QCD

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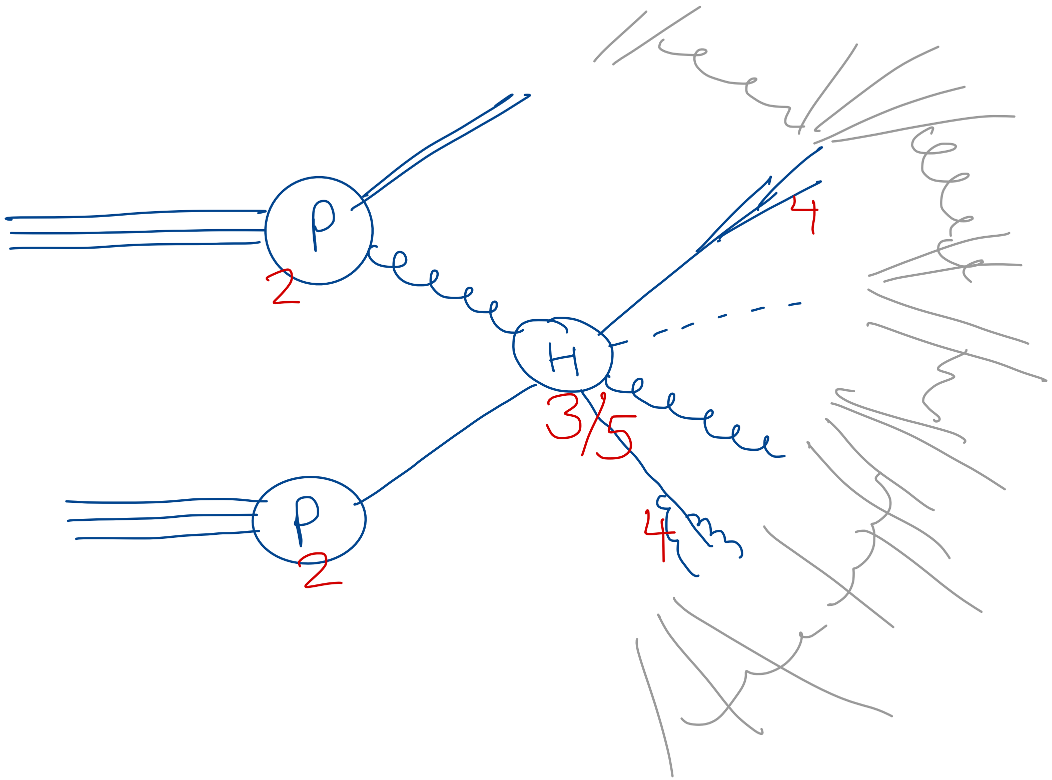
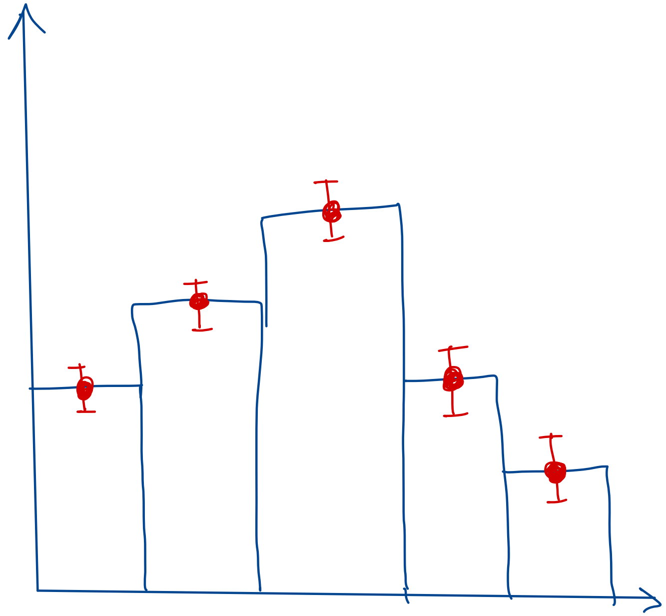
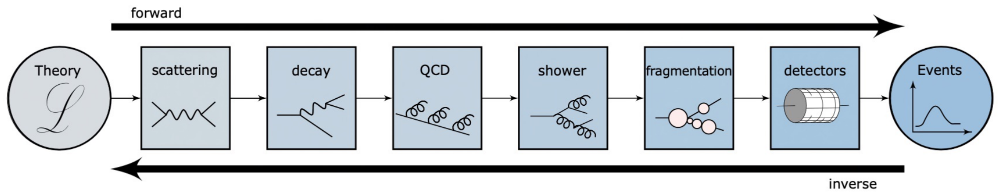




Topics in this course:

1. Intro to QCD
2. Parton Distribution Functions
3. Scattering Amplitudes
4. Parton Showers
5. High Energy Limit



Books :

Buckley, White + White "Practical Collider Physics" IoP

Ellis, Stirling + Webber "QCD and Collider Physics" CUP

Campbell, Houston + Krauss "The Black Book of..." CUP

Edinburgh (Met Office)

Today

16° 11°

Sunrise: 04:51 **Sunset:** 21:46

Cloudy changing to light showers in the afternoon.

M UV L Pollution L Pollen

Tue 18 Jul	Wed 19 Jul	Thu 20 Jul	Fri 21 Jul
14° 11°	18° 11°	16° 10°	16° 11°

FIFA's position on extreme heat

If there is a **WGBT of more than 32° Centigrade (89.6 degrees Fahrenheit)** cooling breaks are mandatory in both halves of a match, around the 30th minute and 75th minute; the decision on whether to suspend or cancel the match is at the discretion of competition organisers.

1 QCD Intro

1.1 Definition

QCD is a non-Abelian SU(3) gauge theory consisting of

- Spin-½ quarks : 6 families (d, u, s, c, b, t)
  Each in 3 'colours'

fermion number

- Spin-1 gluons : $8 = N_c^2 - 1$ massless

Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{a\,\mu\nu} F^{a}_{\mu\nu} + \bar{\psi}_i \left(i \not{D}_{ij} - m\,\delta_{ij} \right) \underbrace{\psi_j}$$

quark field
fund^{l} rep^{n}

where $D^{\mu}_{ij} = \partial^{\mu} \delta_{ij} + i g_s T^{a}_{ij} A^{a\mu}$

$$F^{a}_{\mu\nu} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g_s f^{abc} \underbrace{A^{b}_{\mu} A^{c}_{\nu}}$$

gluon field
adjoint rep^{n}

μ, ν etc Lorentz Indices
a, b adjoint colour
i, j fund. colour

Repeated indices summed over.

Colour matrices T^a_{ij} satisfy $[T^a, T^b] = i f^{abc} T^c$ cf $[\sigma_i, \sigma_j] = i\varepsilon_{ijk}\sigma_k$ for $SU(2)$

"Structure constant"

completely anti-symm.

Useful Identities

$\mathrm{Tr}(T^a) = 0$

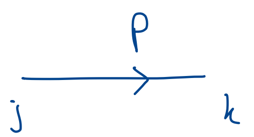
$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$

$T^a_{ij} T^a_{jk} = C_F \delta_{ik}$, $C_F = 4/3$

$f^{abc} f^{abd} = C_A \delta^{ad}$, $C_A = N_c = 3$

We will need Feynman rules (derived from $\mathcal{L}$):

Propagators

q: 

$$\delta_{jk} \frac{i(\not{p}+m)}{p^2 - m^2 + i\varepsilon}$$ = electron propagator + δ in colour space

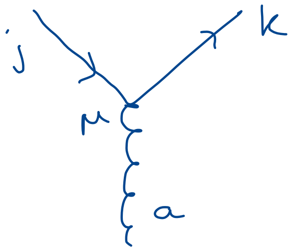
g: 

$$\delta^{ab} \frac{-i g^{\mu\nu}}{p^2 + i\varepsilon}$$

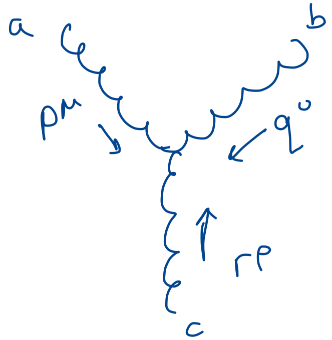
* This is Feynman gauge and is all we will need.

In general, gives unphysical degrees of freedom which are cancelled by "ghosts".

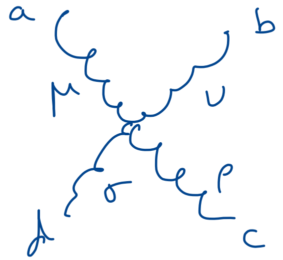
Vertices:



$ig_s \gamma^\mu T^a_{kj}$ (from $ig_s T^a_{ij} A^{a\mu}$ term of $\not{D}$).



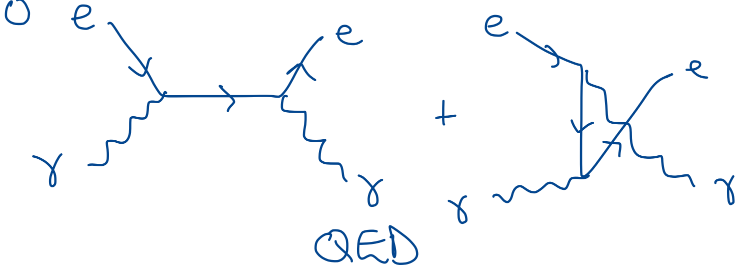
$g_s f^{abc} \left(g^{\mu\nu}(p-q)^\rho + g^{\nu\rho}(q-r)^\mu + g^{\rho\mu}(r-p)^\nu \right)$



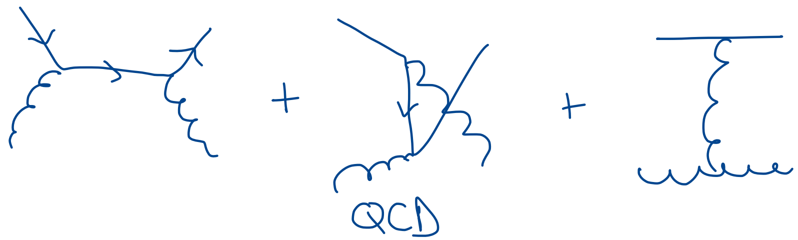
$-ig_s^2 \left(f^{abe} f^{cde} \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho} \right) + \text{perms.} \right)$

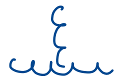
Example Compton scattering ($e\gamma \to e\gamma$) in QED has an extra diagram

so



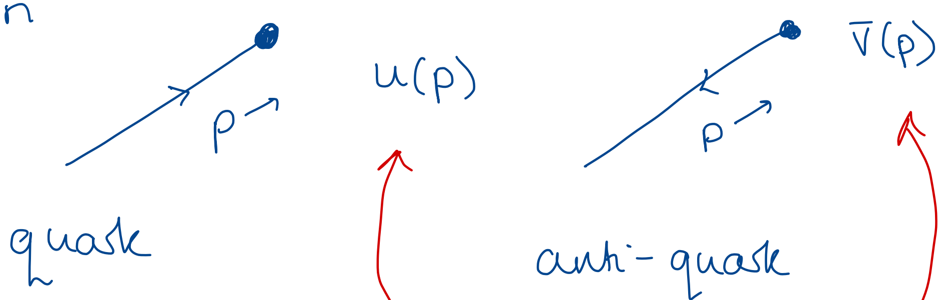
becomes



and the first two are no longer gauge-invariant because gauge piece now multiplied by $[T^a, T^b] \neq 0$, but $[T^a, T^b] \propto f^{abc}$ and is cancelled by .

Finally need incoming/outgoing particles:

In



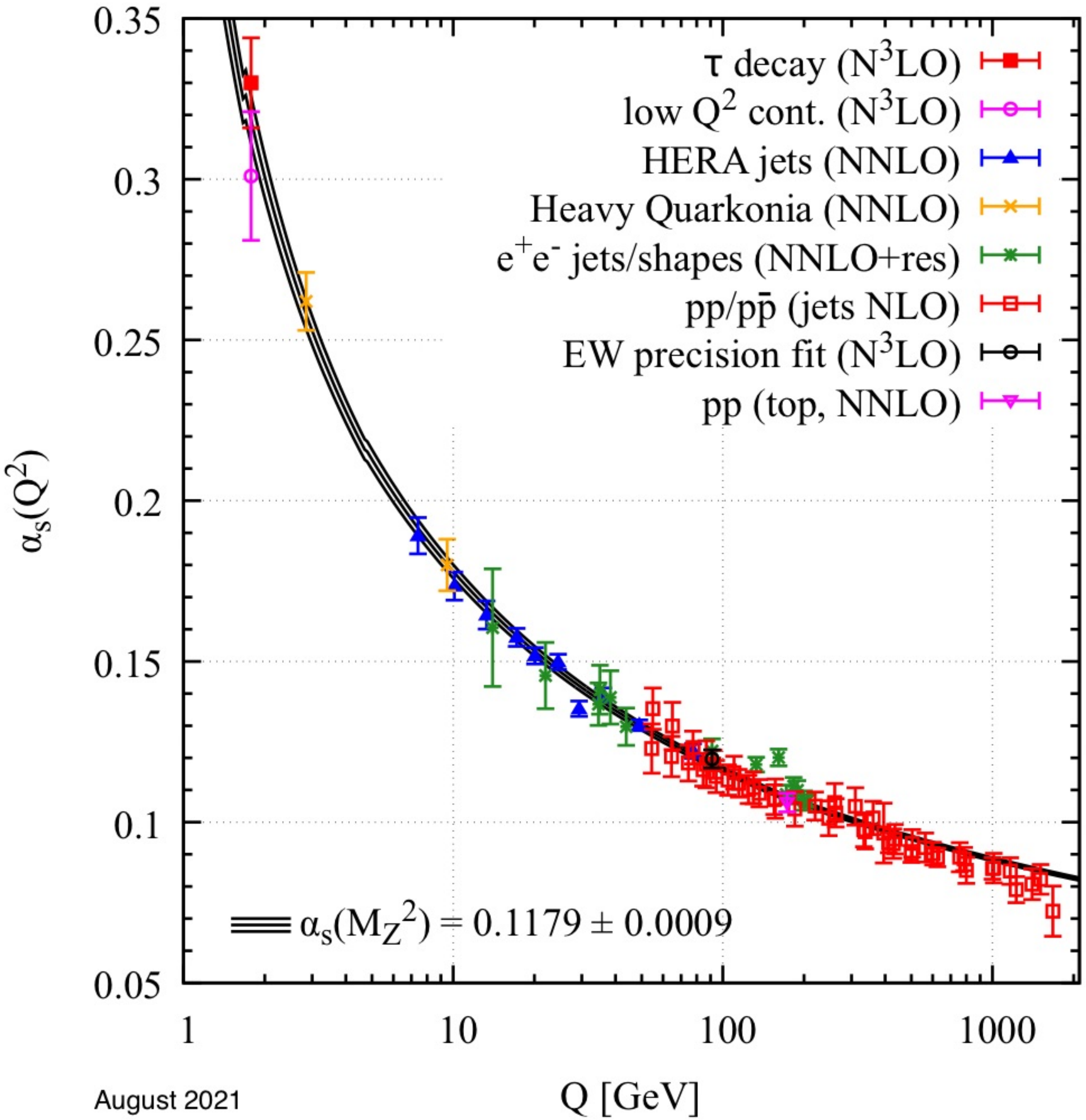
$$\not{p} = p^\mu \gamma_\mu$$

Dirac spinors
Solutions of Dirac Eqn $(\not{p} - m)u(p) = 0$.

They are vectors / co-vectors in spinor space, which we treat separately to colour space.

Any process = $\sum$ Feynman diags = $\sum_{\text{diags}}$ colour factor × kinematics.

* Can choose nice basis for colour factors to simplify intermediate steps.


τ decay (N3LO)
low Q2 cont. (N3LO)
HERA jets (NNLO)
Heavy Quarkonia (NNLO)
e+e- jets/shapes (NNLO+res)
pp/pp̄ (jets NLO)
EW precision fit (N3LO)
pp (top, NNLO)
αs(MZ2) = 0.1179 ± 0.0009
αs(Q2)
Q [GeV]
0.35
0.3
0.25
0.2
0.15
0.1
0.05
1
10
100
1000
August 2021

Coupling constant is g_s or often $\alpha_s = \frac{g_s^2}{4\pi}$

It is not constant, but evolves with scale of the process. We calculate this (in renormalisation) through the β-function:

$$\beta(\alpha_s) = \mu^2 \frac{\partial \alpha}{\partial \mu^2}$$

which is calculated perturbatively, currently to 5-loops. see Tut-Wed.

For QCD, the β-f^n is negative.

→ Usually quoted at $Q^2 = M_Z^2$ where it's value is ~ 0.1.

→ Result means α_s large at small energies → non-pert.

↳ why we only see colour-neutral states day-to-day.

α_s small at large energies. "Asymptotic freedom"

1.2 QCD Cross Sections

At an e^+e^- collider, the cross section for $(p_{e^+}\, p_{e^-} \to p_1 + p_2 + \ldots + p_n)$ is given by

$$\sigma_{ee} = \frac{1}{F} \prod_{i=1}^{n} \underbrace{\left(\int \frac{d^3 p_i}{2E_i (2\pi)^3} \right)} (2\pi)^4 \delta^{(4)}\left(p_{e^-} + p_{e^+} - \sum_{j=1}^{n} p_j\right) \overline{|M|}^2$$

flux factor gives norm^n?

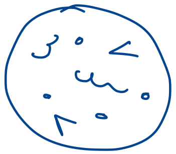
Lorentz - Invariant Phase Space (LIPS)

$$= \int \frac{d^4 p_i}{(2\pi)^4} \, 2\pi \, \delta(p_i^2 - m_i^2)$$

Total mom^m conservation

matrix-element squared, summed + averaged over helicity, colours.

This is straight-forward as the exact momenta of e^+ and e^- are known.



Asymptotic freedom tells us at H.E., coupling weak and we can treat them independently.

Fair to assume they travel with fraction of total mom^m P

$p_i^\mu = x_i P^\mu$ for each component where x_i is the momentum fraction of i.

Picture confirmed by detailed experiments, see Tues tutorial.

We use a parton distribution function (pdf) $f_i(x)$ to connect a partonic cross section to the proton collision, so

$$\sigma_{ep}(e p \to p_1 \cdots p_n) = \sum_{i \in \{q, \bar{q}, g\}} \int_0^1 dx \, f_i(x) \, \hat{\sigma}_i(e\, i \to p_1, \cdots p_n) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

With two incoming protons,

$$\sigma_{pp}(pp \to p_1 \cdots p_n) = \sum_{i,j \in \{q,\bar{q},g\}} \int_0^1 dx_1 \int_0^1 dx_2 \; \underbrace{f_i(x_1) f_j(x_2)}_{\text{non-perturb.}} \; \underbrace{\hat{\sigma}_{ij}(ij \to p_1 \cdots p_n)}_{\text{partonic cross section, perturbative.}} + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

Notes

- This has an appealing interpretation where the pdf gives the probability of finding a particular flavour of parton with mom^m fraction x_i
- It has been analytically shown to be valid for DIS and for Drell-Yan ($p\overset{(-)}{p} \to e^+e^-$). Not true at all orders for every final state.
- There is a correction term of $O\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$ where Λ^2_{QCD} is scale where QCD becomes non-perturb. and Q^2 is scale of the hard-process.
- Pdfs cannot yet be calculated explicitly (some hope from lattice)
- Can be large source of th. unc. at the LHC.