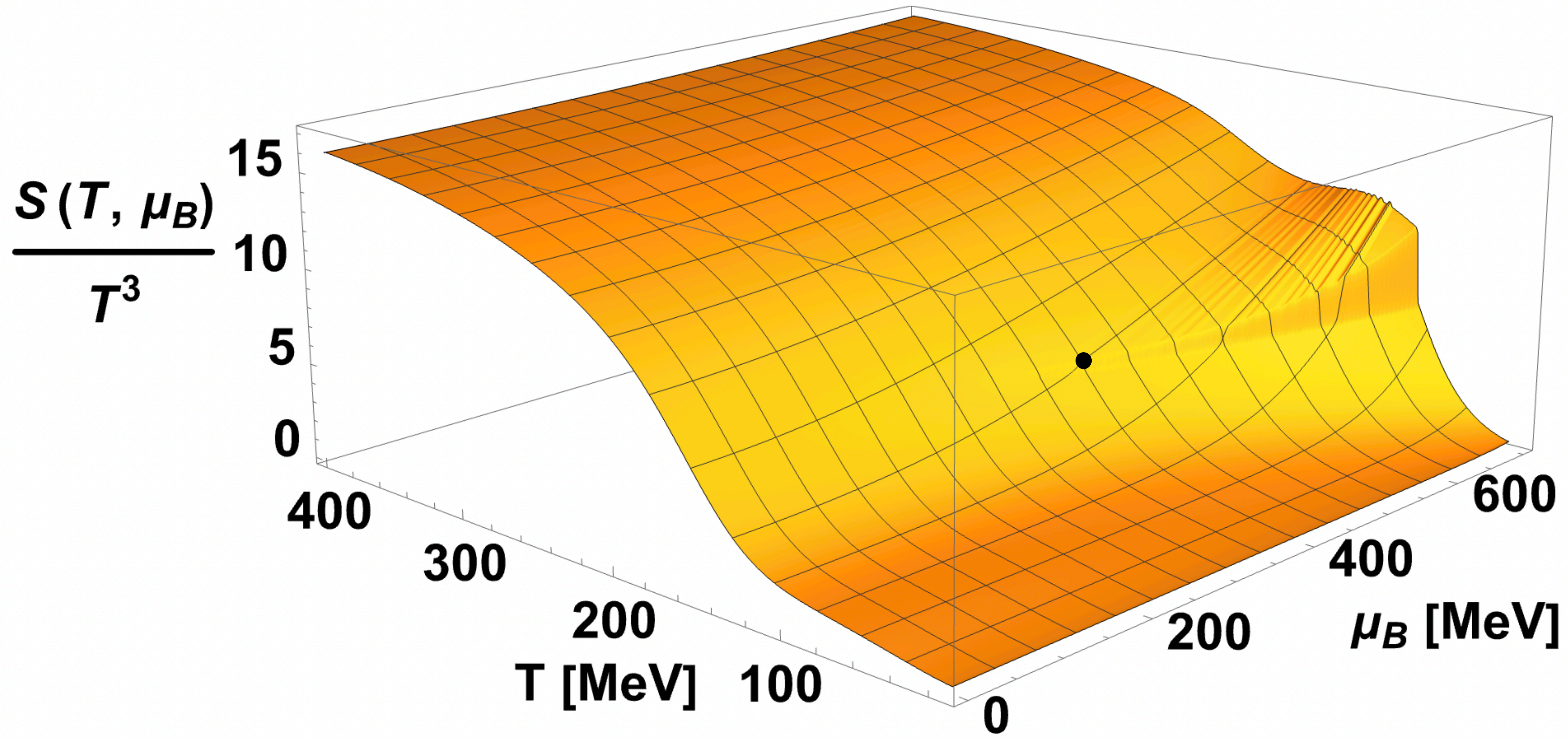
Pressure



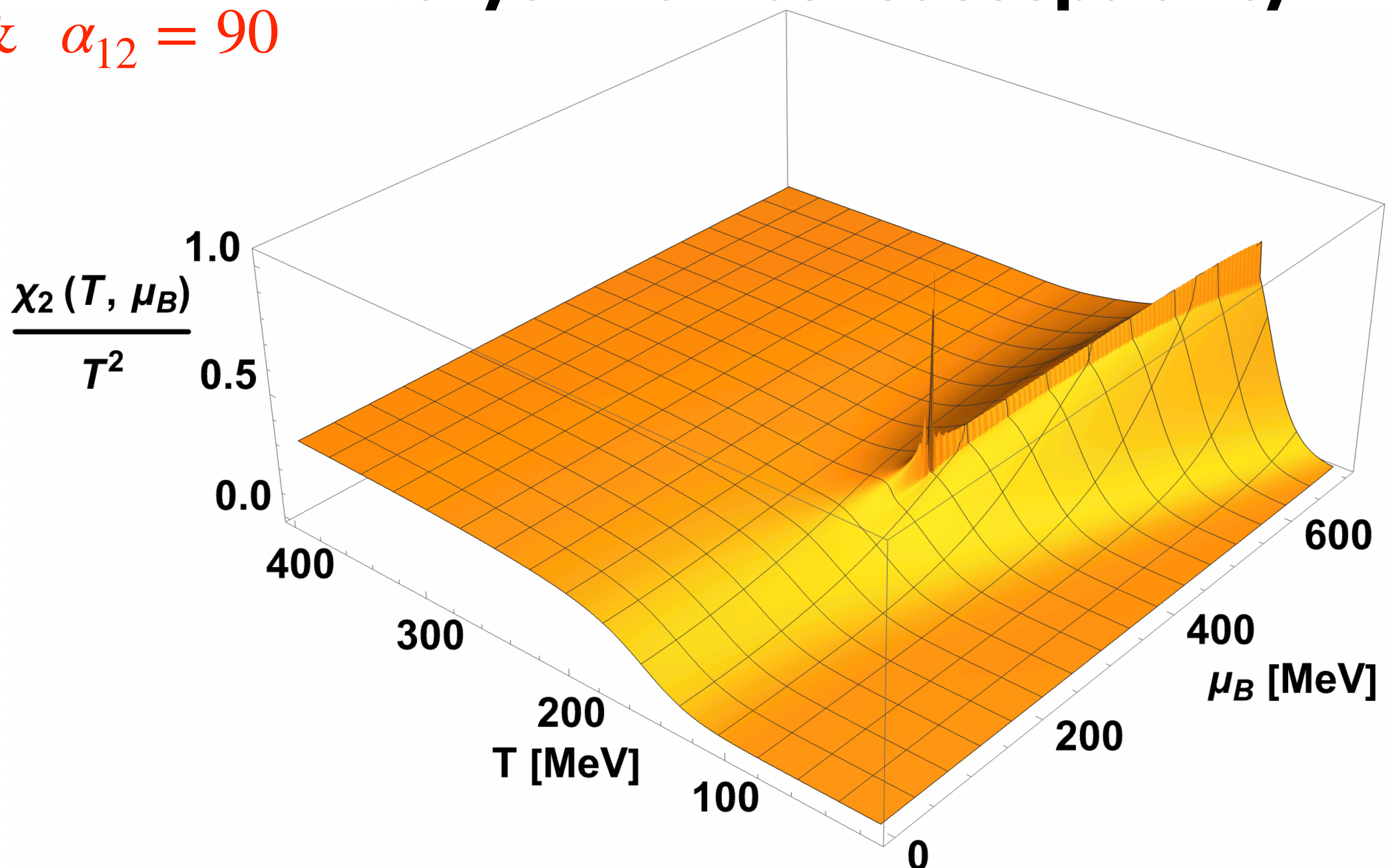
● Critical Point

$w = 2 \;,\; \rho = 2 \;\&\; \alpha_{12} = 90$

Entropy Density

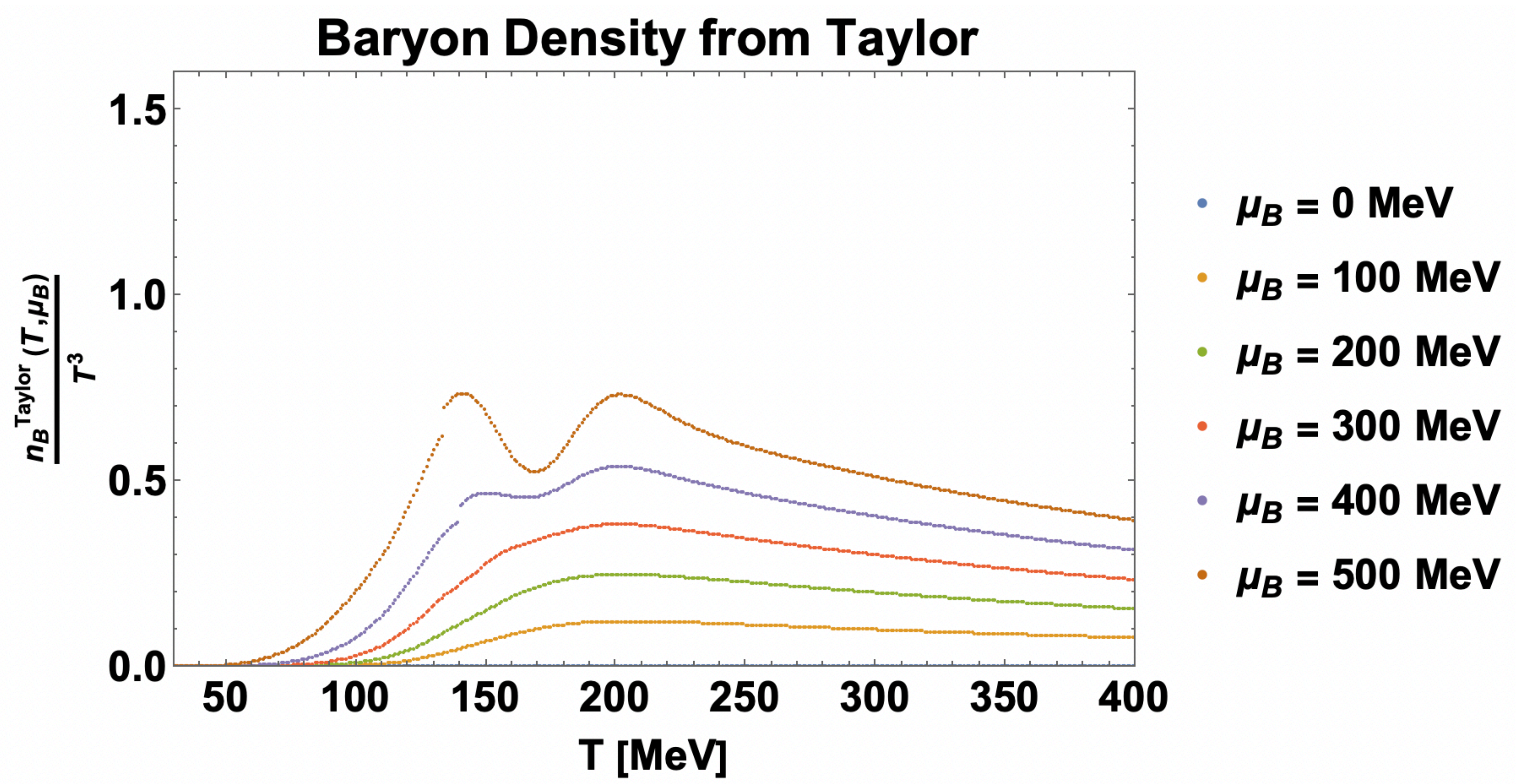


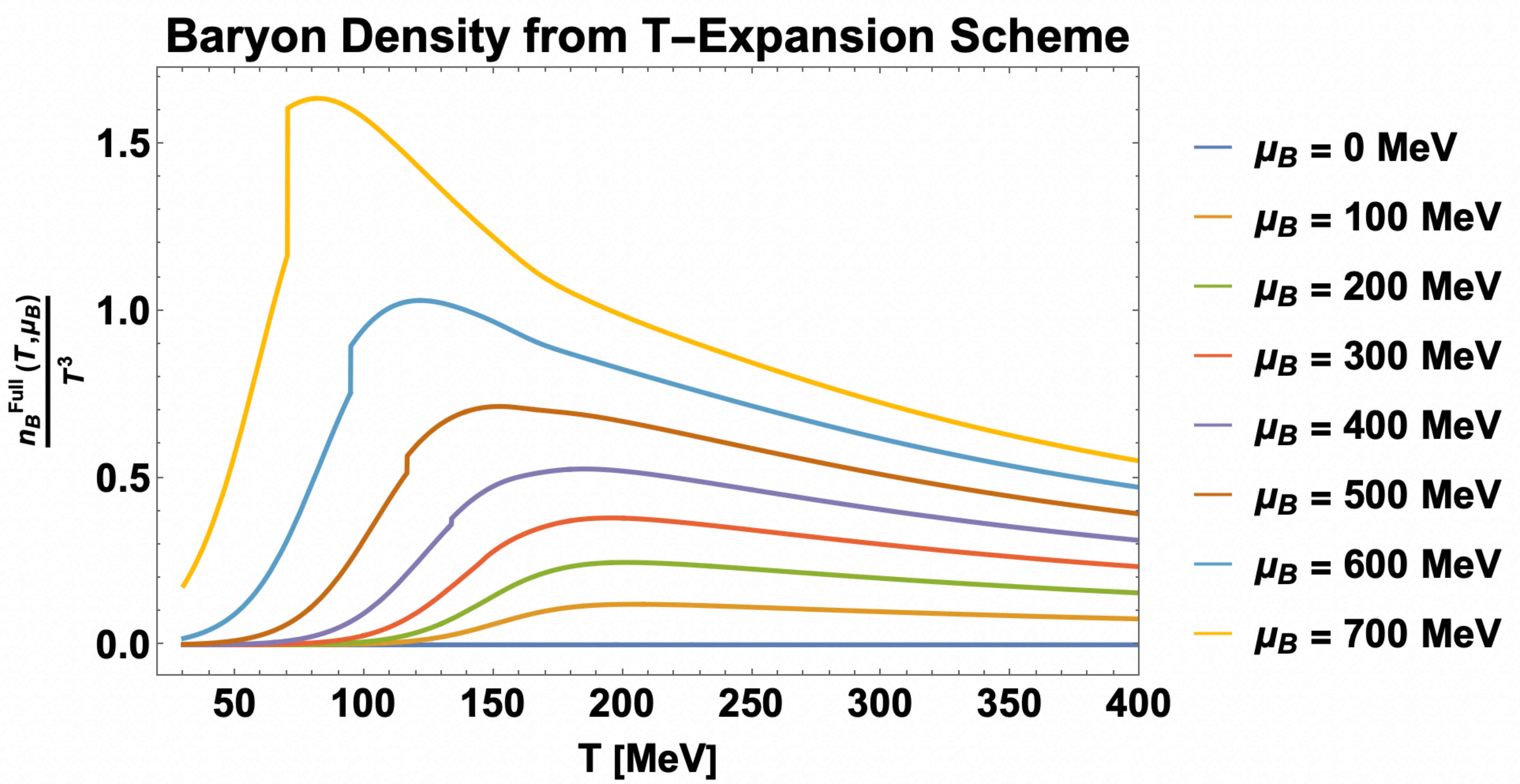
Baryon Number Susceptibility



Summary

$\mu_B = 350$ [MeV], $\alpha_{12} = 90$, $w = 3$, $\rho = 2$





- **A more physical EoS that captures a large part of the Phase diagram is required.**
- **We provide a family of EoS with a correct Critical point up** $\mu_B = 700\ MeV$.
- **Our EoS allows users to change parameters and compare with the data from the Experiment (Beam Energy Scan II)**

Thanks For Your Attention!