QCD Lecture 5

Last time:

- Integrated real correction and saw same functional form as virtual
- Parton Showers for enhanced soft/collinear emissions
- Merging with fixed order

5. The High Energy Limit

Have just seen higher order terms in α_s can be important, esp. in certain regions of phase space. Many other examples e.g. low p_T for W, Z; thresholds, ...

Here we will discuss 'high energy' logs in $\left(\frac{\hat{s}}{p_\perp^2}\right)$

Increased a lot

Transverse mom^m: Not changed much.

Inclusive 2-jet cross section given by $\underbrace{\int P_2}_{}$ $|M_{2j+}|^2$

Two outer partons not integrated

with $|M_{2j+}|^2 = \alpha_s^2 \left(a_2 \frac{s^2}{t^2} + b_2 \right)$

$+ \alpha_s^3 \left(a_3 \underbrace{\left(\frac{s^2}{t^2}\right)\log\left(\frac{s}{t}\right)}_{} + b_3 \frac{s^2}{t^2} + c_3 \right)$

$+ \alpha_s^4 \left(a_4 \left(\frac{s^2}{t^2}\right) \log^2\left(\frac{s}{t}\right) + b_4 \left(\frac{s^2}{t^2}\right) \log\left(\frac{s}{t}\right) + \ldots \right)$

$+ \ldots$

We calculated these!

Fixed order take contribs. horizontally. LO = 1st line, NLO = 1st 2 lines, ...

Log-order takes terms vertically, LL = $a_k \alpha_s^k \left(\frac{s^2}{t^2}\right) \log^{k-2}\left(\frac{s}{t}\right)$

$\alpha_s \log\left(\frac{s}{t}\right) \sim \mathcal{O}(1)$.

Have seen logs arise from virtual + real corrections.

Worth checking size and it can be significant (could already see at

TeVatron, and 7/8 TeV data)

Increase in c-o-m $\hat{s}$ increases these, but sometimes enhance further (for good reasons!) e.g. VBF/VBS cuts
Exotic searches.

So how do we calculate at all orders? They are the terms which persist in the high energy (HE) limit, defined for $p_1, \ldots, p_n$ partons in the final state,

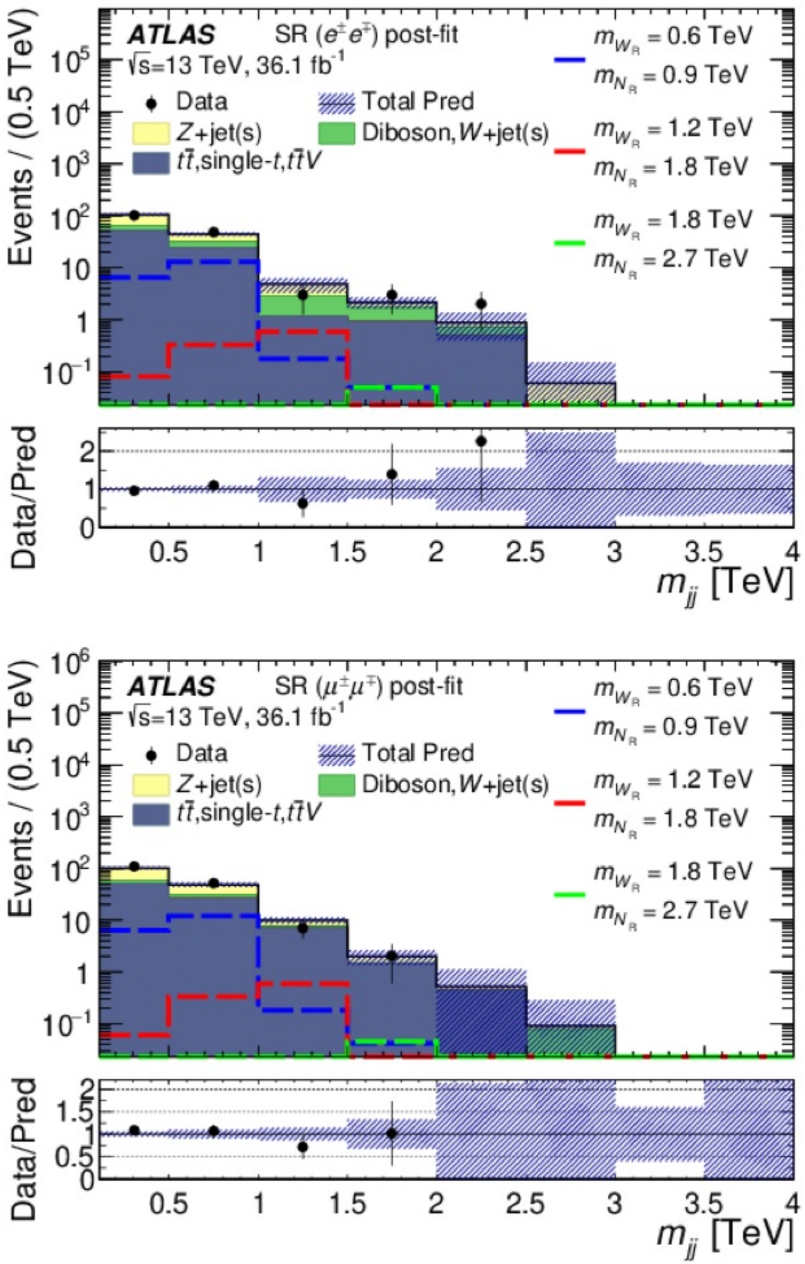
HE limit: $s_{ij} = (p_i + p_j)^2 \longrightarrow \infty \quad \forall i,j \in \{1, \ldots, n\} \quad |p_\perp|_i$ finite

In $2 \to 2$, translates $\hat{s} \to \infty$, $|\hat{t}|$ finite $\Rightarrow \hat{u} \to -\hat{s}$ (as $\hat{s} + \hat{t} + \hat{u} = 0$)

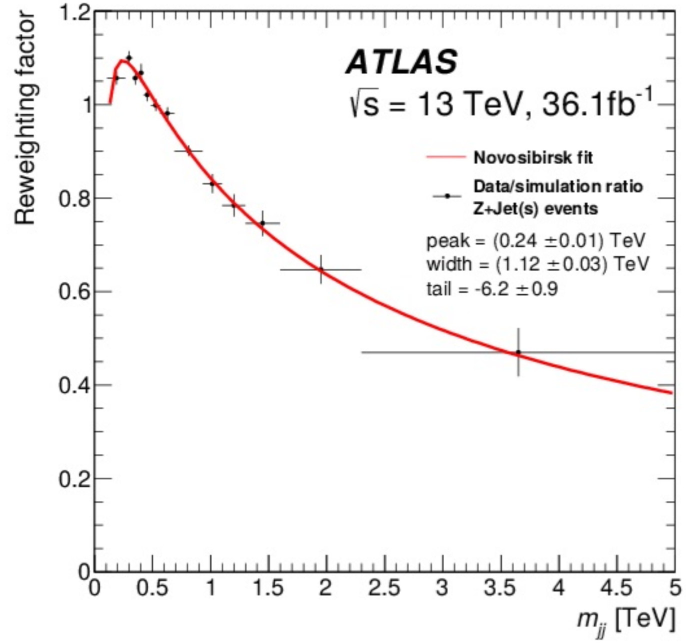
so previous $\dfrac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \to \dfrac{2\hat{s}^2}{\hat{t}^2}$ is part of LL.

Parametrise massless 4-momenta as

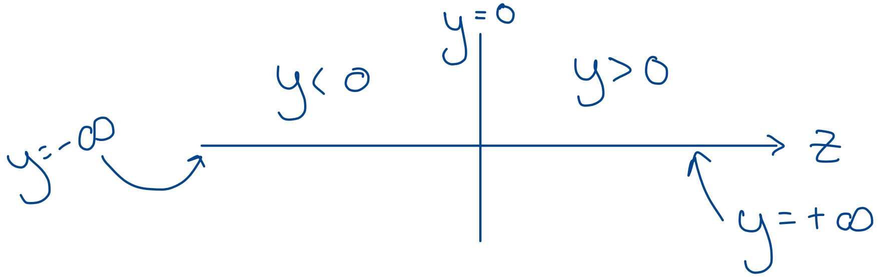
$p^\mu = |p_\perp| (\cosh y; \cos\phi, \sin\phi, \sinh y)$ defines rapidity y



ATLAS arXiv:1809.11105



Had to scale the MC background like this

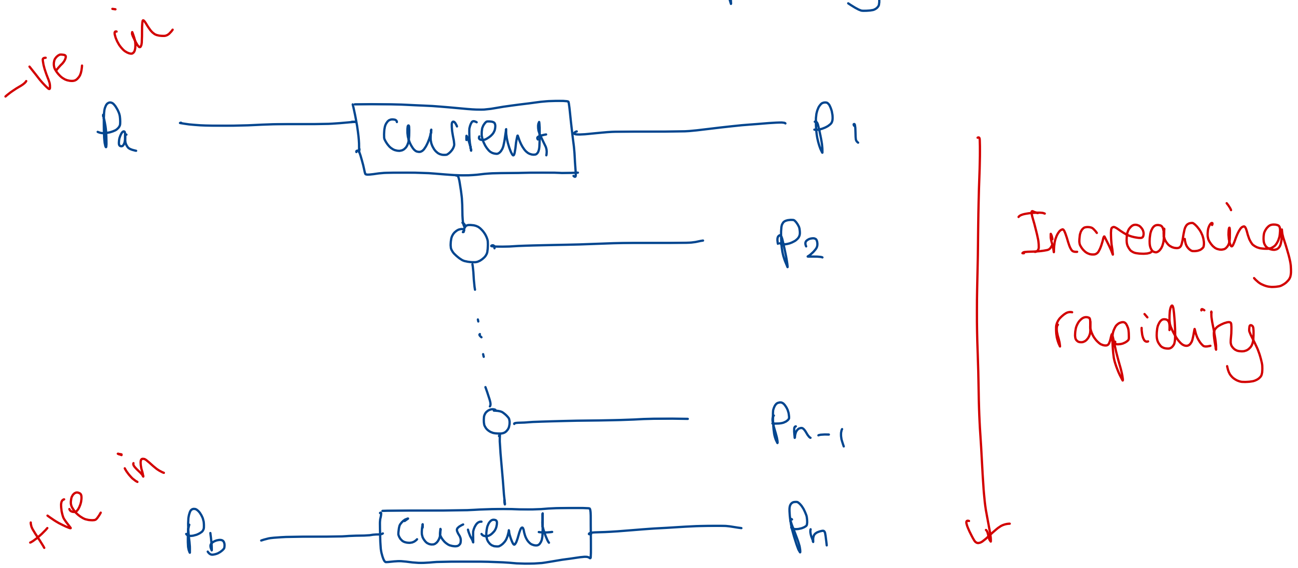


Detector coverage about $|y| \lesssim 4.5$

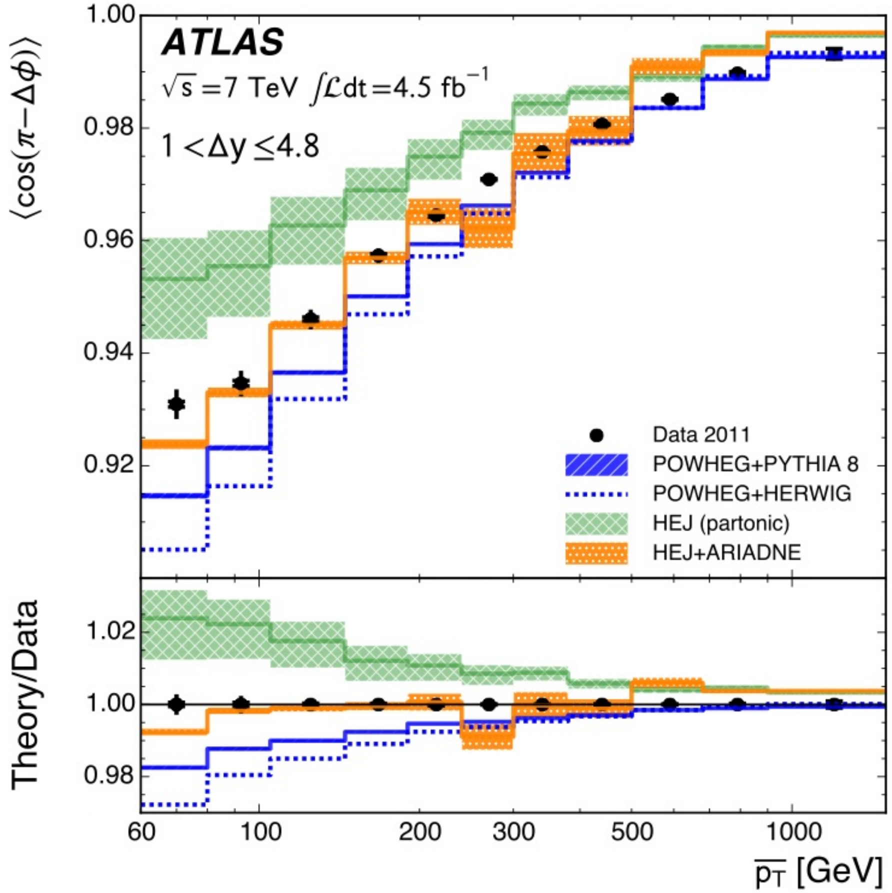
Equiv. HE limit is $|y_i - y_j| \to \infty$ $\forall i,j$ $|p_\perp|_i$ finite

ie Hard/wide-angle radiation as opposed to soft/coll. of parton shower

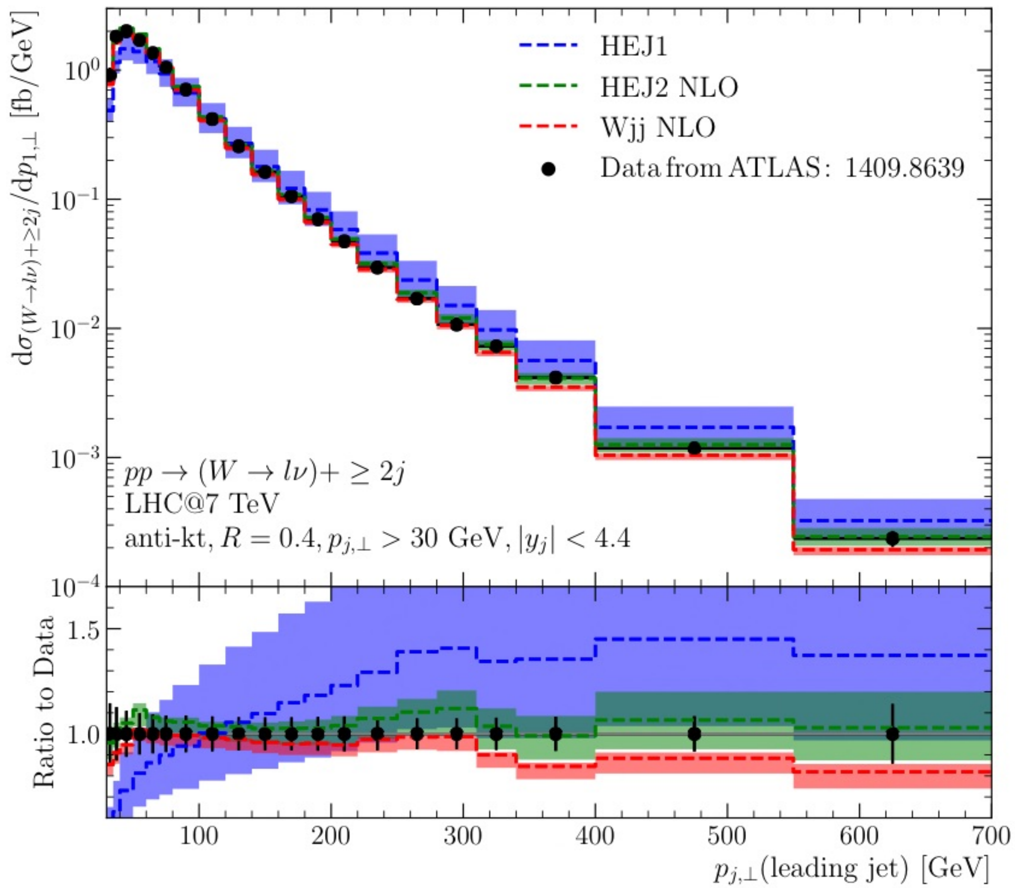
Amplitudes in HE limit "factorise" into independent pieces when strongly ordered in rapidity



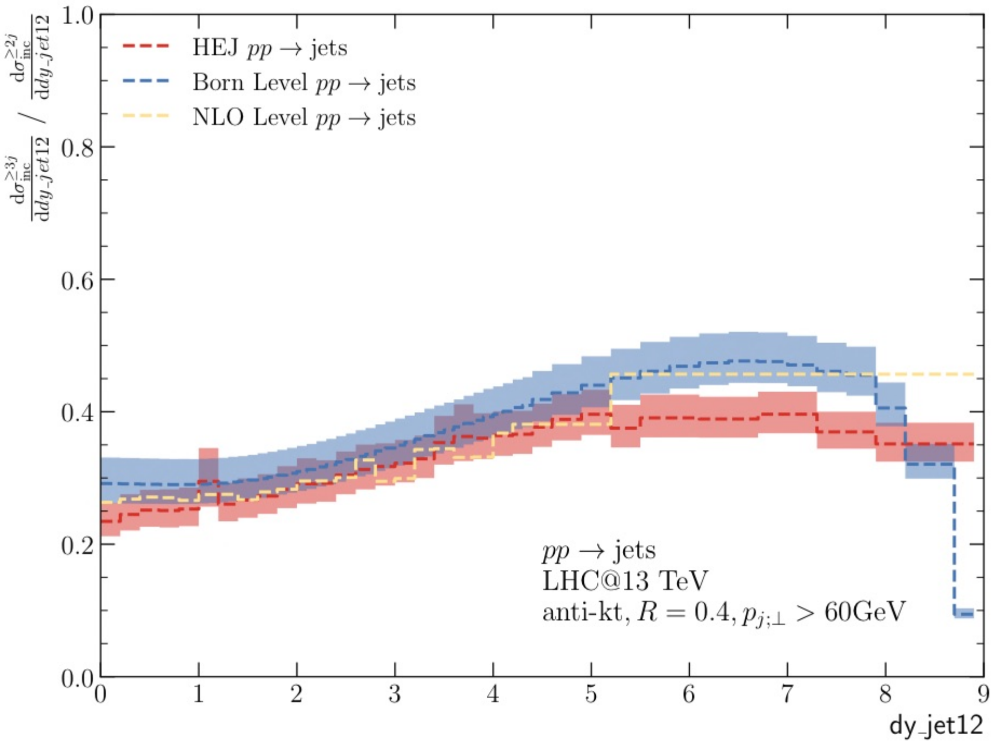
□s, Os multiply, but don't know about others (momenta, # of them, EW particles, ...)



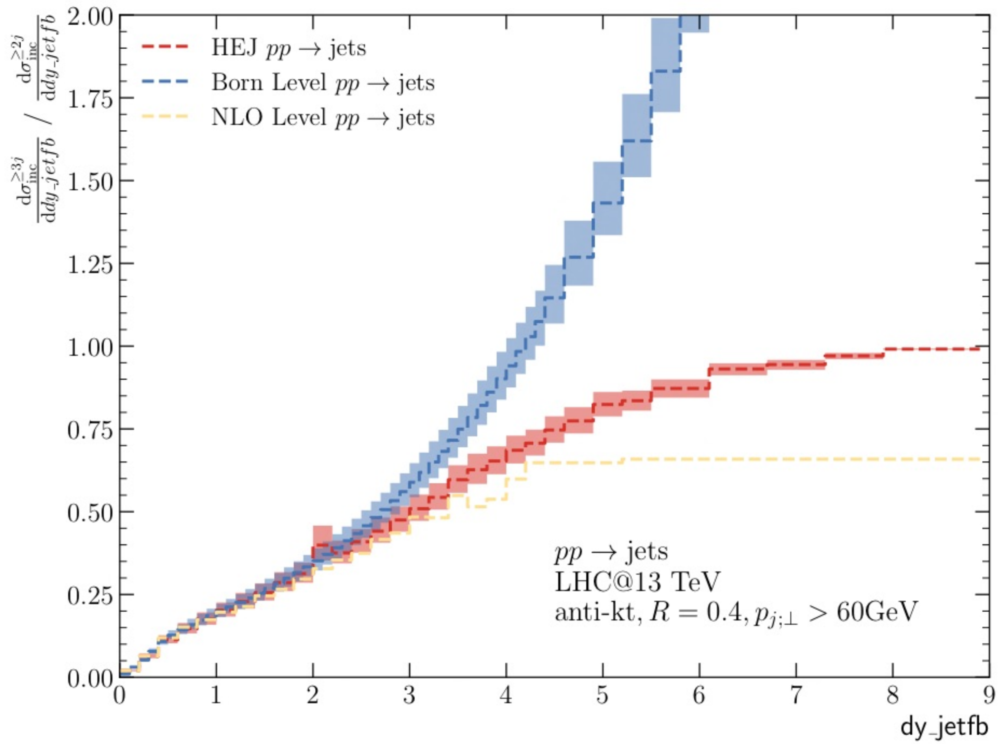
ATLAS arXiv:1407.5756



Andersen et al arXiv:2012.10310

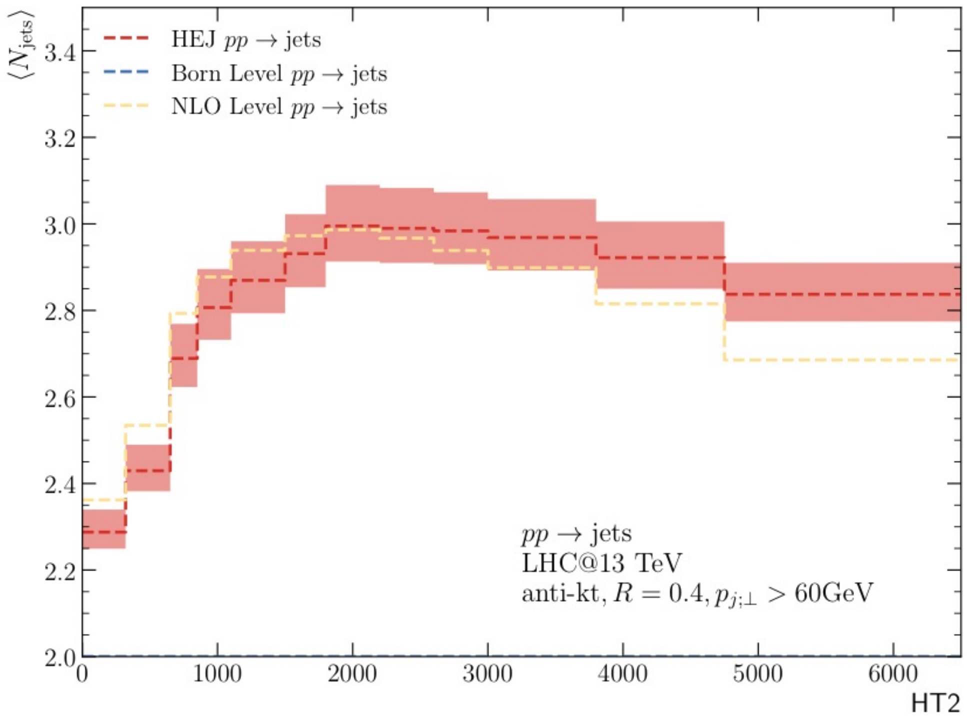


Similar, fairly flat vs Δy_{12}

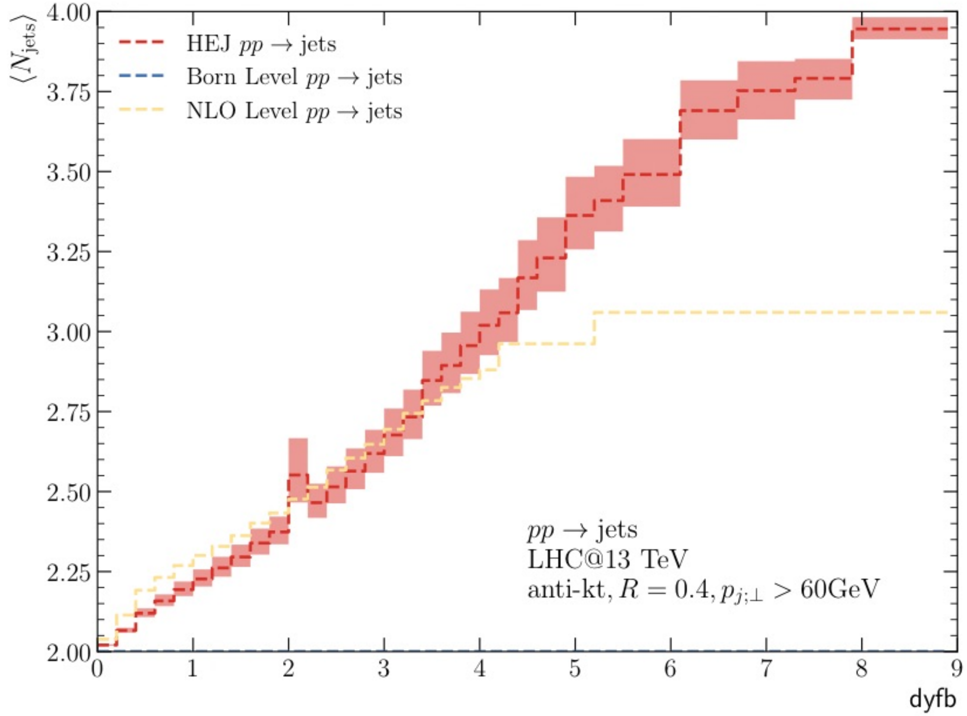


Larger differences vs

Plots by C. Elrick for forthcoming ATLAS analysis



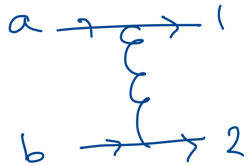
Similar for $p_\perp$ observables



Similar for Δy_{fb} until limit of NLO

Plots by C. Elrick for forthcoming ATLAS analysis

Go back to $q_L Q \to q_L Q$



$$iM^{----} = i g_s^2 \, T^m_{1a} T^m_{2b} \frac{1}{\hat{t}} [\bar{u}(p_a) P_L u(p_b)][\bar{u}(p_2) P_R u(p_1)]$$

Strict limit, $p_1^\mu \simeq p_a^\mu$, $p_2^\mu \simeq p_b^\mu$

$$\longrightarrow i g_s^2 \, T^m_{1a} T^m_{2b} \underbrace{\frac{\hat{s}}{\hat{t}}}$$

$$\propto \frac{p_a^- p_b^+}{\sqrt{(p_a-p_1)^2 (p_b-p_2)^2}}$$

Everything scalar, trivially factorised.

$\underline{p^\pm = E \pm p_z}$.

But misses something, we started with

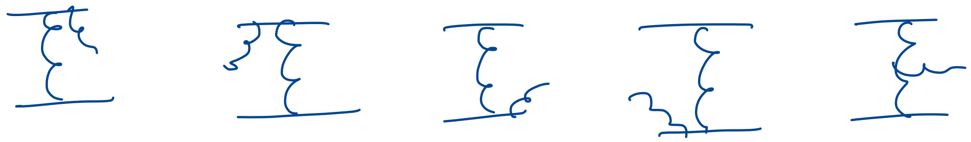
$$iM^{----} = i g_s^2 \, T^m_{1a} T^m_{2b} \frac{1}{\hat{t}} \underbrace{[\bar{u}(p_1) \gamma^\mu P_L u(p_a)]}_{j^\mu(p_1,p_a)} \underbrace{[\bar{u}(p_2) \gamma_\mu P_L u(p_b)]}_{j_\mu(p_2,p_b)}$$ exactly.

Exactly factorised with no approx!

Use currents j^μ instead of scalar components doesn't change LL accuracy, but adds LO accuracy. $\longrightarrow$ Important because $4.5 \neq \infty$.

Exactly true for $qg \to qg$, with modified colour factor.

Let's add a gluon, real correction. Need slight approx. here.



The HE limit of first four leads to same terms as soft limit.
(actually necessary because two limits commute).

4 diagrams give

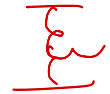
$$iM_{tree} \times \varepsilon_\nu(p_g)\left(\frac{2p_1^\nu}{s_{1g}} T^g_{1i} T^d_{ia} T^d_{2b} + \frac{2p_a^\nu}{-s_{ag}} T^d_{1i} T^g_{ia} T^d_{2b} + \frac{2p_2^\nu}{s_{2g}} T^g_{2i} T^d_{ib} T^d_{1a} + \frac{2p_b^\nu}{-s_{bg}} T^d_{2i} T^g_{ib} T^d_{1a}\right)$$

$$s_{mn} = (p_m+p_n)^2 = 2p_m\cdot p_n$$

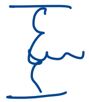
In HE limit, to LL, $p_a^\mu \simeq p_1^\mu$ and $p_b^\mu \simeq p_2^\mu$

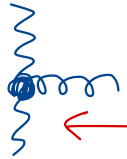
So first two terms are $\simeq \dfrac{p_a^\nu}{p_a\cdot p_g}\left(T^g_{1i}T^d_{ia} - T^d_{1i}T^g_{ia}\right)T^d_{2b} = \dfrac{p_a^\nu}{p_a\cdot p_g}\, i f^{gdc}\, T^c_{1a} T^d_{2b}$

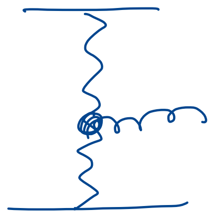
$$\simeq \frac{1}{2}\left(\frac{p_a^{\nu}}{p_a \cdot p_g}+\frac{p_1^{\nu}}{p_1 \cdot p_g}\right) i f^{gdc}\, \underline{T^c_{1a} T^d_{2b}}$$

colour factor of 

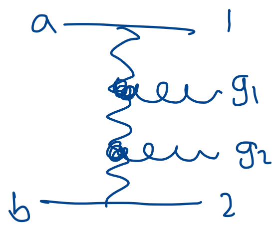
Equiv. to taking colour octet projection in the t-channels.

Last diagram  requires manipulation (replacements above)

gives Lipatov vertex V^μ  ← zigzag as not a single diagram

so $qQ \to qgQ$ described as 

$j^\mu(p_1, p_a)$
$\times V^\nu \varepsilon_\nu(p_g)$
$\times j_\mu(p_2, p_b)$

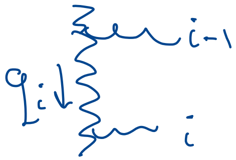
But now $qQ \to qggQ$ described by 

$j^\mu(p_1, p_a)$
$\times V^\nu \varepsilon_\nu(p_{g_1})$
$\times V^\sigma \varepsilon_\sigma(p_{g_2})$
$\times j_\mu(p_2, p_b)$

Same j, same V! continues recursively, solves complexity that

Usually comes with # of emissions.

Factorised structures applied applies to virtual corrections "Lipatov ansatz":



$$\frac{1}{q_i^2} \longrightarrow \frac{1}{q_i^2}\exp\left[\hat{\alpha}(q_i)(y_{i-1}-y_i)\right] \qquad \hat{\alpha}(q_i) = -g_s^2 C_A \frac{\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon}\left(\frac{q_\perp^2}{\mu_R^2}\right)^{\varepsilon}$$

soft div.

No collinear poles in HE limit.

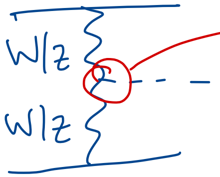
Regularise as for fixed order. Get $\int_{y_{i-1}}^{y_{i+1}} dy\ 1 = y_{i+1}-y_{i-1} = \ln\left(\frac{s_{i-1\,i+1}}{|p_{\perp\, i-1}||p_{\perp\, i+1}|}\right)$

These are the ingredients of the HEJ event generator.

http://hej.hepforge.org

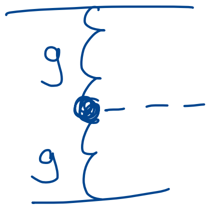
V2.2 – see arXiv.

VBF Higg Production



Study WWH, direct test of EWSB

QCD background

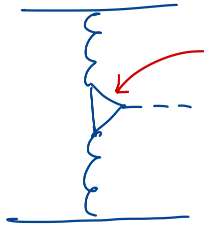


Suppressed with VBF cuts e.g. $\Delta y > 2.8$

$m_{jj} > 400\,\mathrm{GeV}$

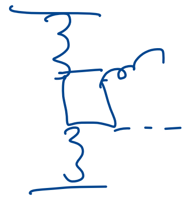
Exactly where logs large!

LO here is 1-loop (actually 1 massive loop)



top loop

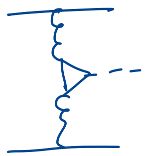
currently have LO for $2j + 3j$



$m_t \to \infty$ NLO for $2j + 3j$ privately

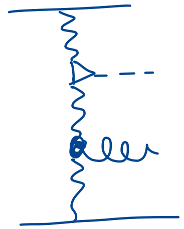
up to $5j$ for LO

HE limit QCD component: $2j$



$j^\mu(p_1, p_a)\, V^H_{\mu\nu}(m_t)\, j^\nu(p_2, p_b)$ exact.

3_j



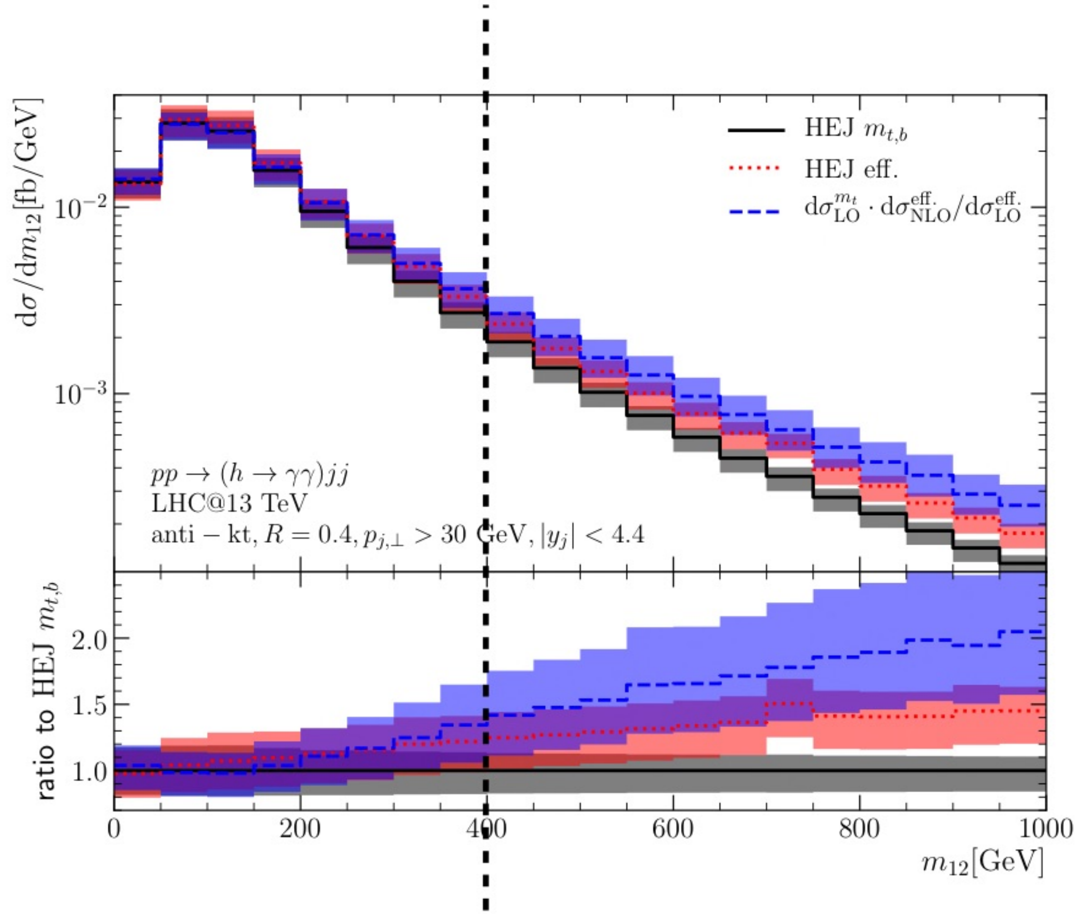
$$j^{\mu}(p_1,p_a)\, V^{H}_{\mu\nu}(m_t)\, V^{\rho}\, \varepsilon_{\rho}(p_g)\, j^{\nu}(p_2,p_b).$$

Del Duca et al showed limits commute.

HE results give description of higher order QCD effects which do not exist even at LO.

In VBF region, significant!

Prediction	xs after VBF cuts
Fixed order	9%
HEJ	4%



Typical VBF cut

Andersen, Cockburn, Heil, Maier, JMS arXiv:1812.08072