Quantum Tops: Entanglement, Bell Inequality, Discord and Steering with Top-Quarks

Foundational tests of Quantum Mechanics at the LHC,
Merton College, Oxford, UK

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Overview

- The Standard Model is a QFT:
  - Special Relativity.
  - Quantum Mechanics.
- Fundamental properties of Quantum Mechanics can be tested via the Standard Model.

- An opportunity to study Quantum Information at High-Energy Colliders.
- This talk is based on:
  - YA, J. R. M. de Nova, EPJP (2021).
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- Three parts are in the talk:
  - Basics: $t\bar{t}$ in hadron colliders.
  - Common Concepts: Entanglement and Bell Inequality.
  - Novel Concepts: Discord and Steering.

First Part: Basics

$t\bar{t}$ in hadron colliders.

Top-Quark

- **Top-quark:**
  - The most massive particle in the Standard Model.
  - Lifetime: $\sim 10^{-25}$ s.
- **General:**
  - Hadronisation: $\sim 10^{-23}$ s.
  - Spin-decorrelation: $\sim 10^{-21}$ s.
- Spin information $\rightarrow$ decay products.
- Spin-correlations between a pair of top-quarks can be measured.
- Considering di-leptonic decays.

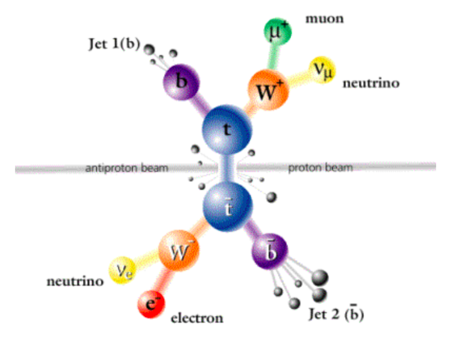
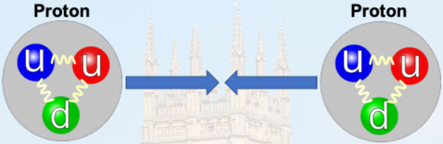


Figure: Di-leptonic decay of a $t\bar{t}$ pair.

Spin-Correlations between Top-Quark Pairs

- Studied extensively theoreticaly.
- Measured by the D0, CDF, ATLAS and CMS collaborations.
- No link between spin-correlations of top-quarks and Quantum Information until recently.
- Spin-Correlations can be a classical property.
  For example, **Spin-Correlations $\neq$ Quantum Entanglement!**
  However, Quantum Entanglement $\subset$ Spin-Correlations.

Collisions at the LHC



- At the LHC, protons are being collided at high energies.
- Proton: quarks and gluons (partons).
- Parton distribution function (PDF): the density of each parton in the proton.

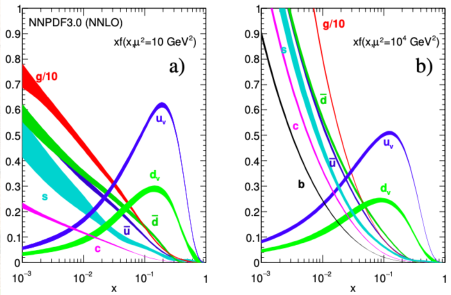
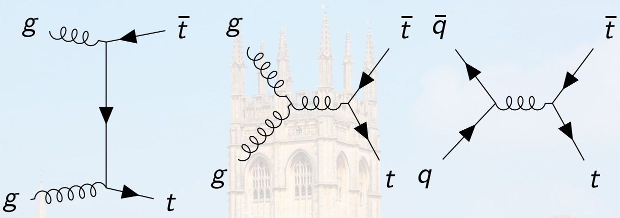


Figure: Parton density at the proton.
Figure is from JHEP 2015, 40 (2015).

Leading-order Analytical Calculation



- Analytical calculation at leading-order. The system is defined by:
  - $\hat{k}$: the direction of the top with respect to the beam axis.
  - The invariant mass $M_{t\bar{t}}$, $\beta = \sqrt{1 - \frac{4 \cdot m_t^2}{M_{t\bar{t}}^2}}$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

How does all of this related to Quantum Information?

Second Part: Common Concepts

Entanglement and Bell Inequality.

Quantum Tomography: One Qubit

- Qubit: quantum system with two states (e.g., spin-1/2 particle).
- Most general density matrix for a qubit:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}$$

- Only 3 parameters $B_i \rightarrow$ Quantum tomography is the measurement of spin polarization $\mathbf{B}$:

$$B_i = \langle \sigma^i \rangle = \mathrm{tr}(\sigma^i \rho)$$

Quantum Tomography: Two Qubits

- Most general density matrix for 2 qubits:

$$\rho = \frac{1 + \sum_i \left(B_i^+ \sigma^i + B_i^- \bar{\sigma}^i \right) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- 15 parameters $B_i^{\pm}, C_{ij}$ $\rightarrow$ Quantum tomography=Measurement of individual spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \rangle \,, \;\; B_i^- = \langle \bar{\sigma}^i \rangle \,, \;\; C_{ij} = \langle \sigma^i \bar{\sigma}^j \rangle$$


Experimental Observables

Quantum Entanglement:

- **Concurrence** $\mathcal{C}[\rho]$: quantitative measurement of entanglement.
- $0 \leq \mathcal{C}[\rho] \leq 1$, $\mathcal{C}[\rho] \neq 0$ iff the state is entangled.
- Here, $\mathcal{C}[\rho] = \max(\Delta, 0)$; $\Delta = \frac{-C_{nn} + |C_{kk} + C_{rr}| - 1}{2}$.

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Bell Inequality:

- A violation of the CHSH inequality: $\left|\mathbf{a}_1^{\mathrm{T}}\mathbf{C}\left(\mathbf{b}_1-\mathbf{b}_2\right)+\mathbf{a}_2^{\mathrm{T}}\mathbf{C}\left(\mathbf{b}_1+\mathbf{b}_2\right)\right| > 2$.
  - $\mathbf{C}$ - spin correlation matrix.
  - $\mathbf{a}_1, \mathbf{a}_2$ $(\mathbf{b}_1, \mathbf{b}_2)$ - axes in which we measure the spin of the top (antitop).
- Maximization: $2\sqrt{\mu_1+\mu_2} \leq 2\sqrt{2}$ where $0 \leq \mu_i \leq 1$ are the eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$.

Loopholes in a Collider Experiment

- Loopholes: experimental tests of Bell inequality may not fulfill all hypotheses of the theorem.
- Collider experiment:
  - Free-will loophole: spin measurement directions should be free, independent from hidden-variables.
  - Detection loophole: only a subset of events is selected for the measurement, which can be biased.
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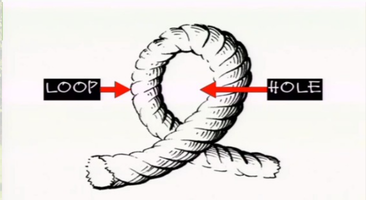
- Collider experiments were not designed to test Bell Inequality!
  $\Rightarrow$ Can only detect a *weak* violation of CHSH (Bell) Inequality.



- **Bell-Inequality $\subset$ Quantum Entanglement.**

Entanglement and Bell Inequality Before Integration

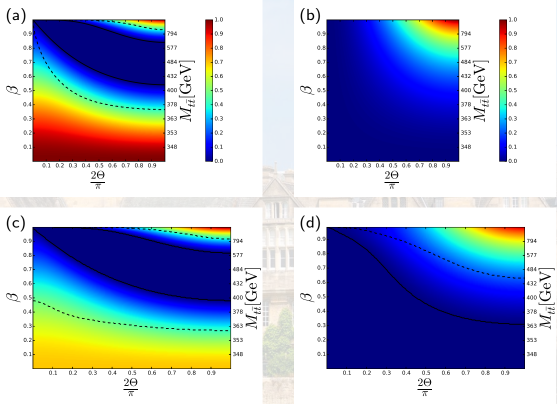
a) $gg \to t\bar{t}$ Concurrence.

b) $q\bar{q} \to t\bar{t}$ Concurrence.

c) Full LHC $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

d) Full Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Concurrence.

- Solid line: entanglement boundary; Dashed line: Bell non-locality boundary.



- It is possible to control the $gg/q\bar{q}$ fraction by further selections ($\beta_{t\bar{t}}$), see Aguilar-Saavedra, Casas, EPJC (2022).

Measurable Entanglement Witness

- Integration only for $[2m_t, M_{t\bar{t}}]$. For high p_T see:
  - Fabbrichesi, Floreanini, Panizzo, PRL (2021).
  - Severi, Boschi, Maltoni, Sioli, EPJC (2022).
- In particular: $\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$ where φ is the angle between the lepton directions in each one of the parent top and antitop rest frames.
- $\Delta > 0 \Leftrightarrow D = \frac{\mathrm{tr}[\mathbf{C}]}{3} < -\frac{1}{3}$.
- Can be achieved by measuring D close to threshold.

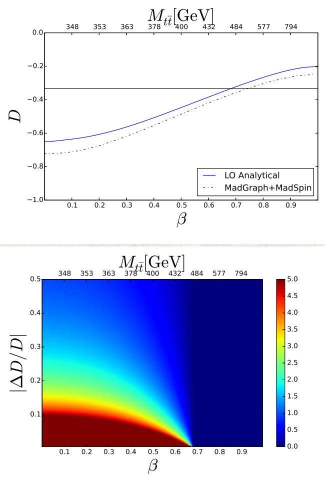


Figure: Up: the value of D; bottom: statistical deviation from the null hypothesis ($D = -1/3$).

- Recently, D was measured with no selection on $M_{t\bar{t}}$ by the CMS collaboration.
- Results:
  $D = -0.237 \pm 0.011 > -1/3$;
  $\Delta D/D = 4.6\%$.
- No evidence of quantum entanglement.
  $\Rightarrow$ **We need a dedicated analysis!**

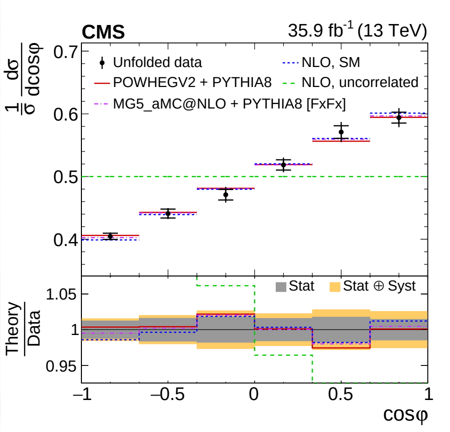


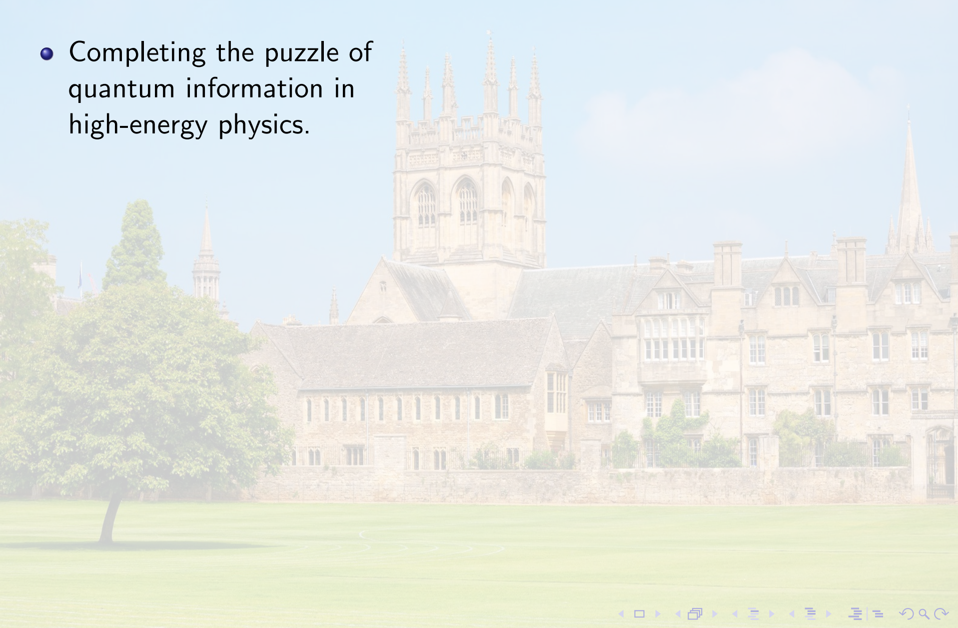
Figure: Distribution of $\cos\varphi$. Figure is from Phys. Rev. D 100, 072002.

Third Part: Novel Concepts

Discord and Steering.

What are Discord and Steering?

- Completing the puzzle of quantum information in high-energy physics.



What are Discord and Steering?

- Completing the puzzle of quantum information in high-energy physics.
- Quantum Discord:
  - The most basic form of quantum correlations.
  - Asymmetric between different subsystems.
- Quantum Steering:
  - Measurements on one subsystem can be used to “steer” the other one.
  - A non-local feature that lies between entanglement and Bell non-locality.

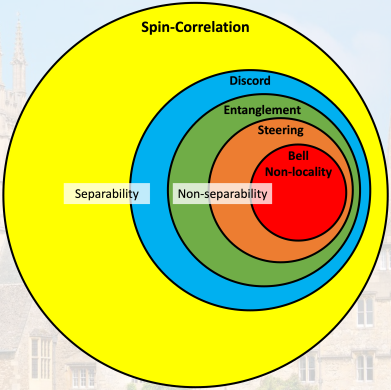


Figure: Schematic description of the relation between the different concepts discussed in the talk.

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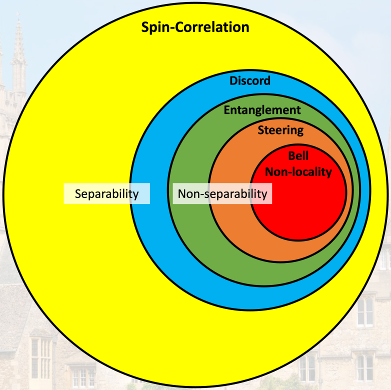


Figure: Schematic description of the relation between the different concepts discussed in the talk.

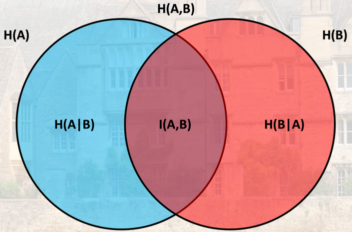
Bell Non-locality $\subset$ *Steering* $\subset$ *Entanglement* $\subset$ *Discord* $\subset$ *Spin-Correlation*

Quantum Discord

- Classically: $I(A,B) = H(A) + H(B) - H(A,B) = H(A) - H(A|B)$, $H(X)$ is the Shannon entropy.
- QM "discord": $\mathcal{D}(A,B) \equiv H(B) - H(A,B) + H(A|B) \neq 0$.
- The condition for discord in a two-qubit system is:
  $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}}) \neq 0$.

with $S(\rho) = -\mathrm{Tr}\,\rho \log_2 \rho$
the Von Neumann entropy.

- Can be asymmetric:
  $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$.
  $\rightarrow$ A test for CP-violation.



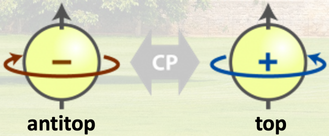


Figure: Schematic description of two subsystems with mutual information.

Steering

- Measurement of how Alice can “steer” the quantum state of Bob.
- Original conception of Schrödinger for the EPR paradox, only well-defined in 2007 (Wiseman, Jones, Doherty, PRL (2007)).

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- Alice performs a spin measurement x and obtains the result $a = \pm$.
- Bob's resulting state is the corresponding conditional states $\rho(a|x)$.
- Bob has to believe that Alice can influence his state, unless local hidden state holds.
- Can be asymmetric.
  $\rightarrow$ A test for CP-violation.

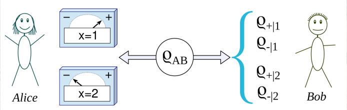
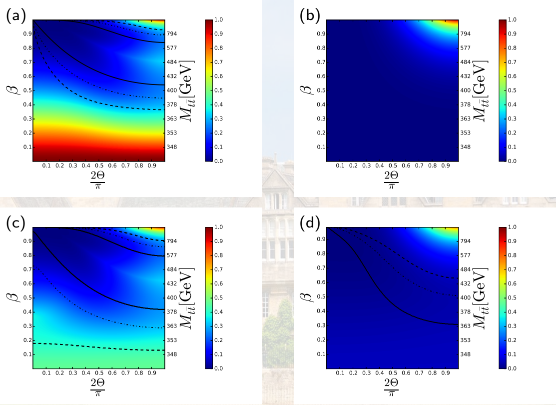


Figure: Schematic description of the steering phenomenon: Figure is from Uola, Costa, Nguyen, Gühne, Rev. Mod. Phys. (2020).

Discord and Steering Before Integration

a) $gg \to t\bar{t}$ Discord.
b) $q\bar{q} \to t\bar{t}$ Discord.
c) Full LHC $\rho(M_{t\bar{t}}, \hat{k})$ Discord.
d) Full Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Discord.

- Solid, dashed-dotted, dashed black lines are the critical boundaries of entanglement, steerability, and Bell non-locality, respectively.



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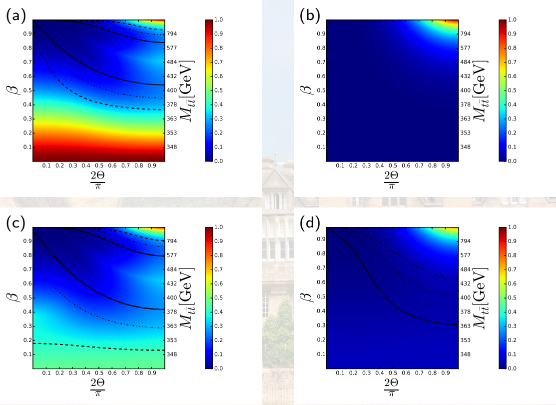


Full picture of quantum correlations in $t\bar{t}$.

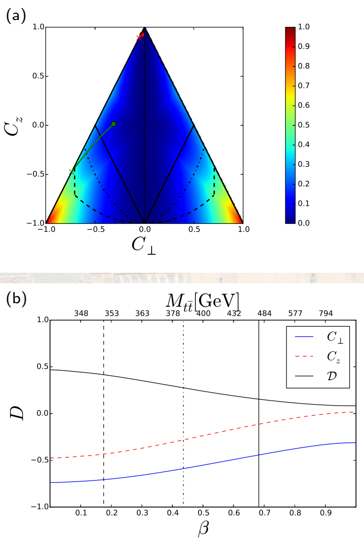
Discord and Steering After Integration

- Integration only for $[2m_t, M_{t\bar{t}}]$.

a) Discord for $C_\perp, C_z$ (symmetry around the beam axis).

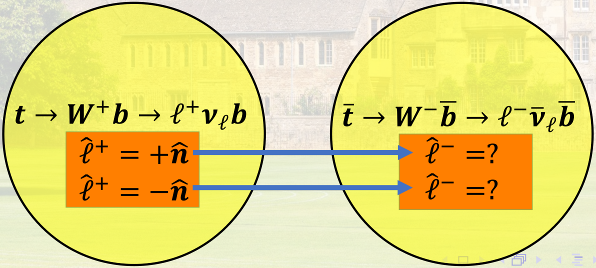
- Green: LHC trajectory;
  Red: Tevatron trajectory.
- Cross: $\beta = 0$; Circle: $\beta = 1$.
- Quantum discord: $C_\perp \neq 0$.
- Solid, dashed-dotted, dashed black lines are the critical boundaries of entanglement, steerability, and Bell non-locality, respectively.

b) Detailed trajectory of green line in the upper panel.



Experimental Measurement - Discord

- The tomography is required for $\rho_{A,B}$, ρ, $\rho_{\hat{\mathbf{n}}}$, $\rho_{-\hat{\mathbf{n}}}$:
  $\mathcal{D}_A = S(\rho_B) - S(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} S(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} S(\rho_{-\hat{\mathbf{n}}}) \neq 0$.
  $\rightarrow$ Can be done by measuring the differential cross-sections.
- One-qubit tomography of $\rho_{\hat{\mathbf{n}}}$ from conditional Bloch vectors $\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}$:
  $p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-}{(4\pi)^2}$
  $p(\hat{\ell}_\pm | \hat{\ell}_\mp = \mp\hat{\mathbf{n}}) = \frac{p(\hat{\ell}_\pm, \hat{\ell}_\mp = \mp\hat{\mathbf{n}})}{p(\hat{\ell}_\mp = \mp\hat{\mathbf{n}})} = \frac{1 \pm \mathbf{B}^{\pm}_{\hat{\mathbf{n}}} \cdot \hat{\ell}_\pm}{4\pi}$.
- Actual discord is evaluated from minimization over $\hat{\mathbf{n}}$.
  $\rightarrow$ Measuring discord according to its very definition.



Experimental Measurement - Steering

- Steering ellipsoid: the set of states to which Bob can steer Alice.
  - Forms an ellipsoid $\mathcal{E}_A$ in Alice's Bloch sphere, containing her Bloch vector **a**.
  - Fundamental object in quantum information.
  - Contains most of the information about system's quantumness.
- Measurement of $\mathbf{B}^{\pm}_{\hat{\mathbf{n}}}$ enables the reconstruction of $t, \bar{t}$ steering ellipsoids.
- Highly-challenging measurements in conventional setups.
  $\rightarrow$ Natural implementation in colliders.

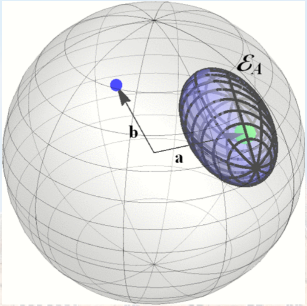


Figure: Ellipsoid representation of a two-qubit state. Figure is from Jevtic, Pusey, Jennings, Rudolph, PRL (2014).

Summary

- Implementation of ABC in Quantum Information at colliders, in particular at the LHC: interdisciplinary, huge potential and great interest.
- Quantum Information perspective: new system to test quantum foundations at the highest-energy scale so far. Genuinely relativistic, exotic symmetries and interactions, fundamental nature.
  Discord (directly from its definition) and Steering ellipsoid, challenging measurements in conventional laboratory setups, can be measured.
- High-Energy perspective: Quantum Information techniques have inspired new approaches to test physics beyond the Standard Model.
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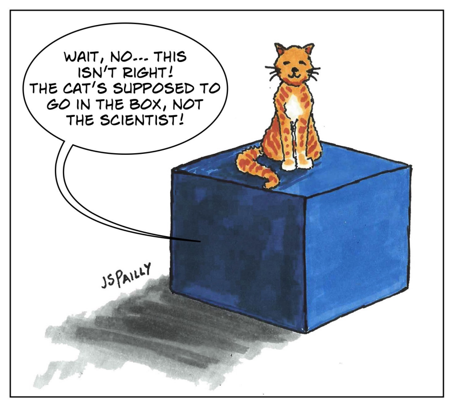
- The first measurements of entanglement between a pair of top-quarks are ongoing within ATLAS and CMS.
- **The results are expected to be public soon - stay tuned!**

Thank You

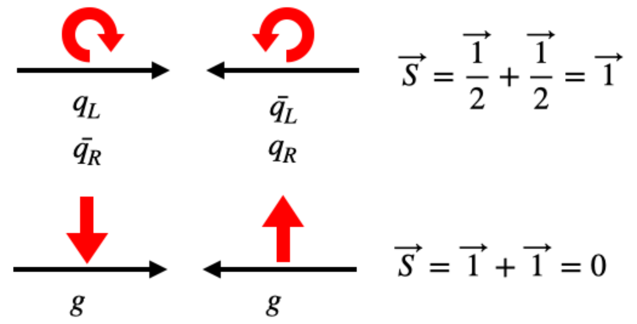

WAIT, NO... THIS ISN'T RIGHT! THE CAT'S SUPPOSED TO GO IN THE BOX, NOT THE SCIENTIST!
JSPAILLY

Backup Slides

Backup

Intuition: Spin States at Threshold

- The state is determined by the initial spins.
- $q\bar{q}$: $\rho^{q\bar{q}} = \left(\left|\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle\left\langle\uparrow_{\hat{p}}\uparrow_{\hat{p}}\right| + \left|\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle\left\langle\downarrow_{\hat{p}}\downarrow_{\hat{p}}\right|\right)/2$.
- gg: $\rho^{gg} = \left|\Psi_0\right\rangle\left\langle\Psi_0\right|$, with $\left|\Psi_0\right\rangle = \left(\left|\uparrow_{\hat{p}}\downarrow_{\hat{p}}\right\rangle - \left|\downarrow_{\hat{p}}\uparrow_{\hat{p}}\right\rangle\right)/\sqrt{2}$.
- $q\bar{q} \to$ correlated, not entangled; $gg \to$ correlated, entangled.



Basis Selection

- Helicity basis: $\{\hat{k}, \hat{r}, \hat{n}\}$:
  - $\hat{k}$ - direction of the top in the $t\bar{t}$ CM frame.
  - $\hat{p}$ - direction of the beam.
  - $\cos\Theta = \hat{k}\cdot\hat{p}$.
  - $\hat{r} = (\hat{p} - \cos\Theta\hat{k})/\sin\Theta$.
  - $\hat{n} = \hat{r}\times\hat{k}$.
- Beam basis: $\{\hat{x}, \hat{y}, \hat{z}\}$:
  - $\hat{z}$ along the beam axis.
  - $\hat{x}, \hat{y}$ transverse directions to the beam.
  - After averaging: $C_x = C_y = C_\perp$.

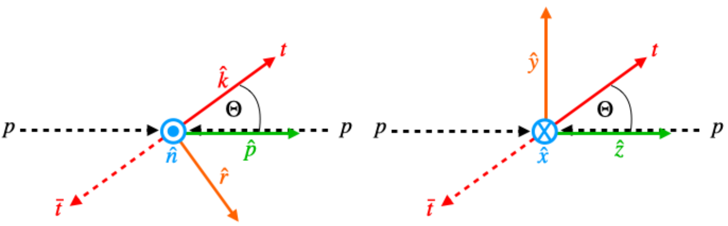


Figure: Helicity and beam bases.

High-Energy Physics Example

- At B-Factories, e^+e^- collisions can be properly adjusted in order to create $\Upsilon(4S)(b\bar{b})$.
- $\Upsilon(4S)(b\bar{b})$ decays to $B^0 + \bar{B}^0$, where we have $|B^0\rangle = |\bar{b}d\rangle, |\bar{B}^0\rangle = |b\bar{d}\rangle$.
- We get an entangled state: $\frac{1}{\sqrt{2}}(|B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle)$.

Quantum Tomography

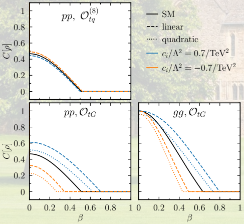
- **Quantum Tomography**: reconstruction of the quantum state from measurement of a set of expectation values.
- Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ can be extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of di-leptonic decay:

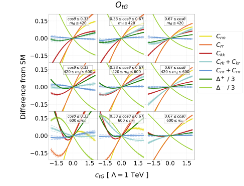
$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left[1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-\right]$$

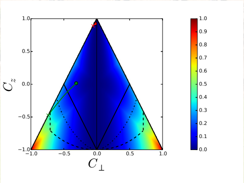
- Symmetry around beam axis:
  - 2 spin correlations $C_\perp, C_z$.
  - 2 individual spin (longitudinal) polarizations $B_z^{\pm}$.
- No assumption on the particular form of the quantum state:
  - 9 spin correlations C_{ij}.
  - 6 individual spin polarizations $B_i^{\pm}$.

Physics Beyond the Standard Model: Examples

- Change in the concurrence (the entanglement marker) with new interactions: Aoude, Madge, Maltoni, Mantani, PRD (2022).
- Change at NLO from the SM value for spin observables for new interactions: Severi, Vryonidou, JHEP (2023).
- Asymmetric quantum correlations (e.g. discord) imply CP-violating new physics. YA, de Nova, 2210.09330.







Physics Beyond the Standard Model: Examples

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu} T^A t)\tilde{\varphi} G^A_{\mu\nu} + \text{h.c.}\,, \qquad (1)$$

$$\mathcal{O}^{(8)}_{tq} = (\bar{t}_R \gamma_\mu T^a t_R)(\bar{q}_L \gamma^\mu T^a q_L)\,, \qquad (2)$$

Quantum State

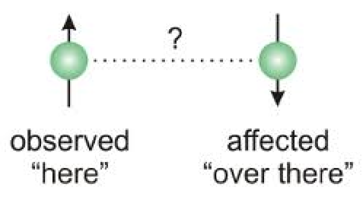
- **Pure state:** can be described by wave-functions $\sum_i \alpha_i \cdot |\psi_i\rangle$.

- **Mixed state:** can be described by a density matrix: $\rho = \sum_i p_i \cdot |\psi_i\rangle \langle\psi_i|$.
  - Example: at the LHC we cannot control the initial state.

- **Quantum Tomography**: reconstruction of the quantum state from measurement of a set of expectation values.

What is Quantum Entanglement?

- Quantum state of one particle cannot be described independently from another particle.
- $\Rightarrow$ **Correlations** of observed physical properties of both systems.
- $\Rightarrow$ **Measurement** performed on one system seems to be influencing other systems entangled with it.



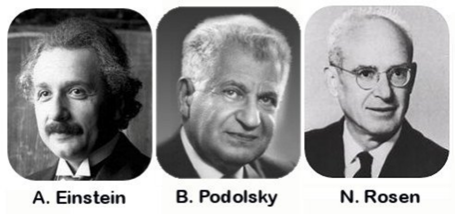
- Observed in photons, atoms, superconductors, mesons, analog Hawking radiation, nitrogen-vacancy centers in diamond and even macroscopic diamond.

Quantum Entanglement

- Two different systems A and B: $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$.
- Separable: $\rho = \sum_n p_n \rho_n^a \otimes \rho_n^b$.
- $\rho_n^{a,b}$ are quantum states in A, B, $\sum_n p_n = 1,\ \ p_n \geq 0$
- Classically correlated state in $\mathcal{H} \to$ can be written in this form.
- Non-separable state is called entangled and hence, it is a non-classical state.

- For two qubits:
  - Separability $\Longleftrightarrow$ Classical probability distribution.
  - Entanglement $\Longleftrightarrow$ No classical probability distribution description.

EPR Paradox



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

EPR Paradox

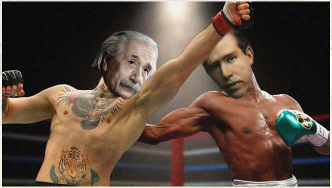
- Entanglement: "spooky action at a distance" (A. Einstein).

- Assuming two particles with spacial distance.
- When a measurement is done on one of the particles, the other one "knows" about it immediately.
- Information travel faster than light?
- Contradicts the theory of relativity.
- **Conclusion:** the theory of Quantum Mechanics is incomplete.

Hidden Variables

- By EPR, each particle "carries" variables that know the state before the measurement.
  ⇒ There are some hidden variables that are missing in order to have a full theory.
- The Copenhagen Interpretation: superposition of states until a measurement was done.
- Bohr Vs. Einstein.

"God does not play at dice with the universe".



"Quit telling God what to do!"

- Who is right?

Bell Inequality

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

- If local hidden variables hold, they should satisfy some inequality.
- $C(x, y)$ are the correlations between different measurements at different detectors.
- The parameters a,b,c are different directions for the measurement.
- Original form: $1 + C(b, c) \geq |C(a, b) - C(a, c)|$.

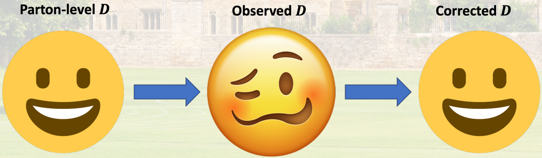
The Nobel Prize in Physics 2022

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science". (link)

Figure: Alain Aspect, John F. Clauser and Anton Zeilinger.

Ongoing Measurement

- Measurements are ongoing within ATLAS and CMS.
- Goal: measure $D < -1/3$ with high statistical significance, preferably more than 5σ.
  - Simple observable.
  - Difficult (narrow) phase space.
- Not all $t\bar{t}$ events enter the (basic) analysis selections:
  - Efficiency.
  - Detector acceptance.
- Detector reconstruction of the objects distort the observable.
- We need to find a way to measure the parton-level value of D.



Calibration Curve

- We can use a calibration curve.
- A link between the observed data and the parton-level value.
- The curve is built from samples with alternative sets the **observed** D and **corrected** D.

  - We measure a value of D from the data (**Observed** D).
  - We match it to the corresponding parton-level value (**Corrected** D).

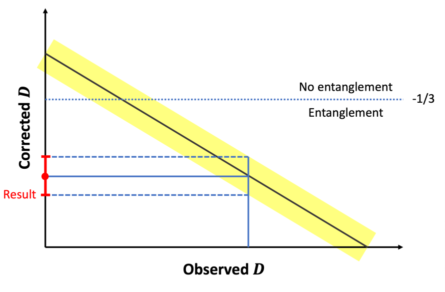
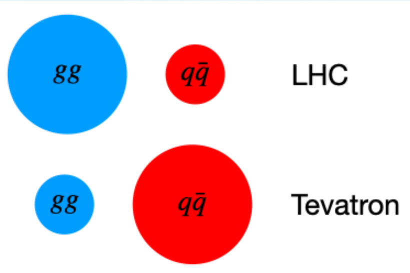


Figure: An illustration of a calibration curve.

- The significance is the difference between the result and the null hypothesis:
  Corrected $D = -1/3$.

Leading-order Analytical Calculation



- Analytical calculation at leading-order. The system is defined by:
  - $\hat{k}$: the direction of the top with respect to the beam axis.
  - The invariant mass $M_{t\bar{t}}$, $\beta = \sqrt{1 - \frac{4 \cdot m_t^2}{M_{t\bar{t}}^2}}$.
- Each one $I = q\bar{q}, gg$ gives rise to $\rho^I(M_{t\bar{t}}, \hat{k})$ with probability $w_I(M_{t\bar{t}}, \hat{k})$, which is PDF dependent.
- The spin density matrix: $\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \hat{k})\rho^I(M_{t\bar{t}}, \hat{k})$.
- The total quantum state:
  $\rho(M_{t\bar{t}}) \equiv \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \; p(M, \hat{k})\rho(M, \hat{k}) = \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \; p(M)\rho_\Omega(M)$

Critical Values After Integration

- We focus on pp interactions.
- Clear motivation to restrict to selected regions of phase space.
- Plot is shown with integration only for $[2m_t, M_{t\bar{t}}]$.
- We focus on the region close to threshold. For high p_T see:
  - Fabbrichesi, Floreanini, Panizzo, PRL (2021).
  - Severi, Boschi, Maltoni, Sioli, EPJC (2022).

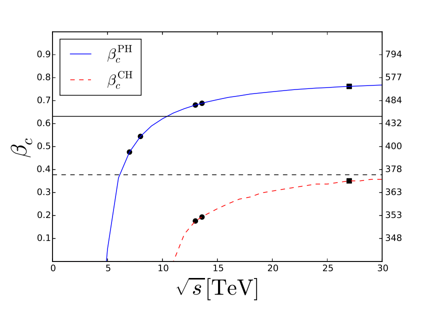


Figure: Critical values below which entanglement and CHSH violation can be observed, for different COM values.