MSSM-inflation

MSSM = Minimal Supersymmetric Standard Model

Linking cosmic inflation to high energy physics

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A2C / CMB team

in collaboration with
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Laurent Duflot, Richard Freiherr von Eckardstein

Based on arXiv:2304.04534

10/05/2023

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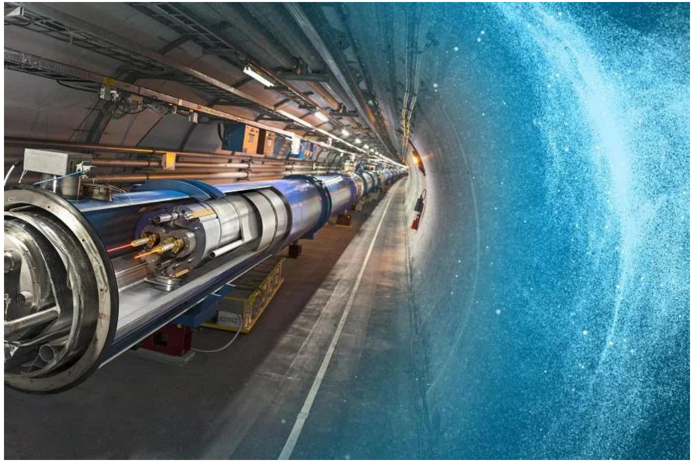
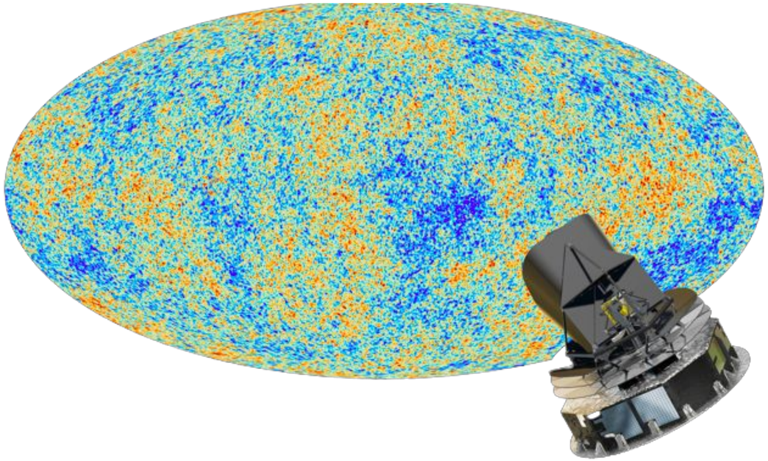
Outline



HEP (eg: at LHC)

Bridge: MSSM ⟷



Cosmology and CMB (eg: Planck satellite, …)

1) The **paradigm**: **Cosmology, Inflation, slow-roll**
2) The **model: MSSM-inflation**
3) The **results: Parameter space**

Based on arXiv:2304.04534

Cosmology: observables (1)

- **Hubble's law** & **cosmic expansion** (1929):

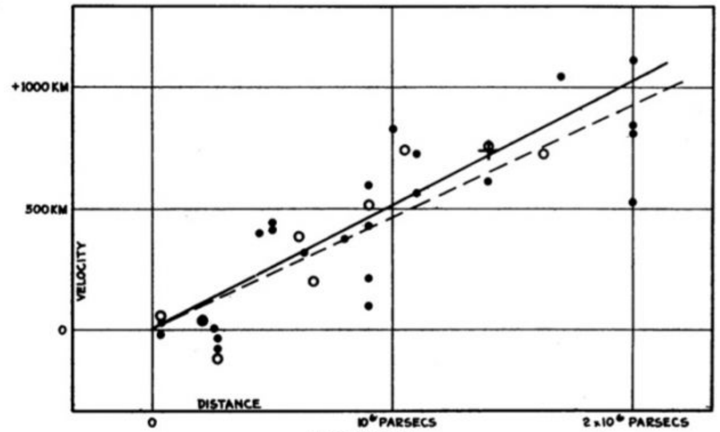
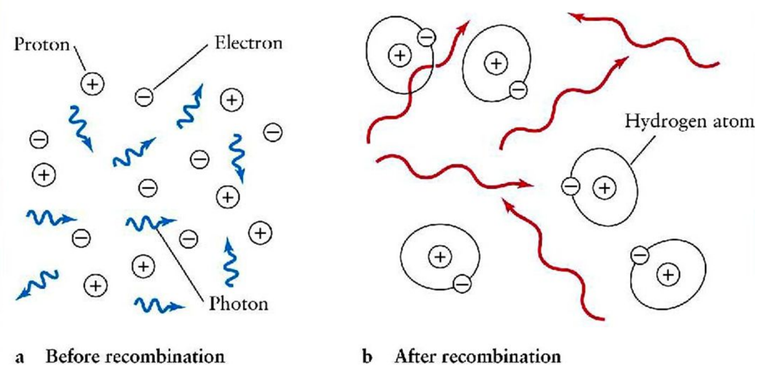


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

=> **High energy** in the **past**.
=> **prediction** of

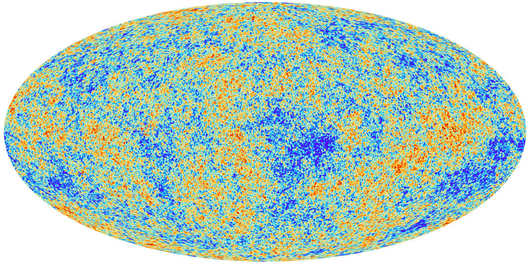
- Primordial **nucleosynthesis**
- **Cosmic microwave background** emission



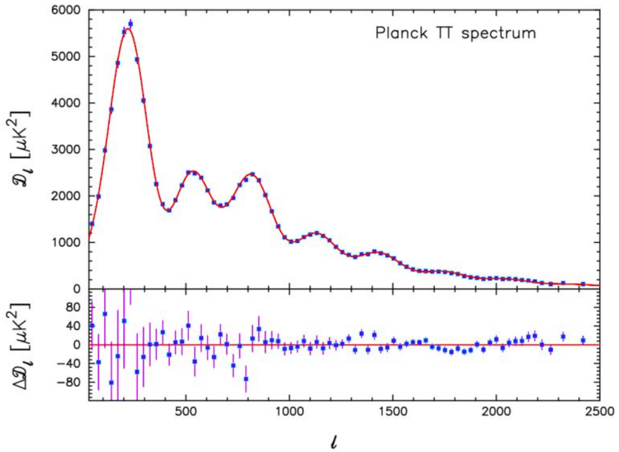
Cosmology: observables (2)

- **Primordial nucleosynthesis**
- **Cosmic microwave background**

TT:

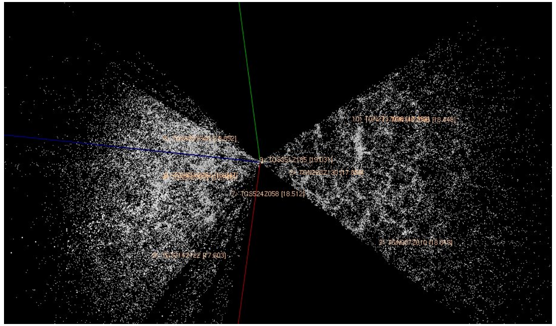


spherical harmonics decomposition

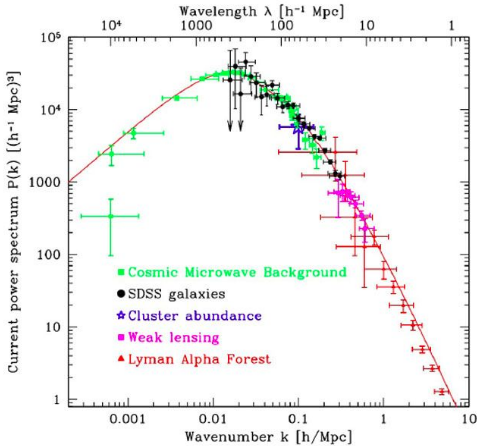


TE:
EE:
BB:

- **Large sky surveys**



3D Fourier transform



other stuff

- **BAO**
- **...**

ΛCDM: 1) ingredients

Einstein equations for the whole universe

=> **Friedman** equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\mathcal{K}}{a^2} + \frac{\Lambda}{3},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3},$$

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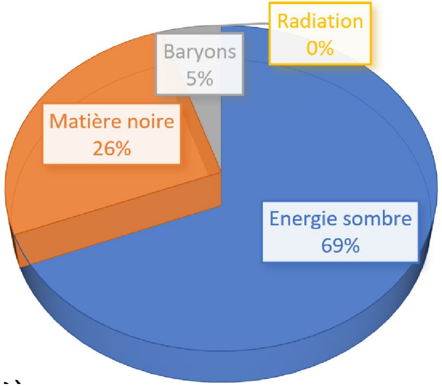
\+ **Initial conditions** for the densities (today)



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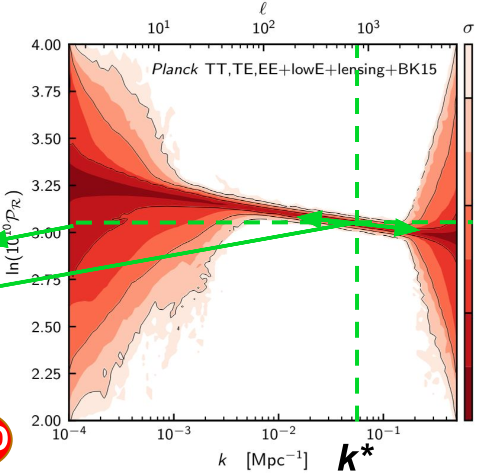
\+ **Initial conditions** for the perturbations

- **scalar** ones:

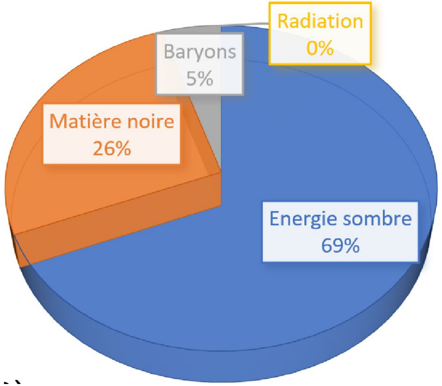
at ***k****:
- amplitude: ***As***
- slope: ***ns***
- running: ***nsrun***

TBD



k*

\+ **Initial conditions** for the densities (today)



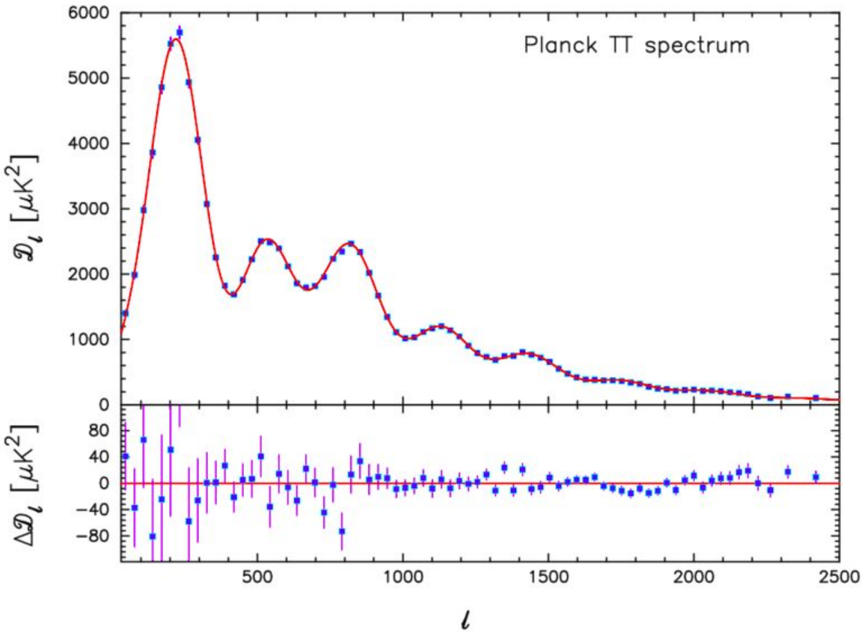
- **tensor** ones:

at ***k****:
- amplitude: ***r*** TBD
- slope: ***nt*** TBD

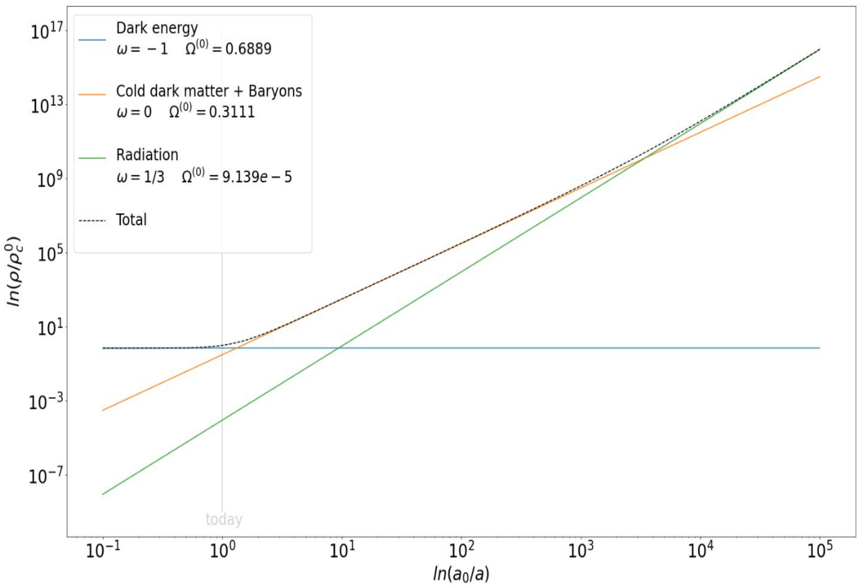
ΛCDM: 2) results

only **6 dof's**

- **3 density** params
- **2** params linked to **IC for perturbations**
- **1 late-time** param
  - that **fits** very well **the data**



- **predicts** unambiguously an **universe history**



ΛCDM: 3) conceptual problems

The **universe today** looks so far **very flat**.
… lambda CDM predicts that in the past this **curvature** was even much low. Is it **natural**?

At **large scales**, we see an **uniform-temperature** CMB.
… lambda CDM do not suggest that. **Conflict**?

The **primordial fluctuations:**
- Why their **power spectrum** looks like a **power law**?
- What is the **mechanism** that generated them?

One **mechanism** to solve them all:
Inflation: A phase of **accelerated expansion** with **ä > 0** in the **very young universe,** which needs to be sufficiently **long** $N = \ln \frac{a_{end}}{a_{in}} > 50$.

Λ gives an example of **ä > 0** universe, **of (quasi)-de-Sitter universe**:
- The **galaxies get diluted exponentially**,
- The **dark energy density** remains **constant** with the expansion,
- Friedmann equations => it is like a **fluid** with $\rho = -P$

Scalar field rolling on its potential

Action of a **single scalar field** minimally coupled to gravity:

$$S_\phi = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

Energy-momentum tensor:

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\partial S_{\mathrm{matter}}}{\partial g_{\mu\nu}}$$

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}\left[-\frac{1}{2}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + V(\phi)\right]$$

T00 is the density
Tii the pressure

In the Friedmann equations:

Density and pressure:

$$\rho = \frac{\dot{\phi}^2}{2} + V\,,$$
$$p = \frac{\dot{\phi}^2}{2} - V\,.$$

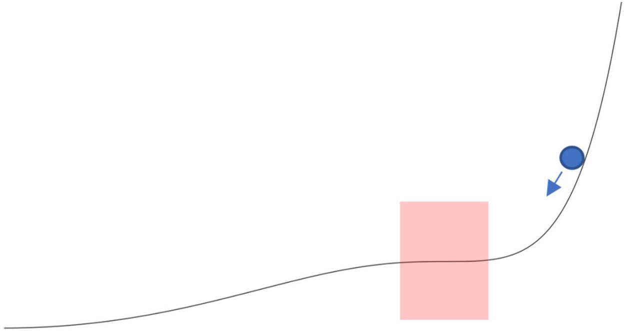
Klein-gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Same equation: **ball rolling down** a **slope**!

Slow-rolling of a scalar field

If the potential has a **flat region**, the scalar field **velocity** phidot will become small and its kinetic energy **negligible** with respect to its **potential energy**.



$$\rho = \cancel{\frac{\dot{\phi}^2}{2}} + V\,,$$
$$p = \cancel{\frac{\dot{\phi}^2}{2}} - V\,.$$

=> $\rho = -P$

The **slow-roll approximation** consists of that alongside with $\cancel{\ddot{\phi}} + 3H\dot{\phi} + V' = 0$

Equivalent to conditions on the potential, that quantify the **deviation from de-Sitter** and the **flatness of the potential**

$$\epsilon_0 \equiv \frac{H_{in}}{H} = H_{in}\sqrt{\frac{3M_{PL}^2}{V}},$$
$$\epsilon_1 \equiv \frac{\mathrm{d}\ln|\epsilon_0|}{\mathrm{d}N} = \frac{M_{PL}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,$$
$$\epsilon_{n+1} \equiv \frac{\mathrm{d}\ln|\epsilon_n|}{\mathrm{d}N} \ll 1.$$

Linking slow-roll to observables

Now we have a **quasi de-Sitter** universe that can **solve** the ΛCDM **opened questions**, and in particular **predicts** the origin of the **perturbations**, and their primordial power spectra, **scalar** $\mathcal{P}_\zeta$ and **tensor** $\mathcal{P}_h$.

<u>How</u>? Add a perturbation to the scalar action, derive the new equation of motions for the perturbation, quantize it, and after some calculus

<u>Result</u>:

<u>Amplitude</u>

$$A_\mathrm{S} \equiv \mathcal{P}_\zeta\Big|_{k_*} \overset{\text{SRLO}}{\simeq} \frac{V_*}{24\pi^2 M_\mathrm{Pl}^4 \varepsilon_{1*}},$$

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_\zeta}\Big|_{k_*} \overset{\text{SRLO}}{\simeq} 16\varepsilon_{1*},$$

<u>Tilt</u>

$$n_\mathrm{S} \equiv 1 + \frac{\mathrm{d}\ln\mathcal{P}_\zeta}{\mathrm{d}\ln k}\Big|_{k_*} \overset{\text{SRLO}}{\simeq} 1 - 2\varepsilon_{1*} - \varepsilon_{2*}$$

$$n_\mathrm{T} \equiv \frac{\mathrm{d}\ln\mathcal{P}_h}{\mathrm{d}\ln k}\Big|_{k_*} \overset{\text{SRLO}}{\simeq} -2\varepsilon_{1*}$$

<u>Running</u>

$$n_\mathrm{S,run} \equiv \frac{\mathrm{d}^2\ln\mathcal{P}_\zeta}{(\mathrm{d}\ln k)^2}\Big|_{k_*} \overset{\text{SRLO}}{\simeq} -2\varepsilon_{1*}\varepsilon_{2*} - \varepsilon_{2*}\varepsilon_{3*}$$

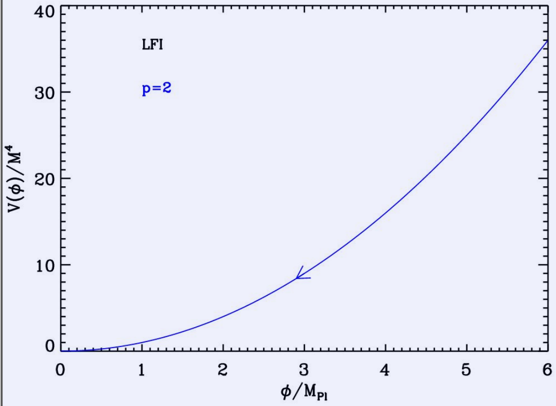
$$n_\mathrm{T,run} \equiv \frac{\mathrm{d}^2\ln\mathcal{P}_h}{(\mathrm{d}\ln k)^2}\Big|_{k_*} \overset{\text{SRLO}}{\simeq} -2\varepsilon_{1*}\varepsilon_{2*}$$

Inferring the potential from the measures

$$\boxed{\phi \rightarrow V(\phi, p)}, \boxed{\ln R_{\rm rad}} \Longrightarrow \phi_* \Longrightarrow \{\varepsilon_i *\} \Longrightarrow \{A_{\rm S},\ n_{\rm S},\ n_{\rm S,run},\ r,\ n_{\rm T},\ n_{\rm T,run},\ N_{\rm e-folds}\ ...\}$$

(Motivated) choice. Eg:

$$V(\phi) = M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$



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In appendix

Inferring the potential from the measures

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$$\varepsilon_1 \overset{\text{SRLO}}{\simeq} \frac{M_{\rm Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2,$$

$$\varepsilon_2 \overset{\text{SRLO}}{\simeq} 2M_{\rm Pl}^2\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right],$$

$$\varepsilon_3 \overset{\text{SRLO}}{\simeq} \frac{2}{\varepsilon_2}M_{\rm Pl}^4\left[\frac{V_{\phi\phi\phi}V_\phi}{V^2} - 3\frac{V_{\phi\phi}}{V}\left(\frac{V_\phi}{V}\right)^2 + 2\left(\frac{V_\phi}{V}\right)^4\right]$$

Inferring the potential from the measures

$$\phi \to V(\phi, p),\ \ln R_{\rm rad} \implies \phi_* \implies \boxed{\{\varepsilon_i *\} \implies \{A_{\rm S},\ n_{\rm S},\ n_{\rm S,run},\ r,\ n_{\rm T},\ n_{\rm T,run},\ N_{\rm e-folds} \dots\}}$$

Amplitude

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Inferring the potential from the measures

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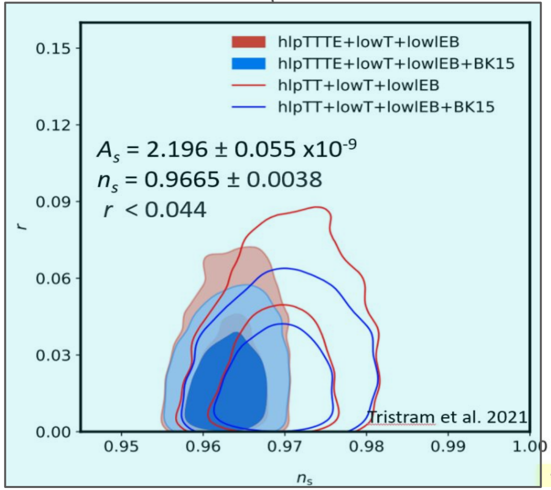
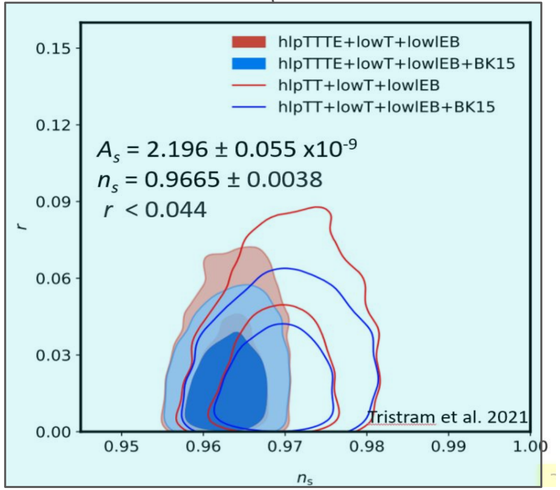


Inferring the potential from the measures

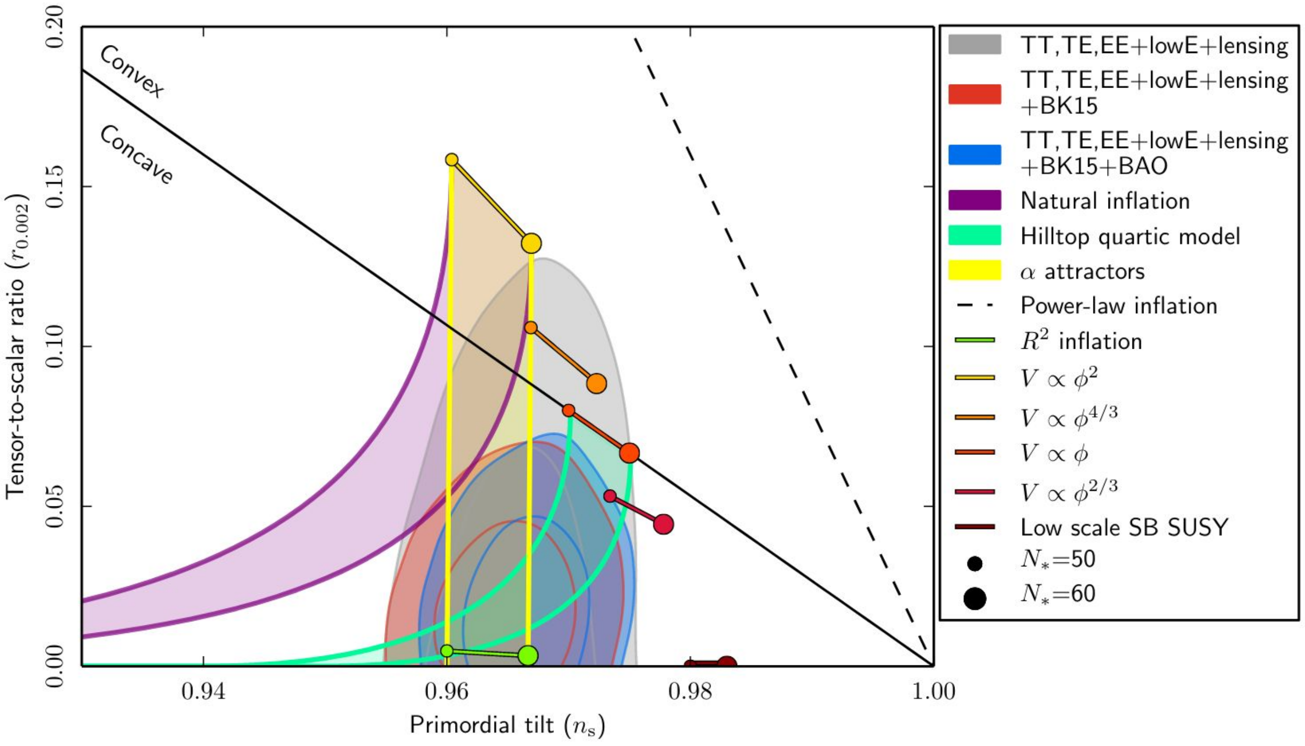
$$\phi \rightarrow V(\phi, \boxed{p}),\ \boxed{\ln R_{\rm rad}} \Longrightarrow \phi_* \Longrightarrow \{\varepsilon_i *\} \Longrightarrow \boxed{\{A_{\rm S},\ n_{\rm S},\ n_{\rm S,run},\ r,\ n_{\rm T},\ n_{\rm T,run},\ N_{\rm e-folds} \ldots\}}$$

for a given potential shape



Given an observation, we can deduce the allowed potential params as done in

J. Martin, C. Ringeval, and V. Vennin, Phys. Dark Univ. 5-6, 75 (2014), ***1303.3787***



Planck collaboration X, Astron. Astrophys. 641, A10 (2020).

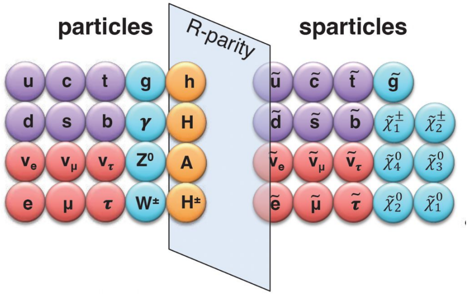
- In this work:

Measurement	Value and error
$\ln 10^{10} \overline{A_S} \pm \sigma(\ln 10^{10} A_S)$	3.047 ± 0.014
$\overline{n_S} \pm \sigma(n_S)$	0.9665 ± 0.0038

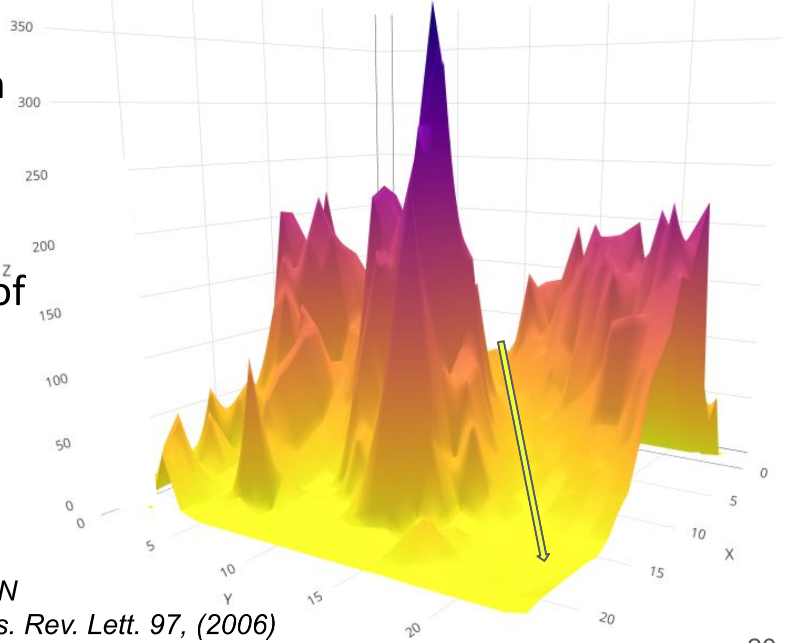
Planck Collaboration VI, Astron. Astrophys. 641, A6 (2020).

- The shape of these **potentials** are theoretically well-**motivated** but still quite **effective**
- **Few** of them come with a **complete study of their embedding** within a **model** of particle physics
- In the following, a **case study: MSSM-inflation**

MSSM-inflation



- **MSSM** = **SUperSYmmetric** extension of the HEP SM.
  - Naturally provides a **WIMP** that can explain the measured $\Omega_{\rm cdm}h^2$.
  - Only a **small fraction** of its parameter space is **excluded** by LHC data.



- **Inflaton** = **scalar field**, evolves with the Klein-Gordon equation in the **MSSM scalar potential** along its **valleys ("flat directions")**.

- We focus on two of its **flat-directions** combinations of **scalar fields:**
  - ***"LLe"***
  - ***"udd"***

<u>Studied previously in (not exhaustive)</u>:

K. Enqvist and A. Mazumdar, Physics Reports 380, 99 (2003), ISSN
R. Allahverdi, K. Enqvist, J. Garcia-Bellido, and A. Mazumdar, Phys. Rev. Lett. 97, (2006)
C. Boehm, J. Da Silva, A. Mazumdar, and E. Pukartas, Phys. Rev. D 87, 023529 (2013)
....

MSSM-inflation potential

The **potential** for ***LLe*** and ***udd:*** $$V_{\rm tree}(\phi) = \frac{1}{2}m_\phi^2\phi^2 - \sqrt{2}A_6\frac{\lambda_6\phi^6}{6M_{\rm Pl}^3} + \lambda_6^2\frac{\phi^{10}}{M_{\rm Pl}^6}$$

where ϕ is the real **field value** associated to the inflaton, m_ϕ its **mass**. m_ϕ and A_6 are **linked** to the underlying **supersymmetric parameters**.

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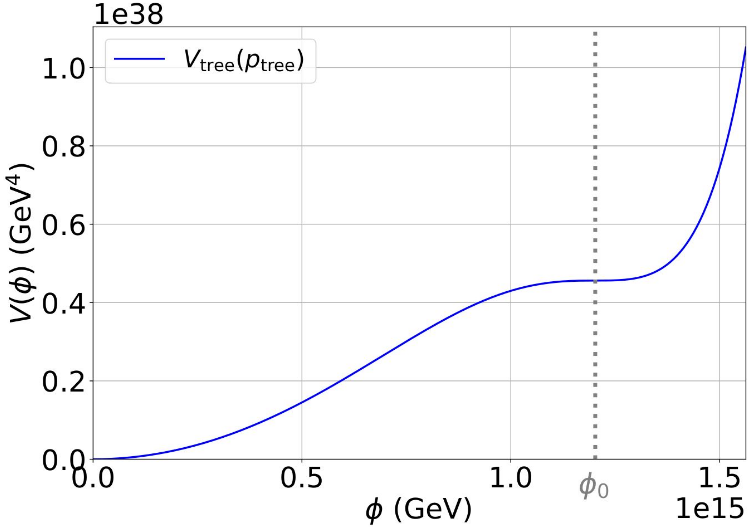
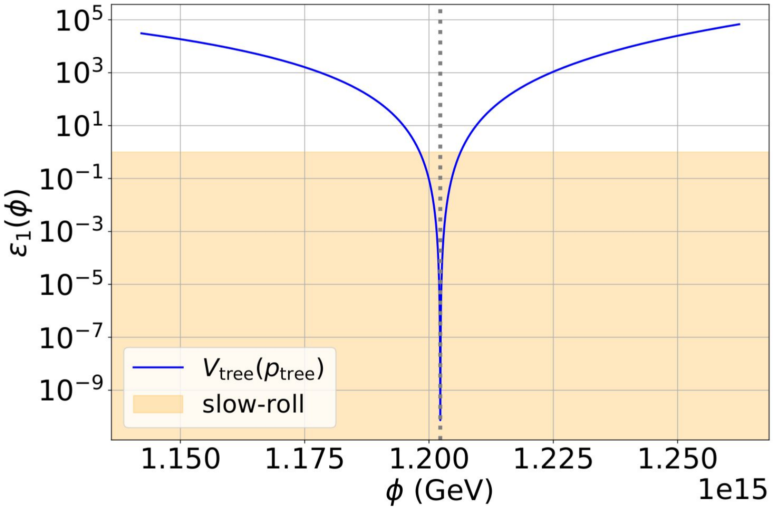
Recipe =>





- **Narrow slow-roll** region ($|\phi - \phi_0| \sim \frac{\phi_0^3}{60 M_P^2}$) & **very-close-from-flat inflection point** ϕ_0.

Radiative corrections impact on the potential

NEW $V_{\rm RGE}$ whose parameters depend on ϕ.

$$m_\phi^2 = m_\phi^2(\phi)$$

$$A_6 = A_6(\phi).$$

$$\lambda_6 = \lambda_6(\phi)$$

- **Radiative corrections**
  - fully computable with the **Renormalization Group Equations**
  - functions of the **gaugino masses** and the **gauge couplings** at **GUT**.
  - vary whether the inflaton is along ***udd*** *or* ***LLe***

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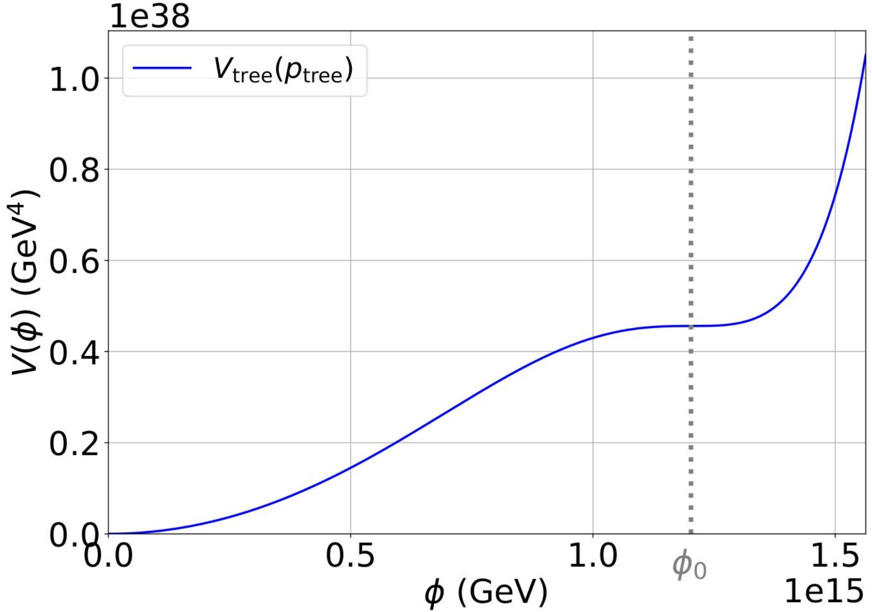

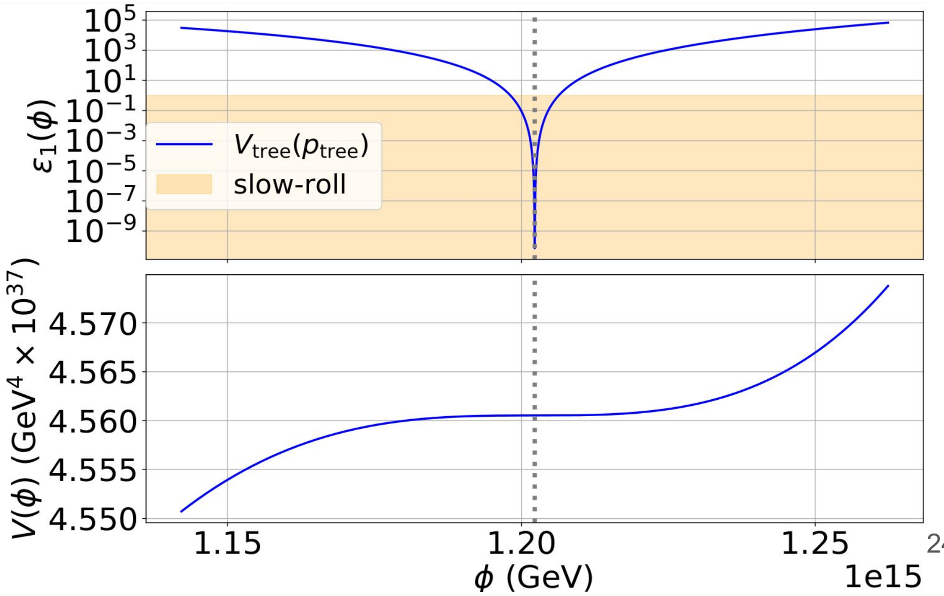

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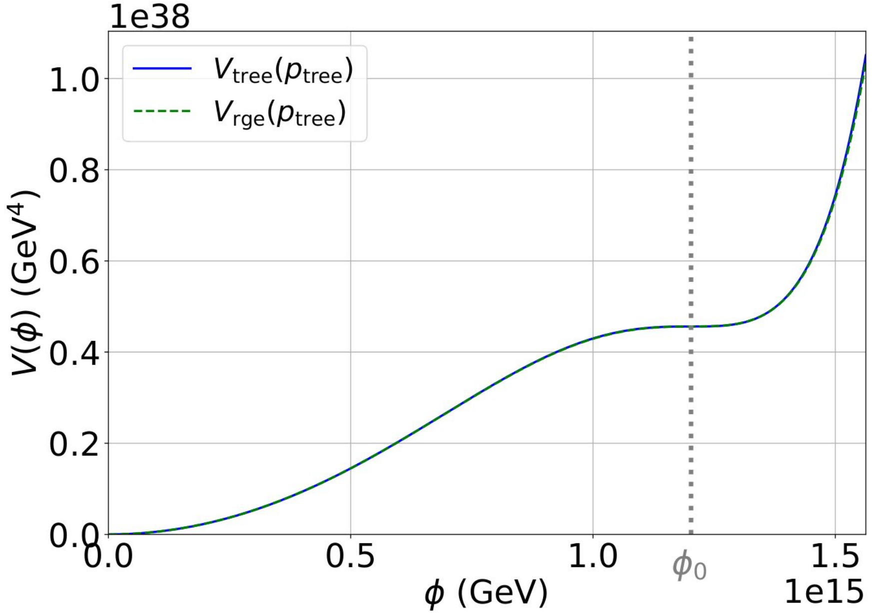

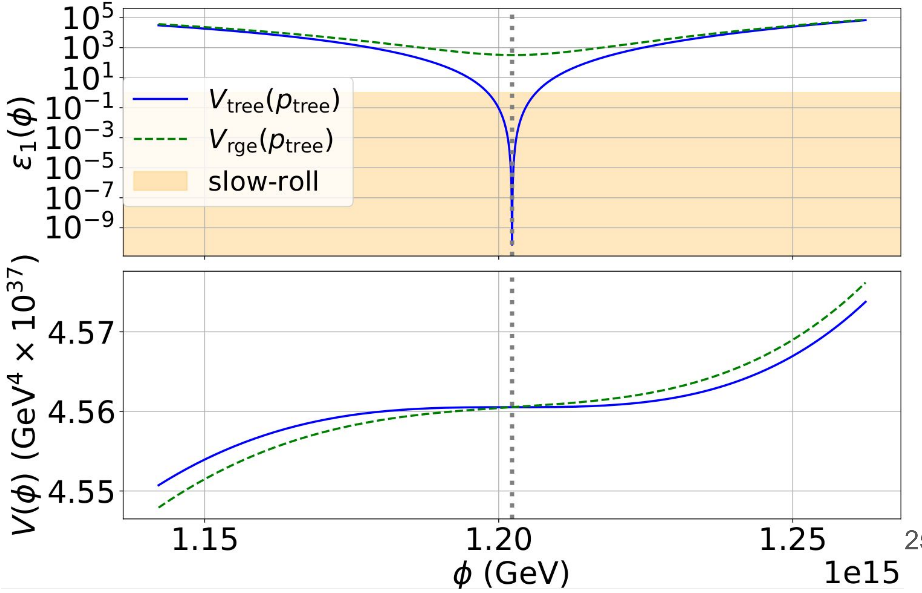

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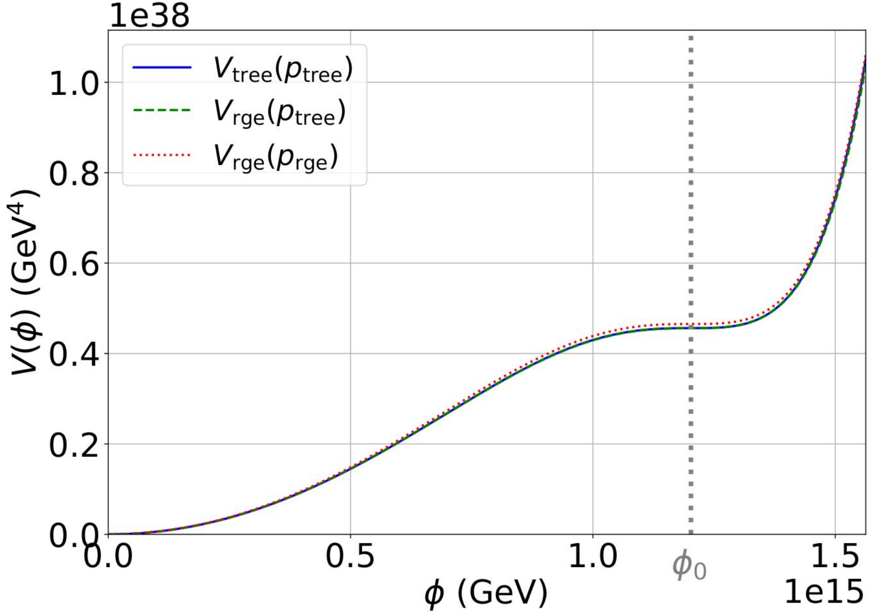
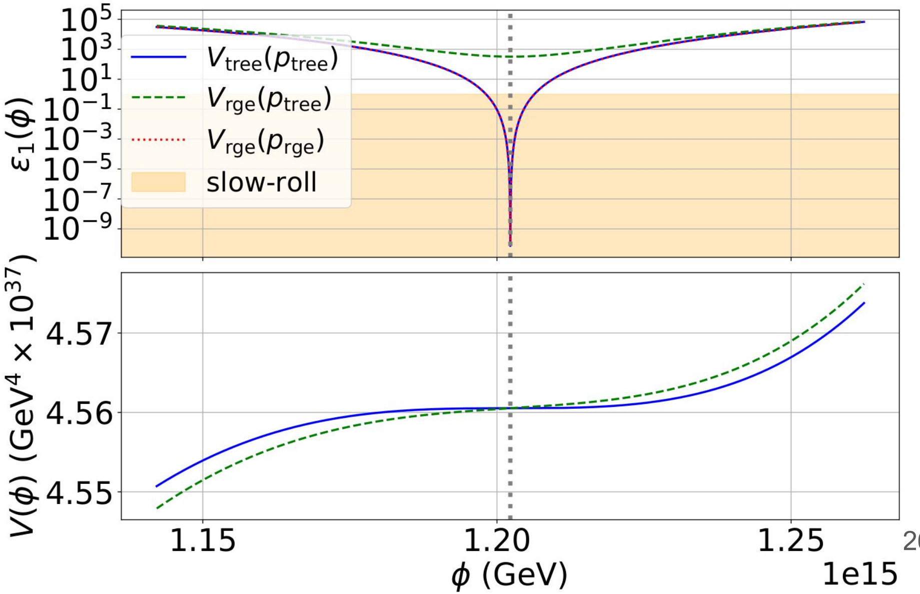
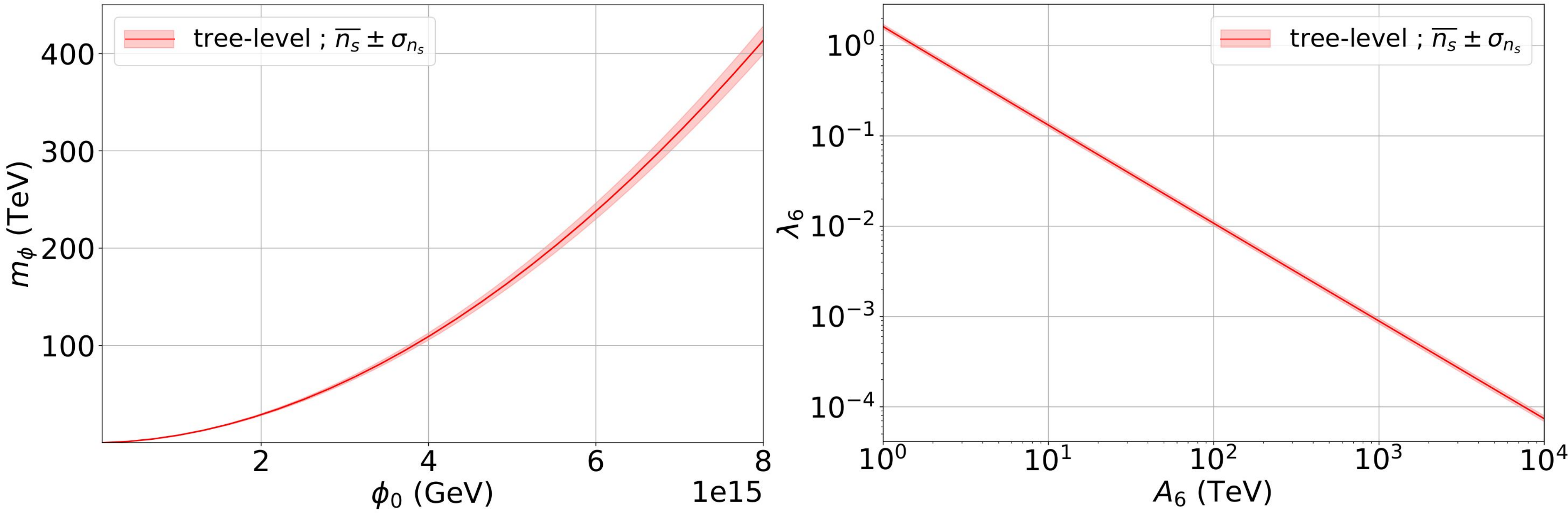




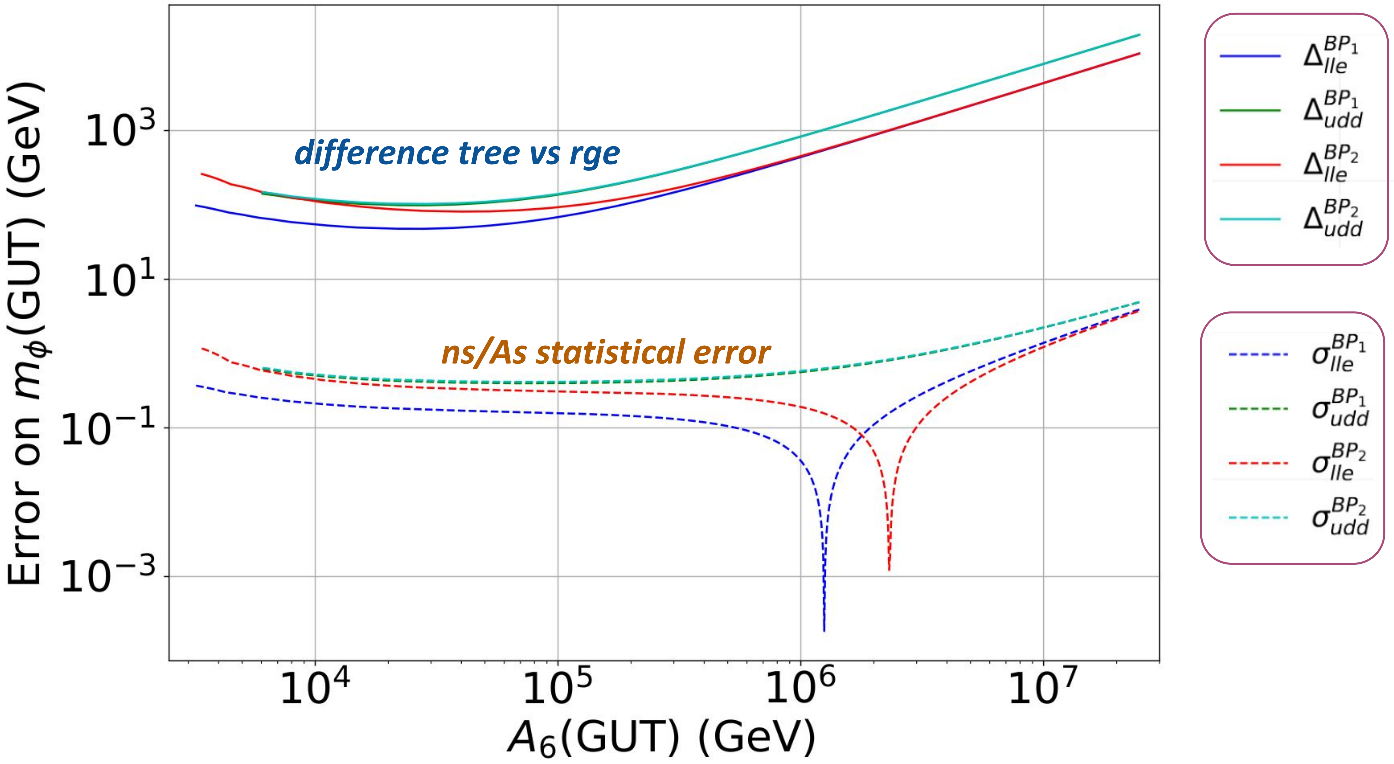
Radiative corrections impact on the parameters

- 3 potential **parameters** - 2 CMB **constraints =** 1 **d.o.f**. Choice: Sample on ϕ_0 or on A6.



- How do these contours **change** beyond tree-level?

Radiative corrections impact on the parameters



- **Not** taking properly **into account** the **RGE corrections** induces a ***systematic bias***:
  - of order **100-1000 GeV** depending on the inflation scale!
  - **well above the *ns/As statistical error***!

Crucial RGE impact!

[*] S. Henrot-Versillé, et al. Phys. Rev. D 89, 055017 (2014)

Toward a global fit

We have identified **some points** [*] for **various dark matter annihilation processes** satisfying:

HEP Constraints:
- Higgs mass (and BR)
- LHC SUSY searches

… not exhaustive

Cosmo constraints:
- $\Omega_{\mathrm{cdm}}h^2$

NEW As, ns

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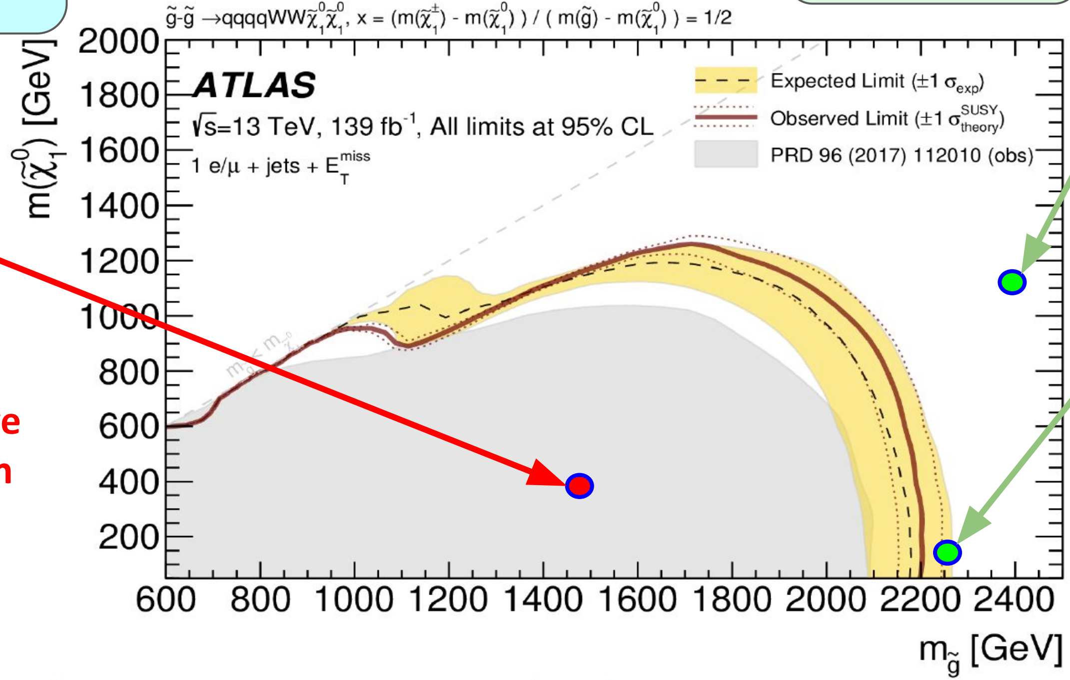
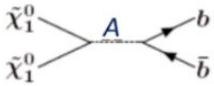
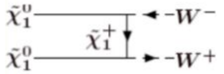


ϕ_0 $9.58\ 10^{14}$ GeV
$m_\phi(\phi_0)$ 6601.5 GeV

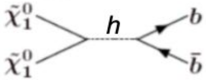


EXCLUDED

ϕ_0 $1.57\ 10^{15}$ GeV
$m_\phi(\phi_0)$ 16954 GeV

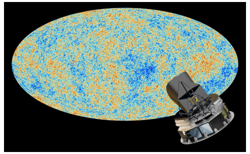


ϕ_0 $1.41\ 10^{15}$ GeV
$m_\phi(\phi_0)$ 13933 GeV



HEP measurements give constraints on inflation

Cosmology restricts the MSSM parameter space



Next step: full **exploration** of the parameter space!

[*] S. Henrot-Versillé, et al. Phys. Rev. D 89, 055017 (2014)

Conclusions & Outlooks

This work:
arXiv:2304.04534

We have revisited the **MSSM-inflation,** a **rigorous** and **computable** framework which builds a bridge between two worlds, **HEP** and **inflation,** thanks to a **slow-roll flat-inflection potential**.

We have shown that cosmology is sensitive to **RGE corrections**: in our study-case, they **cannot be neglected**, and should be accounted for precisely.

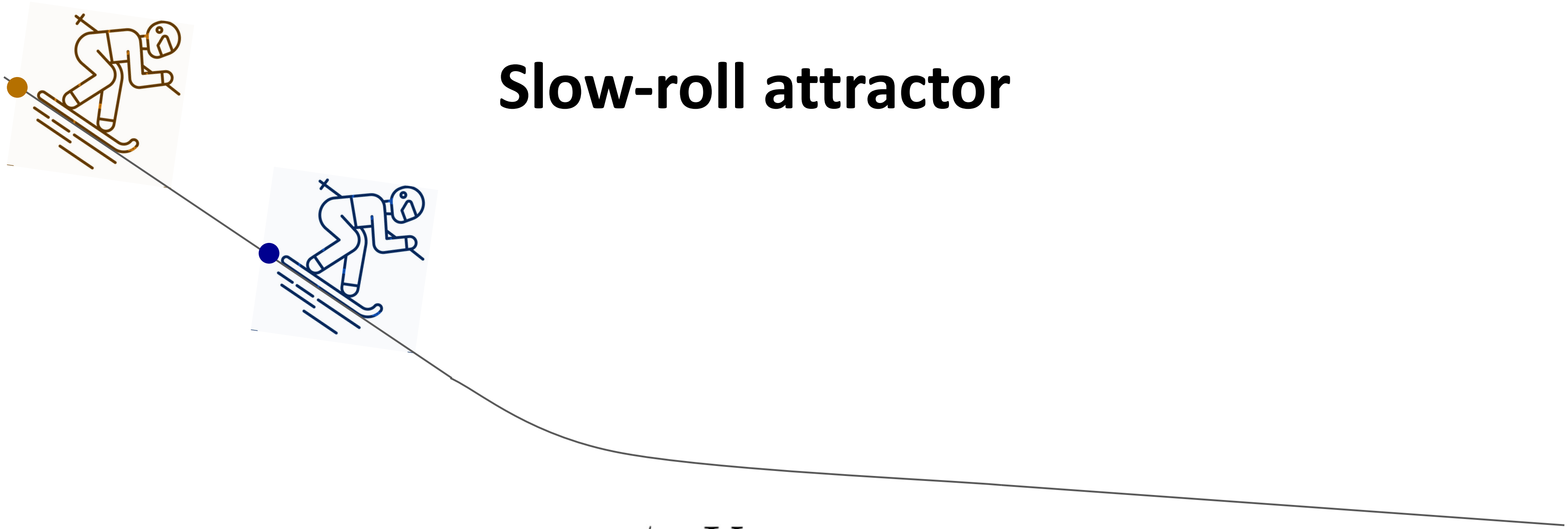
This will be even more true in the coming years (with the expected **improvement** of **ns** & **nsrun**, and with the **new LHC runs**).

Future:

- More systematic **exploration** of the **parameter space**
- Inclusion of the **reheating duration** computation
- Beyond MSSM : **NMSSM**, **Multi-field** inflation…

BACKUPS

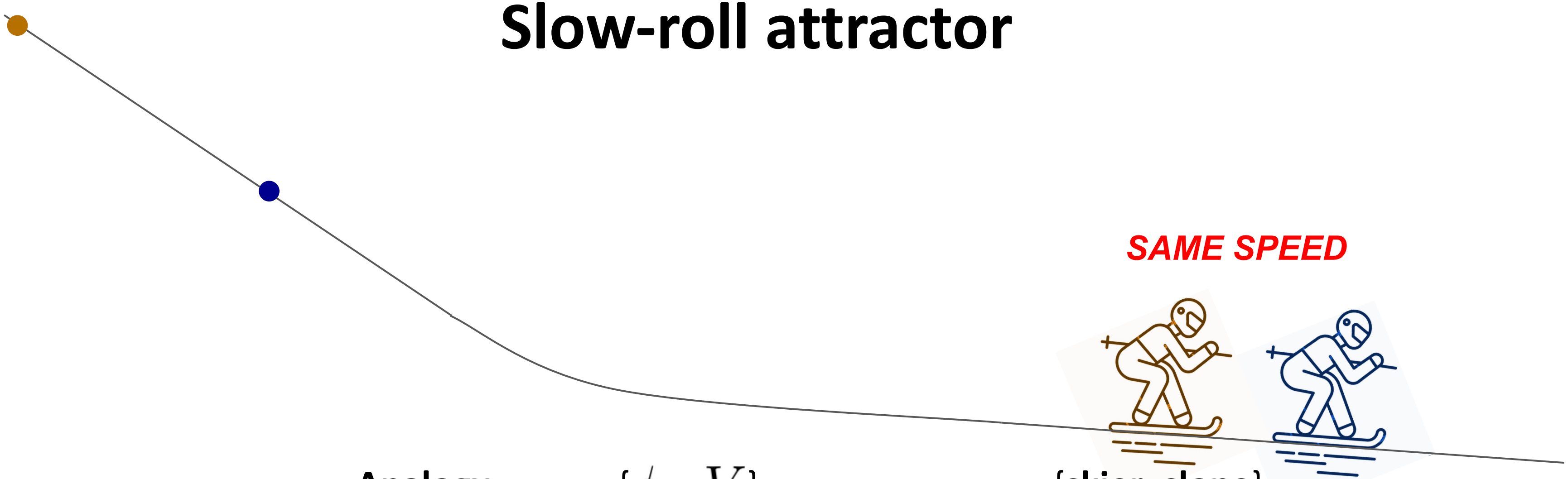
Slow-roll attractor



Analogy: $\{\phi,\ V\}$ **{skier, slope}**

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \qquad ma + fv + mg = 0$$

Slow-roll attractor



Analogy: $\{\phi,\ V\}$ $\longleftrightarrow$ **{skier, slope}**

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \qquad ma + fv + mg = 0$$

Convenient for inflation:

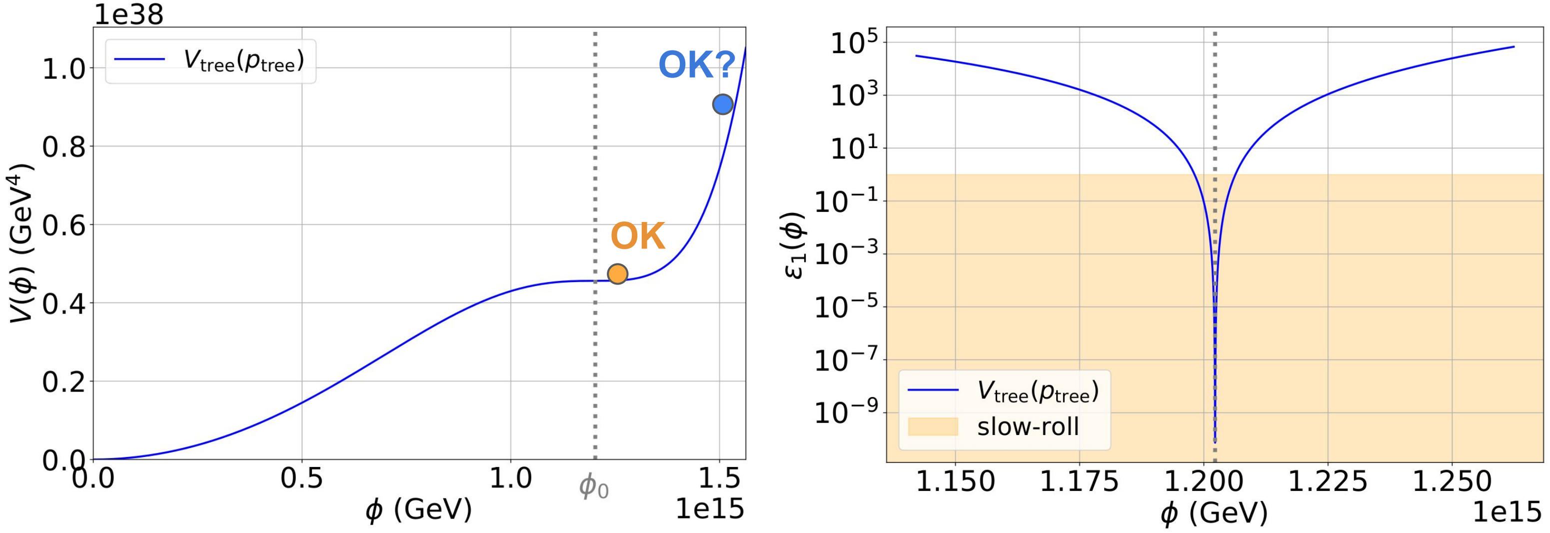
If the slow-roll region (where the potential is quasi-flat) is sufficiently long, **then** all the **trajectories** end being attracted into the low-velocity **slow-roll regime**. **No** need to **worry** about the initial conditions $(\phi, \dot{\phi})$.

MSSM-inflation potential

The **potential** for ***LLe*** and ***udd:*** $$V_{\rm tree}(\phi) = \frac{1}{2}m_\phi^2\phi^2 - \sqrt{2}A_6\frac{\lambda_6\phi^6}{6M_{\rm Pl}^3} + \lambda_6^2\frac{\phi^{10}}{M_{\rm Pl}^6}$$

where ϕ is the real **field value** associated to the inflaton, m_ϕ its **mass**. m_ϕ and A_6 are **linked** to the underlying **supersymmetric parameters**.

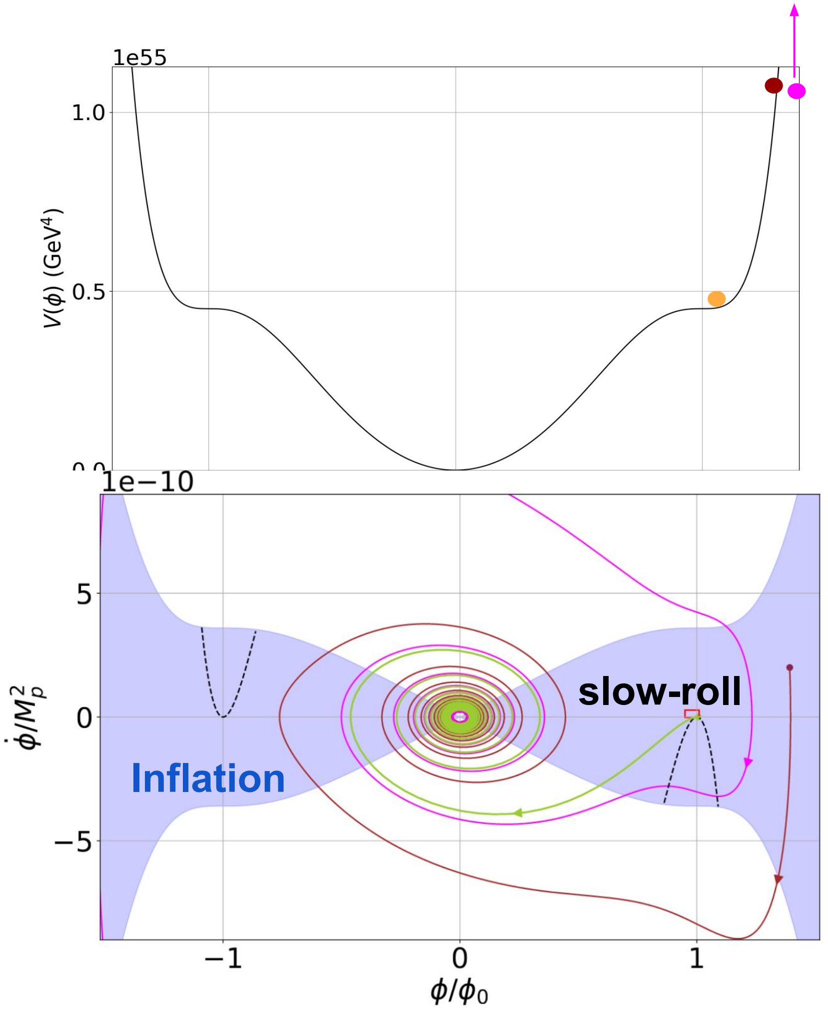
Recipe =>



- **Narrow slow-roll** region ($|\phi - \phi_0| \sim \frac{\phi_0^3}{60M_{\rm P}^2}$) & **very-close-from-flat inflection point** ϕ_0.

Phase space of the inflaton

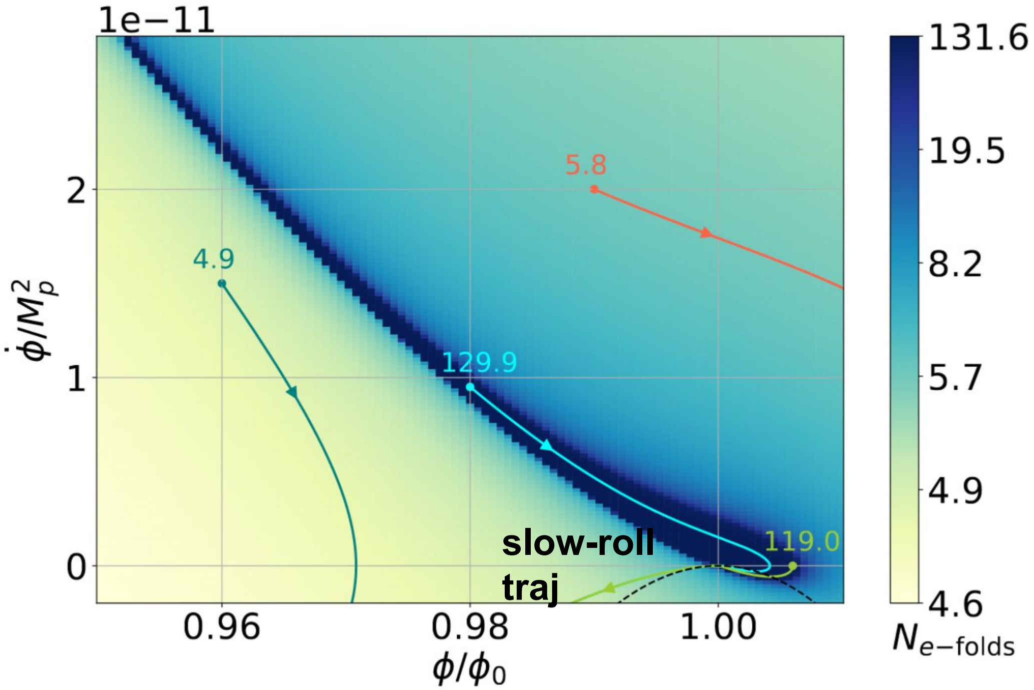
To study that, go to phase space



Already one sees that the **trajectories** beginning **too far from flat inflection point do not follow the SR** trajectory (black-dashed line)

Robustness of slow-roll attractor

- **Very narrow slow-roll** region implies for various **trajectories**:



- Only few **trajectories are attracted into the slow-roll** regime

… **Slow-roll NOT independent of the initial conditions** … ***not usual*** *for a single field slow-roll*

How to get ϕ_*?

$$\Delta N_* \overset{\text{SRLO}}{\simeq} \int_{\phi_{\text{end}}}^{\phi_*} \frac{V(\phi)}{V_\phi(\phi)} \mathrm{d}\phi$$

… how to get ΔN_* then?

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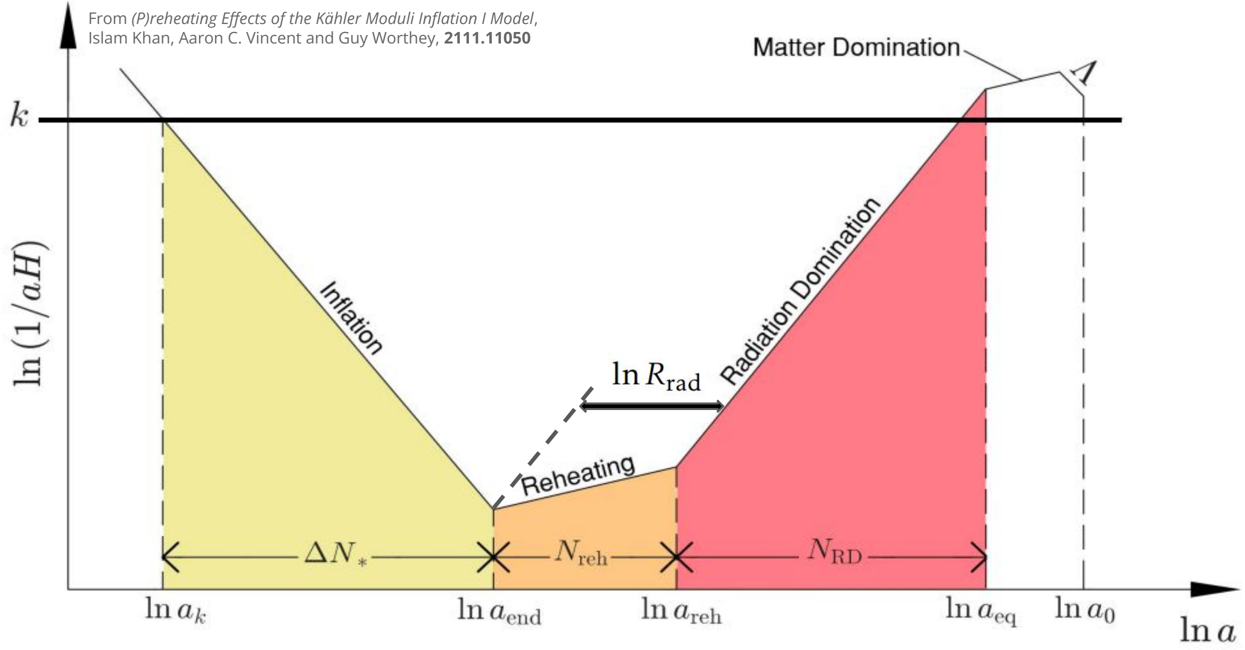
From *(P)reheating Effects of the Kähler Moduli Inflation I Model*, Islam Khan, Aaron C. Vincent and Guy Worthey, **2111.11050**



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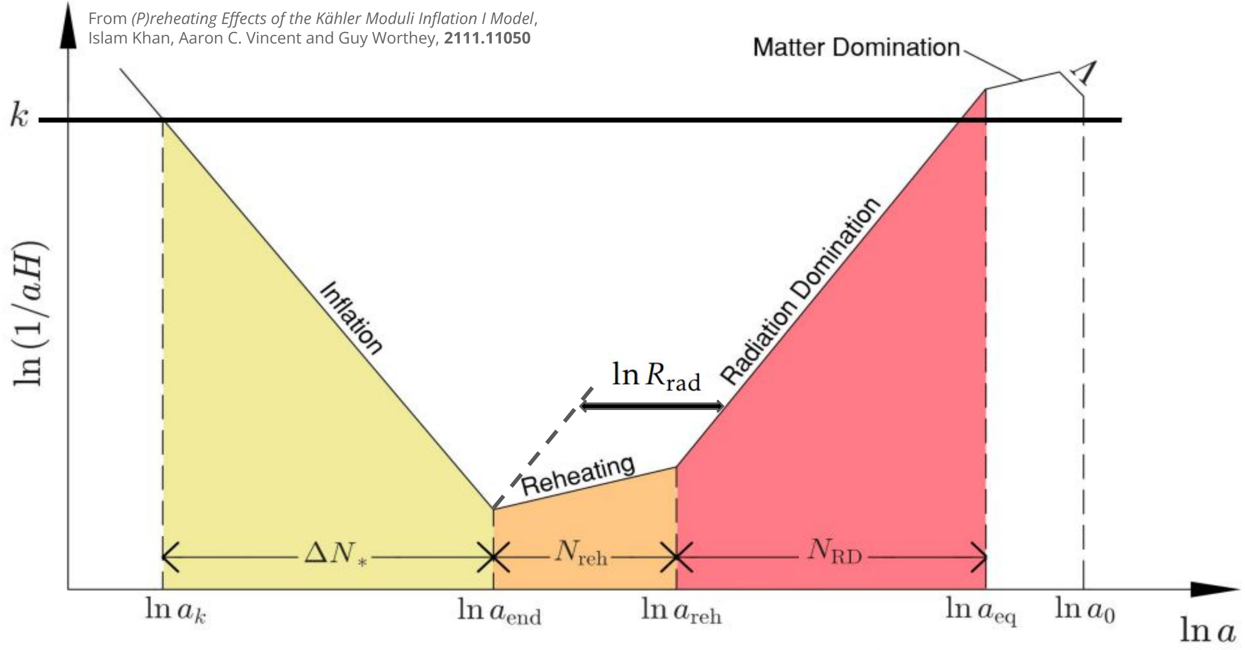


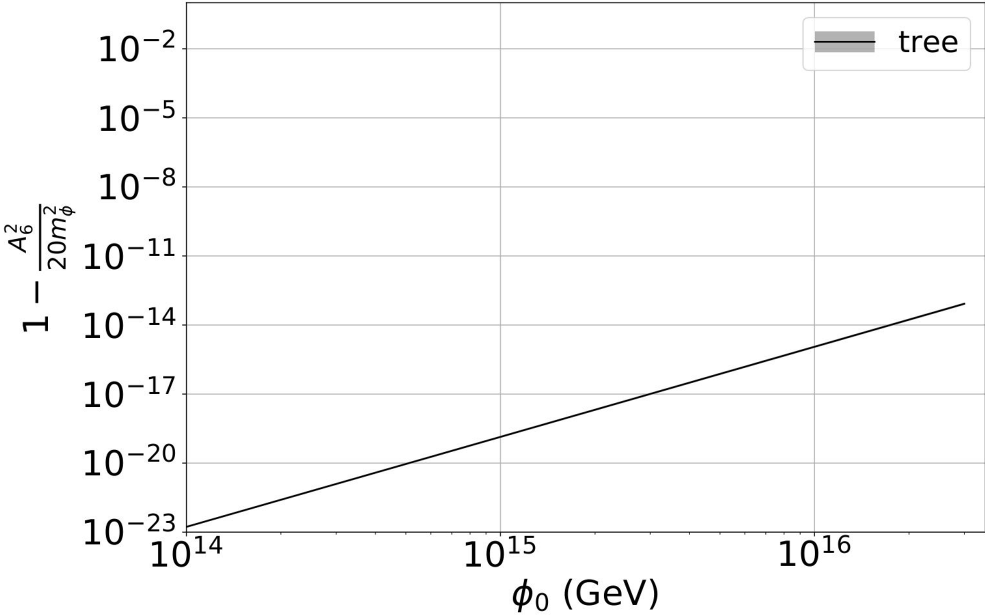
$$\Delta N_* \overset{\text{SRLO}}{\simeq} \ln R_{\text{rad}} - \ln\left(\frac{k_*}{a_0 \tilde{\rho}_\gamma^{1/4}}\right) - \frac{1}{4}\ln\left[\frac{9V_{\text{end}}}{\varepsilon_{1*}(3-\varepsilon_{1\text{end}})V_*}\right] + \frac{1}{4}\ln(8\pi^2 A_S)$$

Radiative corrections impact on the fine-tuning

$$\begin{cases} V_{\phi\phi}(\phi_0) = 0 \\ V_\phi(\phi_0) = \nu \simeq 0 \end{cases}$$

- The potential at **inflection point** ϕ_0 has to be very close to flat.
- At **tree level**:

$$0 < 1 - \frac{A_6^2}{20 m_\phi{}^2} \ll 1 \ .$$



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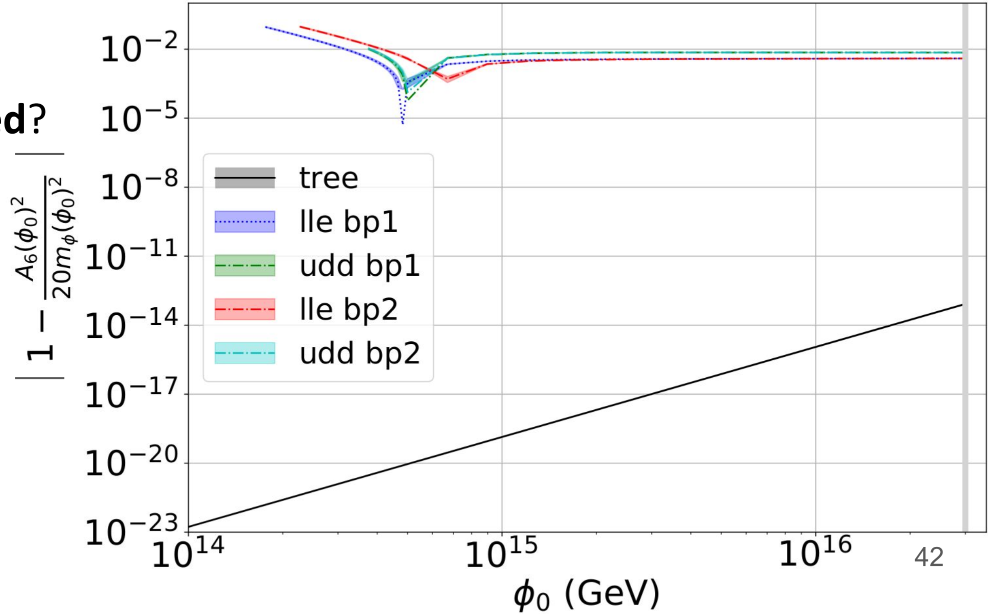

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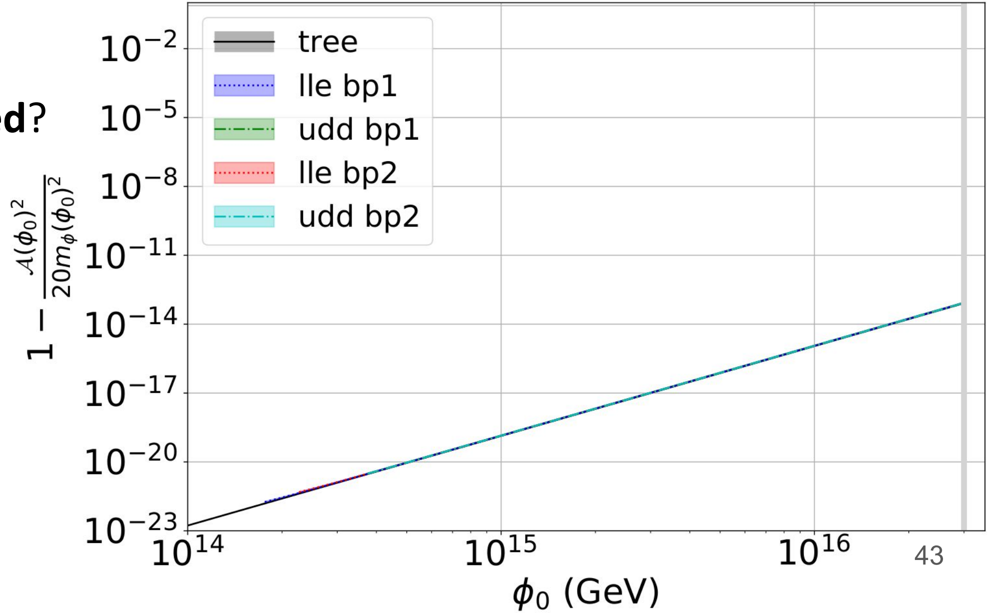
$$0 < \; 1 - \frac{A_6^2}{20{m_\phi}^2} \ll 1 \quad .$$

- **RGE**: Does it mean the fine tuning is **relaxed**?

We showed that

NEW

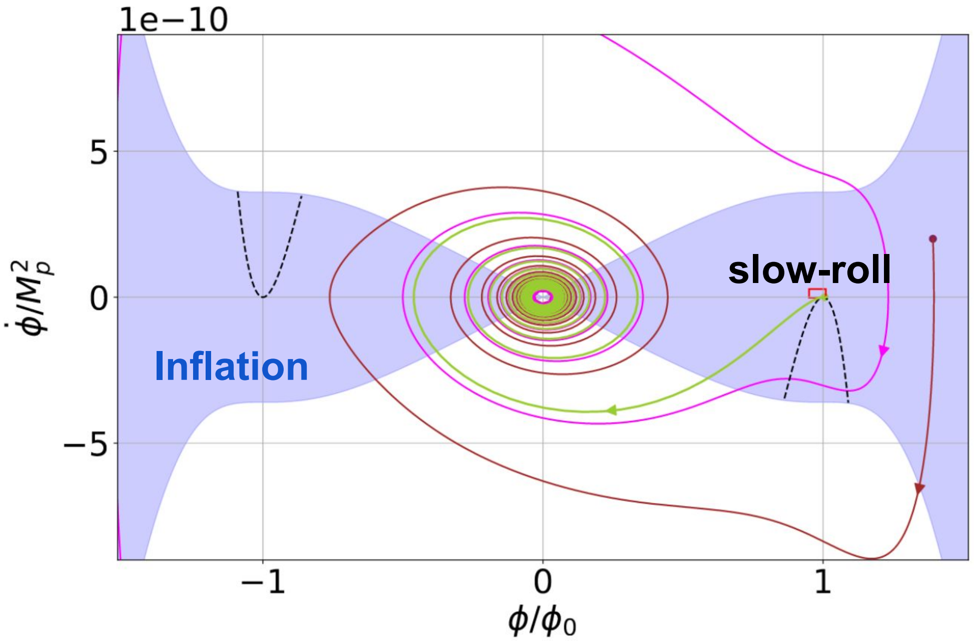
$$0 < \; 1 - \frac{\mathcal{A}^2(\phi_0)}{20 m_\phi^2(\phi_0)} \ll 1$$



- No! Actually, **same fine-tuning**

Robustness of slow-roll attractor

- **Very narrow slow-roll** region implies for various **trajectories**:



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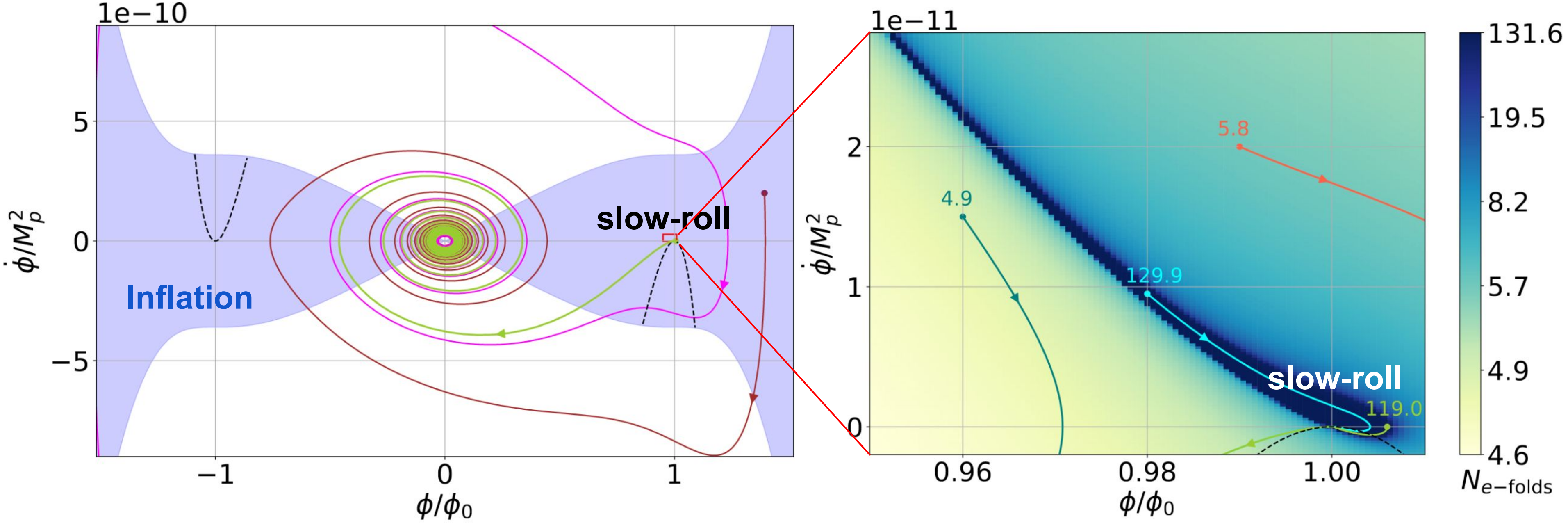
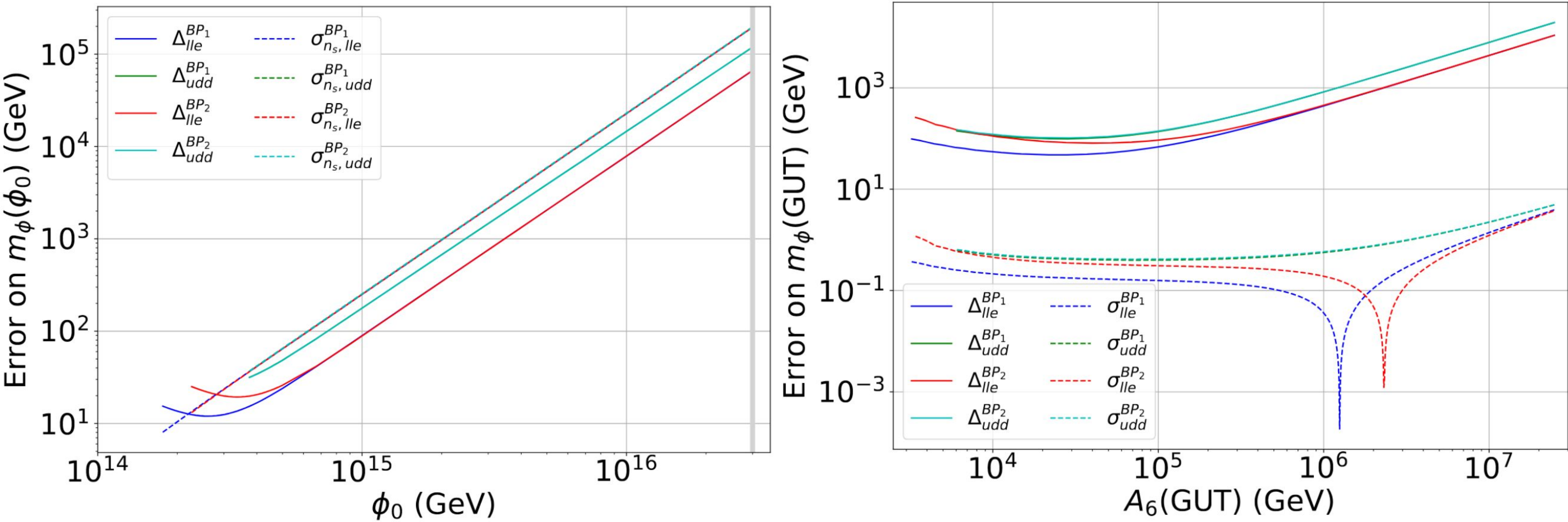


- The **trajectories attracted into the slow-roll** regime are rare

… **Not a very good attractor!** … ***not usual*** *for a single field slow-roll*

RGE impact on the parameter space

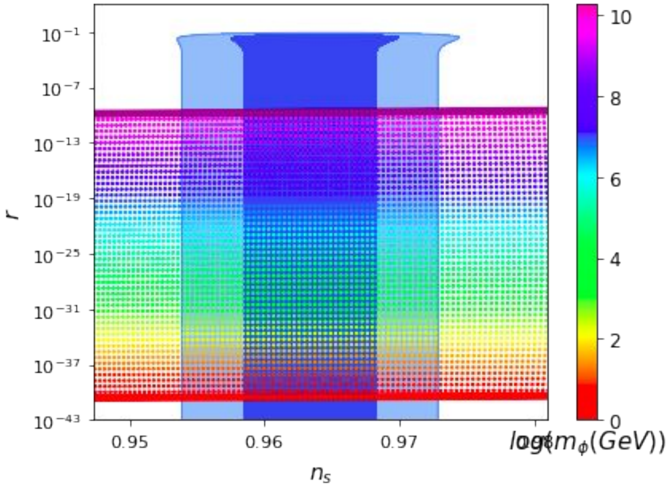
- Two types of error, either versus ϕ_0 or $A_6(\mathrm{GUT})$.
  - A **statistical** error on ns/As: $\sigma^{\mathrm{BP}_j}_{n_S,i}[p] \;=\; \frac{1}{2}\left|p^{\mathrm{RGE}}(n_S=\overline{n_S}+\sigma_{n_S})-p^{\mathrm{RGE}}(n_S=\overline{n_S}-\sigma_{n_S})\right|$
  - Impact of corrections $\Delta^{\mathrm{BP}_j}_i[p] \;=\; p^{\mathrm{RGE}}(n_S=\overline{n_S})-p^{\mathrm{tree}}(n_S=\overline{n_S})$



- The **systematic error** is not **negligible!**

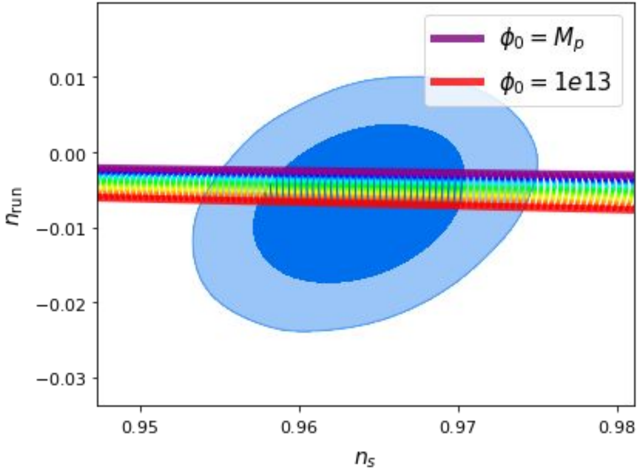
Cosmology beyond As and ns

- r



- ns_run: potential constraining power in a near future

for phi0 = 1e15 (<= mphi at lhc), ns_run = **4.4e-3**



- nt and nt_run: beyond experimental reach
- No induced gravitational waves
- No non gaussianities

Robust forecasts on fundamental physics from the foreground-obscured, gravitationally-lensed CMB polarization

Josquin Errard,[*,a,b,c,d] Stephen M. Feeney,[*,e] Hiranya V. Peiris,[f] and Andrew H. Jaffe[e]

We show that in the case of CMB, synchrotron
and dust, and after delensing and marginalization over foreground residuals, the best pre-2020 instruments in combination with Planck can reach $\sigma(r) \sim 3 \times 10^{-3}$, $\sigma(n_t) \sim 0.2$, $\sigma(n_s) \sim 2.2 \times 10^{-3}$, $\sigma(\alpha_s) \sim 3 \times 10^{-3}$, $\sigma(M_\nu) \sim 55$ meV, $\sigma(w) \sim 0.16$, $\sigma(w_0) \sim 0.36$, $\sigma(w_a) \sim 0.71$, $\sigma(N_{eff}) \sim 0.05 - 0.06$ and $\sigma(\Omega_k) \sim 2.5 \times 10^{-3}$ when delensing using the CMB×CIB method. Post-2020 instruments, in particular the combination of the ground-based Stage-IV and a space mission, could reach constraints $\sigma(r) \sim$ **1.3×10^{-4}**, $\sigma(n_t) \sim 0.03$, **$\sigma(n_s) \sim 1.8 \times 10^{-3}$**, **$\sigma(\alpha_s) \sim 1.7 \times 10^{-3}$**, $\sigma(M_\nu) \sim 31$ meV, $\sigma(w) \sim 0.09$, $\sigma(w_0) \sim 0.25$, $\sigma(w_a) \sim 0.50$, $\sigma(N_{eff}) \sim 0.024$ and $\sigma(\Omega_k) \sim 1.5 \times 10^{-3}$

Slow-roll inflation

Previous inflation talk(s):

“- **Inflation**: solution the

Slow-roll:

$$V \gg \dot{\phi}^2$$

- On its simplest versions, only needs to introduce a **single scalar field *slow-rolling* on a quasi-flat potential** *V*.

Deviation from **de-Sitter**:

$$\left.\begin{aligned} \varepsilon_1 &\overset{\text{SRLO}}{\simeq} \frac{M_{\text{Pl}}^2}{2}\left(\frac{V_\phi}{V}\right)^2 \\ \varepsilon_i &\overset{\text{SRLO}}{\simeq} \varepsilon_i\Big(V(\phi), V_\phi(\phi), \ldots\Big) \end{aligned}\right\} \ll 1$$

- On top of that, allows one to explain the density **fluctuations** origin and to predict their primordial power spectra, **scalar** $\mathcal{P}_\zeta$ and **tensor** $\mathcal{P}_h$, around the **CMB scale k***”

$$\mathcal{P}_\zeta(k \simeq k_*) \overset{\text{SRLO}}{\simeq} \mathcal{P}_\zeta(V_*, \{\varepsilon_{i*}\})$$

$$\mathcal{P}_h(k \simeq k_*) \overset{\text{SRLO}}{\simeq} \mathcal{P}_h(V_*, \{\varepsilon_{i*}\})$$

$$\phi_* \overset{\text{SRLO}}{\simeq} \phi_*(V, k_*)$$

Potential <-> Power spectra

- **Ingredients**:
    - The **potential** $\phi \to V(\phi, p)$
    - Two necessary conditions so **slow-roll takes place**
        - $\varepsilon_1 < 1$
        - **Trajectories reach** and **stay** in the SR region

- **Recipe:**
    - **Potential parameters --> power spectra parameters**

$$\phi \to V(\phi, p), \Longrightarrow \phi_* \Longrightarrow \{\varepsilon_{i*}\} \Longrightarrow \{A_S,\ n_S,\ n_{S,\mathrm{run}},\ r,\ n_T,\ n_{T,\mathrm{run}},\ N_{\mathrm{e-folds}} \dots\}$$

J. Martin, C. Ringeval, and V. Vennin, *Phys. Dark Univ.* 5-6, 75 (2014), ***1303.3787***

MSSM-inflation potential

- The **potential** for ***LLe*** and ***udd*** reduces to

$$V(\phi) = \frac{1}{2} m_\phi^2(\phi)\phi^2 - \sqrt{2} A_6(\phi) \frac{\lambda_6(\phi)\phi^6}{6M_{\rm Pl}^3} + \lambda_6(\phi)^2 \frac{\phi^{10}}{M_{\rm Pl}^6}$$

where ϕ is the real **field value** associated to the inflaton, m_ϕ its **mass**, A_6 its trilinear **coupling** and anoth λ_6 **coupling** of order **1**. They are linked to the MSSM spectrum.

- Approximation usually done
  - ***Tree-level*** approximation $V_{\rm tree}$ =>

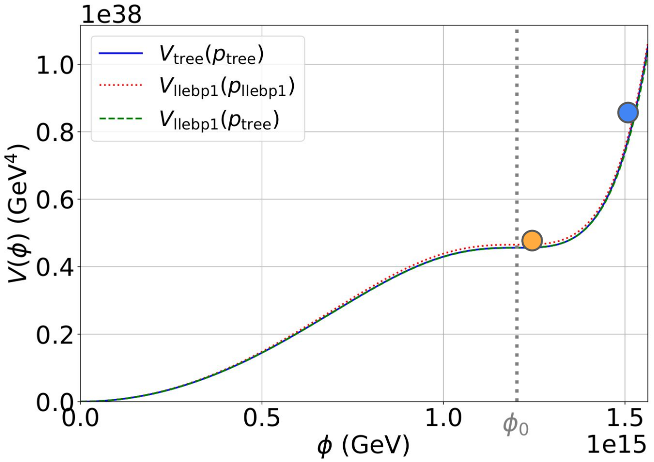
$$m_\phi^2(\phi) = m_\phi^2$$
$$A_6(\phi) = A_6$$
$$\lambda_6(\phi) = \lambda_6$$

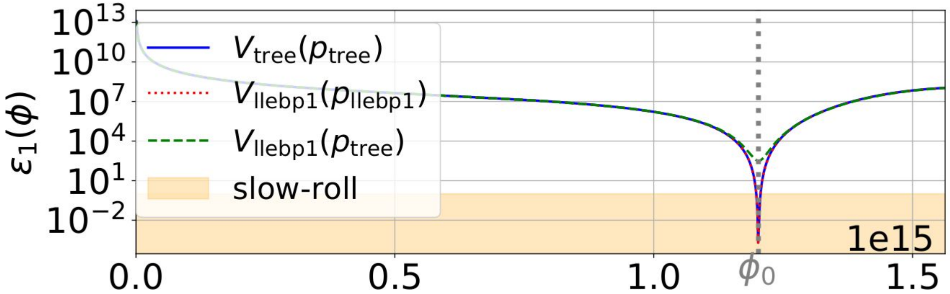
- ***NEW RGE*** potential: $V_{\rm RGE}$ whose parameters depend on ϕ, on the gaugino masses and gauge couplings (well defined through the **Renormalization Group Equations (RGE))**. Varies whether the inflaton is along ***udd** or **LLe**.*

BP1 resp BP2:
given gaugino masses
& gauge couplings

Come-back to our hypothesis

- *Remember slide 3…*
    - Two necessary conditions so **slow-roll takes place**
        - $\varepsilon_1 < 1$
        - **Trajectories get to** and **stay** in the SR region ▲



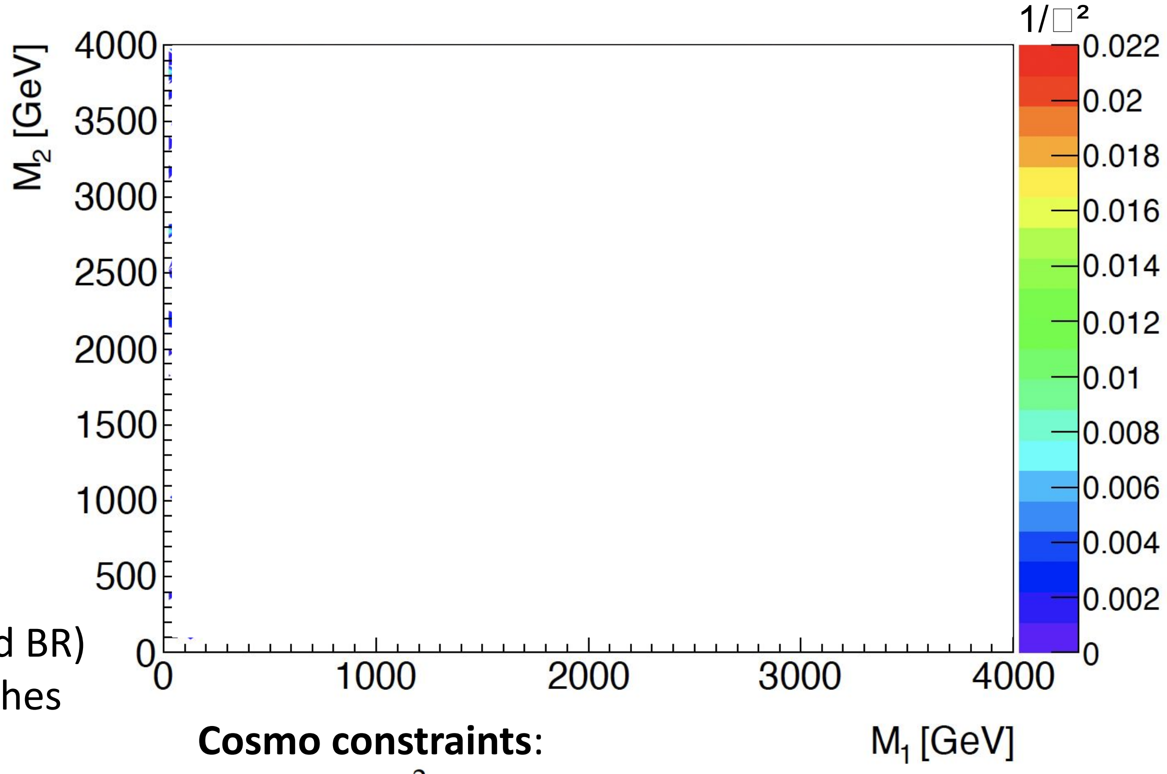


- Initial condition 🟠 should be ok but what about 🔵?
- Is the **slow-roll** region to **thin Deltaphi/phi ~ phi0^2/Mp^2** to act like an attractor?

Constrained MSSM parameters space

Dark matter:

neutralino $\tilde{\chi}_1^0$



HEP Constraints:

- Higgs mass (and BR)
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- Xenon1T
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… not exhaustive

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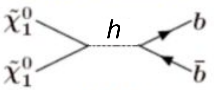
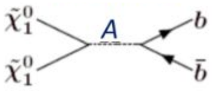
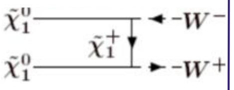
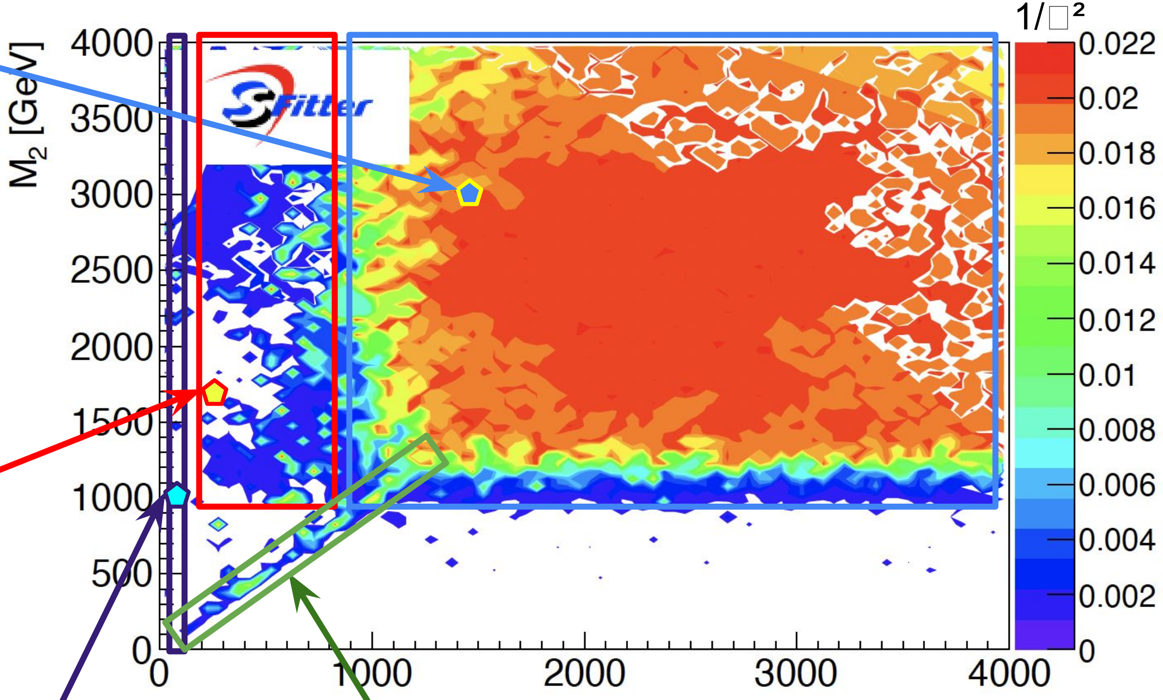
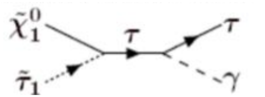
	Higgs funnel	HEP Spectrum	udd inflaton
		$\tilde g$ 2422 $\tilde\chi_1^0$ 60.0	ϕ_0 $1.41\ 10^{15}$ $m_\phi(\phi_0)$ 13933 $A_6(\phi_0)$ 44204 $\lambda_6(\phi_0)$ 0.0159
satisfies	$\Omega_{\rm cdm}h^2$, *Xenon1T*	m_h, *LHC, …*	A_s, n_s, $n_{s,\rm run}$, r

	A funnel	HEP Spectrum	lle inflaton
		$\tilde g$ 1464 $\tilde\chi_1^0$ 359	ϕ_0 $9.64\ 10^{14}$ $m_\phi(\phi_0)$ 6590 $A_6(\phi_0)$ 20857 $\lambda_6(\phi_0)$ 0.0347
satisfies	$\Omega_{\rm cdm}h^2$, *Xenon1T*	m_h	A_s, n_s, $n_{s,\rm run}$, r
excluded		*LHC (gluinos)*	

	Higgsino	HEP Spectrum	udd inflaton
		$\tilde g$ 2444 $\tilde\chi_1^0$ 1110	ϕ_0 $1.57\ 10^{15}$ $m_\phi(\phi_0)$ 16954 $A_6(\phi_0)$ 53787 $\lambda_6(\phi_0)$ 0.0128
satisfies	$\Omega_{\rm cdm}h^2$, *Xenon1T*	m_h, *LHC, …*	A_s, n_s, $n_{s,\rm run}$, r



Stau-coanihilation



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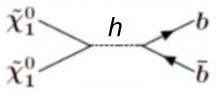
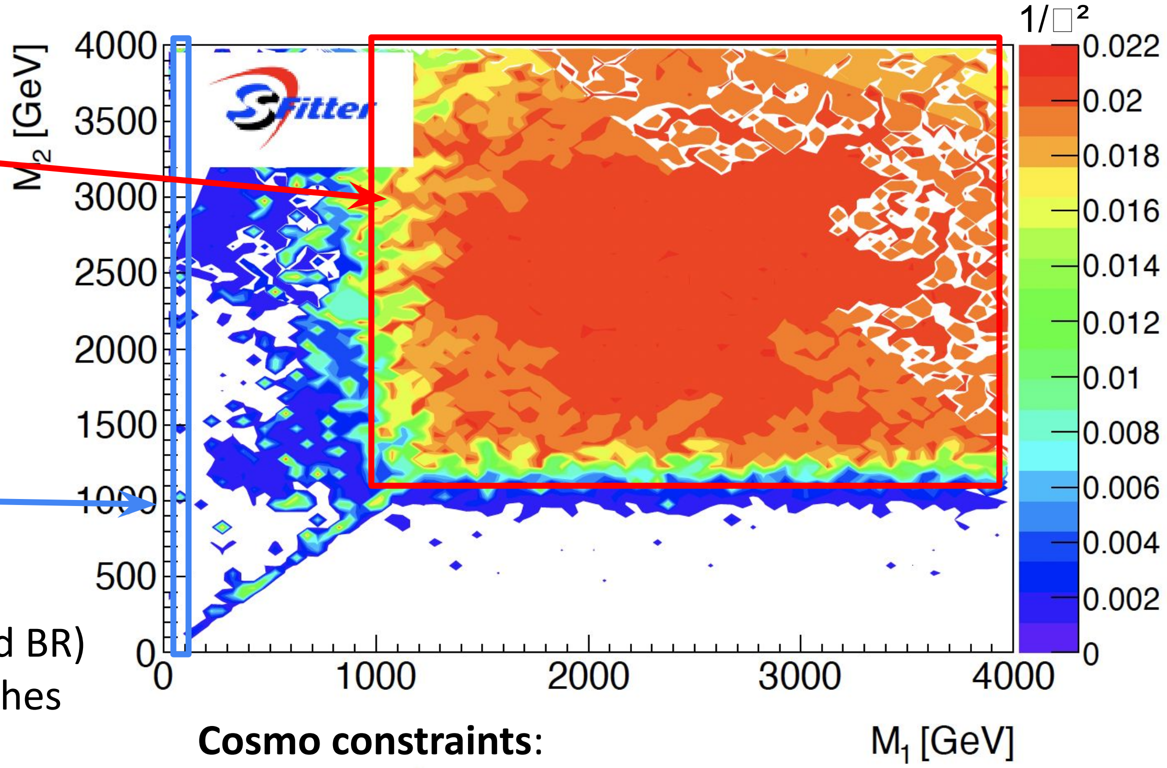
*S. Henrot-Versillé, R. Lafaye, T. Plehn, M. Rauch, D. Zerwas, S. Plaszczynski, B. Rouillé d'Orfeuil, and M. Spinelli, Phys. Rev. D 89, 055017 (2014), **1309.6958***

A funnel

Higgs funnel





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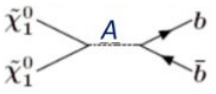
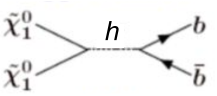
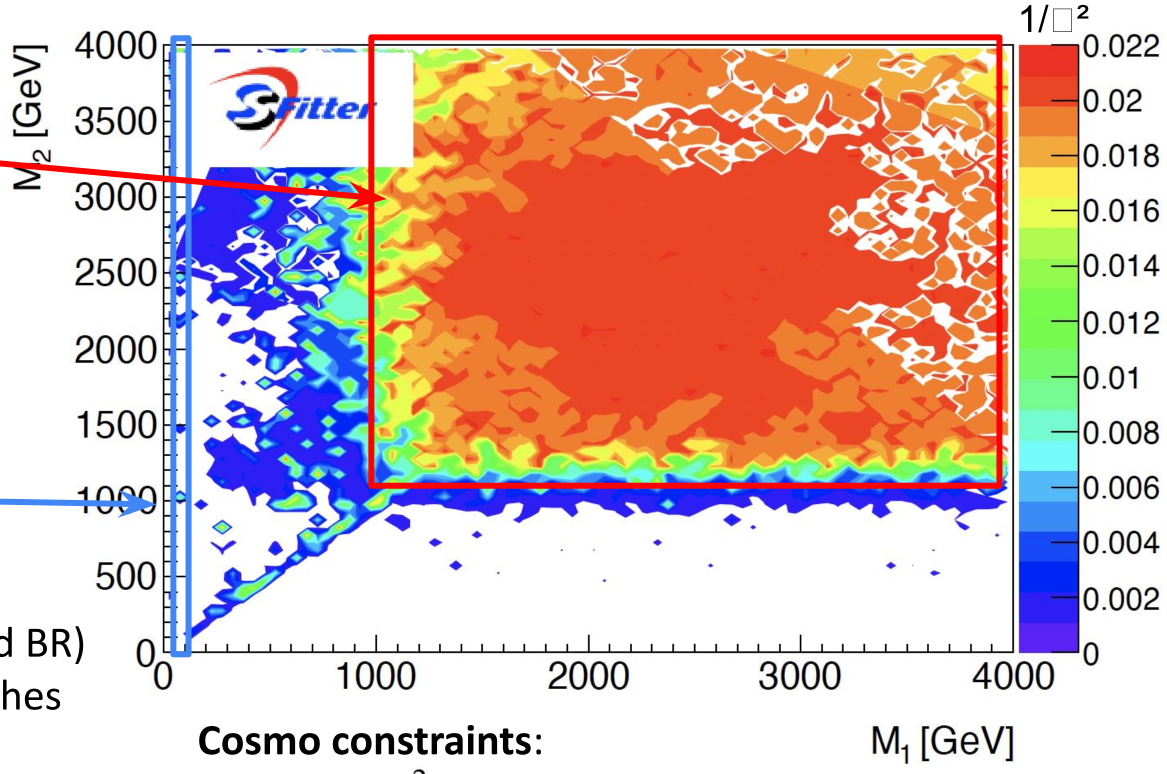


Higgs funnel





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NEW **ns, As**

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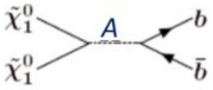
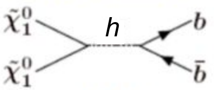
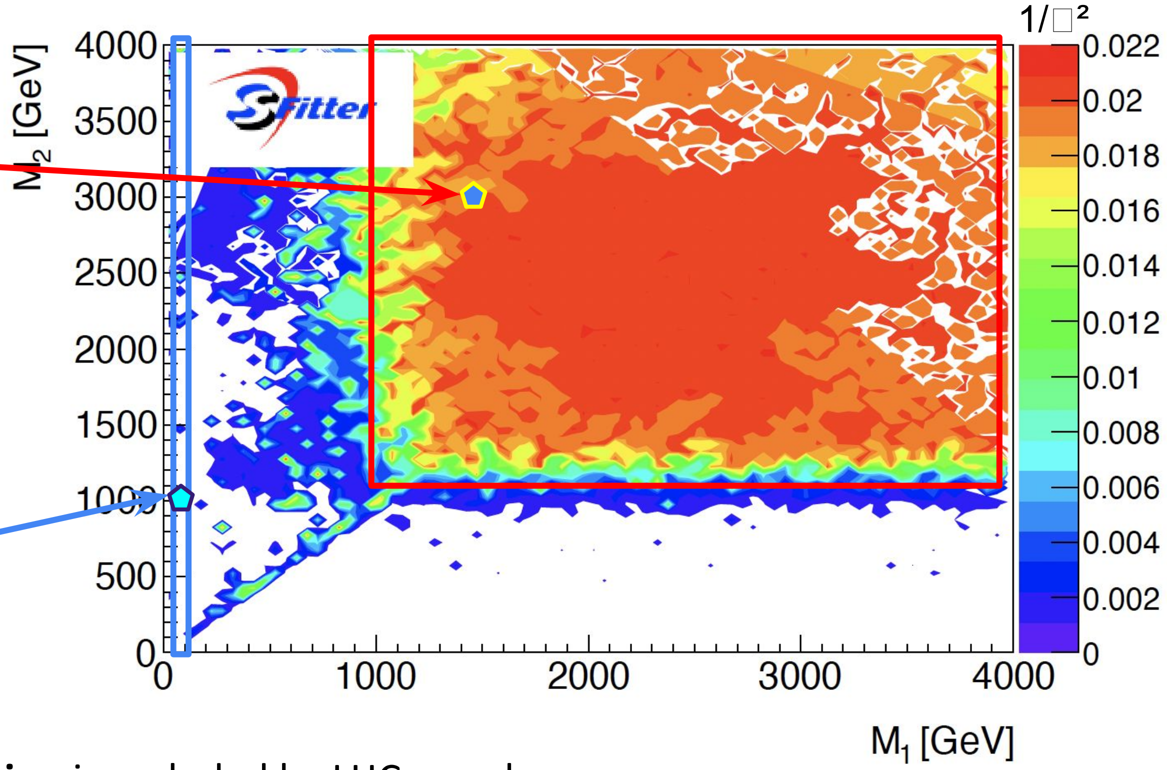
neutralino $\tilde{\chi}_1^0$

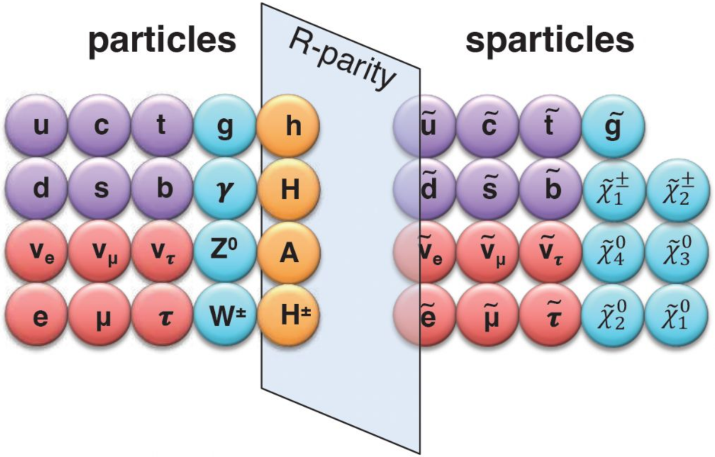
	A funnel	*HEP Spectrum*	*lle inflaton*
		$\tilde{g}$ 1464 $\tilde{\chi}_1^0$ 359	ϕ_0 $9.64\ 10^{14}$ $m_\phi(\phi_0)$ 6590 $A_6(\phi_0)$ 20857 $\lambda_6(\phi_0)$ 0.0347
satisfies	$\Omega_{\rm cdm}h^2$, *Xenon1T*	m_h	A_s, n_s, $n_{s,\rm run}$, r
excluded		*LHC (gluinos)*	

	Higgs funnel	*HEP Spectrum*	*udd inflaton*
		$\tilde{g}$ 2422 $\tilde{\chi}_1^0$ 60.0	ϕ_0 $1.41\ 10^{15}$ $m_\phi(\phi_0)$ 13933 $A_6(\phi_0)$ 44204 $\lambda_6(\phi_0)$ 0.0159
satisfies	$\Omega_{\rm cdm}h^2$, *Xenon1T*	m_h, *LHC, …*	A_s, n_s, $n_{s,\rm run}$, r

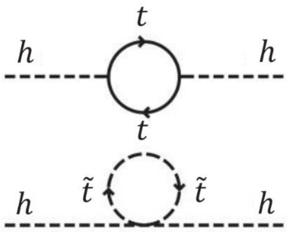


- This specific **A-funnel configuration** is excluded by LHC searches
  => constraints on inflation from HEP

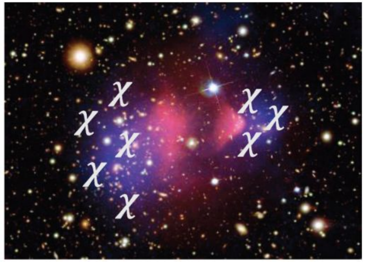
SUSY Overview



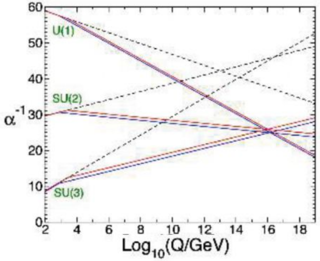
hierarchy problem



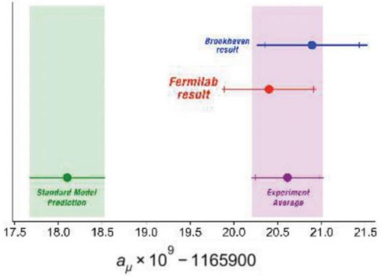
dark matter



gauge coupling unification



g-2



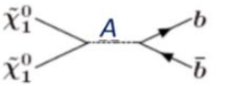
- SUSY can explain disparate phenomena and SM theoretical shortcomings

Global fit: first results

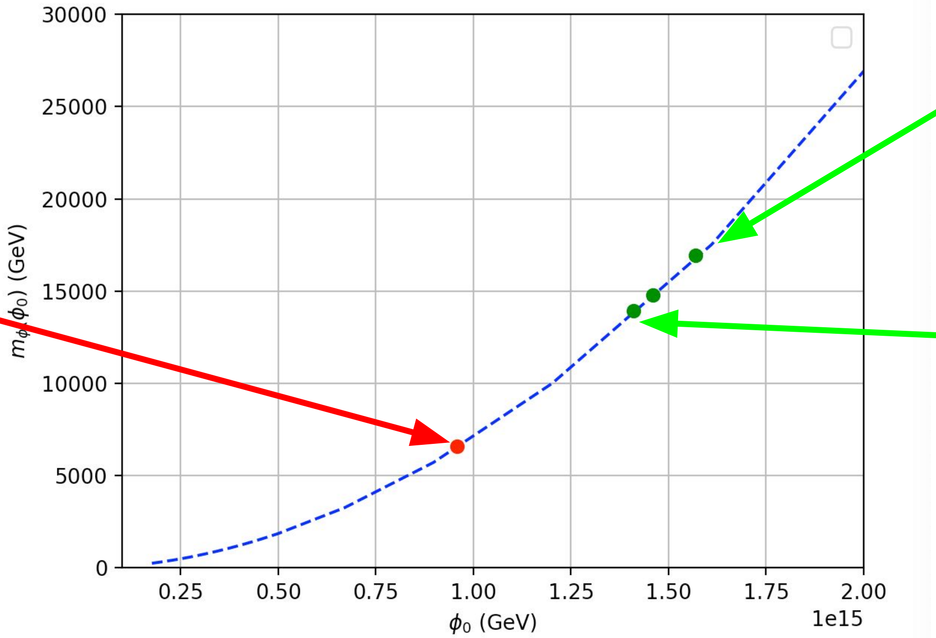
We have identified some points (motivated by [*]) compatible with m(Higgs), $\Omega_{\mathrm{cdm}} h^2$ and (As,ns) for various dark matter annihilation channels

[*] *S. Henrot-Versillé, et al. Phys. Rev. D 89, 055017 (2014),* ***1309.6958***

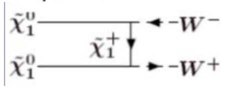
ϕ_0	$9.58\ 10^{14}$	GeV
$m_\phi(\phi_0)$	6601.5	GeV



EXCLUDED by LHC searches



ϕ_0	$1.57\ 10^{15}$	GeV
$m_\phi(\phi_0)$	16954	GeV



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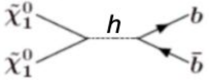
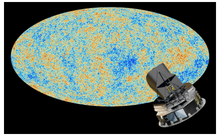


=> LHC searches give constraints on inflation

=> (As,ns) restrict the MSSM parameter space



=> First steps toward a full exploration of the parameter space including all constraints

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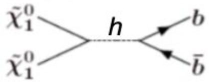
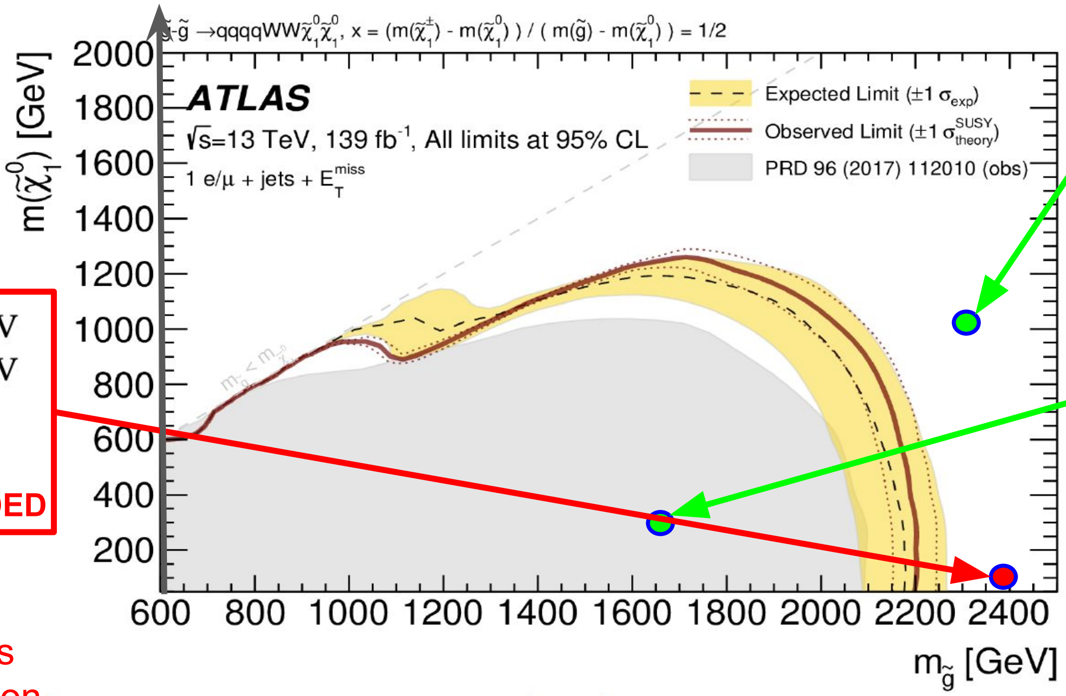


EXCLUDED



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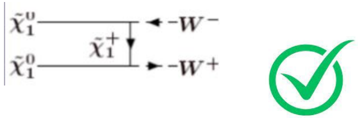

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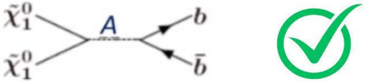

=> here HEP measurements gives constraints on inflation

=> here cosmology restricts the MSSM parameter space

=> First steps toward a full exploration of the parameter space including all constraints

How to get phi*?...

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$$\Delta N_* \overset{\text{SRLO}}{\simeq} \int_{\phi_{\text{end}}}^{\phi_*} \frac{V(\phi)}{V_\phi(\phi)} \mathrm{d}\phi$$

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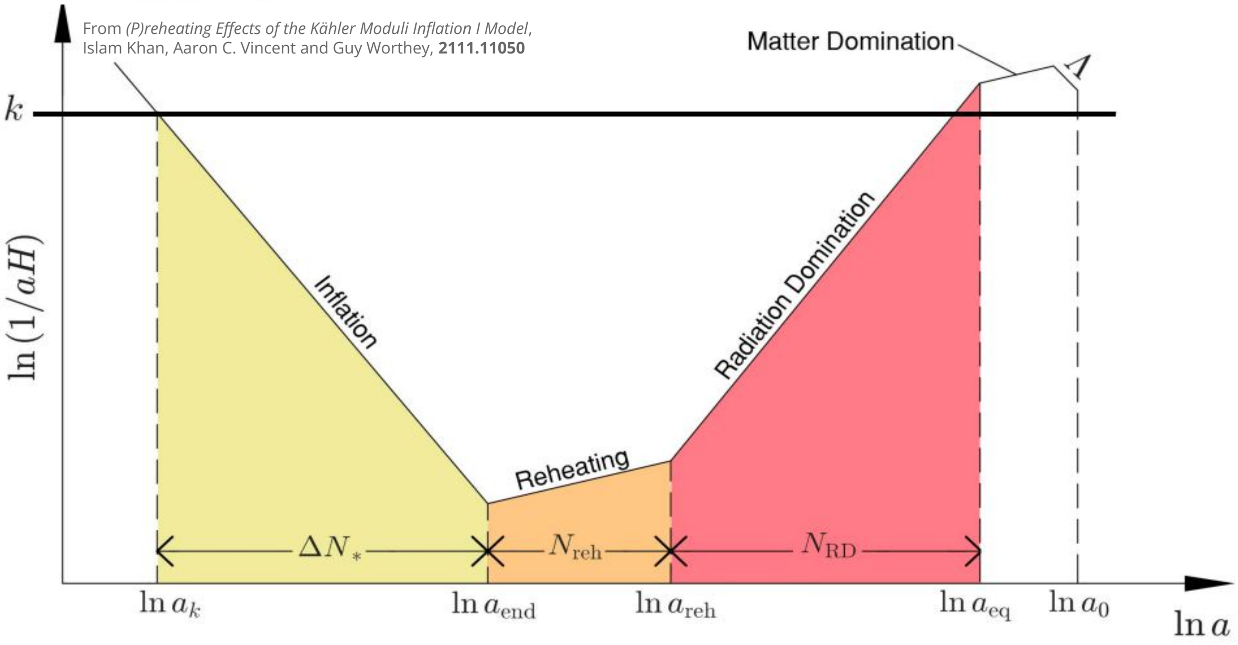
From *(P)reheating Effects of the Kähler Moduli Inflation I Model*, Islam Khan, Aaron C. Vincent and Guy Worthey, **2111.11050**



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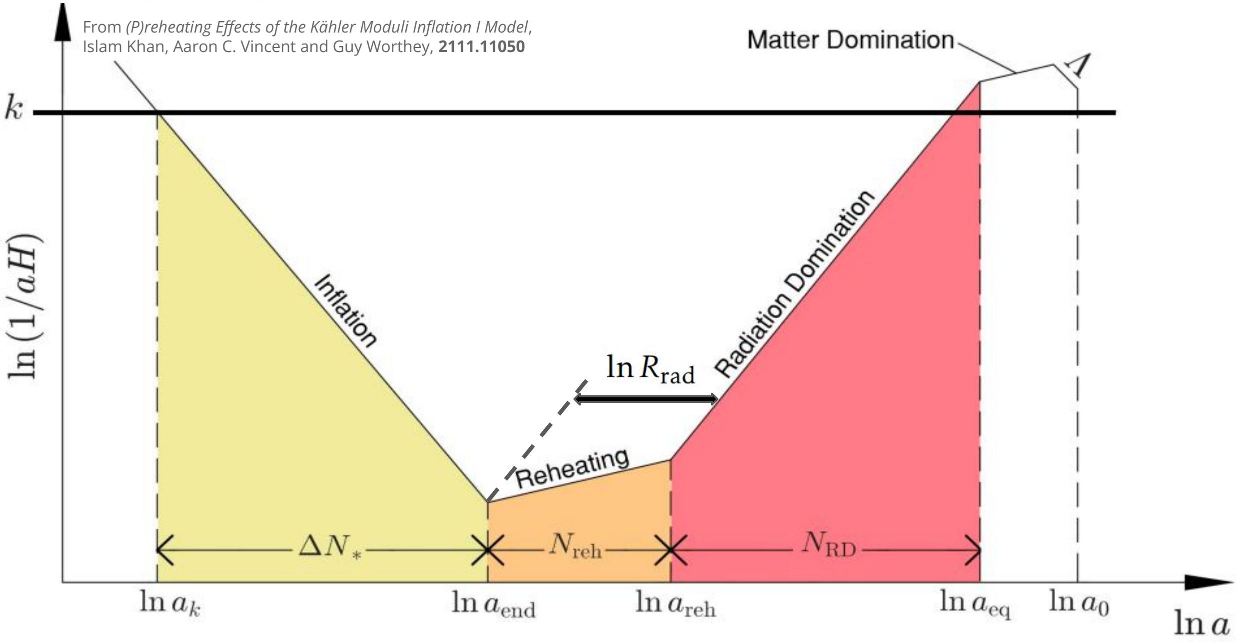

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$$\Delta N_* \overset{\text{SRLO}}{\simeq} \ln R_{\text{rad}} - \ln\left(\frac{k_*}{a_0 \tilde{\rho}_\gamma^{1/4}}\right) - \frac{1}{4}\ln\left[\frac{9V_{\text{end}}}{\varepsilon_{1*}(3-\varepsilon_{1\text{end}})V_*}\right] + \frac{1}{4}\ln(8\pi^2 A_{\text{S}})$$

Important assumption

Slow-roll has to occur in a first place!

It is usually the case when there is a wide region of the potential where ε_1 < 1, independently on the initial conditions $(\phi, \dot{\phi})$ because slow-roll acts as an attractor.

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$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0 \xrightarrow{\text{attracted to}} \dot{\phi}^{\text{SRLO}} \simeq -M_{\text{Pl}} \frac{V_\phi}{\sqrt{3V}}$$

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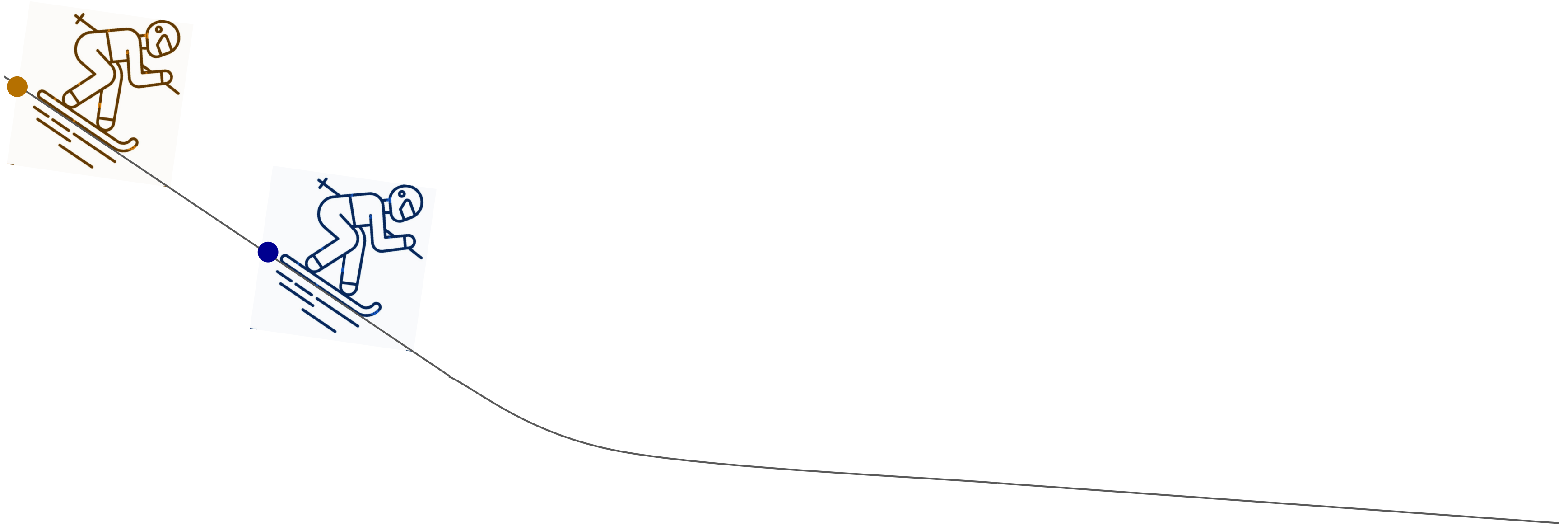

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Now, we have all the ingredients to predict the cosmological observables of any potential:

$$\phi \to V(\phi, p),\ \ln R_{\rm rad} \implies \phi_* \implies \{\varepsilon_i *\} \implies \{A_{\rm S},\ n_{\rm S},\ n_{\rm S,run},\ r,\ n_{\rm T},\ n_{\rm T,run},\ N_{\rm e-folds}\ ...\}$$

Conversely, given an observation, we can deduce the allowed potentials as done

J. Martin, C. Ringeval, and V. Vennin, Phys. Dark Univ. 5-6, 75 (2014), ***1303.3787***

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Examples of potentials

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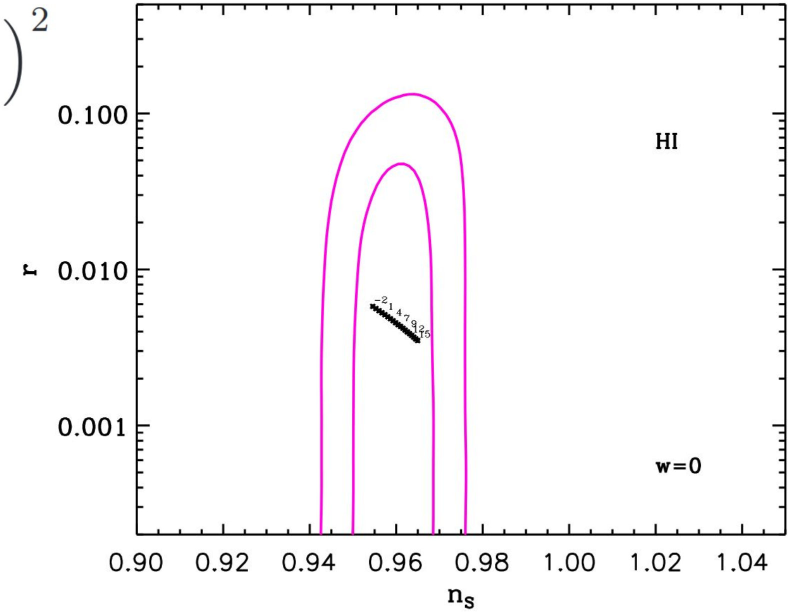

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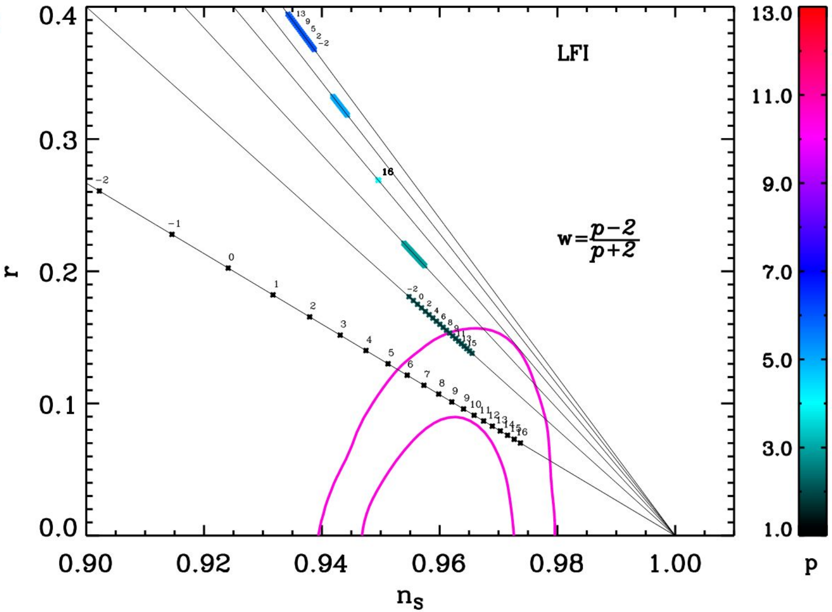

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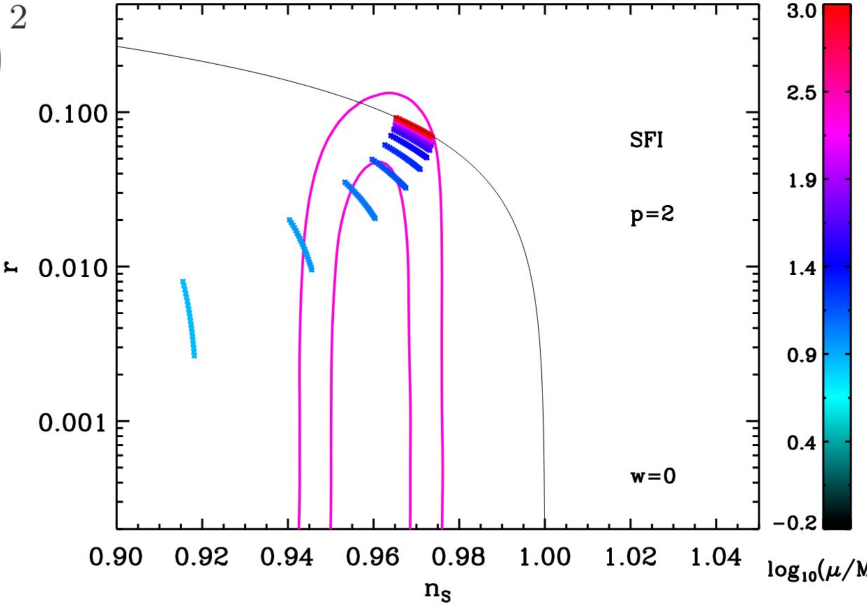
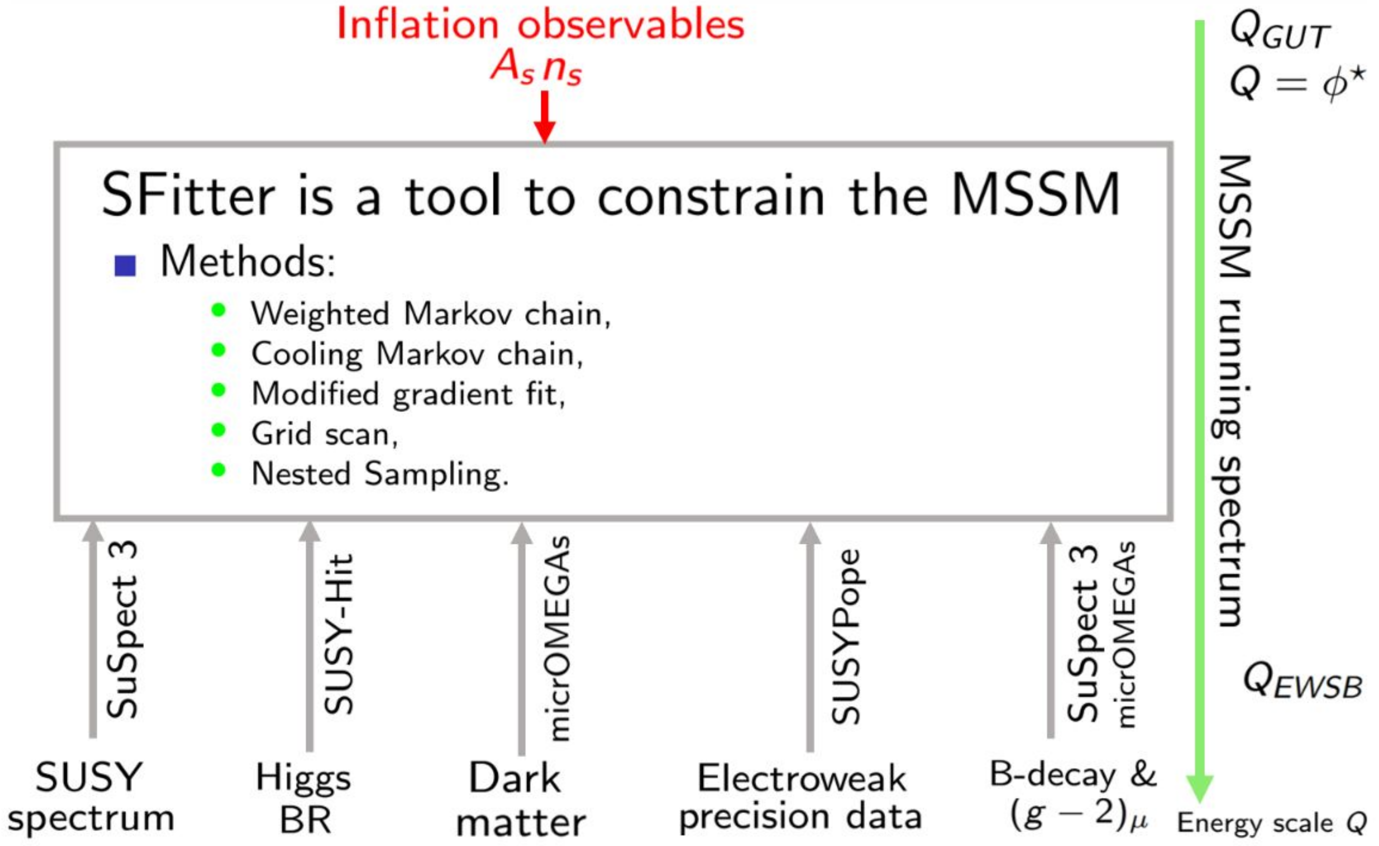
- Small field inflation: $$V(\phi) = M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$



Join the constraint from CMB to the other observables



Conclusions

- is **finely-tuned flat** (which propagates to the parameters)
- **does not attract** all the pre-inflation trajectories

needs to account for radiative **RGE corrections** at the level **of the potential**

MSSM-inflation

is able to generate a **CMB-like power spectrum**

- **ns** improvements => propagated **experimental errors** will fall **below** the **systematic errors**
- **ns_run** improvement => will begin to constrain the **inflation scale of the model**
- **r** detection would **exclude** the model

will be more constrained with **next LHC runs**

ns, As and **HEP observables constrain jointly** the same region of the parameter space.

bridge between two worlds,
HEP and **inflation**

This work:
soon on arXiv

Future:

- More systematic **exploration** of the **parameter space**
- Inclusion of the **reheating duration** computation
- Beyond MSSM : **eNMSSM**, **Multi-field** inflation…