

The new theory pipeline pineline and its applications

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Acknowledgement: This project has received funding from the European Unions Horizon 2020 research and innovation programme under grant agreement number 740006.



Outline

1. The new theory pineline [in preparation]

1.1 Motivation

1.2 PineAPPL [JHEP12.108]

1.3 EKO [EPJC82.976]

1.4 yadism [in preparation]

1.5 Outlook

2. First Applications of the pineline

2.1 Evidence for Intrinsic Charm in the Proton
[Nature608.483]

2.2 Towards N3LO PDFs [in preparation]

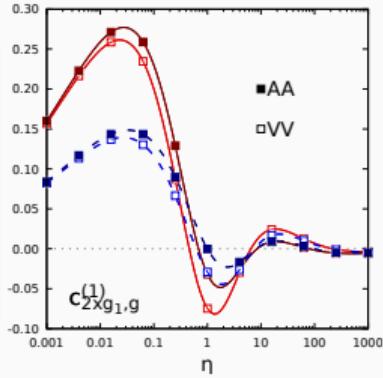
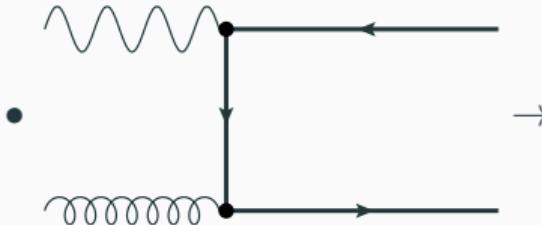
2.3 Parton Distributions and New Physics
Searches: the Drell-Yan Forward-Backward
Asymmetry as a case study [EPJC82.1160]

1. The new theory pipeline [in preparation]

1.1. Motivation

Including New Computation

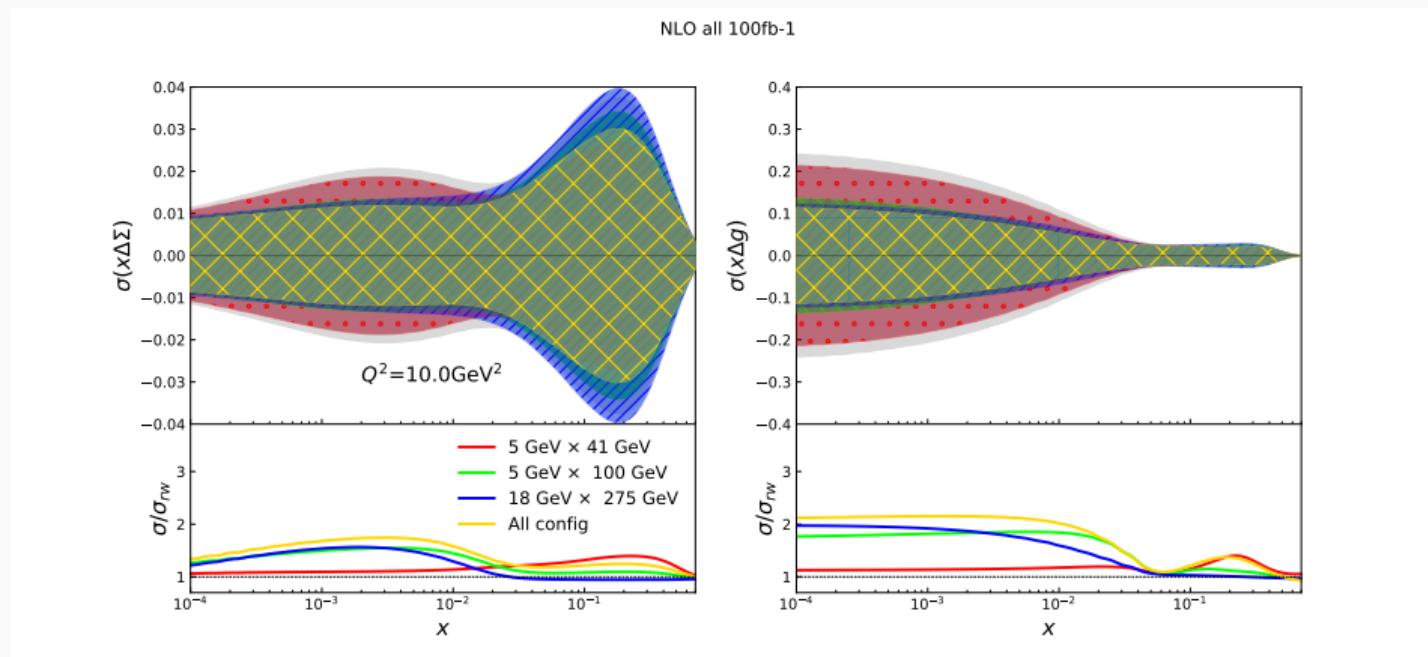
- Computing new observables is expensive both in runtime (days/weeks) and development time (month/years)
 - E.g. NLO heavy quark production in polarized DIS [PRD98.014018] [1910.01536] [PRD104.016033]



- How to measure the actual impact on PDFs?

Reweighting

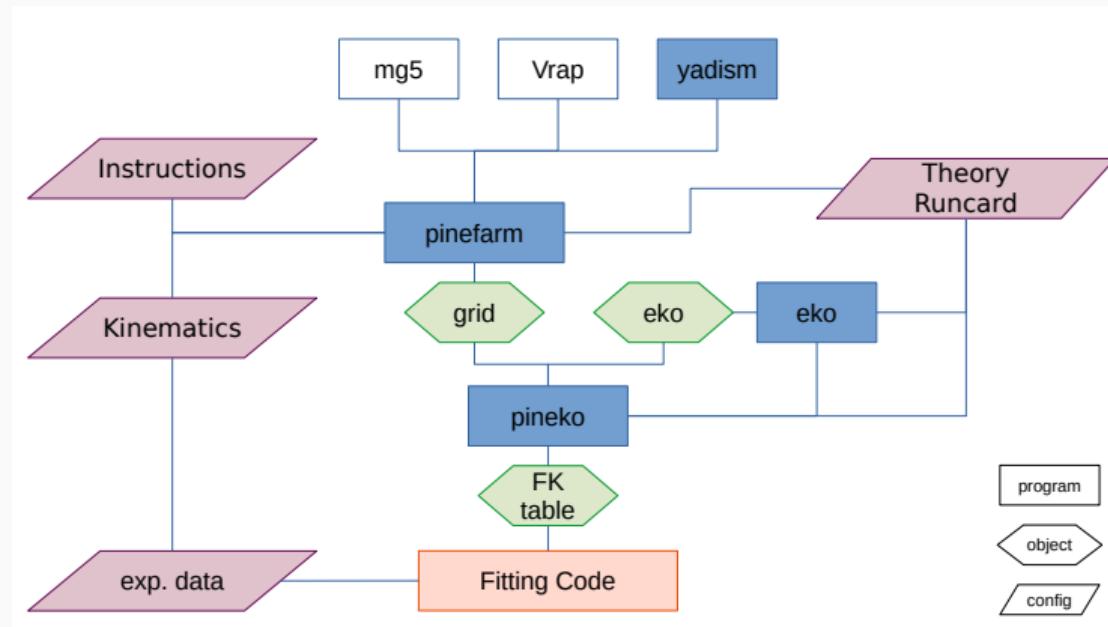
Reweighting is possible [PRD104.114039] - and even needed for the EIC



but a new PDF fit would be better!

New Theory Prediction Pipeline

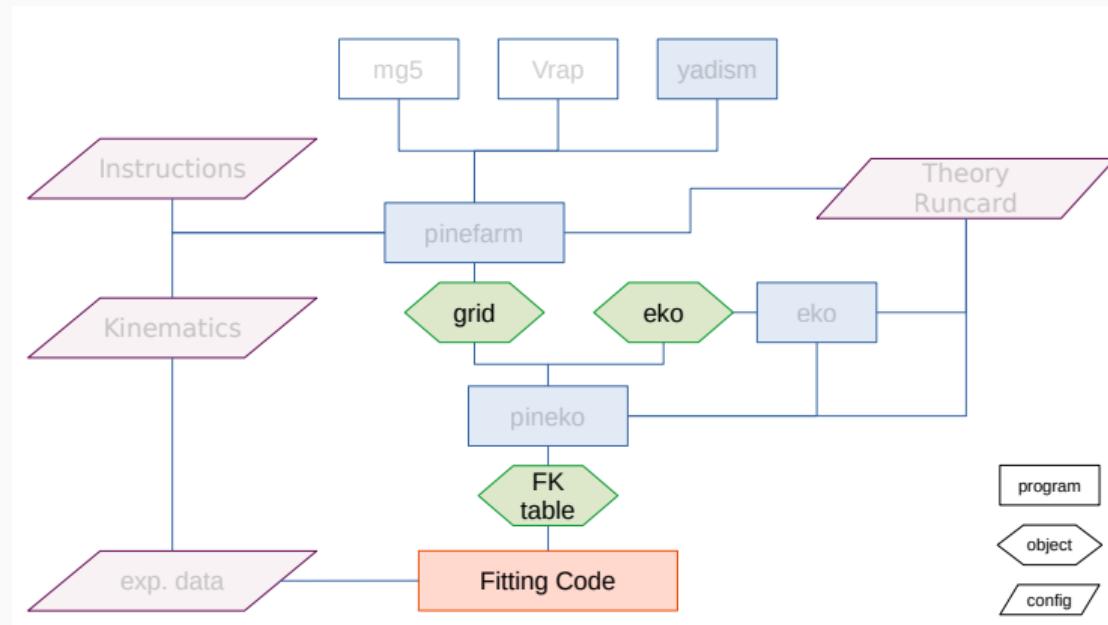
Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

New Theory Prediction Pipeline

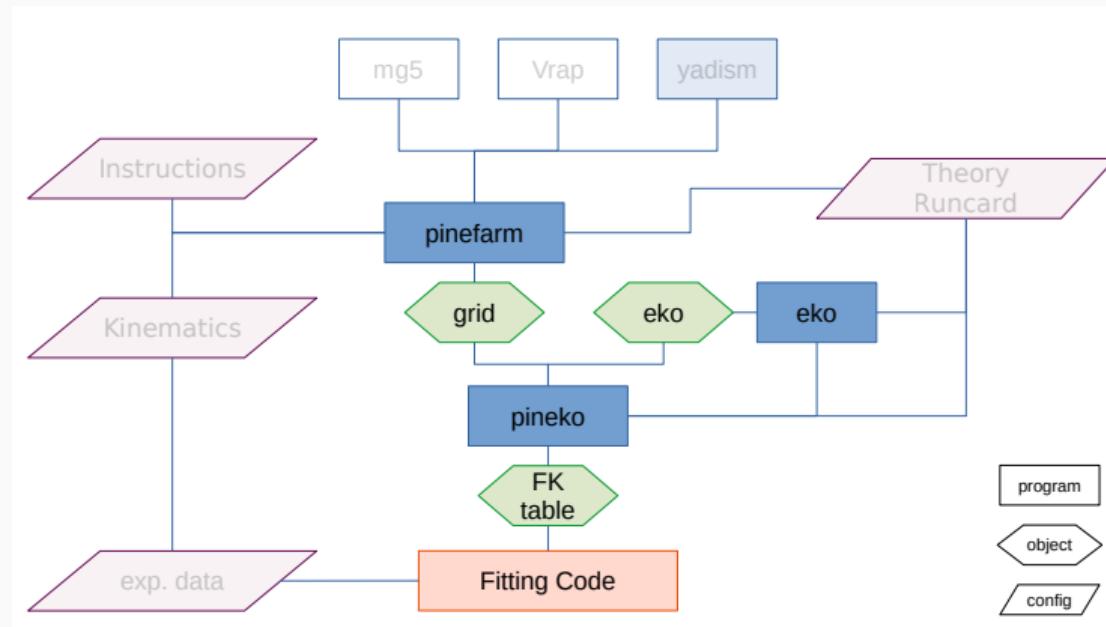
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New Theory Prediction Pipeline

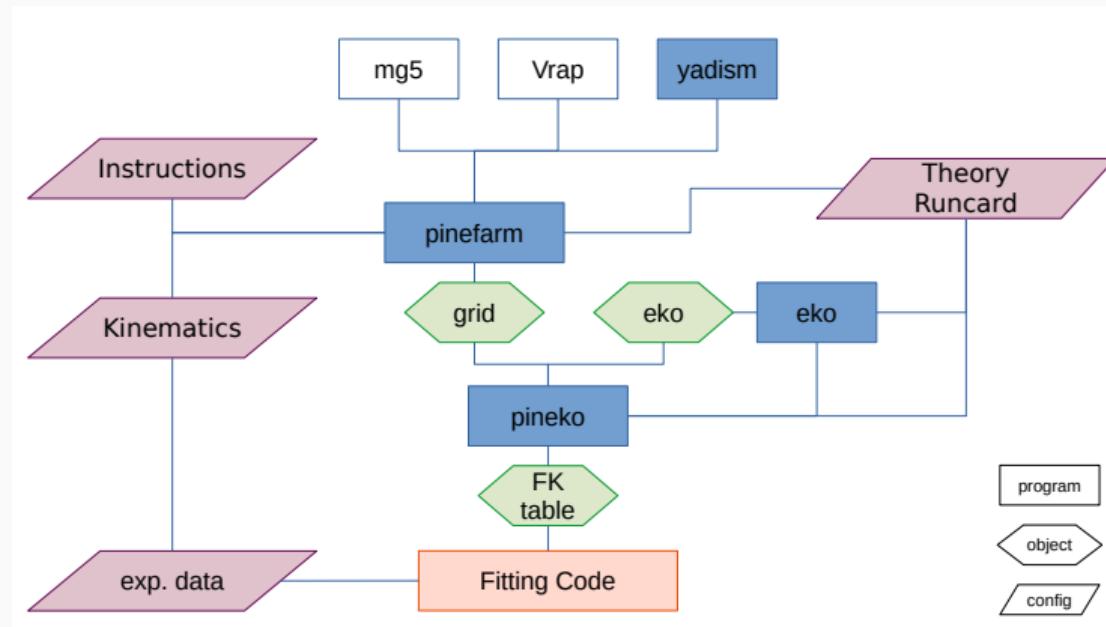
Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

New Theory Prediction Pipeline

Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

1.2. PineAPPL [JHEP12.108]

About PineAPPL

PineAPPL is a fast interpolation grid library that

- extends to arbitrary orders in QCD and EW coupling
- provides a very good CLI
- provides several interfaces: Rust, Python, C/C++, Fortran
- can convert APPLgrid [EPJC66.503] and FastNLO [DIS12.217]

Check the documentation: <https://n3pdf.github.io/pineappl/>

The Command Line Interface

The CLI (pineappl) serves for the everyday life questions:

- `convolute` - get the predictions for any PDF set including uncertainties
- `channels` - split the predictions into luminosity channels
- `orders` - split the predictions into perturbative orders
- `info` - access meta data
- `plot` - generate a (customizable) python plot script

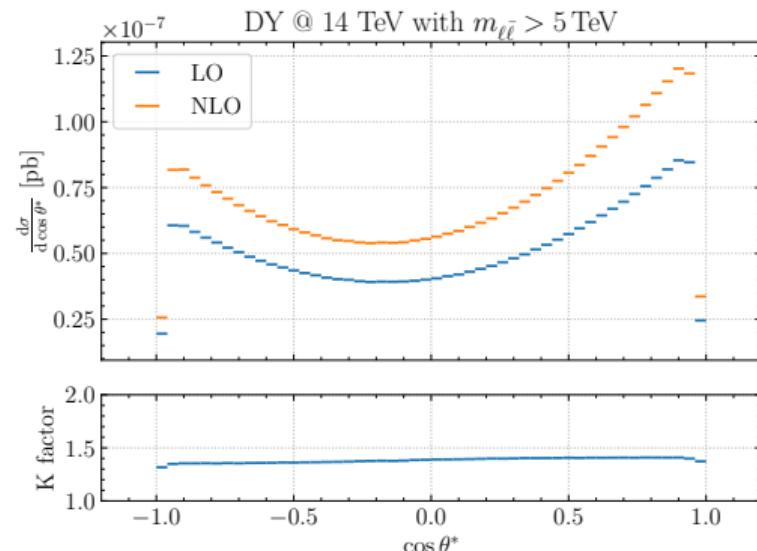
The Command Line Interface

```
$ pineappl convolute CMS_DY_14TEV_MLL_5000_COSTH.pineappl.lz4 \
    NNPDF40_nnlo_as_01180
```

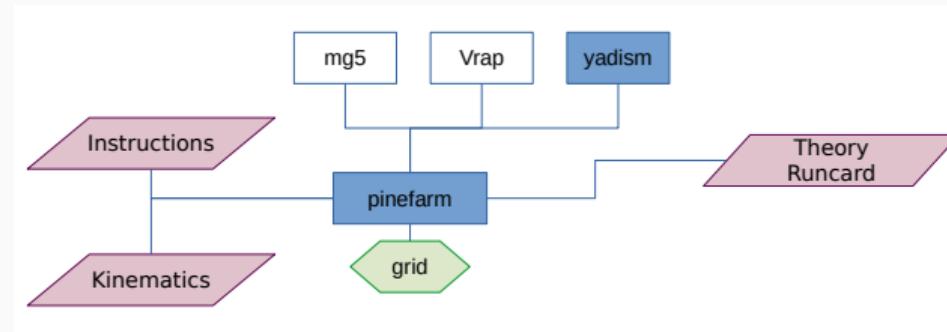
b	costh	dsig/dcosth	scale	uncertainty
	[]	[pb]		[%]
0	-1	-0.96 5.0382145e-8	-4.73	4.32
1	-0.96	-0.92 1.6366674e-7	-4.98	4.62
2	-0.92	-0.88 1.6611145e-7	-5.06	4.70
3	-0.88	-0.84 1.5983761e-7	-5.08	4.74
4	-0.84	-0.8 1.5374426e-7	-5.09	4.75
5	-0.8	-0.76 1.4800320e-7	-5.10	4.76
6	-0.76	-0.72 1.4238050e-7	-5.10	4.76
7	-0.72	-0.68 1.3708378e-7	-5.10	4.76
8	-0.68	-0.64 1.3191722e-7	-5.12	4.78
[...]				

The Python Interface

```
1 import lhapdf
2 import pineappl
3 # load PDF
4 pdf = lhapdf.mkPDF("NNPDF40_nnlo_as_01180",0)
5 # load grid
6 grid = pineappl.grid.Grid.read("CMS_DY_14TeV_MLL_5000_COSTH.
    pineappl.lz4")
7 # convolute
8 print(grid.convolute_with_one(2212, pdf.
    xfxQ2, pdf.alphasQ2))
9 # prints the same list of numbers
```



Interface to Other Programs: pinefarm



```
*****
* W E L C O M E t o M A D G R A P H 5 _ a M C @ N L O
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*
*           *
*           *   *   *
*           * * * * 5 * * * *
*           *       *   *
*           *           *
*
* The MadGraph5_aMC@NLO Development Team - Find us at
* https://server06.fynu.ucl.ac.be/projects/madgraph
* and
*         http://amcatnlo.cern.ch
*
*           Code download from:
*         https://launchpad.net/madgraph5
*
* Please refer to: MadGraph5_aMC@NLO paper
*                   J. Alwall et al.
* arXiv:1405.0301, JHEP 1407 (2014) 079
```



[in preparation]

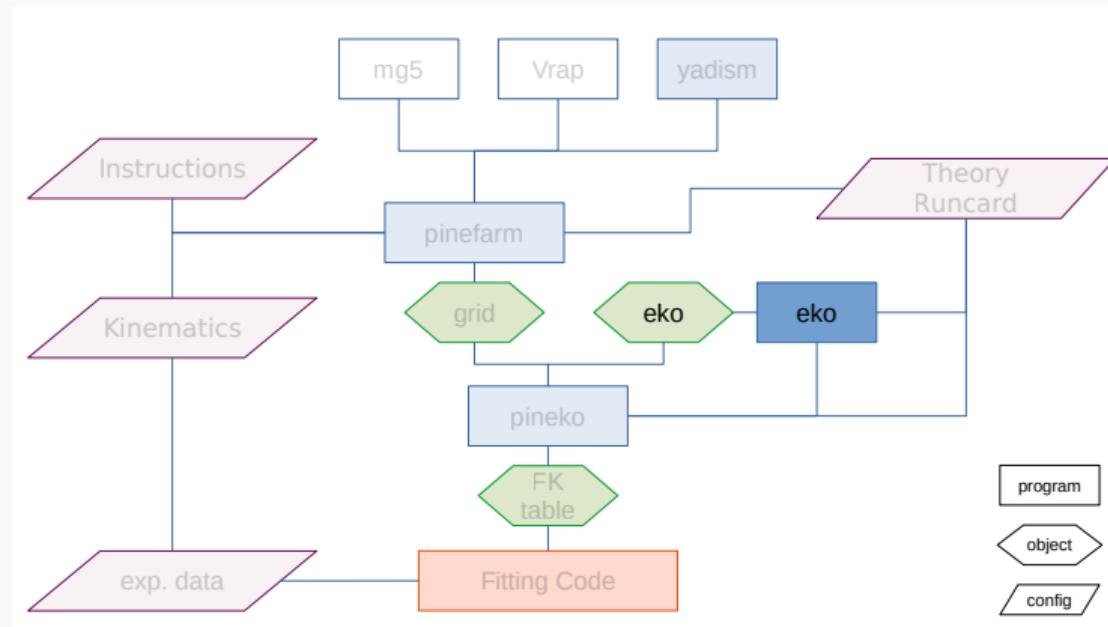
Vrap [PRD69.094008]

more MC interfaces coming
soon (e.g. MATRIX
[EPJC78.537])

1.3. EKO [EPJC82.976]

New Theory Prediction Pipeline

Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL



DGLAP:

$$\mu_F^2 \frac{d\mathbf{f}}{d\mu_F^2}(\mu_F^2) = \mathbf{P}(a_s(\mu_R^2), \mu_F^2) \otimes \mathbf{f}(\mu_F^2)$$

as operator equation for the evolution kernel operator (EKO) **E**:

$$\mu_F^2 \frac{d}{d\mu_F^2} \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2) = \mathbf{P}(a_s(\mu_R^2), \mu_F^2) \otimes \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2)$$

with

$$\mathbf{f}(\mu_F^2) = \mathbf{E}(\mu_F^2 \leftarrow \mu_{F,0}^2) \otimes \mathbf{f}(\mu_{F,0}^2)$$

- independent of boundary condition → PDF fitting
- Mellin (N -) space solution, but momentum (x -) space delivery via piecewise Lagrange-interpolation
- Intrinsic heavy quark distributions → see IC
- Backward VFNS evolution (i.e. across thresholds and with intrinsic) → see IC

EKO Project Management

The screenshot shows the GitHub repository page for EKO. It features a sidebar with navigation links like Code, Issues, Pull requests, Discussions, Articles, Projects, Wiki, Security, Insights, and Settings. The main area displays a list of commits from the master branch, with one commit by AleCamido labeled "Merge branch 'release/0.5.7'". Below the commits is a "Releases" section with a "Paper" link and a "Contributors" section with a "GitHub Pages" link.

The screenshot shows the EKO documentation site at eko.readthedocs.io. The page title is "Interpolation". It includes a search bar, a sidebar with categories like Overview, Features, Examples, Indices and tables, Theory, Interpolation, and Implementation, and a main content area with text about interpolation theory and a mathematical formula for the grid points.

In order to obtain the operators in an PDF independent way we use approximation theory. Therefore, we define the basis grid

$$\mathbb{G} = \{x_j : 0 < x_j \leq 1, j = 0, \dots, N_{\text{grid}} - 1\}$$

from which we define our interpolation

$$f(x) \sim \tilde{f}(x) = \sum_{j=0}^{N_{\text{grid}}-1} f(x_j)p_j(x)$$

Thus each grid point x_j has an associated interpolation polynomial $p_j(x)$ (represented by `eko.interpolation.BasisFunction`). We interpolate in $\ln(x)$ using Lagrange interpolation among the nearest $N_{\text{degree}} + 1$ points, which renders the $p_j(x)$: polynomials of order $O(\ln^{N_{\text{degree}}}(x))$.

Algorithm

First, we split the interpolation region into several areas (represented by `eko.interpolation.Area`), which are bound by the grid points:

$$A_j = (x_j, x_{j+1}], \quad \text{for } j = 0, \dots, N_{\text{grid}} - 2$$

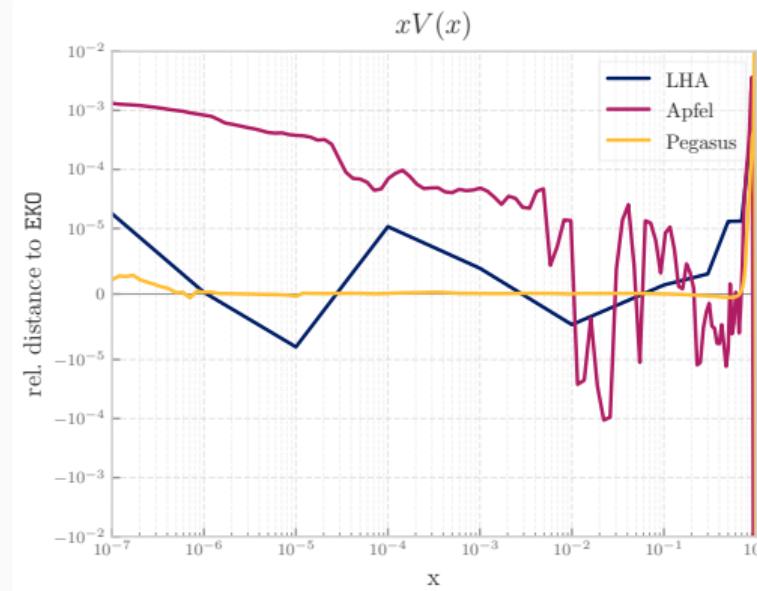
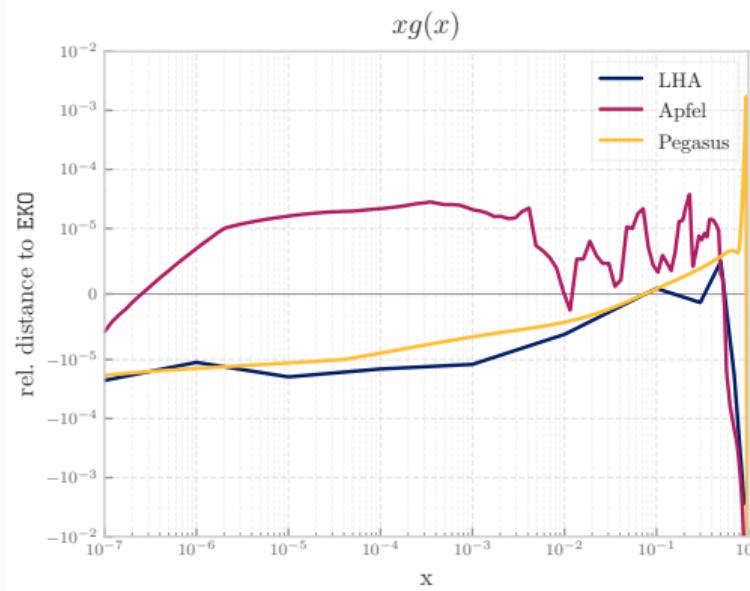
Note, that we include the right border point into the definition, but not the left which keeps all areas disjoint. This assumption is based on the physical fact, that PDFs do have a fixed upper bound ($x = 1$), but no fixed lower bound.

Second, we define the interpolation blocks, which will build the interpolation polynomials and contain the needed amount of points:

- Fully open source: <https://github.com/N3PDF/eko>
- Written in Python
- Fully documented: <https://eko.readthedocs.io/>

EKO Benchmarks

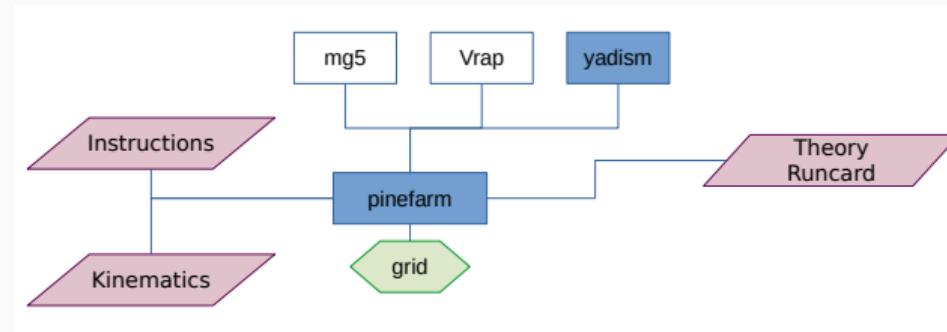
LHA benchmark [0204316][0511119]:



⇒ EKO is working!

1.4. yadism [in preparation]

Interface to Other Programs: pinefarm



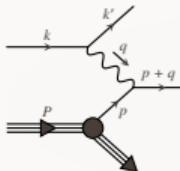
```
*****
* W E L C O M E t o M A D G R A P H 5 _ a M C @ N L O
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*
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*           *   *   *
*           * * * * 5 * * * *
*           *       *   *
*           *           *
*           *
*           *
* The MadGraph5_aMC@NLO Development Team - Find us at
* https://server06.fynu.ucl.ac.be/projects/madgraph
* and
* http://amcatnlo.cern.ch
*
*           Code download from:
*           https://launchpad.net/madgraph5
*
* Please refer to: MadGraph5_aMC@NLO paper
*                   J. Alwall et al.
* arXiv:1405.0301, JHEP 1407 (2014) 079
```



[in preparation]

Vrap [PRD69.094008]

more MC interfaces coming
soon (e.g. MATRIX
[EPJC78.537])



- DIS coefficient function database
- independent of boundary condition → PDF fitting
- separate features: TMC, FNS
- constant benchmark against APFEL

same improvement in terms of project management as EKO!

⌚ <https://github.com/NNPDF/yadism>

Ξ <https://yadism.readthedocs.io>

Coefficient Functions

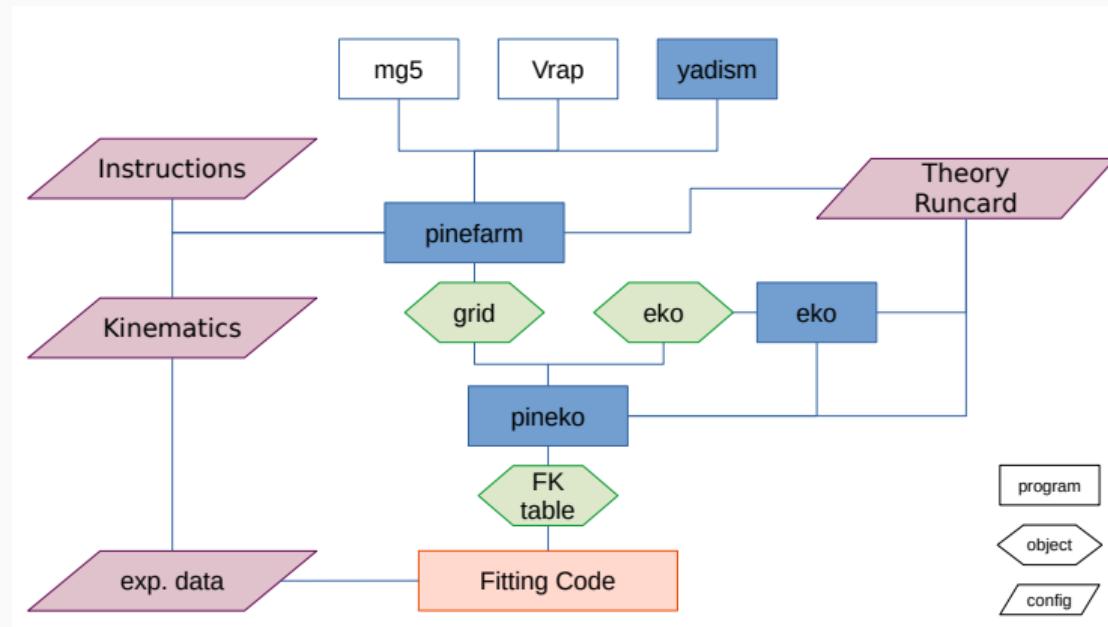
- implemented coefficient functions:

	light	heavy	intrinsic
NC	$O(a_s^2)$ [VVM05,MVV05,MV00]	$O(a_s^2)$ [Hek19]	$O(a_s)$ [KS98]
CC	$O(a_s^2)$ [MRV08,MVV09]	$O(a_s)$ [GKR96]	$O(a_s)$ [in prep.]

- implemented flavor number schemes: FFNS, ZM-VFNS, FONLL

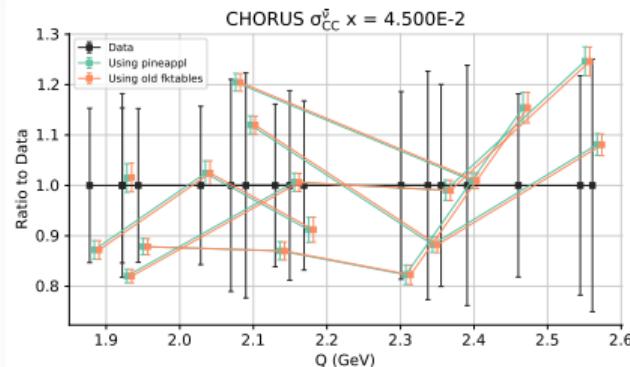
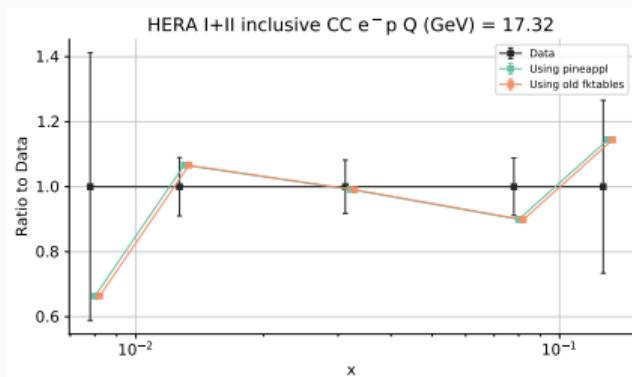
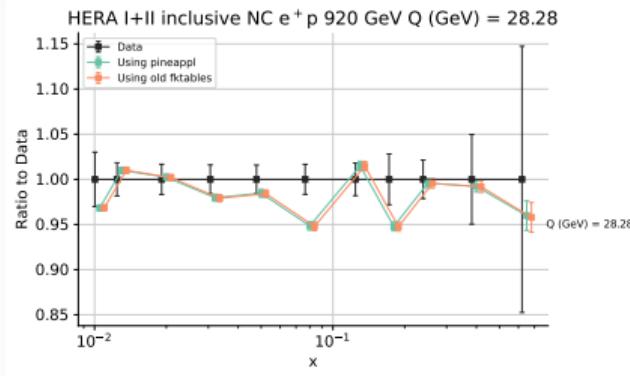
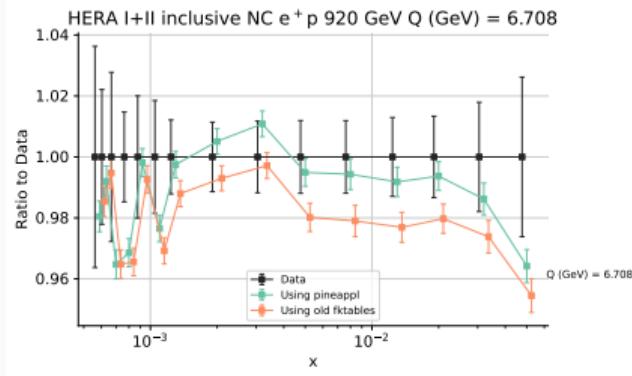
New Theory Prediction Pipeline

Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

Comparison yadism against APFEL



green, "pineappl" = yadism vs. orange, "old" = APFEL

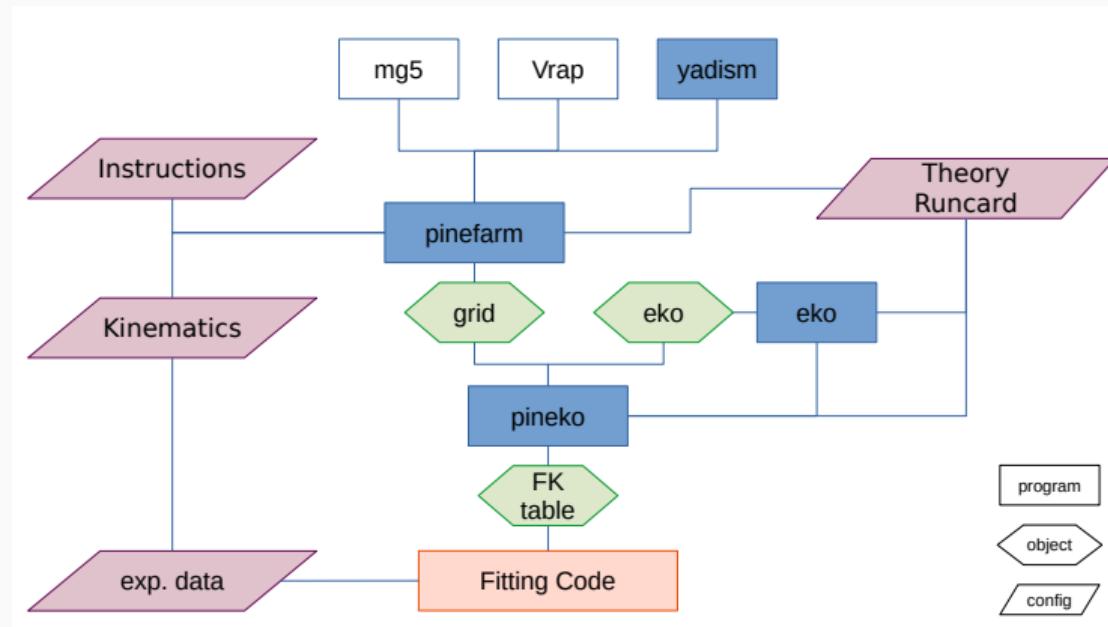
1.5. Outlook

Outlook

- extend to N3LO → Giacomo Magni (VU) → see applications 2
- include MHOU → Andrea B.
- include QED corrections → Niccolò L.
- add polarized setup → Adrienne Schaus (VU)
- extend to fragmentation function → Tanishq Sharma (UNITO)
- ...

New Theory Prediction Pipeline

Produce FastKernel (FK) tables!



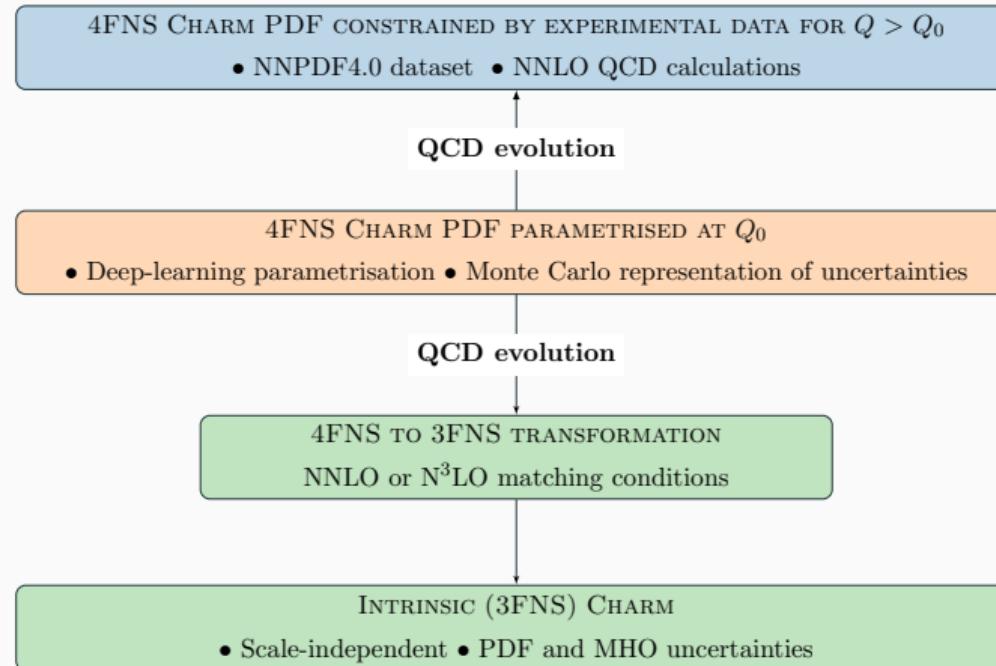
The workhorse in the background: PineAPPL

2. First Applications of the pineline

2.1. Evidence for Intrinsic Charm in the Proton [Nature608.483]

Intrinsic Charm: Strategy

based on NNPDF4.0 [EPJC82.428]



Matching Conditions and Backward Evolution

For (forward) evolution across a matching scale μ_h^2 :

$$\tilde{\mathbf{f}}^{(n_f+1)}(\mu_{F,1}^2) = \tilde{\mathbf{E}}^{(n_f+1)}(\mu_{F,1}^2 \leftarrow \mu_h^2) \mathbf{R}^{(n_f)} \tilde{\mathbf{A}}^{(n_f)}(\mu_h^2) \tilde{\mathbf{E}}^{(n_f)}(\mu_h^2 \leftarrow \mu_{F,0}^2) \tilde{\mathbf{f}}^{(n_f)}(\mu_{F,0}^2) \quad (1)$$

with $\mathbf{R}^{(n_f)}$ a flavor rotation matrix and $\tilde{\mathbf{A}}^{(n_f)}(\mu_h^2)$ the operator matrix elements (partially known up to N³LO)

Matching Conditions and Backward Evolution

For (forward) evolution across a matching scale μ_h^2 :

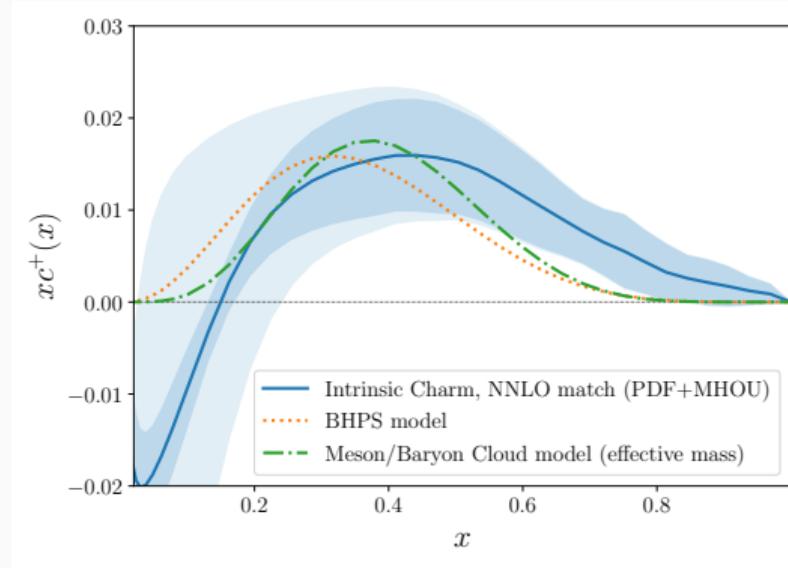
$$\tilde{\mathbf{f}}^{(n_f+1)}(\mu_{F,1}^2) = \tilde{\mathbf{E}}^{(n_f+1)}(\mu_{F,1}^2 \leftarrow \mu_h^2) \mathbf{R}^{(n_f)} \tilde{\mathbf{A}}^{(n_f)}(\mu_h^2) \tilde{\mathbf{E}}^{(n_f)}(\mu_h^2 \leftarrow \mu_{F,0}^2) \tilde{\mathbf{f}}^{(n_f)}(\mu_{F,0}^2) \quad (1)$$

with $\mathbf{R}^{(n_f)}$ a flavor rotation matrix and $\tilde{\mathbf{A}}^{(n_f)}(\mu_h^2)$ the operator matrix elements (partially known up to N³LO)

for backward evolution:

- invert $\tilde{\mathbf{E}}^{(n_f)}$: simple
- invert $\mathbf{R}^{(n_f)}$: simple
- invert $\tilde{\mathbf{A}}^{(n_f)}$: expanded or exact

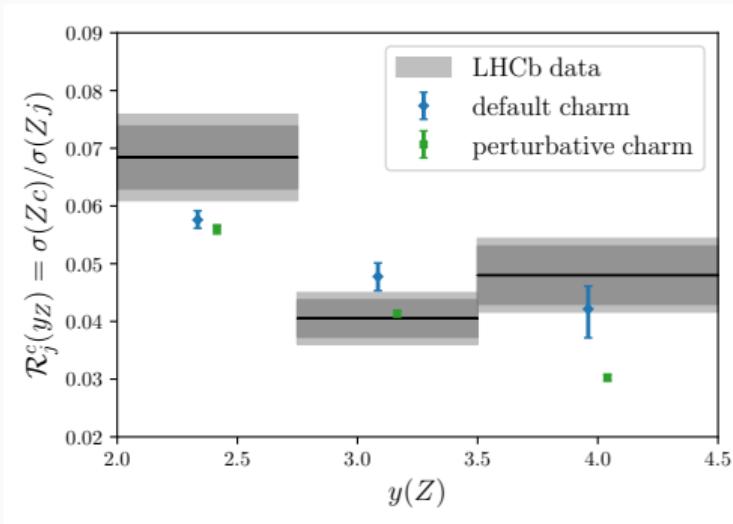
Intrinsic Charm: PDF plot



[BHPS] or [Meson/Baryon Cloud Model]

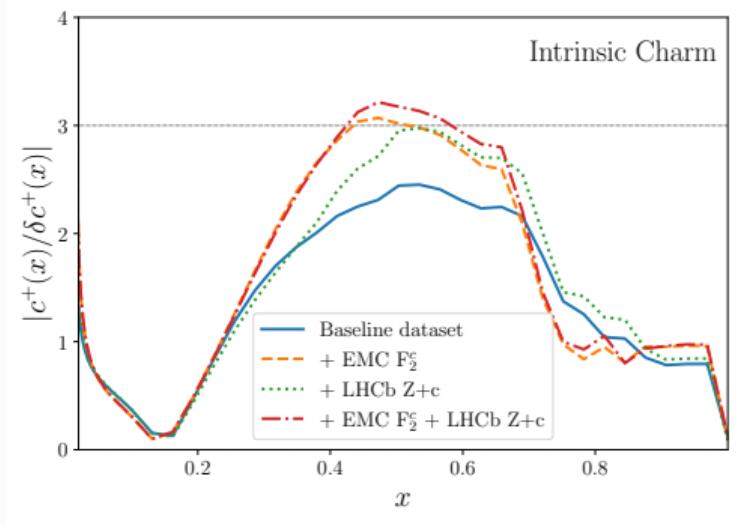
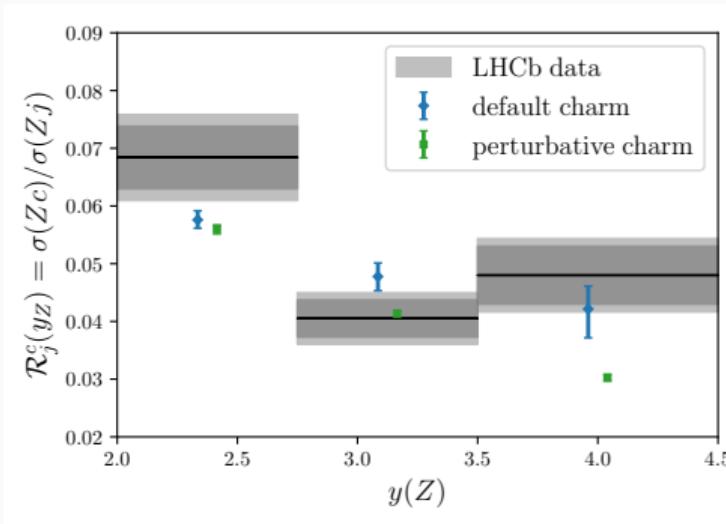
- in **3FNS** a valence-like peak is present
- for $x \leq 0.2$ the perturbative uncertainties are quite large
- the carried momentum fraction is within **1%**

Intrinsic Charm: LHCb and Significance



- match better recent **LHCb** $Z+c$ measurement [PRL128.082001]

Intrinsic Charm: LHCb and Significance



- match better recent **LHCb** Z+c measurement [PRL128.082001]
- we find a **3σ** evidence of intrinsic charm
- result is **stable** with mass variation, dataset variation

2.2. Towards N3LO PDFs [in preparation]

Towards N3LO PDFs

- N3LO DGLAP evolution: splitting functions approximation
- N3LO DIS light coefficients (*+ approximation for the massive contributions*)
- Hadronic observables @ NNLO (*+ K-factors*)
- Inclusion of theory uncertainties both from scale variations and N3LO accuracy.

N3LO singlet sector

Analytical calculations of the complete N3LO splitting functions are not available yet.
Restricting to the singlet sector the known limits are:

large- n_f

- Davies, Vogt, Ruijl, Ueda, and Vermaseren. Large- n_f contributions to the four-loop splitting functions in QCD. [\[arXiv:1610.07477\]](https://arxiv.org/abs/1610.07477)

small-x

- Bonvini and Marzani. Four-loop splitting functions at small-x. [\[arXiv:1805.06460\]](https://arxiv.org/abs/1805.06460)
- Davies, Kom, Moch, and Vogt. Resummation of small-x double logarithms in QCD: inclusive deep-inelastic scattering. 2 2022. [arXiv:2202.10362](https://arxiv.org/abs/2202.10362).

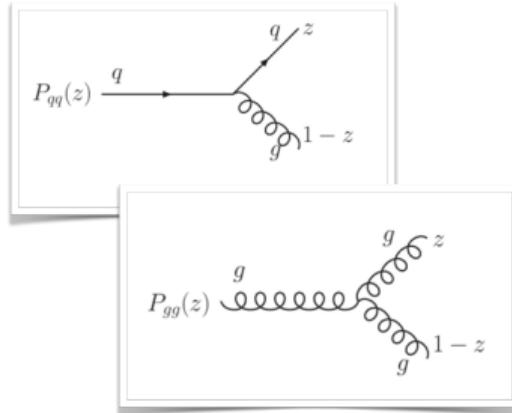
large-x

- Duhr, Mistlberger, and Vita. Soft integrals and soft anomalous dimensions at N3LO and beyond. [\[arXiv:2205.04493\]](https://arxiv.org/abs/2205.04493).
- Henn, Korchemsky, and Mistlberger. The full four-loop cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills and QCD. [\[arXiv:1911.10174\]](https://arxiv.org/abs/1911.10174).
- Soar, Moch, Vermaseren, and Vogt. On Higgs-exchange DIS, physical evolution kernels and fourth-order splitting functions at large x. [\[arXiv:0912.0369\]](https://arxiv.org/abs/0912.0369).

Moments

- Moch, Ruijl, Ueda, Vermaseren, and Vogt. Low moments of the four-loop splitting functions in QCD. [\[arXiv:2111.15561\]](https://arxiv.org/abs/2111.15561).

- Theoretical inputs are not enough to determine the full expressions analytically.
- Need to parametrise the unknown part with sub-leading contributions.
- Uncertainties from this determination has to be taken into account during the fit.



Singlet

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{gg}^{(3)}$	✓	✓	✓	✓
$\gamma_{q\bar{q}}^{(3)}$	✓	✓	✓	✓
$\gamma_{q\bar{q}}^{(3)}$		✓	✓	✓
$\gamma_{q\bar{q},ps}^{(3)}$		✓	✓	✓

Approximation of $P_{gg}(x)$

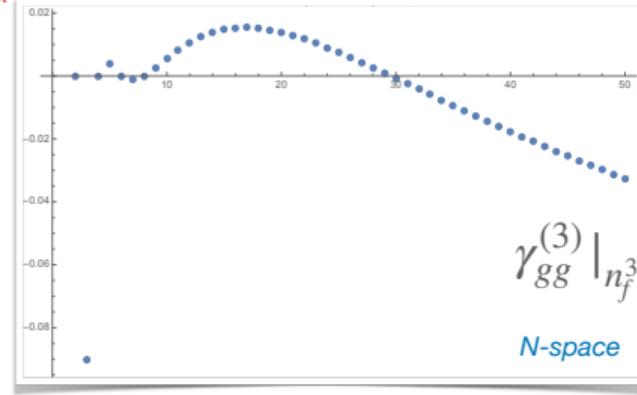
$$\tilde{f}(N) = \int_0^1 x^{N-1} f(x) dx$$

Rule of thumb:
 small- $N \rightarrow$ small- x ,
 large- $N \rightarrow$ large- x

The approximation procedure is performed in Mellin space for each n_f part independently:

1. Parametrise the difference between the 4 known moments and known limits with 4 functions $f_i(N)$.
2. Varying the sub-leading unknown $f_i(N)$ to produce a large set of parameterisation candidates (≈ 70).
3. Reduce the number of samples discarding too wiggly parameterisations and looking at the most representative cases.

Comparison w.r.t. known analytical part (%)



In $P_{gg}(x)$:

Theoretical constrain include:

- large- N :

$$\gamma_{gg}^{(3)}(N \rightarrow \infty) \approx \Gamma_A S_1(N) + B_{gg} + \mathcal{O}\left(\frac{\ln(N)}{N}\right)$$

- small- N pole at $N = 0$, and $N = 1$ (*leading contribution*):

$$\gamma_{gg}^{(3)}(N \rightarrow 1) \approx C_4 \frac{1}{(N-1)^4} + C_3 \frac{1}{(N-1)^3} + \mathcal{O}((N-1)^{-2})$$

- 4 lowest moments $N = \{2, 4, 6, 8\}$

Solve the constrain given by the 4 known Mellin moments with many different candidates $\{f_1, f_2, f_3, f_4\}$:

$$f_1 = \frac{S_1(N)}{N}, \quad f_2 = \frac{1}{(N-1)^2}$$

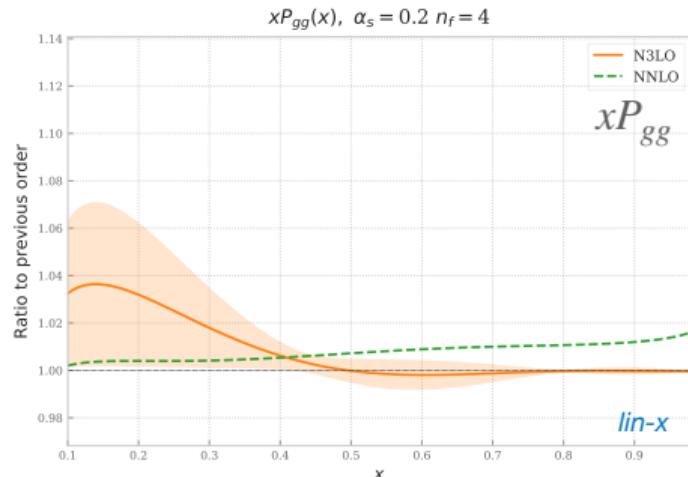
$$f_3 = \left\{ \frac{1}{(N-1)}, \frac{1}{N} \right\}$$

$$f_4 = \left\{ \frac{1}{(N-1)}, \frac{1}{N^4}, \frac{1}{N^3}, \frac{1}{N^2}, \frac{1}{N}, \frac{1}{(N+1)^3}, \frac{1}{(N+1)^2}, \right.$$

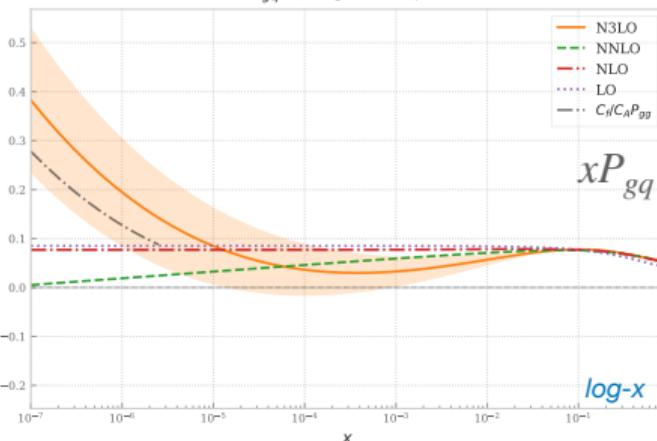
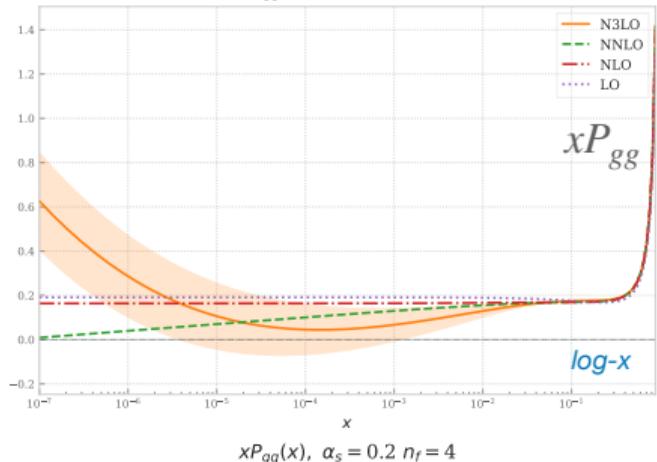
$$\left. \frac{1}{N+1}, \frac{1}{N+2}, \mathcal{M}[\ln(1-x)], \mathcal{M}[(1-x)\ln(1-x)], \frac{S_1(N)}{N^2} \right\}$$

N3LO singlet sector

PRELIMINARY RESULTS

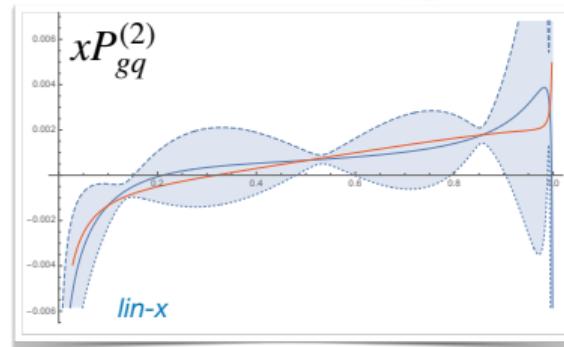
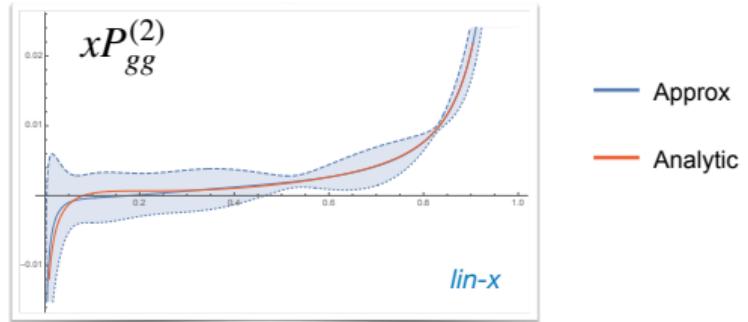
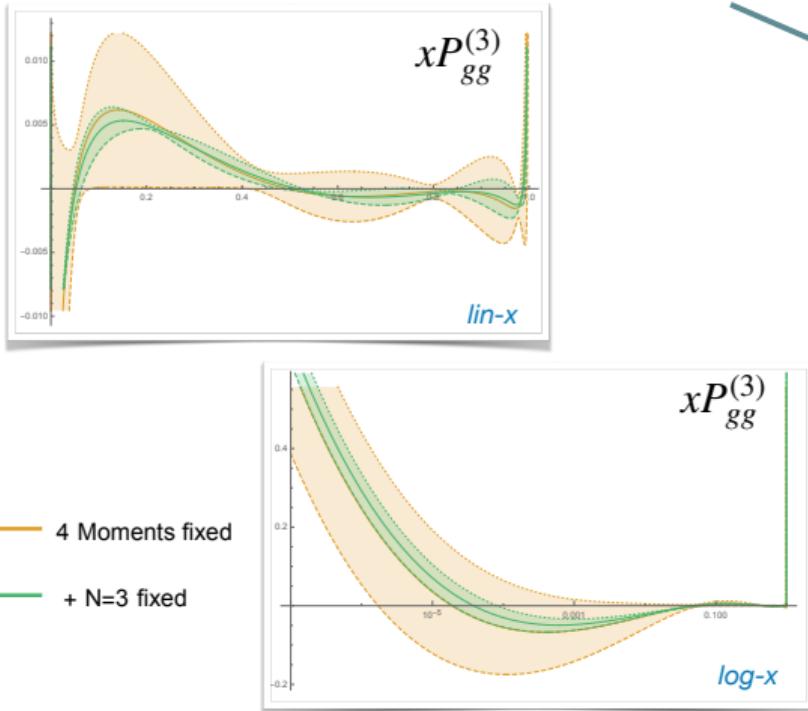


- Singlet approximated splitting functions are less constrained by the known limits. The coefficients of $1/x \ln^2(x)$, $1/x \ln(x)$ play a crucial role in the small- x region.
- Uncertainty arising from the approximation is not negligible.
- Off diagonal terms P_{qg} , P_{gq} are more difficult to estimate (large- N goes to 0).
- Only theoretical inputs are considered.
- All the implemented approximations respect momentum sum rules.



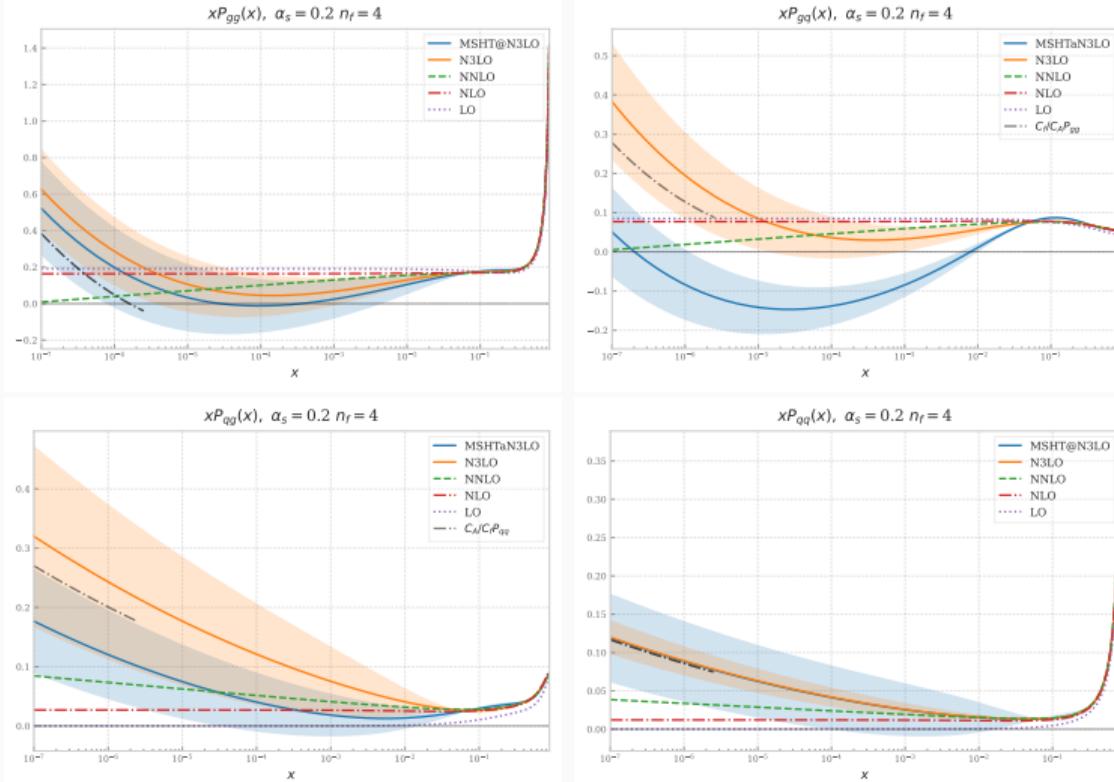
Approximation checks

1. A possible way to validate the procedure is to **reproduce the known NNLO singlet splitting functions** using the very similar constrain that we have right now on the N3LO ones.

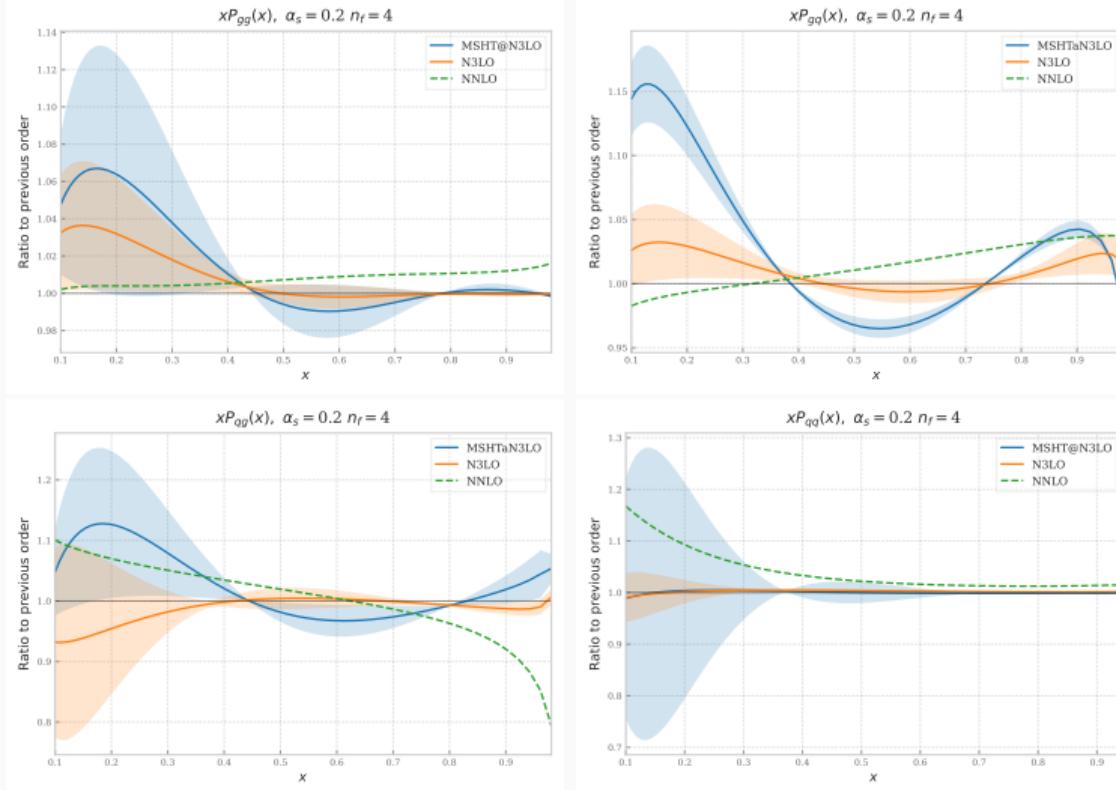


2. Another way to validate the results is to **interpolate the known moments**, and construct a more constrained parametrisation now including 5/6 moments. If the procedure is working (the samples are varied enough) the uncertainty band obtained in this way should be small than the default one.

Comparison with MSHT [2207.04739] at small-x - PRELIMINARY!



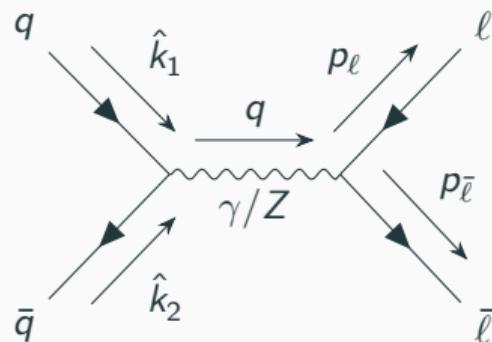
Comparison with MSHT [2207.04739] at large-x - PRELIMINARY!



2.3. Parton Distributions and New Physics Searches: the Drell-Yan Forward-Backward Asymmetry as a case study

[EPJC82.1160]

The Forward-Backward Asymmetry



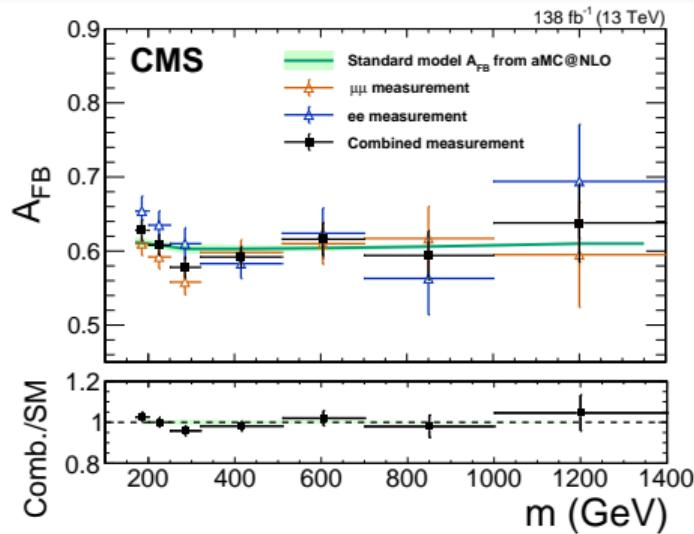
$$A_{fb}(\cos \theta^*) = \frac{\frac{d\sigma}{d \cos \theta^*}(\cos \theta^*) - \frac{d\sigma}{d \cos \theta^*}(-\cos \theta^*)}{\frac{d\sigma}{d \cos \theta^*}(\cos \theta^*) + \frac{d\sigma}{d \cos \theta^*}(-\cos \theta^*)} \quad (2)$$

advantages:

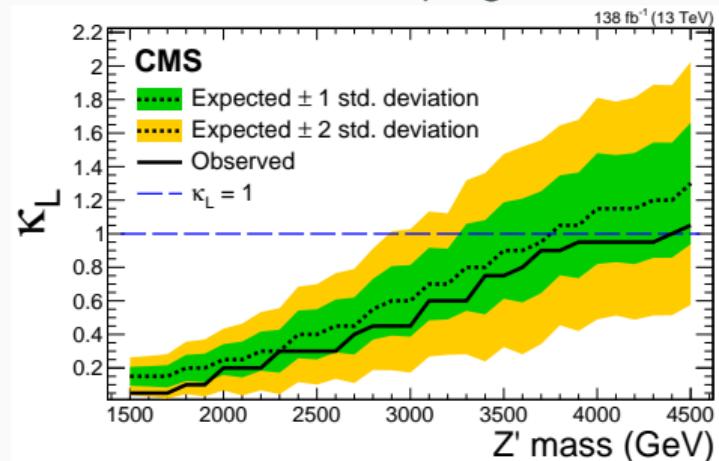
- theory: asymmetry sensitive to chiral BSM couplings
- experiment: several systematics cancel in ratio

New Physics Searches

e.g. at CMS [JHEP08.063]

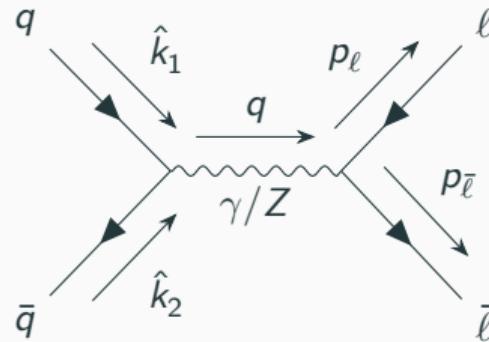


limit on SSM Z' LH coupling



$$A_{fb} = \int_0^1 d \cos \theta^* A_{fb}(\cos \theta^*) \quad (3)$$

Kinematics



Define the Collins-Soper angle θ^* by:

$$\cos \theta^* = \text{sign}(y_{\ell\bar{\ell}}) \cos \theta, \quad (4)$$

$$\cos \theta \equiv \frac{p_\ell^+ p_{\bar{\ell}}^- - p_\ell^- p_{\bar{\ell}}^+}{m_{\ell\bar{\ell}} \sqrt{m_{\ell\bar{\ell}}^2 + p_{T,\ell\bar{\ell}}^2}}, \quad p^\pm = p^0 \pm p^3 \quad (5)$$

at LO:

$$x_1 = \frac{m_{\ell\bar{\ell}}}{\sqrt{s}} \exp(y_{\ell\bar{\ell}}), \quad x_2 = \frac{m_{\ell\bar{\ell}}}{\sqrt{s}} \exp(-y_{\ell\bar{\ell}}) \Rightarrow x_1 x_2 = \frac{m_{\ell\bar{\ell}}^2}{s} \quad (6)$$

A_{fb} at LO

The cross-section is

$$\frac{d^3\sigma}{dm_{\ell\bar{\ell}} dy_{\ell\bar{\ell}} d\cos\theta^*} = \frac{\pi\alpha^2}{3m_{\ell\bar{\ell}} s} \left((1 + \cos^2(\theta^*)) \sum_q S_q \mathcal{L}_{S,q} + \cos\theta^* \sum_q A_q \mathcal{L}_{A,q} \right) \quad (7)$$

with luminosities

$$\mathcal{L}_{S,q} = f_q(x_1, m_{\ell\bar{\ell}}^2) f_{\bar{q}}(x_2, m_{\ell\bar{\ell}}^2) + f_q(x_2, m_{\ell\bar{\ell}}^2) f_{\bar{q}}(x_1, m_{\ell\bar{\ell}}^2) \quad (8)$$

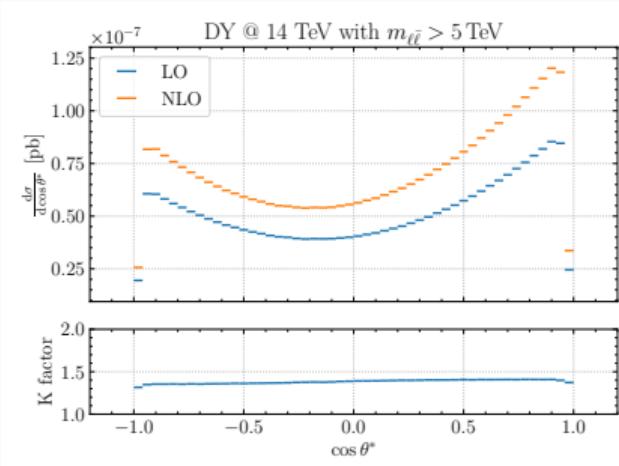
$$\mathcal{L}_{A,q} = \text{sign}(y_{\ell\bar{\ell}}) \left(f_q(x_1, m_{\ell\bar{\ell}}^2) f_{\bar{q}}(x_2, m_{\ell\bar{\ell}}^2) - f_q(x_2, m_{\ell\bar{\ell}}^2) f_{\bar{q}}(x_1, m_{\ell\bar{\ell}}^2) \right) \quad (9)$$

and the couplings S_q and A_q

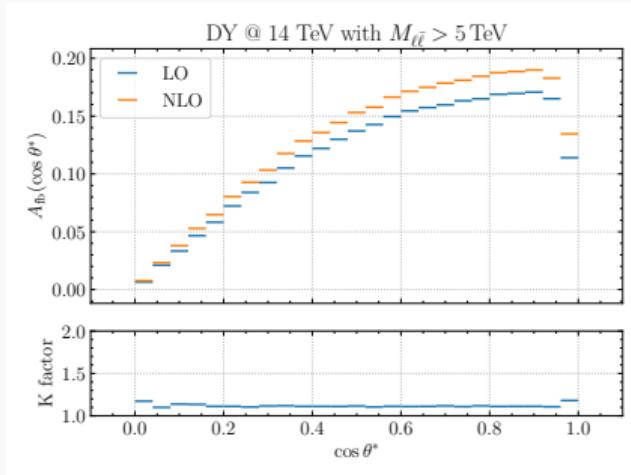
$A_{\text{fb}} \Leftrightarrow$ asymmetric part, i.e. $\mathcal{L}_{A,q}$

A_{fb} at NLO

cross section



asymmetry

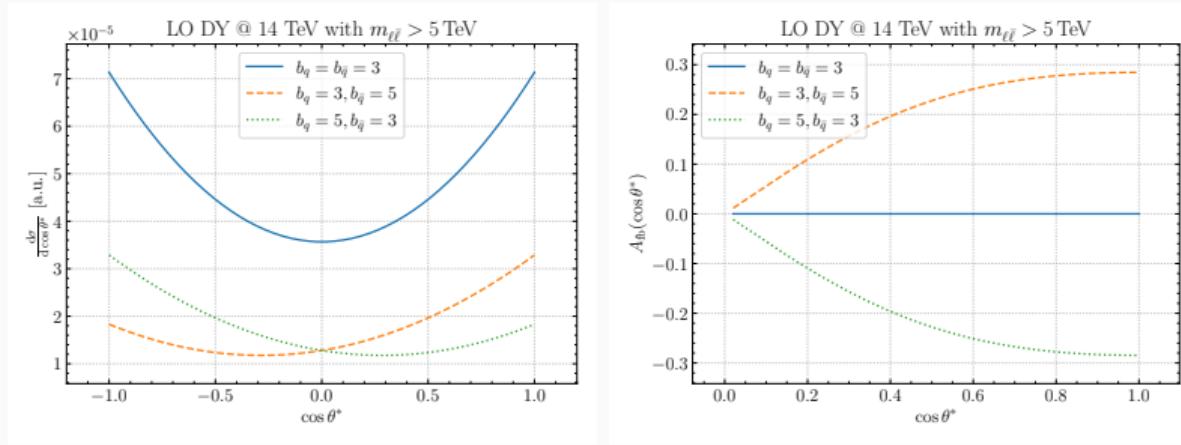


- NLO K-Factor almost θ^* -independent
- at LO: effective couplings determined by PDF luminosity

$$A_{\text{fb}} = \frac{\cos\theta^*}{1 + \cos^2\theta^*} \underbrace{\frac{\sum_q g_{A,q}}{\sum_{q'} g_{S,q'}}}_{R_{\text{fb}}} \quad g_{A,q} \propto \int \frac{dm_{\ell\bar{\ell}}}{m_{\ell\bar{\ell}}} dy_{\ell\bar{\ell}} A_q \mathcal{L}_{A,q} \quad (10)$$

Toy Model

$$xf_q = x^{-1}(1-x)^{b_q}, xf_{\bar{q}} = x^{-1}(1-x)^{b_{\bar{q}}} \Rightarrow xf_q^{\pm} = xf_q \pm xf_{\bar{q}} \quad (11)$$

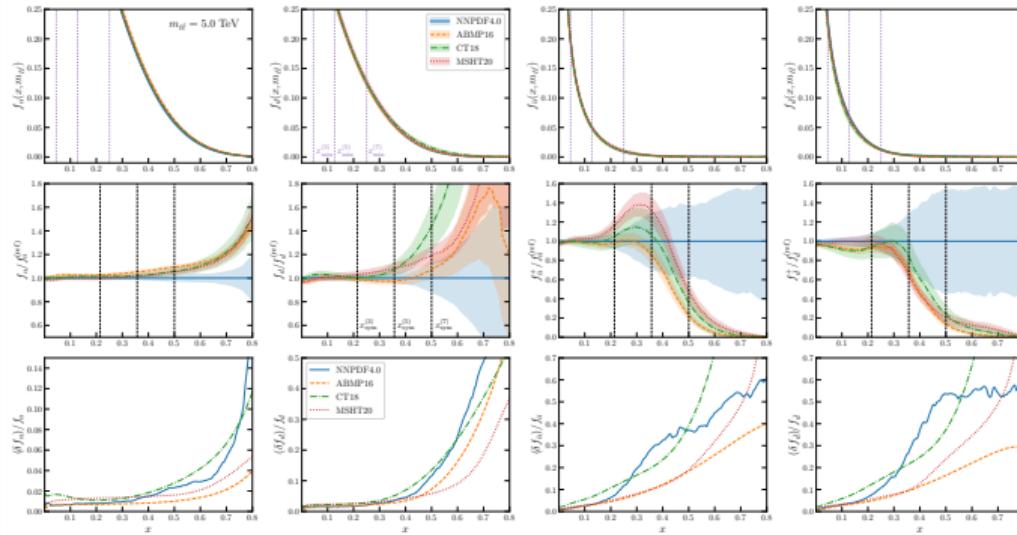


- Toy: sign of valence \Leftrightarrow sign of asymmetry
- however, cannot have negative valence, but only drop-off valence/sea matters

$$\text{sign} [\mathcal{L}_{A,q}] = \text{sign} \left[\frac{f_q(x_2)}{f_q(x_1)} - \frac{f_{\bar{q}}(x_2)}{f_{\bar{q}}(x_1)} \right] = \text{sign} \left[\frac{f_q^+(x_2)}{f_q^+(x_1)} - \frac{f_q^-(x_2)}{f_q^-(x_1)} \right], \quad x_1 > x_2 \quad (12)$$

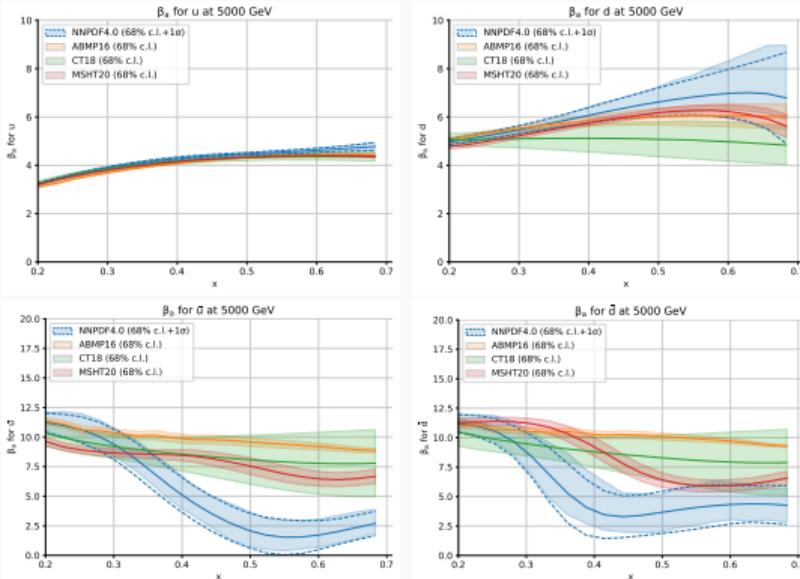
How About Real PDFs?

dominant contribution: up and down quarks, antiquarks $x_1 x_2 = \frac{m_{\ell\bar{\ell}}^2}{s}$



- $M_{Z'} \lesssim 3$ TeV: data region, all PDF sets agree
- $M_{Z'} \gtrsim 5$ TeV: extrapolation region: NNPDF disagrees
 - different central value
 - larger uncertainties

The Effective Exponent [EPJC76.383]

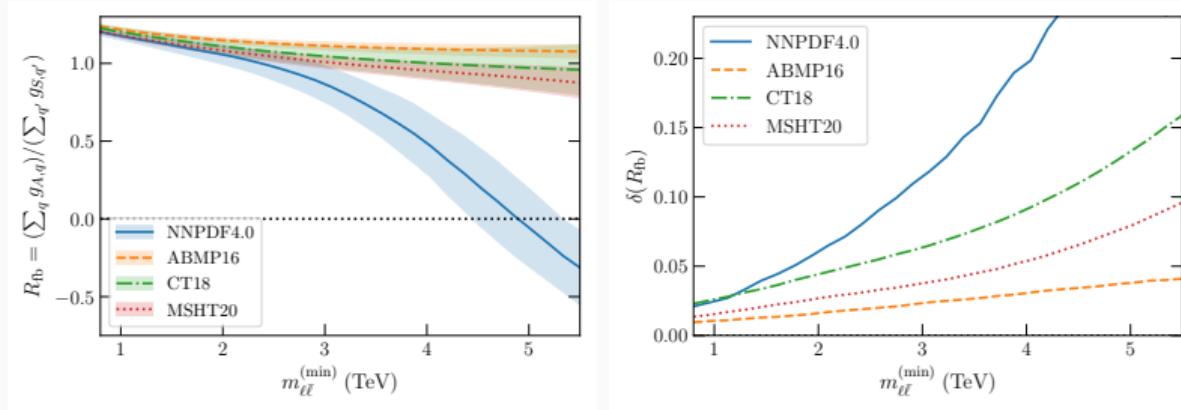


$$\beta_a(x) = \frac{\partial \ln |xf(x)|}{\partial \ln(1-x)} \quad (13)$$

- CT, MSHT, ABMP: $f \sim x^\alpha (1-x)^\beta P(x) \Rightarrow$ for large x : $\beta_a(x) \rightarrow \beta$
- NNPDF: $f \sim NN(x) \Rightarrow \beta_a$ not fixed by parametrization

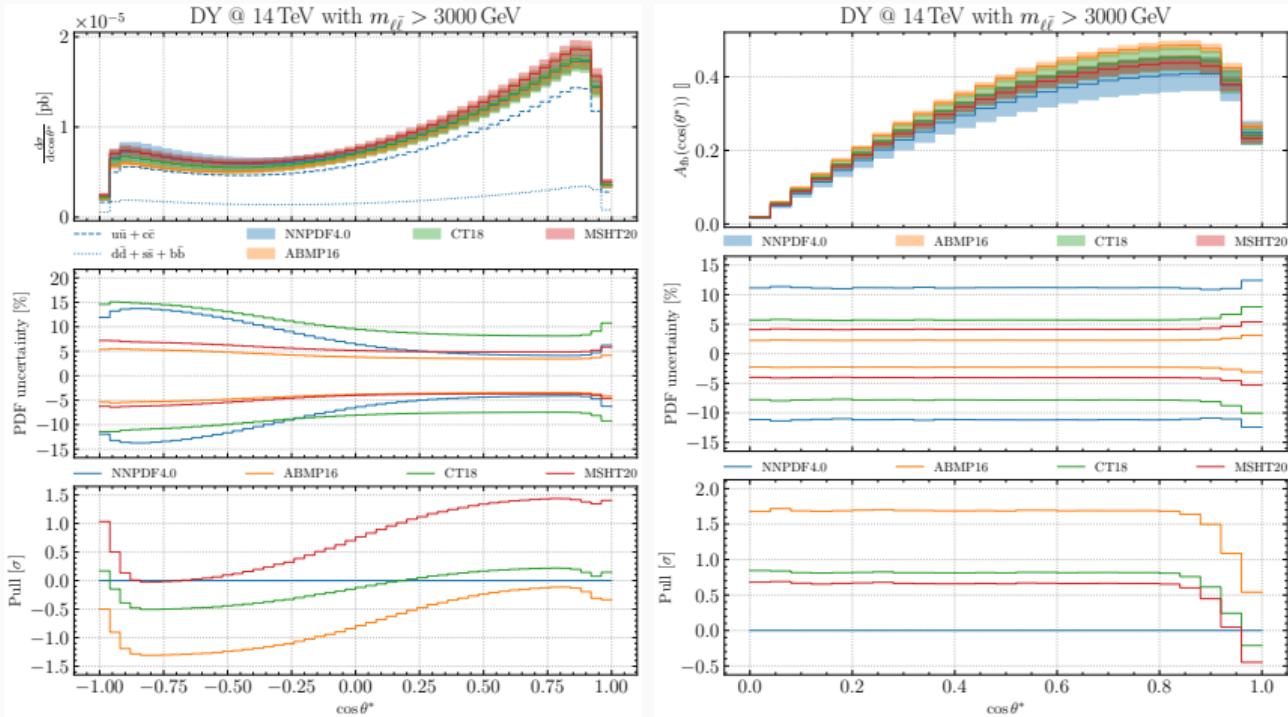
The Effective Coupling

$$A_{\text{fb}} = \frac{\cos \theta^*}{1 + \cos^2 \theta^*} \underbrace{\frac{\sum_q g_{A,q}}{\sum_{q'} g_{S,q'}}}_{R_{\text{fb}}} \quad g_{A,q} \propto \int \frac{dm_{\ell\bar{\ell}}}{m_{\ell\bar{\ell}}} \frac{dx_1}{x_1} A_q \mathcal{L}_{A,q} \quad (14)$$



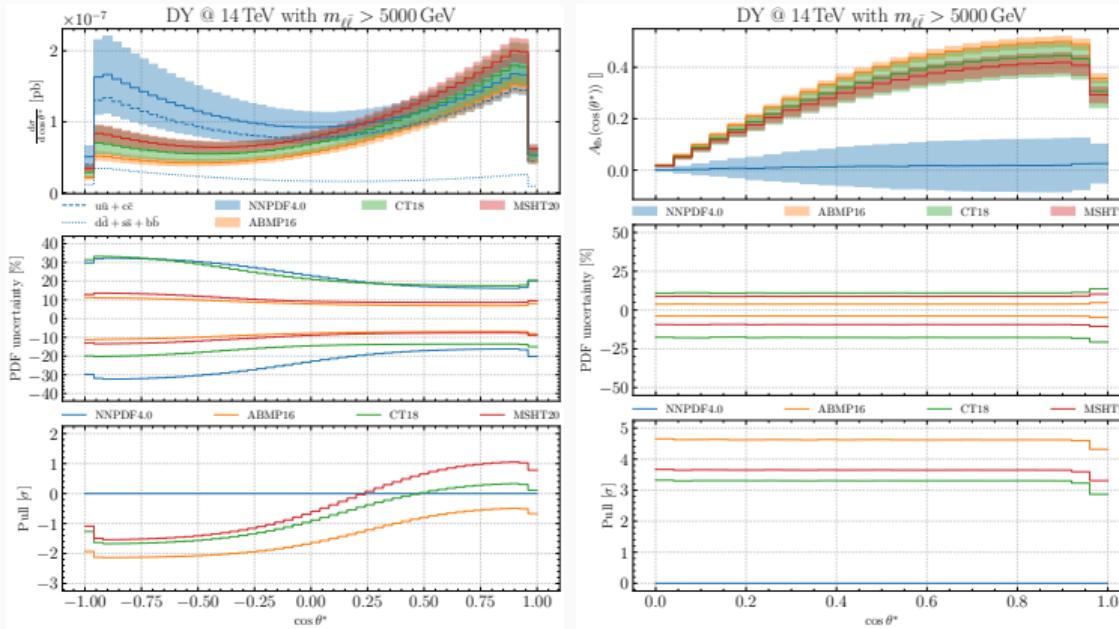
- as scale increases, larger x probed
- CT, MSHT, ABMP: coupling is approximate scale independent
- NNPDF: coupling depends on scale, larger uncertainty

A_{fb} at not so High Masses



for $M_{Z'} \geq 3$ TeV: data region, all PDF sets agree

A_{fb} at High Masses



for $M_{Z'} \geq 5 \text{ TeV}$:

- CT, MSHT, ABMP: asymmetry unchanged with increasing scale
- NNPDF: asymmetry disappears as scale increases

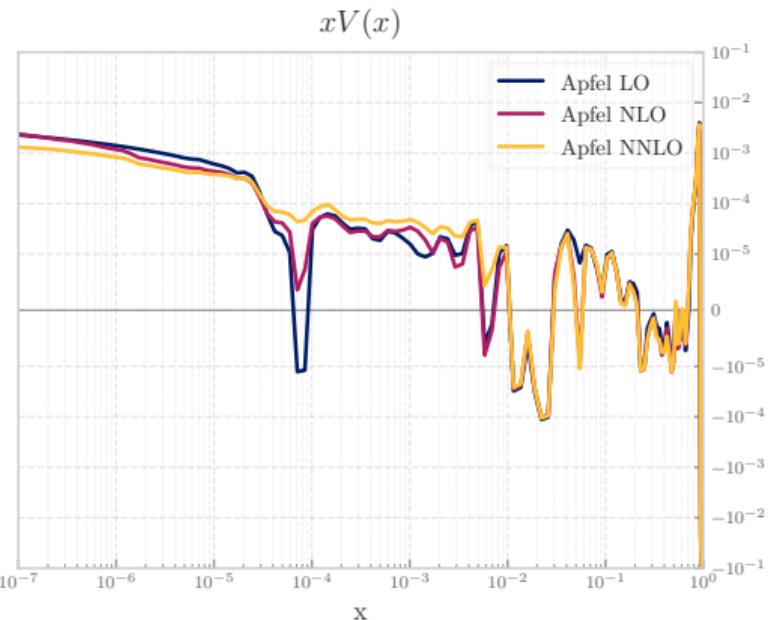
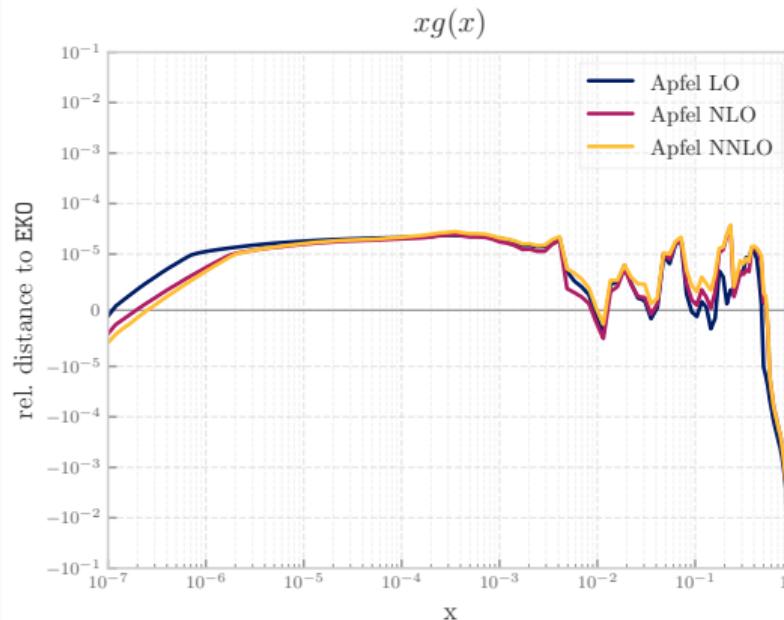
Summary

- PDFs largely unconstrained in the high-mass discovery region
- fixed-parametrization PDFs overly restrictive:
 - over-constrained extrapolation
 - under-estimated uncertainties
- flexible parametrization required for reliable results
- future Drell-Yan measurements important in order to constrain PDFs

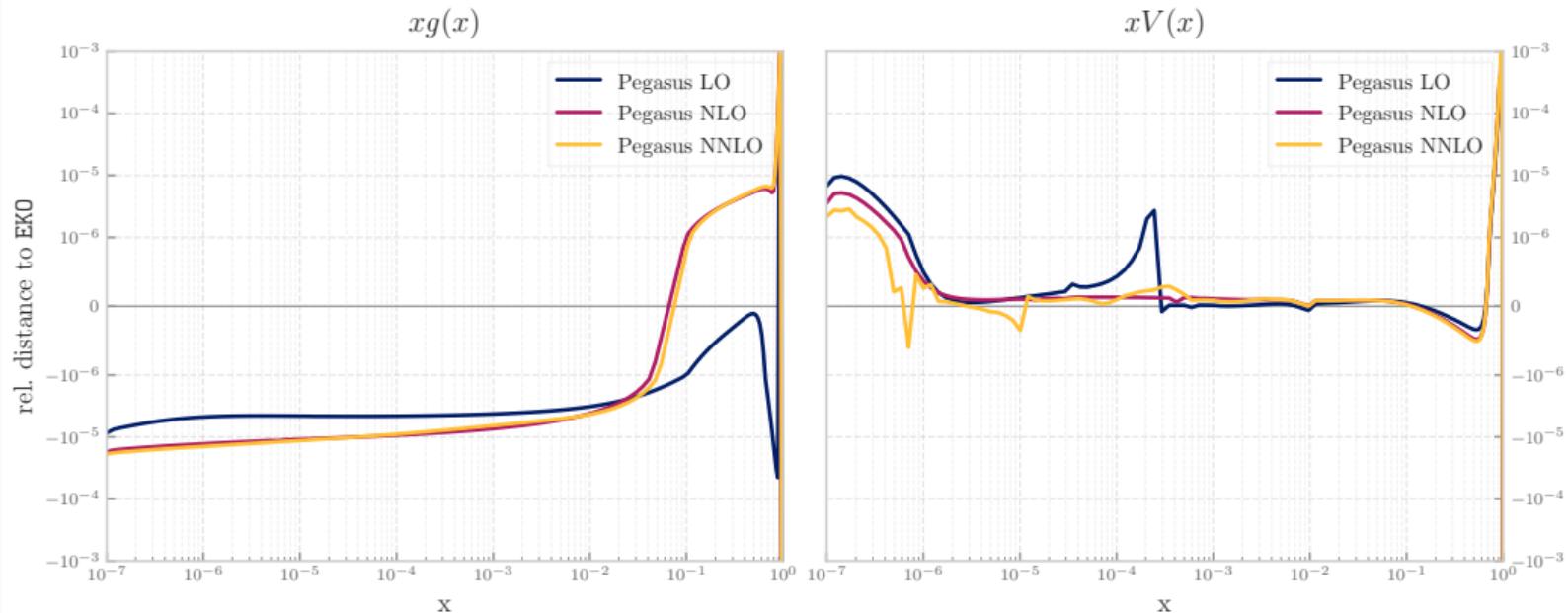
Thank you!

3. Backup slides

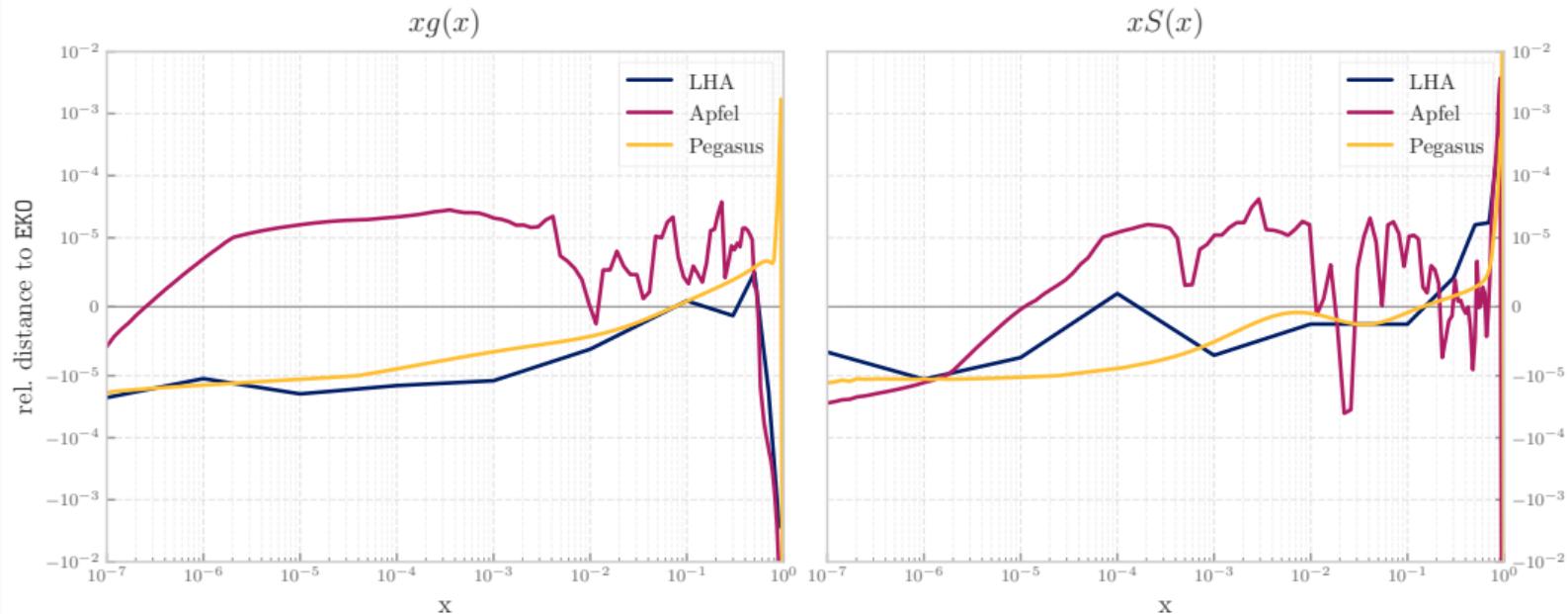
EKO APFEL benchmark



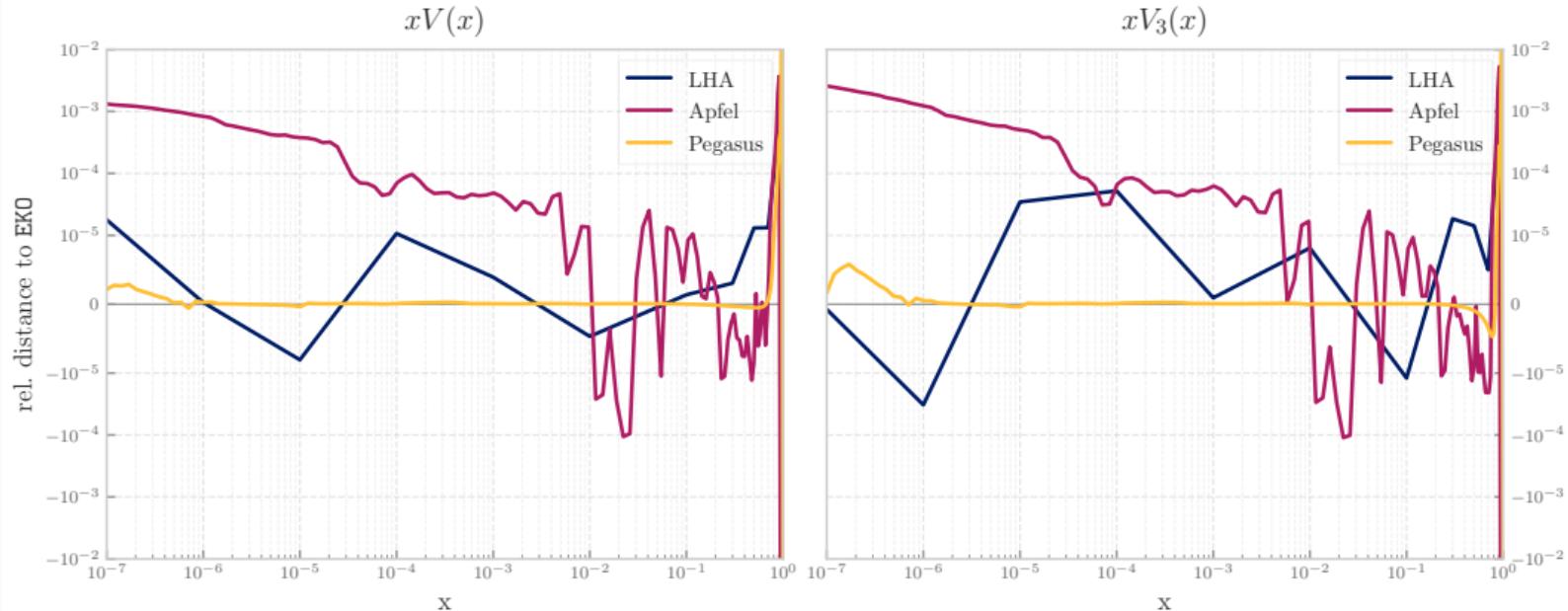
EKO PEGASUS benchmark



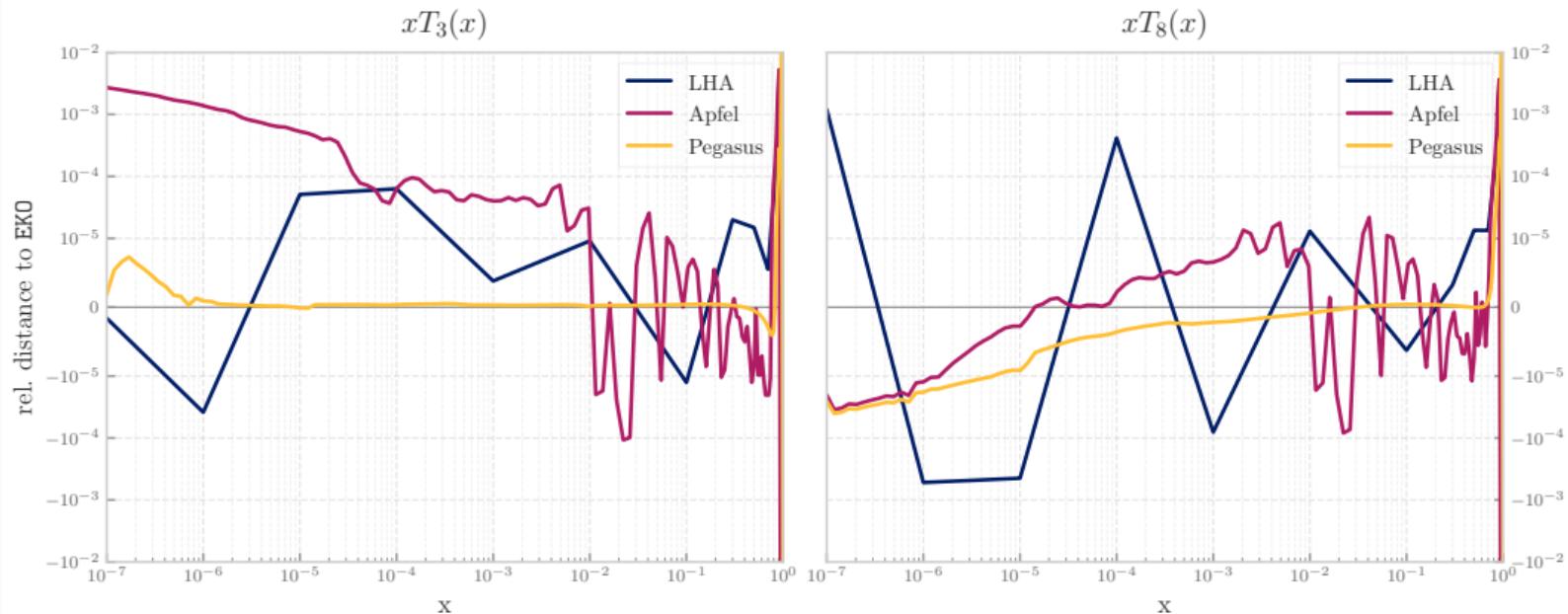
EKO LHA benchmark: g and Σ



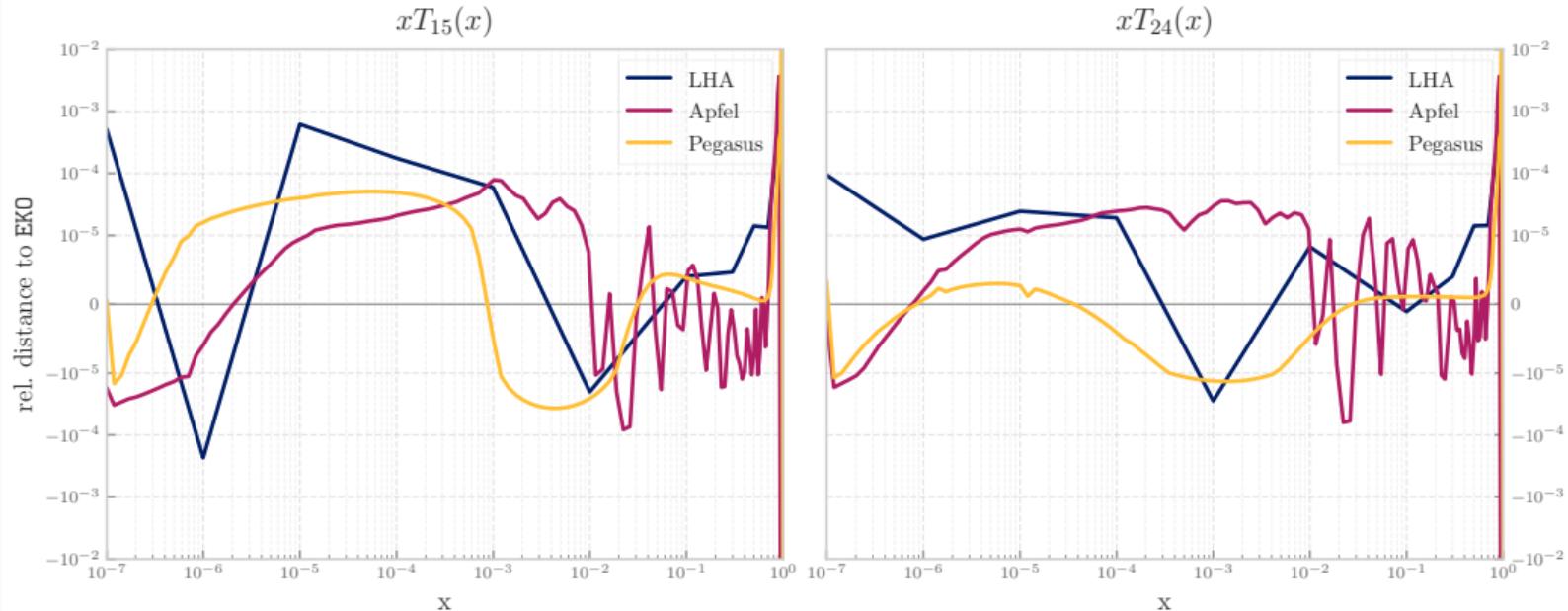
EKO LHA benchmark: V and V_3



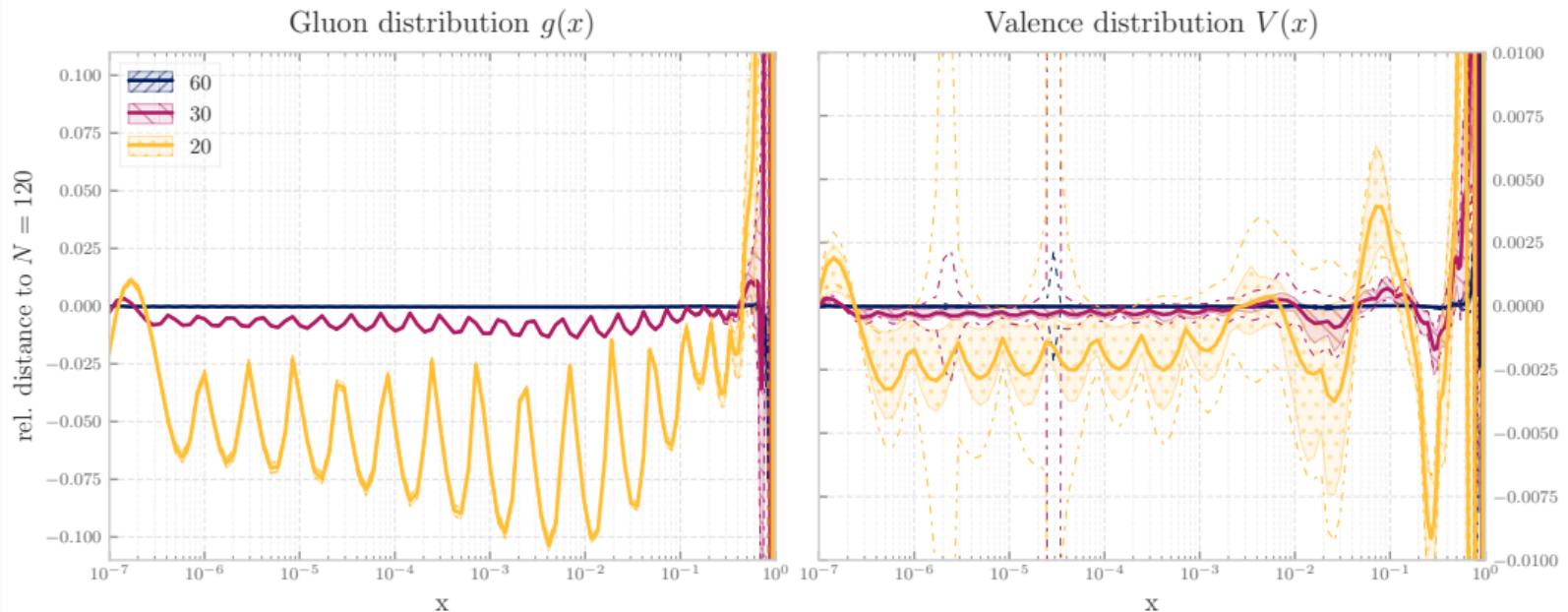
EKO LHA benchmark: T_3 and T_8



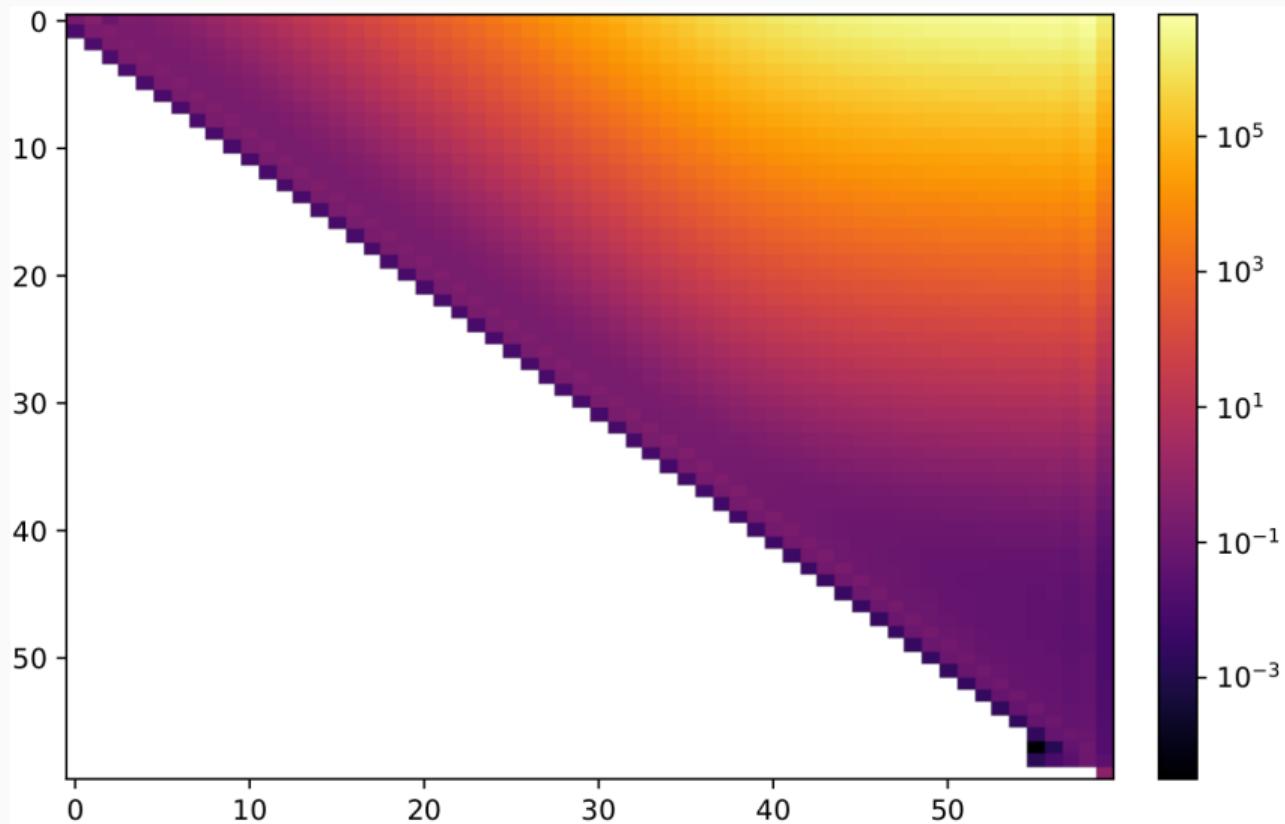
EKO LHA benchmark: T_{15} and T_{24}



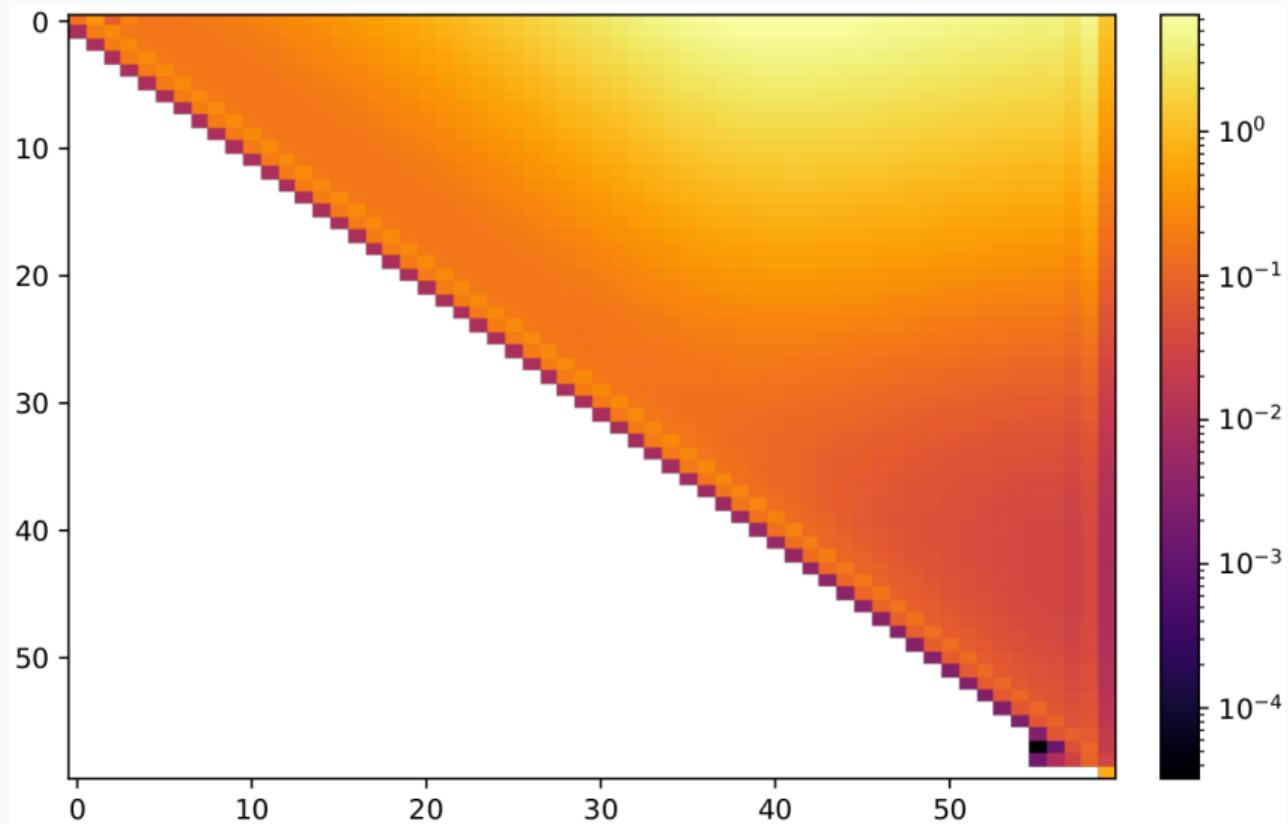
EKO Interpolation Error



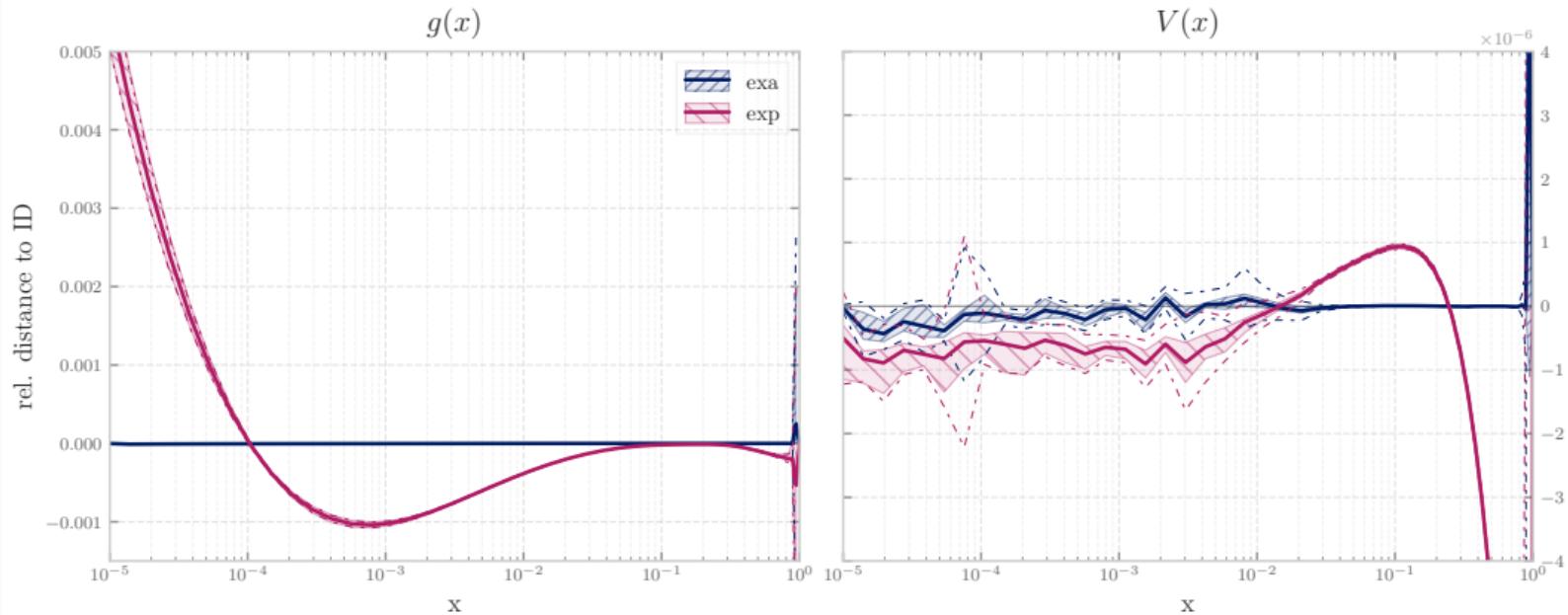
EKO Snapshot $S \leftarrow S$



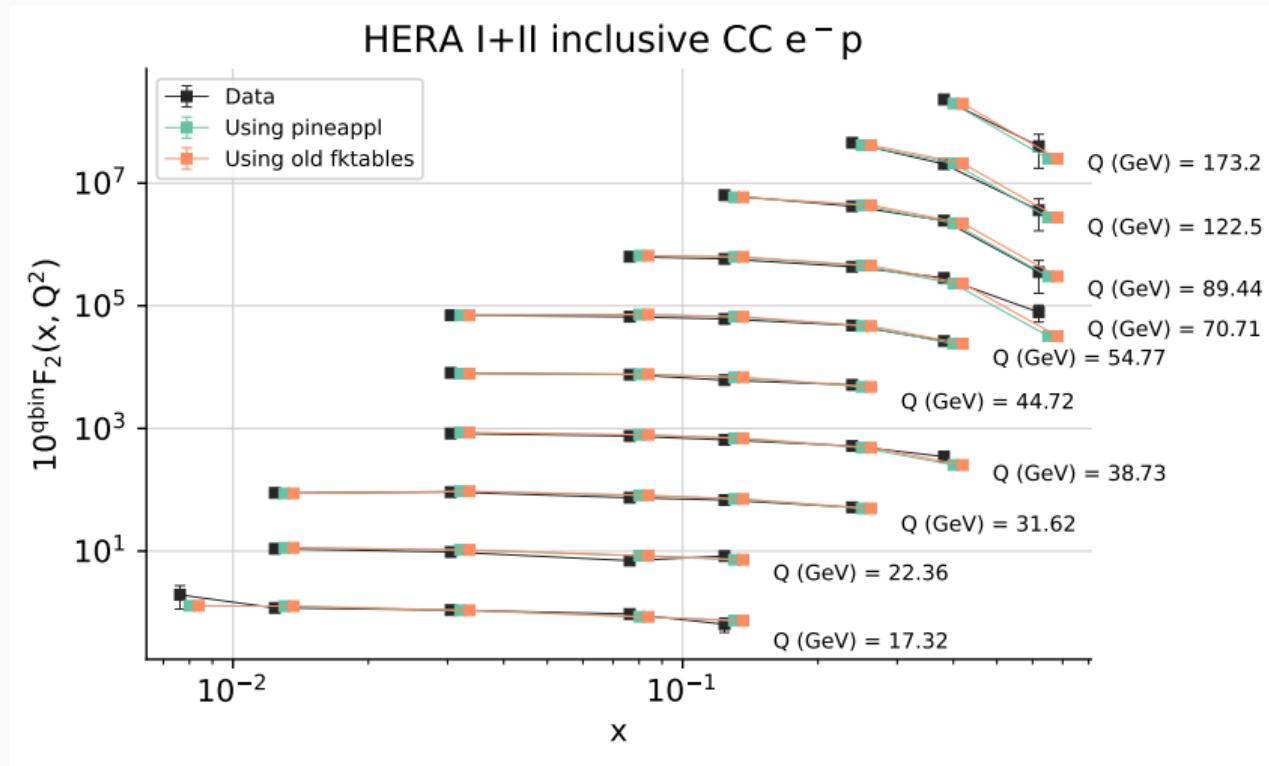
EKO Snapshot $V \leftarrow V$



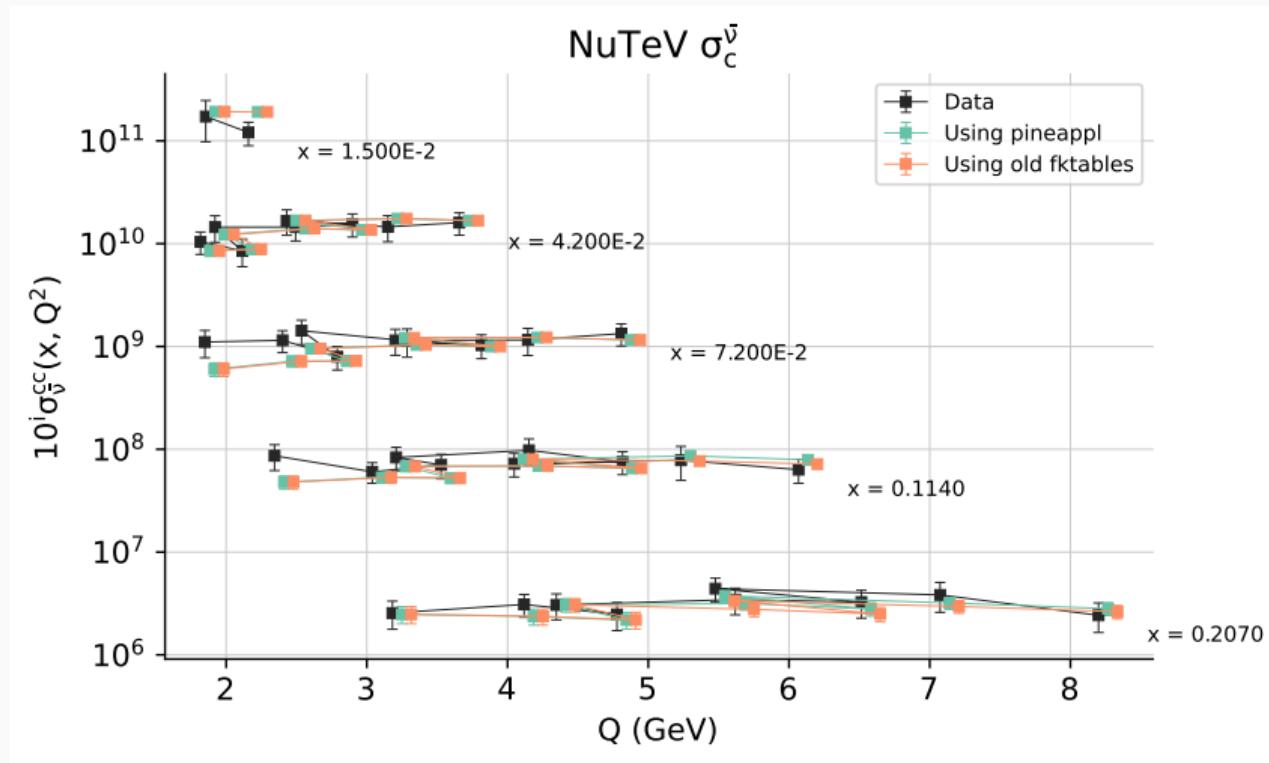
EKO Backward Evolution



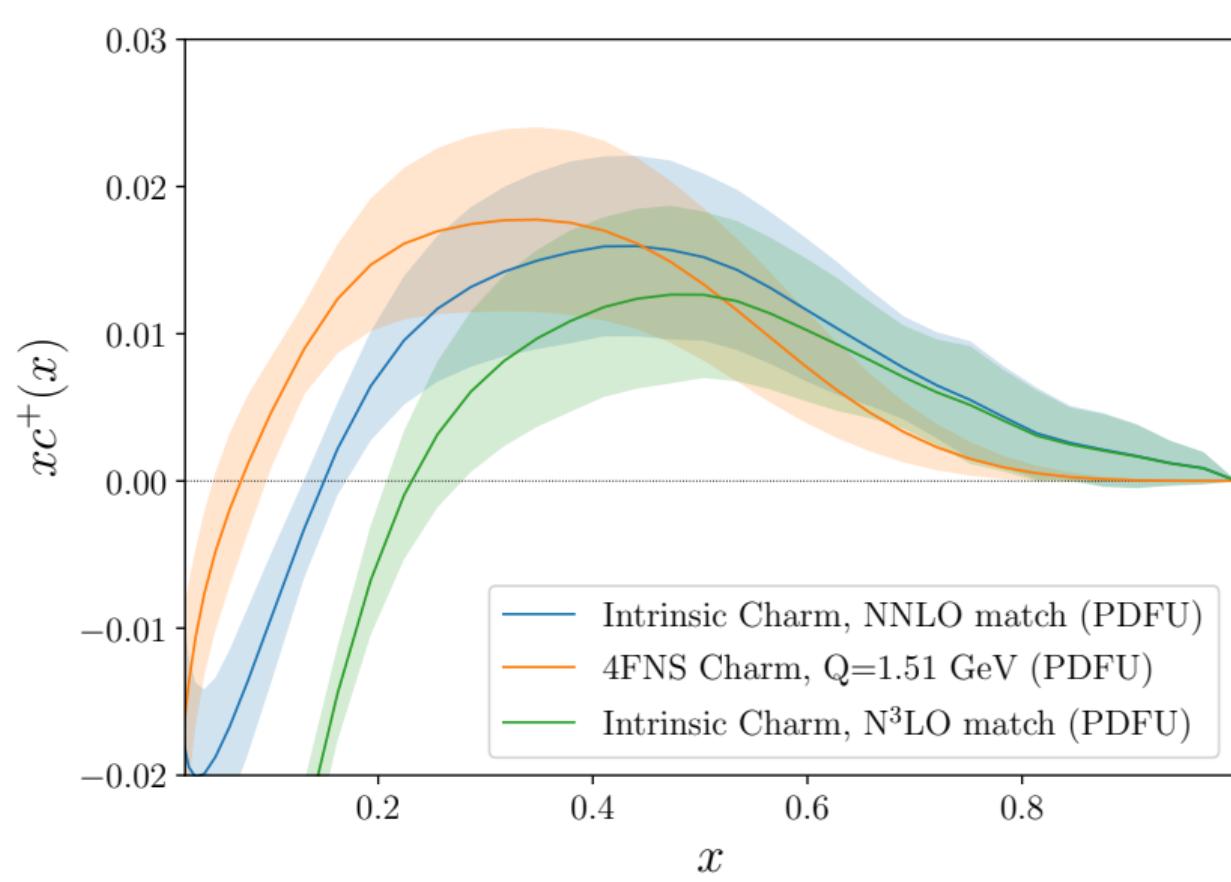
Comparison yadism against APFEL



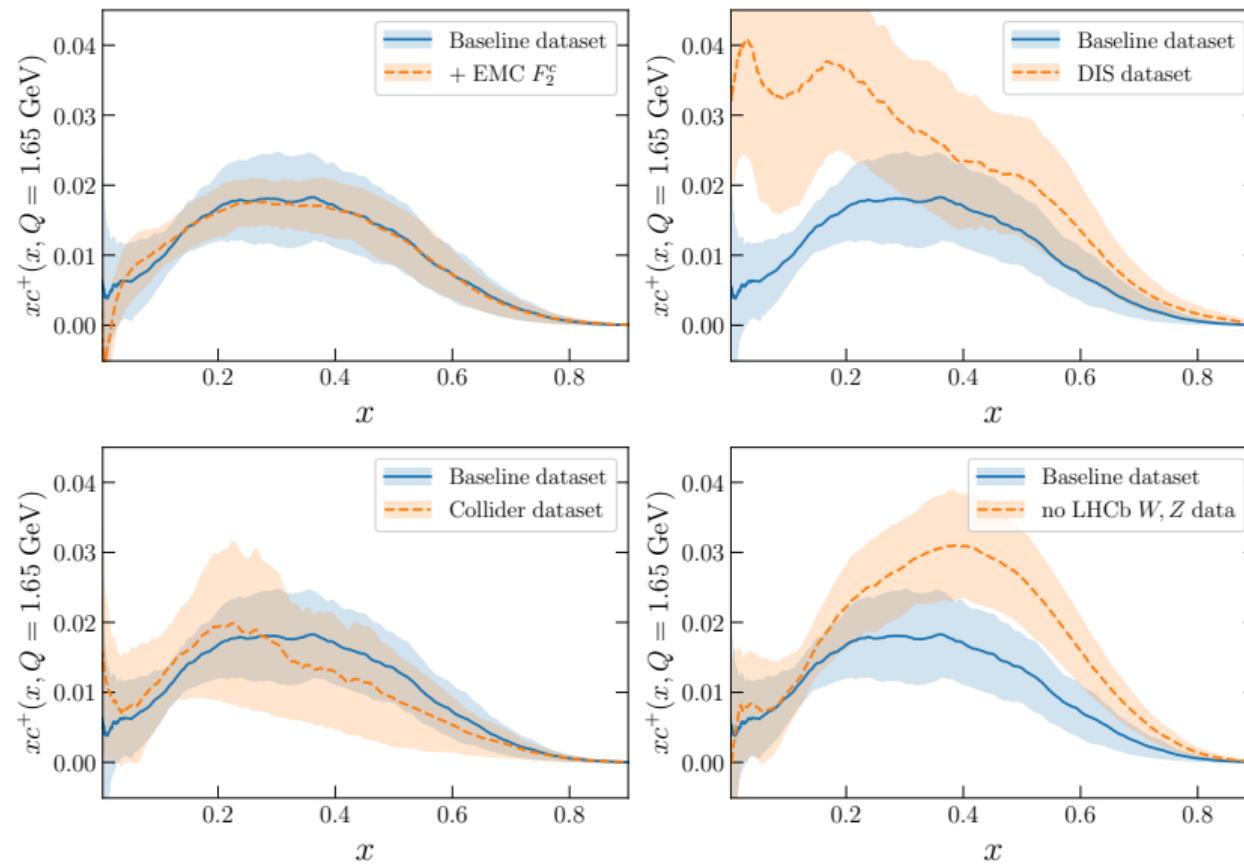
Comparison yadism against APFEL



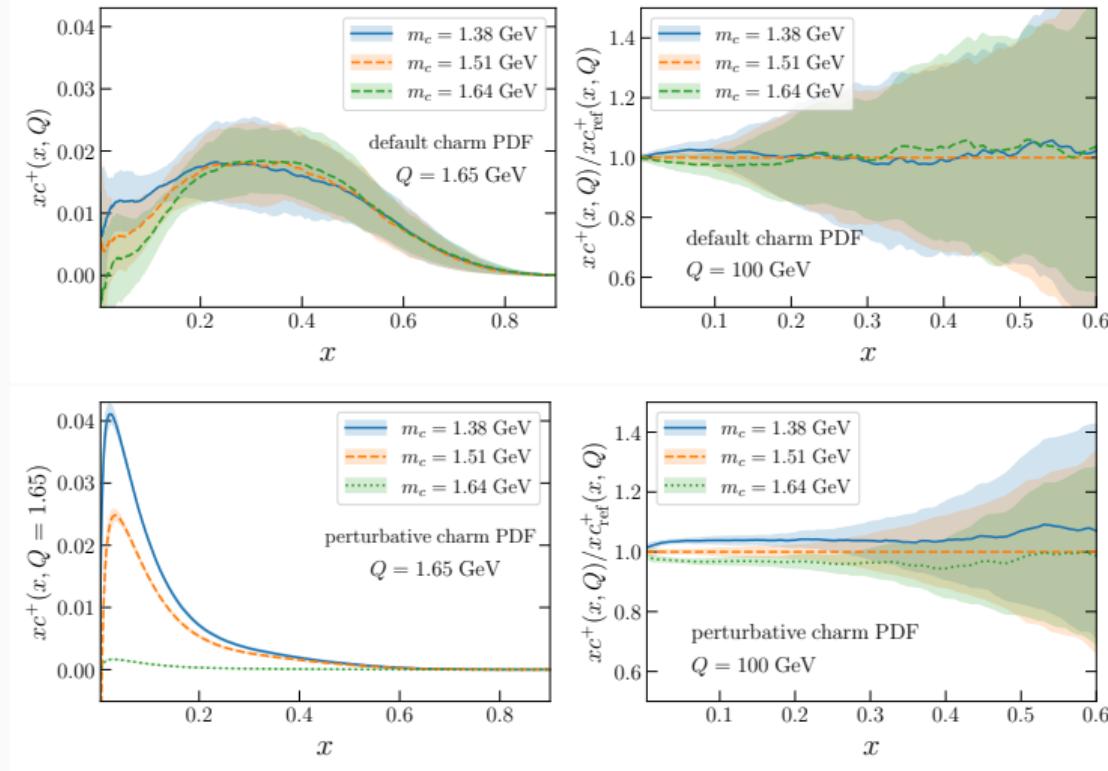
IC - uncertainties splitted



IC - dataset variation



IC - mass variation



N3LO non singlet sector

non-singlet 4-loop Anomalous Dimensions

	n_f^0	n_f^1	n_f^2	n_f^3
$\gamma_{ns,-}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,+}^{(3)}$	✓	✓	✓	✓
$\gamma_{ns,s}^{(3)}$		✓	✓	

- Estimation of the N3LO anomalous dimensions is based on the best available theoretical constraints:

- large-N: $\gamma_{ns}^{(3)}(N \rightarrow \infty) \approx \Gamma_f S_1(N) + B + C \frac{S_1(N)}{N} + D \frac{1}{N} + \mathcal{O}\left(\frac{\ln(N)}{N^2}\right)$

- small-N:

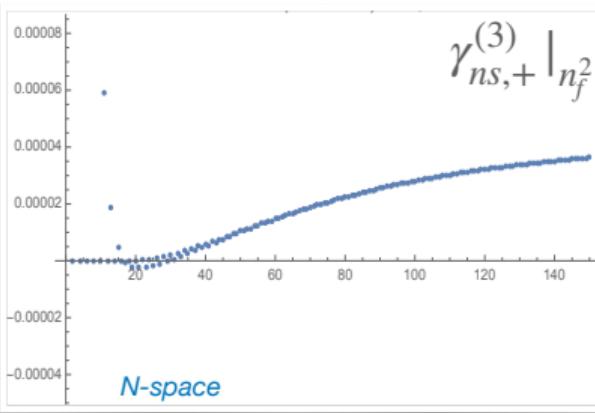
$$\gamma_{ns}^{(3)}(N \rightarrow 0) \approx \sum_{i=1}^7 C_i \frac{1}{N^i}$$

- 8 lowest Mellin moments

- For more details on the procedure used see [EKO N3LO ad documentation](#)

- Non singlet approximated splitting functions are compatible with the known analytical (and much more complex) parts within numerical accuracy.

Comparison w.r.t. known analytical part (%)

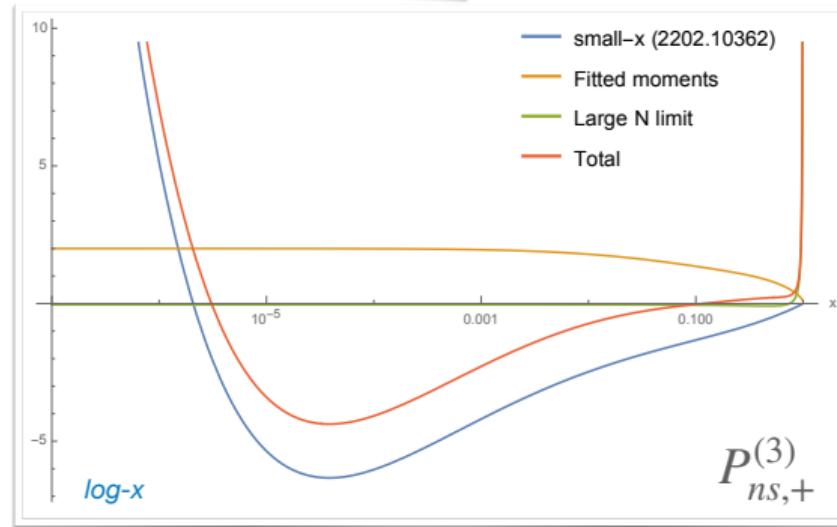


N-space

Main references:

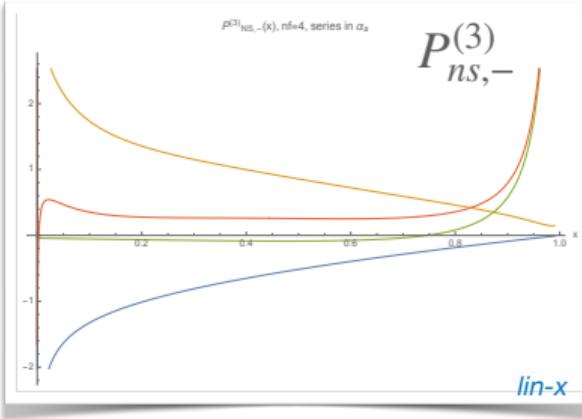
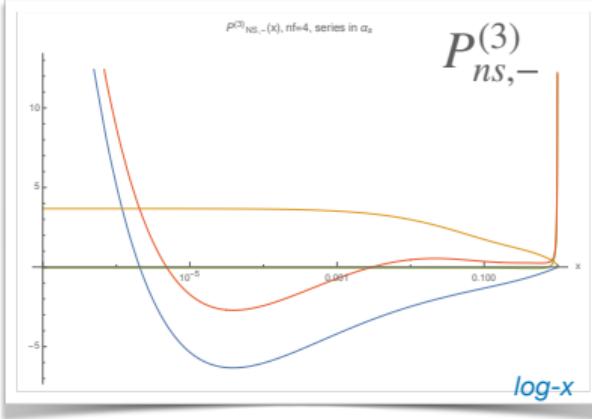
- Moch, Ruijl, Ueda, Vermaseren, Vogt [arXiv:1707.08315].
- Davies, Vogt, Ruijl, Ueda, Vermaseren. [arXiv:1610.07477]
- Davies, Kom, Moch, Vogt . [arXiv:2202.10362].

Rule of thumb:
small-*N* → small-*x*,
large-*N* → large-*x*



$P_{ns,+}^{(3)}$

N3LO non singlet



- small-x (2202.10362)
- Fitted moments
- Large N limit
- Total

