

Superconducting thermometers for the direct detection of light dark matter

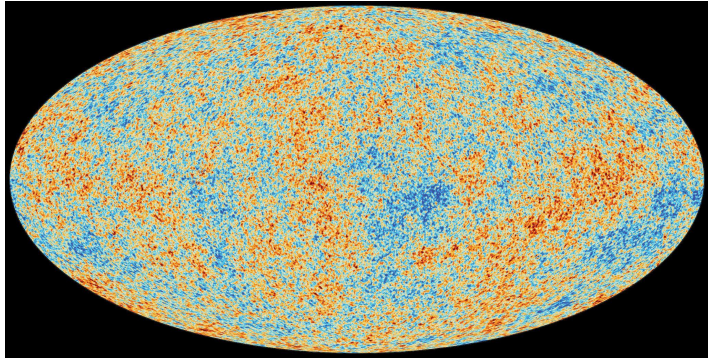
Felix Wagner

Institute of High Energy Physics of the Austrian Academy of Sciences

Quantum technologies lecture series, CERN, June 2023

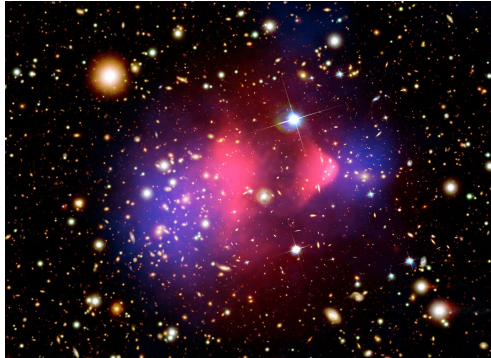
Dark matter

cosmic microwave background (CMB)



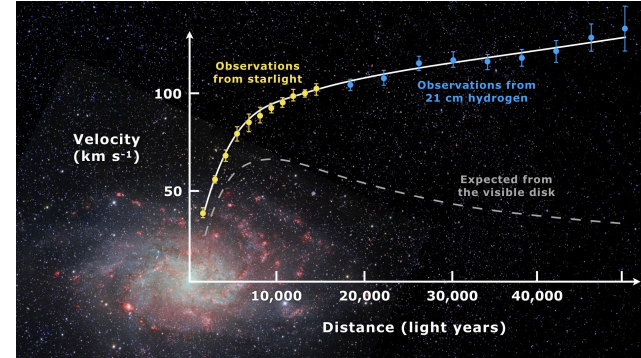
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bullet cluster (1E 0657-56)



© X-ray: NASA/CXC/CfA/M.Markevitch, Optical and lensing map: NASA/STScI, Magellan/U.Arizona/D.Clowe, Lensing map: ESO WFI

galactic rotation curve (Messier M33)



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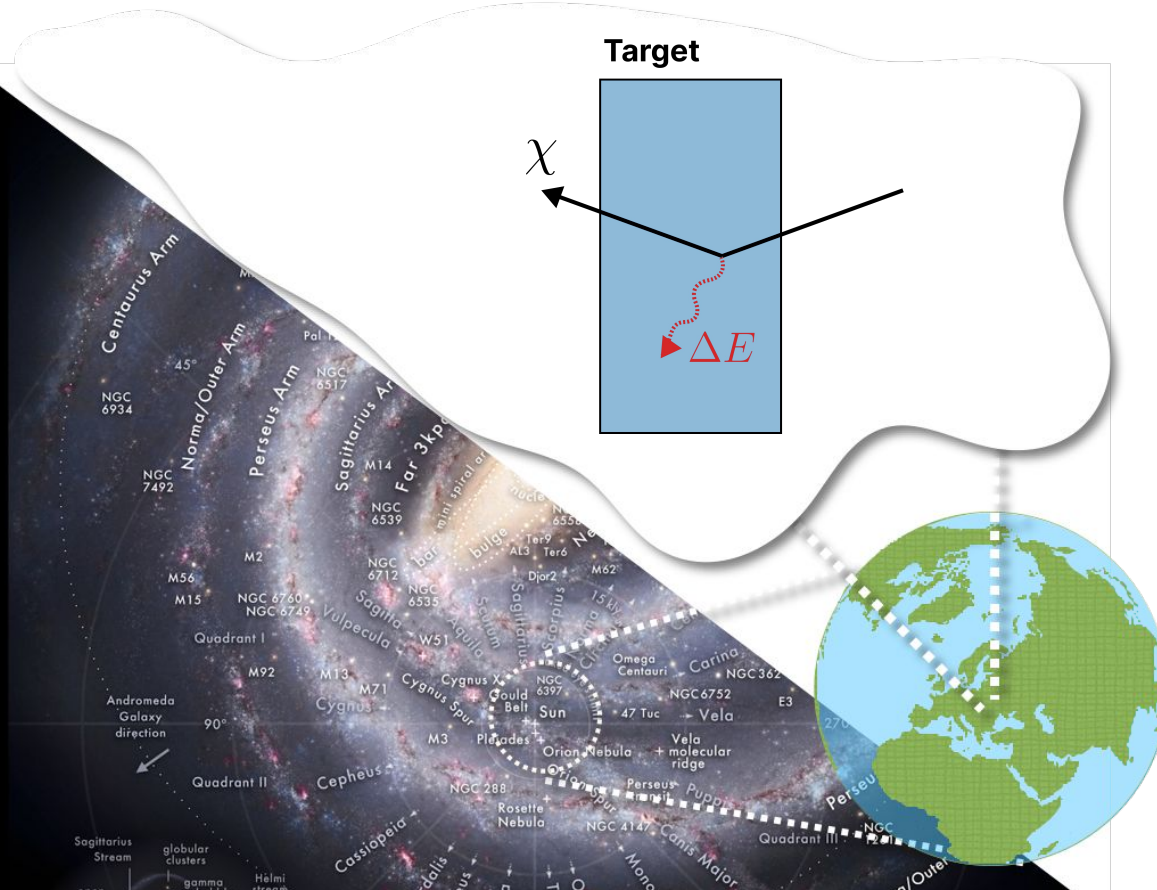
We observe overwhelming gravitational evidence for the existence of non-luminous, stable, cold, non-baryonic “dark” matter (DM) in our universe.

Best estimate from CMB: **(83.9 +- 1.5)%* of all matter in our universe is DM.**

Many particle candidates: (generalized) WIMPs, Axions, SIMPs, ...

The mass scale of a hypothetical DM particle is not fixed.

Direct detection



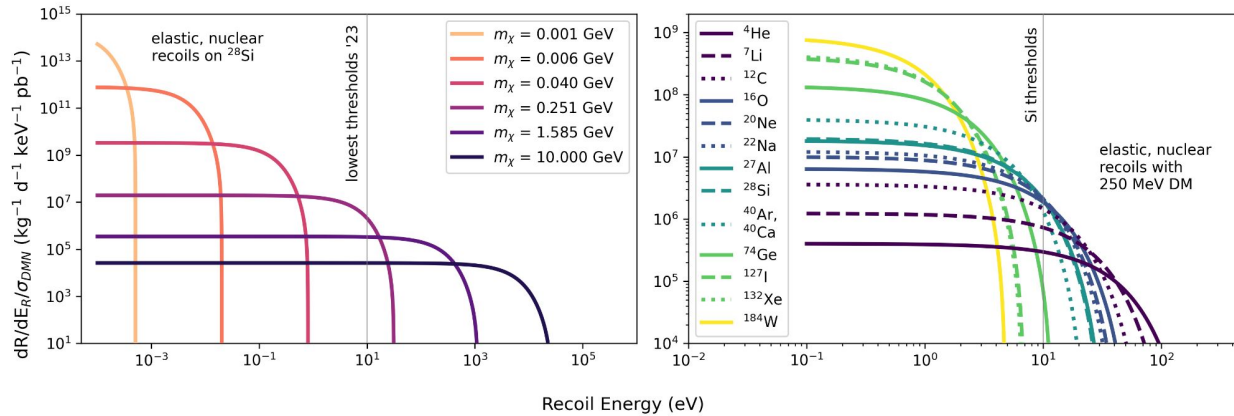
DM velocity distribution in Milky Way can be approximated with Maxwellian distribution.

Density along the solar circle is $\sim 0.4 \text{ GeV}/c^2/\text{cm}^3$.

Direct detection experiments test DM models via DM-target scattering.

Likelihoods depend on assumed DM mass and interaction cross section σ with target.

Expected scattering rate



Massive DM particles scatter elastically and spin-independently off target nuclei.

Objectives of (ideal) direct DM search: low energy threshold, high target mass.

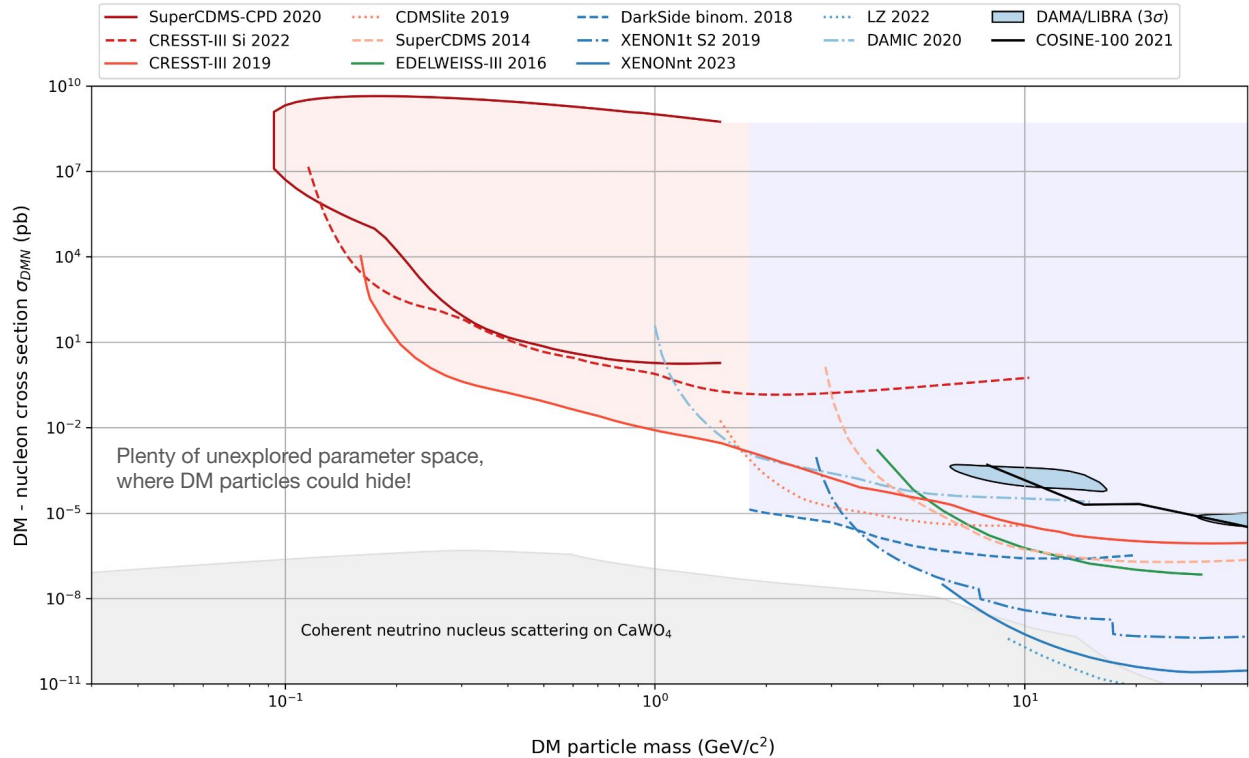
Light DM and heavy nuclei reduce expected recoil energy \Rightarrow experiments use

- high target mass and heavy nuclei for heavy DM searches,
- **low threshold and light nuclei for light DM searches.**

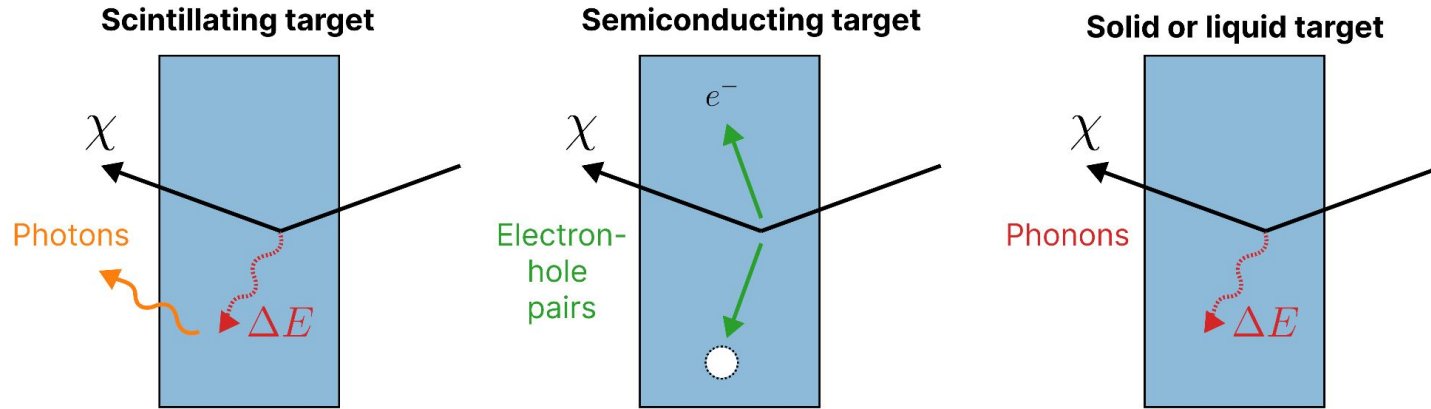
Exclusion limits on DM-nucleus elastic scattering

Cryogenic calorimeters with superconducting thermometers reach the best energy thresholds and sensitivity for sub-GeV/c² DM.

For higher DM masses time projection chambers (TPCs) with liquid noble gases are most sensitive.



Detection channels



Signal channels to detect target scattering:

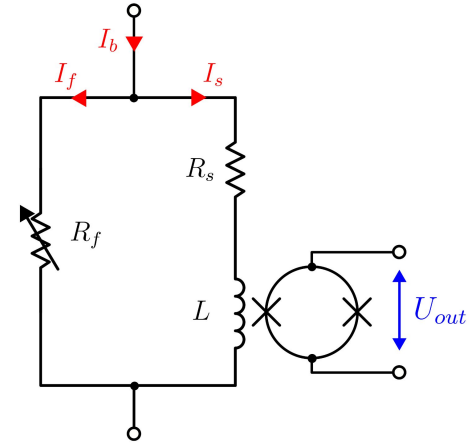
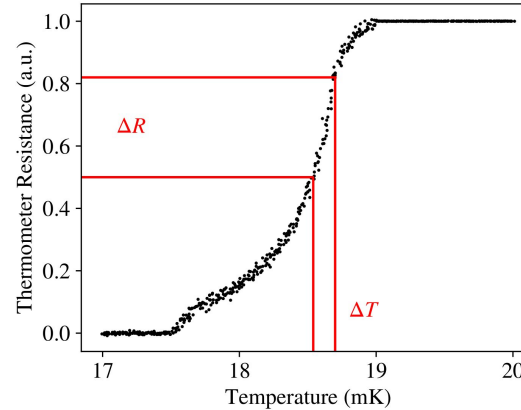
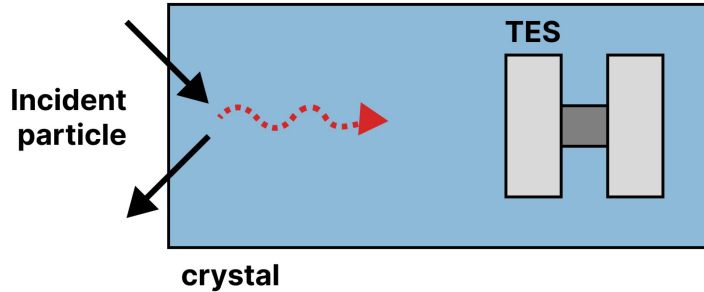
- scintillation (photons) measurable with scintillating crystals, liquids or gases,
- ionisation (electron-hole pairs) measurable with semiconductors, certain liquids, gases,
- **lattice vibrations (phonons) measurable (mostly) with single crystals.**

} Measured recoil energy depends on scattering particle

← Particle-independent energy scale!

Often multiple signal channels used for event discrimination, e.g. scintillating crystals.

Superconducting thermometers



Transition edge sensor (TES): superconducting film operated in the transition from its normal to superconducting state. Operation in low-noise SQUID-based readout circuit turns temperature/resistance fluctuations into measurable quantities.

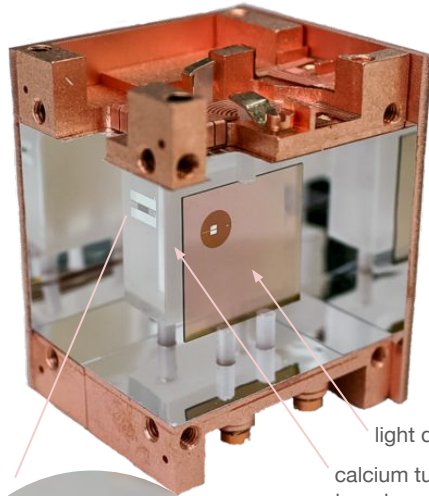
⇒ **thermometer sensitive to $\approx \mu\text{K}$.**

Film is thermally coupled to target crystal, collecting phonons from particle recoils in target.

⇒ **recoils down to 10 eV recoil energy detectable**.**

Technology developed in mid 90's* and continuously improved for DM searches by experiments (**CRESST, CDMS**, et al.).

DM detectors with superconducting thermometers



light detector
calcium tungstate target

CRESST-III DetA

Figs. on the courtesy of the CRESST collaboration

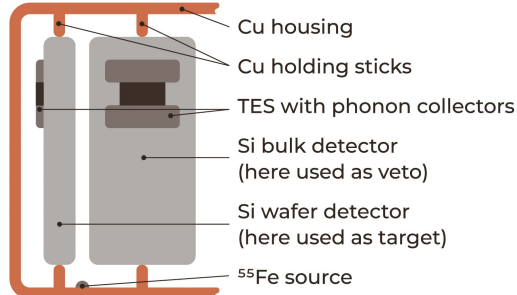
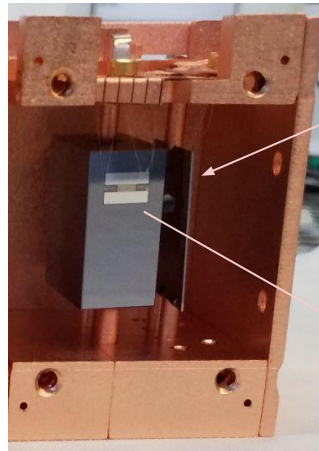
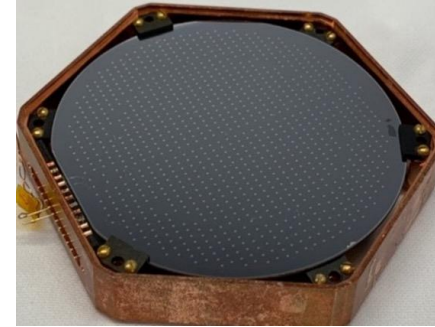


Fig. used in [arXiv:2212.12513](https://arxiv.org/abs/2212.12513)

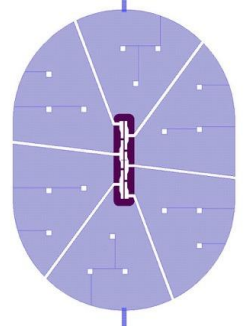
CRESST-III Si



silicon wafer target



SuperCDMS-CPD



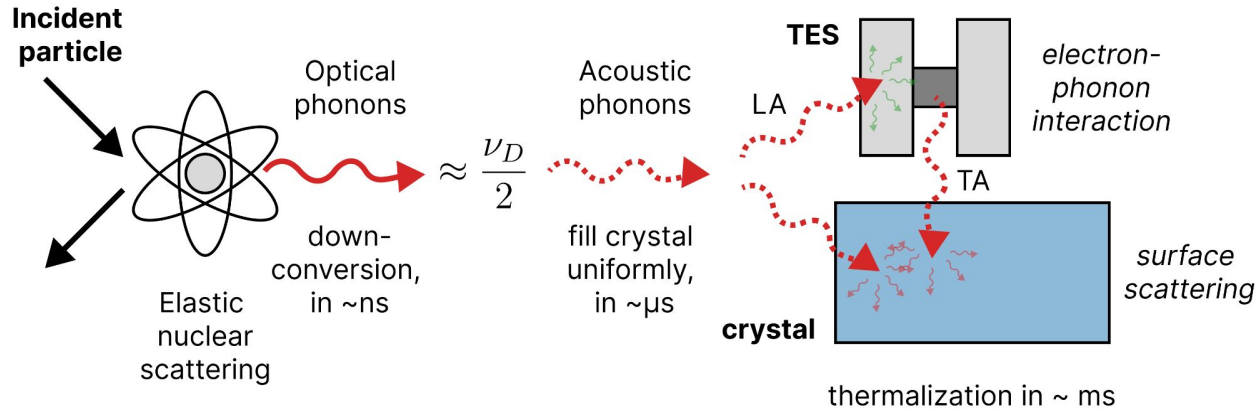
design called "quasiparticle trap assisted electrothermal feedback TES" (QET)

Fig. on the courtesy of Sam Watkins and the SuperCDMS-CPD collaboration, used in [Appl. Phys. Lett. 118, 022601 \(2021\)](https://doi.org/10.1088/0953-4075/118/2/022601)



TES on main crystal

Physics of particle recoils



Particle recoil produces population of high energy phonons that down-convert rapidly.

Crystal is after $\sim\mu\text{s}$ uniformly filled, phonons thermalize within $\sim\text{ms}$.

Share ϵ of athermal phonons are collected by the superconducting film and thermalize there, $(1-\epsilon)$ in the crystal.

Electrothermal response model

System can be modeled with ordinary differential equations (ODEs), as interaction of thermal and electrical circuits:

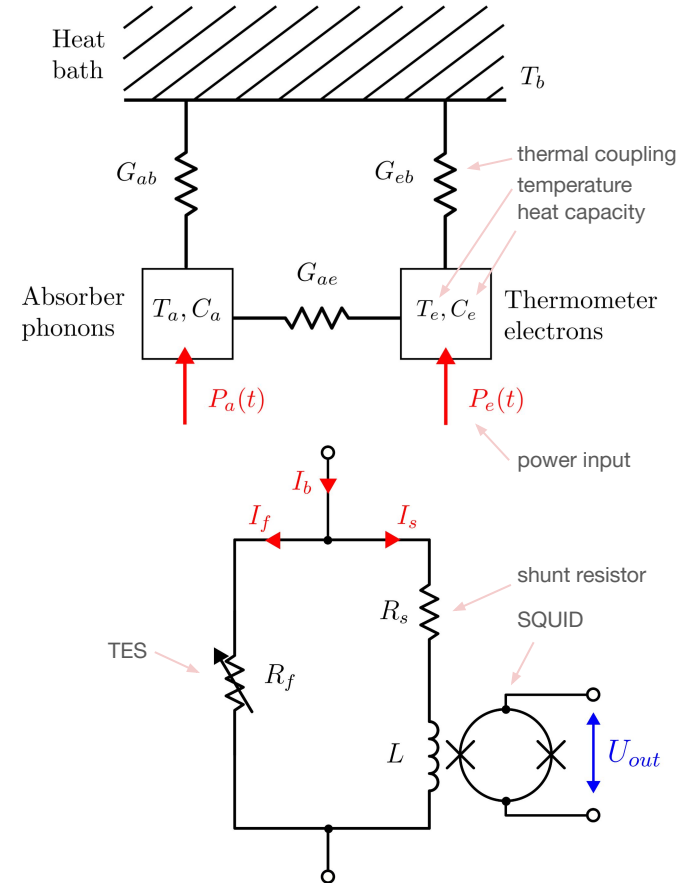
thermal
ODEs

$$C_e \frac{dT_e}{dt} + (T_e - T_a) G_{ea} + (T_e - T_b) G_{eb} = P_e(t),$$

$$C_a \frac{dT_a}{dt} + (T_a - T_e) G_{ea} + (T_a - T_b) G_{ab} = P_a(t),$$

electrical
ODE

$$L \frac{dI_f}{dt} + R_s I_b - (R_f(T_e) + R_s) I_f = 0.$$



Pulse shape

System can be simplified to describe the temperature rise from a small energy deposition $\Delta E \propto \Delta P$:

$$\frac{d}{dt} \begin{pmatrix} \Delta T_e \\ \Delta T_a \end{pmatrix} = - \begin{pmatrix} \frac{G_{ea} + G_{eff}}{C_e} & -\frac{G_{ea}}{C_e} \\ -\frac{G_{ea}}{C_a} & \frac{G_{ea} + G_{ab}}{C_a} \end{pmatrix} \begin{pmatrix} \Delta T_e \\ \Delta T_a \end{pmatrix} + \begin{pmatrix} \frac{\Delta P_e}{C_e} \\ \frac{\Delta P_a}{C_a} \end{pmatrix}$$

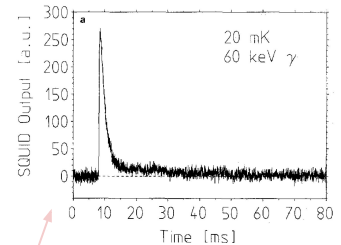
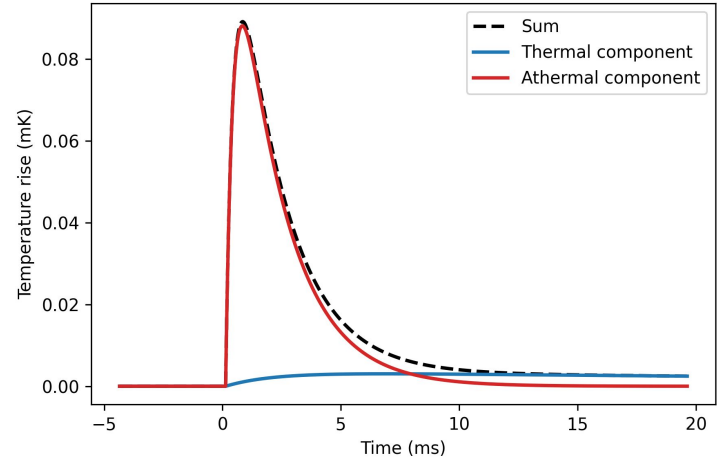
Solutions for temperature rise in thermometer are pulses of ≈ 1 -100 ms length, with two components and three time constants:

$$\Delta T_e(t) = \Theta(t) \left[A_n \left(e^{-t/\tau_n} - e^{-t/\tau_{in}} \right) + A_t \left(e^{-t/\tau_t} - e^{-t/\tau_n} \right) \right]$$

(formulas in appendix)

crystal relaxation time
phonon thermalization time

thermometer relaxation time



measured pulse for comparison

Thermal properties

Debye model (phononic system):

$$G_{ab} \propto T^3$$

$$C_{ph} = \frac{12\pi^4}{5} n_a k_B \left(\frac{T}{\Theta_D} \right)^3$$

Debye temperature

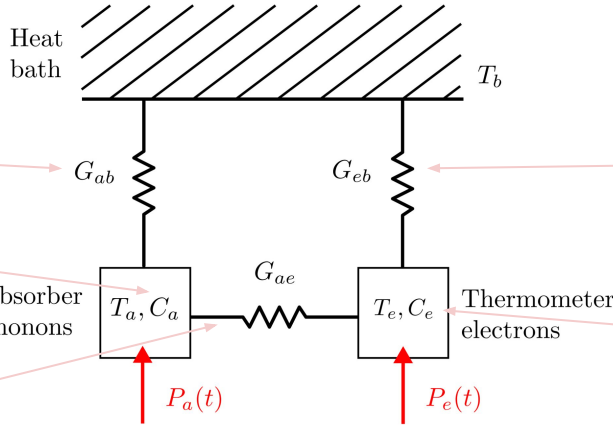
$$G_{ea} = \left(\frac{1}{G_{ep}} + \frac{1}{G_K} \right)^{-1}$$

$$G_{ep} \propto T^5$$

$$G_K \propto T^3$$

electron-phonon coupling (dominating)

Kapitza resistance



Wiedemann Franz law (electronic system):

$$G_{eb} \propto T$$

Sommerfeld model (electronic system):

$$C_e = \frac{\pi^2}{2} n_e k_B \frac{T}{T_F} = \gamma T$$

Sommerfeld constant

$$P_a(t) = (1 - \epsilon) \frac{\Delta E}{\tau_n} \exp\left(\frac{t}{\tau_n}\right)$$

$$P_e(t) = \epsilon \frac{\Delta E}{\tau_n} \exp\left(\frac{t}{\tau_n}\right)$$

Effective phonon thermalization time: $\tau_n = \left(\frac{1}{\tau_{\text{film}}} + \frac{1}{\tau_{\text{crystal}}} \right)^{-1}$

Athermal phonon collection efficiency: $\epsilon = \frac{\tau_{\text{crystal}}}{\tau_{\text{crystal}} + \tau_{\text{film}}}$

Competing thermalization times in film and crystal:

$$\tau_{\text{film}} = \frac{\tau_0}{\bar{\eta}}$$

$$\tau_{\text{crystal}} \propto \frac{V_a}{A_a \Theta_D}$$

$$\tau_0 = \frac{2V_a}{A \langle v_{\perp} \alpha \rangle}$$

Ideal film thermalization:

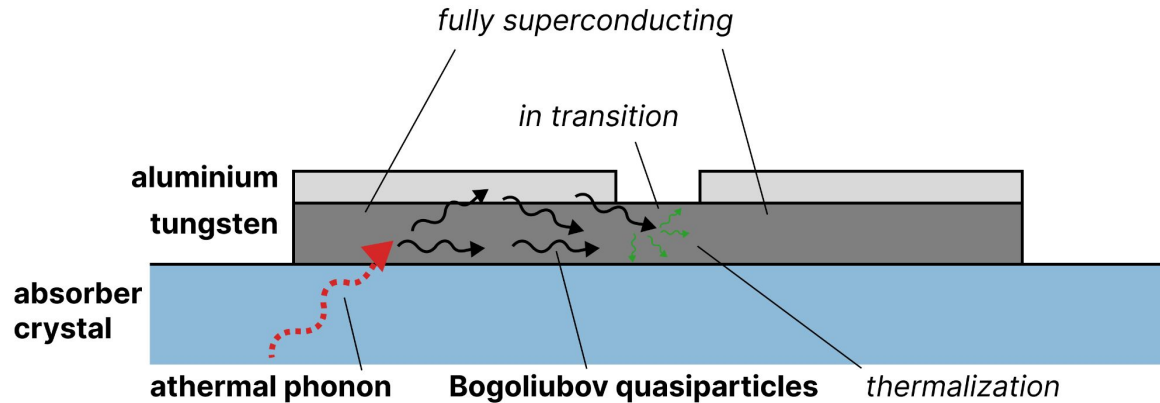
Mode-average phonon velocity, transmission c.:

$$\langle v_{\perp} \alpha \rangle \propto \Theta_D$$

Absorption coeff.:

$$\bar{\eta} \propto T$$

Optimizing athermal phonon collection



$$T \approx 15 \text{ mK} \approx T_{c,W} \ll T_{c,Al}$$

For optimized detectors, sensitivity (pulse height and area) is

$$\propto \varepsilon \Delta E / C_e$$

Collection efficiency ε can be increased without adding heat capacity, through phonon collectors.

Aluminium/tungsten bilayer is kept through proximity effect in superconducting state (exponentially suppressed heat capacity). Scattering **athermal phonons break cooper pairs** that travel diffusively to the uncovered tungsten film.

Noise model

Similarly to the pulse shape model, a model for noise contributions can be derived, depending on the occurring power and voltage fluctuations in the circuit, and the bias heating P_J :

$$C_e \frac{dT_e}{dt} + (T_e - T_b)G_{eb} = \Delta P_{e,noise} + P_J,$$
$$-L \frac{dI_f}{dt} + R_S I_b - (R_f(T_f) + R_S)I_f = \Delta U_{noise},$$

Solution of the system is a transition matrix S , which translates fundamental fluctuations to observable currents in the readout circuit I_f :

$$\begin{pmatrix} \Delta T_e \\ \Delta I_f \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} s_{11}(w) & s_{12}(w) \\ s_{21}(w) & s_{22}(w) \end{pmatrix} \begin{pmatrix} \Delta P_e \\ \Delta U \end{pmatrix}$$

(formulas in appendix)

Fundamental sensor noise contributions

1) Electrical, Johnson noise from movement of electrons in shunt with temperature T_s and film in superconducting transition with resistance R_{f0} :

$$|\Delta U|_{J_s}^2 = 4k_B T_s R_s \quad |\Delta U|_{J_f}^2 = 4k_B T_e R_{f0}$$

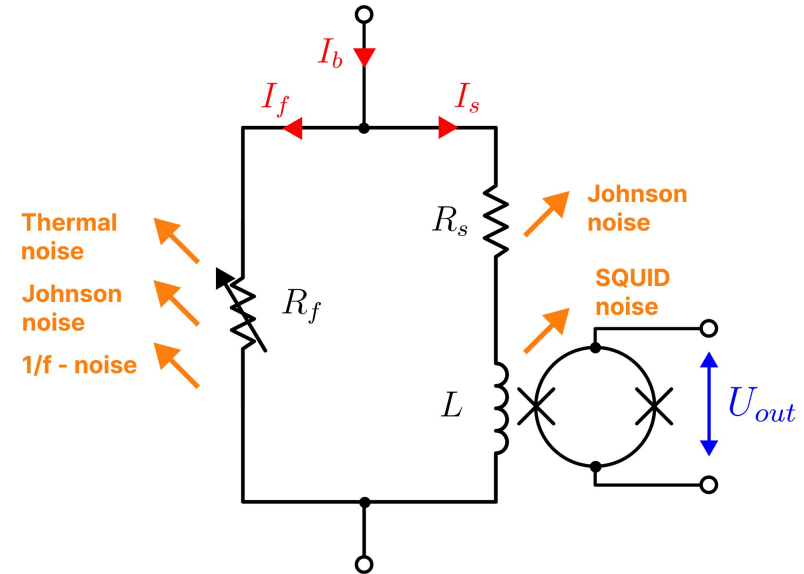
2) Phonon noise from thermal fluctuations between TES and heat bath:

$$|\Delta P_e|_{ph}^2 = 4k_B T_e^2 G_{eb} \frac{2}{5} \frac{1 - \left(\frac{T_b}{T_{e0}}\right)^5}{1 - \left(\frac{T_b}{T_{e0}}\right)^2}$$

3) TES-intrinsic 1/f excess-flicker noise:

$$|\Delta P_e|_{flicker}^2 = \frac{\left(\frac{\Delta R_{f,flicker}}{R_{f0}}\right)^2 R_{f0}^2}{\omega^\alpha} I_{f0}^2$$

4) “White” SQUID noise.



Intrinsic excess noise

Internal Thermal Fluctuation Noise (ITFN): thermometer is spatially extended object, consists therefore of a system of connected heat capacities \Rightarrow thermal fluctuations exist between them. Higher normal resistance can make it worse.

Excess electrical noise*: spectral shape similar to Johnson noise, larger for lower normal resistance and operation low in the transition, affected by homogeneity of film and magnetic field.

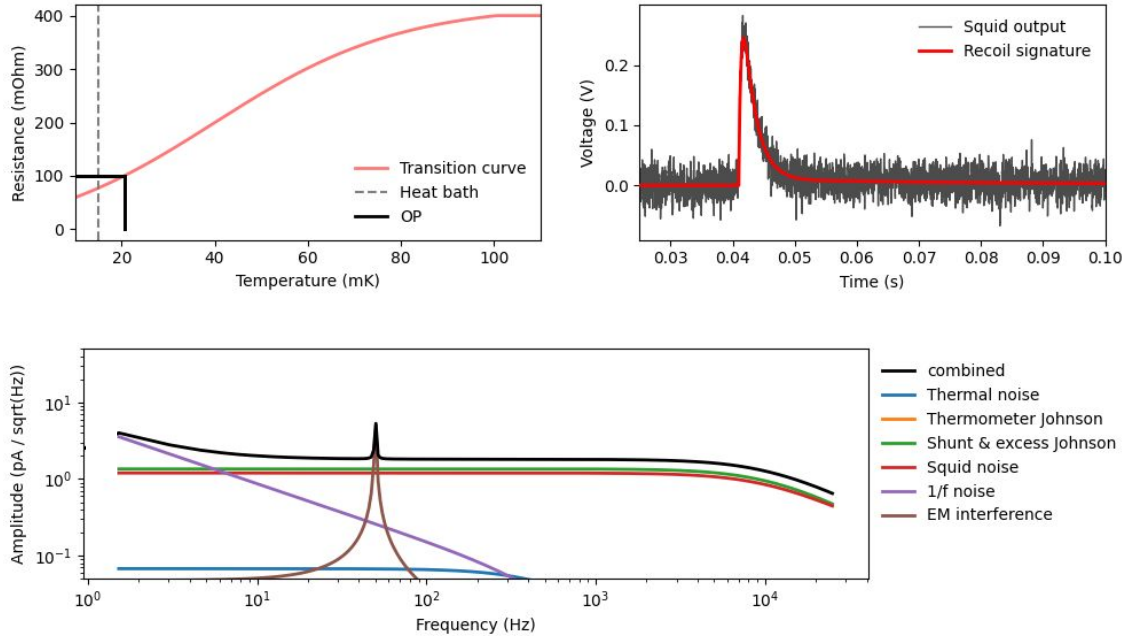
1/f-noise*: spectral shape rises steeply towards lower frequencies, cutoff $\ll 1$ Hz, often correlated with excess electrical noise.

Telegraph noise*: associated with certain bias values and points in transition curve. Possibly due to structure reordering in metallic film.

Possible sources: non-equilibrium effects, fundamental effects of superconductivity or electron-phonon interaction.

***Origin of this noise is not fully understood.**

Exemplary detector response

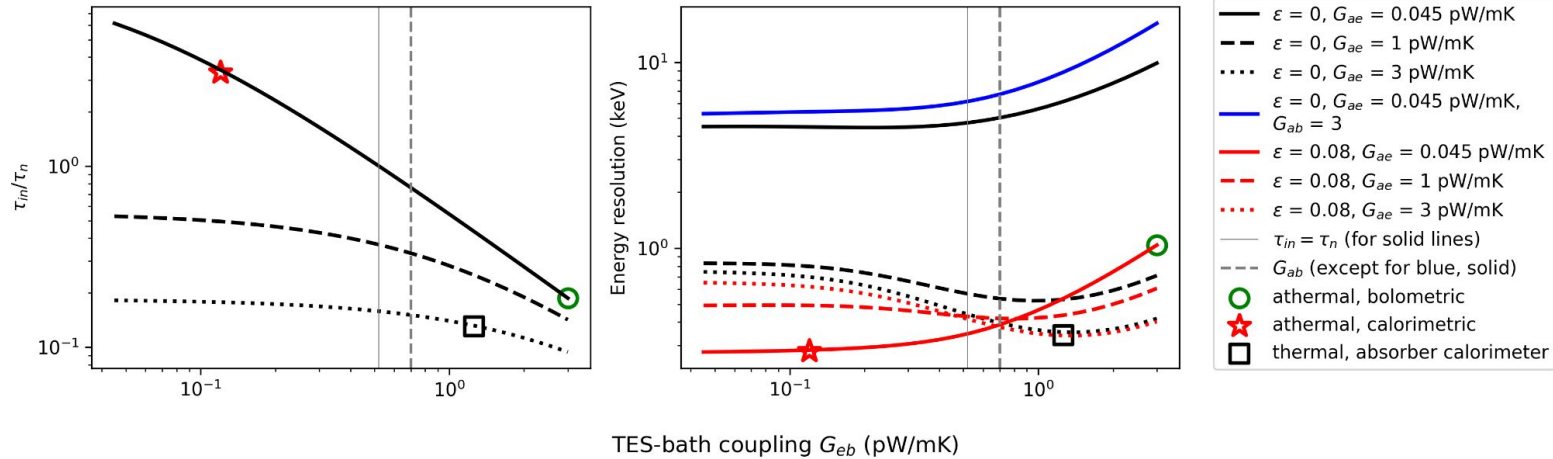


Simulation of detector with silicon target and iridium/gold bilayer TES, based on introduced response and noise models.

Transition curves of modern devices are much steeper, with a width of ≈ 1 -2 mK.

Optimizing thermal couplings

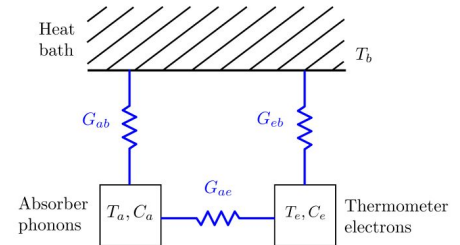
variable: \bar{G}_{eb} , \bar{G}_{ae} , ε ; fixed: $C_e = 0.001$ pJ/mK, $C_a = 0.038$ pJ/mK, $\bar{G}_{ab} = 0.7$ pW/mK (except for blue, solid)



Ratio of thermometer relaxation time τ_{in} to phonon thermalization time τ_n separates **bolometric** (measuring power) and **calorimetric** (measuring energy) modi.

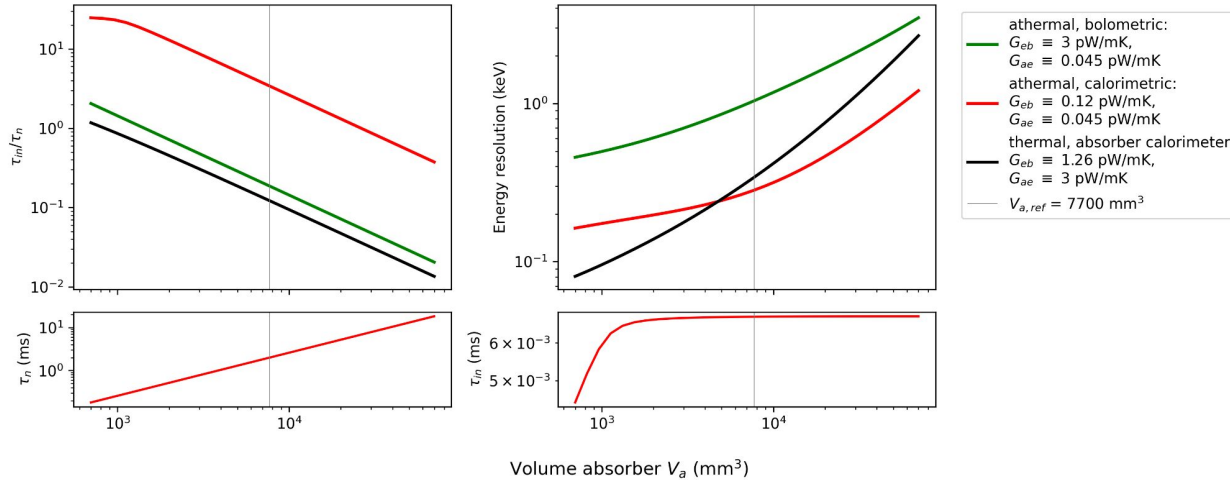
Most modern devices are **athermal calorimeters**.

Small, thermal detectors can reach comparable thresholds, but scale differently.



Scaling absorber volume

variable: $\tau_n \propto V_a$, $C_a \propto V_a$; fixed: G_{ae} , G_{eb} , $C_e = 0.001$ pJ/mK, $G_{ab} = 0.7$ pW/mK, $C_a(V_{a,ref}) = 0.038$ pJ/mK, $\tau_n(V_{a,ref}) = 2$ ms, $\epsilon = 0.08$

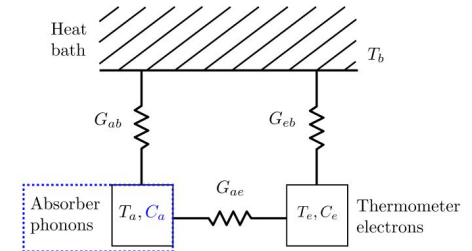


Threshold of **thermal** detector scales $\propto V_a$,

athermal detector scales $\propto V_a^{2/3}$.

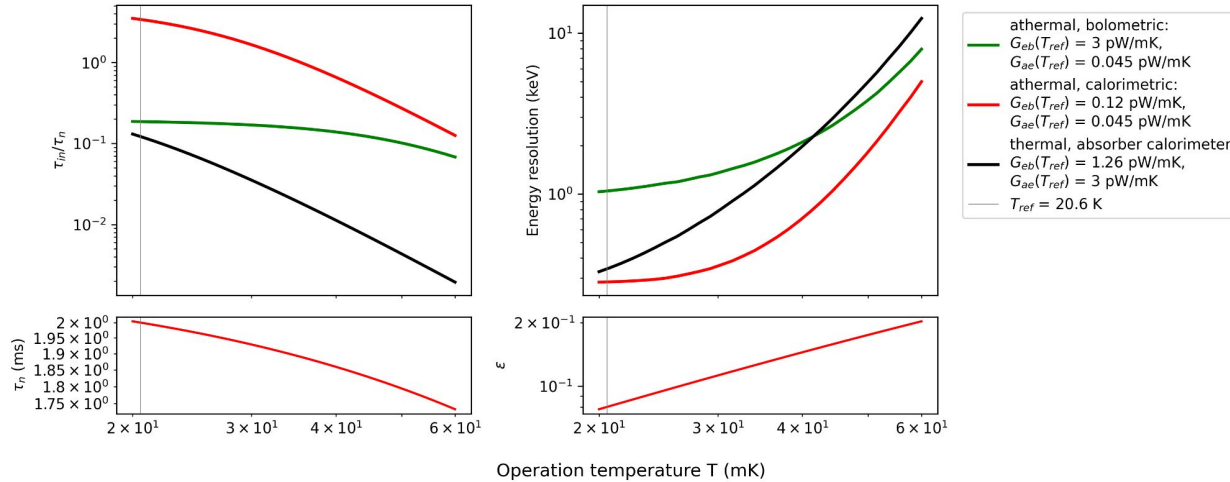
⇒ Operating large target with low threshold challenging,

lower threshold achievable by consequently **lowering target mass!**



Scaling operation temperature

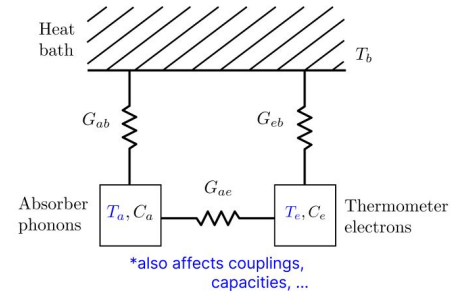
$$\tau_n(T_{ref}) = 2 \text{ ms}, C_a(T_{ref}) = 0.038 \text{ pJ/mK}, C_e(T_{ref}) = 0.001 \text{ pJ/mK}, G_{ab}(T_{ref}) = 0.7 \text{ pW/mK}, \epsilon(T_{ref}) = 0.08$$



Thermal couplings scale strongly with temperature: $G_{ae} \propto T^5$.

Calorimetric modus only possible **at lowest $\approx 10\text{-}30 \text{ mK}$ temperatures!**

Heat capacities decrease with temperature \Rightarrow sensitivity improves.



Low energy excess events

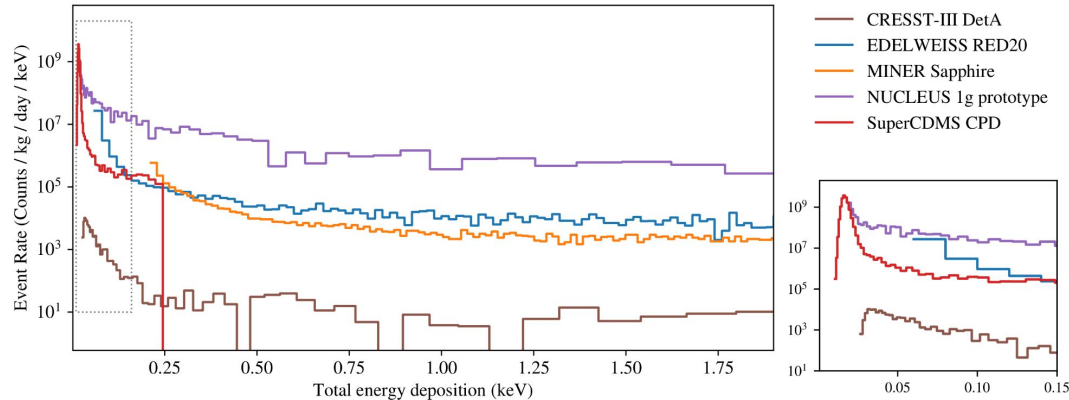


Fig. from EXCESS workshop summary [SciPost Phys. Proc. 9, 001 \(2022\)](#)

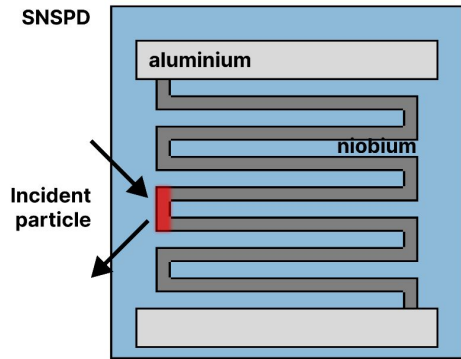
Experiments observe steeply rising non-ionising background towards lower energies $\approx < 100$ eV. Event rate ...

- ... apparently uncorrelated with detector mass, surface, surroundings and temperature.
- ... decays after cool down of experiment, on time scale of $O(\text{months})^*$.

Currently most probable hypothesis: **stress effect**, e.g. on crystal-thermometer interface.

Ongoing community-organized workshop series to discuss the excesses: [EXCESS workshop](#).

Relations to other quantum technologies

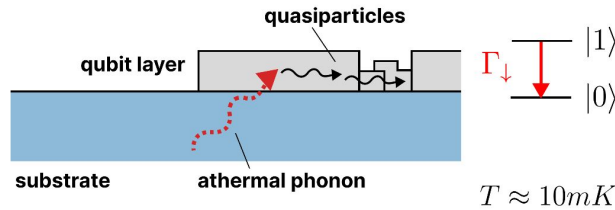
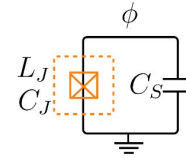


$T \approx 1 - 10K$

Superconducting nanowire single-photon detector (SNSPD):

- Wire is operated slightly below T_c .
- Difference to TES: sensor is target.
- Recoil needs to warm wire only locally for measurable effect.
- Can reach lower energy resolution.
- Cannot collect significant exposure.

Transmon qubit



$T \approx 10mK$

Radiation and quasiparticle poisoning are harmful error source for superconducting qubits.

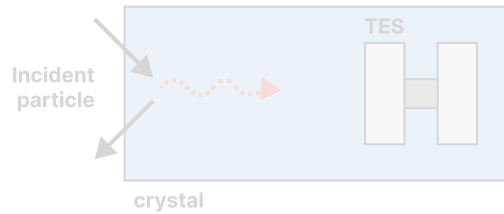
Phonon population in substrate, induced by radiation or possibly the low energy excess, can break cooper pairs in qubit layer.

Similar effect as in phonon collectors of TES.

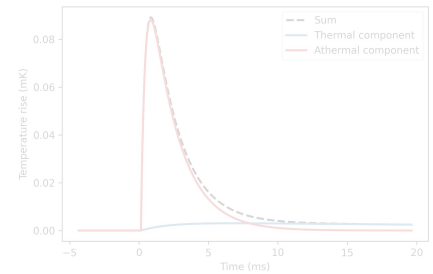
Fig. adapted from [Nature Physics volume 18, pages 107–111 \(2022\)](#)

Qubit errors through radiation discussed in [Nature volume 594, pages 369–373 \(2021\)](#)
[Nature Communications volume 12, Article number: 2733 \(2021\)](#)
[Nature Physics volume 18, pages 107–111 \(2022\)](#)
[The European Physical Journal C volume 83, Article number: 94 \(2023\)](#)

Qubit errors through quasiparticle excess discussed in [Nature Physics volume 18, pages 145–148 \(2022\)](#)
[arXiv:2208.02790](#)



Summary



- Cryogenic detectors with superconducting thermometers are currently the **most sensitive devices for sub-GeV/C² DM searches.**
- Lowest achieved nuclear recoil energy threshold is **10 eV.**
- Sensitivity in calorimetric mode is dominated by capability to **collect athermal phonons** from target.
- Similar challenges in fabrication and background mitigation with other superconducting technologies: SNSPDs, transmon qubits, ...
- Devices are continuously improved, new phenomena continue to be discovered and research is done to understand them: **low energy excess events and excess noise.**

Appendices

Formulas for pulse shape model

$$= -I_f^2 \frac{(R_{f0} - R_s)}{(R_{f0} + R_s)} \frac{dR_f}{dT_e} \Big|_{T_{e0}} \quad G_{eff} = G_{eb} + G_{ETF}$$

$$\stackrel{\text{def}}{=} -G_{ETF}.$$

$$\lambda_{1,2} = \frac{-a \mp \sqrt{a^2 - 4b}}{2} \stackrel{\text{def}}{=} -\frac{1}{\tau_{1,2}} \quad \left(\stackrel{\text{def}}{=} -\frac{1}{\tau_{in,t}} \stackrel{\text{def}}{=} -s_{in,t} \right),$$

with

$$a \stackrel{\text{def}}{=} \frac{(G_{ea} + G_{eff})C_a + (G_{ea} + G_{ab})C_e}{C_e C_a}, \quad b \stackrel{\text{def}}{=} \frac{(G_{ea} + G_{eff})(G_{ea} + G_{ab}) - G_{ea}^2}{C_e C_a}.$$

$$\alpha_{1,2} \stackrel{\text{def}}{=} 1 + \frac{G_{ab}}{G_{ea}} + \lambda_{1,2} \frac{C_a}{G_{ea}}.$$

$$A_n \stackrel{\text{def}}{=} \frac{\frac{\Delta E}{\tau_n}}{\left(1 - \frac{\alpha_2}{\alpha_1}\right)} \left(\alpha_2 \frac{(1 - \epsilon)}{C_a} - \frac{\epsilon}{C_e} \right) \left(\frac{1}{\tau_n} - \frac{1}{\tau_1} \right)^{-1}$$

$$= -\frac{P_0(s_{in} - (G_{ab}/C_a))}{\epsilon(s_{in} - s_t)(s_{in} - s_n)} \left(\frac{s_t - (G_{ab}/C_a)}{G_{eff} - (C_e/C_a)G_{ab}} - \frac{\epsilon}{C_e} \right)$$

$$A_t \stackrel{\text{def}}{=} -\frac{\frac{\Delta E}{\tau_n}}{\left(1 - \frac{\alpha_1}{\alpha_2}\right)} \left(\alpha_1 \frac{(1 - \epsilon)}{C_a} - \frac{\epsilon}{C_e} \right) \left(\frac{1}{\tau_n} - \frac{1}{\tau_2} \right)^{-1}$$

$$= \frac{P_0(s_t - (G_{ab}/C_a))}{\epsilon(s_t - s_{in})(s_t - s_n)} \left(\frac{s_{in} - (G_{ab}/C_a)}{G_{eff} - (C_e/C_a)G_{ab}} - \frac{\epsilon}{C_e} \right)$$

Definitions for noise matrix elements

$$\tau \stackrel{\text{def}}{=} \frac{C_e}{G_{eb}},$$

$$\tau_I \stackrel{\text{def}}{=} \frac{\tau}{(1 - \mathcal{L}_I)},$$

$$\tau_{el} \stackrel{\text{def}}{=} \frac{L}{R_{f0} + R_S},$$

thermometer resistance
in operation point

$$\mathcal{L}_I \stackrel{\text{def}}{=} \frac{I_{f0}^2}{G_{eb}} \left. \frac{dR_f}{dT_e} \right|_{T_{e0}},$$

current through
thermometer
in operation point

base temperature of
thermometer
in operation point

Formulas for intrinsic noise matrix elements

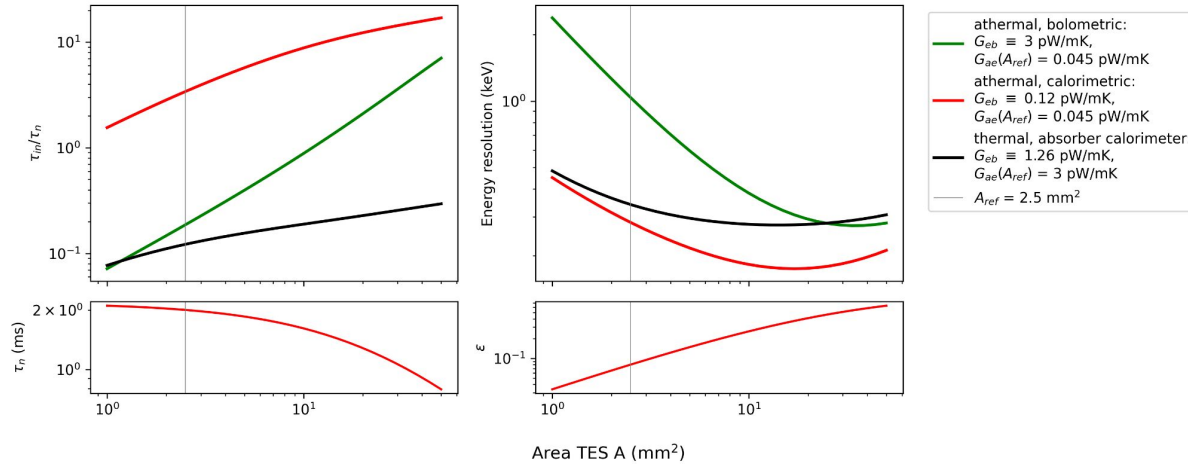
$$\begin{aligned}
 s_{21}^{int}(w) &= \frac{-\frac{G_{eb}\mathcal{L}_I}{I_{f0}}}{\left(\frac{1}{(\tau)_{int}} + 2\pi wi\right) C_e \left(\frac{1}{\tau_{el}} + 2\pi wi\right) L - \frac{G_{eb}\mathcal{L}_I}{I_{f0}} (I_{f0}R_s - I_{f0}R_{f0})_{int}} \\
 &= -\frac{1}{I_{f0}} \left[\frac{L\tau}{(\tau)_{int}\tau_{el}\mathcal{L}_I} - \frac{(I_{f0}R_s - I_{f0}R_{f0})_{int}}{I_{f0}} \right. \\
 &\quad \left. + 2\pi wi \frac{L\tau}{\mathcal{L}_I} \left(\frac{1}{(\tau)_{int}} + \frac{1}{\tau_{el}} \right) - \frac{4\pi^2 w^2 \tau L}{\mathcal{L}_I} \right]^{-1}, \\
 s_{22}^{int}(w) &= \frac{\left(\frac{1}{(\tau)_{int}} + 2\pi wi\right) C_e}{\left(\frac{1}{(\tau)_{int}} + 2\pi wi\right) C_e \left(\frac{1}{\tau_{el}} + 2\pi wi\right) L - \frac{G_{eb}\mathcal{L}_I}{I_{f0}} (I_{f0}R_s - I_{f0}R_{f0})_{int}} \\
 &= -s_{21}^{int}(w) I_{f0} \frac{1}{\mathcal{L}_I} (1 + 2\pi wi\tau).
 \end{aligned}$$

Formulas for external noise matrix elements

$$\begin{aligned}
 s_{21}^{ext}(w) &= \frac{-\frac{G_{eb}\mathcal{L}_I}{I_{f0}}}{\left(\frac{1}{(\tau_I)_{ext}} + 2\pi wi\right) C_e \left(\frac{1}{\tau_{el}} + 2\pi wi\right) L - \frac{G_{eb}\mathcal{L}_I}{I_{f0}}(-2R_{f0}I_{f0})_{ext}} \\
 &= -\frac{1}{I_{f0}} \left[\frac{L\tau}{(\tau_I)_{ext}\tau_{el}\mathcal{L}_I} - \frac{(-2R_{f0}I_{f0})_{ext}}{I_{f0}} \right. \\
 &\quad \left. + 2\pi wi \frac{L\tau}{\mathcal{L}_I} \left(\frac{1}{(\tau_I)_{ext}} + \frac{1}{\tau_{el}} \right) - \frac{4\pi^2 w^2 \tau L}{\mathcal{L}_I} \right]^{-1}, \\
 s_{22}^{ext}(w) &= \frac{\left(\frac{1}{(\tau_I)_{ext}} + 2\pi wi\right) C_e}{\left(\frac{1}{(\tau_I)_{ext}} + 2\pi wi\right) C_e \left(\frac{1}{\tau_{el}} + 2\pi wi\right) L - \frac{G_{eb}\mathcal{L}_I}{I_{f0}}(-2R_{f0}I_{f0})_{ext}} \\
 &= s_{21}^{ext}(w) I_{f0} \frac{\mathcal{L}_I - 1}{\mathcal{L}_I} (1 + 2\pi wi\tau_I).
 \end{aligned}$$

Optimizing thermometer area

variable: $\varepsilon(A)$, $\tau_n(A)$, $C_e \propto A$, $G_{ae} \propto A$; fixed: G_{eb} , $C_a = 0.038$ pJ/mK, $G_{ab} = 0.7$ pW/mK, $C_e(A_{ref}) = 0.001$ pW/mK, $\tau_n(A_{ref}) = 2$ ms, $\varepsilon(A_{ref}) = 0.08$

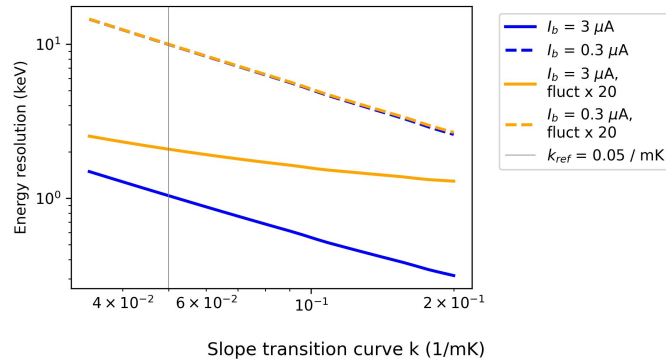


Larger thermometer increases TES heat capacity linearly, collection efficiency (slightly) nonlinearly, and TES relaxation time (almost) linearly,

⇒ for a given detector, a **broad optimum** for it's TES size exists!

Optimizing transition curves

$\tau_n = 2$ ms, $C_a = 0.038$ pJ/mK, $C_e = 0.001$ pJ/mK, $G_{ab} = 0.7$ pW/mK,
 $\varepsilon = 0.08$, $G_{eb} = 3$ pW/mK, $G_{ae} = 0.045$ pW/mK



Steeper transition curve improves signal from TES resistance changes against electrical sensor noise.

⇒ **the steeper, the better** (with some practical stability constraints ...)

However, if TES-intrinsic noise dominates over electrical noise, a steeper curve cannot improve the sensitivity further.