Bell violation by relativistic particles

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- Almost all of these papers used massive spin-$1/2$ particles.

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We have considered EPR experiment with vector bosons in:

P.C., J.R., M.Włodarczyk., Phys. Rev. A 77, 012103 (2008);

P.C., Phys. Rev. A 77, 062101 (2008);

A.J.Barr., P.C., J.R., arXiv:2204.11063 (2022).

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$k^2 = m^2$, m – mass of the particle, $\sigma = -1, 0, 1$.

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The vectors $|k, \sigma\rangle$ can be generated from the standard vector $|\tilde{k}, \sigma\rangle$:
$|k, \sigma\rangle = U(L_k)|\tilde{k}, \sigma\rangle$, $\tilde{k} = m(1, 0, 0, 0)$, $k = L_k \tilde{k}$, $L_{\tilde{k}} = I$.

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The standard Wigner procedure leads to (Λ – Lorentz transformation)

$$U(\Lambda)|k, \sigma\rangle = \mathcal{D}_{\lambda\sigma}(R(\Lambda, k))|\Lambda k, \lambda\rangle,$$

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For $s = 1$ the representation $\mathcal{D}(R)$ is unitary equivalent to R by

$$\mathcal{D}(R) = VRV^\dagger, \qquad V^\dagger V = I, \qquad V = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

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s – spin of a particle,
$\hat{W}^\mu = \frac{1}{2}\epsilon^{\nu\gamma\delta\mu}\hat{P}_\nu \hat{J}_{\gamma\delta}$ – the Pauli-Lubanski four-vector, $(\epsilon^{0123} = 1)$,
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Different choices of $\hat{\mathbf{Q}}$ lead to different spin operators.

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The most popular choice of $\hat{\mathbf{Q}}$: the Newton–Wigner position operator [Rev. Mod. Phys. 21, 400 (1949).]

$$\hat{\mathbf{Q}}_{NW} = -\frac{1}{2}\left[\frac{1}{\hat{P}^0}\hat{\mathbf{K}} + \hat{\mathbf{K}}\frac{1}{\hat{P}^0}\right] - \frac{\hat{\mathbf{P}} \times \hat{\mathbf{W}}}{m\hat{P}^0(m + \hat{P}^0)},$$

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Properties of the Newton-Wigner position operator:

- it is a vector;
- it has commuting components;
- it has self-adjoint components;
- it is defined for arbitrary spin;
- it does not transform in a manifestly covariant way under Lorentz boosts.

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The spin operator related to the Newton-Wigner position operator:

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- it is the only axial vector which is a linear function of the Pauli-Lubanski four-vector components;
- under Lorentz group action it transforms according to $\hat{\mathbf{S}}'_{NW} = R(\Lambda, \hat{P})\hat{\mathbf{S}}_{NW}$, where $R(\Lambda, \hat{P})$ is the corresponding Wigner rotation.

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The spin operator $\hat{\mathbf{S}}_{NW}$ acts on one-particle states according to

$$\hat{\mathbf{S}}_{NW}|k,\sigma\rangle = \mathbf{S}_{\lambda\sigma}|k,\lambda\rangle,$$

where S^i are standard spin-1 matrices:

$$S^1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix},\ S^2 = \frac{i}{\sqrt{2}}\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & -1\\ 0 & 1 & 0\end{pmatrix},\ S^3 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}.$$

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Observable corresponding to the normalized projection of this operator on the direction **a** has the following form:

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Projections of both these spin operators on momentum direction are equal:

$$\hat{\mathbf{S}}_C(\hat{\mathbf{P}}) = \hat{\mathbf{P}}\cdot\hat{\mathbf{S}}_{NW}.$$

Boson field

Boson field

To describe two types of vector bosons, particle and antiparticle, we consider the free field operator $\hat{\phi}^{\mu}(x)$:

$$\hat{\phi}^{\mu}(x) = \frac{1}{(2\pi)^{(3/2)}} \int \frac{d^3\mathbf{k}}{2\omega_k} \left[e^{ikx} e^{\mu}_{\sigma}(k) a^{\dagger}_{\sigma}(k) + e^{-ikx} e^{*\mu}_{\sigma}(k) b_{\sigma}(k) \right],$$

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They fulfill the standard canonical commutation relations

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The Klein-Gordon equation and Lorentz transversality condition imply

$$k^2 = m^2, \quad k_\mu e^\mu_\sigma(k) = 0.$$

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The one-particle and one-antiparticle states are given by

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This condition leads to the explicit form of $e^\mu_\sigma(k)$:

$$e(k) = [e^\mu_\sigma(k)] = \begin{pmatrix} \frac{\mathbf{k}^T}{m} \\ I + \frac{\mathbf{k}\otimes\mathbf{k}^T}{m(m+k^0)} \end{pmatrix} V^T,$$

Covariant states

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We can construct covariant particle/anti-particle states

$$|(\mu, k)\rangle_{a/b} = e^{\mu}_{\sigma}(k)|k, \sigma\rangle_{a/b}.$$

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$$|(\mu, k)\rangle_{a/b} = e^{\mu}_{\sigma}(k)|k, \sigma\rangle_{a/b}.$$

These states transform in a manifestly covariant way under Lorentz transformations

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Finally, a two-particle boson–antiboson covariant state:

$$e^{\mu}_{\lambda}(k) e^{\nu}_{\sigma}(p)|(k, \lambda)_a; (p, \sigma)_b\rangle.$$

Two-particle scalar states

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A general scalar state has the following form

$$|\alpha(k,p)\rangle = g_{\mu\nu}(k,p) e^{\mu}_{\lambda}(k) e^{\nu}_{\sigma}(p) |(k,\lambda);(p,\sigma)\rangle,$$

where

$$g_{\mu\nu}(k,p) = \eta_{\mu\nu} + \frac{c}{(kp)}\big(k_\mu p_\nu + p_\mu k_\nu\big), \quad c \in \mathbb{R}.$$

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Normalization

$$\langle\alpha(k,p)|\alpha(k,p)\rangle = 4k^0 p^0 (\delta^3(\mathbf{0}))^2 A(k,p),$$

with

$$A(k,p) = 2 + \left[c\,\tfrac{m^2}{(kp)} - \tfrac{(kp)}{m^2}(1+c)\right]^2.$$

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Operators which act like a spin on particles a (antiparticles b) whose momenta belong to region Ω in momentum space and gives 0 otherwise:

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The spectral decomposition of the operator $\boldsymbol{\omega}\cdot\hat{\mathbf{S}}^a_\Omega$:

$$\boldsymbol{\omega}\cdot\hat{\mathbf{S}}^a_\Omega = 1\cdot\Pi^{a+}_{\Omega\boldsymbol{\omega}} + (-1)\cdot\Pi^{a-}_{\Omega\boldsymbol{\omega}} + 0\cdot\Pi^{a0}_{\Omega\boldsymbol{\omega}},$$

and analogously for $\boldsymbol{\omega}\cdot\hat{\mathbf{S}}^b_\Omega$.

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The correlation function is defined as

$$C(\mathbf{a}, k; \mathbf{b}, p) = \sum_{\lambda,\sigma=-1,0,1} \lambda\, \sigma\, P(\mathbf{a}, \mathbf{b})_{\lambda\sigma}.$$

Bell-type inequalities for vector bosons

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The CHSH inequality can be written in the form

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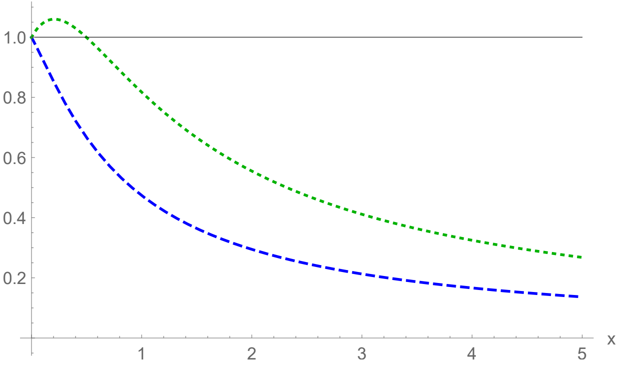

Comparison of the violation of the Mermin inequality in the state $|\xi(k, k^{\pi})\rangle$ (blue, dashed line) and in the state $|\psi(k, k^{\pi})\rangle$ (green, dotted line). The configuration of particles momenta and measurements directions is the following: $\mathbf{n} = (0, 0, 1)$, $\mathbf{w} = (\cos\phi_w \sin\theta_w, \sin\phi_w \sin\theta_w, \cos\theta_w)$, $\mathbf{w} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ and $\theta_a = 1.593$, $\phi_a = 3.236$, $\theta_b = 1.564$, $\phi_b = 1.150$, $\theta_c = 1.514$, $\phi_c = 5.322$.

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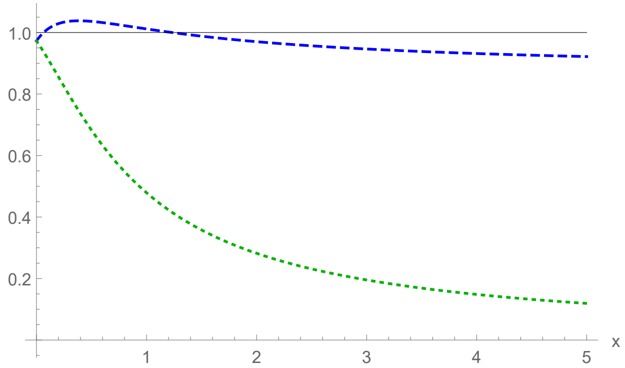

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Denoting $P(A_i = B_j + k) = \sum_{l=0}^{l=2} P(A_i = l, B_j = l + k \mod 3)$ and

$$\begin{aligned}\mathcal{I}_3 = &\big[P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1)\big] \\ -&\big[P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)\big],\end{aligned}$$

the CGLMP inequality can be written in the form

$$\mathcal{I}_3 \leq 2.$$

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We identify spin projections $-1, 0, 1$ with outcomes $0, 1, 2$:

$$-1 \leftrightarrow 0, \quad 0 \leftrightarrow 1, \quad 1 \leftrightarrow 2,$$

and measurements A_1, B_1, A_2, B_2 with spin projections on $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, respectively.

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$$\begin{aligned}\mathcal{I}_3 = C(\mathbf{a},\mathbf{b}) + C(\mathbf{c},\mathbf{d}) + C(\mathbf{a},\mathbf{d}) - C(\mathbf{c},\mathbf{b}) \\ + P_{+-}(\mathbf{a},\mathbf{b}) + P_{+-}(\mathbf{c},\mathbf{d}) + P_{-+}(\mathbf{a},\mathbf{d}) - P_{+-}(\mathbf{c},\mathbf{b}) \\ + P_{00}(\mathbf{a},\mathbf{b}) + P_{00}(\mathbf{c},\mathbf{d}) + P_{00}(\mathbf{a},\mathbf{d}) - P_{00}(\mathbf{c},\mathbf{b}) \\ - \big[P_{0-}(\mathbf{a},\mathbf{b}) + P_{0-}(\mathbf{c},\mathbf{d}) + P_{-0}(\mathbf{a},\mathbf{d}) - P_{0-}(\mathbf{c},\mathbf{b}) \\ + P_{+0}(\mathbf{a},\mathbf{b}) + P_{+0}(\mathbf{c},\mathbf{d}) + P_{0+}(\mathbf{a},\mathbf{d}) - P_{+0}(\mathbf{c},\mathbf{b})\big].\end{aligned}$$

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In [A.J.B., P.C., J.R., arXiv:2204.11063] we have shown that the CGLMP inequality can be violated either in the state $|\psi(k, k^\pi)\rangle$ or in the state $|\xi(k, k^\pi)\rangle$.

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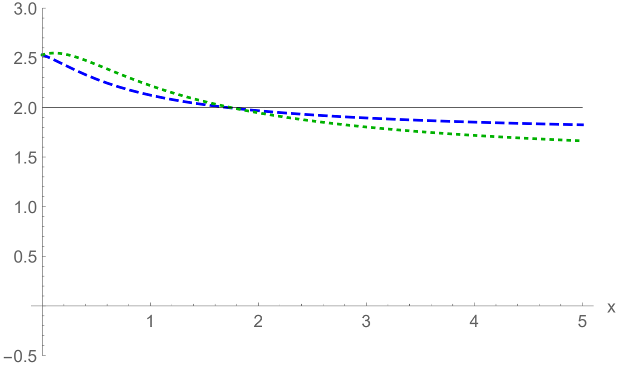

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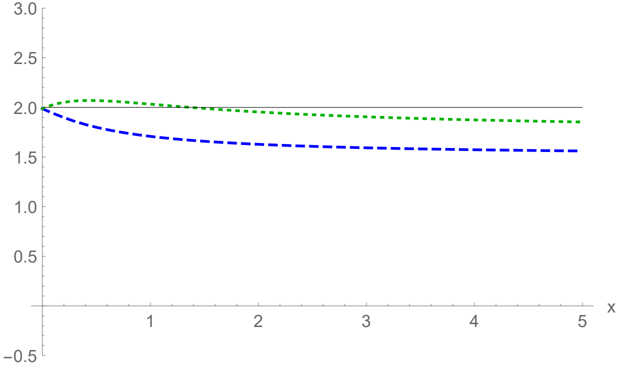

Comparison of the violation of the CGLMP inequality in the state $|\xi(k, k^{\pi})\rangle$ (blue, dashed line) and in the state $|\psi(k, k^{\pi})\rangle$ (green, dotted line). The configuration of particles momenta and measurements directions is the following: $\mathbf{n} = (0, 0, 1)$, $\mathbf{w} = (\cos\phi_w \sin\theta_w, \sin\phi_w \sin\theta_w, \cos\theta_w)$, $\mathbf{w} \in \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ and $\theta_a = 3.141$, $\phi_a = 0$, $\theta_b = 0$, $\phi_b = 0$, $\theta_c = 0.836$, $\phi_c = 5.044$, $\theta_d = 2.754$, $\phi_d = 1.897$.

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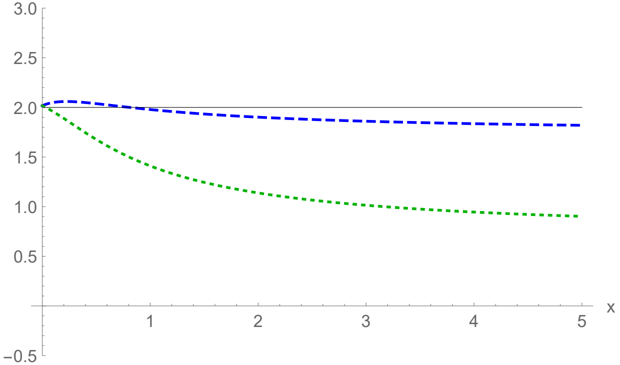

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  - and the state which in the massless limit converges to the scalar two-photon state $|\xi(k, k^{\pi})\rangle$.
- We have shown that both the Mermin and CGLMP inequalities can be violated in both states $|\psi(k, k^{\pi})\rangle$ and $|\xi(k, k^{\pi})\rangle$ and that the degree of violation depends on bosons momenta.

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- In [P.C., J.R., M.W., Phys. Rev. A 79, 014102 (2009)] we compared correlation functions calculated with the help of two different spin operators: $\hat{\mathbf{S}}_{NW}$ and $\hat{\mathbf{S}}_C$.