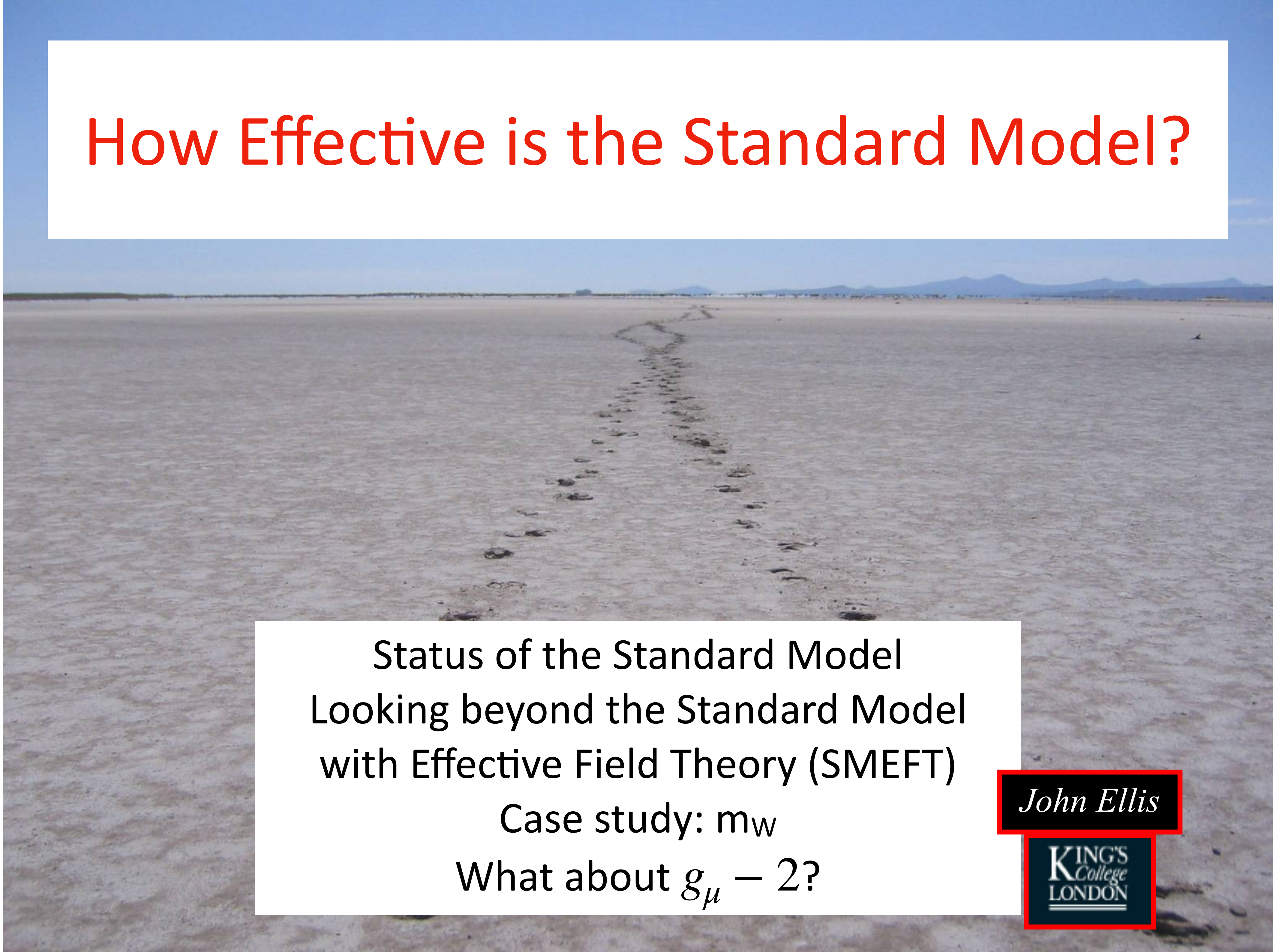
How Effective is the Standard Model?



Status of the Standard Model
Looking beyond the Standard Model
with Effective Field Theory (SMEFT)
Case study: m_W
What about $g_\mu - 2$?

John Ellis
King's College London

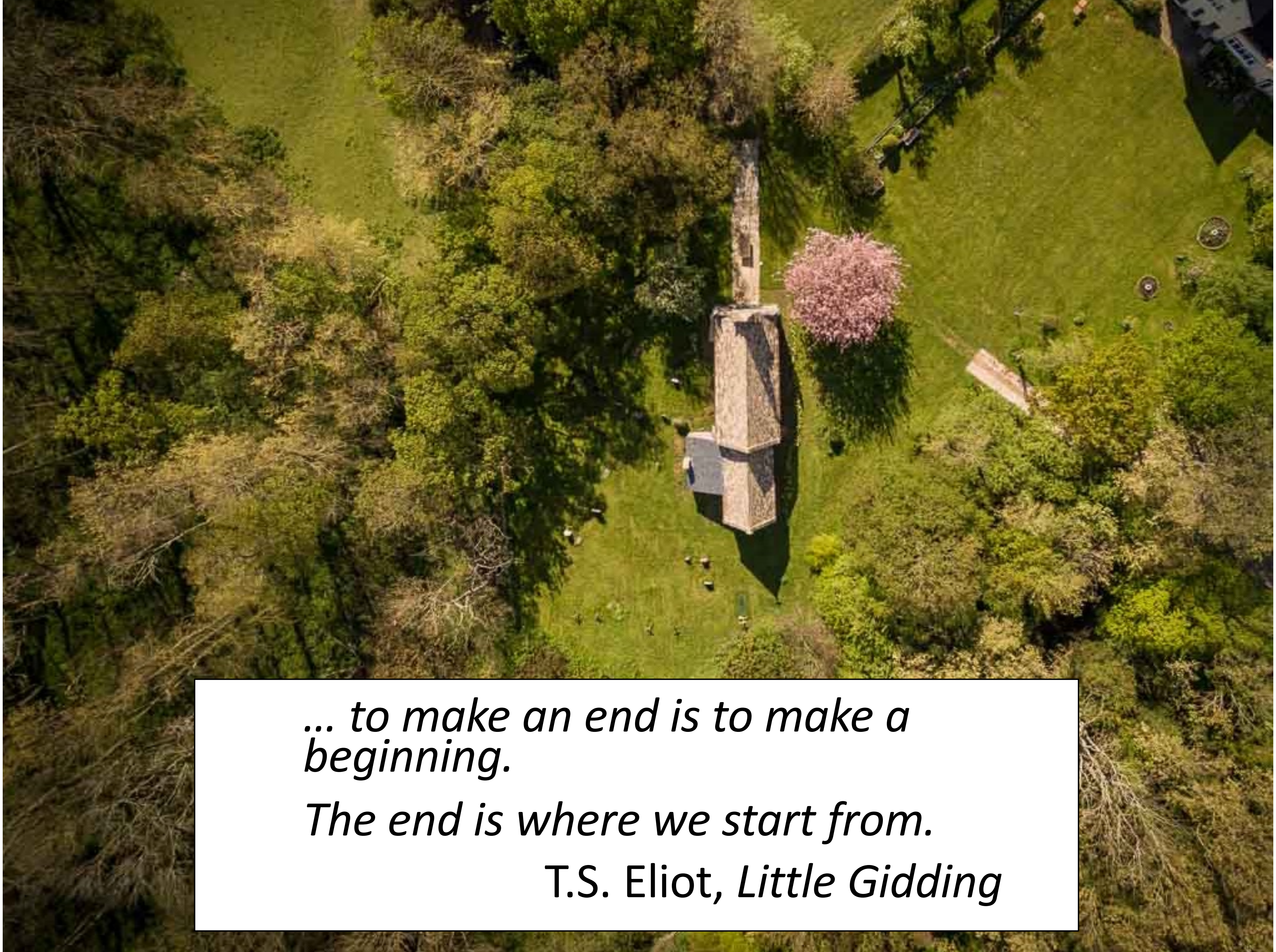
Summary of the Standard Model

- Particles and SU(3) × SU(2) × U(1) quantum numbers:

L_L	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	$(\mathbf{1},\mathbf{2},\text{-}1)$
E_R	$e^-_R \, , \, \mu^-_R \, , \, \tau^-_R$	$(\mathbf{1},\mathbf{1},\text{-}2)$
Q_L	$\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L$	$(\mathbf{3},\mathbf{2},+1/3)$
U_R	$u_R \, , \, c_R \, , \, t_R$	$(\mathbf{3},\mathbf{1},+4/3)$
D_R	$d_R \, , \, s_R \, , \, b_R$	$(\mathbf{3},\mathbf{1},\text{-}2/3)$

- Lagrangian:

$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$	gauge interactions	Tested < 0.1% before LHC
$+ \; i\bar{\psi} \not{D} \psi + h.c.$	matter fermions	
$+ \; \psi_i y_{ij} \psi_j \phi + h.c.$	Yukawa interactions	Testing now in progress
$+ \; \lvert D_\mu \phi \rvert^2 - V(\phi)$	Higgs potential	



… to make an end is to make a beginning.

The end is where we start from.

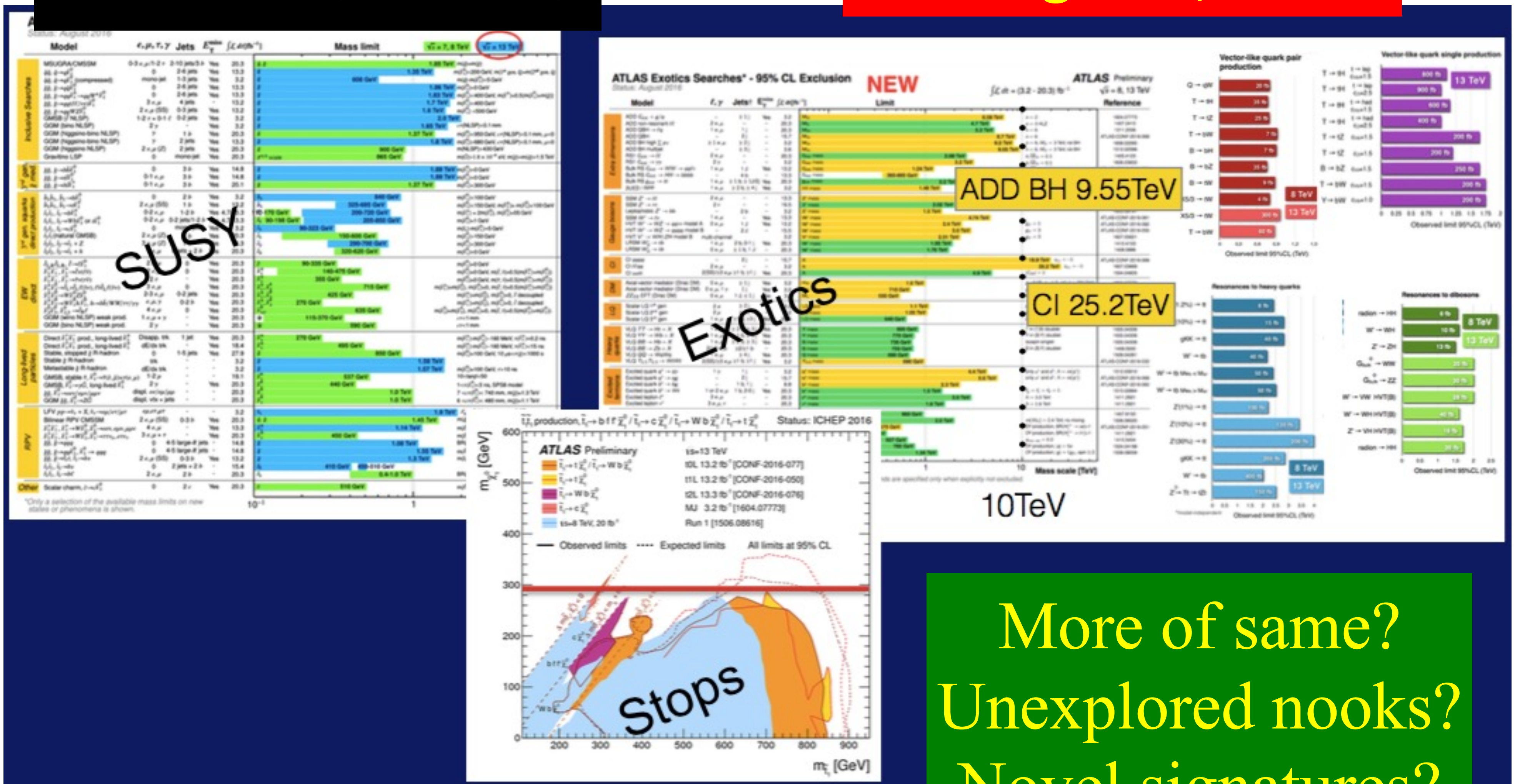
T.S. Eliot, *Little Gidding*

- « Empty » space is unsta LHC
- Dark matter LHC
- Origin of matter LHC
- Sizes of masses LHC
- Masses of neutrinos
- Inflation
- Quantum gravity
- …

Nothing (yet) at the LHC

No supersymmetry

Nothing else, either



More of same?
Unexplored nooks?
Novel signatures?

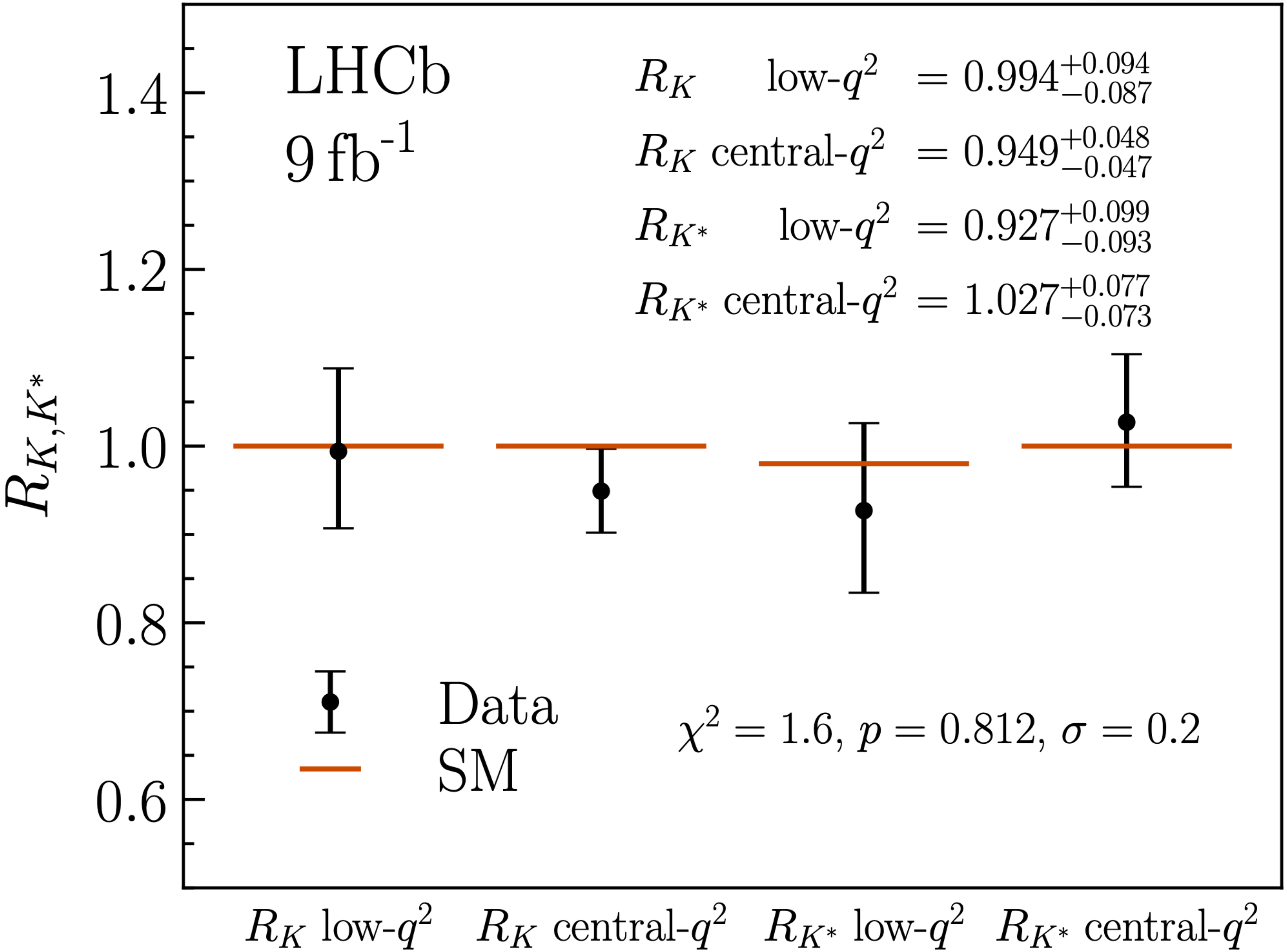
Everything about Higgs is Puzzling

$$\mathcal{L} = yH\psi\overline{\psi} + \mu^2|H|^2 - \lambda|H|^4 - V_0 \quad + \ldots$$

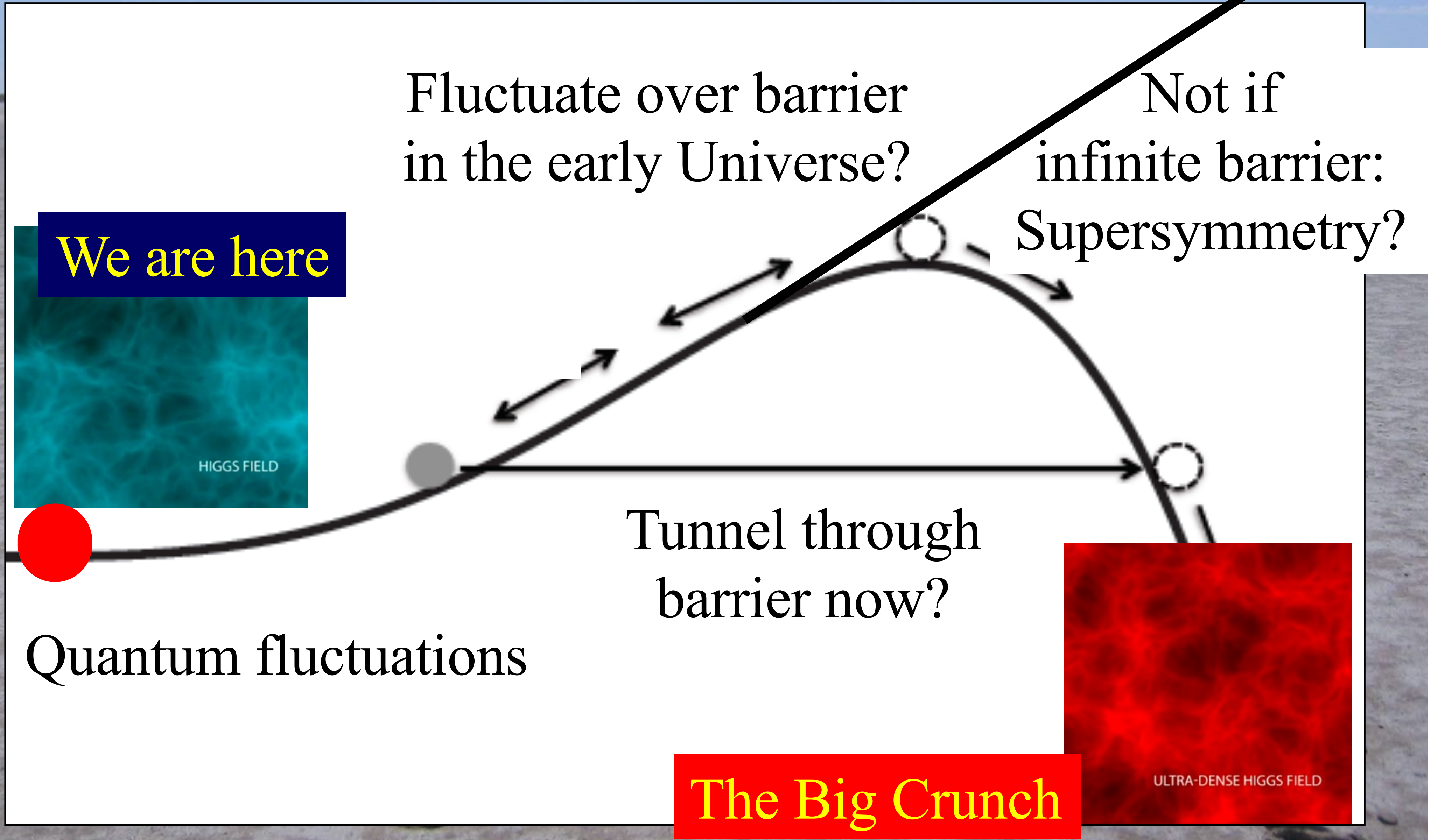
- Pattern of Yukawa couplings y:
  - **Flavour problem**
- Magnitude of mass term μ:
  - **Naturalness/hierarchy problem**
- Magnitude of quartic coupling λ:
  - **Stability of electroweak vacuum**
- Cosmological constant term V_0:
  - **Dark energy**

Higher-dimensional interactions?

Sic Transit Gloria R_K Anomaliae



Will the Universe Collapse? Should it have Collapsed already?



Is Empty Space Unstable?

- Dependence of instability scale on masses of Higgs boson and top quark, and strong coupling:

$$\mathrm{Log}_{10}\frac{\Lambda}{\mathrm{GeV}} = 10.5 - 1.3\left(\frac{m_t}{\mathrm{GeV}} - 172.6\right) + 1.1\left(\frac{m_H}{\mathrm{GeV}} - 125.1)\right) + 0.6\left(\frac{\alpha_s(m_Z) - 0.1179}{0.0009}\right)$$

Buttazzo et al, arXiv:1307.3536;
Franceschini et al, 2203.17197

- New CMS value of m_t : CMS Collaboration, April 2022

$$m_t = 171.77 \pm 0.38\,\mathrm{GeV}$$

- Particle Data Group values:

$$m_H = 125.25 \pm 0.17\,\mathrm{GeV}, \quad \alpha_s(m_Z) = 0.1179 \pm 0.0009$$

- Instability scale:

$$\mathrm{Log}_{10}\frac{\Lambda}{\mathrm{GeV}} = 11.7 \pm 0.8$$

- Dominant uncertainties those in α_s and m_t

Looking Beyond the Standard Model with the SMEFT

"...the direct method may be used...but indirect methods will be needed in order to secure victory...."

"The direct and the indirect lead on to each other in turn. It is like moving in a circle...."

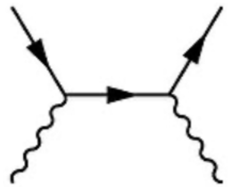
Who can exhaust the possibilities of their combination?"

Sun Tzu

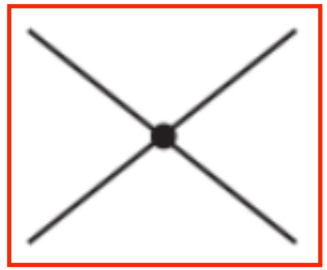
Effective Field Theories (EFTs)

a long and glorious History

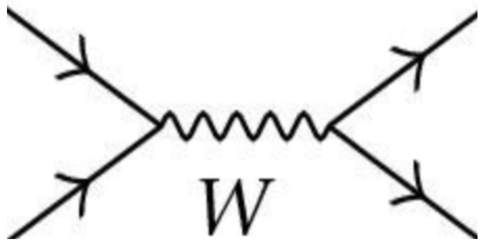
- 1930's: "Standard Model" of QED had d=4



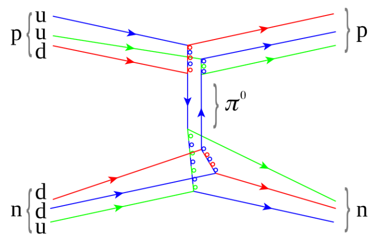
- **Fermi's four-fermion theory of the weak force**



- Dimension-6 operators: form = S, P, V, A, T?
  – Due to exchanges of massive particles?
- V-A ➔ massive vector bosons ➔ gauge theory



- Yukawa's meson theory of the strong N-N force
  – Due to exchanges of mesons? ➔ pions



- Chiral dynamics of pions: (∂π∂π)ππ clue ➔ QCD

Standard Model Effective Field Theory

a more powerful way to analyze the data

- Assume the Standard Model Lagrangian is correct (quantum numbers of particles) but incomplete
- Look for additional interactions between SM particles due to exchanges of heavier particles
- Analyze Higgs data together with electroweak precision data and top data
- Most efficient way to extract largest amount of information from LHC and other experiments
- **Model-independent way to look for physics beyond the Standard Model (BSM)**

Dimension-6 SMEFT Operators

- Including 2- and 4-fermion operators
- Different colours for different data sectors
- Grey cells violate SU(3)^5 symmetry
- Important when including top observables

X^3		H^6 and H^4D^2		ψ^2H^3	
$\mathcal{O}_G$	$f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}_p e_r H)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
$\mathcal{O}_W$	$\varepsilon^{IJK}W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\star(H^\dagger D_\mu H)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$				

X^2H^2		ψ^2XH		ψ^2H^2D	
$\mathcal{O}_{HG}$	$H^\dagger H\, G^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{Hl}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$\mathcal{O}^{(3)}_{Hl}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{HW}$	$H^\dagger H\, W^I_{\mu\nu}W^{I\mu\nu}$		$\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G^A_{\mu\nu}$	$\mathcal{O}_{He}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$		$\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{Hq}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{HB}$	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$		$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$	$\mathcal{O}^{(3)}_{Hq}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$		$\sigma^{\mu\nu}T^A d_r)H\, G^A_{\mu\nu}$	$\mathcal{O}_{Hu}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{HWB}$	$H^\dagger\tau^I H\, W^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dW}$	$\sigma^{\mu\nu}d_r)\tau^I H\, W^I_{\mu\nu}$	$\mathcal{O}_{Hd}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

Anomalous magnetic moments

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}^{(1)}_{qq}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}^{(3)}_{qq}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}^{(1)}_{lq}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}^{(3)}_{lq}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}^{(1)}_{qu}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}^{(1)}_{ud}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}^{(8)}_{qu}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}^{(8)}_{ud}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}^{(1)}_{qd}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}^{(8)}_{qd}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

Flavour anomalies

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(q^{\gamma j}_s)^T C l^k_t\right]$
$\mathcal{O}^{(1)}_{quqd}$	$(\bar{q}^j_p u_r)\varepsilon_{jk}(\bar{q}^k_s d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(u^\gamma_s)^T C e_t\right]$
$\mathcal{O}^{(8)}_{quqd}$	$(\bar{q}^j_p T^A u_r)\varepsilon_{jk}(\bar{q}^k_s T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(q^{\gamma m}_s)^T C l^n_t\right]$
$\mathcal{O}^{(1)}_{lequ}$	$(\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(u^\gamma_s)^T C e_t\right]$
$\mathcal{O}^{(3)}_{lequ}$	$(\bar{l}^j_p\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}^k_s\sigma^{\mu\nu}u_t)$		

Baryon decay

JE, Madigan, Mimasu, Sanz & You, arXiv:2012.02779

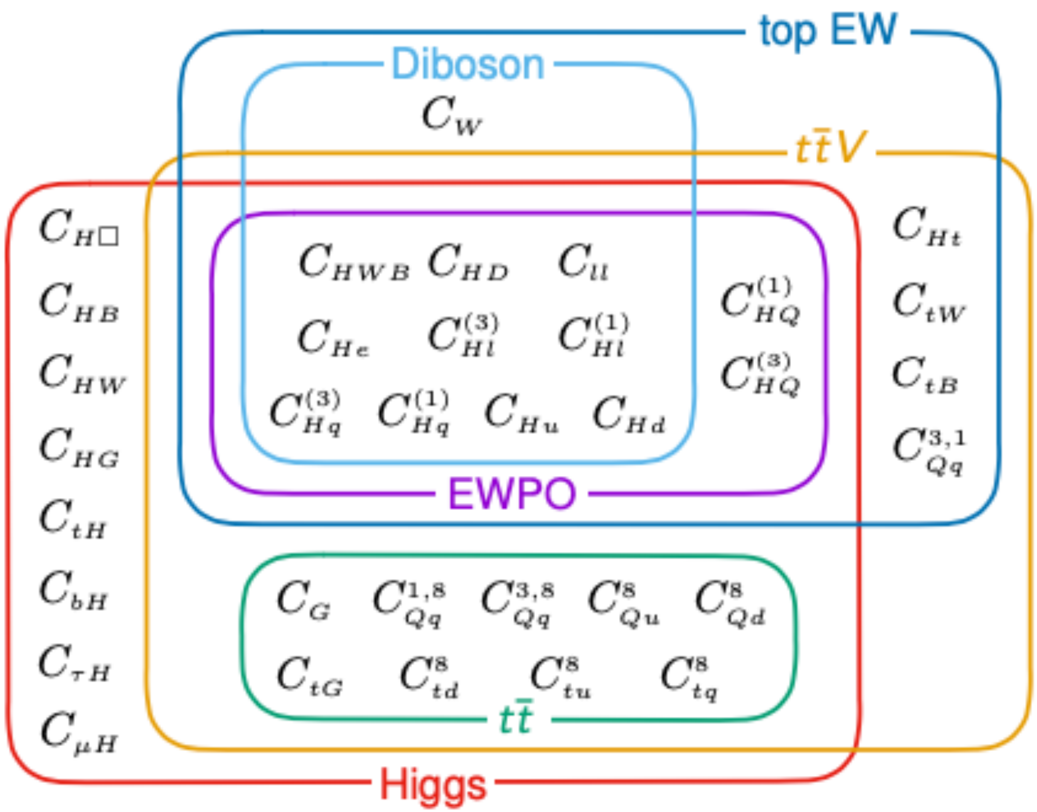
Global SMEFT Fit

to Top, Higgs, Diboson, Electroweak Data

JE, Madigan, Mimasu, Sanz & You, arXiv:2012.02779

- Global fit to dimension-6 operators using precision electroweak data, W^+W^- at LEP, top, Higgs and diboson data from LHC Runs 1, 2
- Search for BSM
- Constraints on BSM
  - At tree level
  - At loop level

341 measurements included in global analysis

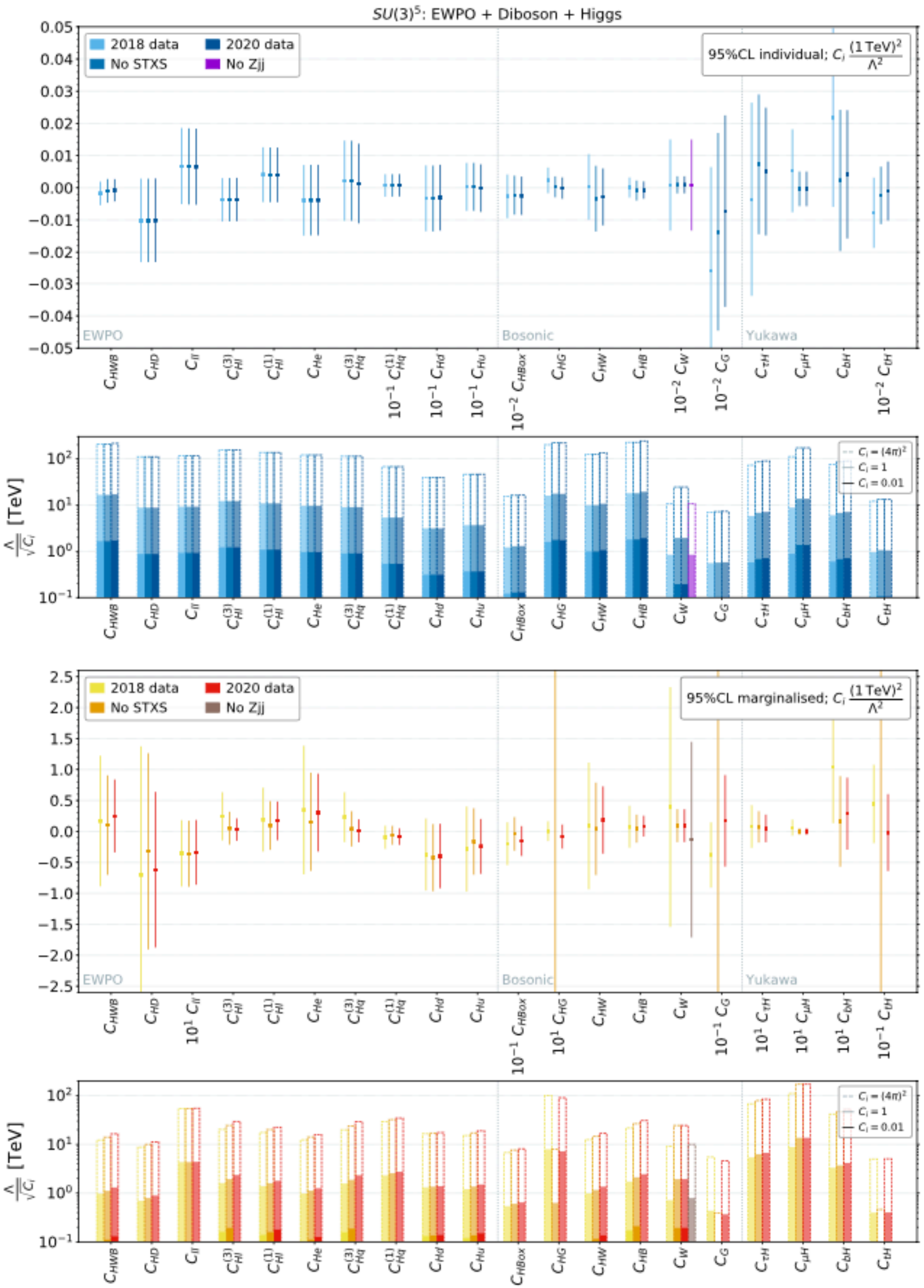


Dimension-6 Constraints with Flavour-Universal SU(3)^5 Symmetry

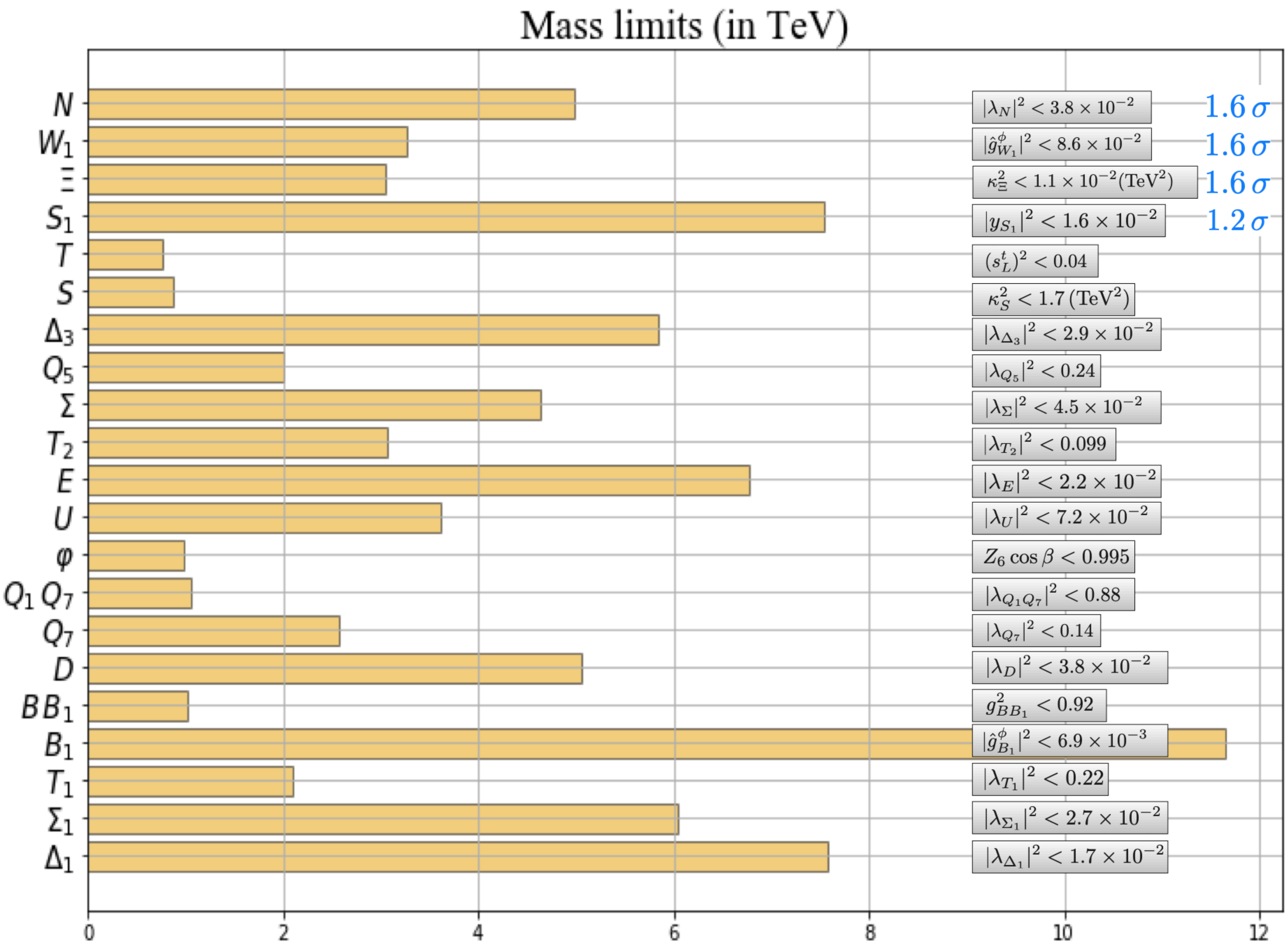
- **Individual operator coefficients**
- **Marginalised over all other operator coefficients**

No significant deviations from SM

JE, Madigan, Mimasu, Sanz & You, arXiv:2012.02779



Single-Field Extensions of the Standard Model

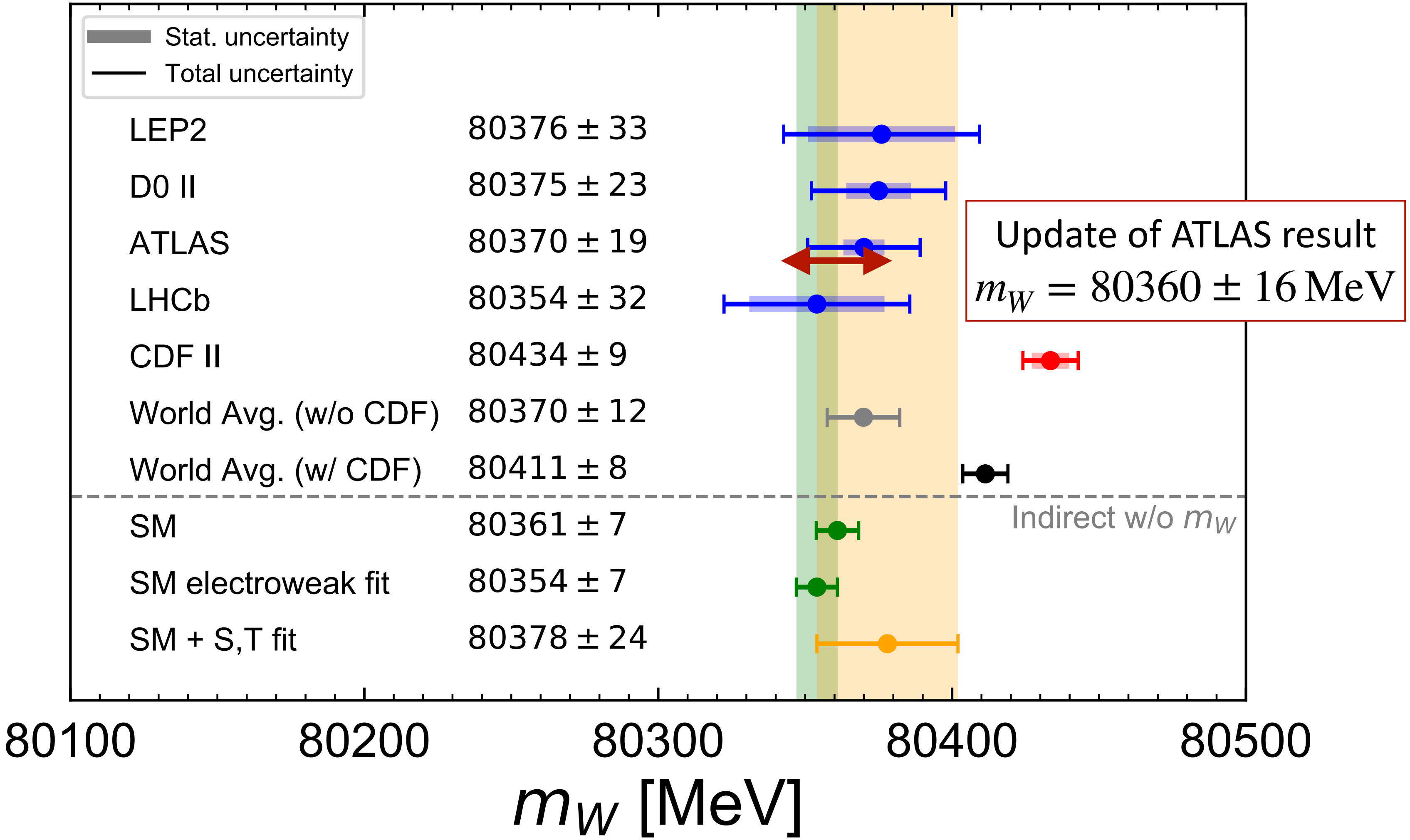


JE, Madigan, Mimasu, Sanz & You, arXiv:2012.02779

What about experimental hints of new physics?

CDF Measurement of m_W

compared with other measurements



Tension: 7-σ discrepancy with Standard Model?

SMEFT Operators that can Contribute to W Mass

- Relevant SMEFT operators

$$\mathcal{O}_{HWB} \equiv H^\dagger \tau^I H\, W^I_{\mu\nu} B^{\mu\nu}, \quad \mathcal{O}_{HD} \equiv \left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right)$$

$$\mathcal{O}_{\ell\ell} \equiv \left(\bar{\ell}_p \gamma_\mu \ell_r\right)\left(\bar{\ell}_s \gamma^\mu \ell_t\right), \quad \mathcal{O}^{(3)}_{H\ell} \equiv \left(H^\dagger i \overleftrightarrow{D}^I_\mu H\right)\left(\bar{\ell}_p \tau^I \gamma^\mu \ell_r\right)$$

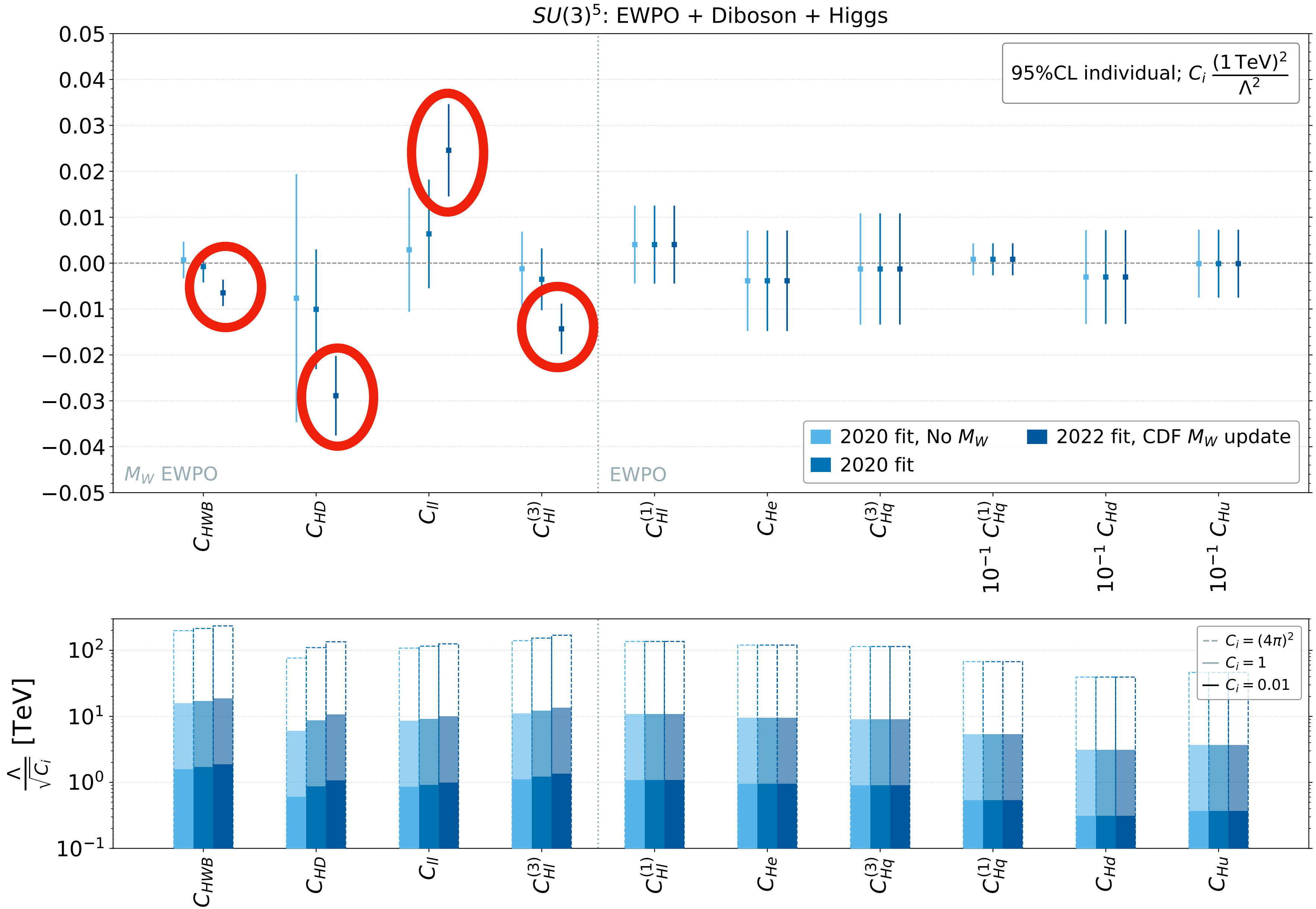
- Contributions to W mass

$$\frac{\delta m_W^2}{m_W^2} = -\frac{\sin 2\theta_w}{\cos 2\theta_w}\frac{v^2}{4\Lambda^2}\left(\frac{\cos\theta_w}{\sin\theta_w} C_{HD} + \frac{\sin\theta_w}{\cos\theta_w}\left(4C^{(3)}_{Hl} - 2C_{ll}\right) + 4C_{HWB}\right)$$

- Contributions to *S* and *T* oblique parameters

$$\frac{v^2}{\Lambda^2} C_{HWB} = \frac{g_1 g_2}{16\pi} S \quad , \quad \frac{v^2}{\Lambda^2} C_{HD} = -\frac{g_1 g_2}{2\pi(g_1^2 + g_2^2)} T$$

SMEFT Fit with the Mass of the W Boson



Non-zero coefficients for any of four operators can fit W mass

Bagnaschi, JE, Madigan, Mimasu, Sanz & You, arXiv:2204.05260

Single-Field Models that can Contribute to W Mass

Model	C_{HD}	C_{ll}	$C_{Hl}^{(3)}$	$C_{Hl}^{(1)}$	C_{He}	$C_{H\square}$	$C_{\tau H}$	C_{tH}	C_{bH}
S_1		X							
Σ			X	$\frac{3}{16}$			$\frac{y_\tau}{4}$		
Σ_1			X	$-\frac{3}{16}$			$\frac{y_\tau}{8}$		
N			$-\frac{1}{4}$	$\frac{1}{4}$					
E			$-\frac{1}{4}$	$-\frac{1}{4}$			$\frac{y_\tau}{2}$		
B_1	X					$-\frac{1}{2}$	$-\frac{y_\tau}{2}$	$-\frac{y_t}{2}$	$-\frac{y_b}{2}$
B	-2						$-y_\tau$	$-y_t$	$-y_b$
Ξ	-2					$\frac{1}{2}$	y_τ	y_t	y_b
W_1	$-\frac{1}{4}$					$-\frac{1}{8}$	$-\frac{y_\tau}{8}$	$-\frac{y_t}{8}$	$-\frac{y_b}{8}$
W	X					$-\frac{1}{2}$	$-y_\tau$	$-y_t$	$-y_b$

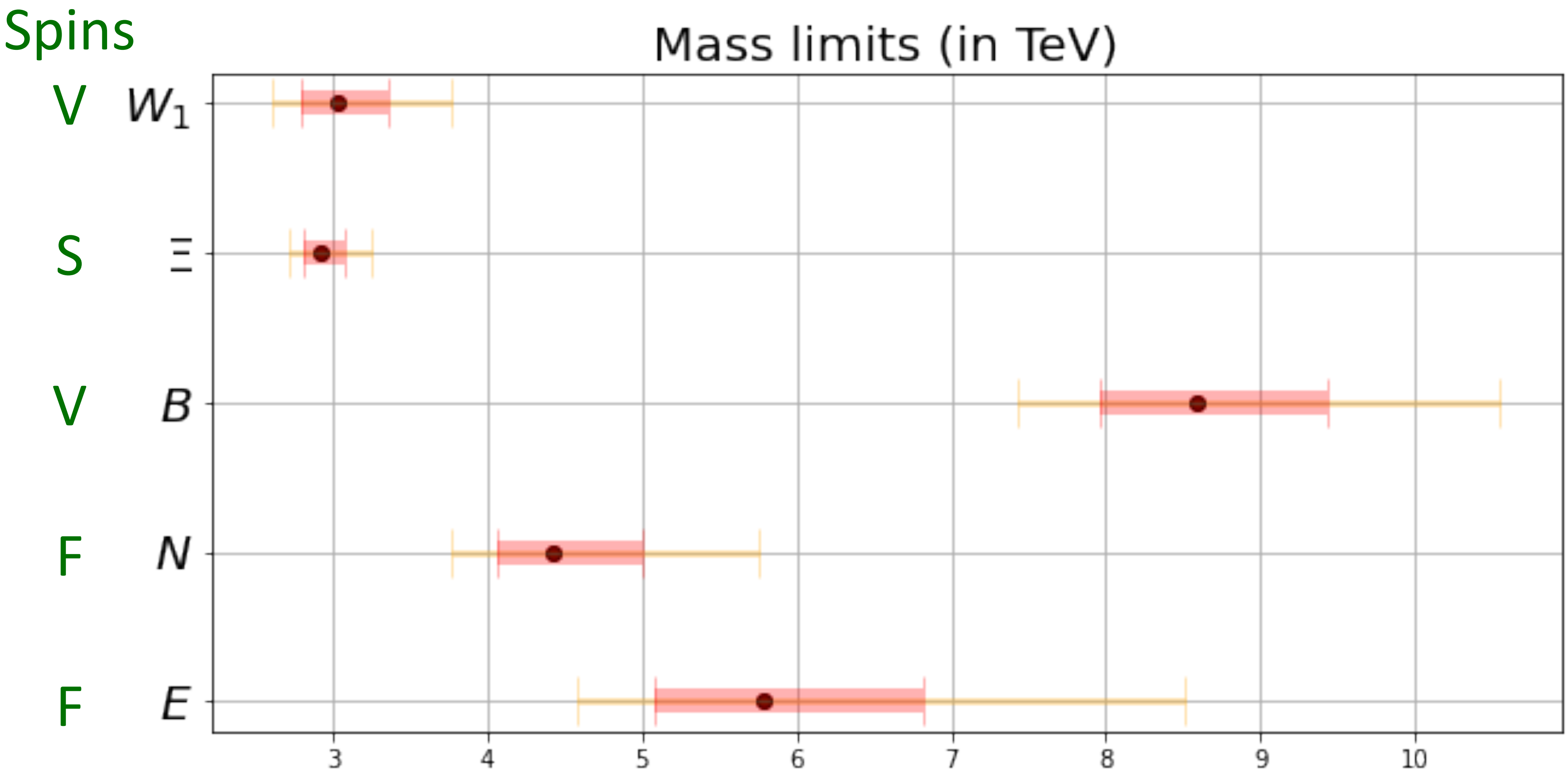
Wrong sign

Right sign

Operators contributing to m_W

Bagnaschi, JE, Madigan, Mimasu, Sanz & You, arXiv:2204.05260

Models Fitting the Mass of the W Boson



68 and 95% CL ranges of masses assuming unit couplings,
mass range proportional to coupling

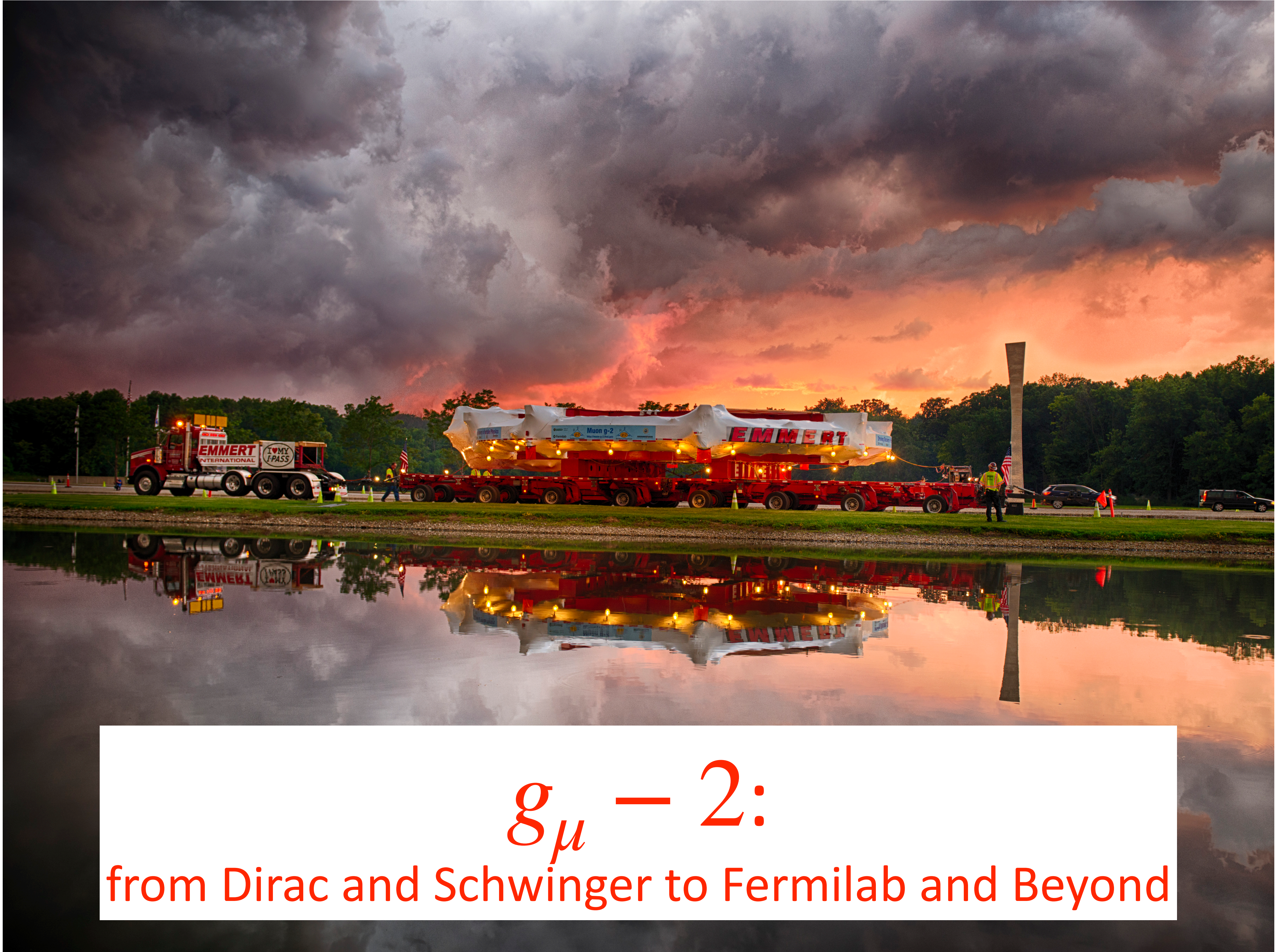
Bagnaschi, JE, Madigan, Mimasu, Sanz & You, arXiv:2204.05260

Quo Vadis m_W?

- The jury is still out concerning the experimental measurement
  - Tension with SM, previous measurements

    "Extraordinary claims require extraordinary evidence"

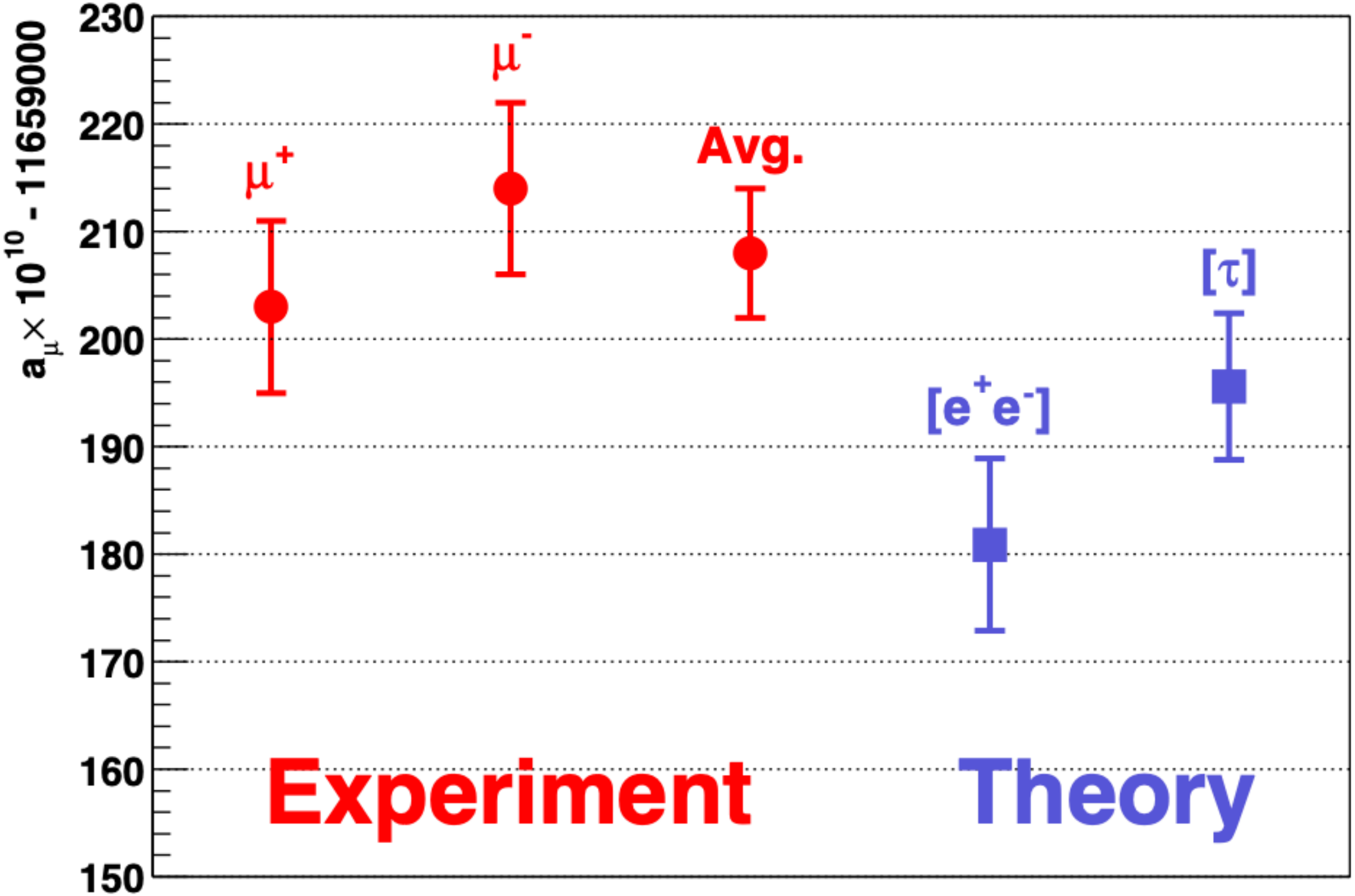
- Nevertheless, much theoretical speculation (> 90 papers!)
- 4 SMEFT operators can increase m_W
- 3 SMEFT operators generated by single field extensions of the SM at tree level
  - Vector bosons W or B, scalar boson Ξ, fermions N, E
- Prospects for the LHC?



$g_\mu - 2$:

from Dirac and Schwinger to Fermilab and Beyond

Possible Discrepancy with Theory?

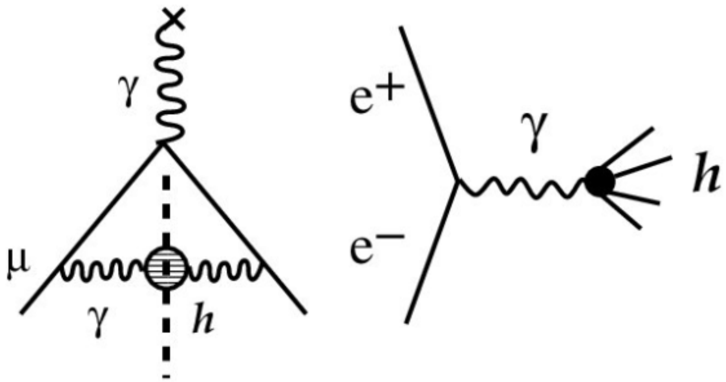


$$\delta a = \pm\, 0.47\ \text{ppm}$$

BNL E821 experiment, 2001 - 2006

Theory Initiative

- Comprehensive review of calculations of the Standard Model contributions to $g_\mu - 2$
- Including discussion of the uncertainties
- Particularly in calculation of leading-order vacuum polarisation



Aoyama et al, arXiv:2006.04822

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The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama [1,2,3], N. Asmussen [4], M. Benayoun [5], J. Bijnens [6], T. Blum [7,8], M. Bruno [9], I. Caprini [10], C.M. Carloni Calame [11], M. Cè [9,12,13], G. Colangelo [14,*], F. Curciarello [15,16], H. Czyż [17], I. Danilkin [12], M. Davier [18,*], C.T.H. Davies [19], M. Della Morte [20], S.I. Eidelman [21,22,*], A.X. El-Khadra [23,24,*], A. Gérardin [25], D. Giusti [26,27], M. Golterman [28], Steven Gottlieb [29], V. Gülpers [30], F. Hagelstein [14], M. Hayakawa [31,2], G. Herdoíza [32], D.W. Hertzog [33], A. Hoecker [34], M. Hoferichter [14,35,*], B.-L. Hoid [36], R.J. Hudspith [12,13], F. Ignatov [21], T. Izubuchi [37,8], F. Jegerlehner [38], L. Jin [7,8], A. Keshavarzi [39], T. Kinoshita [40,41], B. Kubis [36], A. Kupich [21], A. Kupść [42,43], L. Laub [14], C. Lehner [26,37,*], L. Lellouch [25], I. Logashenko [21], B. Malaescu [5], K. Maltman [44,45], M.K. Marinković [46,47], P. Masjuan [48,49], A.S. Meyer [37], H.B. Meyer [12,13], T. Mibe [1,*], K. Miura [12,13,3], S.E. Müller [50], M. Nio [2,51], D. Nomura [52,53], A. Nyffeler [12,*], V. Pascalutsa [12], M. Passera [54], E. Perez del Rio [55], S. Peris [48,49], A. Portelli [30], M. Procura [56], C.F. Redmer [12], B.L. Roberts [57,*], P. Sánchez-Puertas [49], S. Serednyakov [21], B. Shwartz [21], S. Simula [27], D. Stöckinger [58], H. Stöckinger-Kim [58], P. Stoffer [59], T. Teubner [60,*], R. Van de Water [24], M. Vanderhaeghen [12,13], G. Venanzoni [61], G. von Hippel [12], H. Wittig [12,13], Z. Zhang [18], M.N. Achasov [21], A. Bashir [62], N. Cardoso [47], B. Chakraborty [63], E.-H. Chao [12], J. Charles [25], A. Crivellin [64,65], O. Deineka [12], A. Denig [12,13], C. DeTar [66], C.A. Dominguez [67], A.E. Dorokhov [68], V.P. Druzhinin [21], G. Eichmann [69,47], M. Fael [70], C.S. Fischer [71], E. Gámiz [72], Z. Gelzer [23], J.R. Green [9], S. Guellati-Khelifa [73], D. Hatton [19], N. Hermansson-Truedsson [14], S. Holz [36], B. Hörz [74], M. Knecht [25], J. Koponen [1], A.S. Kronfeld [24], J. Laiho [75], S. Leupold [42], P.B. Mackenzie [24], W.J. Marciano [37], C. McNeile [76], D. Mohler [12,13], J. Monnard [14], E.T. Neil [77], A.V. Nesterenko [68], K. Ottnad [12], V. Pauk [12], A.E. Radzhabov [78], E. de Rafael [25], K. Raya [79], A. Risch [12], A. Rodríguez-Sánchez [6], P. Roig [80], T. San José [12,13], E.P. Solodov [21], R. Sugar [81], K. Yu. Todyshev [21], A. Vainshtein [82], A. Vaquero Avilés-Casco [66], E. Weil [71], J. Wilhelm [12], R. Williams [71], A.S. Zhevlakov [78]

[1] Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan
[2] Nishina Center, RIKEN, Wako 351-0198, Japan
[3] Kobayashi–Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya 464-8602, Japan
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[5] LPNHE, Sorbonne Université, Université de Paris, CNRS/IN2P3, Paris, France

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E-mail address: MUON-GM2-THEORY-SC@fnal.gov (G. Colangelo, M. Davier, S.I. Eidelman, A.X. El-Khadra, M. Hoferichter, C. Lehner, T. Mibe, A. Nyffeler, B.L. Roberts, T. Teubner).

https://doi.org/10.1016/j.physrep.2020.07.006

Hadronic Vacuum Polarization

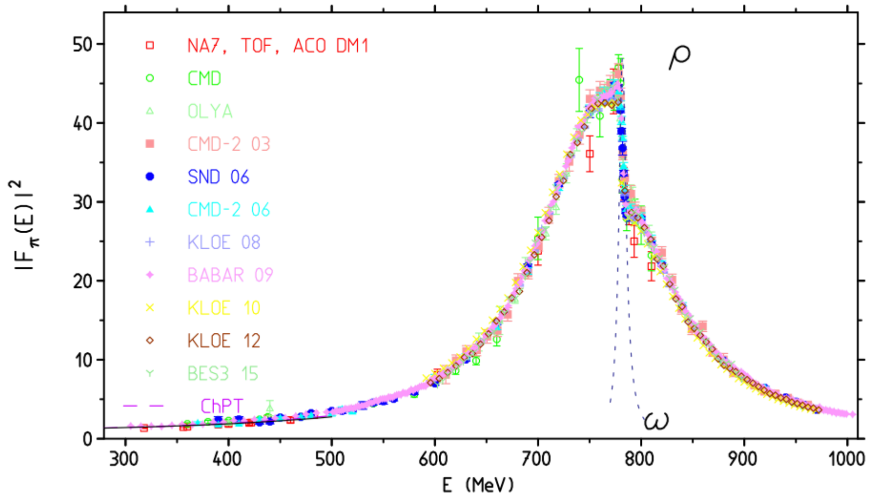
- Most important contribution is from low energies $\lesssim 1$ GeV, dominated by ρ and ω peaks, taking account of interference effects

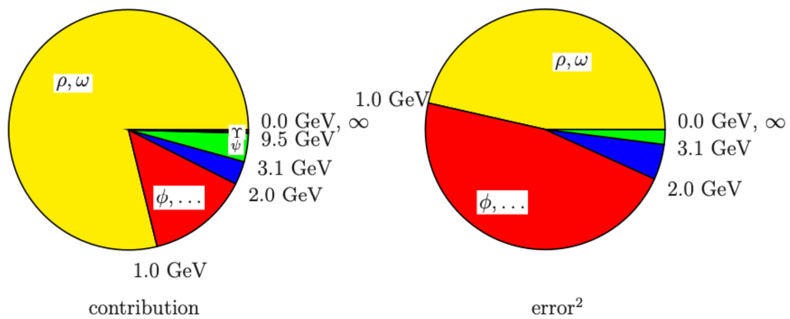
- Uncertainties dominated by ρ and ω region, and by region between 1 and 2 GeV (ϕ, etc.)

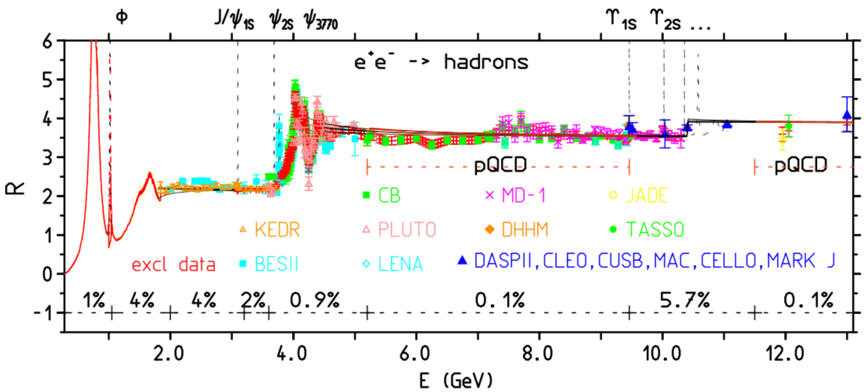
- High energies under good control from perturbative QCD

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}\,.$$

Aoyama et al, arXiv:2006.04822





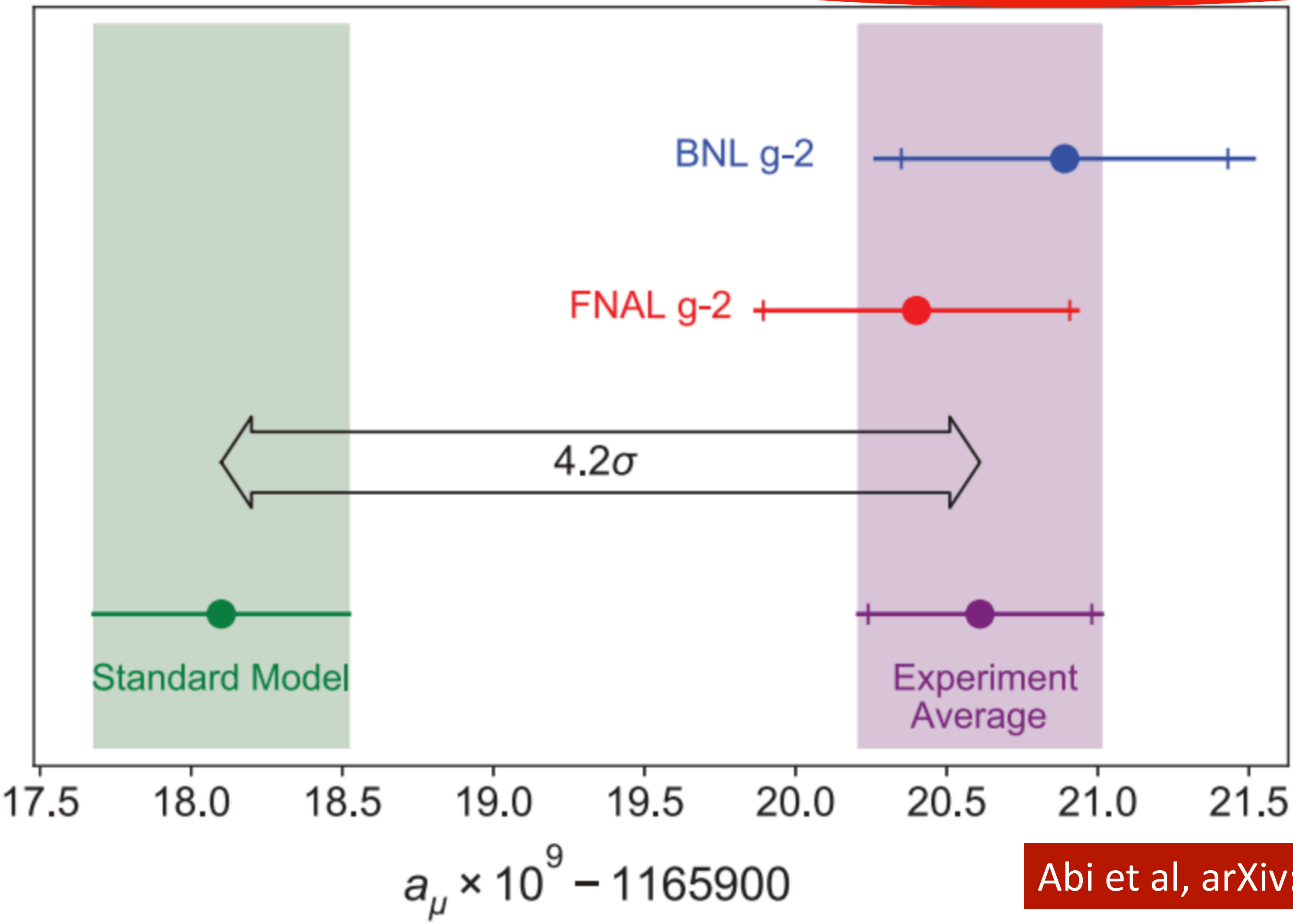


Fermilab Measurement

FNAL result: $a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$ (0.46 ppm)

Combined result: $a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11}$ (0.35 ppm)

Difference from Standard Model: $a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$



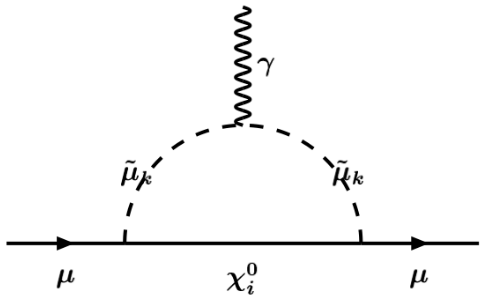
Abi et al, arXiv:2104.03281

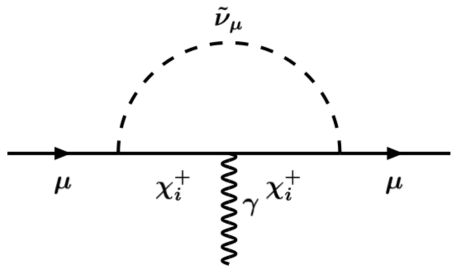
Interpretation Papers

2104.05685	Vector LQ	B	Du	
5656	L_\mu - L_\tau	DM	Borah	
5006	B_q - L_\mu	B	Cen	Leptoquarks
4494	LFV	LFV	Li	
4503	Pseudoscalar	DM, H decays	Lu	Extra U(1)
4456	2HDM	DM	Arcadi	
3542	B-LSSM	H decays	Yang	Extra Higgs
3701	Leptophilic spin 0	H factory	Chun	
3839	SUSY	HL-LHC	Aboubrahim	Supersymmetry
3691	Survey	DM, LHC	Athron	
3705	Seesaw	g_e	Escribano	Axion
3699	Gauged 2HDM	B	Chen	
3239	SUSY	Gravitino DM	Gu	
3284	NMSSM	DM	Cao	
3262	GUT-constrained SUSY	DM, LHC	Wang	
3292	MSSM	CPV	Han	
3296	lepton mass matrix	Flavour	Calibbi	
3280	Z_d	Cs weak charge	Cadeddu	
3334	E_6 3-3-1	H stability	Li	
3242	\mu-\tau-philic H	\tau decays, LHC	Wang	
3259	Anomaly mediation	DM	Yin	
3245	pMSSM	DM, fine-tuning	Van Beekveld	
3274	NMSSM	DM, AMS-02 pbar	Abdughani	
3290	MSSM	DM	Cox	
3367	2HDM	V-like leptons	Ferreira	
3267	Axion	Low-scale	Buen-Abad	
3340	L_\mu - L_tau	AMS-02 positrons	Zu	
3282	ALP	V-like fermions	Brdar	
3301	Lepton portal	DM	Bai	
3276	Dark axion portal	Dark photon	Ge	
3491	GmSUGRA	LHC	Ahmed	
3227	2HDM	LHC	Han	
3302	SUSY	small \mu	Baum	
3238	Scalar	DM, p radius	Zhu	
3489	\mu \nu SSM	B, H decays	Zhang	
3287	pMSSM	ILC	Chakraborti	
3228	DM	B, H decays	Arcadi	

890	Radiative seesaw		Chiang
2103.13991	Scalar LQ	B, H decays	Greljo
2012.11766	DM		D'Agnolo
2012.07894	Axions		Darmé
1812.06851	Charmphilic LQ		Kowalska
2104.04458	GUT-constrained SUSY	DM	Chakraborti
5730	LQ + charged singlet	B, Cabibbo	Marzocca
6320	L-R symmetry		Boyarkin
6858	L_\mu - L_\tau	\nu masses	Zhou
6854	D-brane	U(1), Regge	Anchordoqui
6656	vector LQ	B	Ban
7597	SUSY	LHC, landscape	Baer
7047	3HDM	Fermion masses	Carcamo
7680	Leptophilic Z'	Global analysis	Buras
8289	Custodial symmetry	Light scalar + pseudoscalar	Balkin
9205	U(1)D	Neutrino mass	Dasgupta
8819	Lepton non-universality	Naturalness	Cacciapaglia
8640	2x2x1	Higgses, heavy nus	Boyarkina
8293	Multi-TeV sleptons in FSSM	Extended H, tau decays	Altmannshofer
10114	SO(10)	Yukawa unification	Aboubrahim
7681	U(1)B-L	DUNE	Dev
10324	Gauged lepton number	Dark matter	Ma
10175	2HDM	Lighter Higgs?	Jueid
11229	LQ	Matter unification	Fileviez
15136	U(1)	HE neutrinos, H tension	Alonso
2105.00903	Anomalous 3-boson vertex	W mass	Arbuzov
7655	U(1)T3R	RK(*)	Dutta
8670	Leptoquark	nu mass, LFV	Zhang

$g_\mu - 2$ in Supersymmetry





- Muon ψ_f, 4 neutralinos ψ_i, 2 smuons ϕ_k ($\tilde{\mu}_{L,R}$)

$$-\mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f (K_{ik} \frac{1-\gamma_5}{2} + L_{ik} \frac{1+\gamma_5}{2}) \psi_i \phi_k + H.c.$$

- One-loop contributions from smuon/neutralino loops:
  - Left-right mixing: $a_f^{11} = \sum_{ik} \frac{m_f}{8\pi^2 m_i} Re(K_{ik} L_{ik}^*) I_1(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2})$ ← Most important
  - Unmixed: $a_f^{12} = \sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) I_2(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2})$

Ibrahim & Nath, hep-ph/9910553

$g_\mu - 2$ in Supersymmetry

- One-loop contribution from smuon/neutralino loop

$$\Delta(g-2)_\mu = -ab(\cos\alpha \sin\alpha/4\pi^2)(m_\mu/m_{\tilde{G}})$$
$$\times \{1/(1-\eta_1) + 2\eta_1/(1-\eta_1)^2$$
$$+ [2\eta_1/(1-\eta_1)^3] \log \eta_1 - (\eta_1 \leftrightarrow \eta_2)\},$$

- where $\eta_i \equiv (m^2_{s\mu_i}/m^2_{\tilde{G}})$

- and $\mathcal{L} = a\sqrt{2}\, s_\mu \bar{\mu}_L \tilde{G} + b\sqrt{2}\, t_\mu \bar{\mu}_R \tilde{G}$

(1982)

Volume 116B, number 4 PHYSICS LETTERS

SPIN-ZERO LEPTONS AND THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

John ELLIS, John HAGELIN and D.V. NANOPOULOS
CERN, Geneva, Switzerland

Received 14 June 1982

The anomalous magnetic moment of the muon $(g-2)_\mu$ imposes constraints on the masses and mixing of spin-zero leptons (sleptons). We develop the predictions of models of spontaneous supersymmetry breaking for the slepton mass matrix, and show that they are comfortably consistent with the $(g-2)_\mu$ constraints.

During the present resurgence of interest in supersymmetry broken at low energies [1] new significance is attached to the classical phenomenological playgrounds of gauge theories such as the anomalous magnetic moments of the electron and muon [2], flavour-changing neutral interactions [3–5] parity [6] and *CP* violation [7,8] in the strong interactions. The three latter phenomena make life rather difficult [3,7] for the most general form of soft supersymmetry breaking, whereas simple models [9–11] of spontaneously broken supersymmetry naturally [3,4 7] respect the $\Delta F \neq 0$, *P* and *CP* violation constraints. As for the anomalous magnetic moments of the leptons, it has long been known that they vanish in an exactly supersymmetric theory [12], and Fayet [2] showed that in his model of supersymmetry breaking $(g-2)_\mu$ would be compatible with experiment if the spin-zero muon (smuon) masses were heavier than 15 GeV. Direct experimental searches [13] now exclude the existence of lighter smuons. Fayet's analysis [2] was in the context of a model with a very light photino $\tilde{\gamma}$ (see fig. 1a), and Grifols and Méndez [14] have recently made the interesting observation that his analysis is significantly altered for massive gauginos (see figs. 1b, 1c). They show that there are potentially nontrivial constraints on the smuon masses in models of broken supersymmetry.

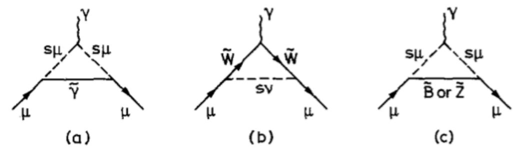


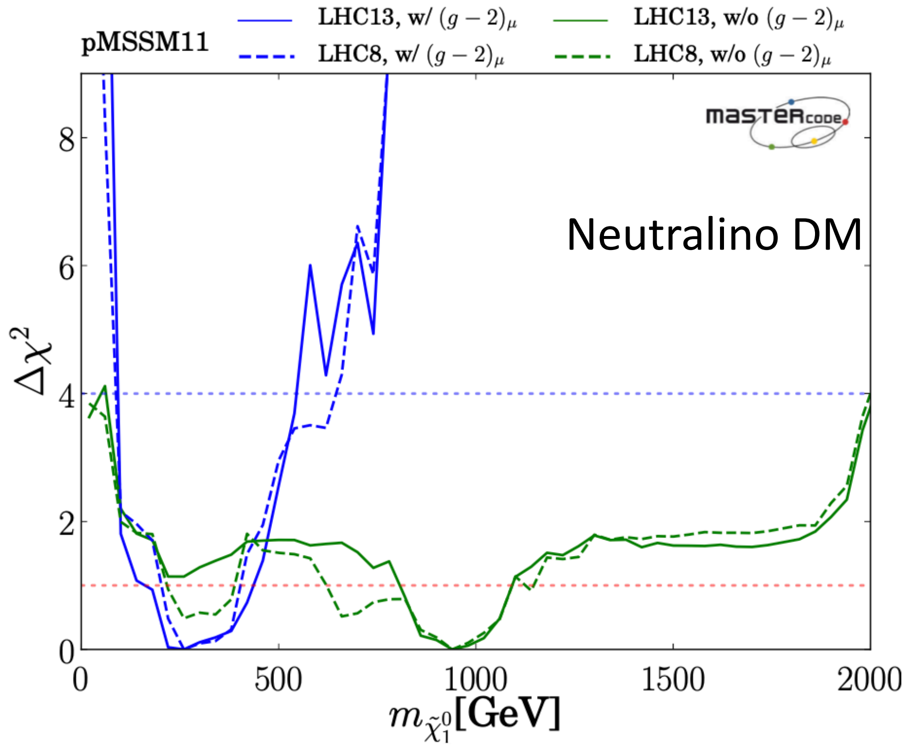
Fig. 1. One-loop diagrams contributing to $(g-2)_\mu$: (a) essentially massless photino ($\tilde{\gamma}$) exchange, (b) $\tilde{W}$ and sneutrino ($s\nu$) exchange, and (c) $\tilde{B}$ or $\tilde{Z}$ exchange.

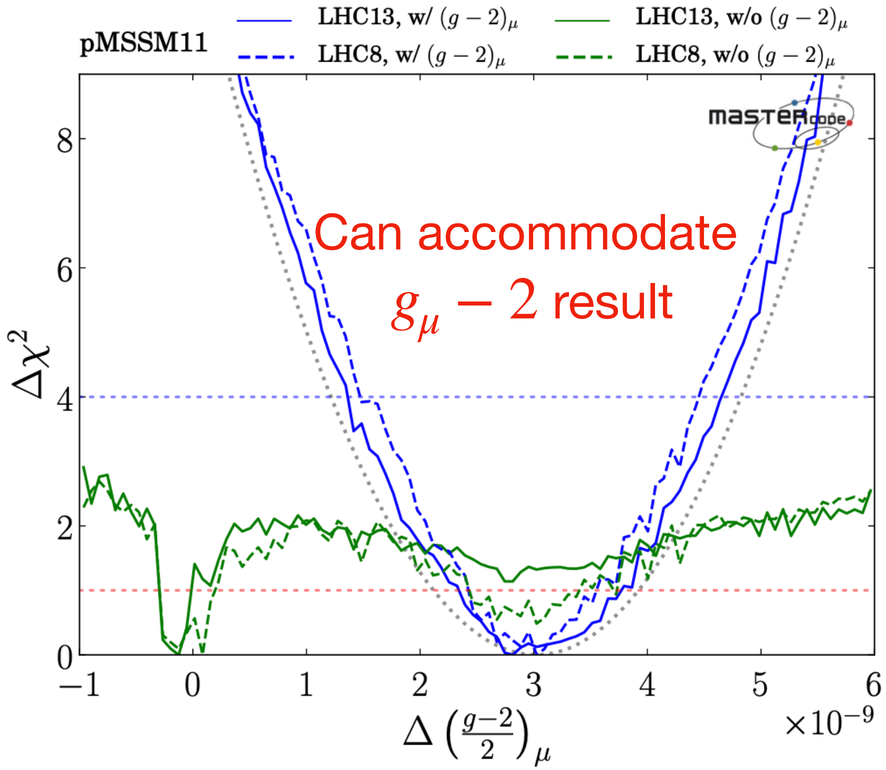
right transition operator there is a GIM [15]-like cancellation between the smuon mass eigenstates in fig. 1c which provides a potential suppression mechanism. We analyze recent models [10,11] of spontaneous supersymmetry breaking originating in the *D* and *F* sectors, respectively. We show that in the former case $(g-2)_\mu$ is suppressed by near degeneracy between the smuon mass eigenstates, while in the latter case $(g-2)_\mu$ is suppressed by small mixing angles between the left- and right-handed smuons. We close with some remarks about $(g-2)_e$ and about parity violation in the strong interactions.

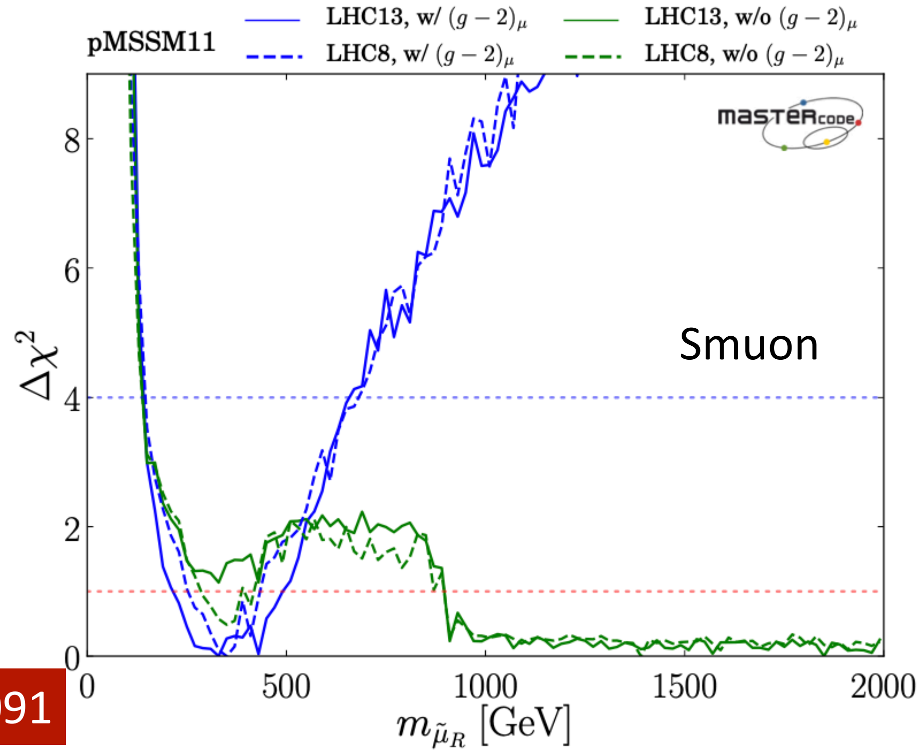
When they examined figs. 1a, 1b and 1c, Grifols and Méndez [14] realized that there was a fundamental difference between the (almost ?) massless $\tilde{\gamma}$ diagram of fig. 1a and the $\tilde{W}$ diagram of fig. 1b as compared to the massive $\tilde{B}$ or $\tilde{Z}$ diagram of fig. 1c. The

$g_\mu - 2$ in Phenomenological Supersymmetry
(pMSSM11)

No relation between squark/gluino masses and slepton/neutralino masses







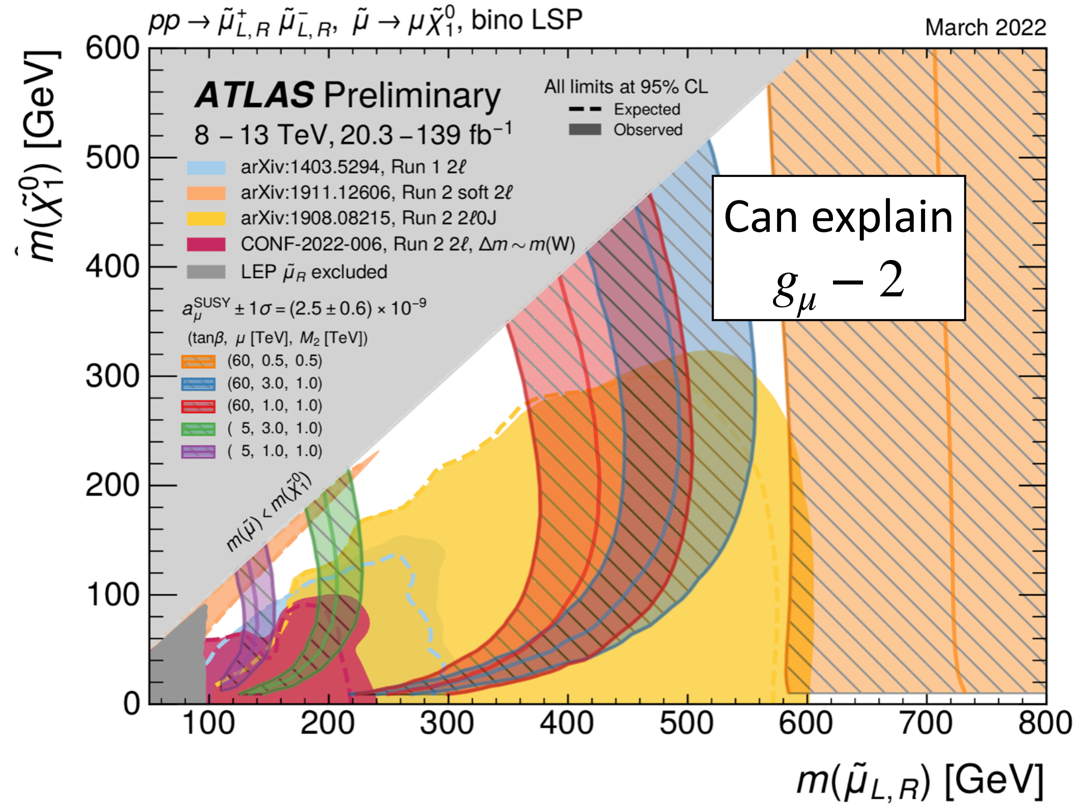
No problem accommodating BNL/FNAL result

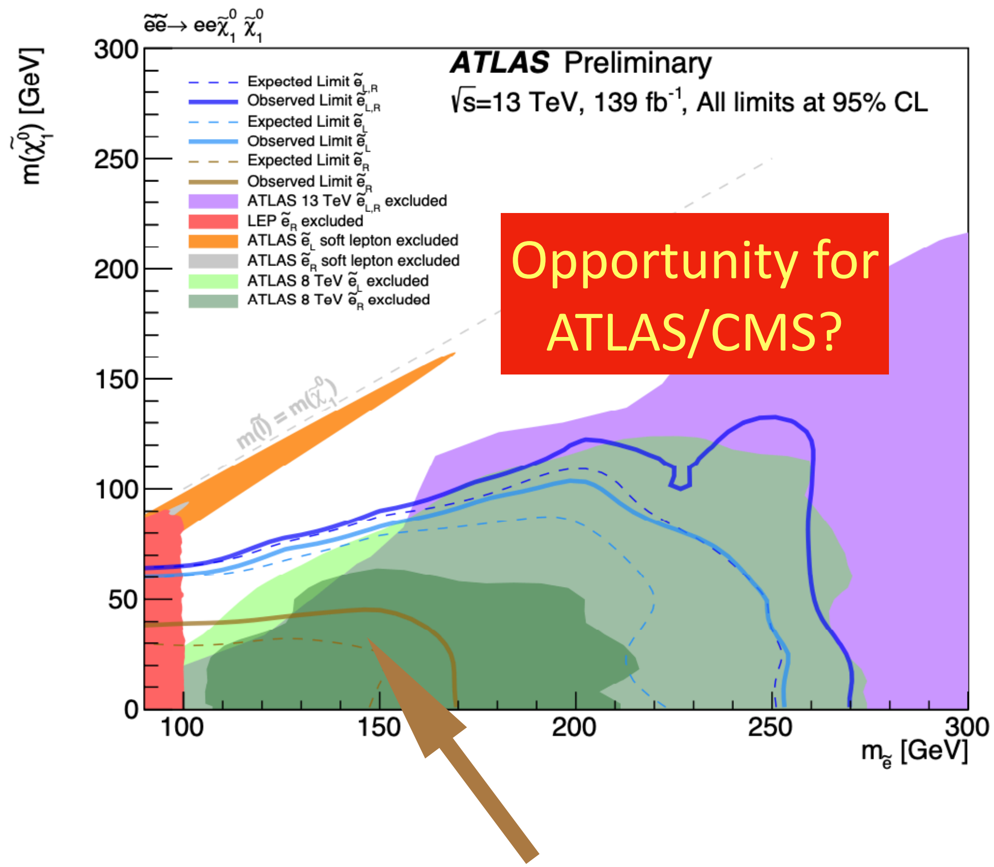
Neutralino DM, smuon masses $\sim 300/400$ GeV

MasterCode, E. Bagnaschi, …, JE et al, arXiv:1710.11091

LHC vs Supersymmetry

- LHC favours squarks & gluinos > 2 TeV (but loopholes)
- Does not exclude lighter electroweakly-interacting particles, e.g., sleptons

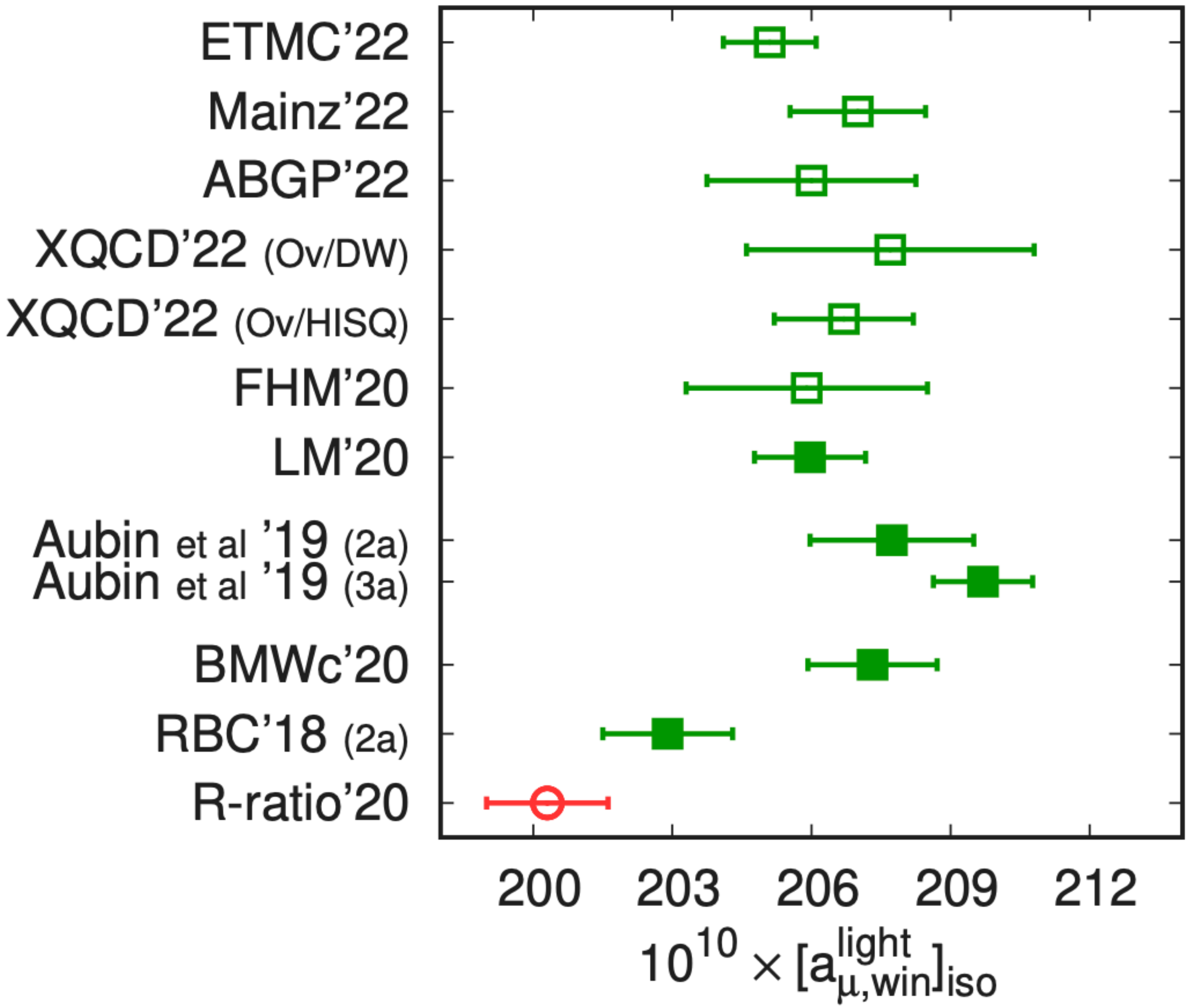




- Most models have $m_{\tilde{\mu}_L} > m_{\tilde{\mu}_R}$ but $m_{\tilde{\mu}_R} \simeq m_{\tilde{e}_R}$: **relevant constraint**

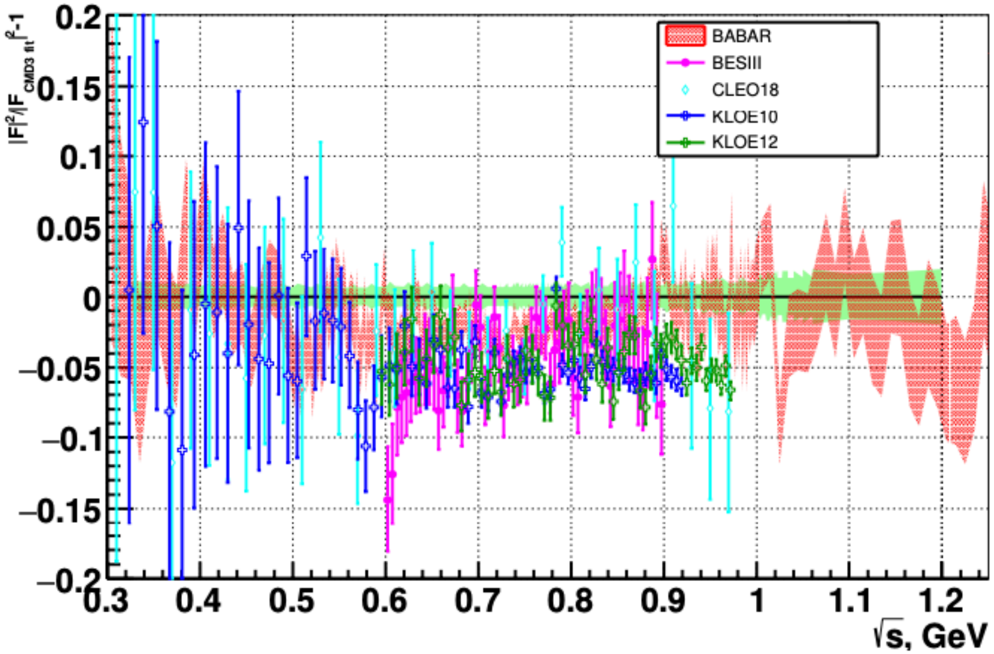
ATLAS Collaboration

Recent Lattice Calculations

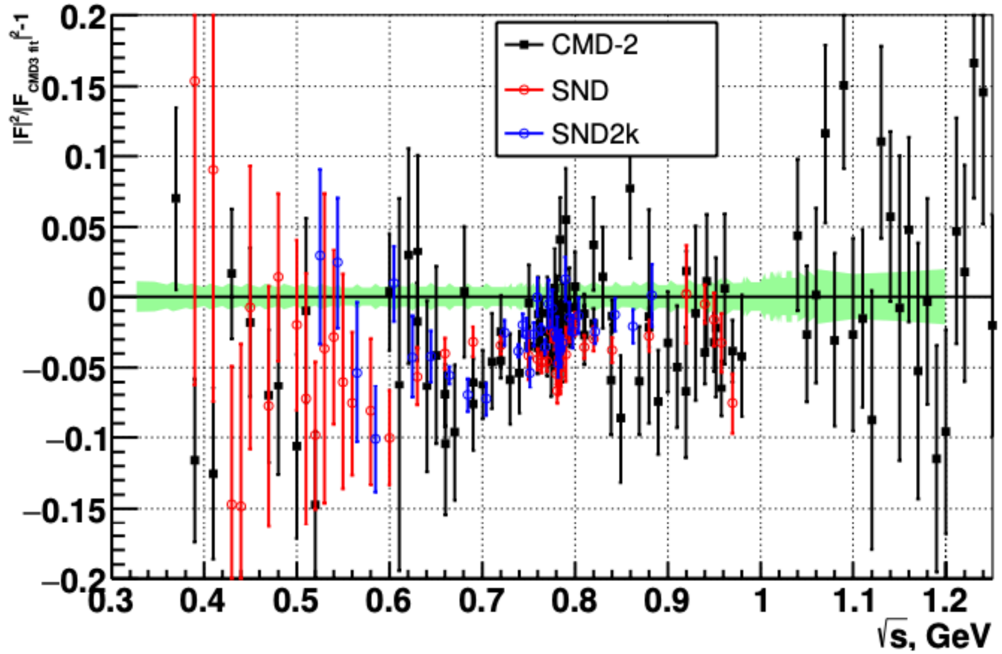


New CMD-3 Measurement of HVP

Comparison with radiative return (ISR) measurements



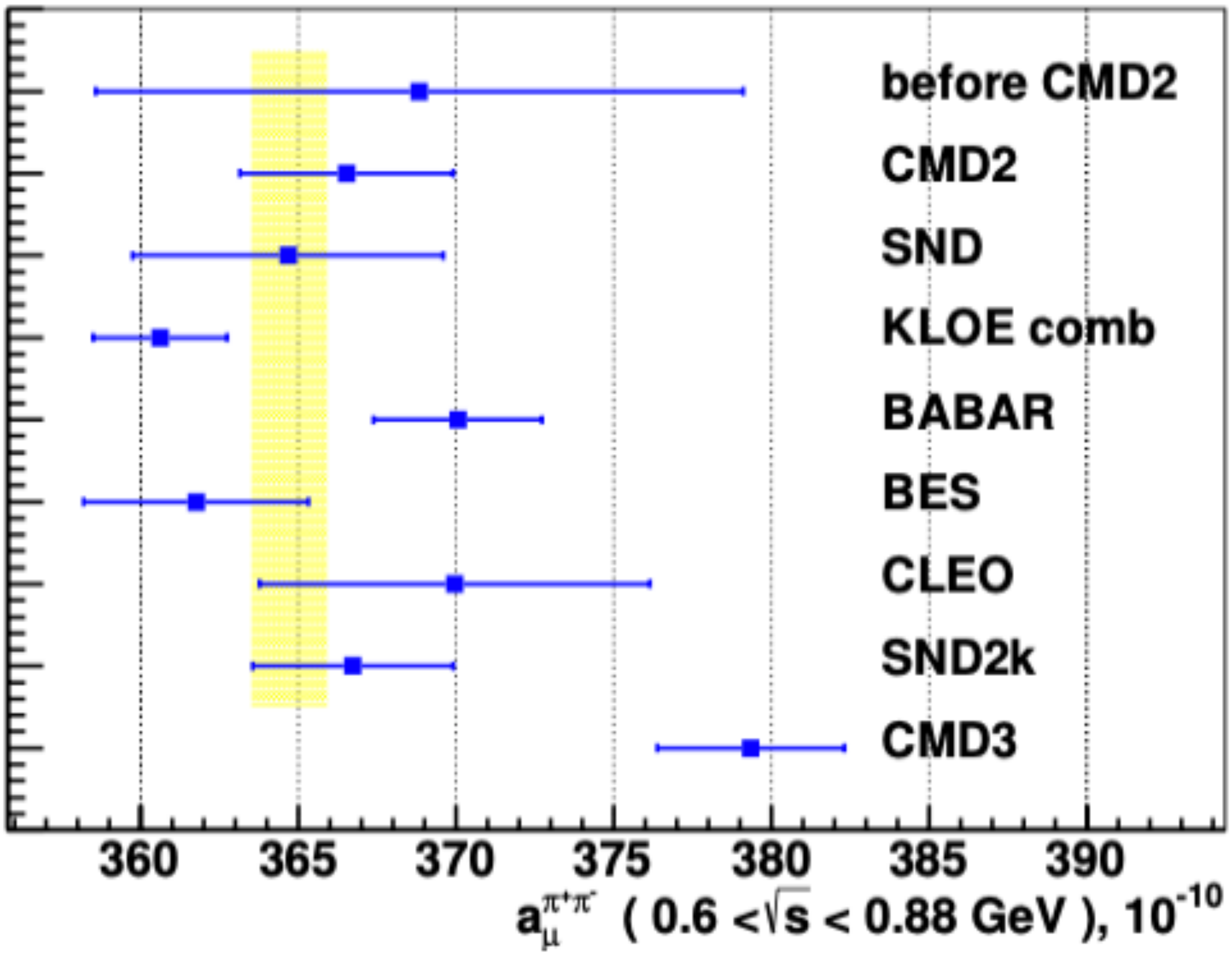
Comparison with previous energy scan measurementss



A task here for Belle II?

CMD-3 Collaboration, arXiv:2302.08834

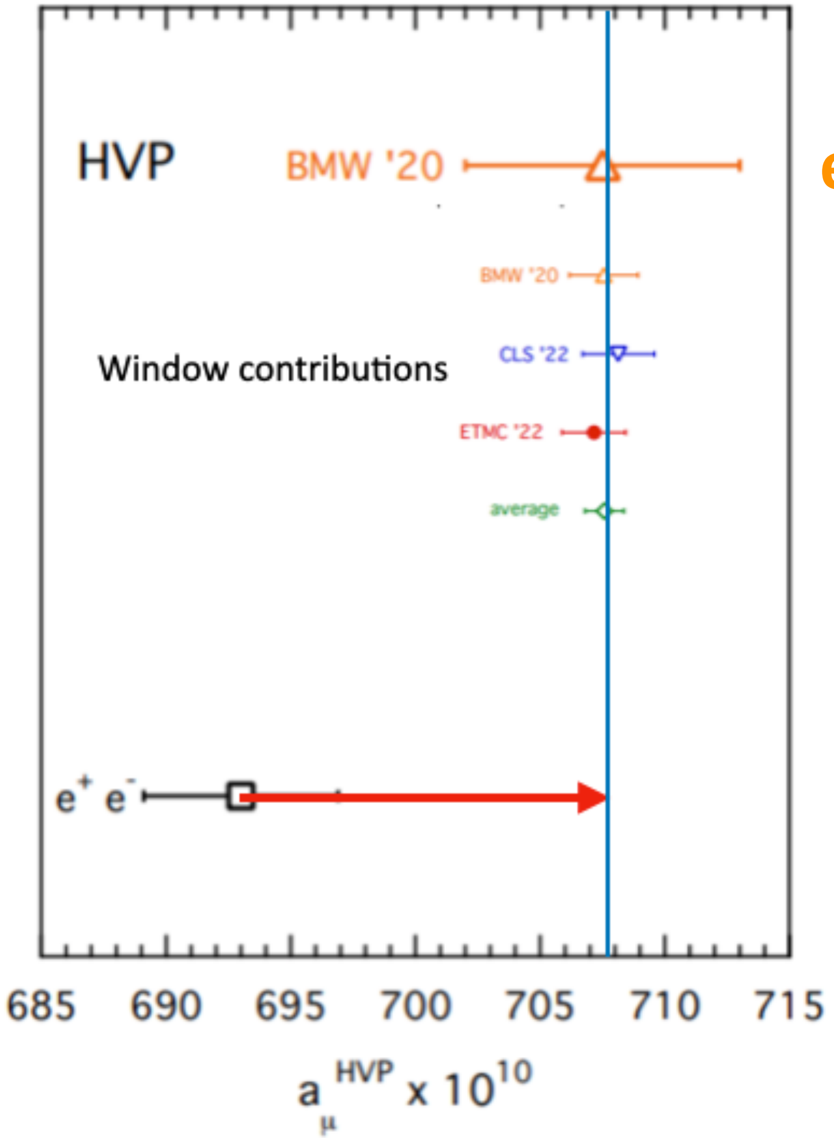
New CMD-3 Measurement of HVP



Discrepancy $\Delta a_\mu \sim 15 \times 10^{-10}$

CMD-3 Collaboration, arXiv:2302.08834

Update on Hadronic Vacuum Polarization



BMW including "window" + extrapolations to small/large distances

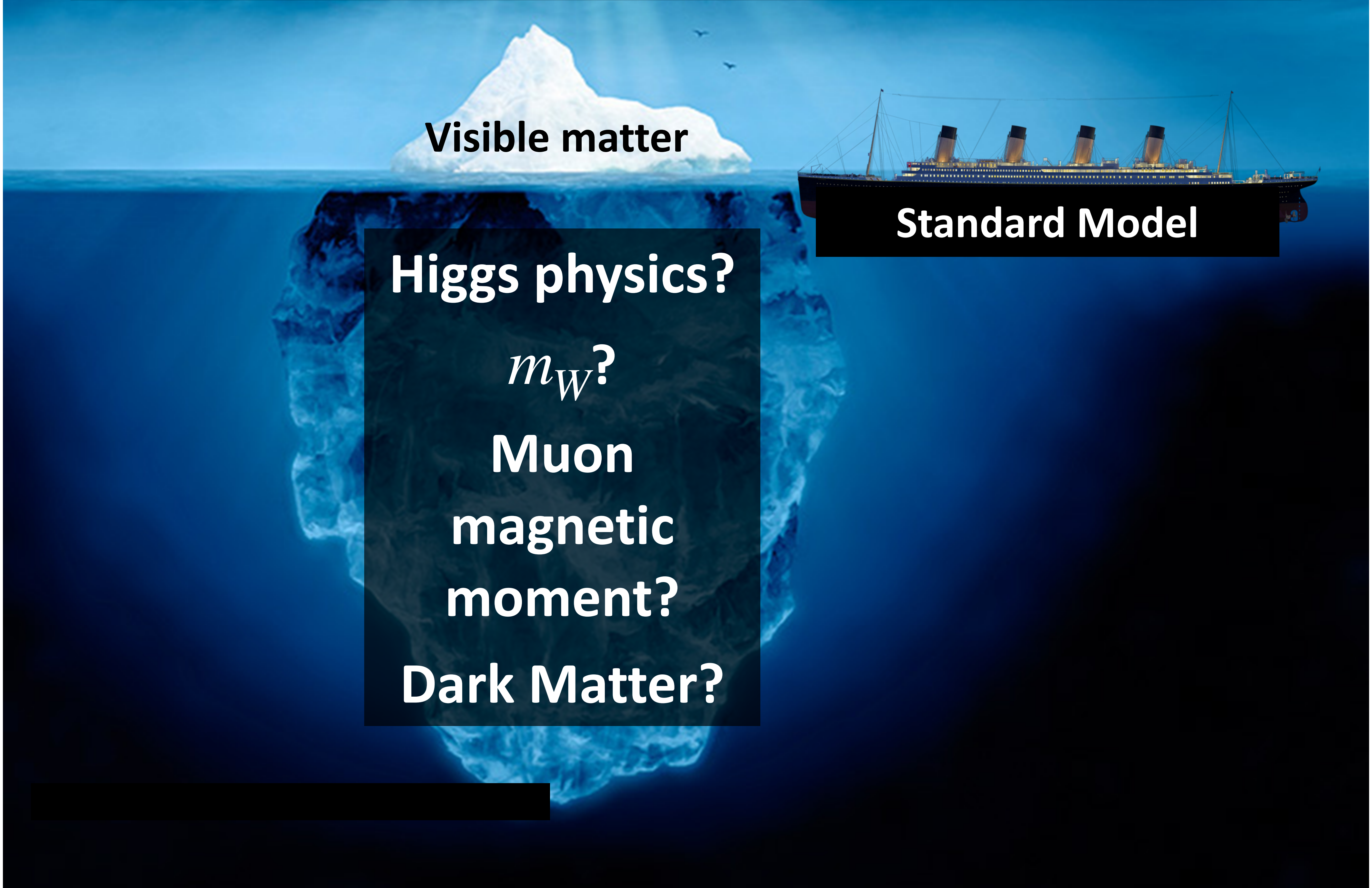
New lattice values of "window" contribution
from intermediate scales

Previous HVP world average
Difference between CMD-3 and previous data

Quo Vadis $g_\mu - 2$?

- The jury is still out concerning the theoretical calculation: https://muon-gm2-theory.illinois.edu/
- Tension with experiment reduced by lattice calculations
- Also by CMD-3 measurement of hadronic vacuum polarization
- Still some scope for new physics
- Prospects for the LHC?
- New results 6pm today from Fermilab: https://fnal.zoom.us/j/93860521626?pwd=K0JpMWFVMjlxbE1yRVA4a2NIWWdrZz09
- J-PARC experiment in preparation, also MuonE at CERN

Summary



[redacted]