Zero Casimir-Force in Axion Electrodynamics

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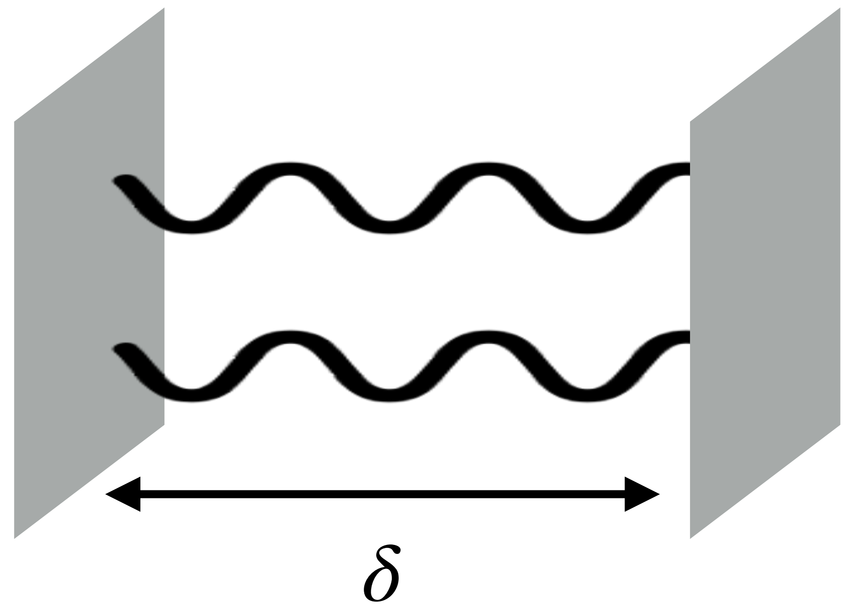
Based on **2302.14676** with M. Hazumi, H. Iizuka, K. Mukaida, K. Nakayama

Casimir force

- Casimir force: force between bodies from vacuum fluctuation. [Casimir 48]



$$\text{Dispersion relation : } \omega_n^2 = k_x^2 + k_y^2 + \left(\frac{\pi n}{\delta}\right)^2 \text{ with } n \in \mathbb{Z}.$$

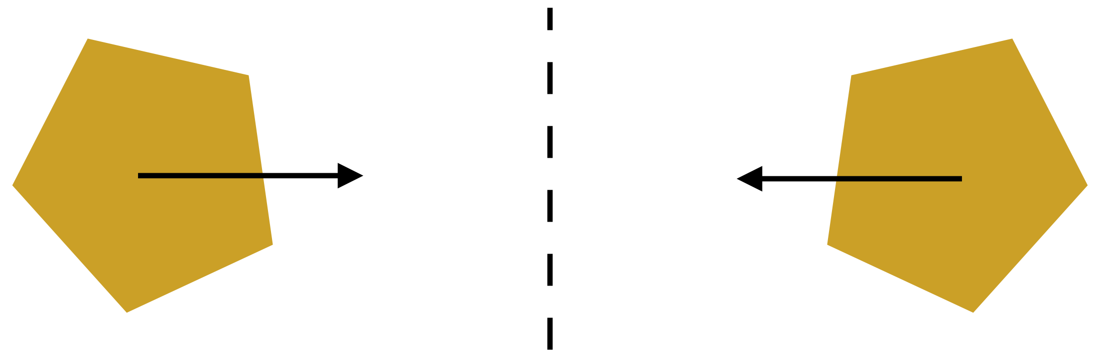
$$F = -\frac{\partial E}{\partial \delta} = -\frac{\partial}{\partial \delta}\int \frac{dk_x dk_y}{(2\pi)^2}\sum_n \frac{\omega_n}{2} = -\frac{\pi^2}{240\delta^4}.$$

(i.e. $E \propto -\delta^{-3}$)

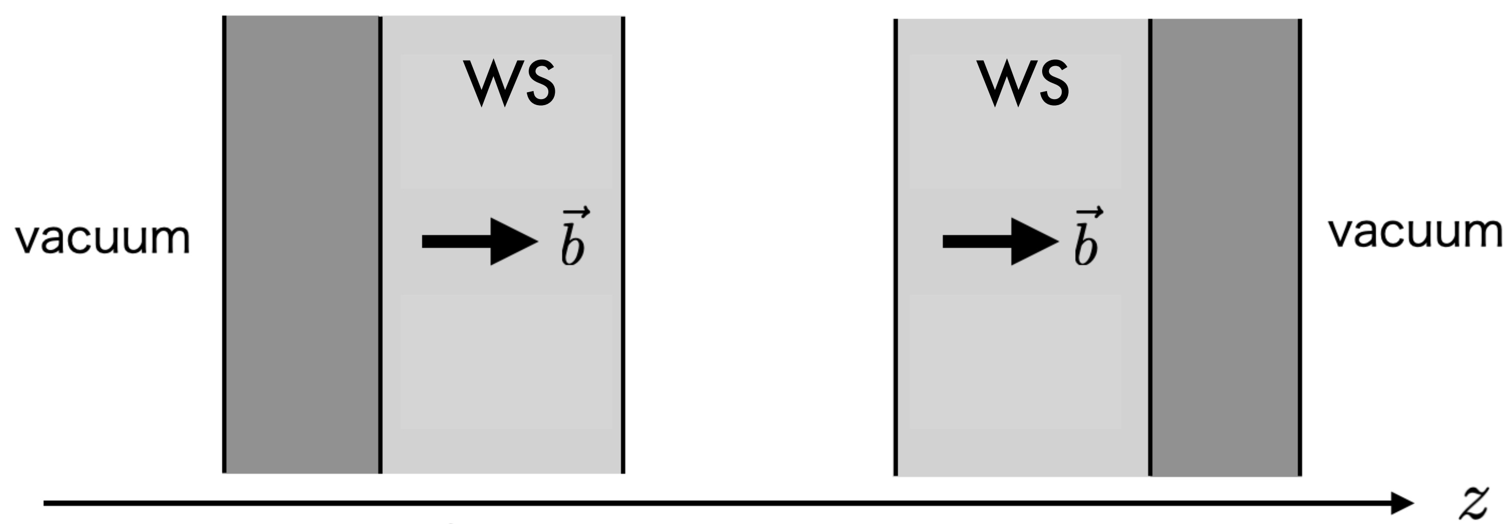
- Always attractive independent of δ in this simplest setup.

Repulsive Casimir force

- Two bodies related by reflection always attract: [Kenneth, Klich 06]



- Weyl semimetal has a "direction" = axion gradient.



[Wilson+ 15; YE, Hazumi, Iizuka, Mukaida, Nakayama 23]

- Stable point exits between repulsive and attractive region.

➡ Potentially suppress "background" for new force search at $\mathcal{O}(\mu\mathrm{m})$.

Outline

1. Casimir force with Weyl semimetal

2. New force search

3. Summary

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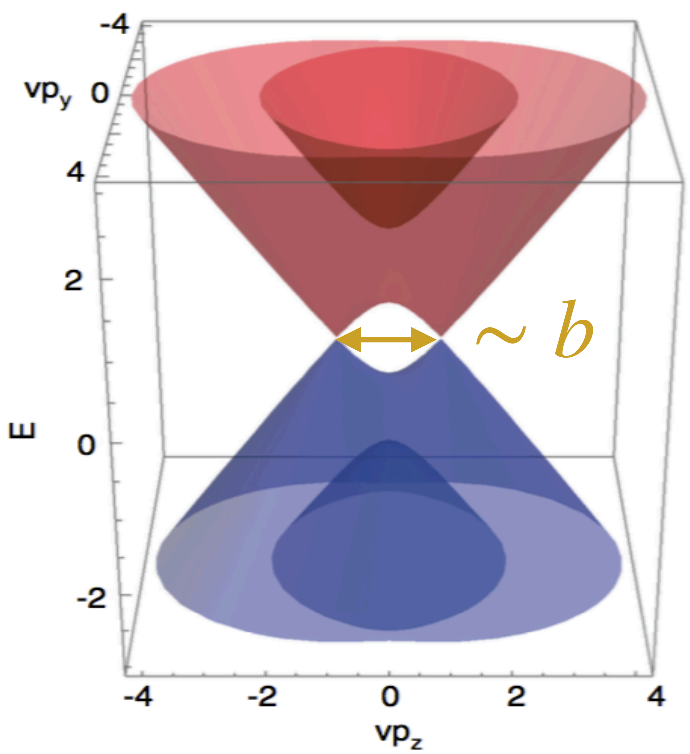
Weyl semimetal

- Weyl semimetal (WS) effectively provides $\theta = \overrightarrow{b} \cdot \overrightarrow{x}$.



From [Amitage+ 17]

Each node has opposite chirality

$$\mathscr{L}_{\text{eff}} \sim b_i \bar{\psi} \gamma^i \gamma_5 \psi$$

$$\mathscr{L}_{\text{eff}} = -\frac{1}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} \text{ with } \theta = \overrightarrow{b} \cdot \overrightarrow{x}.$$

- Described by axion electrodynamics with spatial gradient:

$$\begin{cases} \overrightarrow{\nabla} \cdot \overrightarrow{E} + \overrightarrow{b} \cdot \overrightarrow{B} = 0, \quad \dot{E} - \overrightarrow{\nabla} \times \overrightarrow{B} + \overrightarrow{b} \times \overrightarrow{E} = 0, \\ \dot{\overrightarrow{B}} + \overrightarrow{\nabla} \times \overrightarrow{E} = 0, \quad \overrightarrow{\nabla} \cdot \overrightarrow{B} = 0, \end{cases}$$

Dispersion relation

- Plane wave $\vec{E} = \vec{\epsilon}\, e^{-i\omega t + i\vec{k}\cdot\vec{x}}, \quad \vec{B} = \frac{\vec{k}\times\vec{\epsilon}}{\omega} e^{-i\omega t + i\vec{k}\cdot\vec{x}}.$

$$\Rightarrow \quad 0 = \begin{pmatrix} \omega^2 - k_z^2 & -i\omega b & k_x k_z \\ i\omega b & \omega^2 - k^2 & 0 \\ k_x k_z & 0 & \omega^2 - k_x^2 \end{pmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{pmatrix} \quad \text{with } \vec{b} = (0,0,b),\ \vec{k} = (k_x, 0, k_z)\,.$$

- Dispersion relation modified by b.

$$\omega^2 = k_T^2 + \left(\sqrt{k_z^2 + \frac{b^2}{4}} \pm \frac{b}{2}\right)^2 \quad \text{or} \quad k_z^2 = (k_z^\pm)^2 = \kappa_z(\kappa_z \pm b), \quad \kappa_z = \sqrt{\omega^2 - k_T^2}\,.$$

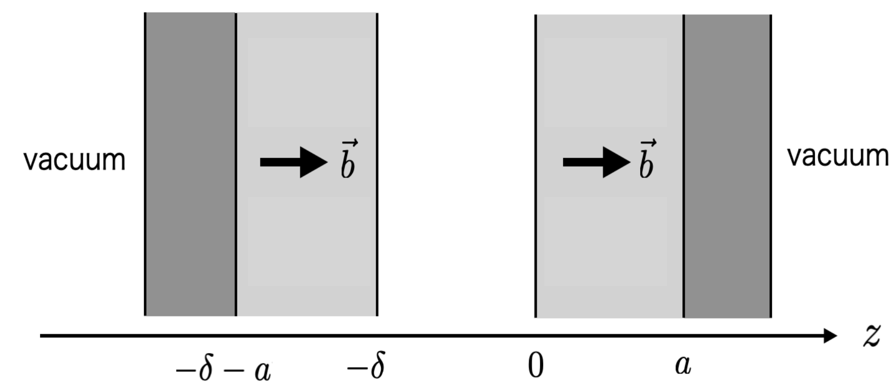
- Polarization vector also modified by b.

$$\vec{\epsilon}^{\,(\lambda)}_{\pm} = \frac{1}{\sqrt{2}N}\left(\lambda\kappa_z^2,\ i\lambda\omega\kappa_z,\ -k_x k_z^\pm\right) \quad \text{where } \lambda = +/- \text{ for right/left movers.}$$

(Here $\pm$ corresponds to the solutions with $k_z^\pm$.)

Lifshitz formula

- Consider WS between metals and vacuum.



Spectrum given by solutions of

$$1 - e^{2i\kappa_z\delta} r_\lambda^2(\omega) = 0 \quad \text{where} \quad r_\lambda = \frac{k_z^\lambda - \kappa_z + (k_z^\lambda + \kappa_z)e^{2iak_z^\lambda}}{k_z^\lambda + \kappa_z + (k_z^\lambda - \kappa_z)e^{2iak_z^\lambda}} \,.$$

- Casimir free energy per unit area given by

$$\mathscr{F} = \int \frac{dk_\parallel^2}{(2\pi)^2} \sum_\lambda \sum_{n\geq 0}{}' \left[\frac{\omega_{\lambda,n}}{2} + T \log\left(1 - e^{-\omega_{\lambda,n}/T}\right) \right] = T \int \frac{d^2k_\parallel}{(2\pi)^2} \sum_\lambda \sum_{n,l\geq 0}{}' \log\left(\xi_l^2 + \omega_{\lambda,n}^2\right) \quad \text{where } \xi_l = 2\pi T l.$$

- By complex analysis this is given by

$$\mathscr{F} = T \int \frac{d^2k_\parallel}{(2\pi)^2} \sum_\lambda \sum_{l\geq 0}{}' \log\left[1 - r_\lambda^2(i\xi_l)e^{-2q_l\delta}\right] \quad \text{where } q_l = \sqrt{\xi_l^2 + k_\parallel^2} \,.$$

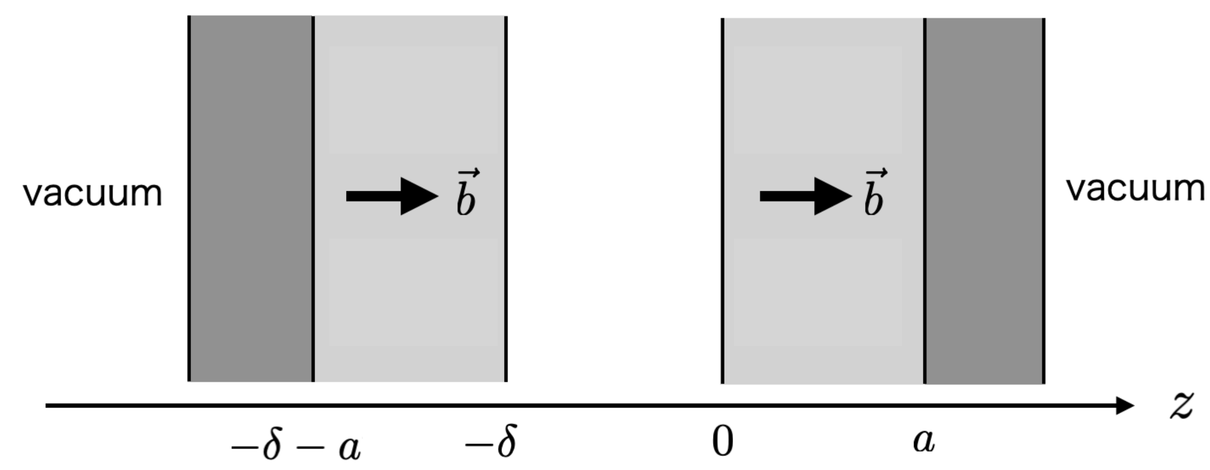
(Picking up poles of $\frac{d}{d\omega} \log\left(1 - r_\lambda^2(\omega)e^{2i\kappa_z\delta}\right)$ gives the original summation.)

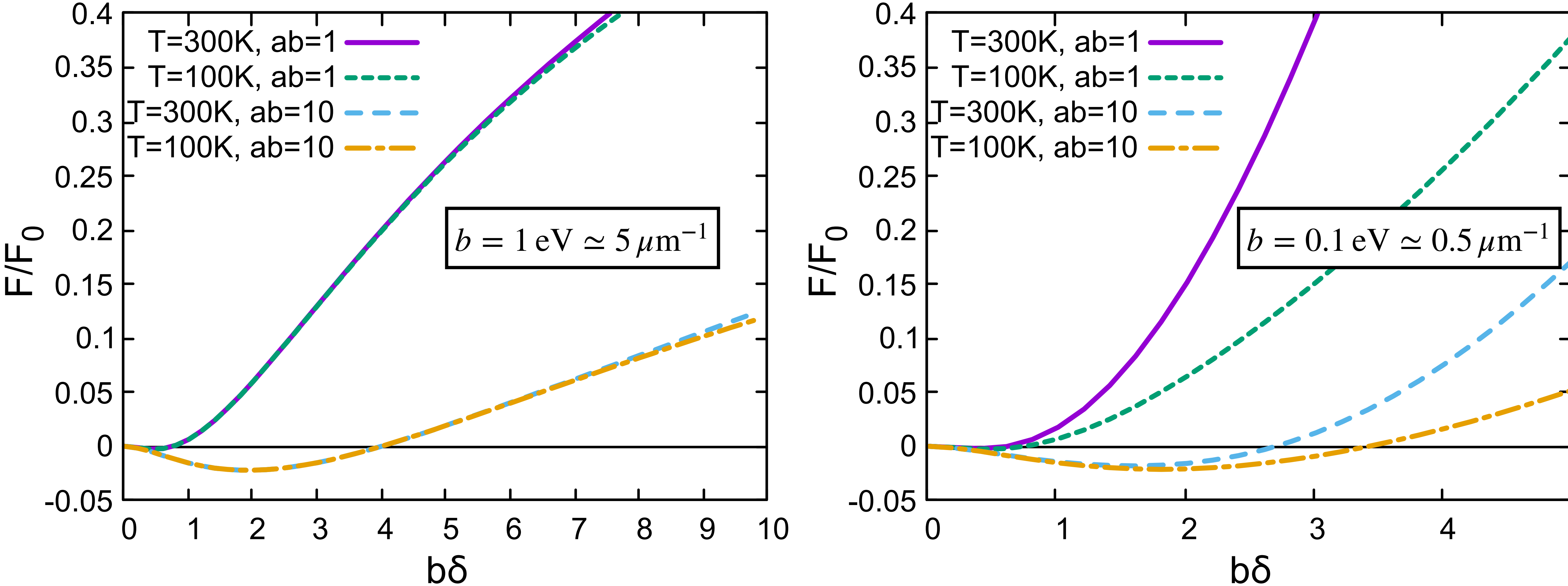
Casimir force with Weyl semimetal

- Casimir force with WS: repulsive to attractive by increasing δ.

[Wilson+ 15; YE, Hazumi, Iizuka, Mukaida, Nakayama 23]

Zero force point: stable.





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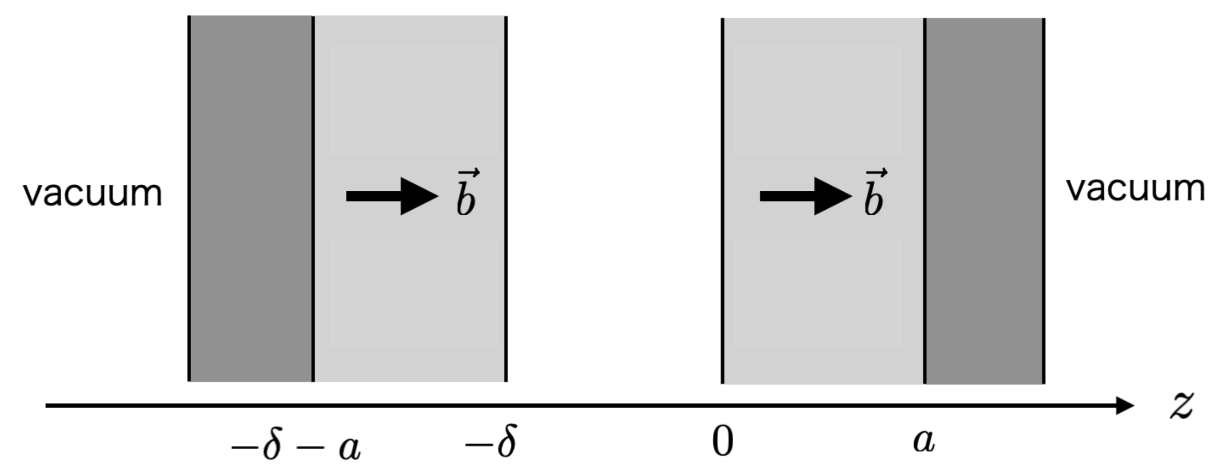

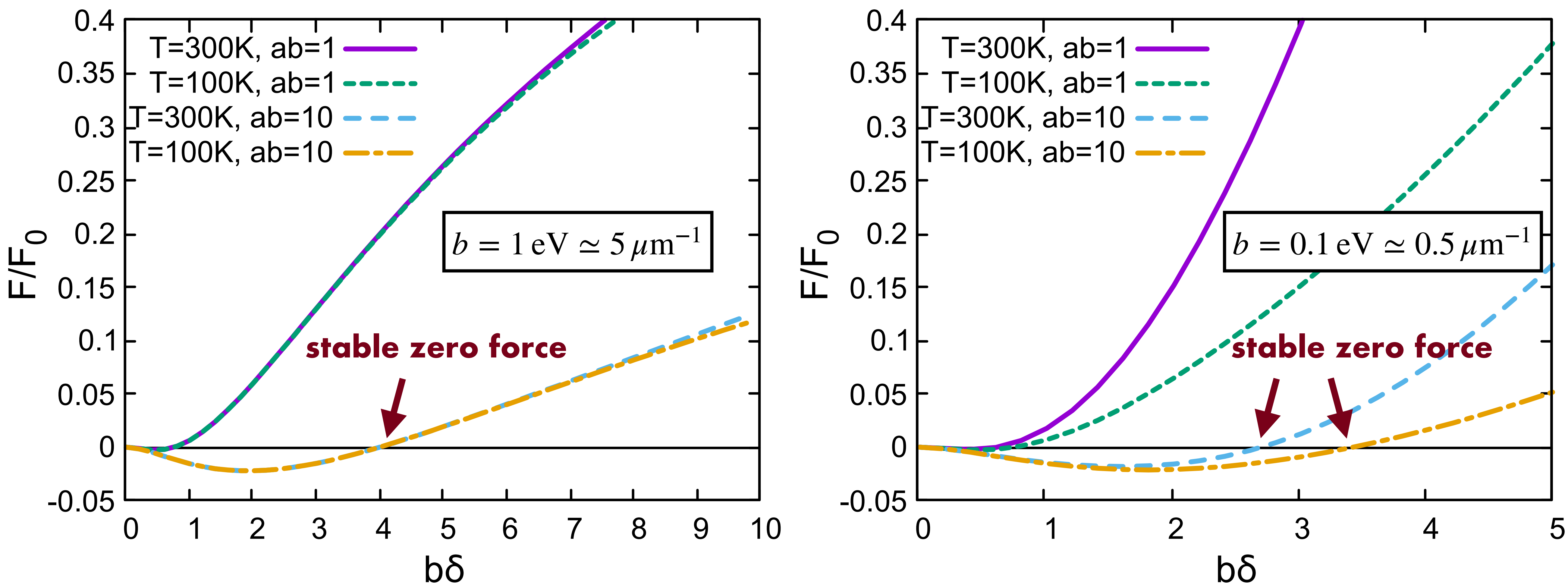

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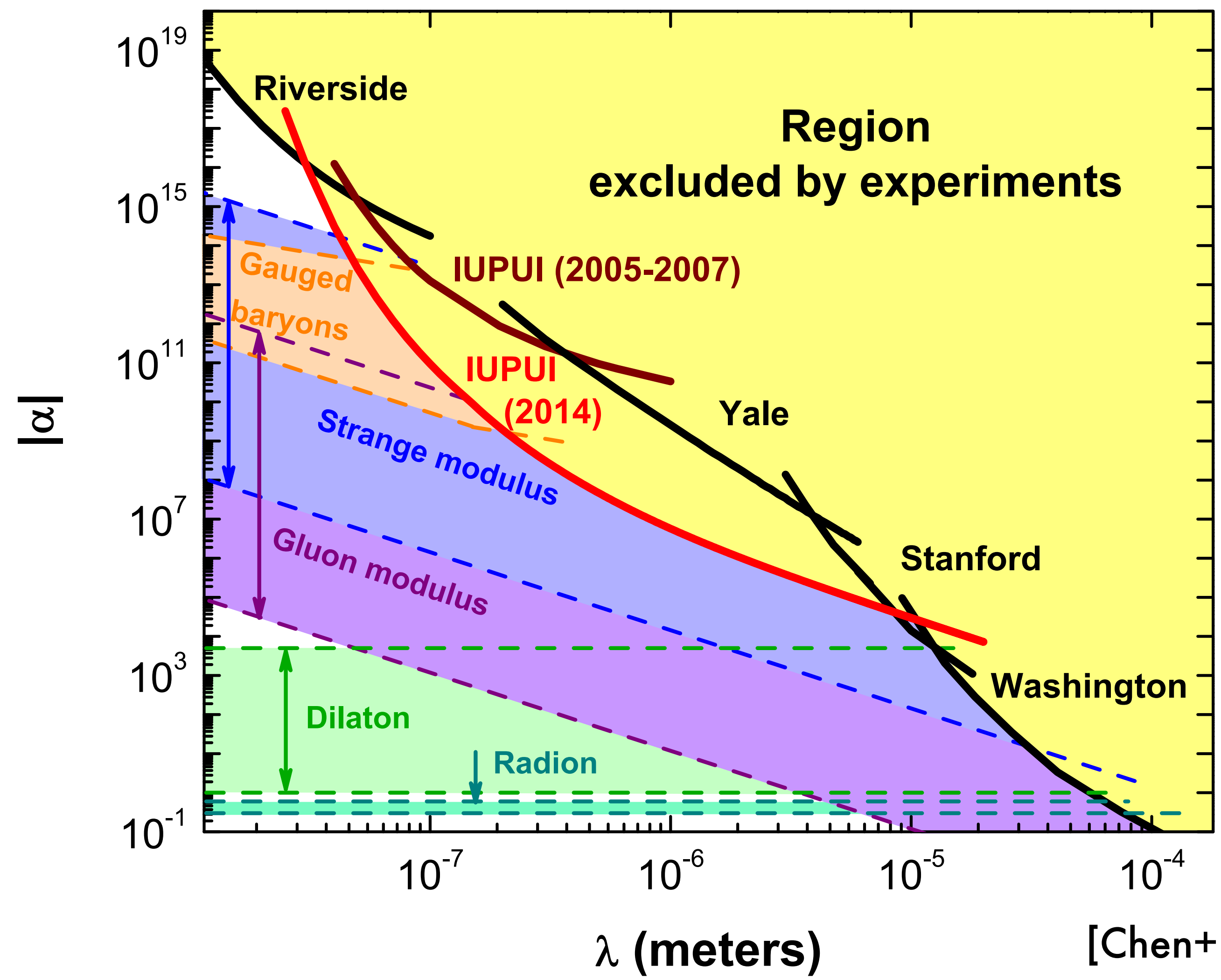
New force search

- New long-range force search done for different scales, including $\mathcal{O}(\mu\mathrm{m})$.

$$V_{\mathrm{new}} \equiv \alpha \times \frac{G m_1 m_2}{r} e^{-r/\lambda}.$$



[Chen+ 14]

- Casimir force: "background" for new force search at $\mathcal{O}(\mu\mathrm{m}) \sim \mathcal{O}(\mathrm{eV})$.

Zero Casimir force offers a new possibility.

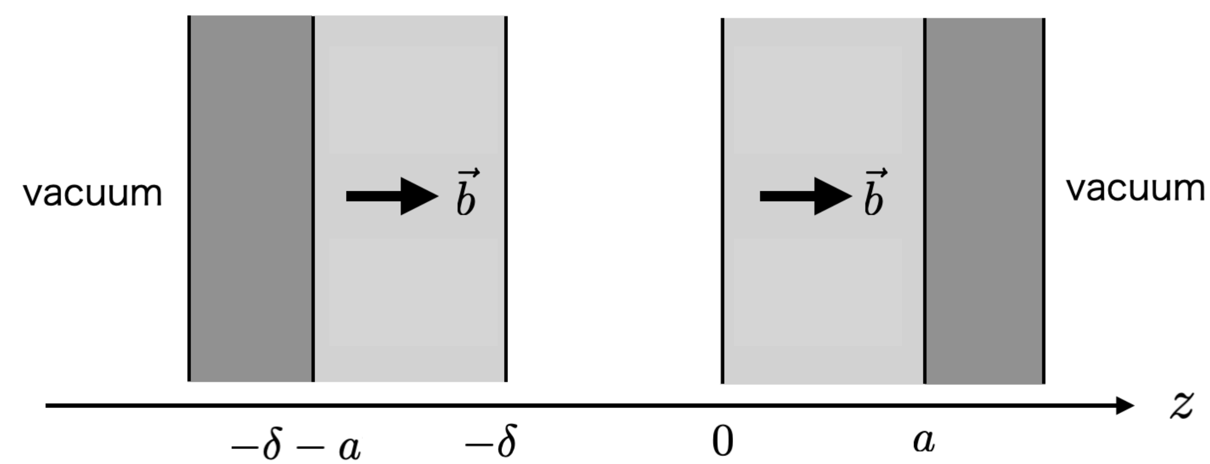
[YE, Hazumi, Iizuka, Mukaida, Nakayama 23]

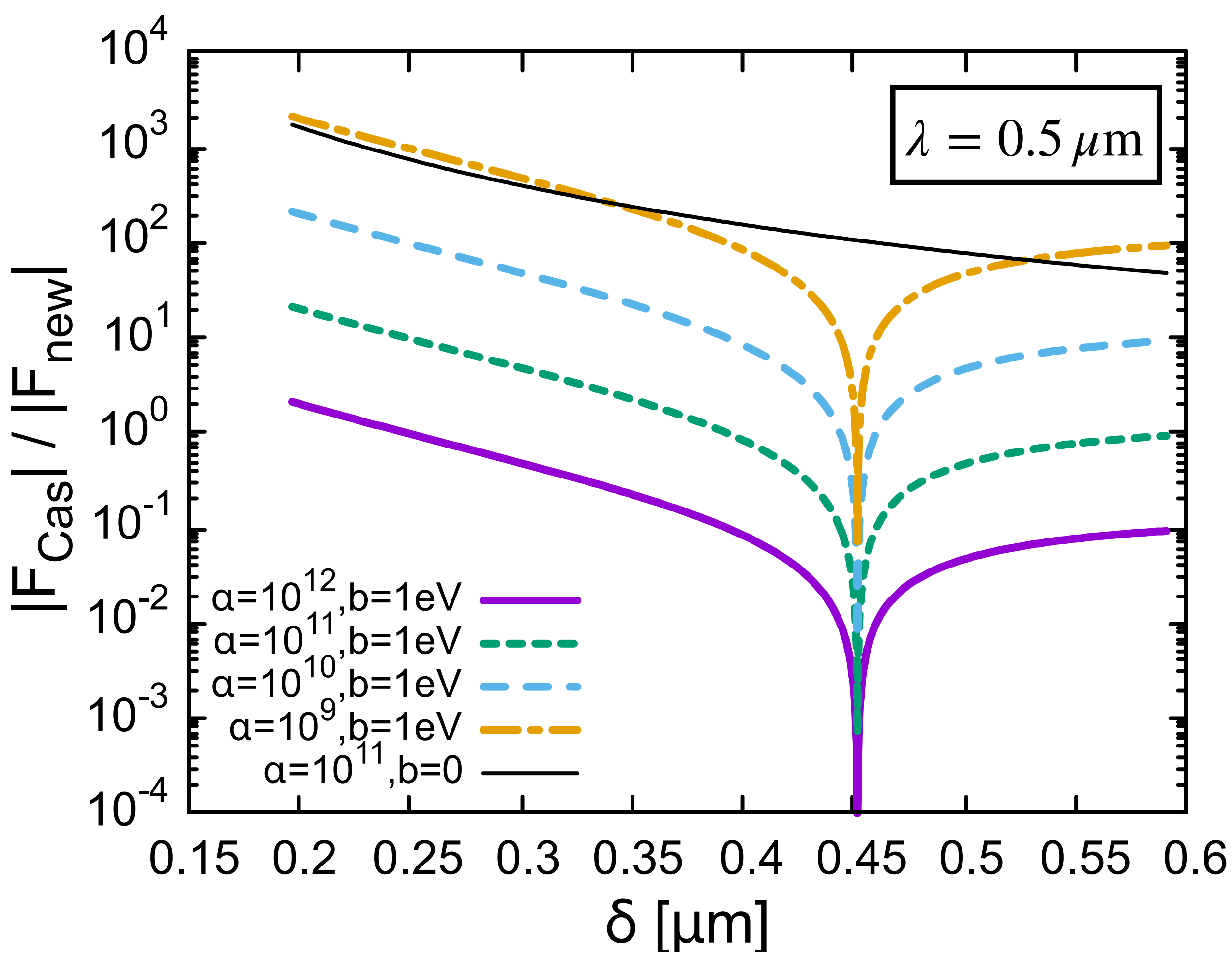
Stable point for new force search

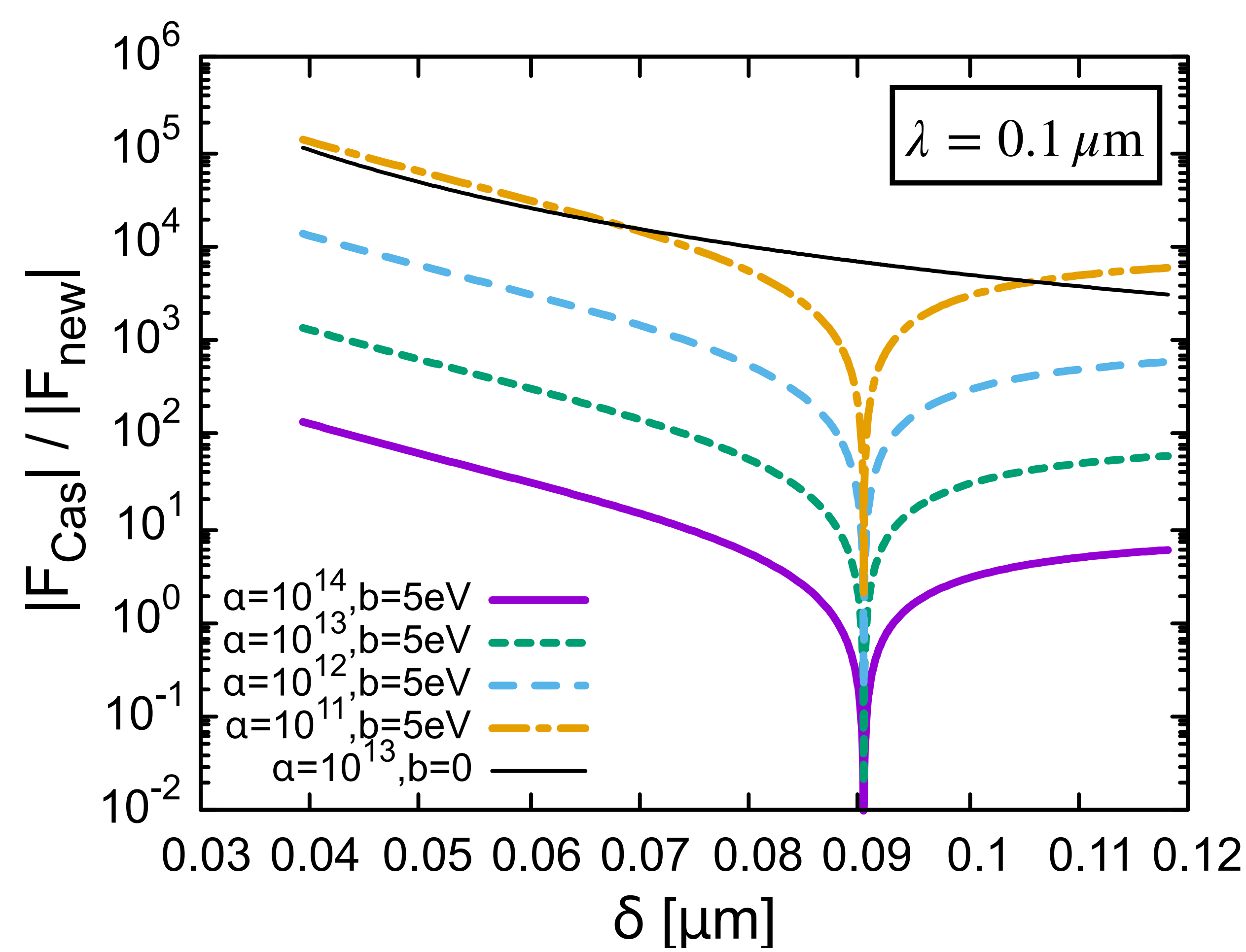
- Casimir force can be suppressed with the Weyl semimetals.

[YE, Hazumi, Iizuka, Mukaida, Nakayama 23]

$$V_{\rm new} = \alpha \times \frac{G m_1 m_2}{r} e^{-r/\lambda}.$$







$a = 2/b$, $T = 300\,{\rm K}$, $\rho_{\rm Au} = 19.3\,{\rm g/cm^3}$, $d_{\rm Au} = 0.2\,\mu{\rm m}$, $\rho_{\rm WS} = 10\,{\rm g/cm^3}$

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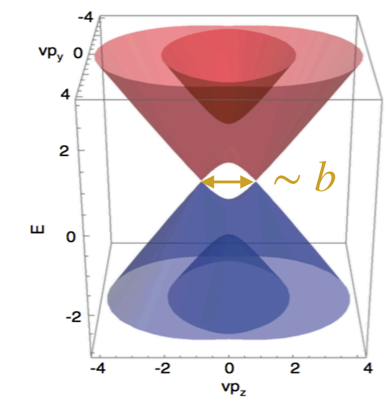
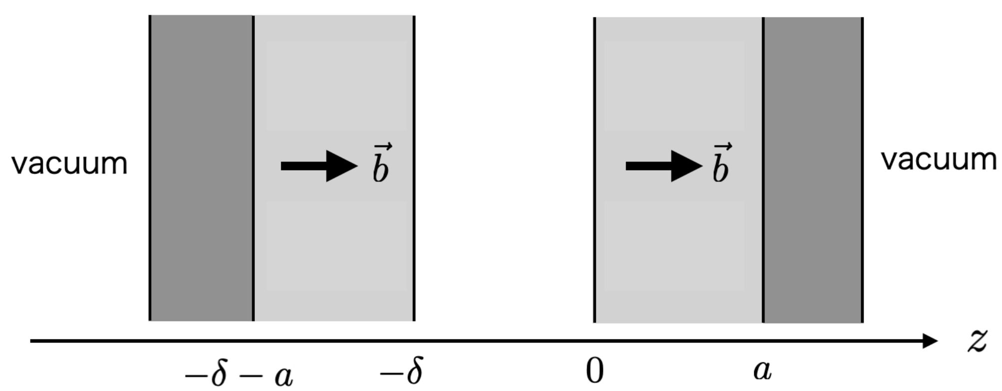
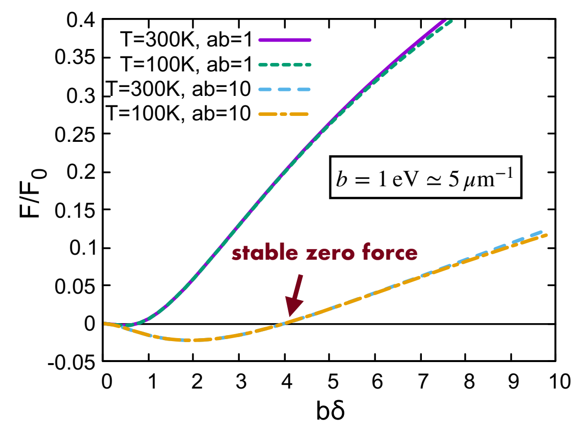
Summary

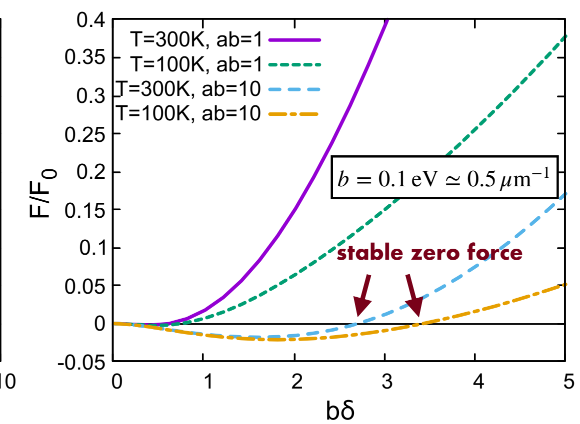
- Weyl semimetal provides axion electrodynamics with $\theta = \overrightarrow{b} \cdot \overrightarrow{x}$.



- Casimir force has stable zero point with Weyl semimetal.







- Potentially offering a new way of long-range force search at $\mathcal{O}(\mu\mathrm{m})$ (and more?).