QCD Lecture 4

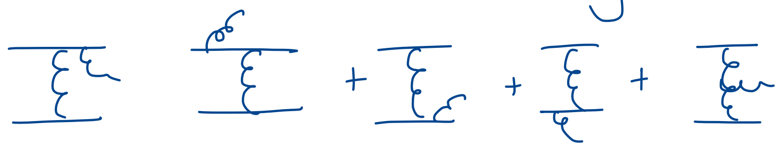
Last time: Hard Scatter Part with $qQ \to qQ$ as an example

- LO
- Found a virtual contribution:

$$\text{constants} \times \overline{|M_{tree}|}^2 \times \left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \log\left(-\frac{\mu_R^2}{\hat{s}}\right) + \mathcal{O}(\varepsilon^0) \right)$$

full thing Ellis + Sexton 1986

Real cont'd



$$iM_1 \xrightarrow{p_g \to 0} \underbrace{\varepsilon_{g\nu} \frac{2p_1^\nu}{2p_1 \cdot p_g}}_{\text{Eikonal}} \times \underbrace{[\bar{u}(p_1)\gamma^\mu p_L u(p_a)][\bar{u}(p_2)\gamma_\mu p_L u(p_b)] \times \frac{1}{\hat{t}}}_{\text{kinematic tree.}} \times \text{colour}$$

colour: changed by soft.

Other q-g vertex diagrams have same form, different colour factors.

When we sum over gluon helicity,

$$\sum_{h=+,-} \varepsilon^h(p_g)^\mu \varepsilon^{h\nu}(p_g) = -g^{\mu\nu} \quad \text{(F. gauge.)}$$

$\overline{|M|}^2$ then has cross-terms of the form $\frac{p_1 \cdot p_2}{p_1 \cdot p_g \; p_2 \cdot p_g}$, ...

We need to integrate over p_g^μ.

$s_{ij} = (p_i + p_j)^2 = 2 p_i \cdot p_j$

We define IR regions of phase space are where s_{ij} small.

Soft: s_{ig} small for all i, $\quad s_{ig} < s_{min}$ for all $i \in [a, b, 1, 2]$.

Coll: one of s_{ig} small, others not.

The phase space also factorises (Giele + Glover 1992) in $d = 4 - 2\varepsilon$,

Full $\int dP_3 \xrightarrow{p_g \to 0} \int dP_2 \, \frac{(4\pi^2 \mu^2)^\varepsilon}{16\pi^2 \, \Gamma(1-\varepsilon)} \, \frac{d|s_{1g}| \, d|s_{2g}|}{s_{12}} \left(\frac{|s_{1g}||s_{2g}|}{s_{12}} \right)^{-\varepsilon} \Theta(s_{min} > s_{1g}, s_{2g})$

This '$-$' missing in the live version

Can take $\overline{|M_{tree}|}^2$ out of integrals over $|s_{1g}| + |s_{2g}|$ so are left with

$$\frac{(4\pi \mu_R^2)^\varepsilon}{8\pi^2 \, \Gamma(1-\varepsilon)} \int_0^{s_{min}} \frac{d|s_{1g}| \, d|s_{2g}|}{\cancel{s_{12}}} \left(\frac{|s_{1g}||s_{2g}|}{s_{12}} \right)^{-\varepsilon} \left(\frac{\cancel{s_{12}}}{s_{1g} \, s_{2g}} \right)$$

$$= \frac{(4\pi)^\varepsilon \, (\mu_R^2 \, s_{12})^\varepsilon}{8\pi^2 \, \Gamma(1-\varepsilon)} \left(\int_0^{s_{min}} dx \, \frac{1}{x^{1+\varepsilon}} \right)^2$$

$$= \frac{(4\pi)^\varepsilon}{8\pi^2\,\Gamma(1-\varepsilon)} \frac{1}{\varepsilon^2} \left(\frac{\mu_R^2}{S_{min}}\right)^\varepsilon \left(\frac{S_{12}}{S_{min}}\right)^\varepsilon$$

$$= \frac{(4\pi)^\varepsilon}{8\pi^2\,\Gamma(1-\varepsilon)} \left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\log\left(\frac{\mu_R^2}{S_{min}}\right) + \log\left(\frac{\hat{s}}{S_{min}}\right)\right) + O(\varepsilon^0)\right)$$

$$x^\varepsilon = e^{\varepsilon \log x}$$

$$= 1 + \varepsilon \log x + \tfrac{1}{2}\varepsilon^2 \log^2 x + \ldots$$

$\frac{1}{\varepsilon^2}$ as contains soft and collinear

separate similar treatment for collinear starts at $1/\varepsilon$

Note similar form to the virtual. Can see how cancellation could occur, and it does if you sum correctly.

Practically, organising the cancellation is key. Above is 'phase space slicing'.

Most common are Catani-Seymour dipoles and Frixione-Kunszt-Signer (FKS), automated.

$$\text{NLO} = \int dP_2 \; |M_{tree}|^2 + |M_{virt}|^2 + F_2 + \int dP_3 \; |M_{real}|^2 - f_3$$

s.t. $\int d\Phi_2 F_2 = \int d\Phi_3 f_3$ and $\int d\Phi_2 (|M_{virt}|^2 + F_2)$, $\int \Phi_3 |M_{real}|^2 - f_3$ both finite.

$$|M_{virt}|^2 = 2\mathrm{Re}(M_{virt} \times M_{tree}^*)$$

Need finite contribs to 2j and 3j! Not separately guaranteed to be positive which leads to a problem of 'negative weights'.

All prescriptions introduce new params → th. unc. Should check independence of final result.

4. Parton Showers

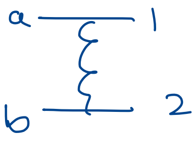
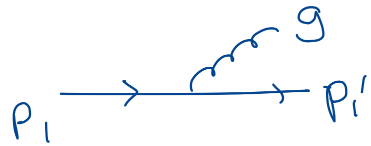
Calculating at fixed order in α_s makes sense when each term in the series is smaller than the last

$$\sigma = \alpha_s^2 C_2 + \alpha_s^3 C_3 + \alpha_s^4 C_4 + \dots$$

so far taken α_s small to be guiding principle.

There is a problem when the 'c_i's increase with i.

An important example is collinear emissions. We cancel the '∞'s, but nearby is still large.

Take  with p_1 splitting 

As for initial collinear,

$$d\hat{\sigma}_3 = d\hat{\sigma}_2 \int \frac{dt}{t} \int \frac{dz}{z} P_{qq}(z)$$

previously $|k_\perp|$

same regularised DGLAP splitting f^n

For outgoing, define $t = p_1^2$ = Lorentz Invariant <u>virtuality</u>.

Can iterate with more and more emissions

$$d\hat{\sigma}_{n+1} = d\hat{\sigma}_n \int \frac{dt_{n-1}}{t_{n-1}} \int \frac{dz_{n-1}}{z_{n-1}} P_{i_n i_{n+1}}(z) \ldots$$

After each emission, parton is closer to on-shell $p_1 \to 0$ so virtualities ordered

$t_1 > t_2 > t_3 > \ldots$

Use $|M|^2$ for hard scatter, then <u>evolve</u> down like this to small scales, to

Of course gives an approx to full $2 \to n$ result, but if n high enough, don't have a full result.

Similar for soft. An extra soft parton not so easily associated to 1 (or 2) hard partons. "Colour coherence" means well-modelled if instead of t you order emissions by angle θ.

A <u>parton shower</u> uses these probabilities to produce a set of events with varying numbers of extra emissions faithful to that distⁿ.
To get this right, need a "no emission probability".
Define Sudakov factor, $\Delta_i(t_0, t)$, to be the probability that no resolvable emission has been produced from parton i between two scales t_0 and t.

$$\Delta_i(t_0, t+\delta t) = \Delta_i(t_0, t) \times \left[1 - \sum_j \int_t^{t+\delta t} \frac{dt}{d} \int dz \; P_{ji}(z)\right]$$

Limit $\delta t \to 0$ gives

$$\frac{d\Delta_i(t_0,t)}{dt} = -\frac{1}{t}\sum_j \overbrace{\int dz\, P_{ji}(z)\, \Delta_i(t_0,t)}^{\text{Prob}^{y} \text{ there was an emission}}$$

$$\Rightarrow \quad \Delta_i(t_0,t) = \exp\left(-\sum_j \int_{t_0}^{t} \frac{dt'}{t'} \int dz\, P_{ji}(z)\right)$$

$\therefore$ Cascade/shower of n emissions with flavour i_k, mom^{m} fraction z_k, virtuality t_k has prob^{y}

$$P_{i_1,\dots,i_n}(\{z_i\},\{t_i\}) = \underbrace{\left[\prod_{m=1}^{n} \Delta_{i_m}(t_m, t_{m+1})\right]}_{\text{Sudakov for each separation in scales}}\left[\prod_{m=1}^{n} \int dz_{m+1}\, P_{i_{m+1} i_m}(z_{m+1})\right]$$

Splitting f^{n} for each emission.

<u>Notes</u>

- Only mentioned real emissions. Virtual corrections are implicitly included in the Sudakov.

- We required probabilities sum to 1, and if you sum above eqⁿ over all final states, you get 1.

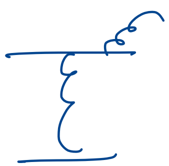
  $\implies$ Total cross section should be unchanged by showering, just moved in jet bins / distributions.

In practice, it will change slightly due to cuts, jet algorithms, ...

- Similar treatment for soft/collinear emissions from incoming colour-charged particles, <u>but</u> • must exclude emissions in pdfs.
  - Sudakov includes a ratio of pdfs.

- Need to run shower on each colour-charged particle, and the ones you produce. The ordering and t_0 mean it will be time-consuming.

<u>Merging</u>

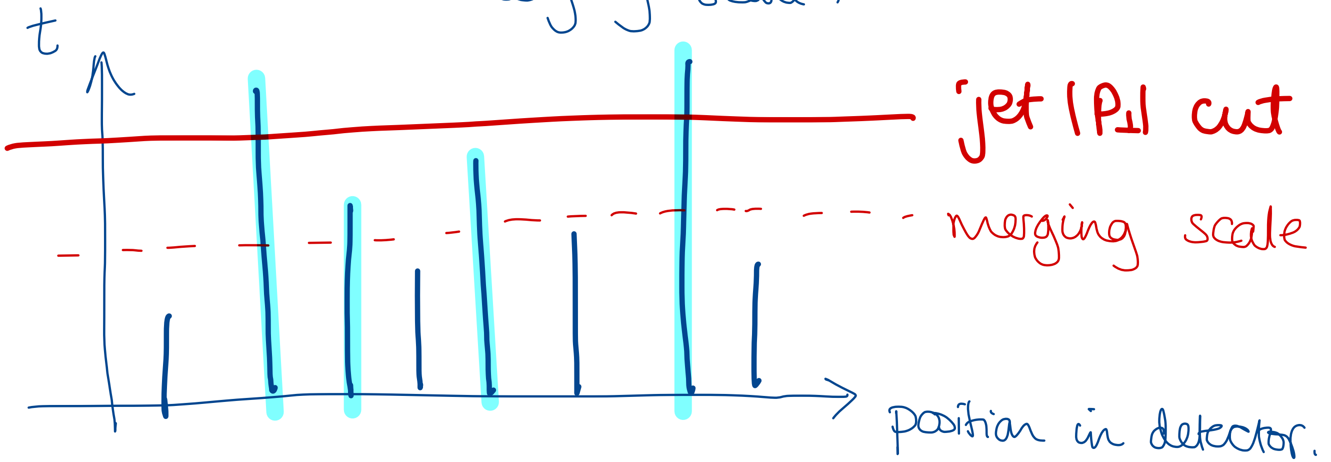


Wish to combine the best of our fixed orders with the best shower.

Merging means doing that, merging FO at different orders.

→ means improve accuracy and only ~~to~~ approx where necessary.

Most public showers do this + avoid double-counting with a merging scale.



Each line is a parton.

describe with MEs

others from shower.

Merging scale source of unc. (designed to be smooth step).
Same true to combine NLO, NNLO, ..