LIMITED FREQUENCY BANDWIDTH ON THE INVERSE SCATTERING SERIES AND THE EFFECTS ON SEISMIC PROCESSING GOALS

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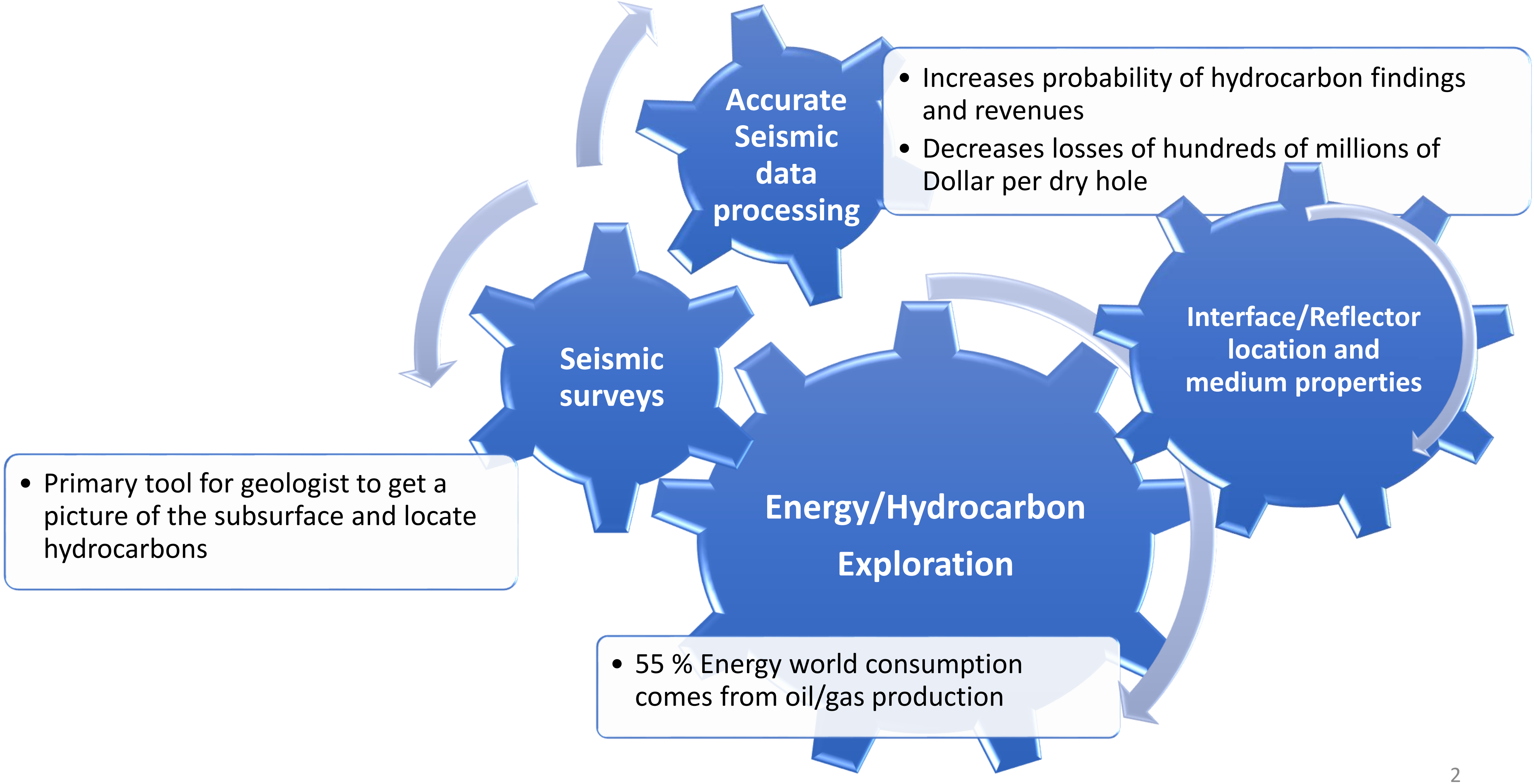
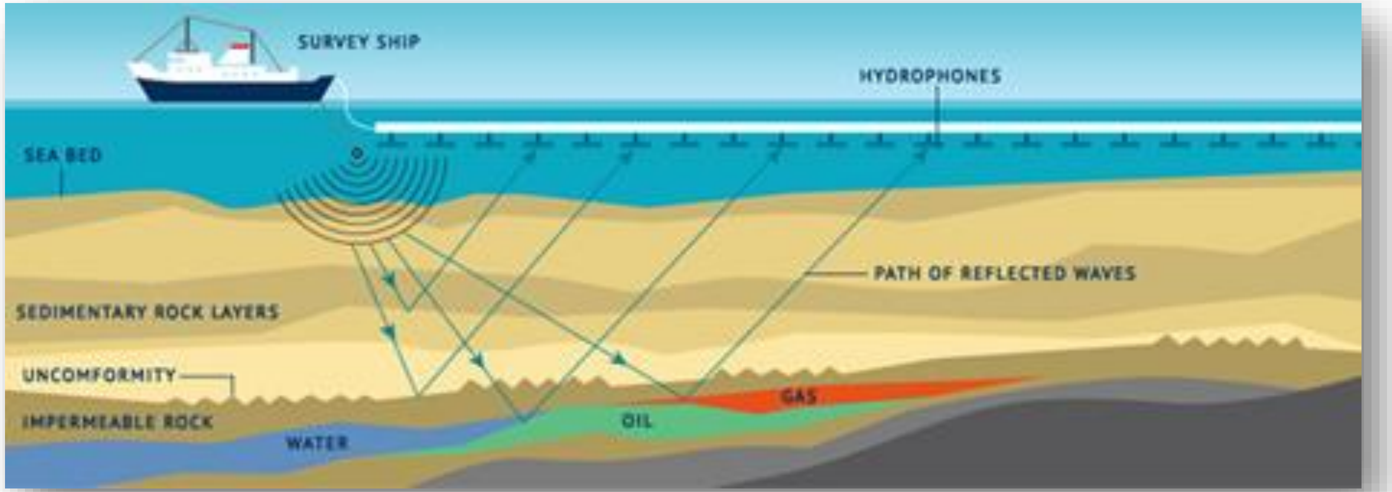
Background


Accurate Seismic data processing
• Increases probability of hydrocarbon findings and revenues
• Decreases losses of hundreds of millions of Dollar per dry hole
Seismic surveys
• Primary tool for geologist to get a picture of the subsurface and locate hydrocarbons
Interface/Reflector location and medium properties
Energy/Hydrocarbon Exploration
• 55 % Energy world consumption comes from oil/gas production

Background



- Locate Hydrocarbons
- Seismic Data Processing
- α: perturbation parameter
- Inverse Scattering Theory

$$\alpha(\vec{r}) = \alpha_1(\vec{r}) + \alpha_2(\vec{r}) + \alpha_3(\vec{r}) + \ldots$$

Statement of the problem

Objective

Determine the effects of limited frequency bandwidth in the inverse scattering series and examine the impact of improved bandwidth in achieving seismic processing goals.

Inverse Scattering Series

$$(\psi_s)_{ms} = (G_o^+ V_1 P_o)_{ms}$$
$$(G_o^+ V_2 P_o)_{ms} = -(G_o^+ V_1 G_o^+ V_1 P_o)_{ms}$$
$$(G_o^+ V_3 P_o)_{ms} = -(G_o^+ V_1 G_o^+ V_2 P_o)_{ms} - (G_o^+ V_2 G_o^+ V_1 P_o)_{ms}$$
⋮

$$V_1(\vec{r}) = k^2 \alpha_1(\vec{r})$$
$$V_2(\vec{r}) = k^2 \alpha_2(\vec{r})$$
$$V_3(\vec{r}) = k^2 \alpha_3(\vec{r})$$
⋮

$$\alpha(\vec{r}) = \alpha_1(\vec{r}) + \alpha_2(\vec{r}) + \alpha_3(\vec{r}) + \ldots$$

α_1 α_2

* **Assumption:** Homogeneous, acoustic medium (no absorption or dispersion) with constant density, varying velocity.

Analytical method /numerical evaluation (Python)

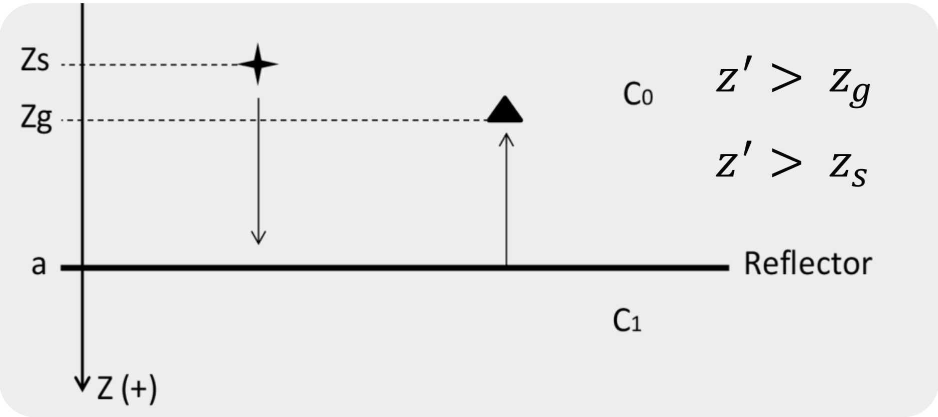
* One interface
* Assuming plane waves propagating in one-dimension, normal incidence
* Reference medium with defined velocity Co.

- All frequencies available
- Low frequencies missing (LFM)
- High frequencies missing (HFM)
- Limited bandwidth frequency ($w_1 \rightarrow w_2$)

→ **Error analysis**

Weglein et al, 1981.
Weglein et al, 2003.

Determining α_1 analytically for an active source with all frequencies available



According to inverse scattering series the recorded data for the linear inverse problem at Z = Zg is:

$$\mathrm{DATA} = \int_{-\infty}^{\infty} G_0^{+}(z', z_g, w)\, V_1(z', w) P_0(z', z_s, w)\, dz'$$

$G_0^{+}(z', z_g, w)$ is the causal Green's Function $\dfrac{e^{i\frac{w}{c_0}|z'-z_g|}}{2ik}$

$V_1(z', w) = k^2 \alpha_1(z')$

Considering Z > Zs, the wave field traveling away from the source to the reflector is given:

$$P_0(z, w) = e^{iw\frac{(z-z_s)}{c_0}}$$

The reflected wave field at Z = Zg is:

$$P_R(z_s, z_g, w) = Re^{i\frac{w}{c_0}(2a-z_s-z_g)} = \mathrm{DATA}$$

Fourier transforming from Z to Kz

$$\widetilde{\alpha_1}(k_z) = -\frac{4i}{k_z} Re^{-ik_z a}$$

Inverse Fourier transforming from Kz to Z

$$\alpha_1(z) = 4R\, H(z-a)$$

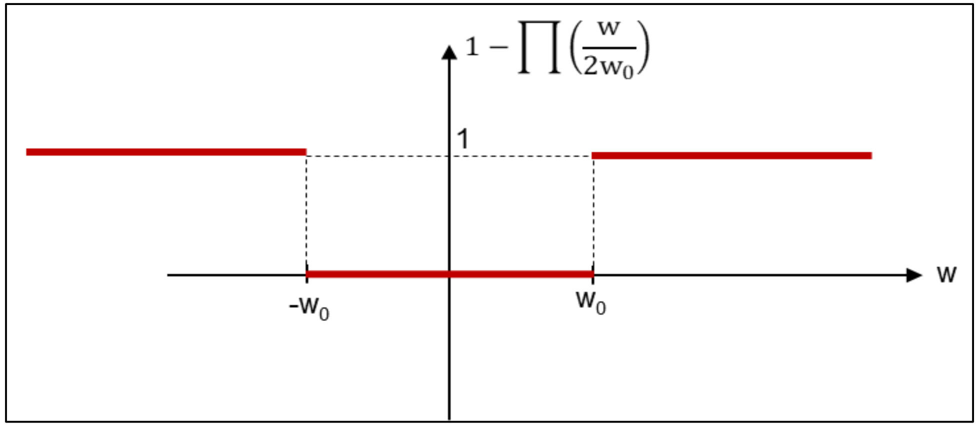
Weglein et al, 1981.
Weglein et al, 2003.

Determining α_1 analytically for an active source with Low frequencies Missing (LFM)

$$\mathrm{DATA} = \int_{-\infty}^{\infty} G_0^{+}(z', z_g, w)\, V_1(z', w) P_0(z', z_s, w) dz'$$

The wave field coming from the source at z = zs is given by:

$$P_0(z = z_s, w) = 1 - \prod\left(\frac{w}{2w_0}\right)$$



$$P_R(z_s, z_g, w) = R e^{i\frac{w}{c_0}(2a - z_s - z_g)} \left[1 - \prod\left(\frac{w}{2w_0}\right)\right] = \mathrm{DATA}$$

Fourier transforming from Z to Kz

$$\widetilde{\alpha_1}(k_z)\left[1 - \prod\left(\frac{w}{2w_0}\right)\right] = -\frac{4i}{k_z} R e^{-ik_z a}\left[1 - \prod\left(\frac{w}{2w_0}\right)\right]$$

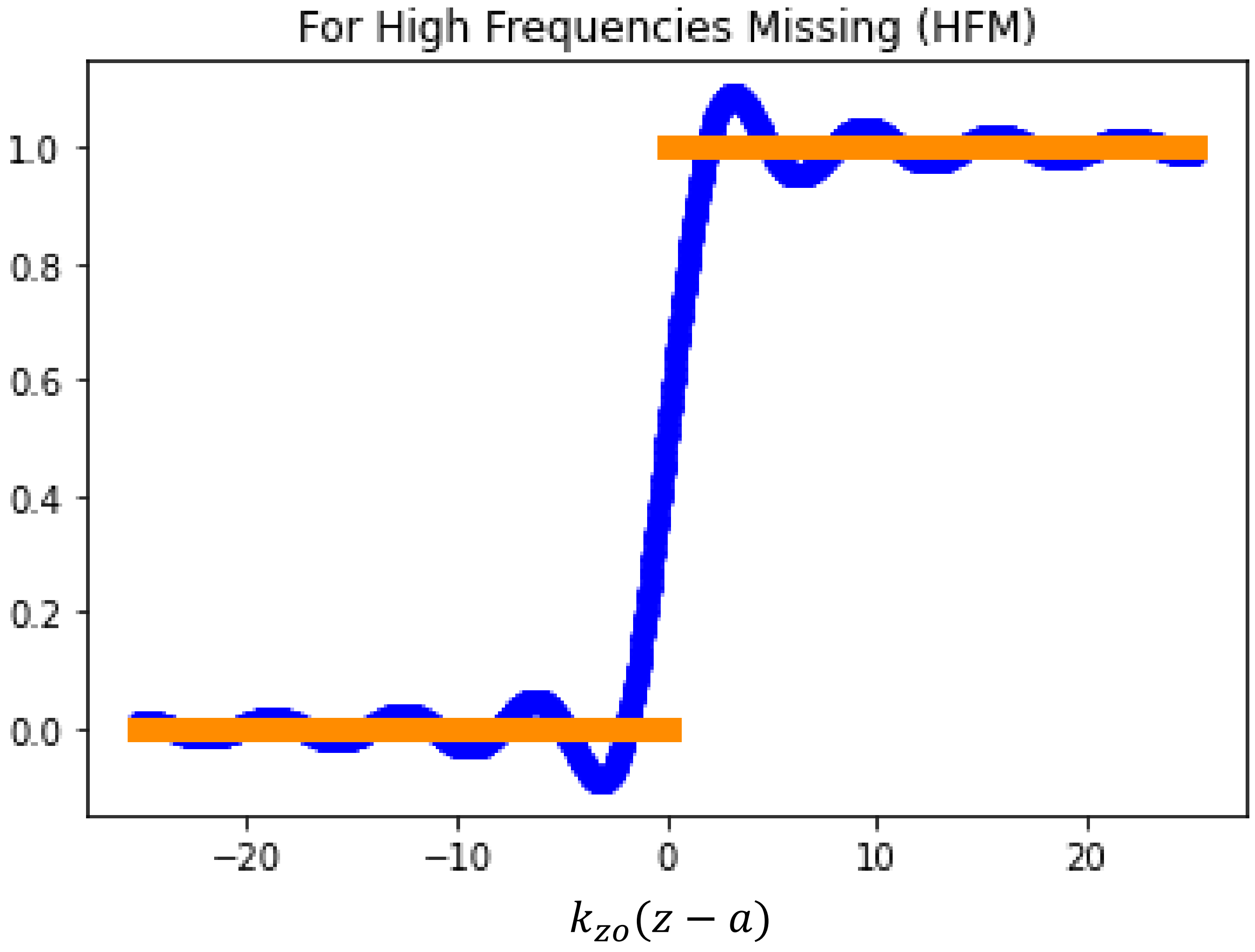
$$\widetilde{\alpha_1}(k_z)\left[1 - \prod\left(\frac{w}{2w_0}\right)\right] = \widetilde{\alpha_1}(k_z)\mathbf{estimated} = \widetilde{\alpha_1}(k_z)_{est}$$

Inverse Fourier transforming from Kz to Z

$$\frac{\alpha_1(k_{zo} z)_{est}}{4R} = H(z) - \frac{1}{2} - \frac{1}{\pi}\int_0^{z} \frac{\sin k_{zo} z'}{z'} dz'$$

$$\frac{\alpha_1(z)_{est}}{4R} = H(z-a) - \frac{1}{2} - \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

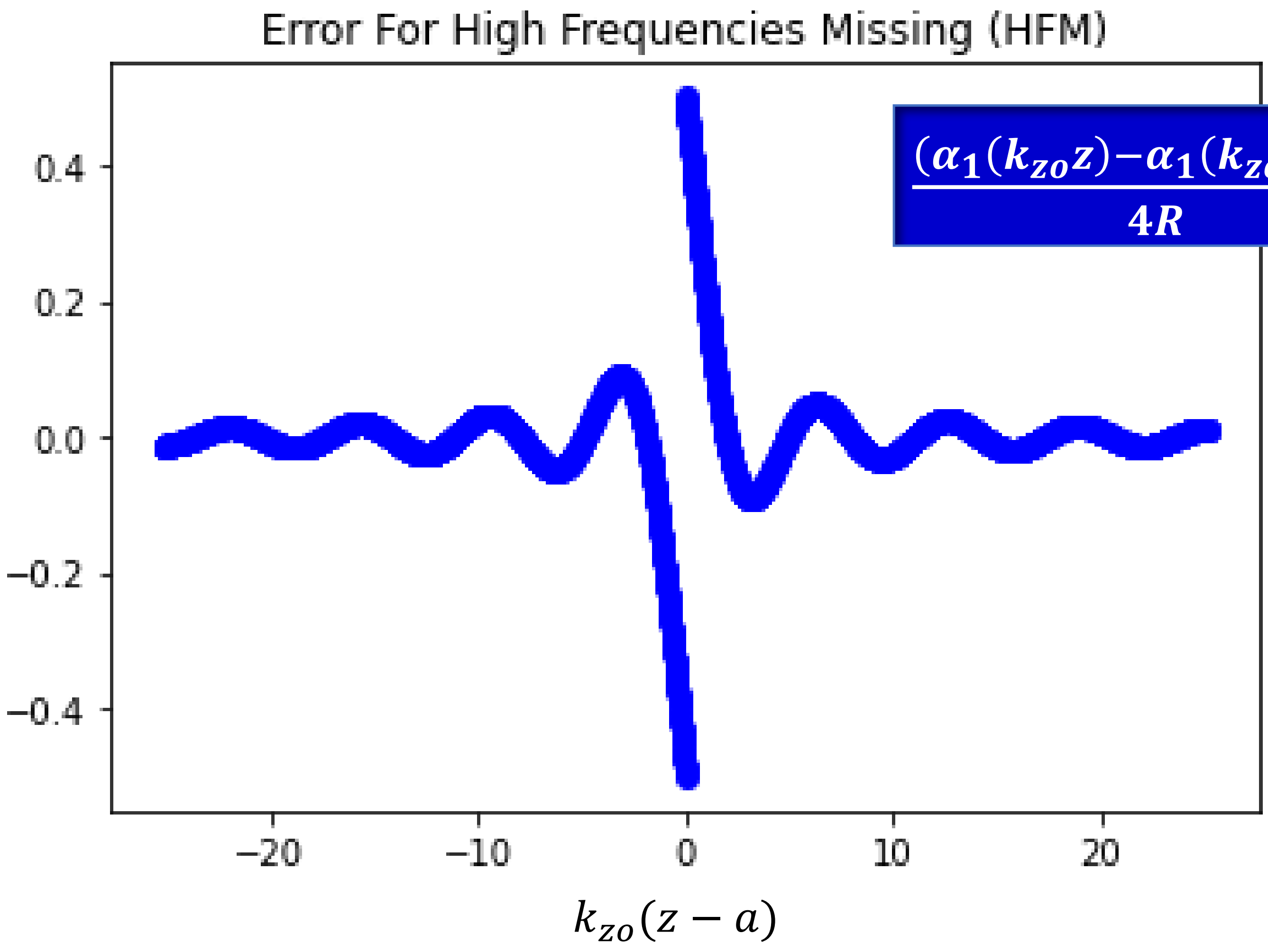
Analysis for α_1 with an active source with HFM



$$\frac{\alpha_1(z)}{4R} = \mathrm{H}(z-a)$$

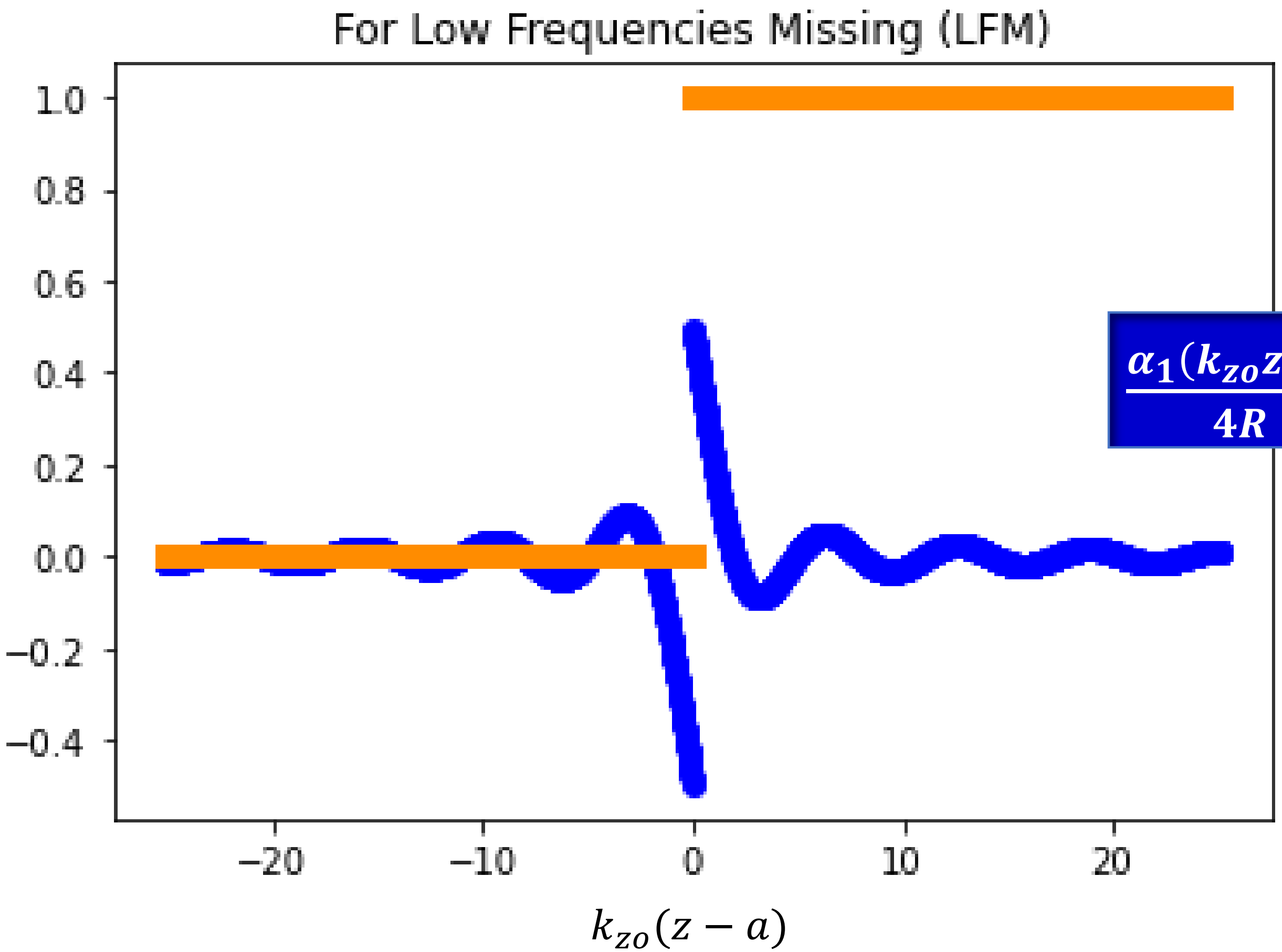
$$\frac{\alpha_1(z)_{est}}{4R} = \frac{1}{2} + \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z\prime}{z\prime} dz'$$

Analysis for α_1 with an active source with HFM



$$\frac{(\alpha_1(k_{zo}z)-\alpha_1(k_{zo}z)_{est})}{4R} = H(z-a) - \frac{1}{2} - \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

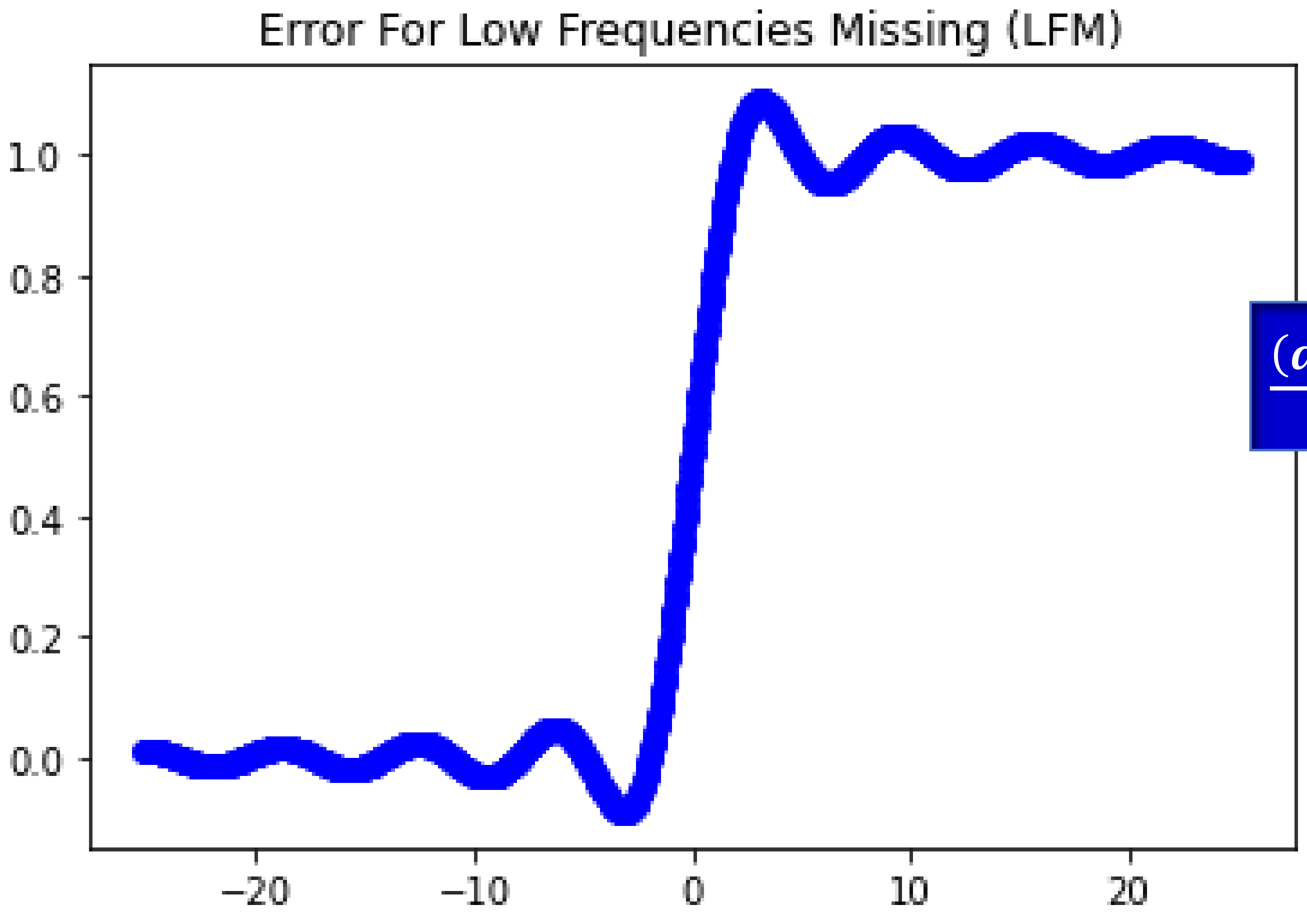
Analysis for α_1 with an active source with LFM



$$\frac{\alpha_1(z)}{4R} = \mathrm{H}(z-a)$$

$$\frac{\alpha_1(k_{zo}z)_{est}}{4R} = H(z-a) - \frac{1}{2} - \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

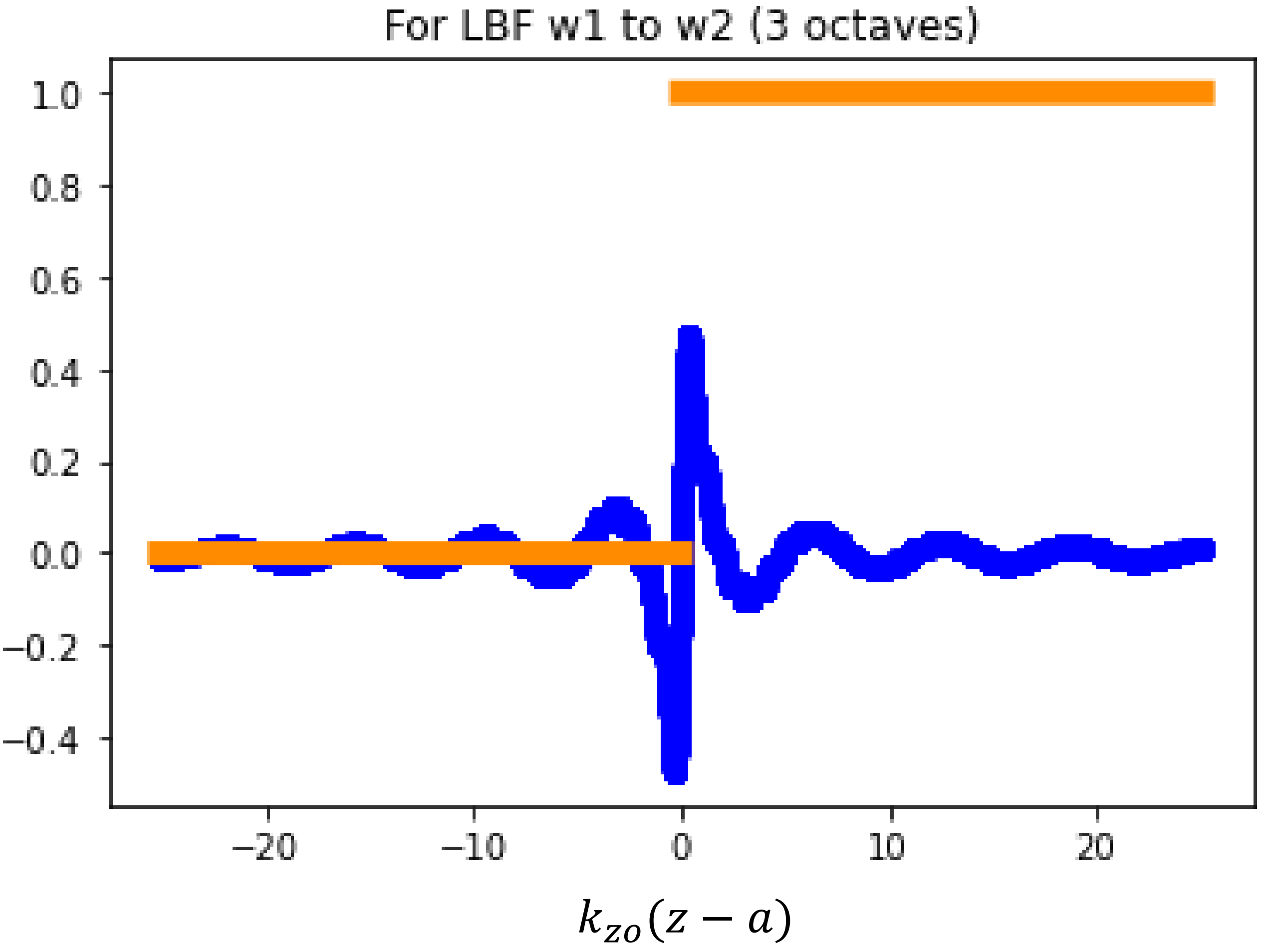
Analysis for α_1 with an active source with LFM



$k_{zo}(z-a)$

$$\frac{(\alpha_1(k_{zo}z)-\alpha_1(k_{zo}z)_{est})}{4R} = \frac{1}{2} + \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'}\,dz'$$

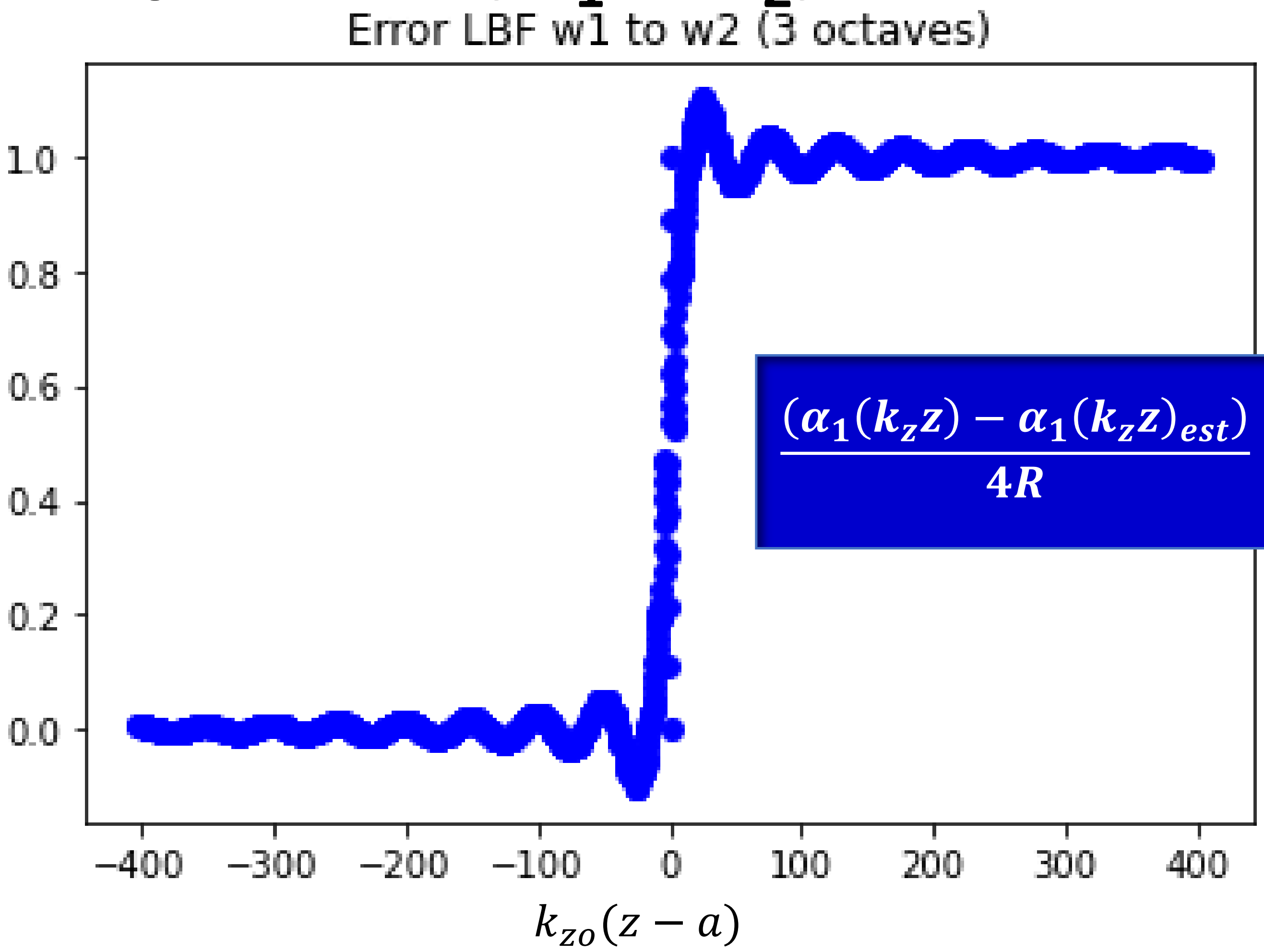
Analysis for α_1 with an active source with limited bandwidth frequencies ($w_1 - w_2$), 3 octaves



$$\frac{\alpha_1(z)}{4R} = \mathrm{H}(z-a)$$

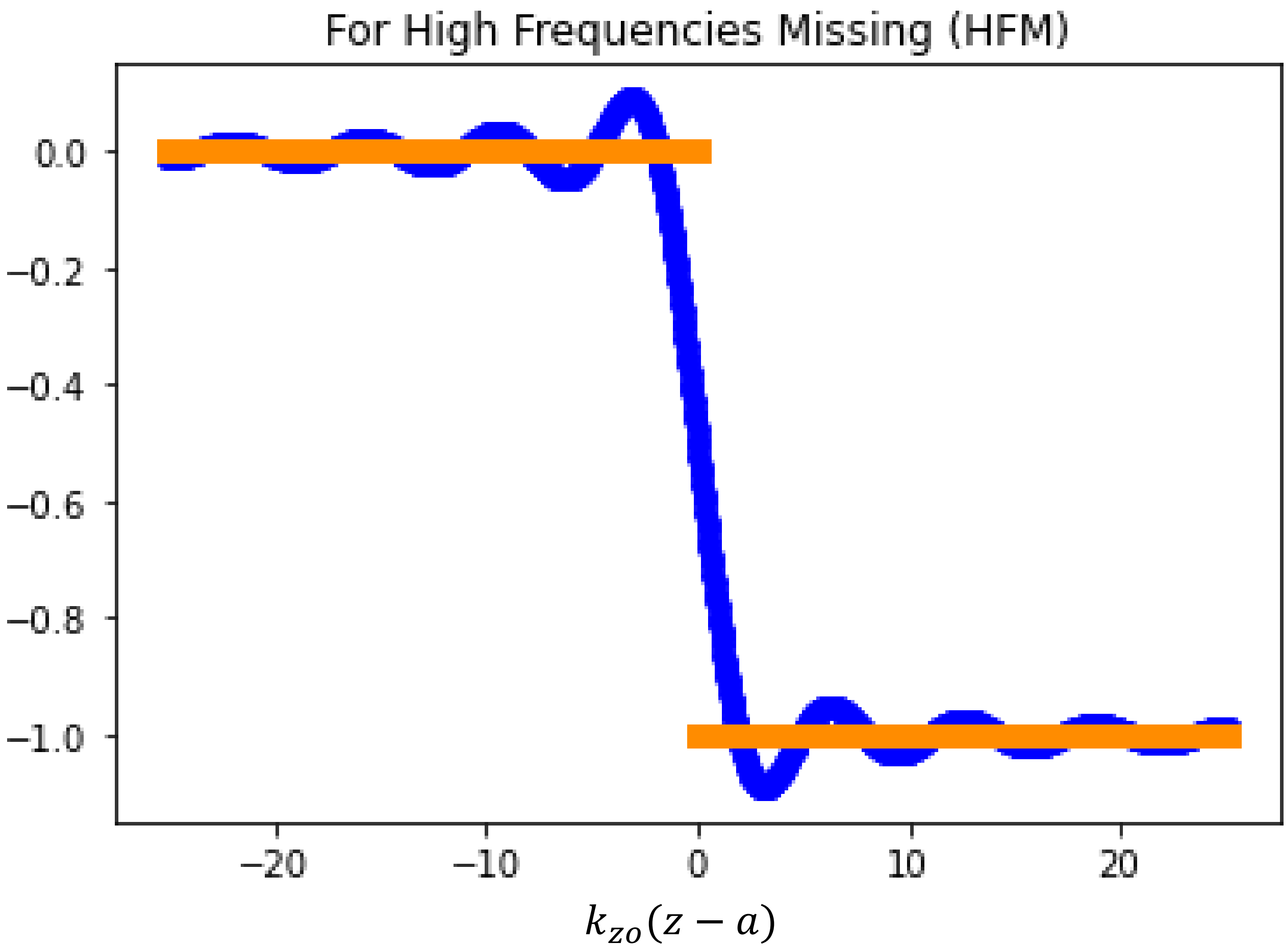
$$\frac{\alpha_1(k_z z)_{est}}{4R} = \frac{1}{\pi}\left[\int_0^{k_{z2}(z-a)} \frac{\sin z'}{z'}\mathrm{dz}' - \int_0^{k_{z1}(z-a)} \frac{\sin z'}{z'}\mathrm{dz}'\right]$$

Analysis for α_1 with an active source with limited bandwidth frequencies $(w_1 - w_2)$



$$\frac{(\alpha_1(k_z z) - \alpha_1(k_z z)_{est})}{4R} = H(z-a) - \frac{1}{\pi}\left[\int_0^{k_{z2}(z-a)} \frac{\sin z'}{z'} dz' - \int_0^{k_{z1}(z-a)} \frac{\sin z'}{z'} dz'\right]$$

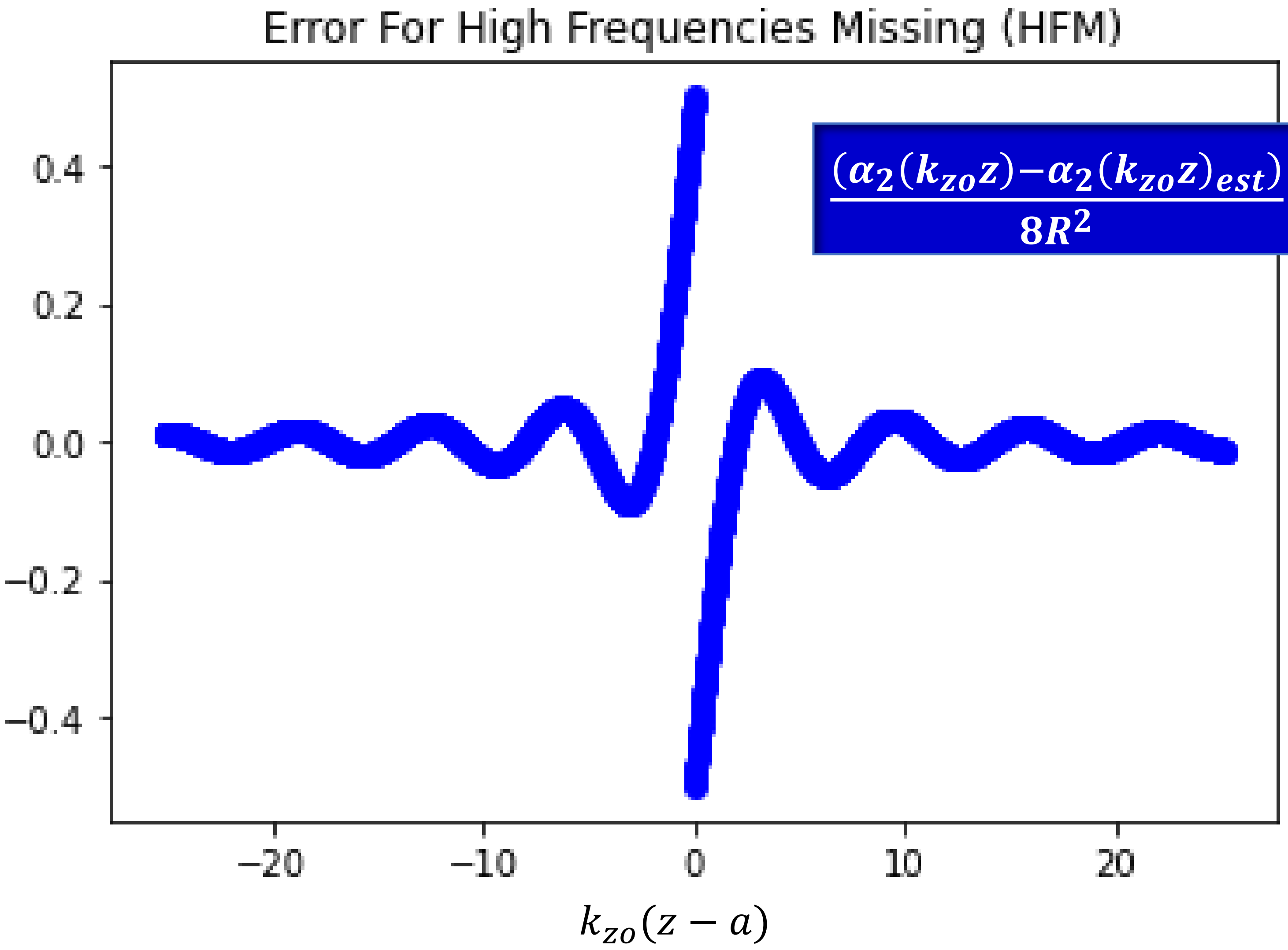
Analysis for α_2 with an active source with HFM



$$\frac{\alpha_2(z) =}{8R^2} - H(z-a)$$

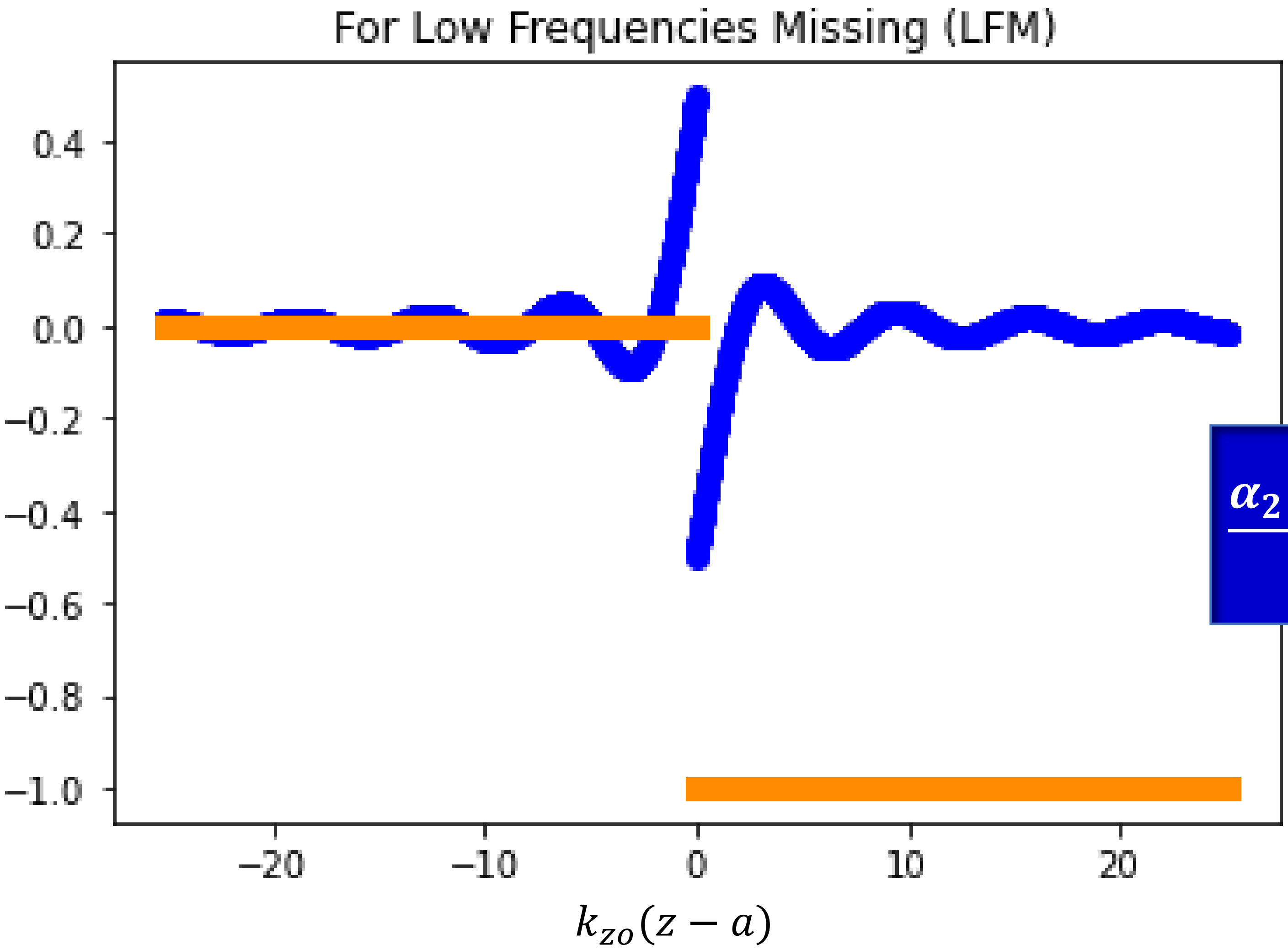
$$\frac{\alpha_2(k_{zo}z)_{est}}{8R^2} = -\frac{1}{2} - \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

Analysis for α_2 with an active source with HFM



$$\frac{(\alpha_2(k_{zo}z)-\alpha_2(k_{zo}z)_{est})}{8R^2} = -H(z-a) + \frac{1}{2} + \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

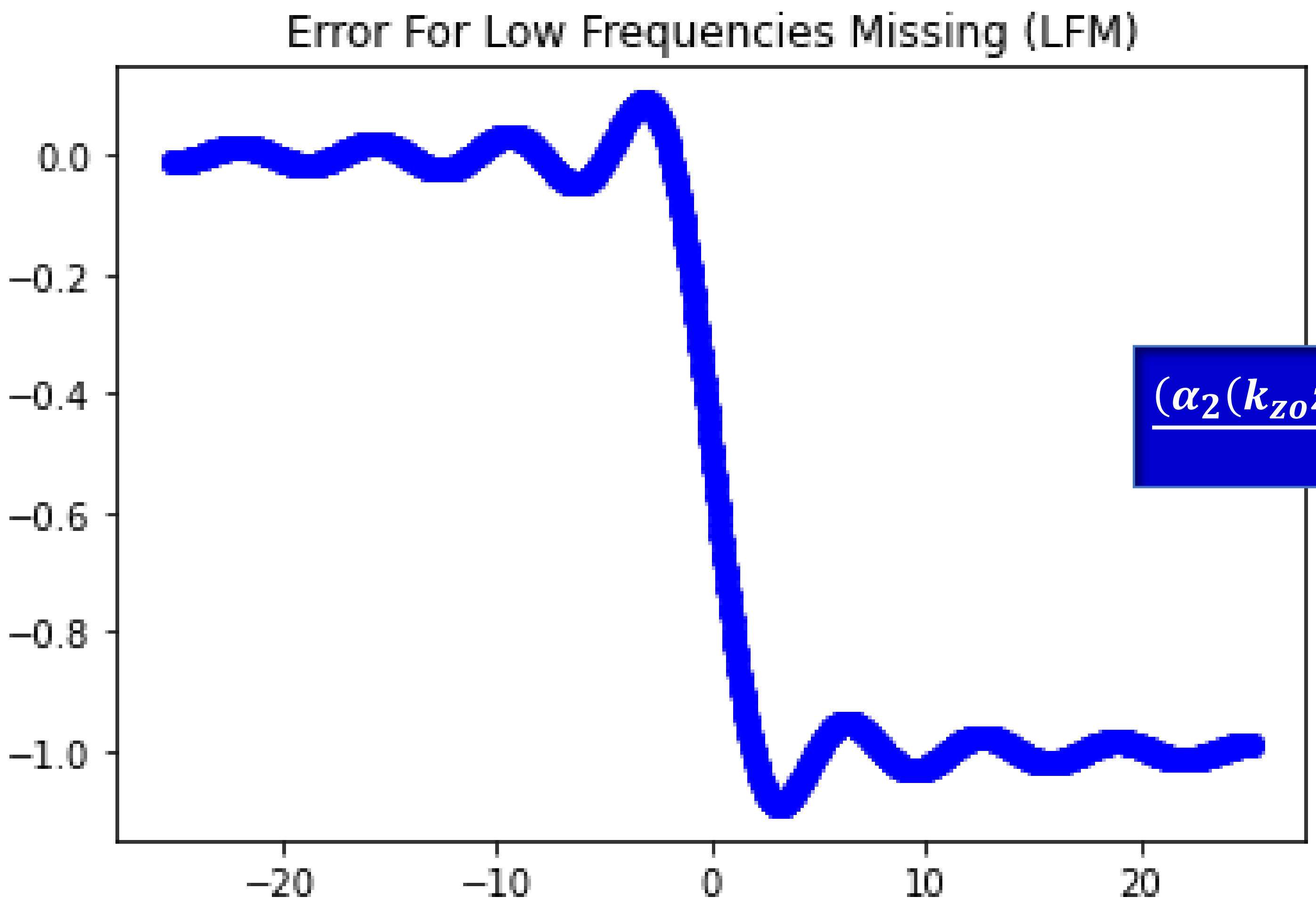
Analysis for α_2 with an active source with LFM



$$\frac{\alpha_2(z)}{8R^2} = -H(z-a)$$

$$\frac{\alpha_2(k_{zo}z)_{est}}{8R^2} = -H(z-a) + \frac{1}{2} + \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'}dz'$$

Analysis for α_2 with an active source with LFM



$k_{zo}(z-a)$

$$\frac{(\alpha_2(k_{zo}z)-\alpha_2(k_{zo}z)_{est})}{8R^2} = -\frac{1}{2} - \frac{1}{\pi}\int_0^{k_{zo}(z-a)} \frac{\sin z'}{z'} dz'$$

Summary

For an acoustic model medium with constant density and varying velocity, 1-D normal incidence plane wave, and one interface:

- α_1 and α_2 cannot be fully determined when frequencies are missing, but portions of α_1 and α_2 can be determined which are defined as α_{1est} and α_{2est}.

- The error is defined as the difference between α_1 and α_{1est}, and as the difference between α_2 and α_{2est}, respectively; and for both:
  - if only high frequencies are missing, the error is largest near the interface and decreases going away from the interface,
  - if low frequencies are missing, the error is increasing approaching the interface and remains large going deeper from the interface,
  - the same behavior is observed for three octave bandwidth.

- A band limited source restricts solutions to α_{1est} and α_{2est} affecting the accuracy of seismic inversion. Improving low frequency bandwidth has the greatest benefit. Though still under study, it appears band limit effects for seismic imaging and internal multiple removal are considerably less and are absent for surface multiple removal.

References

[1] Moses, H. E., 1956. Calculation of scattering potential from reflection coefficients. *Phys. Rev.* 102 559–67

[2] Prosser, R. T., 1969. Formal solutions of inverse scattering problems *J. Math. Phys.* 10 1819–22

[3] Razavy,M.,1975. Determination of the wave velocity in an inhomogeneous medium from reflection data *J. Acoust.Soc. Am.* 58 956–63

[4] Weglein, A. B., Boyse, W. E. and Anderson, J. E., 1981. Obtaining three-dimensional velocity information directly from reflection seismic data: an inverse scattering formalism. *Geophysics* 46 1116–20

[5] Stolt, R. H. and Jacobs, B., 1980. Inversion of seismic data in a laterally heterogeneous medium. *SEP Rep.* 24 135–52

[6] Weglein, A. B., 2021. Scattering Theory Spring course. University of Houston.

[7] Weglein, A. B., Araujo, F. V., Carvalho, P. M., Stolt, R. H., Matson, K. H., Coates, R. T., Corrigan, D., Foster, D. J., Sgaw, S. A. and Zhang, H., 2003. Inverse Scattering series and seismic exploration. Inverse problems, 19: R27-R83.

[8] Weglein, A. B., Gasparotto, F. A., Carvalho, P. M. and Stolt, R. H., 1997. An inverse scattering series method for attenuating multiples in seismic reflection data. *Geophysics* **62** 1975–89

[9] Bracewell, R.N. (1978). The Fourier Transform and Its Applications. Second Edition, McGraw-Hill Book Company, New York, 444 p.

[10] Shaw, S. A., Weglein, A. B., Foster, D. J., Matson, K. H. and Keys, R. G., 2004. Isolation of a leading order depth imaging series and analysis of its convergence properties. J. Seismic Explor., 13: 157-195.

[11] Clayton, R. W. and Stolt, R. H., 1981. A Born–WKBJ inversion method for acoustic reflection data. *Geophys. Soc.Explor. Geophys.* 46 1559–67

[12] Stolt, R. H. and Weglein, A. B., 1985. Migration and inversion of seismic data. *Geophysics* 50 2458–72