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Quantum entanglement and Indistinguishability

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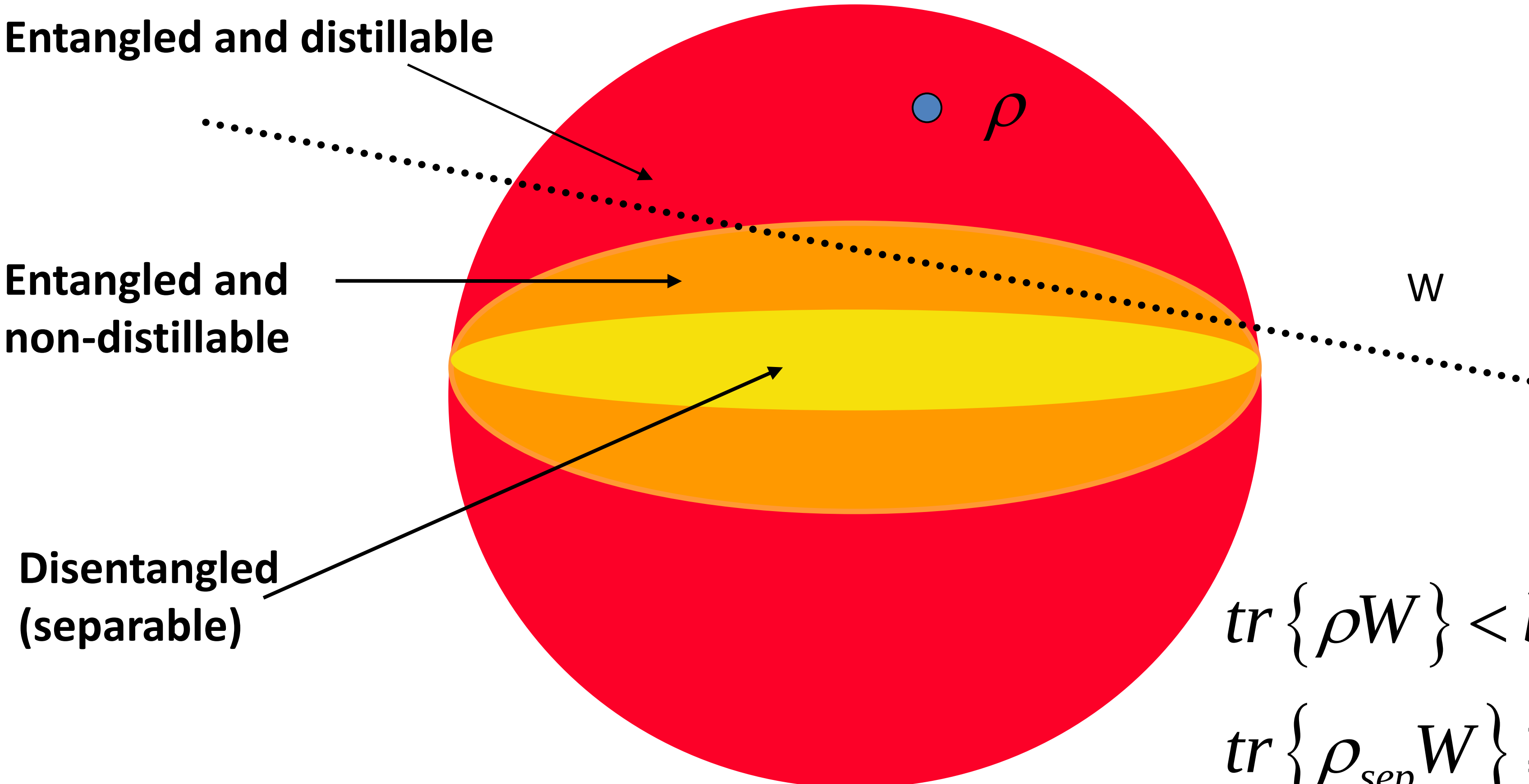
Entangled versus Separable

$$\rho \stackrel{?}{=} \sum_i p_i \rho_1^i \otimes \rho_2^i \otimes \ldots \otimes \rho_n^i$$

States like these can be created by LOCC (sometimes also stated that they can never lead to any information processing advantage over classical systems – but this is not clear)

Modi, Brodutch, Cable, Paterek, and Vedral, Rev. Mod. Phys. (2012).

Witnessing Entanglement



$$tr\left\{\rho W\right\} < b$$

$$tr\left\{\rho_{sep} W\right\} \geq b$$

Bipartite purity based witnesses

For separable states purity of the reduced states A,B is bigger than of the whole state AB:

$\mathrm{Tr}\,\sigma^2_{AB} <= \mathrm{Tr}\,\sigma^2_{A}$ & $\mathrm{Tr}\,\sigma^2_{AB} <= \mathrm{Tr}\,\sigma^2_{A}$

L. Amico, Rosario Fazio, A. Osterloh, V. Vedral, Rev. Mod. Phys. (2008)

Indistinguishability and entanglement

- First quantisation is misleading

- Mode entanglement the right way to go (modes are the relevant subsystems)

- Note: even the coupling between internal degrees is an effect of particle statistics

Libby Heaney and Vlatko Vedral, Phys. Rev. Lett. **103**, 200502 2009

First quantisation confusion

$$\left(\left|\psi_1\psi_2\right\rangle - \left|\psi_2\psi_1\right\rangle\right)\otimes\left|\uparrow\uparrow\right\rangle$$

There is no spatial entanglement here!

$$a^{\dagger}{}_{\psi_1\uparrow}\,a^{\dagger}{}_{\psi_2\uparrow}\left|0\right\rangle$$

In other words, any operator correlation: <AB>=<A><B>

Second Quantisation

In order to study entanglement of identical particles need to talk about modes – second quantisation.

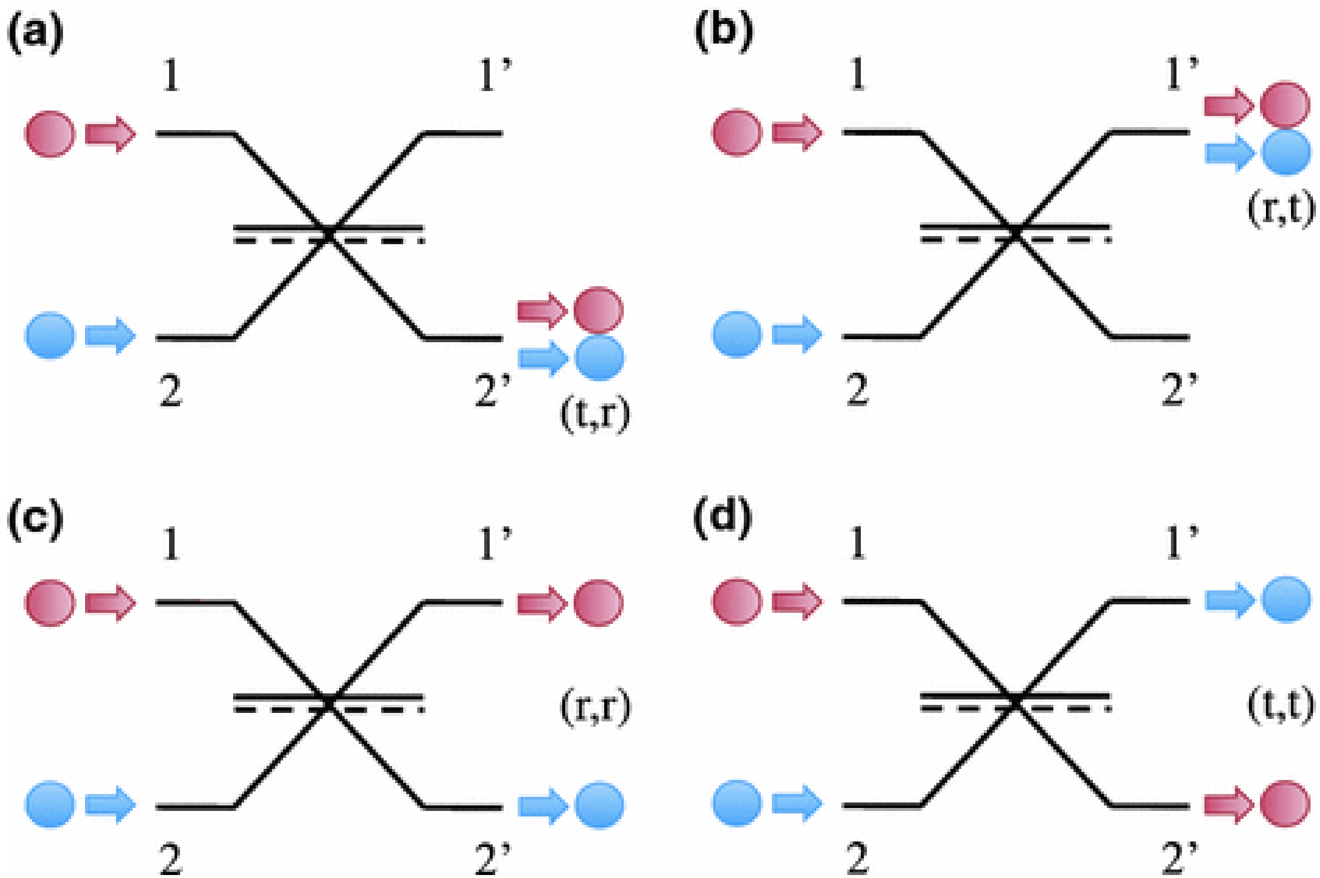
$$\left|0\right\rangle = \left|0\right\rangle_{k_1} \left|0\right\rangle_{k_2} \left|0\right\rangle_{k_3} \left|0\right\rangle \left|0\right\rangle \left|0\right\rangle \ldots \Rightarrow$$

$$(a^{\dagger}{}_{k_i} + a^{\dagger}{}_{k_j} + \ldots)\left|0\right\rangle$$

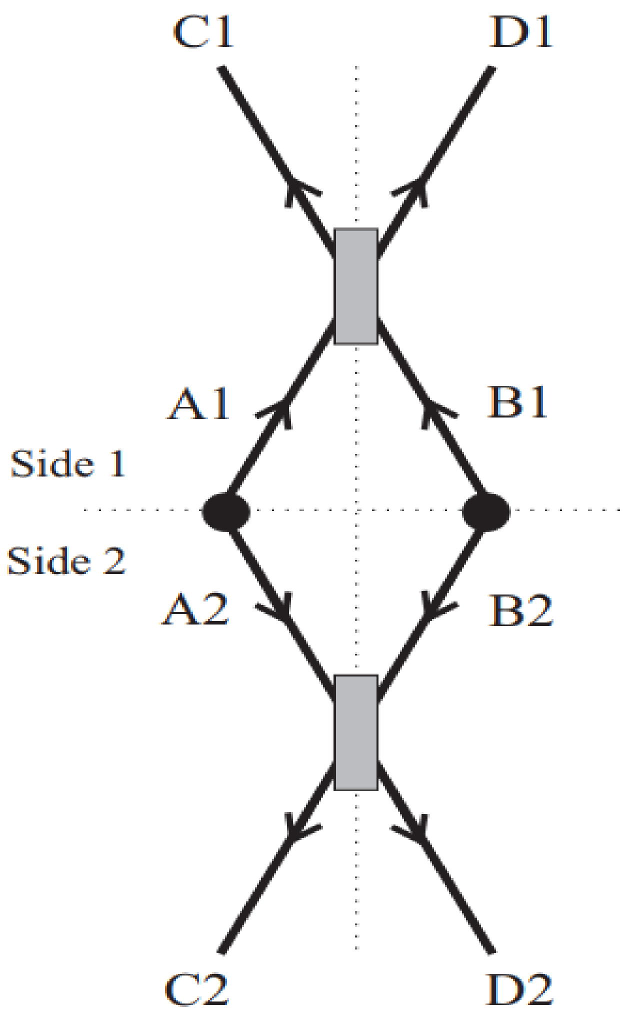
But, we need to be careful about superselections!
(this is different to symmetrisation, anti-symmetrisation where there is no entanglement to access in the first place)

Jacob Dunningham and Vlatko Vedral, Phys. Rev. Lett. **99**, 180404 2007

Hong Ou Mandel



Photon Entanglement

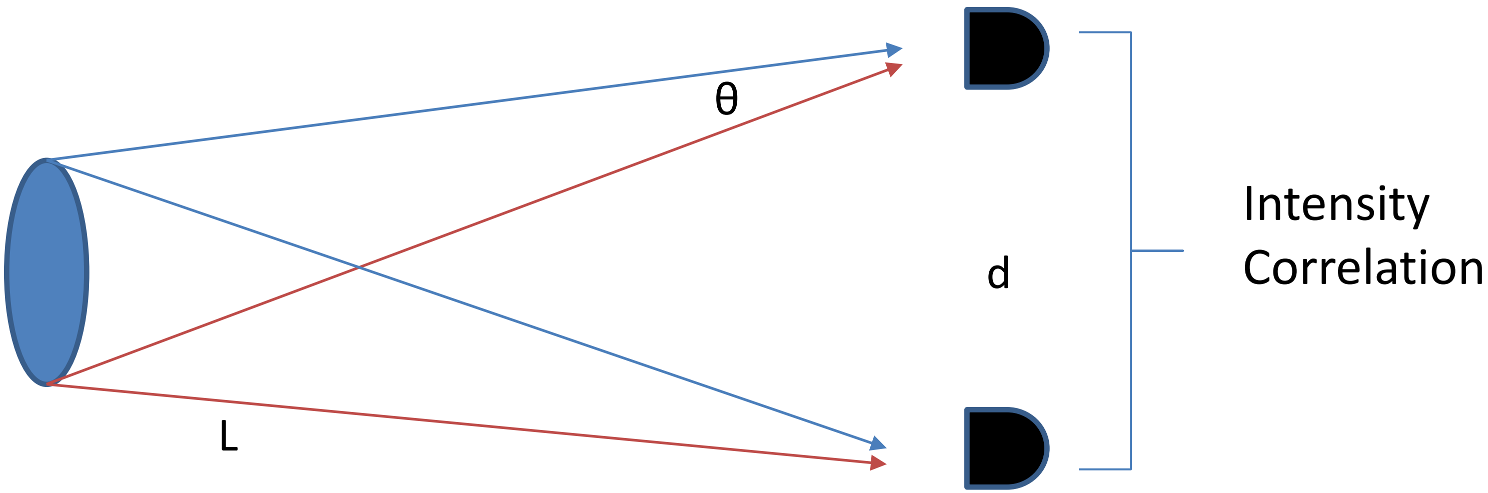


A1 and A2 initially entangled (internal degrees) as well as B1 and B2. Triplet (A1 and B1; A2 and B2) leads to bunching, singlet to antibunching.

Looking at correlations between bunching and anti-bunching on the sides 1 and 2 gives us complete information about all purities.

N. Paunković, Y. Omar, S. Bose, and V. Vedral, Phys. Rev. Lett. **88** 2002

Hanbury-Brown Twiss

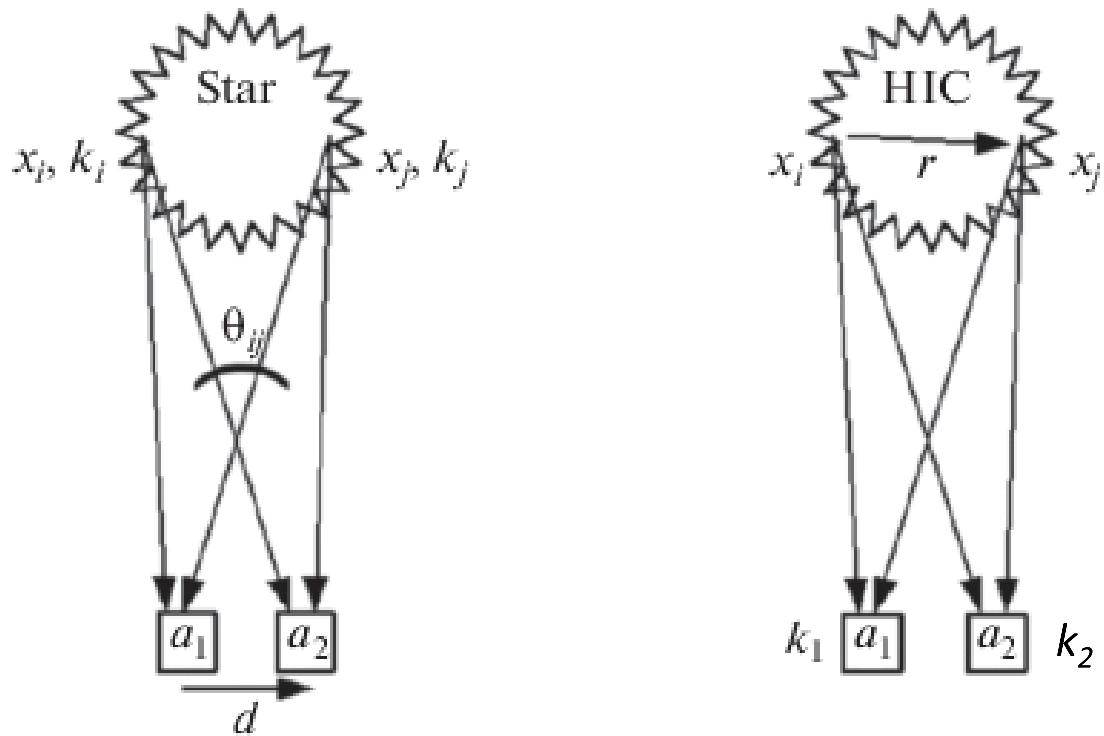


Originally proposed to measure stellar angular width

$\theta = d/L = \lambda/l_c$

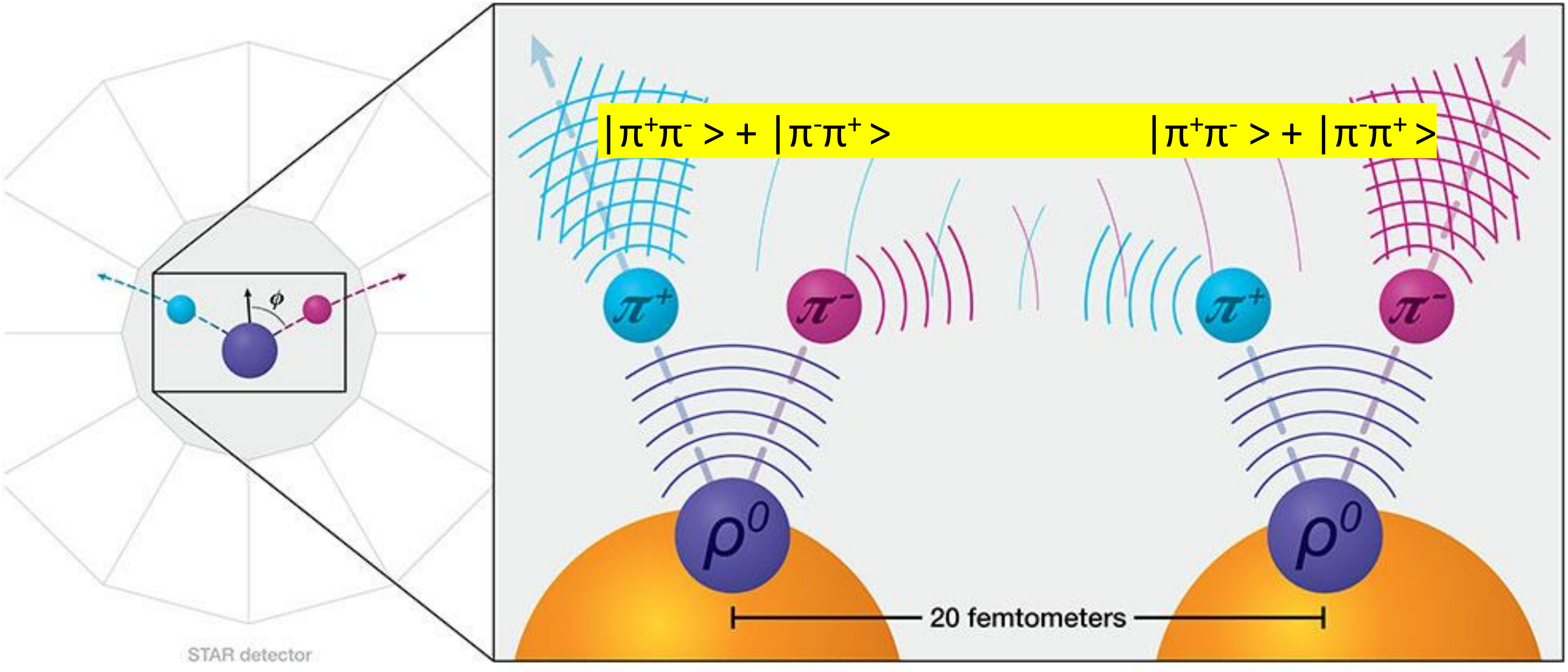
Femtoscopy

But can be used for nuclear size measurements
(25 orders of magnitude difference!)



Michael Annan Lisa, Scott Pratt, Ron Soltz, Urs Wiedemann**, Annual Review of Nuclear and Particle Science** 55:357-402 (2005)

Pion Entanglement – HOM meets HBT



Entanglement is needed to explain the statistics of detection correlations.

STAR Collaboration, Sci. Adv. 9, eabq3903 (2023)

The crux of the matter

The interference between detectors occurs when π^+ and π^- are detected in each detector.

There are two indistinguishable ways in which this can happen (unlike when the pions have the same charge).

$$|A(\pi^+\pi^-;\pi^-\pi^+) + A(\pi^-\pi^+;\pi^+\pi^-)|^2 = 1 + \cos(k\,\Delta r)$$

G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Bev. 120, 300 (1960).

Summary

- Particle Statistics can be used to create/witness entanglement

- Confirming entanglement this way, rules out separable states, but of course, does not rule out all hidden variables.

- The problem in many systems is to test concepts related to locality (to close loopholes).