Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future lepton colliders

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Entanglement at Colliders

$$|A\rangle_L = \sum_{m_a} c_{m_a} |m_a\rangle$$
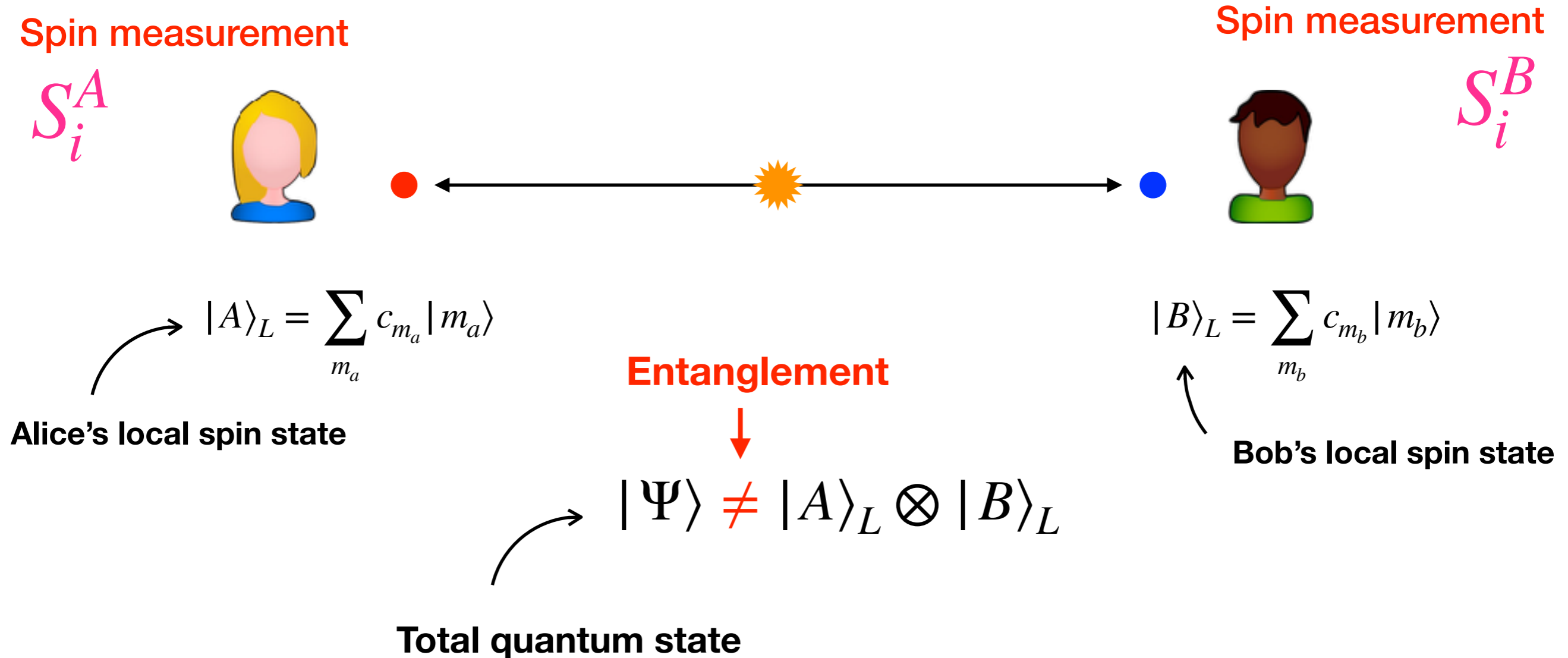
Alice's local spin state

Entanglement

$$|\Psi\rangle \neq |A\rangle_L \otimes |B\rangle_L$$

Total quantum state

$$|B\rangle_L = \sum_{m_b} c_{m_b} |m_b\rangle$$

Bob's local spin state

Entanglement at Colliders

Spin measurement

$$S_i^A$$

Spin measurement

$$S_i^B$$

$$|A\rangle_L = \sum_{m_a} c_{m_a} |m_a\rangle$$

Alice's local spin state

$$|B\rangle_L = \sum_{m_b} c_{m_b} |m_b\rangle$$

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Entanglement

$$|\Psi\rangle \neq |A\rangle_L \otimes |B\rangle_L$$

Total quantum state

Spin correlation $C_{ij} = \langle S_i^A S_j^B \rangle$ **contains the information of entanglement**
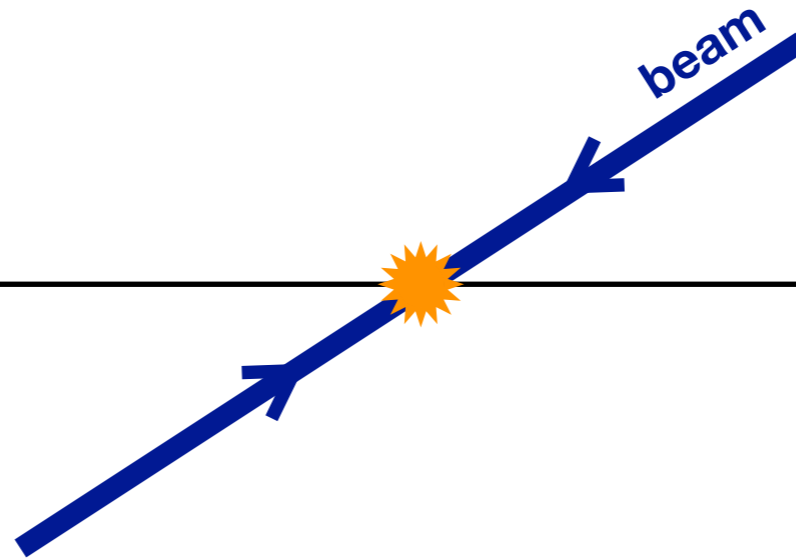
Entanglement at Colliders

Spin measurement

S_i^A

beam

Spin measurement

S_i^B

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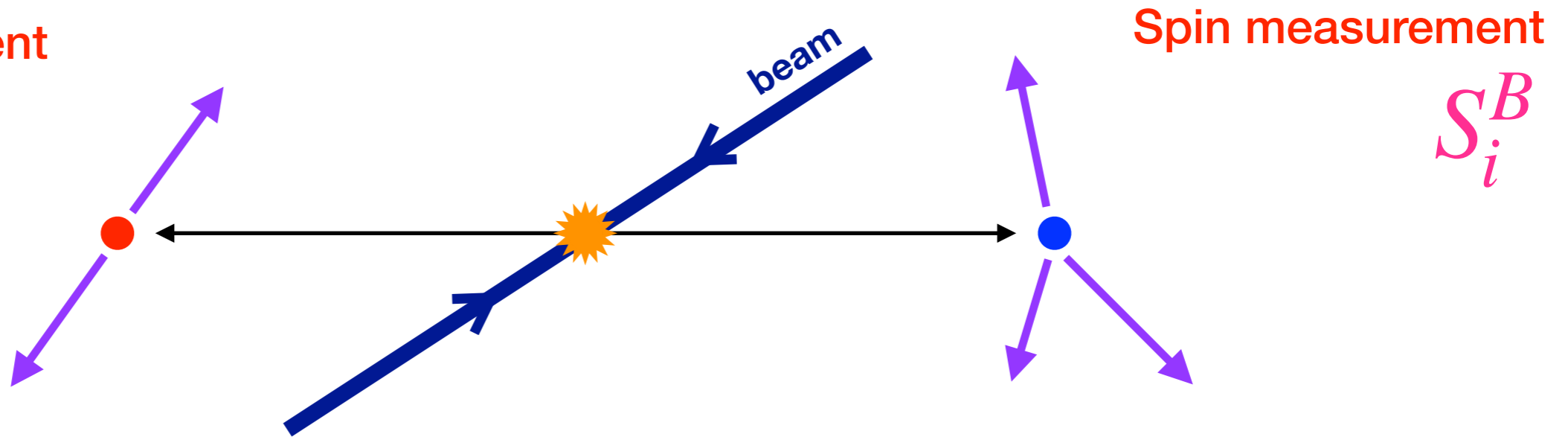
Entanglement at Colliders

Spin measurement

S_i^A

Spin measurement

S_i^B

spin analyzing
power $\alpha \in [-1,1]$

beam

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha \cdot (\hat{\mathbf{u}} \cdot \hat{\mathbf{S}})$$

Spin can only be inferred from the decay

direction of a decay product

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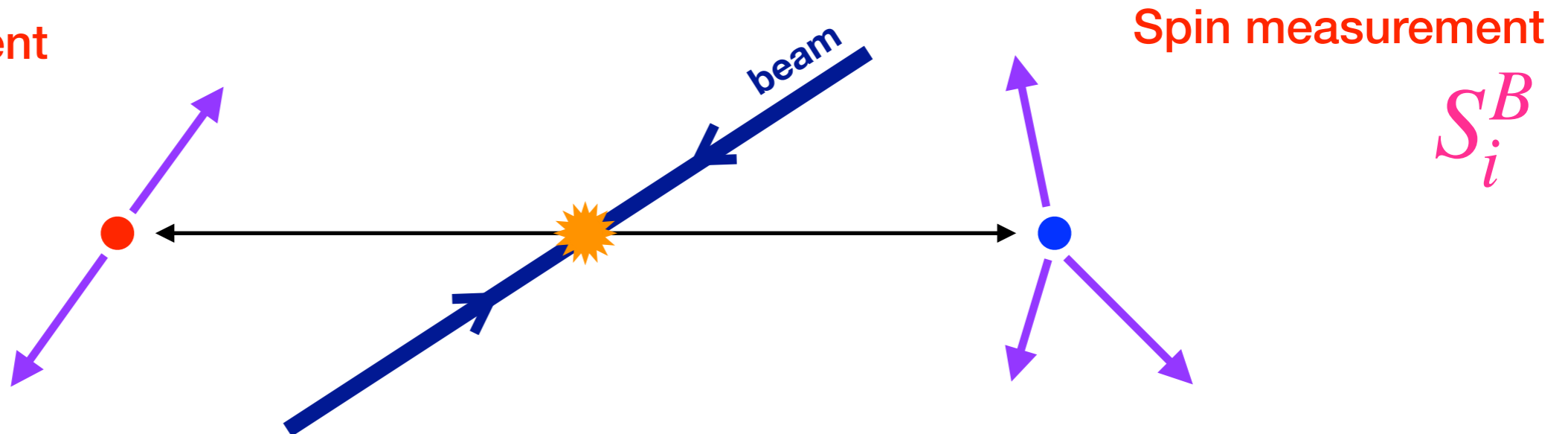
Entanglement at Colliders

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direction of a decay product

beam

Spin measurement

S_i^B

Spin can only be inferred from the decay

→ Condition on produced particles:

- have non zero spin

- their decay must be analyzable

$$W^\pm, \; Z^0, \; t, \; \tau$$

Entanglement at Colliders
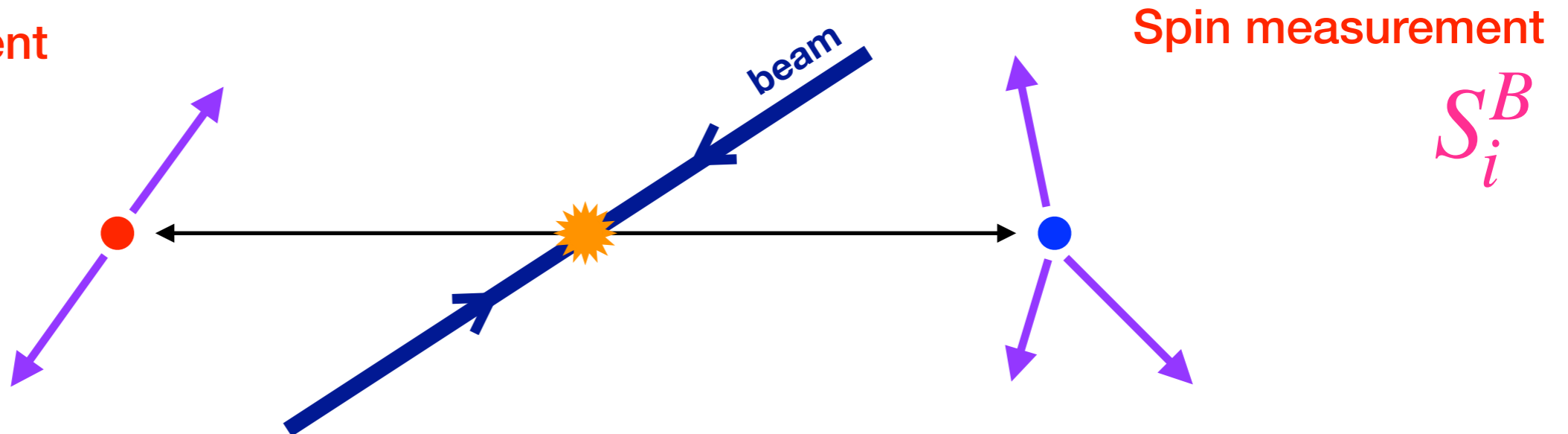

Spin measurement

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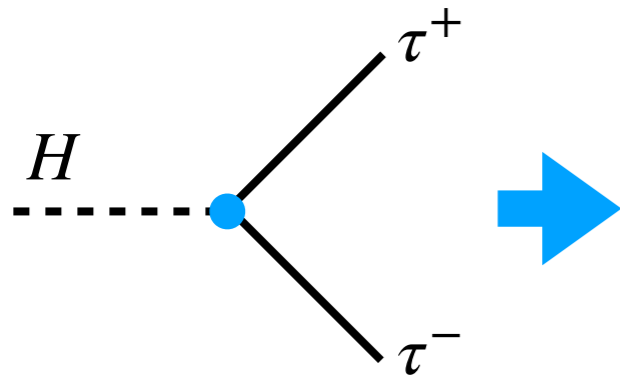- their decay must be analyzable

$$W^{\pm}, Z^0, t, \tau$$

tau is special

$$m_\tau \ll M_{\text{weak}}$$ ➡ experimental challenge

$H \to \tau^+ \tau^-$

SM: $(\kappa, \delta) = (1, 0)$

$$\mathcal{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}} \kappa H \bar{\psi}_\tau (\cos \delta + i\gamma_5 \sin \delta)\, \psi_\tau$$

$$\mathcal{M}^{m\bar{m}} = c\, \bar{u}^m(p)(\cos \delta + i\gamma_5 \sin \delta) v^{\bar{m}}(\bar{p})$$

density matrix:

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}} \mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}} |\mathcal{M}^{m\bar{m}}|^2} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$H \to \tau^+ \tau^-$

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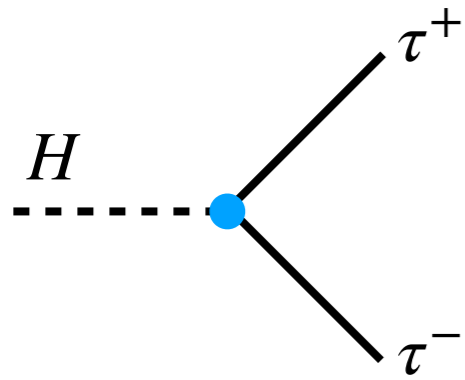
$H \to \tau^+, \tau^-$

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$$|\Psi_{H \to \tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$
(CP even)

$$|+-\rangle + |-+\rangle$$

$\delta = \pi/2$ (CP odd)

$$|+-\rangle - |-+\rangle$$

$H \to \tau^+ \tau^-$

$$\mathscr{L}_{\text{int}} = -\frac{m_\tau}{v_{\text{SM}}} \kappa H \bar{\psi}_\tau (\cos \delta + i\gamma_5 \sin \delta) \psi_\tau$$

H
τ^+
τ^-

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$$\boxed{|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle}$$
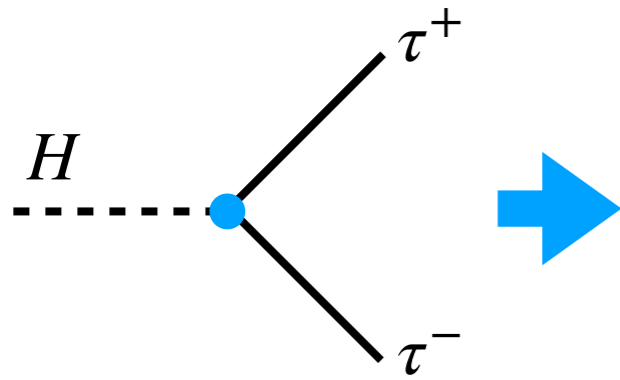
$\delta = 0$
(CP even)

$$|\Psi^{(s=1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ |+-\rangle + |-+\rangle \\ |--\rangle \end{pmatrix}$$

Parity: $P = (\eta_f \eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f \eta_{\bar{f}} = -1$:

$$J^P = \begin{cases} 0^+ \implies & l = s = 1 \\ 0^- \implies & l = s = 0 \end{cases}$$

$\delta = \pi/2$ (CP odd)

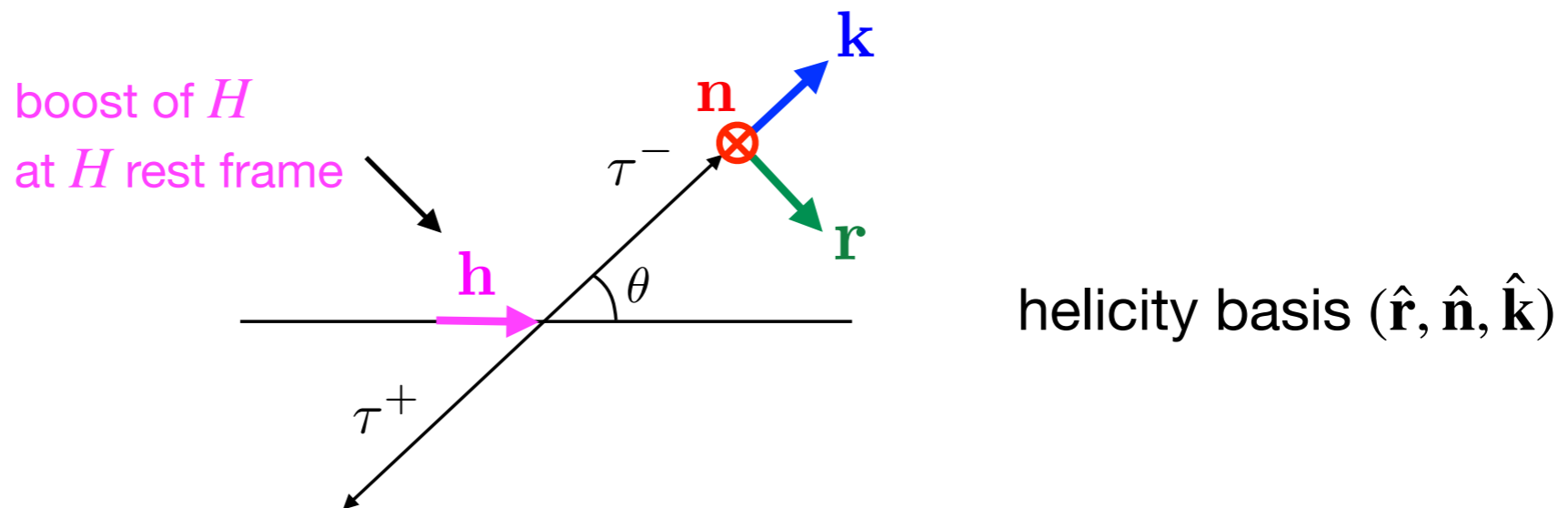$$|\Psi^{(0,0)}\rangle \propto |+-\rangle - |-+\rangle$$

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$$\rho = \frac{1}{4}\left[\mathbf{1}_4 + B_i\cdot(\sigma_i\otimes\mathbf{1}) + \overline{B}_i\cdot(\mathbf{1}\otimes\sigma_i) + C_{ij}\cdot(\sigma_i\otimes\sigma_j)\right]$$

$$\langle\hat{S}_i^{\tau^-}\rangle = \mathrm{Tr}[(\sigma_i\otimes\mathbf{1})\rho] = B_i = 0 \qquad \langle\hat{S}_i^{\tau^+}\rangle = \mathrm{Tr}[(\mathbf{1}\otimes\sigma_i)\rho] = \overline{B}_i = 0$$

$$\langle\hat{S}_i^{\tau^-}\hat{S}_j^{\tau^+}\rangle = \mathrm{Tr}[(\sigma_i\otimes\sigma_j)\rho] = C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

boost of H
at H rest frame

τ^-

$\mathbf{n}$

$\mathbf{k}$

$\mathbf{r}$

$\mathbf{h}$

θ

τ^+

helicity basis $(\hat{\mathbf{r}}, \hat{\mathbf{n}}, \hat{\mathbf{k}})$

Once $C_{ij} = \langle \hat{S}_i^{\tau-} \hat{S}_j^{\tau+} \rangle$ are measured/computed, it is straightforward to obtain:

❖ **Entanglement (Concurrence):**

$$\mathcal{C}[\rho] = \max\left[0, \frac{D_+ + C_{kk} - 1}{2}, \frac{D_- - C_{kk} - 1}{2}\right] > 0$$
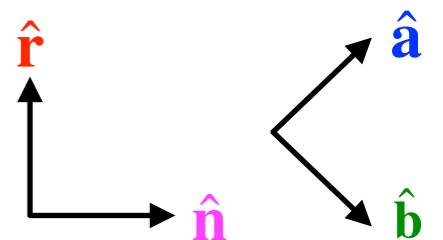
$$D_\pm \equiv \sqrt{(C_{rn} \pm C_{nr})^2 + (C_{rr} \mp C_{nn})^2}$$

❖ **Bell non-locality**

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left|\langle \hat{S}_r^{\tau-} \hat{S}_a^{\tau+} \rangle - \langle \hat{S}_r^{\tau-} \hat{S}_b^{\tau+} \rangle + \langle \hat{S}_n^{\tau-} \hat{S}_a^{\tau+} \rangle + \langle \hat{S}_n^{\tau-} \hat{S}_b^{\tau+} \rangle\right| > 1$$

❖ **Steerability**

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\hat{\mathbf{n}}} \sqrt{\hat{\mathbf{n}}^T C^T C \hat{\mathbf{n}}} > 1$$

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$$H \to \tau^+ \tau^-$$

$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
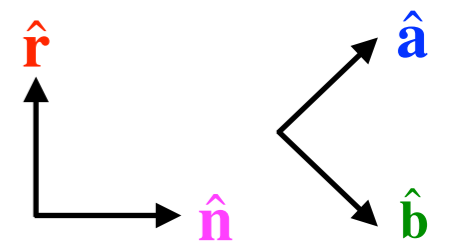
prediction

❖ **Bell non-locality**

$$R_{\mathrm{CHSH}} \equiv \frac{1}{2}\left| \langle \hat{S}_r^{\tau^-} \hat{S}_a^{\tau^+} \rangle - \langle \hat{S}_r^{\tau^-} \hat{S}_b^{\tau^+} \rangle + \langle \hat{S}_n^{\tau^-} \hat{S}_a^{\tau^+} \rangle + \langle \hat{S}_n^{\tau^-} \hat{S}_b^{\tau^+} \rangle \right| \quad = \sqrt{2} \quad > 1$$

$\hat{r}$ $\hat{a}$

$\hat{n}$ $\hat{b}$

❖ **Steerability**

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\hat{n}} \sqrt{\hat{n}^T C^T C \hat{n}} \quad = 2 \quad > 1$$

independent of CP phase δ

How can we measure spin correlation?

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha \cdot (\hat{\mathbf{u}}^{\pi^{\pm}} \cdot \hat{\mathbf{S}}^{\tau^{\pm}})$$

$\alpha = 1$ for $\tau^- \to \pi^- \nu$

- *At the rest frame* of $\tau^{\pm}$, the spin can be inferred from the $\pi^{\pm}$ direction

➡ $$\langle \hat{S}_i^{\tau^-} \hat{S}_j^{\tau^+} \rangle = C_{ij} = -9 \cdot \langle (\hat{\mathbf{u}}^{\pi^-} \cdot \hat{\mathbf{e}}_i)(\hat{\mathbf{u}}^{\pi^+} \cdot \hat{\mathbf{e}}_j) \rangle$$

→ Reconstruction of the tau rest frames (i.e. neutrino reconstruction) is necessary

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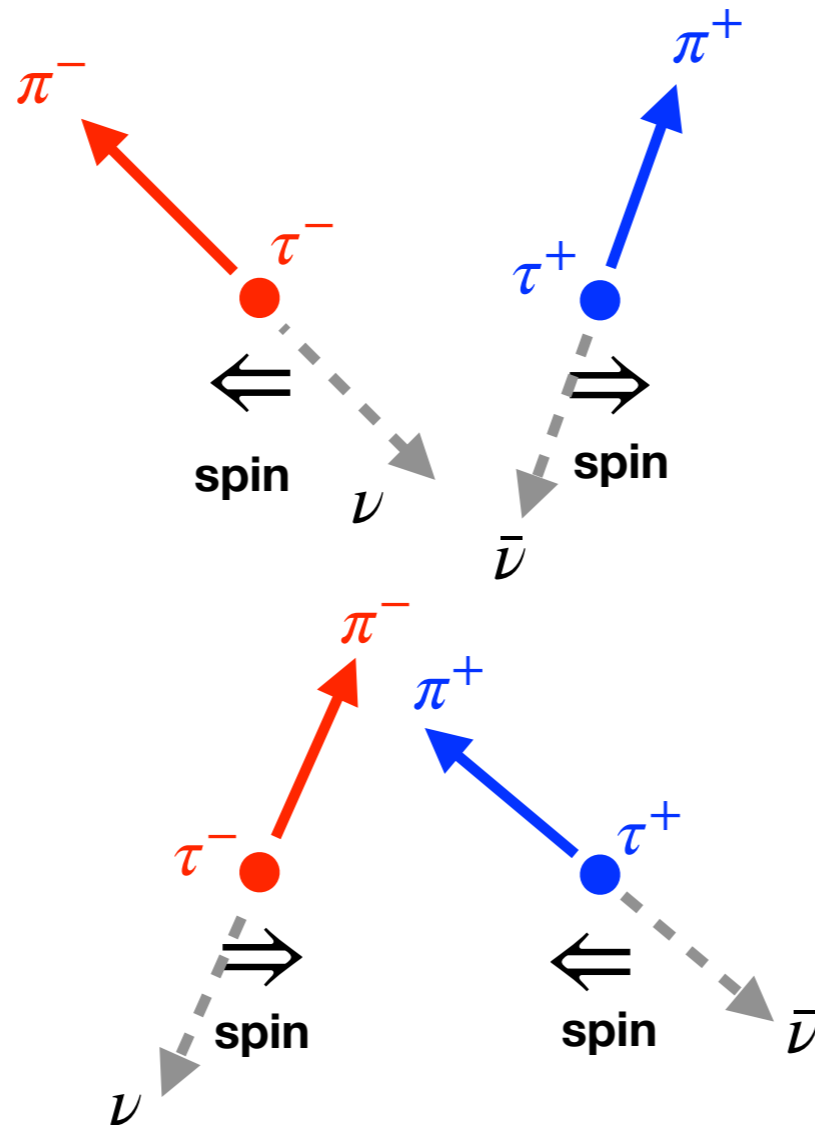
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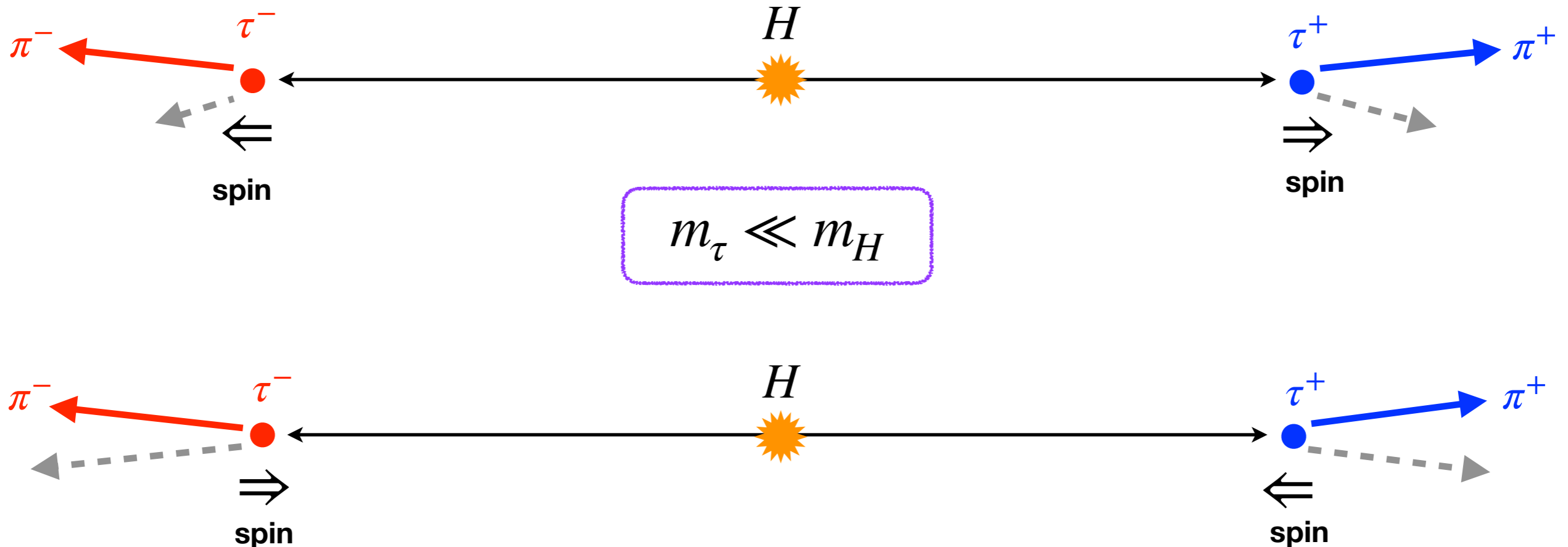
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- *At the rest frame* of $\tau^\pm$, the spin can be inferred from the $\pi^\pm$ direction

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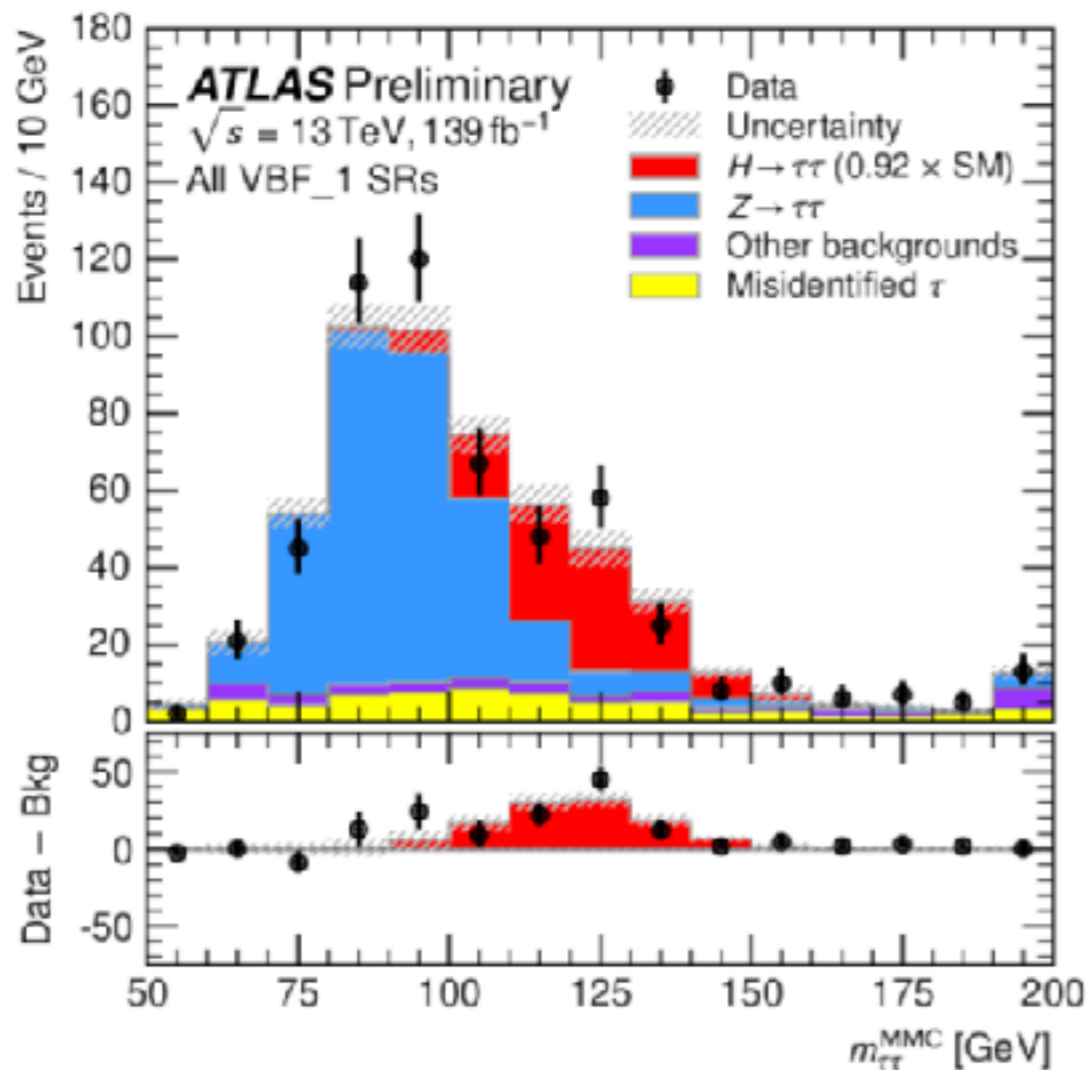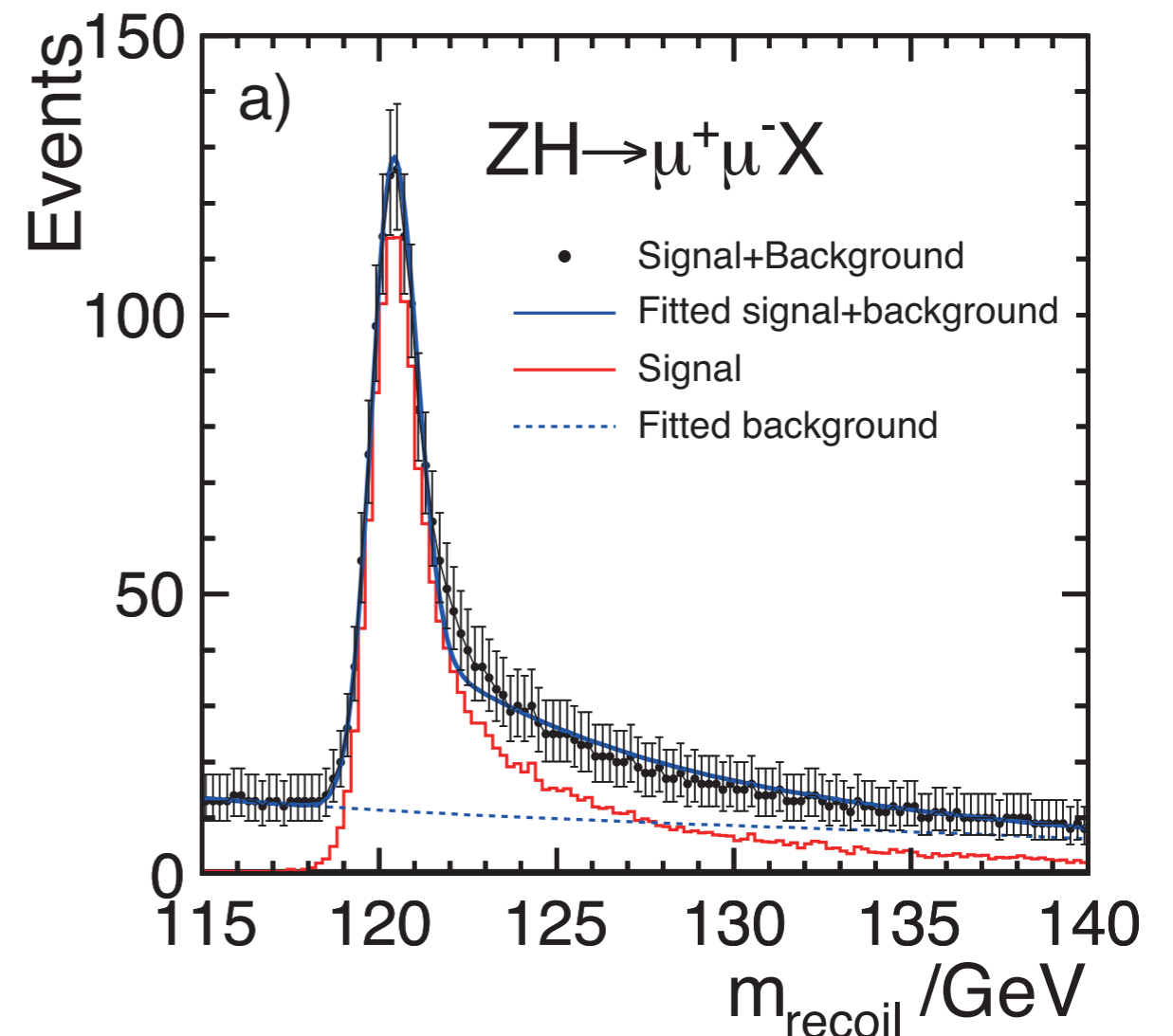taus are highly boosted → Very accurate event reconstruction is required

$$m_\tau \ll m_H$$

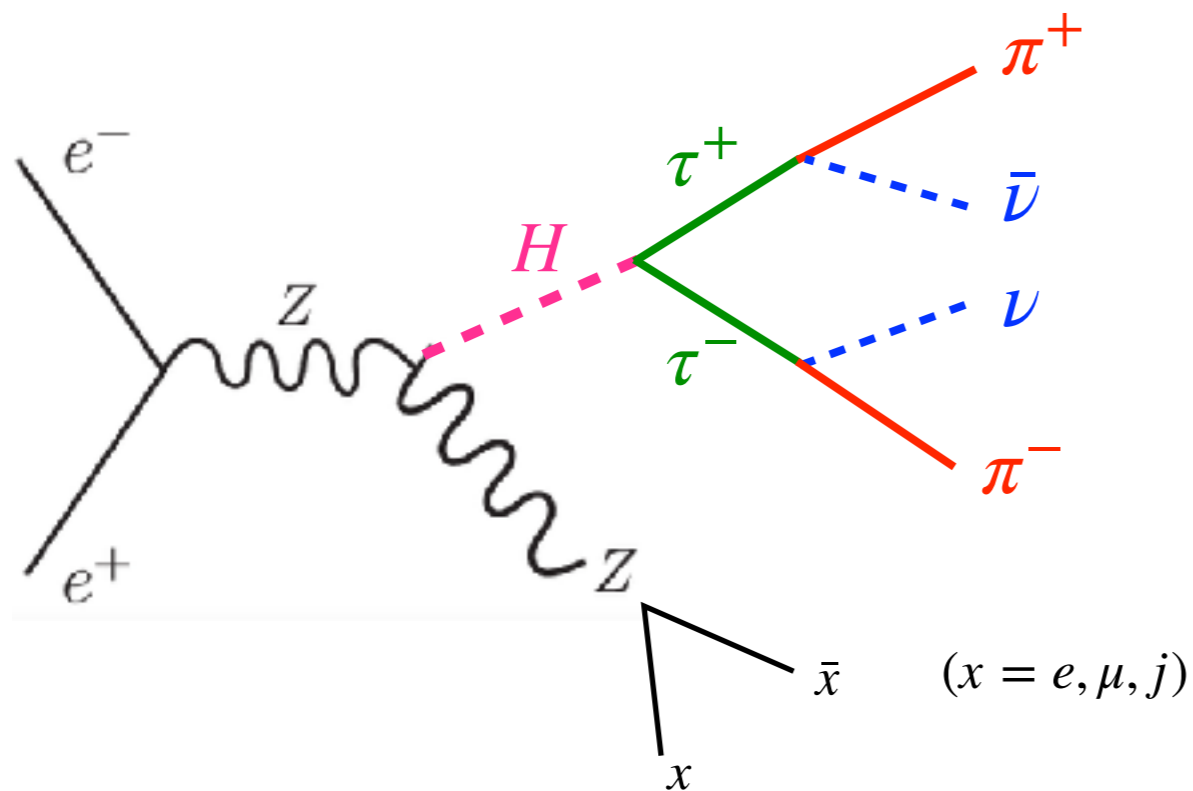# $H \to \tau^+\tau^-$ @ lepton colliders

- For precise event reconstruction and for much smaller background, we consider lepton colliders.

LHC

ILC

$$(P_H^{\text{reco}})^\mu \equiv P_{e^+e^-}^\mu - P_{Z\to x\bar{x}}^\mu \qquad M_{\text{recoil}}^2 \equiv (P_H^{\text{reco}})^2$$

Event selection: $\quad |M_{\text{recoil}} - 125\,\text{GeV}| < 5\,\text{GeV}$

$(x = e, \mu, j)$

$e^+e^- \to Z + (Z^*/\gamma^*) \to f\bar{f} + \tau^+\tau^-$

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \text{BR} \cdot L \cdot \epsilon$)	385	663
# of background ($\sigma \cdot \text{BR} \cdot L \cdot \epsilon$)	20	36

- Generate the SM events $(\kappa, \delta) = (1, 0)$ with **MadGraph5**.

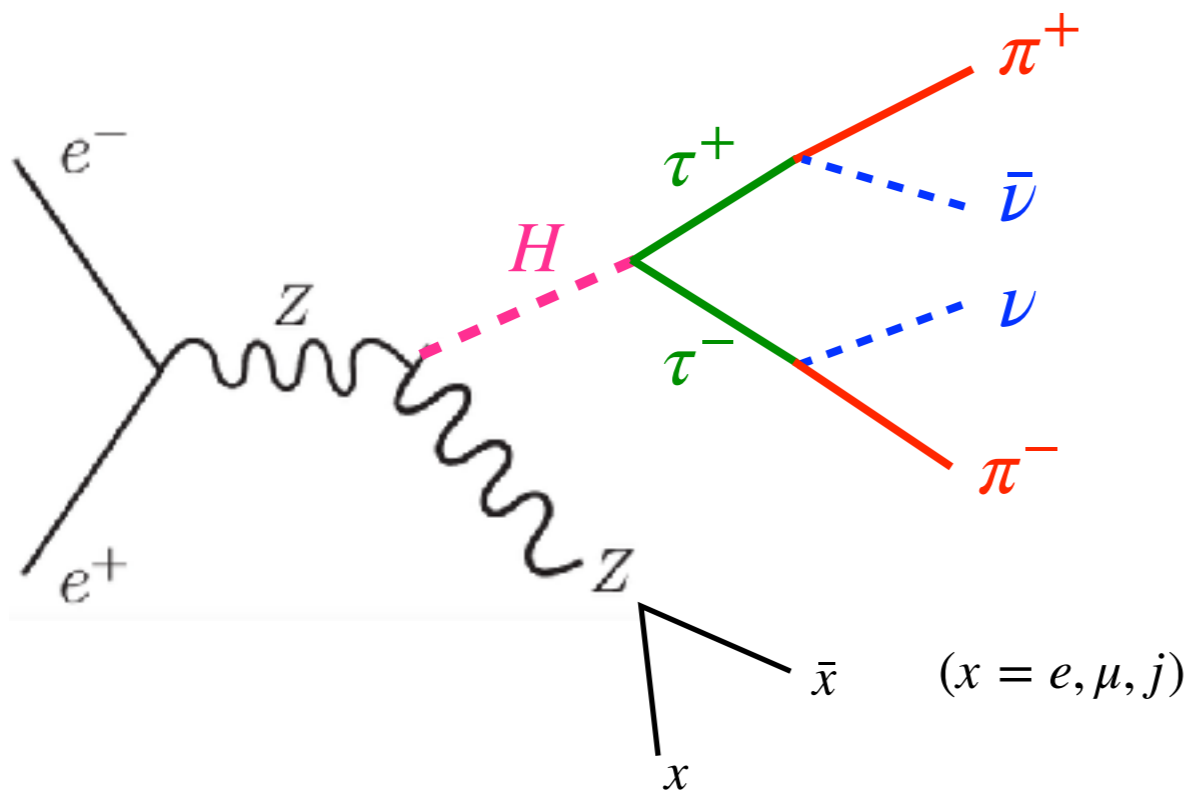- **100 pseudo-experiments** to estimate the statistical uncertainties

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- Generate the SM events $(\kappa, \delta) = (1,0)$ with **MadGraph5**.

- **100 pseudo-experiments** to estimate the statistical uncertainties

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^\nu, p_y^\nu, p_z^\nu)$, $(p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.

- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_\nu)^2$$

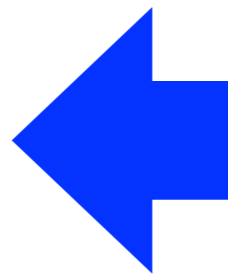$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu$$

=> 2-fold solutions.

$$C_{ij}^{\mathrm{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$
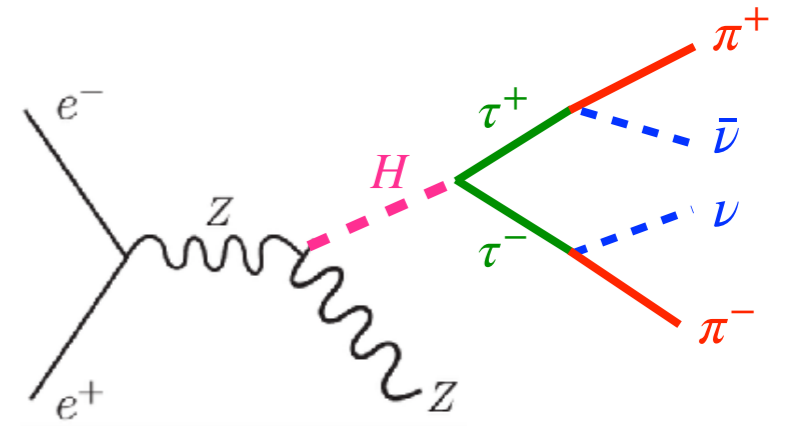
$$\mathscr{C}_{\mathrm{SM}}[\rho] = 1$$

$$\mathscr{S}_{\mathrm{SM}}[\rho] = 2$$

$$R_{\mathrm{CHSH}}^{\mathrm{SM}} = \sqrt{2}$$

reproduced very accurately in the simulation

→ false solutions somehow enjoy the correct correlations(?)

Effect of momentum mismeasurement

$$E_i^{\text{true}} \to E_i^{\text{obs}} = (1 + \sigma_E \cdot \omega) \cdot E_i^{\text{true}} \qquad \sigma_E = 0.03 \qquad (i = \pi^\pm, e^\pm, \mu^\pm, j)$$

random number drawn from the normal distribution

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} -0.600 \pm 0.210 & 0.003 \pm 0.125 & 0.020 \pm 0.149 \\ 0.003 \pm 0.125 & -0.494 \pm 0.190 & 0.007 \pm 0.128 \\ 0.048 \pm 0.174 & 0.0007 \pm 0.156 & 0.487 \pm 0.193 \end{pmatrix}$	$\begin{pmatrix} -0.559 \pm 0.143 & -0.010 \pm 0.095 & -0.014 \pm 0.122 \\ -0.010 \pm 0.095 & -0.494 \pm 0.139 & -0.002 \pm 0.111 \\ 0.012 \pm 0.124 & 0.020 \pm 0.105 & 0.434 \pm 0.134 \end{pmatrix}$
E_k	-1.057 ± 0.385	-0.977 ± 0.264
$\mathcal{C}[\rho]$	0.030 ± 0.071	0.005 ± 0.023
$\mathcal{S}[\rho]$	1.148 ± 0.210	1.046 ± 0.163
R_{CHSH}^*	0.769 ± 0.189	0.703 ± 0.134

$$C_{ij}^{\text{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\text{SM}}[\rho] = 1 \qquad \mathscr{S}_{\text{SM}}[\rho] = 2 \qquad R_{\text{CHSH}}^{\text{SM}} = \sqrt{2}$$

Momentum smearing spoils the previous good result…

Use impact parameter information

$c\tau \sim 100\,\mu m$

$\tau^{\pm}$

$\Theta_{\pm}$

$\pi^{\pm}$

ν

Primary Vertex

$\vec{b}_{\pm}$

IP

$(\sigma_{b_T} = 2\mu m,\quad \sigma_{b_z} = 5\mu m)$
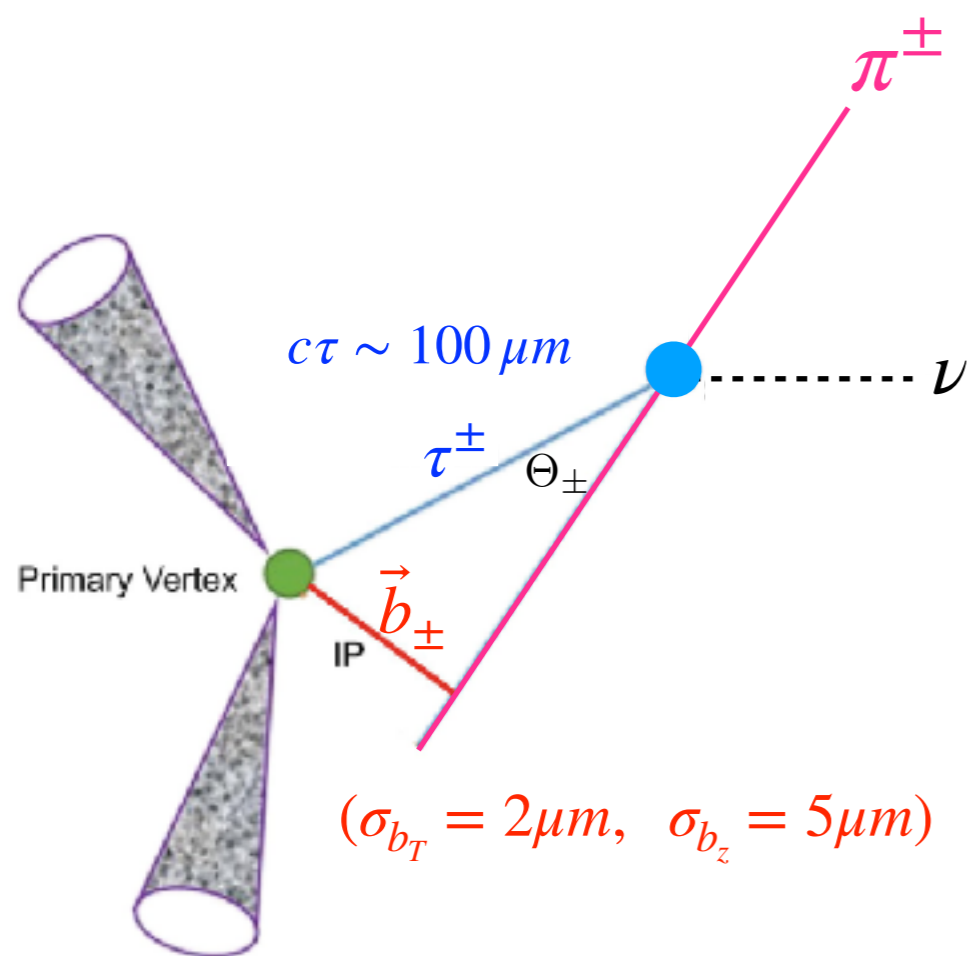
Goal:

$$E_i^{\text{true}} \to E_i^{\text{obs}} \to E_i^{\text{true}} \quad (i = \pi^{\pm}, e^{\pm}, \mu^{\pm}, j)$$

What we do:

- modify E_i^{obs} for some amount by δ

$$E_i^{\text{obs}} \to E_i(\delta_i) = (1 + \delta_i\,\sigma_E) \cdot E_i^{\text{obs}}$$

Use impact parameter information

$\pi^\pm$

$c\tau \sim 100\,\mu m$

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Primary Vertex

$\vec{b}_\pm$
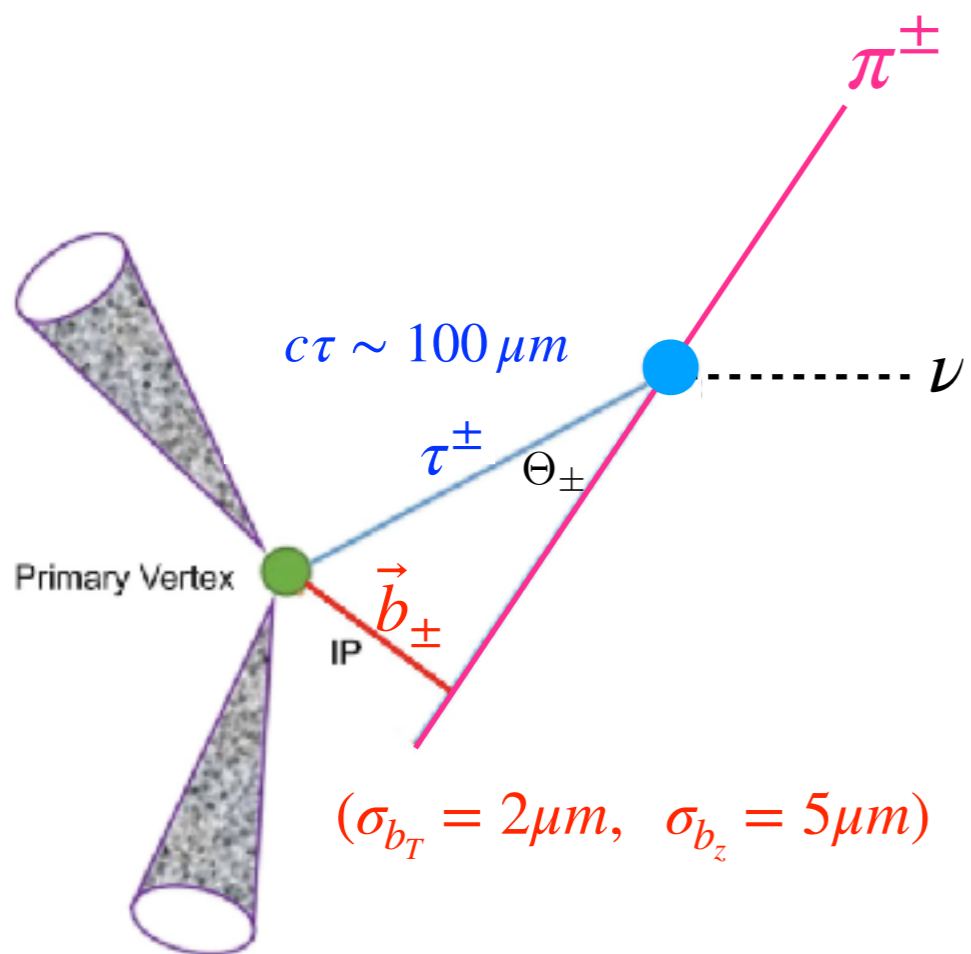
IP

$(\sigma_{b_T} = 2\mu m, \quad \sigma_{b_z} = 5\mu m)$

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$$E_i^{\text{obs}} \to E_i(\delta_i) = (1 + \delta_i\,\sigma_E) \cdot E_i^{\text{obs}}$$

- solve tau direction $\mathbf{e}_{\tau^\pm}(\boldsymbol{\delta})$

→ lets us calculate $\vec{b}_\pm$ as functions of δ

$$\vec{b}_\pm^{\text{reco}}(\mathbf{e}_{\tau^\pm}) = |\vec{b}_\pm| \cdot \left[\mathbf{e}_{\tau^\pm} \cdot \sin^{-1}\Theta_\pm - \mathbf{e}_{\pi^\pm} \cdot \tan^{-1}\Theta_\pm \right]$$

Use impact parameter information

Goal:

$$E_i^{\text{true}} \to E_i^{\text{obs}} \to E_i^{\text{true}} \quad (i = \pi^\pm, e^\pm, \mu^\pm, j)$$

$\pi^\pm$

$c\tau \sim 100\,\mu m$

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Primary Vertex

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What we do:
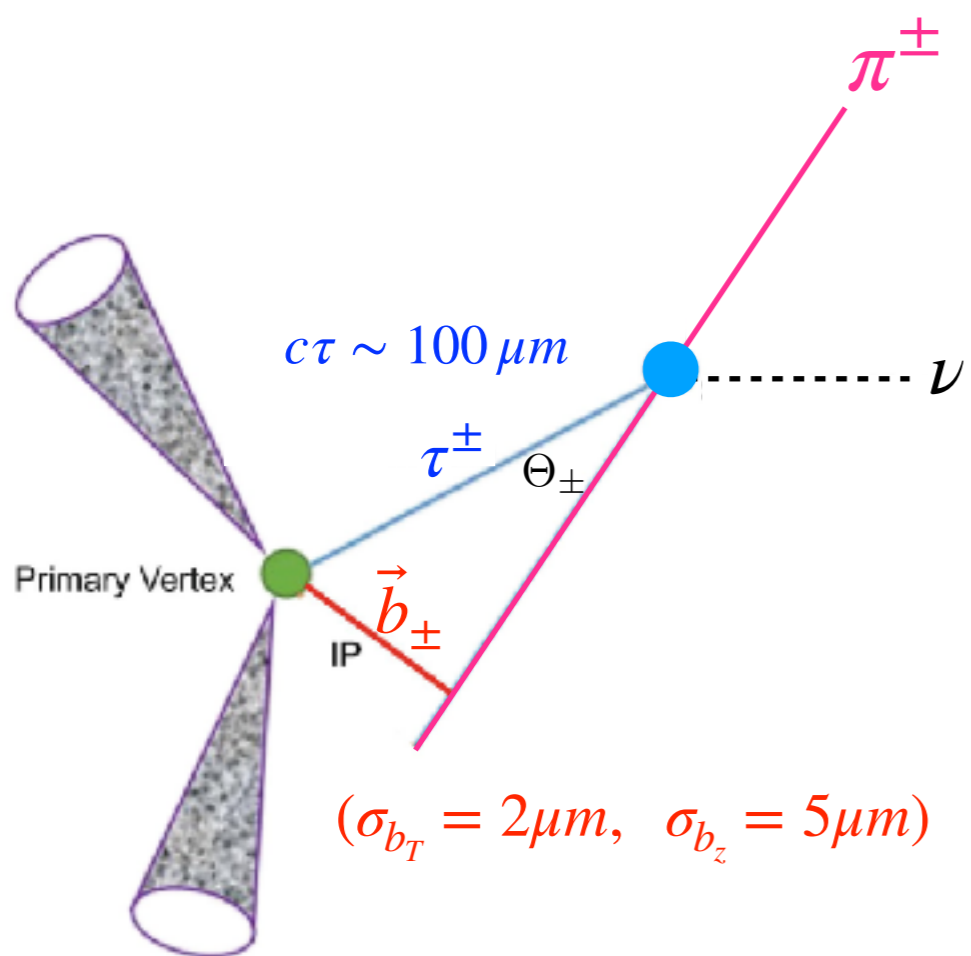
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$\to$ lets us calculate $\vec{b}_\pm$ as functions of δ

$$\vec{b}_\pm^{\text{reco}}(\mathbf{e}_{\tau\pm}) = |\vec{b}_\pm| \cdot \left[\mathbf{e}_{\tau\pm} \cdot \sin^{-1}\Theta_\pm - \mathbf{e}_{\pi\pm} \cdot \tan^{-1}\Theta_\pm \right]$$

- compare the calculated $\vec{b}_\pm^{\text{reco}}(\delta)$ and measured $\vec{b}_\pm^{\text{obs}}$ and construct the likelihood function

2 fold solutions: $i_s = 1,2$

$$\vec{\Delta}_{b_\pm}^{i_s}(\delta) \equiv \vec{b}_\pm - \vec{b}_\pm^{\text{reco}}\left(\mathbf{e}_{\tau^+}^{i_s}(\delta) \right)$$

$$L_\pm^{i_s}(\delta) = \frac{[\Delta_{b_\pm}^{i_s}(\delta)]_x^2 + [\Delta_{b_\pm}^{i_s}(\delta)]_y^2}{\sigma_{b_T}^2} + \frac{[\Delta_{b_\pm}^{i_s}(\delta)]_z^2}{\sigma_{b_z}^2} + \delta_{\pi^+}^2 + \delta_{\pi^-}^2 + \delta_x^2 + \delta_{\bar{x}}^2 \,.$$

minimizing $L^{i_s}(\delta)$ would give us the correct set of δs and solution i_s

Result

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279	2.696 ± 0.215
$\mathcal{C}[\rho]$	0.778 ± 0.126	0.871 ± 0.084
$\mathcal{S}[\rho]$	1.760 ± 0.161	1.851 ± 0.111
R^*_{CHSH}	1.103 ± 0.163	1.276 ± 0.094

$$C_{ij}^{\mathrm{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\mathrm{SM}}[\rho] = 1 \qquad \mathcal{S}_{\mathrm{SM}}[\rho] = 2 \qquad R_{\mathrm{CHSH}}^{\mathrm{SM}} = \sqrt{2}$$

Result

	ILC	FCC-ee
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E_k	2.567 ± 0.279 $\sim 5\sigma$	2.696 ± 0.215 $\gg 5\sigma$
$\mathcal{C}[\rho]$	0.778 ± 0.126 $\sim 5\sigma$	0.871 ± 0.084 $\gg 5\sigma$
$\mathcal{S}[\rho]$	1.760 ± 0.161 $\sim 3\sigma$	1.851 ± 0.111 $\sim 5\sigma$
R^*_{CHSH}	1.103 ± 0.163	1.276 ± 0.094 $\sim 3\sigma$

$$C_{ij}^{\text{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\text{SM}}[\rho] = 1 \qquad \mathcal{S}_{\text{SM}}[\rho] = 2 \qquad R_{\text{CHSH}}^{\text{SM}} = \sqrt{2}$$

— 68.3% CL		ILC (250 GeV, 3 ab^{-1})
--- 95.5% CL		FCC-ee (240 GeV, 5 ab^{-1})
—·— 99.7% CL		

10^3

10^2

Result

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279 $\quad \sim 5\sigma$	2.696 ± 0.215 $\quad \gg 5\sigma$
$\mathcal{C}[\rho]$	0.778 ± 0.126 $\quad \sim 5\sigma$	0.871 ± 0.084 $\quad \gg 5\sigma$
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R^*_{CHSH}	1.103 ± 0.163	1.276 ± 0.094 $\quad \sim 3\sigma$

$$C_{ij}^{\mathrm{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\mathrm{SM}}[\rho] = 1 \qquad \mathcal{S}_{\mathrm{SM}}[\rho] = 2 \qquad R^{\mathrm{SM}}_{\mathrm{CHSH}} = \sqrt{2}$$

Superiority of FCC-ee over ILC is due to a better beam resolution

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP measurement

- Under CP, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$

- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \geq 0$$

- Observation of $A \neq 0$ immediately confirms CP violation.

- From our simulation, we observe

$$A = \begin{cases} 0.204 \pm 0.173 & \text{(ILC)} \\ 0.112 \pm 0.085 & \text{(FCC-ee)} \end{cases} \quad \longleftarrow \quad \text{consistent with absence of CPV}$$

- This model independent bounds can be translated to the constraint on the CP-phase δ

$$\mathscr{L}_{\text{int}} \propto H\bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\psi_\tau \implies C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \implies A(\delta) = 4\sin^2 2\delta$$

CP measurement

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$|\delta| < \begin{cases} 8.9^o & \text{(ILC)} \\ 6.4^o & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$$\Delta\delta \sim 11.5^o \quad \text{(HL-LHC)} \qquad \text{[Hagiwara, Ma, Mori 2016]}$$

$$\Delta\delta \sim 4.3^o \quad \text{(ILC)} \qquad \text{[Jeans and G. W. Wilson 2018]}$$

Summary

- The quantum state of $H \to \tau^+ \tau^-$ is simple but measuring quantum properties is challenging even at lepton colliders since taus are highly boosted.

- Very accurate event reconstruction is required, which can be achieved by using the impact parameters.

- ILC and FCC-ee are able to see entanglement, steering and violation of BI.

- Spin correlation is sensitive to CP-phase and we can measure the CP-phase as byproduct of the quantum property measurement.

	Entanglement	Steering	Bell-inquality	CP-phase
ILC	$\sim 5\sigma$	$\sim 3\sigma$		8.9^o
FCC-ee	$\gg 5\sigma$	$\sim 5\sigma$	$\sim 3\sigma$	6.4^o
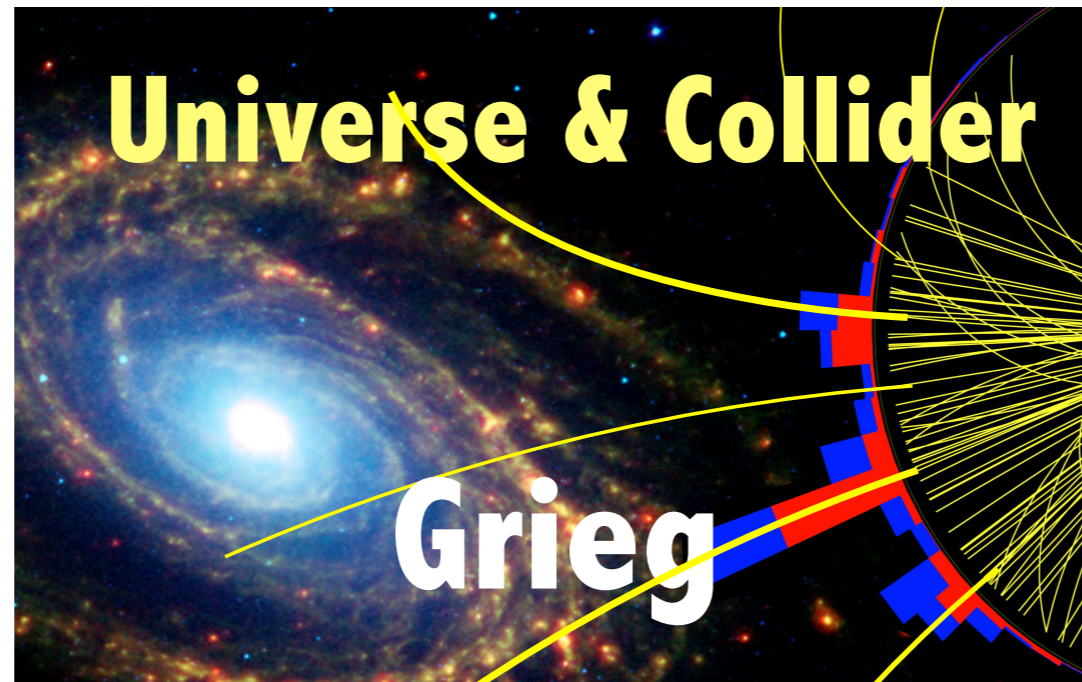
**Understanding the Early Universe:
interplay of theory and collider experiments**

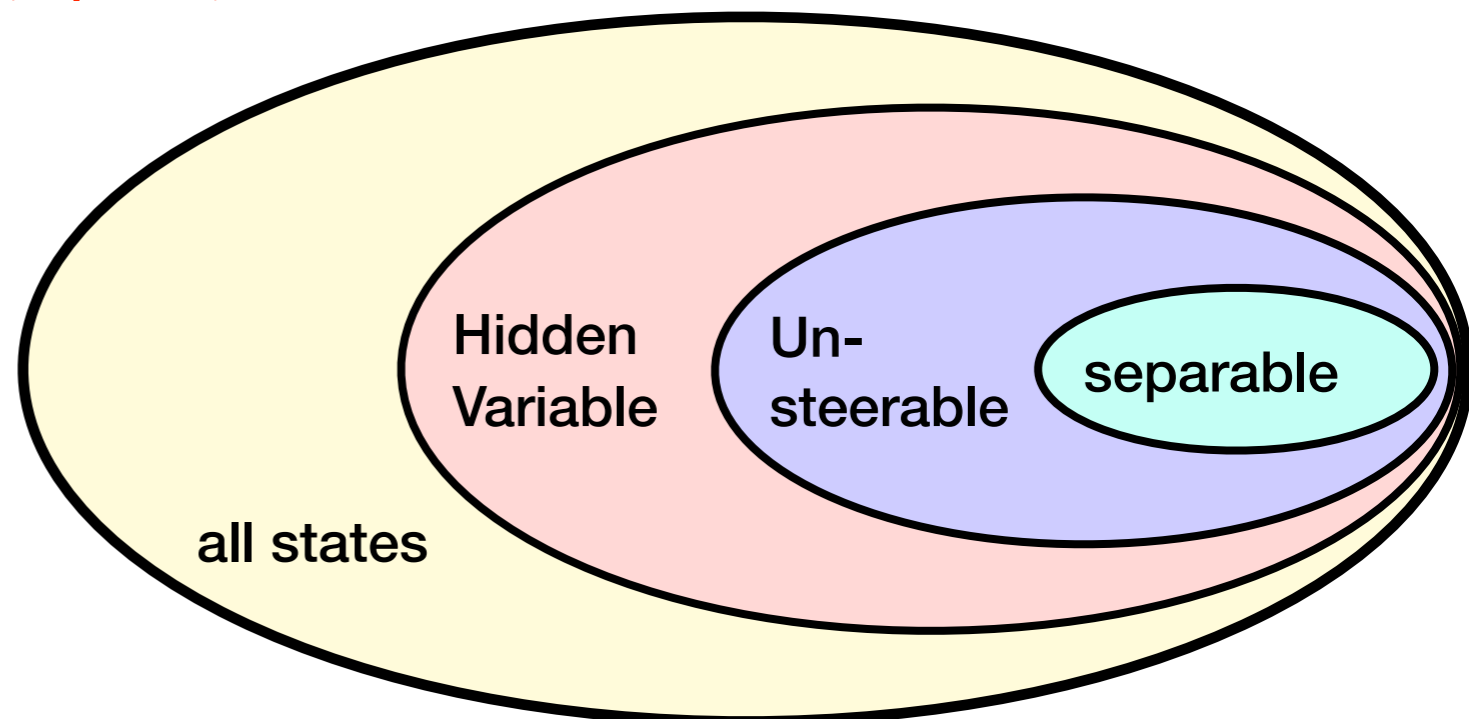Joint research project between the University of Warsaw & University of Bergen

Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_{\lambda} p_{\lambda} \langle a\,|\,\rho_{\lambda}^{\alpha}\,|\,a\rangle \cdot \langle b\,|\,\rho_{\lambda}^{\beta}\,|\,b\rangle \qquad \longleftarrow \qquad \rho = \sum_{\lambda} p_{\lambda}\,\rho_{k}^{\alpha} \otimes \rho_{\lambda}^{\beta}$$

Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b\,|\,A,B) = \sum_{\lambda} p_{\lambda}\,P_{\alpha}(a\,|\,A,\lambda) \cdot P_{\beta}(b\,|\,B,\lambda)$$

arbitrary conditional
probabilities

Hidden
Variable

Un-
steerable

separable

all states

Separable state (compliment of entangled state):

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda \langle a|\rho_\lambda^\alpha|a\rangle \cdot \langle b|\rho_\lambda^\beta|b\rangle \quad \longleftarrow \quad \rho = \sum_\lambda p_\lambda\,\rho_k^\alpha \otimes \rho_\lambda^\beta$$

Un-steerable state (not-steerable by Alice):

[Jones, Wiseman, Doherty 2007]

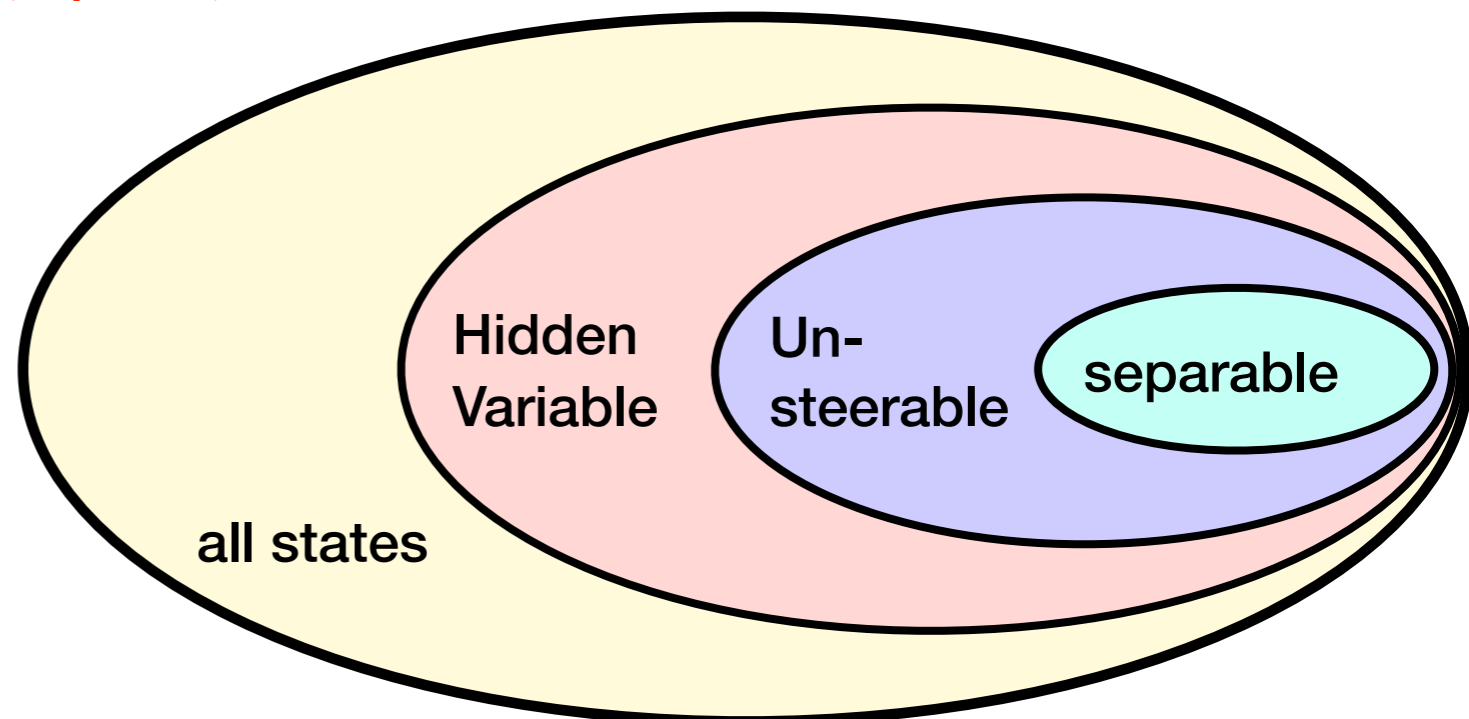$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda\,P_\alpha(a\,|\,A,\lambda) \cdot \langle b|\rho_\lambda^\beta|b\rangle \quad \longleftarrow$$

If this description is possible, Alice cannot influence (`steer") Bob's local state

Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda\,P_\alpha(a\,|\,A,\lambda) \cdot P_\beta(b\,|\,B,\lambda)$$

arbitrary conditional probabilities

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Separable state (compliment of entangled state):

$$P(a, b \mid A, B) = \sum_\lambda p_\lambda \langle a \mid \rho_\lambda^\alpha \mid a \rangle \cdot \langle b \mid \rho_\lambda^\beta \mid b \rangle \quad \longleftarrow \quad \rho = \sum_\lambda p_\lambda \, \rho_k^\alpha \otimes \rho_\lambda^\beta$$

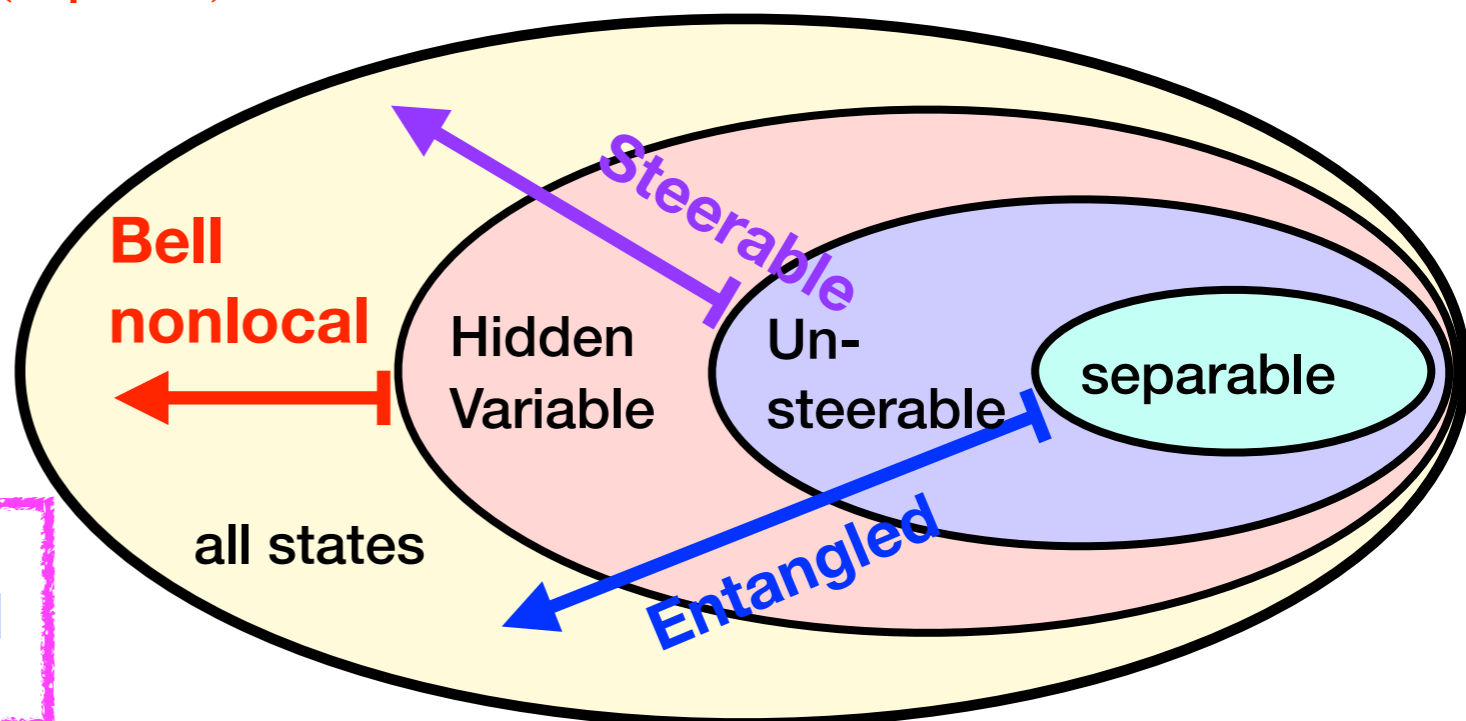**Un-steerable state** (not-steerable by Alice):

[Jones, Wiseman, Doherty 2007]

$$P(a, b \mid A, B) = \sum_\lambda p_\lambda \, P_\alpha(a \mid A, \lambda) \cdot \langle b \mid \rho_\lambda^\beta \mid b \rangle \quad \longleftarrow$$

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Hidden Variable state (complement of Bell nonlocal state):

$$P(a, b \mid A, B) = \sum_\lambda p_\lambda \, P_\alpha(a \mid A, \lambda) \cdot P_\beta(b \mid B, \lambda)$$

arbitrary conditional probabilities

Bell nonlocal $\subset$ **Steerable** $\subset$ **Entangled**