QCD Lecture 3

Last time: Pdfs including
- sum rules,
- singularities,
- evolution
- fitting

3. Scattering Amplitudes

Example $qQ \to qQ$ observed as part of $pp \to jj$

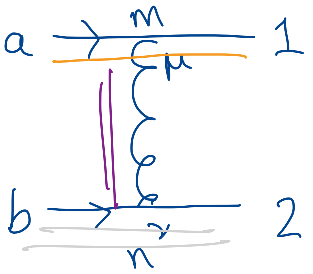
↑ Different flavours

2 jets with $|p_\perp| = 100$ GeV with rapidity ±2 at LHC c.o.m 13.6 TeV

→ x values $\simeq 0.1$.

3.1 Tree Level / Leading Order (LO)

Choose all −ve helicity



projects -ve helicity.

$$i\mathcal{M}^{----} = \underline{T^m_{1a}\,[\bar u(p_1)\, i g_s \gamma^\mu P_L\, u(p_a)]}$$

$$\times\ \underline{\frac{-i g_{\mu\nu}}{(p_a-p_1)^2}\ \delta^{mn}}$$

$[\ ]_s$

$$\times\ \underline{\underline{T^n_{2b}\,[\bar u(p_2)\, i g_s \gamma^\nu P_L\, u(p_b)]}}$$

$$= i g_s^2\, T^m_{1a}\, T^m_{2b}\ \frac{1}{\hat t}\ [\bar u(p_1)\gamma^\mu P_L u(p_a)][\bar u(p_2)\gamma_\mu P_L u(p_b)]$$

using Mandelstam invariants $\hat t = (p_a-p_1)^2$, $\hat s = (p_a+p_b)^2$, $\hat u = (p_a-p_2)^2$.

Need $\overline{|\mathcal{M}|^2}$ for σ, converts spinor strings to traces, but neater to use a spinor identity for $m=0$:

$$[\bar u(p_1)\gamma^\mu P_L\, u(p_a)][\bar u(p_2)\gamma_\mu P_L\, u(p_b)] = [\bar u(p_a)\overset{P_L}{\downarrow} u(p_b)][\bar u(p_2) P_R u(p_1)].$$

final $\overline{|M|^2} = \underbrace{\frac{1}{N_c^2}}_{\text{average in-colours}} \underbrace{\frac{1}{4}}_{\text{average in-hels!}} \underbrace{\sum_{a,1,b,2}}_{\sum \text{colours}} \underbrace{\left(|M^{----}|^2 + |M^{-+-+}|^2 + |M^{+-+-}|^2 + |M^{++++}|^2 \right)}_{\text{" non-zero } \sum \text{ helicities}}$

All these have same colour factor $\sum_m T^m_{1a} T^m_{2b}$

Sum is $\displaystyle\sum_{\substack{a,1,b,2,\\ m,n}} \left(T^m_{1a} T^m_{2b}\right) \left(\underbrace{T^n_{a1}}_{\text{reverse order of indices}} \overset{\text{new dummy index}}{T^n_{b2}}\right)$

$$= \sum_{m,n} Tr(T^m T^n)\, Tr(T^m T^n)$$

Matrices A,B

$Tr(AB) = A_{ij} B_{ji}$

$$= \sum_{m,n} \left(\frac{1}{2}\delta^{mn}\right)\left(\frac{1}{2}\delta^{mn}\right)$$

$$= \sum_m \frac{1}{4}\delta^{mm}$$

$$= \underline{\underline{\frac{1}{4}(N_c^2-1)}}.$$

Need to square kinematic parts.

E.g. $|\bar{u}(p_a) P_R u(p_b)|^2 = [\bar{u}(p_a) P_R u(p_b)][\bar{u}(p_b) P_L u(p_a)] = \underline{\underline{2 p_a \cdot p_b}} = \hat{s}$

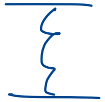
so get $g_s^4 \frac{1}{\hat{t}^2} \times (2 p_a \cdot p_b) \times 2(p_1 \cdot p_2) = g_s^4 \frac{\hat{s}^2}{\hat{t}^2}$

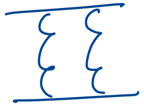
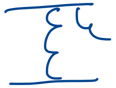
This is the same for ++++. Similar calculation for +-+-, -+-+ gives $g_s^4 \frac{\hat{u}^2}{\hat{t}^2}$.

Finally with colour get $g_s^4 \frac{N_c^2-1}{4N_c^2 \times 4} \times \frac{2(\hat{s}^2+\hat{u}^2)}{\hat{t}^2} = \underline{\underline{g_s^4 \frac{C_F}{C_A} \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}}} = \overline{|M|^2}$

3.2 Next-to-Leading Order

LO was α_s^2



NLO corrections are  -type and  -type

$$M = \alpha_s M_{tree} + \alpha_s^{3/2} M_{real} + \alpha_s^2 M_{virtual} + O(\alpha_s^{5/2})$$

We want

$$|M|^2 = \alpha_s^2 |M_{tree}|^2 + \underline{\alpha_s^3 |M_{real}|^2} + \alpha_s^4 |M_{virtual}|^2 + \underline{2\alpha_s^3 Re(M_{tree}^* \times M_{virtual})} + \overset{\mathcal{O}(\alpha_s^4)}{\dots}$$

No $Re(M_{tree} \times M_{real})$ as different final states.

$\Rightarrow$ So NLO is α_s^3 terms.

[Squared diagram is the same

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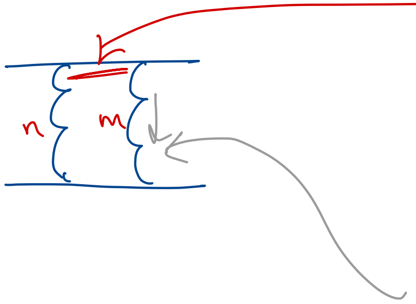
Virtual

When you have a loop, lose definition of momenta in each propagator.



If you choose to have momenta l, the rest are fixed up to l.

Have to integrate over all possible l.

$$iM^{----} = \int \frac{d^4 l}{(2\pi)^4} \, T^m_{1c} \, T^n_{ca} \, \left[\bar{u}(p_1) \, i g_s \gamma^\mu \, \frac{i \not{l}}{l^2 + i\varepsilon} \, i g_s \gamma^\rho P_L u(p_a) \right]$$

$$\times \, \frac{-i g_{\mu\nu}}{(l-p_1)^2 + i\varepsilon} \times \frac{-i g_{\rho\sigma}}{(l-p_a)^2 + i\varepsilon}$$

$$\times \, T^m_{2d} \, T^n_{db} \, \left[\bar{u}(p_2) \, i g_s \gamma^\nu \, \frac{i(\not{l} - \not{p}_1 - \not{p}_2)}{(l-p_1-p_2)^2 + i\varepsilon} \, i g_s \gamma^\sigma P_L u(p_b) \right]$$

Multiplying by conjugate of the tree gives

$$\frac{i g_s \, Tr(T^m T^n T^p)^2}{4 N_c^2} \; \frac{1}{\hat{t}} \int \frac{d^4 l}{(2\pi)^4} \; \frac{Tr\left(\gamma^\mu \not{l} \gamma^\rho \not{p}_a \gamma^\tau \not{p}_1\right) Tr\left(\gamma_\mu (\not{l} - \not{p}_1 - \not{p}_2) \gamma_\rho \not{p}_b \gamma_\tau \not{p}_2\right)}{l^2 \, (l-p_1)^2 \, (l-p_a)^2 \, (l-p_1-p_2)^2}$$

Using FORM (Vermaseren), traces reduce to give

$$\int \frac{d^4 l}{(2\pi)^4} \; \frac{a \, l^2 + b \cdot l + c_{\mu\nu} l^\mu l^\nu}{l^2 \, (l-p_1)^2 \cdots} \qquad ✪$$

where a, b_μ and $c_{\mu\nu}$ are functions of external momenta.

First term immediately simplifies to give $a \int \frac{d^4 l}{(2\pi)^4} \frac{\boxed{1}}{(l-p_1)^2 (l-p_a)^2 (l-p_1-p_2)^2}$ ← scalar.

Passarino-Veltman reduction is prescription to reduce all to scalar integrals. e.g. $p_1 \cdot l = -\frac{1}{2}(l-p_1)^2 + \frac{1}{2} l^2$.

Nowadays rare to use P-V. Instead use 'unitarity method' which is complex analysis + physics knowledge to find coefficients.

These integrals are not finite! They have UV + IR divergences. You may treat the two together by altering the number of dimensions $4 \to d = 4 - 2\varepsilon$ [We finally take $\varepsilon \to 0$] Divs appear as poles in ε.

$$\int \frac{d^4 l}{(2\pi)^4} \longrightarrow \mu_R^{2\varepsilon} \int \frac{d^{4-2\varepsilon} l}{(2\pi)^{4-2\varepsilon}}$$

↑ actually comes from renormalised coupling

μ_R is renormalisation scale. It should be chosen to be 'typical' but it's another source of theory uncertainty. You would remove this if you calculate to all order.

Estimate sensitivity by varying by factor of 2.

It is not an error bar, or even a σ.

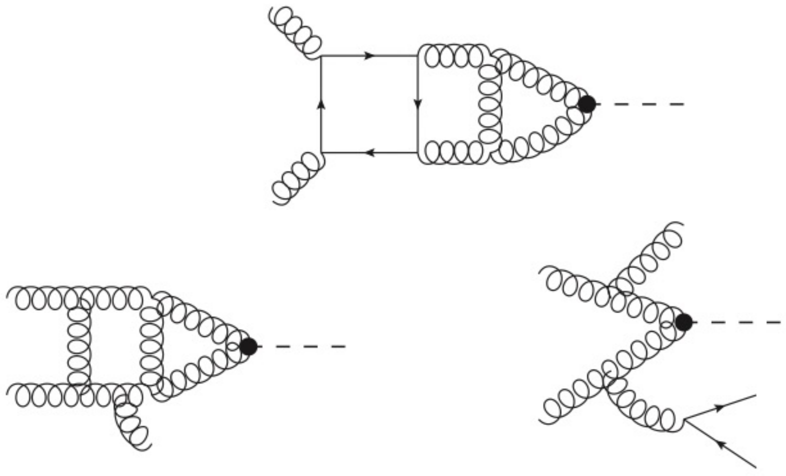
$$C_\Gamma = \frac{\Gamma(1+\varepsilon)\,\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

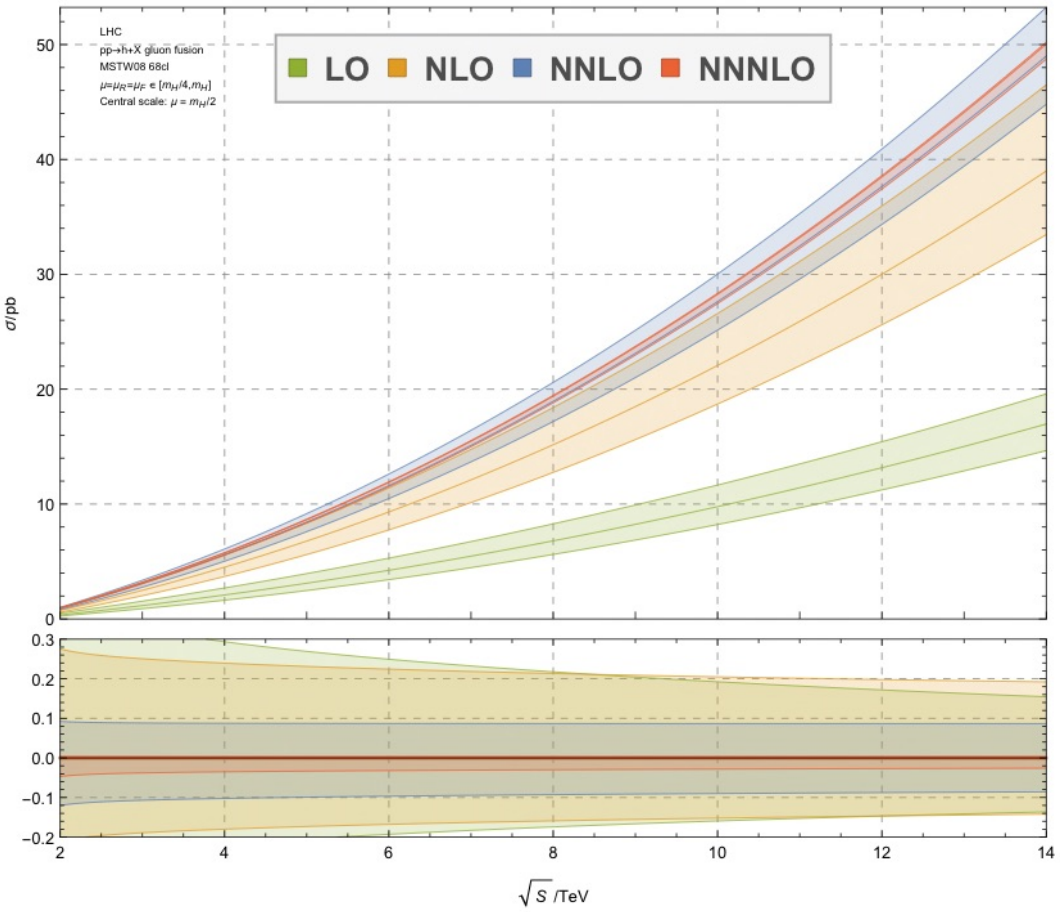
Our integral ⊛ is

$$\frac{8 g_s^6}{N_c^2}\, \mathrm{Tr}(T^m T^n T^p)^2\, \frac{\boxed{C_\Gamma}}{(4\pi)^{2-\varepsilon}} \underbrace{\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right)}_{\text{Tree-level } |\overline{M}|^2} \left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\log\left(\frac{-\mu_R^2}{\hat{s}}\right) + O(\varepsilon^0)\right)$$

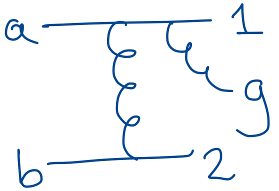
Scalar integrals in $d=4-2\varepsilon$ are mostly known, see e.g. QCDLoop. Intro to QFT for examples.

Real Corrections





Anastasiou, Duhr, Dulat, Herzog, Mislberger arXiv:1503.06056



+ 3 similar +

This is similar to real corrections in DY, and we encounter denominators like $p_1 \cdot p_g$, $p_a \cdot p_g$, ...

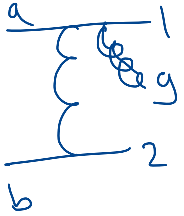
Again we must integrate over all p_g^μ in $d = 4 - 2\epsilon$ dimensions, and find divergences when $p_g \to 0$ and/or p_g collinear to an external particle.

Saw collinear splitting functions yesterday. There are similar universal soft functions, called Eikonal functions.

$$iM_1 = (ig_s)^3 \, T^g_{1i} \, T^d_{ia} \, T^d_{2b} \, \epsilon^\pm_\nu(p_g)$$

Should be in F. rules

External gluons have polarisation vector $\epsilon^\pm_\nu(p)$, $p_g \cdot \epsilon^\pm = 0$.



$u\bar{u}$

$$\times \left[\bar{u}(p_1) \gamma^\nu (\not{p}_g + \underset{\uparrow}{\not{p}_1}) \gamma^\mu P_L u(p_a)\right] \left[\bar{u}(p_2) \gamma_\mu P_L u(p_b)\right] \times \frac{1}{(p_1+p_g)^2} \times \frac{1}{(p_b-p_2)^2}$$

use $\not{p}_1 = \sum_{h=+,-} u^h(p_1)\bar{u}^h(p_1)$ and $\bar{u}(k)\gamma^\mu u(k) = 2k^\mu$ for any k^μ.

$$iM_1 \xrightarrow{p_g\to 0} \underbrace{\varepsilon_{g_\nu}\frac{2p_1^\nu}{2p_1\cdot p_g}}_{\text{Eikonal}} \times \underbrace{[\bar{u}(p_1)\gamma^\mu P_L u(p_a)][\bar{u}(p_2)\gamma_\mu P_L u(p_b)] \times \frac{1}{\hat{t}}}_{\text{kinematic tree.}} \times \text{colour}$$