Data Analysis and Bayesian Methods Lecture 2

Maurizio Pierini











Data analysis Bayesian inference Bayesian inference beyond hypothesis





A <u>multiple-step process</u>











- Typically, HEP data analysis follows a top-down supervised problem
 - One starts with a specific process in mind
 - A theoretical framework allows to predict the experimental signature (qualitatively and quantitatively) through a Monte Carlo simulation
 - The data analysis is tailored on the process
- PROS: maximise sensitivity (e.g., can work on
 improving background rejection)
- CONS: poor generalization. Performance loss if the signal is different
- A supervised approach is ideal when you have a target in mind, e.g., Higgs@LHC, WIMP underground, a precision measurement of a Standard Model process
- For searches, one might need some additional tool with a different perspective (see Friday lecture)

A supervised problem





The program for these lectures

• STEP 1: make sure that potentially interesting events enter your dataset (aka the trigger)

• STEP 2: define an event selection that selects a subset of your data, potentially enhanced with signal

• STEP 3: define a procedure to estimate the amount of residual background events in your selected sample

• STEP 4: extract the signal component (aka the *measurement*)

• STEP 5: use the measurement to learn something about nature (aka phenomenology)

Monday

Tuesday

Wednesday and Thursday

Friday: a degression on signal "agnostic" analyses









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STEP 1: trigger selection





- Ideally, one would like to be able to store and analyse each individual events
- In practice this is never done typically because of limited resources (storage, bandwidth, etc)

• Sometimes this is harmless

- In a clean environment, one just needs to reject the obvious background (e+e- colliders, underground experiments, ...)
- Sometimes it's a challenge
 - At the LHC, one cannot store everything
 - Difficult choices have to be made very early

The need of a trigger

R. Tenchini "Trigger @LEP"

- All systems, except TPC, delivered signals suitable for L1 trigger purpose (i.e much faster than $11 \mu s$)
- L1 criterion was to require the presence of at least one single particle candidate, charged or neutral, from one or more system



At LEP, the trigger consisted in a set of algorithms requiring some activity, to reject beamgas interactions and to discard/tag obvious noise







much to be stored

Reduced to 5x(google)

• Can only store 5% of those

How big is big?





• STEP 1: make sure that potentially interesting events enter your dataset (aka the trigger)

STEP 2: define an event selection that selects a subset of your data, potentially enhanced with signal

These are actually two aspects of the same problem

 Both consists in applying a set of requirements to select a subset of the events

They differ in scope and for practical reasons

• Efficiency vs Purity

Accuracy vs Speed







• Efficiency is the fraction of signal events that would pass your selection

• aka True positive rate, recall, etc.

• Purity is a measure of how large is the fraction of signal events in the selected dataset

• Measured as $S/B, S/\sqrt{B}, S/\sqrt{S+B}$, depending on the context

• Maximizing efficiency (what one would do in a trigger) implies a loose selection

• Maximizing purity (what one would do in a data analysis offline) implies a tight selection

Efficiency vs Purity









• When applying a selection, one pays for the limited detector resolution

• The amount of background leaking in the selected region is inflated by poor resolution

• When working in real time one has *limited resources*

Timited computing power

calculations

• Detector performance are not exploited at best

• coarser algorithms, limited input information (e.g., no particle tracking)





• You have a discriminating quantity x and two estimates of it

 $\bigcirc X_{off}$: an accurate measurements of x

• Xon: a coarser measurement of x

• A turn-on curve models how the distribution of x_{off} is affected by a cut on Xon

 A typical analysis would work beyond
 the turn-on

• constant efficiency loss with measurable uncertainty

• some analysis could try to model efficiency (ɛ) along the curve and reweigh events by $1/\varepsilon$







STEP 2: data selection





X2 • S vs B separation depends on the discriminating quantity you use

Depending on the quantity on axis, selection can be more or less efficient

• How to cut

Inear vs non-linear cuts

• Where to cut

• trade-off between efficiency and purity

Uhat you need to decide











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Intervs non-linear cuts

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Uhat you need to decide











• An event selection is a multidimensional problem

• Several quantities can discriminate S vs B for different reasons

• Example:

- Signal: long-lived particles at LHC decaying to electrons
- Signature: electrons displaced from collision point

Backgrounds

- Events with real electrons from SM processes
- Fake events (random association of a track to a calorimeter deposit)

Uhat to cut on









• This is the easiest case

• one can define two quantities which are (to a large extent) independent

• Track displacement from primary vertex typically relies on the tracker

An electron ID score uses the calorimeter information



Uhat to cut on



0.008

0.012

0.016

0.020

 $\sigma_{i\eta i\eta}$

0.004

0





 Real problem and state-of-art
 solutions are typically more complicated

• There is no magic quantity that gives optimal separation (but you can try to build one)

• Several quantities can be defined, based on same inputs and correlated

• Just using the quantity with best discrimination might not be optimal

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• Using more quantities can *improve separation*

I-Dim VS M-Dim Cut







• Cut-based selection

• Select a portion of the N dim space, through a set cuts on each quantity

MVA-based selection

• Combine quantities in a single discriminator

N-dim likelihood (when correlations are known)

Machine learning e.g., BDT, NN, etc (when they are not) 22

Different approaches

	Variable	Barrel (tight WP)	Endcaps (tight W
	$\sigma_{i\eta i\eta}$	< 0.010	< 0.035
	$ \Delta \eta_{\rm in}^{\rm seed} $	< 0.0025	< 0.005
1	$ \Delta \phi_{\rm in} $	<0.022 rad	<0.024 rad
_	H/E	$< 0.026 + 1.15 \text{GeV} / E_{\text{SC}}$	< 0.019 + 2.06 GeV
of		$+0.032\rho/E_{\rm SC}$	$+0.183 \rho / E_{\rm SC}$
01	$I_{\text{combined}}/E_{\text{T}}$	$< 0.029 + 0.51 \text{GeV} / E_{\text{T}}$	< 0.0445 + 0.963 GeV
	1/E - 1/p	$< 0.16 { m GeV}^{-1}$	$< 0.0197 { m GeV}^{-1}$
	Number of missing hits	≤ 1	≤ 1
	Pass conversion veto	Yes	Yes







A new approach: end-to-end

- on physics intuition
- engineer high-level quantities
- fake separation)



• Traditional approaches exploits high-level features (hlf) built based

• With Deep learning, it is now possible to start from raw data and

• The hlf definition is optimized together with the task (electron vs









Multi-task problem with single train

 cluster energy deposits into photon/electron candidates

• build hlf quantities from the cluster

• maximise the separation between signal and background

A new approach: end-to-end















Typically, one puts a "flat cut" on each quantity

This quantity could be a physicsinspired function

• Or an MVA score

When doing so, one has to take care of the impact this has on other quantities

• Example:

- jet tagging: identify which kind of particle started a jet
- Jet mass can do it, but the jet substructure provides extra information
- Substally one b builds an MVA, e.g., a Neural Network





A dijet resonance search

- Typically, one puts a "flat cut" on each quantity
 - This quantity could be a physicsinspired function
 - Or an MVA score
- When doing so, one has to take care of the impact this has on other quantities
- Example:
 - jet tagging: identify which kind of particle started a jet
 - Jet mass can do it, but the jet substructure provides extra information
 - Usually one b builds an MVA, e.g., a Neural Network









- Very often, a NN learns information that is exploited in physics inspired quantities
 - A cut on the MVA score can learn kinematic and alert it
 - This can affect background distribution. For instance, a jet ID cut can change the distribution of dijet mass, creating a bump in the background
 - This can make a search for a new resonance more complicated

Mass sculpting





Designed decorrelated taggers

- CMS Simulation Preliminary <t2[→] pT = 500-650 GeVpT = 650-800 GeV pT = 800-950 GeV pT = 950-1100 GeV pT = 1100-1250 GeV 0.8 inspired jet substructure 0.6 used in the search to extract the background 0.4 0.2 typical search ($\rho > -8$) CMS Simulation Preliminary pT = 500-650 GeV pT = 650-800 GeV pT = 800-950 GeV by hand, trading ρ for $\rho^{DDT} = \log\left(\frac{m}{n-m}\right)$ $(\mu = 1 GeV)$ pT = 950-1100 GeV pT = 1100-1250 GeV 0.8 $\tau_{21}^{DDT} = \tau_{21} - k \times \rho$ 0.4 0.2 2 28 ρ^{DDT}

- A two step approach
- The problem is visible even w/o machine learning • Jet substructure before NNs was done with physics-• Jet substructure shows a correlation with $\rho = \log(m^2/p_T^2)$, • The correlation is critical in the regime of a • Most of the dependence is linear and can be corrected • The residual dependent is removed trading τ_{21} for • At that point, a flat cut on τ_{21}^{DDT} is applied









• DDT approaches can be generalized to non-linear dependencies using a *learnable non-linear function*

- One can use machine learning to find the contour defined by $y > y_T(x)$
- $y_T(x)$ is learned through a neural network taking x as input
- The training is performed using a <u>specific loss function</u>

 $L(y_i^p, y_i) = max[q(y_i - y_i^p), (q - 1)(y_i - y_i^p)]$

N-dim generalization: The big advantage of this approach (as usual with neural networks) is that x can actually be a vector of input quantities

Quantile Regression







• When your quantity y is a trained algorithm you have a further option wrt quantile regression, DDT etc.

• You can prevent y from learning x through an adversarial learning at training time, minimizing the correlation between x and y while constructing y

• BE CAREFUL:

- this might alleviate the problem but not remove it
- Adversarial trainings can be tricky: two terms of the likelihood fighting against each other could introduce a training instability

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G. Loupe et al. https://arxiv.org/pdf/1611.01046.pdf





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Adversarial learning



E. Moreno et al. https://arxiv.org/abs/1909.12285





• The cut threshold is chosen maximizing some figure of merit of purity.

• Usually people set cuts that maximizes S/\sqrt{B} , when optimizing for a discovery

• Where is this coming from?

Uhere to cut







- Given a population of N data entries (events)
- Apply a cut with efficiency ε , one expect *\varepsilon N* events surviving
- The number of observed events is distributed according to a Poisson distribution
 - centred at $S = \varepsilon_s N_s$ for signal
 - centred at $B = \varepsilon_B N_B$ for background
- The total number of events is centred around S+B
- A Poisson converges quickly to a Gaussian with mean λ and RMS $\sqrt{\lambda}$

Counting Experiment







• When we talk about an observation, we quantify its strength in number of sigmas

- The difference between the observed and expected yield, in units of the uncertainty
- The uncertainty is that of a bkg-only distribution
- One is minimizing the probability of a background-only distribution to mimic a signal
- Chances to discover are maximal when the signal would induced the largest possible excess wrt bkg-only distribution
 - Expected yield in presence of a signal: S+B Two main issues:
 - Expected yield in absence of a signal: B
 - $\sigma_B = \sqrt{Var[k_B]} = \sqrt{B}$ is the RMS in absence of a signa1

Number of sigmas



- Bad behavior for small exerted background
- Computing the expected significance requires to specify a signal cross section (but the optimal can be found w/o)

G. Punzi, arXiv:0308063





• A given threefold defines the following qualities True-positives: Class-1 events above the threshold • True-negatives: Class-0 events below the threshold • False-positives: Class-0 events above the threshold • False-negatives: Class-1 events below the threshold







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Χ





• A given threefold defines the following qualities True-positives: Class-1 events above the threshold • True-negatives: Class-0 events below the threshold • False-positives: Class-0 events above the threshold • False-negatives: Class-1 events below the threshold



Χ







• Starting ingredients are true positive (TP) and true negative (TN) rates • Accuracy: (TP+TN)/Total • The fraction of events correctly classified • Sensitivity: TP/(Total positive) • AKA signal efficiency in HEP • Specificity: TN/(Total negative) • AKA mistag rate in HEP • Depending on which quantity you prioritise, you would cut at a different place



Receiver operating characteristic

CERN







- One can use the ROC curve to quantify a selection power without a specific cut choice
 - Can be used to compare with N-dim cut based algorithms
 - Can be used to compare different algorithms (architectures, input features, etc)
- In practice, one selects a working point to use for a specific search
 - Where to cut depends on the specific case
 - Custom figures of merits are used to choose

Selection power







• A ROC curve and the area under the curve (arc) are often used to comprare classifiers

• This is an unquestionable criterion when there is separation

• This is extremely misleading when the separation is less obvious (e.g., crossing lines)

• What matters is which classifier is better where you intend to cut not in average

Think Beyond the ROC







Background Estimate



Back to our counting experiment

- The full likelihood including systematics has three terms
 - The "real" likelihood
 - The constraint on the signal expected yield (typically from MC or similar data, e.g., $Z \rightarrow ee$ for $H \rightarrow \gamma \gamma$) (*)
 - The constraint on the background expected yield. This is where troubles start

$$\mathscr{L} = P(n \mid \lambda_B + \lambda_S)G$$

(*) In the following slides I will drop the signal, since the discussion is about controlling the bkg uncertainty 44



 $\overline{G}(\lambda_{S} | \lambda_{S}, \sigma_{\lambda_{S}}) \overline{G}(\overline{\lambda_{B}} | \lambda_{B}, \sigma_{\lambda_{R}})$





MC-based background estimate

Monte Carlo simulation

- One could predict the background with simulation. Not reliable per se (uncertainty on simulation accuracy difficult to estimate).
- A good baseline for a more accurate prediction

• Fitting the uncertainties

- Sometime one can estimate uncertainties of the simulation and model them through nuisance parameters
- Data/MC agreement is then improved with profiling while fitting for the signal
- Incertainties on nuisances propagates to uncertainties on signal through correlation









A typical plots shown by experiment is the "impact plot"

• It shows the pull of the nuisance parameters

• And the impact that the nuisance variation has on the parameter of interest *r* (e.g., the signal yield)

Nuisance pulls







A typical plots shown by experiment is the "impact plot"

• It shows the pull of the nuisance parameters

And the impact that the nuisance variation has on the parameter of interest *r* (e.g., the signal yield)

Nuisance pulls







• A control region:

• One can use a bkg control region bridging the observed yield to the signal region using simulation

• A 2D sideband

One can use two independent quantities to define the signal region and scale background from nearby sideband (ABCD) Inverted µ

isolation

• Connecting the bins

• One can fit the background isolation across adjacent bins with a smooth function

Data-driven methods









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• Given a background function f(x)describing the background shape distribution in x, one can predict the expected background

• The function has parameters a that one has to determine

• Cannot trust Monte Carlo in a data driven method

• Can use profile the α in the fit

One has to choose a robust function and attach some systematic uncertainty to the choice

Smooth fit



$$E[n_i] = f(x_i \mid \alpha)$$

$$\hat{\mathscr{L}} = \max_{\alpha} \prod_{i} P(n_i | f(x_i | \alpha))$$







\bigcirc **PROS**:

Simple to use (e.g., in bump hunts)

• Very little use of MC simulation (typically top test function choice, but data control regions can be used for that

\bigcirc CONS:

• Modeling the un certainty on the functional choice not trivial

• Choice of function tend to be problematic on tail

 With large statistics, poor goodness-of-fit for bkg-only
 hypothesis can compromise analysis robustness







Typically used in searches (e.g., SUSY)

• Several control regions enriched of specific backgrounds

• $Z \rightarrow 11$ for $Z \rightarrow nn$ bkg

 1+jets without b-tag
 jets for W+jets

 1+jets with b-tag
 jets for tt

MC-assisted prediction









- (CRs)
- Monte Carlo samples are added as additional control regions
- corresponding MC expected yields
- The profiles likelihood is obtained maximizing over the λs
- - $\mathscr{L} = \prod P_{SR}(n_{SR,i} | \lambda_{SR,i}) P_{CR}(n_{CR,i} | \lambda_{CR,i})$



• Common likelihood is defined for signal region (SR) and control regions

• expected yields in various data regions are connected, using functions of

• One has to add signal and its uncertainties, as discussed already $\lambda_{SR,i} = \lambda_{CR,i} \frac{\lambda_{SR,i}^{MC}}{\lambda_{CR,i}^{MC}}$

> $\mathscr{L} \to \mathscr{L} = \max_{\lambda} \prod_{i} P_{SR}(n_{SR,i} | \lambda_{CR,i} \times \lambda_{SR,i}^{MC} / \lambda_{CR,i}^{MC})$ $P_{CR}(n_{CR,i} \mid \lambda_{CR,i}) \times P_{SR}^{MC}(n_{SR\,i}^{MC} \mid \lambda_{SR\,i}^{MC}) P_{CR}^{MC}(n_{CR\,i}^{MC} \mid \lambda_{CR\,i}^{MC})$ 52









 \bigcirc **PROS**:

yields are used

• Generalizes very well to multi-bin fits

 \bigcirc CONS

can limit the precision

low-momentum to high-momentum)

• more robust vs MC simulation since only ratios of MC

- Available MC statistics becomes a crucial factor that
- MC modeling can be different if extrapolation is across regions with different kinematic properties (e.g., from









• Similar to our initial counting experiment, but 2D

• Two quantities defining ABCD plane are independent for the background

• Selection factorizes, so one can obtain a bkg prediction from data using three sidebands

ABCD method



 $\mathscr{L} = P(n_A | \lambda_A) P(n_B | \lambda_B) P(n_C | \lambda_C) P(n_D | \lambda_D)$

 $\lambda_A = \lambda_C \times k_{C \to A} = \lambda_C \times k_{D \to B} = \lambda_C \frac{\lambda_B}{2}$

 $\hat{\mathscr{L}}(n_A) = \max_{\lambda_B, \lambda_C, \lambda_D} P(n_A \mid \lambda_C \frac{\lambda_B}{\lambda_D}) P(n_B \mid \lambda_B) P(n_C \mid \lambda_C) P(n_D \mid \lambda_D)$

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 \bigcirc **PROS**

• In principle very clean methods

• Easy to n-bin generalization (ABCD per bin)

\bigcirc CONS

- In practice, difficult to find two uncorrelated quantities (but notable cases exist)
- Residual correlation hard to
 model (with MC?) and associated systematic can be limiting factor

• transfer factor can depend on other quantities (e.g., p_T etc.)

ABCD method



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Fail ID1

Pass ID1





• SM process measurements typically use MC-based profile likelihood fits

• With background control regions added to make fit robots

• Searches for new resonances typically use bump hunts with functional fit of the background

• Systematic on function choice is becoming factor, so the other methods are becoming popular here too

• Searches for exotics (e.g., long-lived particles) use ABCD a lot

• Usually exploiting exotic-signature ID for ABCD plance

• Searches with traditional objects in final states (leptons, jets, MET) use MC-assisted data-driven predictions

• SUSY, Dark Matter searches, etc.

 All methods have hidden assumptions and associated
 systematics

• Robustness comes from using (and comparing) different methods









 We reviewed how to define an event selection
 • what to cut on • how to cut • where to cut • We saw the implications of online vs offline selection • We discuss a few of the most popular background prediction methods





Backup





Efficiency, cross section, luminosity

• At collider, the expected number of events (S or B) is the product between

• The number of produced events for a given process $N = \sigma \mathscr{L}$, where \mathscr{L} is the luminosity and σ is the cross section

• The probability of a sample from a given process to pass the cuts (i.e., the efficiency ε we defined before)

• In other experimental setups, the luminosity is traded for the corresponding time (e.g., time of exposure of a target of a given size, etc.)

 $\odot \sigma$ is computed from theory, \mathscr{L} is measured at the experiment

 \odot But what if σ is not known (e.g., in searches for new physics)?

