Quantum State-Channel Duality applied to Particle Physics*

Clelia Altomonte** (she/her)

Foundational tests of Quantum Mechanics at the LHC

20th-22nd March 2023

Merton College, Oxford

*C. Altomonte, and A.J. Barr, "Quantum State-Channel Duality applied to Particle Physics"(2022).

**MMathPhys dissertation project; King's College London, TPPC probationary PhD student.

Outline

Motivation: quantum information theory applied to high energy systems

Colliders as quantum computing implementations

State-channel duality

A toy model: e+e− $\rightarrow t\bar{t}$ electroweak process with polarised beams

Remarks and findings

Motivation

Quantum State-Channel Duality applied to Particle Physics

Clelia Altomonte,[1] and Alan J. Barr[2]
[1)]*Department of Physics, Strand, King's College London, WC2R 2LS*
[2)]*Department of Physics, Keble Road, University of Oxford, OX1 3RH*
Merton College, Merton Street, Oxford, OX1 4JD

(*Electronic mail: clelia.altomonte@kcl.ac.uk)

(Dated: 6 October 2022)

Abstract: Recent instances of successful application of quantum information techniques to particle physics problems invite for an analysis of the mathematical details behind such connection. In this paper, we identify the Choi-Jamiolkowski isomorphism, or state-channel duality, as a theoretical principle enabling a systematic application of the theory of quantum information to high energy physics.

[C. Altomonte, A.J. Barr, 2022], Oxford MSc Mathematical and Theoretical Physics Dissertation

Colliders as quantum computing implementations

Four conditions for quantum computing implementation [Nielsen and Chuang, 2010]:

Condition	How satisfied by Colliders
a. Robustly representing quantum information ☑	• PP observables (e.g. spin) and qudits Bloch vectors share group theoretic structure
b. Performing a universal family of unitary transformations. ⚠	• State-Channel duality applied to PP (the topic of this talk)
c. Preparing a fiducial initial state. ☑	• Current colliders [Y. Afik and J. R. M. de Nova, 2021] • Future Colliders with polarised beams (e.g. CLIC) [H. Abramowicz et al., 2019]
d. Measuring the output result. ☑	• ATLAS

State-Channel duality

- In quantum information theory, the **initial** and **final states** of a qudit are **connected** through operations called **quantum channels**. (One or more quantum operations (where B(H) is the bounded operator B on H)
- Can we find quantum channels connecting the density matrices describing the initial and final states of **particle physics processes**?
- YES, because of **State-Channel duality** (or Choi-Jamiolkowski isomorphism)

Quantum Channel	State (Choi-matrix)
A **quantum channel** = linear, completely positive, trace preserving map E : B(H) → B (H ’), connecting an input state space H of dimension d and an output state space H ’ of dimension d’	*a* **bipartite state** of the tensor product of the input state space H and output state space H ’ (Choi-Matrix)
$\mathcal{E}(\hat{\rho}) = \mathcal{E}\left(\sum_{ij} \hat{\rho}_{ij} \lvert i\rangle\langle j\rvert\right) = \sum_{i} \hat{\rho}_{ij}\mathcal{E}(\lvert i\rangle\langle j\rvert)$	$\tilde{\mathcal{E}} = \begin{bmatrix} \mathcal{E}(\lvert 0\rangle\langle 0\rvert) & \mathcal{E}(\lvert 0\rangle\langle 1\rvert) & \mathcal{E}(\lvert 0\rangle\langle 2\rvert) & \cdots \\ \mathcal{E}(\lvert 1\rangle\langle 0\rvert) & \mathcal{E}(\lvert 1\rangle\langle 1\rvert) & \mathcal{E}(\lvert 1\rangle\langle 0\rvert) & \cdots \\ \mathcal{E}(\lvert 2\rangle\langle 0\rvert) & \mathcal{E}(\lvert 2\rangle\langle 1\rvert) & \mathcal{E}(\lvert 2\rangle\langle 2\rvert) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$
Properties: • **Completely positive** iff it admits an operator-sum decomposition of the form: $\mathcal{E}(\rho) = \sum_k E_k \rho E_k^{\dagger}$. • **Trace preserving**: $\sum_k E_k^{\dagger} E_k = 1$ • caveat: operator-sum decomposition is not unique.	Related quantum channel is **physically realisable** iff Choi-matrix is a **density matrix**: $\tilde{\mathcal{E}} \geqslant 0$ Which implies: • $\mathcal{E}(\rho) \geqslant 0$ whenever $\rho \geqslant 0$ • $tr\mathcal{E}(\rho) = tr\rho$ for all ρ

is isomorphic to →

[A. Ekert, 2022]

A toy model: $e^+e^- \rightarrow t\bar{t}$ electroweak process with polarised beams

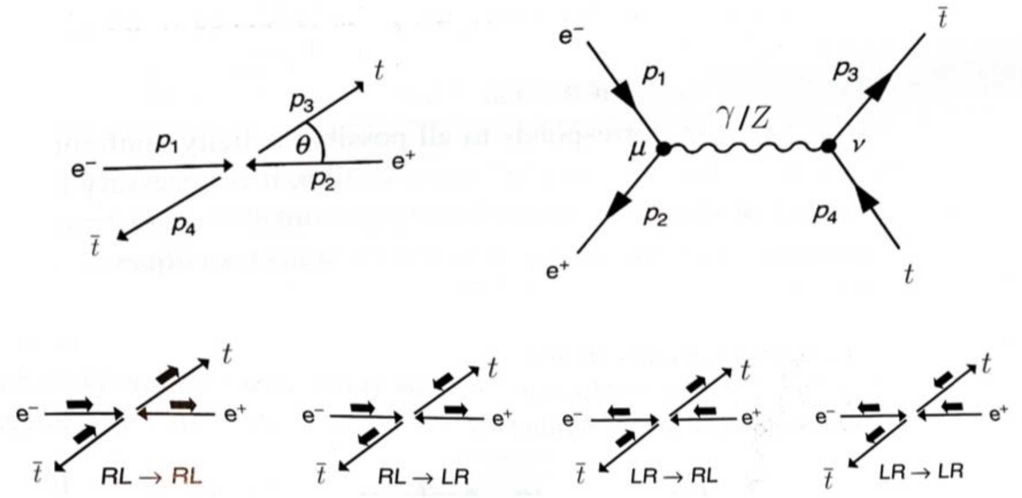


FIG. 1. The $e^+e^- \rightarrow t\bar{t}$ electroweak process viewed in the centre-of-mass frame, corresponding lowest-order Feynman diagmam, and four of the 16 possible helicity combinations (adapted from ref.[19])

[M. Thomson, 2021]

PP toy-model	Equations
Cross-section σ at l.o. in the electroweak coupling constants	$\sigma \propto \lvert\mathscr{M}\rvert^2 = \lvert\mathscr{M}_\gamma + \mathscr{M}_Z\rvert^2$
Consider, e.g. $\mathcal{M}_\gamma$	$M_\gamma = -\frac{e^2}{q^2} g_{\mu\nu} [\bar{v}(p_2)\gamma^\mu u(p_1)] [\bar{u}(p_3)\gamma^\nu v(p_4)] = -\frac{e^2}{s} j_e \cdot j_t$
Construct matrix with all 16 possible helicity combinations (suppose using polarised beams, so do not use trace techniques)	$\lvert M_\gamma\rvert^2 \propto \begin{bmatrix} \lvert M_{RL\to RL}\rvert^2 & \lvert M_{RL\to RR}\rvert^2 & \lvert M_{RL\to LL}\rvert^2 & \lvert M_{RL\to LR}\rvert^2 \\ \lvert M_{RR\to RL}\rvert^2 & \lvert M_{RR\to RR}\rvert^2 & \lvert M_{RR\to LL}\rvert^2 & \lvert M_{RR\to LR}\rvert^2 \\ \lvert M_{LL\to RL}\rvert^2 & \lvert M_{LL\to RR}\rvert^2 & \lvert M_{LL\to LL}\rvert^2 & \lvert M_{LL\to LR}\rvert^2 \\ \lvert M_{LR\to RL}\rvert^2 & \lvert M_{LR\to RR}\rvert^2 & \lvert M_{LR\to LL}\rvert^2 & \lvert M_{LR\to LR}\rvert^2 \end{bmatrix}$
Fully electric interaction	$\lvert M_\gamma\rvert^2 \propto \begin{bmatrix} (1+\cos\theta)^2 & 0 & 0 & (1-\cos\theta)^2 \\ (\sin\theta)^2 & 0 & 0 & (\sin\theta)^2 \\ (\sin\theta)^2 & 0 & 0 & (\sin\theta)^2 \\ (1-\cos\theta)^2 & 0 & 0 & (1+\cos\theta)^2 \end{bmatrix}$
Fully weak interaction	$\lvert M_Z\rvert^2 \propto \begin{bmatrix} (c_R^e c_R^t)^2 (1+\cos\theta)^2 & 0 & 0 & (c_R^e c_L^t)^2 (1-\cos\theta)^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (c_L^e c_R^t)^2 (1-\cos\theta)^2 & 0 & 0 & (c_L^e c_L^t)^2 (1+\cos\theta)^2 \end{bmatrix}$

PP	State-channel duality	Quantum computing

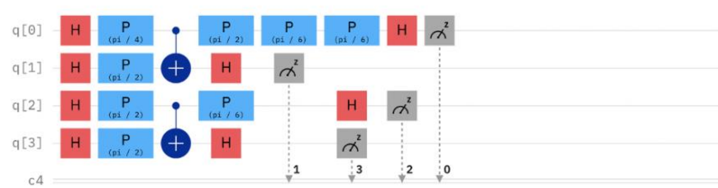
PP density matrix

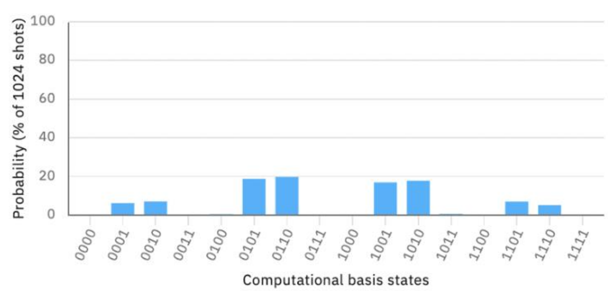
$$|M_\gamma|^2 \propto \begin{bmatrix} |M_{RL\to RL}|^2 & |M_{RL\to RR}|^2 & |M_{RL\to LL}|^2 & |M_{RL\to LR}|^2 \\ |M_{RR\to RL}|^2 & |M_{RR\to RR}|^2 & |M_{RR\to LL}|^2 & |M_{RR\to LR}|^2 \\ |M_{LL\to RL}|^2 & |M_{LL\to RR}|^2 & |M_{LL\to LL}|^2 & |M_{LL\to LR}|^2 \\ |M_{LR\to RL}|^2 & |M_{LR\to RR}|^2 & |M_{LR\to LL}|^2 & |M_{LR\to LR}|^2 \end{bmatrix}$$

Identify as Choi-matrix

$$\widetilde{\mathscr{E}} = \begin{bmatrix} \mathscr{E}(|0\rangle\langle 0|) & \mathscr{E}(|0\rangle\langle 1|) & \mathscr{E}(|0\rangle\langle 2|) & \cdots \\ \mathscr{E}(|1\rangle\langle 0|) & \mathscr{E}(|1\rangle\langle 1|) & \mathscr{E}(|1\rangle\langle 0|) & \cdots \\ \mathscr{E}(|2\rangle\langle 0|) & \mathscr{E}(|2\rangle\langle 1|) & \mathscr{E}(|2\rangle\langle 2|) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Analyse: it corresponds to a physically realisable quantum channel





Construct a quantum channel that **outputs** PP density matrix [IBM quantum]

with **arbitrary precision** (cf. Solovay-Kitaev theorem [M. Nielsen and I. Chuang, 2010]

Constructing quantum channels

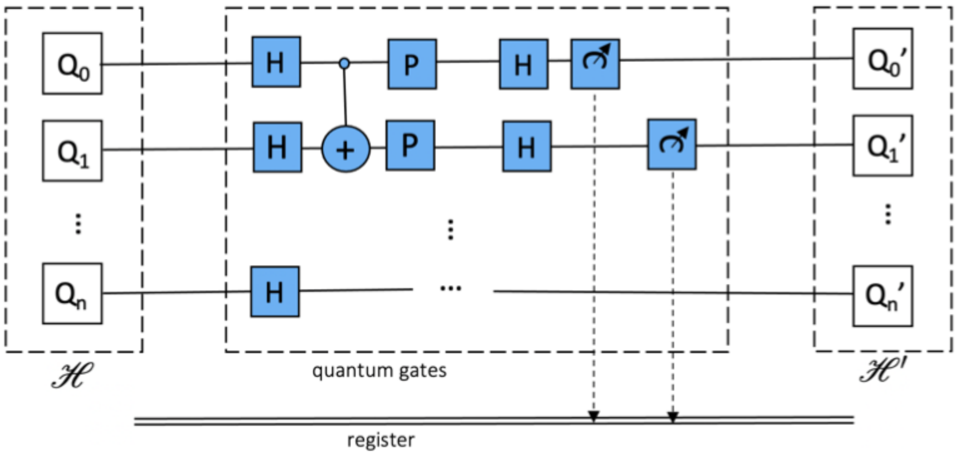


FIG. 2. Schematic representation of a quantum channel comprising n qubits, Hadamart (H), C-NOT (+), Phase (P), and measurement gates.

- **Translate obervables**: (R, L) → Boolean computational basis (0,1)
- Characterise each possible initial-final helicity state by a **string** of four digits.
- **Tunable precision**: Solovay-Kitaev theorem [M. Nielsen and I. Chuang, 2010]

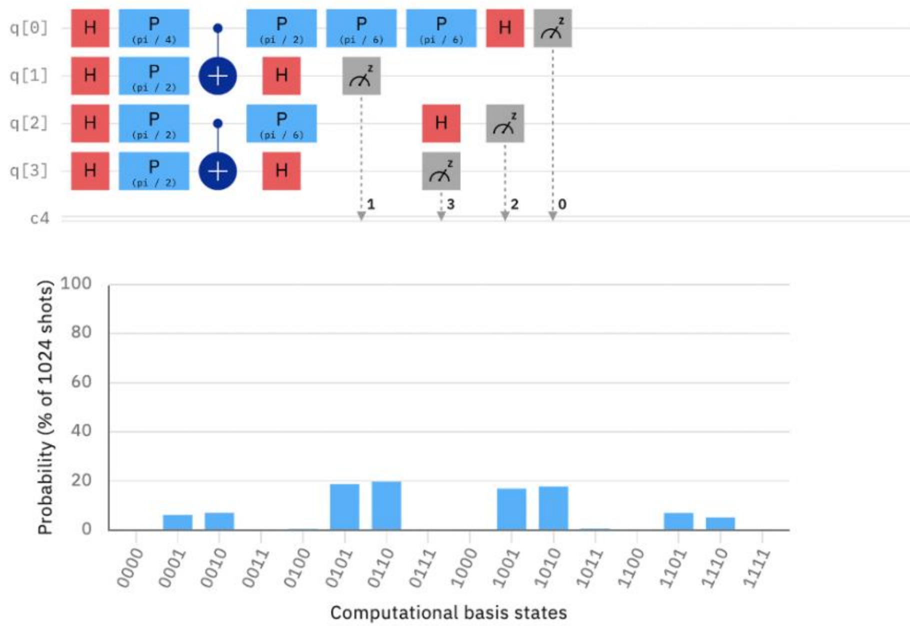
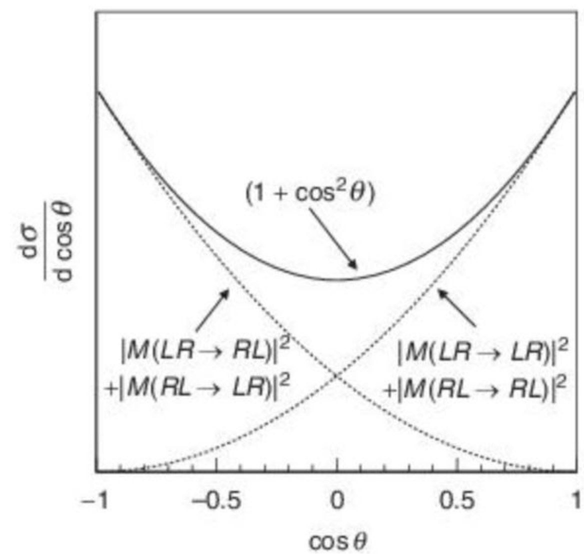


FIG. 3. Quantum channel simulating $|\mathscr{M}_\gamma|^2$ (eq. 6) and related histograms showing a P-preserving process (simulated using the IMB Quantum Lab (https://quantum-computing.ibm.com).

[C. Altomonte and A.J. Barr, 2022]



[M. Thomson, 2021]

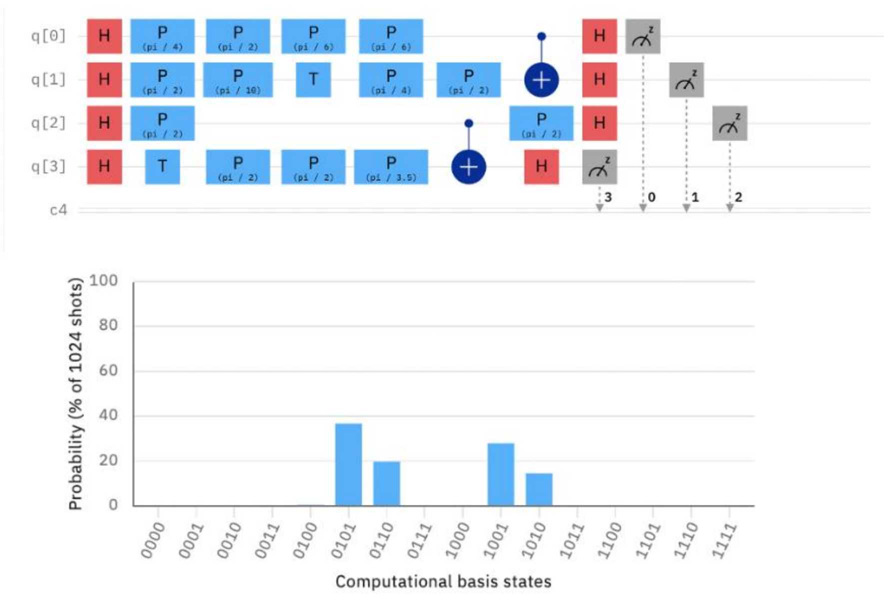
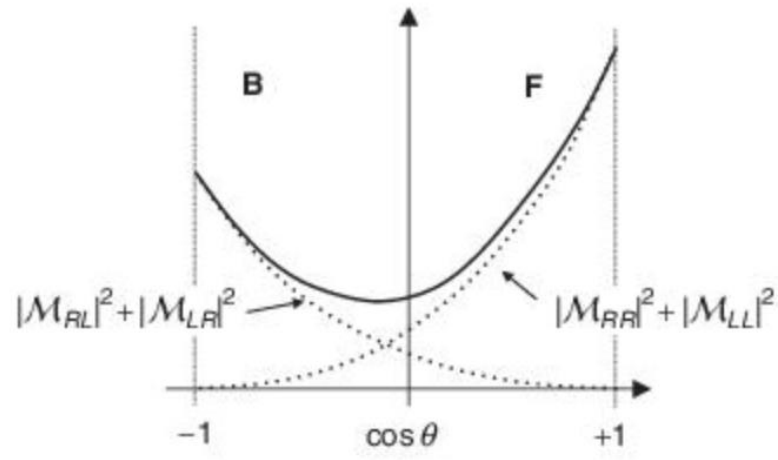


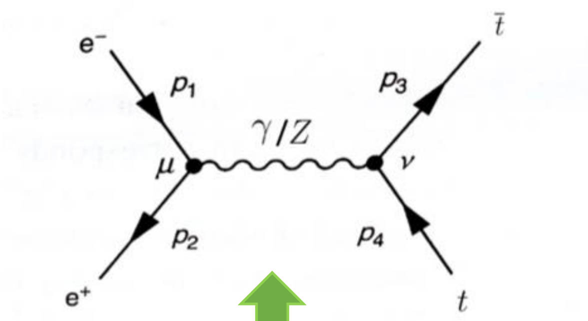
FIG. 4. Quantum channel simulating $|\mathscr{M}_Z|^2$ (eq. 7) and related histograms showing a P-violating process (simulated using the IMB Quantum Lab (https://quantum-computing.ibm.com).

[C. Altomonte and A.J. Barr, 2022]



[M. Thomson, 2021]

Remarks and findings: constructing a dictionary translating between PP and QI

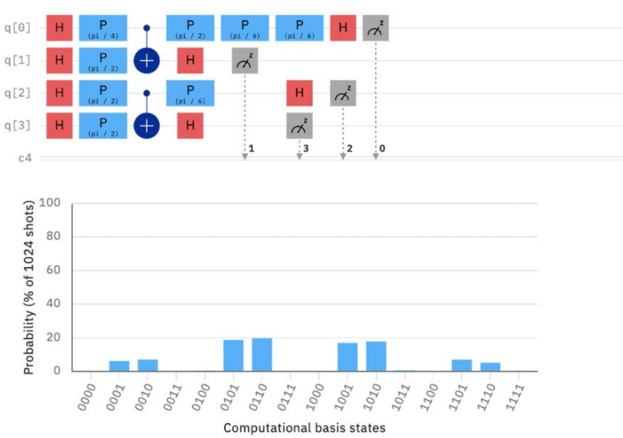


Derive

$$|M_\gamma|^2 \propto \begin{bmatrix} |M_{RL\to RL}|^2 & |M_{RL\to RR}|^2 & |M_{RL\to LL}|^2 & |M_{RL\to LR}|^2 \\ |M_{RR\to RL}|^2 & |M_{RR\to RR}|^2 & |M_{RR\to LL}|^2 & |M_{RR\to LR}|^2 \\ |M_{LL\to RL}|^2 & |M_{LL\to RR}|^2 & |M_{LL\to LL}|^2 & |M_{LL\to LR}|^2 \\ |M_{LR\to RL}|^2 & |M_{LR\to RR}|^2 & |M_{LR\to LL}|^2 & |M_{LR\to LR}|^2 \end{bmatrix}$$

Identify

Identify



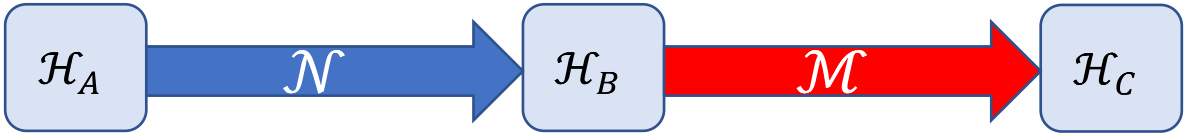
Derive

$$\tilde{\mathscr{E}} = \begin{bmatrix} \mathscr{E}(|0\rangle\langle 0|) & \mathscr{E}(|0\rangle\langle 1|) & \mathscr{E}(|0\rangle\langle 2|) & \cdots \\ \mathscr{E}(|1\rangle\langle 0|) & \mathscr{E}(|1\rangle\langle 1|) & \mathscr{E}(|1\rangle\langle 0|) & \cdots \\ \mathscr{E}(|2\rangle\langle 0|) & \mathscr{E}(|2\rangle\langle 1|) & \mathscr{E}(|2\rangle\langle 2|) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Serial Concatenation (QI)

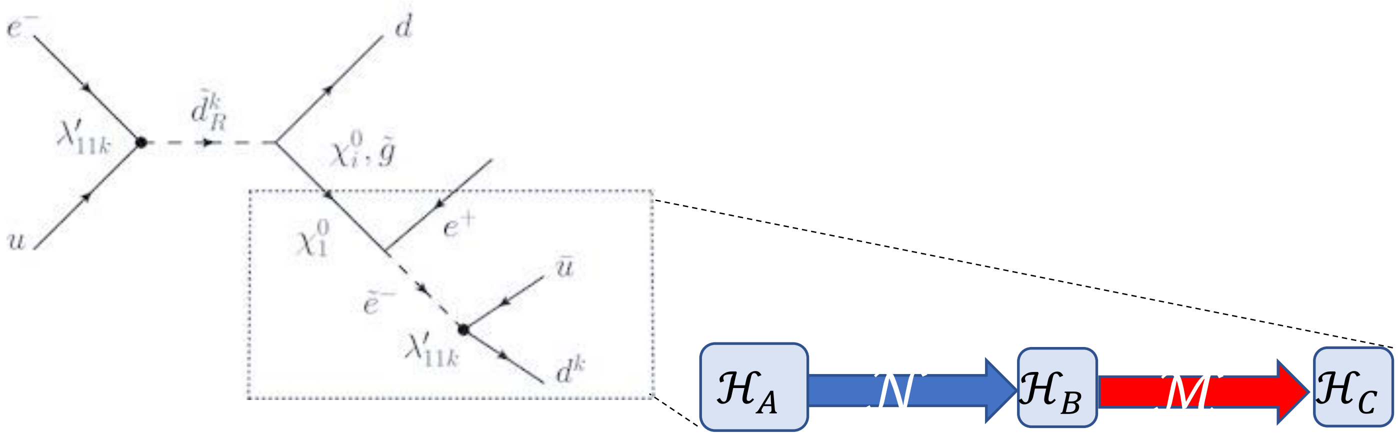
It submits the quantum state to subsequent quantum channels [M. Wilde, 2017].

- quantum channels $\mathcal{N}: \mathcal{B}(\mathcal{H}_A) \rightarrow \mathcal{B}(\mathcal{H}_B)$ $\quad$ $\mathcal{M}: \mathcal{B}(\mathcal{H}_B) \rightarrow \mathcal{B}(\mathcal{H}_C)$
- first channel's output $\mathcal{N}_{A \rightarrow B}(\rho_A) \equiv \sum_k N_k \rho_A N_k^{\dagger}$
- second channel's output $(\mathcal{M}_{B \rightarrow C} \circ \mathcal{N}_{A \rightarrow B})(\rho_A) = \sum_k M_k \mathcal{N}_{A \rightarrow B}(\rho) M_k^{\dagger} =$
  $= \sum_{k,k'} M_k N_{k'} \rho_A N_{k'}^{\dagger} M_k^{\dagger}$



Serial Concatenation (PP)

PP interpretation: applicable to modelling sequential processes such as cascade decays and jets.

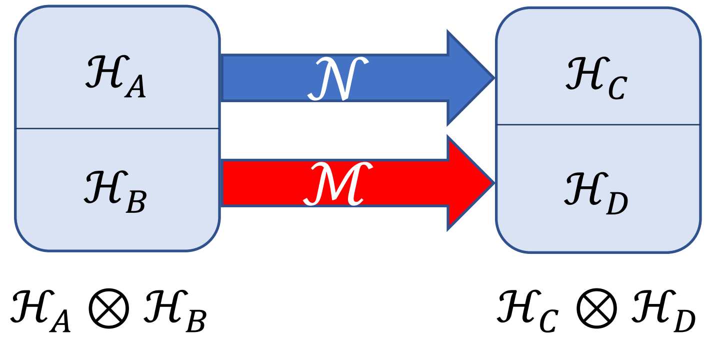


- Cf. classical algorithm (eq. 1) in [I. Knowles, 1990].
- A more complex hybrid quantum-classical algorithm has been developed by [G. Gustafson, S. Prestel, M. Spannowsky, and S. Williams, 2022]

Parallel Concatenation (QI)

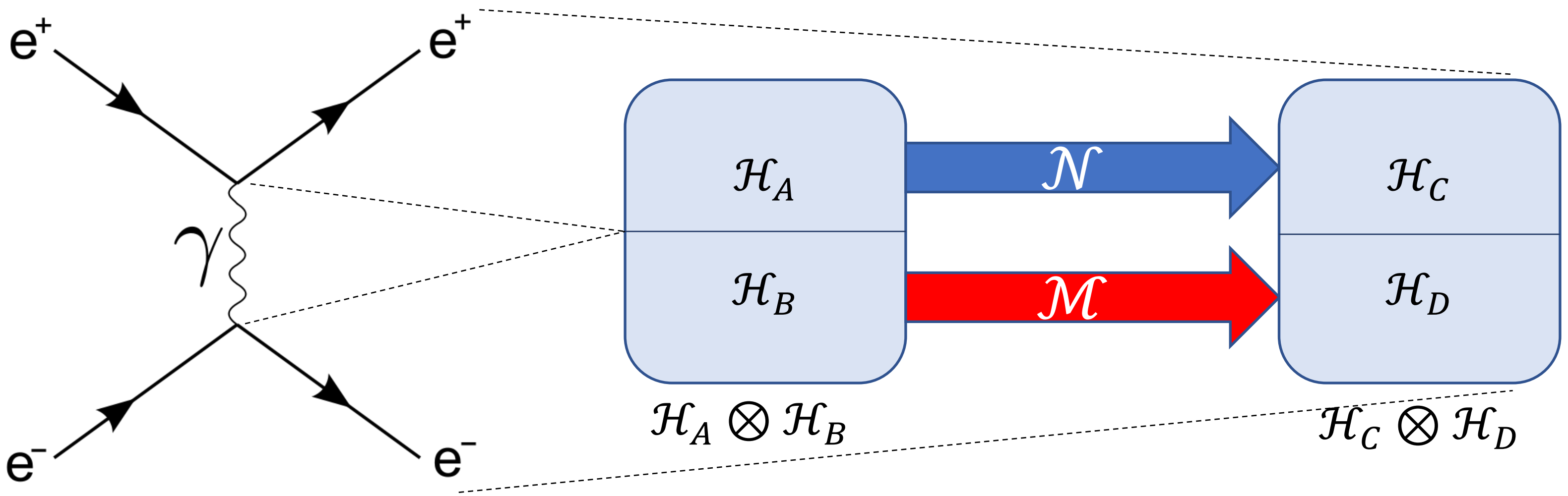
Each subsystem A,B undergoes a different quantum channel [M. Wilde, 2017].

- quantum channels $\mathcal{N}: \mathcal{B}(\mathcal{H}_A) \rightarrow \mathcal{B}(\mathcal{H}_C)$ $\mathcal{M}: \mathcal{B}(\mathcal{H}_B) \rightarrow \mathcal{B}(\mathcal{H}_D)$
- input density operator: $\rho_{AB} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_B)$
- parallel concatenation: $(\mathcal{N}_{A\rightarrow C} \otimes \mathcal{M}_{B\rightarrow D})(\rho_{AB}) =$
  $=\sum_{k,k'} (N_k \otimes M_{k'})(\rho_{AB})(N_k \otimes M_{k'})^{\dagger}$



Parallel Concatenation (PP)

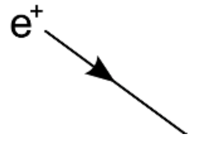
PP interpretation: applicable where subsystems or many-body interactions are present.



Preparation channel

It prepares a quantum system A in a given state $\rho_A \in \mathcal{D}(\mathcal{H}_A)$ [M. Wilde, 2017].

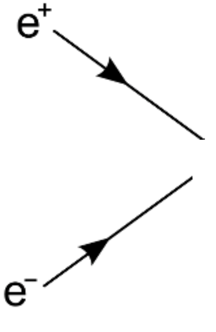
PP interpetation: draw an open incoming line in a Feynman diagram

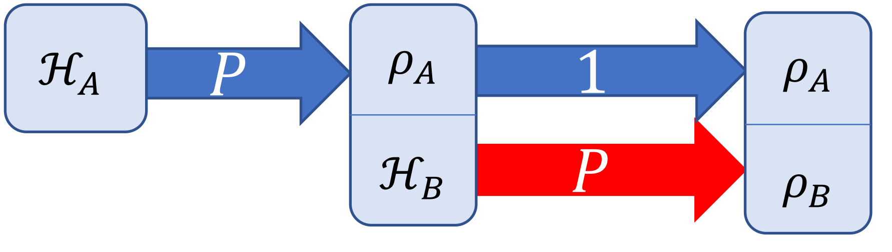


Appending channel

It is the parallel concatenation of the identity channel and a preparation channel [M. Wilde, 2017].

PP interpetation: add a parallel line in a Feynman diagram





Isometry: Stinespring's dilation and Kraus's ambiguity – work in progress

Isometry: Operation that encodes a quantum state of one system into a quantum state of a larger system [A. Ekert, 2022]

Let $\mathcal{H}$ and $\mathcal{H}'$ be Hilbert spaces such that dim $\mathcal{H} \leqslant$ dim $\mathcal{H}'$. An isometry is a linear map $V: \mathcal{H} \rightarrow \mathcal{H}'$ such that $V^{\dagger}V = 1_{\mathcal{H}}$

Stinespring's dilation: $\mathcal{E}$ arises from unitary evolution of dilated system, i.e:

$$\rho \longmapsto \rho' = \mathrm{tr}_{\mathcal{A}} V \rho V^{\dagger}$$

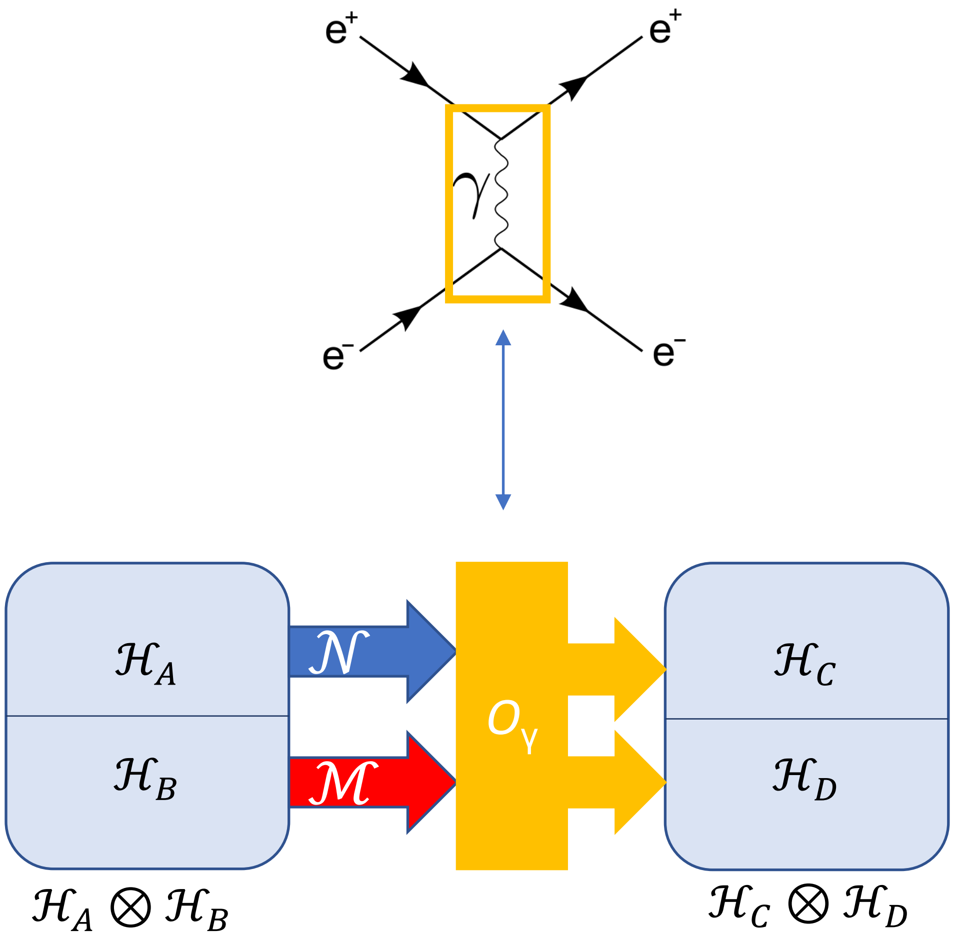
where suitably chosen ancilla $\mathcal{A}$ is traced out.

Kraus representation (operator-sum decomposition): operators E_i must satisfy normalisation (completeness) condition $\sum_i E_i^{\dagger} E_i = 1$, but depend on choice of basis for ancilla $\mathcal{A}$, i.e.:

$$\rho \longmapsto \rho' = \sum_i E_i \rho E_i^{\dagger}$$

Operator-Sum Ambiguity (PP) - work in progress

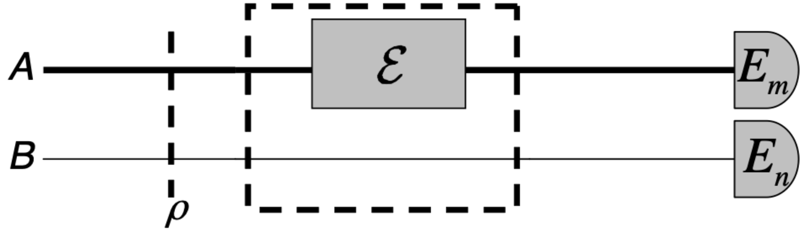
PP interpretation: internal propagators may correspond to 'equivalence classes' of quantum circuits.



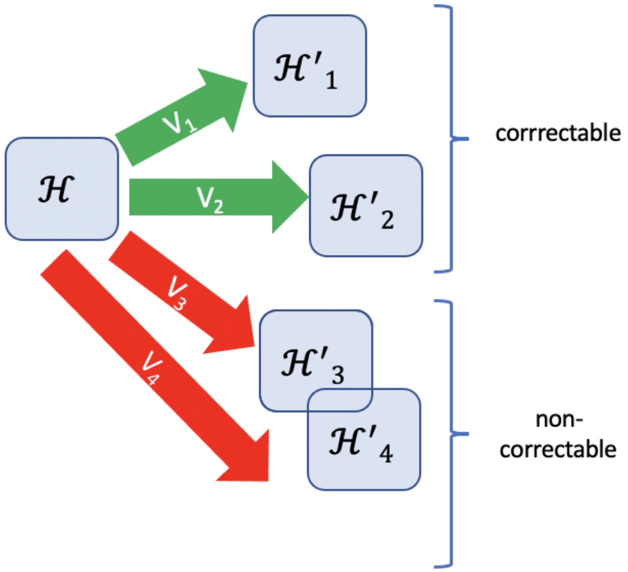
Stinespring Dilation and Quantum Error Correction (PP) – work in progress

PP interpretation: applicable to **current PP research** to probe **confined** or otherwise hardly accessible degrees of freedom (c.f. flavour physics, dark matter searches, etc.)

"information on the underlying quantum process is mapped onto the state of some probe quantum system(s), and the process is reconstructed via quantum state tomography on the output states." [M. Mohseni, A. T. Rezakhani, and D. A. Lidar, 2008]

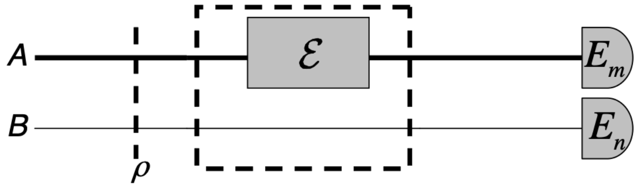


[M. Mohseni, A. T. Rezakhani, and D. . Lidar, 2008]

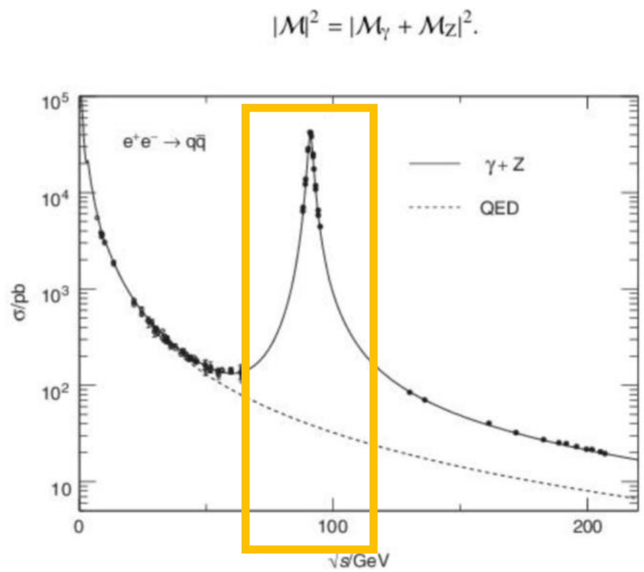


Stinespring's dilation and Quantum Error Correction (PP) – work in progress

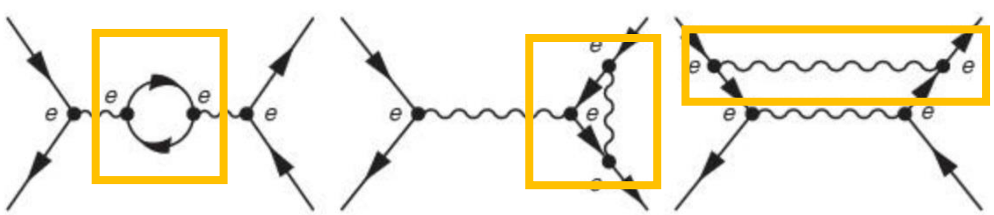
PP interpretation: higher-order corrections and competing processes.



[M. Mohseni, A. T. Rezakhani, and D. . Lidar, 2008]



[M. Thomson, 2021]



[M. Thomson, 2021]

Final remarks

Quantum information measurements such as entanglement acquire a new importance and meaning in PP problems.

- Hybrid quantum-classical algorithms working on existing Noisy Intermediate-Scale Quantum devices (NISQ):
  - dynamics **simulation** [G. Gustafson, S. Prestel, M. Spannowsky, and S. Williams, 2022];
  - unsupervised training of generative models for **synthetic data** of high-energy physics processes [A. Delgado and K. E. Hamilton, 2022];
  - quantum variational **classifier** method to recognise signatures from experimental data from LHC [S. L. Wu et al, 2021]

- Colliders as quantum information processes **observatories**: [H. Abramowicz et al 2019], [C. Altomonte and A.J. Barr]: explore features of high energy physics systems (entanglement, invariant quantities, etc.) by analysing their corresponding quantum channels.
  - e.g. **Bell inequalities violation**: [R. Ashby-Pickering, A. J. Barr, and A. Wierzchucka, 2022], [A. J. Barr, P. Caban, and J. Rembielin ́ski, 2022], [A. J. Barr, 2022], [M. Fabbrichesi, R. Floreanini, and G. Panizzo, 2021], [Y. Afik and J. R. M. de Nova, 2021], 2022 Nobel Prize in Physics.

PP features acquire a new importance and meaning in quantum computing

References

- H. Abramowicz *et al.*, “Top-quark physics at the CLIC electron- positron linear collider,” Journal of High Energy Physics 2019 (2019), 10.1007/jhep11(2019)003.
- Y. Afik and J. R. M. de Nova, “Entanglement and quantum tomography with top quarks at the LHC,” The European Physical Journal Plus 136 (2021), 10.1140/epjp/s13360-021-01902-1.
- C. Altomonte, and A.J. Barr, “State-channel duality applied to Particle Physics”(2022) [and the references therein], https://ora.ox.ac.uk/objects/uuid:305b89aa-959c-4544-af43-8969fc4bdb31
- R. Ashby-Pickering, A. J. Barr, and A. Wierzchucka, “Quantum state tomography, entanglement detection and Bell violation prospects in weak decays of massive particles,” (2022).
- A. J. Barr, “Testing Bell inequalities in Higgs boson decays,” Physics Letters B 825, 136866 (2022).
- A. J. Barr, P. Caban, and J. Rembielin ́ski, “Bell-type inequalities for sys- tems of relativistic vector bosons,” (2022), arXiv:2204.11063 [quant-ph].
- A. Delgado and K. E. Hamilton, “Unsupervised quantum circuit learning in high energy physics,” (2022).
- A. Ekert, “’Introduction to quantum information’ course page,” (2022) https://zhenyucai.com/post/intro_to_qi/. .
- M. Fabbrichesi, R. Floreanini, and G. Panizzo, “Testing Bell Inequalities at the LHC with Top-Quark Pairs,” Physical Review Letters 127 (2021), 10.1103/physrevlett.127.161801.
- M. Jiang, S. Luo, and S. Fu, “Channel-state duality,” Phys. Rev. A 87, 022310 (2013).
- M. Mohseni, A. T. Rezakhani, and D. A. Lidar, “Quantum-process tomography: Resource analysis of different strategies,” Physical Review A 77 (2008), 10.1103/physreva.77.032322.
- M. Nielsen and I. Chuang, *Quantum computation and quantum information* (Cambridge university press, 2010).
- M. Thomson, *Modern Particle Physics* (Cambridge University Press, 2021).
- M. Wilde, *Quantum Information theory* (Cambridge University Press, 2017).
- S. L. Wu, et al, “Application of quantum machine learning using the quantum variational classifier method to high energy physics analysis at the LHC on IBM quantum computer simulator and hardware with 10 qubits,” Journal of Physics G: Nuclear and Particle Physics 48, 125003 (2021)
- IBM Quantum https://quantum-computing.ibm.com/lab