## Towards testing beyond-quantum theories in particle physics

Proc. R. Soc. A. 478:20210806 (2022), arXiv:2103.12000 Michał Eckstein ${ }^{1,2}$ \& Paweł Horodecki ${ }^{2,3}$
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JAGIELLONIAN UNIVERSITY IN KRAKÓW


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## Bell nonlocality - the black box approach

2 parties (Alice and Bob) - 2 inputs $(x, y)-2$ outputs $(a, b)$

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P(a, b \mid x, y)
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The experimental (frequency) correlation function
[Sandu Popescu, Nature Physics 10, 264 (2014)]


Quantum Mechanics [Cirelson (1980)]

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- Alice and Bob can communicate,
- or their settings are pre-correlated.


## But can we do it assuming:

- no-signalling:

- freedom of choice: $P(x, y \mid \lambda)=P(x) \cdot P(y)$ ? Yes!

otherwise,

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S_{\mathrm{PR}}=4 .
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No-signalling boxes [Popescu, Rohrlich (1994)]

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P(a, b \mid x, y)=\left\{\begin{array}{ll}
\frac{1}{2}, & \text { if } a \oplus b=x y, \\
0, & \text { otherwise },
\end{array} \quad S_{\mathrm{PR}}=4\right.
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## Beyond-quantum theories

(1) Beyond-quantum correlations

- No-signalling boxes
[N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani,
S. Wehner, Rev. Mod. Phys. 86, 419 (2014)]
- 3-party monogamy violation
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Purely operational 'theories' - model-independent approach

## Objective collapse models


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Collapse models involve deviations from unitarity and linearity.

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- Is QFT only an effective description of Nature at small scales?


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$\overline{C_{q s}} \subsetneq C_{q c} \quad$ [Z. Ji, A. Natarajan, T. Vidick, J. Wright, H. Yuen, arXiv:2001.04383]

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## Quantum-data boxes

- We regard a chosen physical system as a Q-data box, which can be probed with quantum information.
- Quantum mechanics is valid outside the box, but not necessarily inside.

- The pure input state is prepared, $P: x \rightarrow \psi_{\mathrm{in}}$
- The output state is reconstructed via quantum tomography from the outcomes of projective measurements $M: \rho_{\text {out }} \rightarrow a$.
- $n$ are classical parameters (e.g. scattering kinematics)


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## Quantum-data tests



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- For every input state $\%$ one needs to perform the full tomography of pout
- A Q-data test yields a dataset $\left\{\psi_{\text {in }}^{(k)}, p^{(\ell)} ; \rho_{\text {out }}^{(k, \ell)}\right\}_{k}$

A particular instance of a Q-data test is the quantum process tomography

- QM implies that any $\operatorname{man} \mathcal{E}: \mathcal{H}_{\text {in }} \rightarrow S\left(\mathcal{H}_{\text {out }}\right)$ is CPTP
- CPTP map can be reconstructed from $\left\{\psi_{\text {in }}^{(k)} ; \rho_{\text {out }}^{(k)}\right\}_{k}, k=1, \ldots,(\operatorname{dim} \mathcal{\mathcal { L }} \text { in })^{2}$
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## An example - the Helstrom test

- Suppose that we have two available inputs $\psi_{\text {in }}^{(1)}, \psi_{\text {in }}^{(2)}$.
- We choose randomly the input (with probability $1 / 2$ )
- The task is to guess, which of the two states was input.
- Define the success rate: $P_{\text {succ }}\left(v_{\text {in }}^{(1)}, \psi_{\text {in }}^{(2)}\right):=\frac{1}{2} \sum_{k=1}^{2} P\left(a=k \mid \psi_{\text {in }}^{(k)}\right)$
- In quantum theory $P_{\text {succ }}$ cannot exceed the Helstrom bound

- Make a Q-data test with $\left\{\psi_{\text {in }}^{(k)} ; \rho_{\text {out }}^{(k)}\right\}_{k=1,2}$
- If $P_{\text {succ }}\left(P_{\text {out }}^{(1)}, P_{\text {out }}^{(2)}\right)>P_{\text {succ }}\left(\psi_{i}^{(1)}, \psi^{(2)}\right)$ then the $Q$-data box is not quantum
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## Main idea:

(1) Prepare a 'quantum-programmed' particle carrying $\psi_{\text {in }}$, e.g. electron's spin or photon's polarization
(2) Scatter it on a nucleonic target.
(3) Perform projective measurements
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(4) Reconstruct the output state $\rho_{\text {out }}$

Challenges:

- Need to prepare the quantum state of GeV particles
- Abundance of projectiles in high-energy collisions
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> Proc. R. Soc. A. 478:20210806 (2022), arXiv:2103.12000

Take-home messages:

- Quantum mechanics can be probed from an 'outside' perspective.
- Whenever we are doing a Bell-type test, we are testing QM against both local hidden variables and beyond-quantum correlations.
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