QCD Lecture 2

Last time: Intro to QCD including
- Feynman rules
- Running Coupling
- Cross sections
- Pdfs to link partonic cross sections to hadronic cross sections

2. PDFs

2.1 Properties

Universality: they are independent of final state and collider.

Measure in one place allows prediction in many others.

Sum Rules

Electric charge of proton is +1 attributed to $2 \times u + 1 \times d$.

Actually more complicated with gluons (zero charge) and $q\bar{q}$ pairs "the sea".

Electrical charge conserved requires following "sum rules"

$$\int_0^1 dx \quad [f_u(x) - f_{\bar{u}}(x)] = 2 .$$

$$\int_0^1 dx \quad [f_d(x) - f_{\bar{d}}(x)] = 1$$

$$\int_0^1 dx \quad [f_i(x) - f_{\bar{i}}(x)] = 0 \qquad i \in \{s, c, b\}$$ ← maybe not?

Momentum conservation gives $\int_0^1 dx \quad x \sum_i f_i(x) = 1 .$

Positivity

Technically, pdfs do not need to be positive everywhere, they just need to always lead to positive (differential) cross sections.

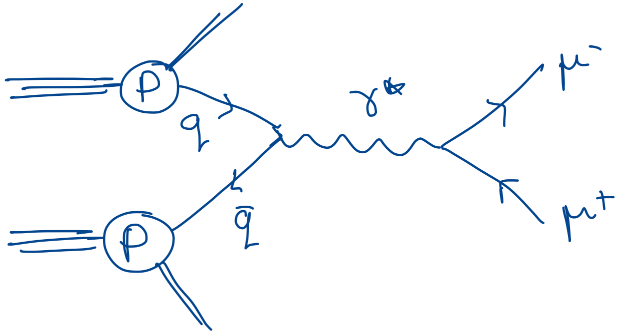
Continuity

No good physics reason to expect discontinuities or other non-analytic behaviour, so usually 'smooth'.

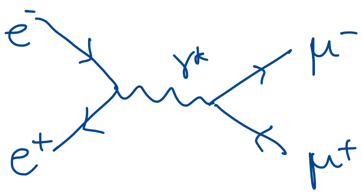
2.2 Singularities

Example

Consider Drell-Yan production of a colour-neutral vector boson



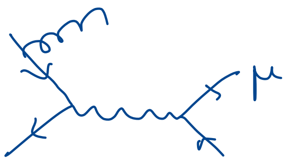
The perturbative part is closely related to the QED process $e^+e^- \to \mu^+\mu^-$



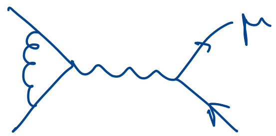
At LO, the difference is only in electric charge + colour factor:

$$\overline{|M_{DY}|}^2 = \frac{Q_q^2}{N_c}\,\overline{|M_{QED}|}^2$$

At next order in α_s: generate corrections like

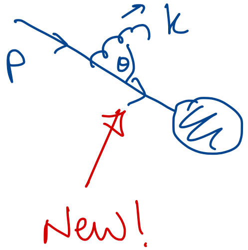


or



"real" extra on-shell particle produced

"virtual" gluon is emitted + absorbed.



Emission creates an extra propagator which contributes a denominator $\frac{1}{(p-k)^2} = \frac{-1}{2p\cdot k}$

In centre-of-mass frame, we may choose

$$p^\mu = (|p|; 0, 0, |p|) \qquad k^\mu = (|k|; |k|\sin\theta, 0, |k|\cos\theta)$$

$$\Rightarrow \frac{-1}{2p\cdot k} = \frac{-1}{2|p||k|(1-\cos\theta)}$$

We need to integrate over all k^μ, (ie all $|k|$ and θ), but the amplitude clearly diverges if $|k| \to 0$ and/or $\theta \to 0$

"soft" sing^y "collinear" sing^y

These are genuine divergences. Mistake is to treat separately to virtual because if $|k|=0$, or if $\theta=0$ identically, what you measure is indistinguishable.

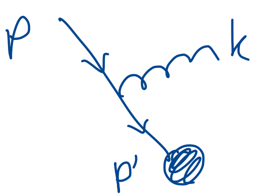
In QED, Bloch-Nordsieck theorem guarantees that combination of real +

virtual is finite.

In QCD, the Kinoshita, Lee + Nauenberg (KLN) also guarantees finite results <u>but</u> only with all possible initial states (e.g. $q\bar{q}g \to \mu^+\mu^-$)

KLN guarantees cancellation of soft sing^ies^, virtual sing^ies^ and collinear sing^ies^ with outgoing particles among themselves, but leaves uncancelled collinear with incoming particles.

Collinear Limit



$p'^\mu = zp^\mu, \quad k^\mu = (1-z)p^\mu + k_\perp$ assumed to be small

Then $$\hat{\sigma}_{q\bar{q}g} \xrightarrow{k \parallel p} \int_0^1 dz \int \frac{d|k_\perp|^2}{|k_\perp|^2} \hat{P}_{qq}(z)\, \hat{\sigma}_{q\bar{q}}(zp, p_b)$$

other inc. leg

2→2 partonic cross section, now with zp.

Unregularised DGLAP splitting f^n^

$$\hat{P}_{qq}(z) = \frac{\alpha_s C_F}{2\pi} \frac{1+z^2}{1-z}$$

<u>Same</u> for any q-initiated process.

But integral over $\hat{P}_{qq}(z)$ diverges as $z\to1$. There is a corresponding virtual self-energy diagram

which cancels it at the point $z=1$.

Adding it gives "regularised splitting function"

$$P_{qq}(z) = \frac{\alpha_s C_F}{2\pi}\left(\frac{1+z^2}{1-z} + \delta(1-z)\left(\frac{3}{2} - \frac{2}{1-z}\right)\right)$$

more usually written with a "plus distribution"

$$\int_0^1 dx\,\frac{f(x)}{(1-x)_+} = \int_0^1 dx\,\frac{f(x)-f(1)}{1-x}$$

so $$P_{qq}(z) = \frac{\alpha_s C_F}{2\pi}\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right).$$

But $\hat{\sigma}_{q\bar{q}g}$ still divergent at $|k_\perp|\to0$. Energy scale very low, unpert. region. So should be in hadronic description.

Independent of hard scatter so valid to include in pdf.

Collinear factorisation

Choose dividing scale, the "factorisation scale", μ_F

For $|k_\perp| < \mu_F \longrightarrow$ pdf.

$|k_\perp| > \mu_F \longrightarrow$ perturb. part

The new modified pdf is

$$\tilde{f}_q(x, \mu_F) = f_q(x) + \sum_j \int_0^{\mu_F^2} \frac{d|k_\perp|^2}{|k_\perp|^2} \int_x^1 \frac{dz}{z} \sum_j P_{qj}(z) f_j\left(\frac{x}{z}\right)$$

after change of variable $x \to \frac{x}{z}$ to factor out $\hat{\sigma}(p, p')$.

Finite $\overline{|M|}^2$ ✓ BUT new dependence on μ_F.

Original $f_q(x)$ indt. of μ_F, so $\frac{d f_q(x)}{d\mu_F^2} = 0$ gives

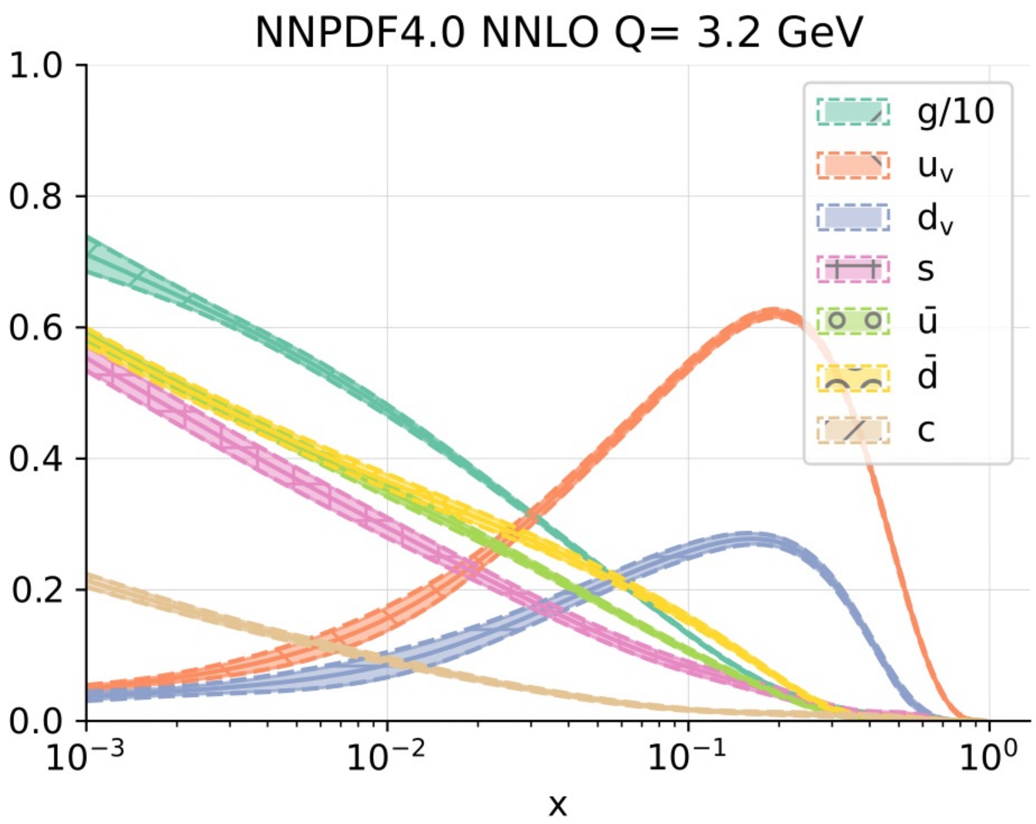
$$\mu_F^2 \frac{\partial \tilde{f}_q(x, \mu_F^2)}{\partial \mu_F^2} = \sum_j \int_x^1 \frac{dz}{z} P_{qj}(z) \tilde{f}_j\left(\frac{x}{z}\right)$$

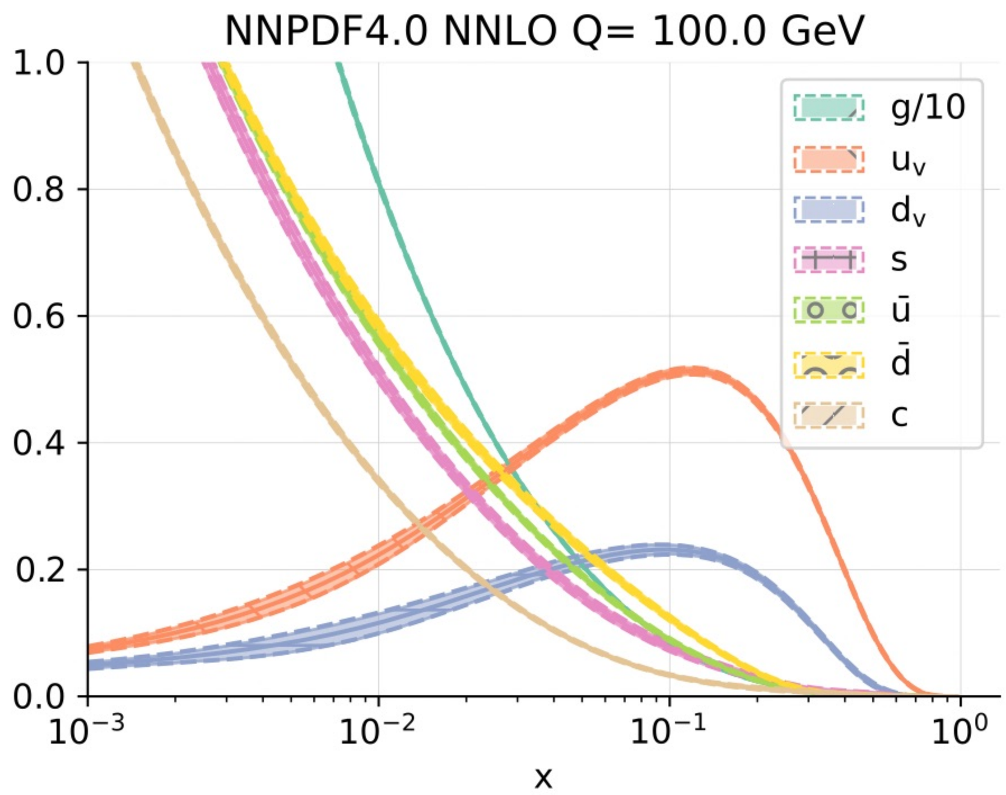
so solved for evolution, like α_s!!

Choice of μ_F undetermined $\longrightarrow$ th. unc.

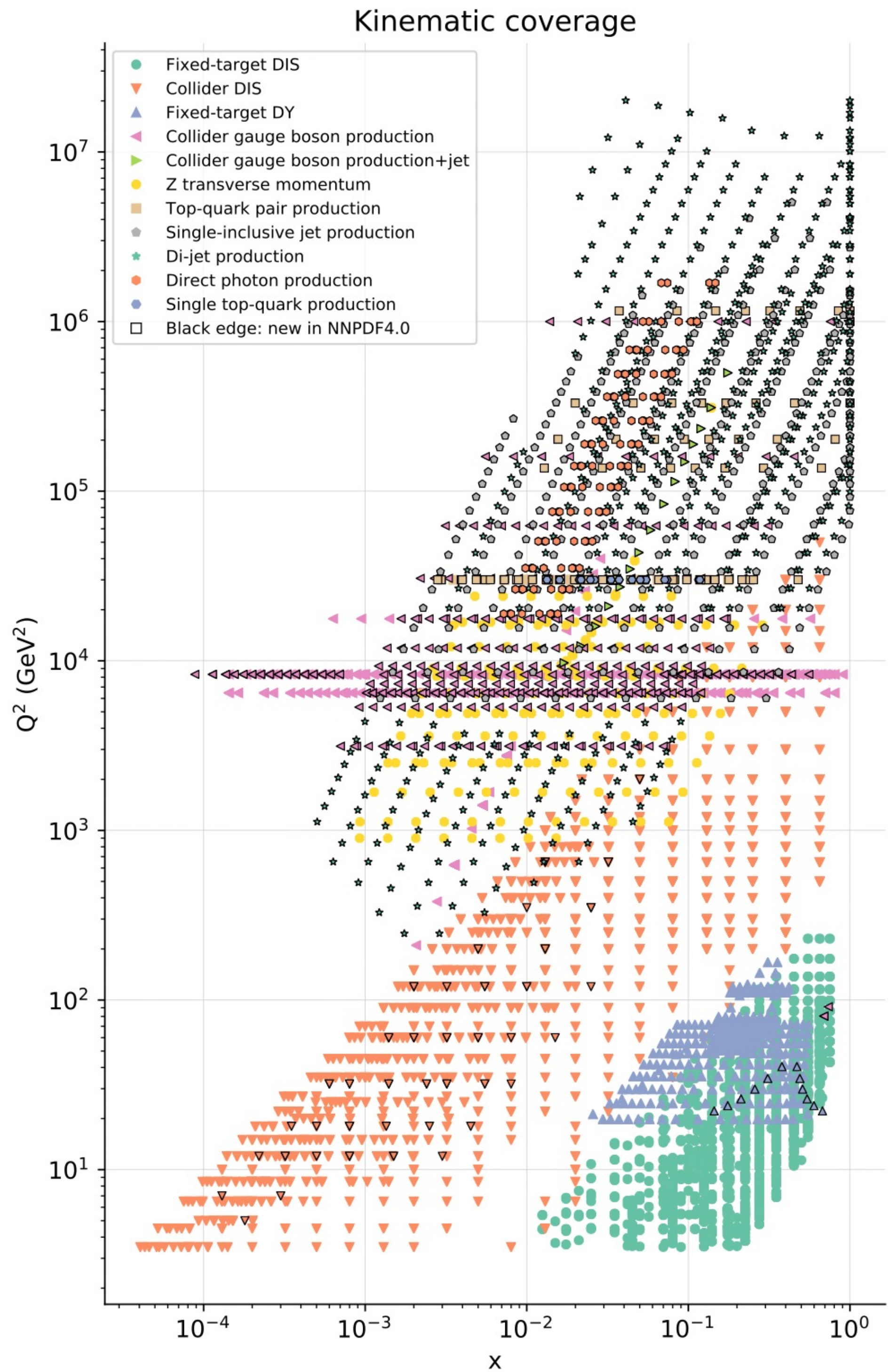
Not improved by better data in the fits!

Plots Here!





Plots taken from NNPDF arXiv:2109.02653



Kinematic coverage
Fixed-target DIS
Collider DIS
Fixed-target DY
Collider gauge boson production
Collider gauge boson production+jet
Z transverse momentum
Top-quark pair production
Single-inclusive jet production
Di-jet production
Direct photon production
Single top-quark production
Black edge: new in NNPDF4.0
Q² (GeV²)
x

* Data points do not give you a pdf directly! Perturbative calcs are the link.
  Their accuracy is pdf accuracy
  And accuracy of ME in prediction dictates accuracy of your pdf.

* Potential circularity between fitting pdfs and searching for new physics.