High energies. Why?

hep-ph

Baltic Summer School - 3
Palanga

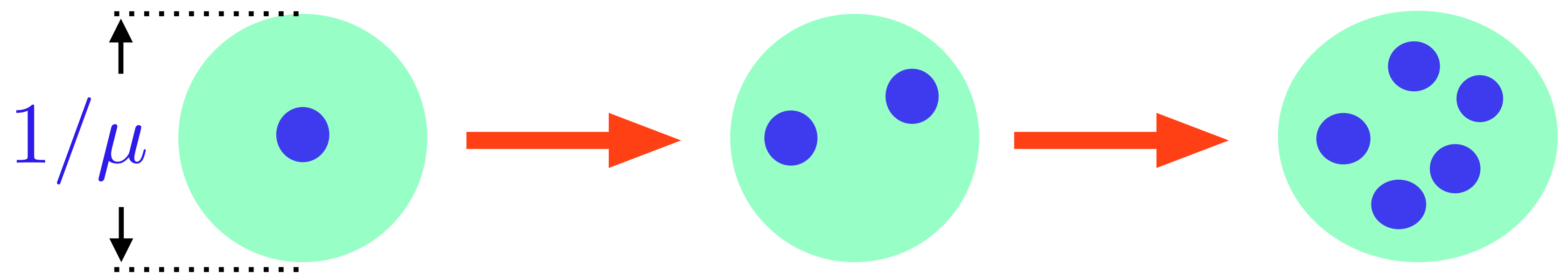
August 2023

Yuri Dokshitzer

growing interaction radius

Because it is profitable to have *multiple peripheral interactions* replacing *a head-on collision* it is not the initial hadron itself but one of its virtual offspring that hits the target.

Let us see what happens in the impact parameter plane (x,y) transversal to the collision axis



The transversal displacement at each step of the cascade remains finite:

$$|\Delta\rho| \sim \Delta t\, |\mathbf{v}_\perp| \sim \frac{x(1-x)p}{\mu^2} \cdot \left|\frac{\mathbf{k}_\perp}{xp}\right| \sim \frac{1}{\mu}$$

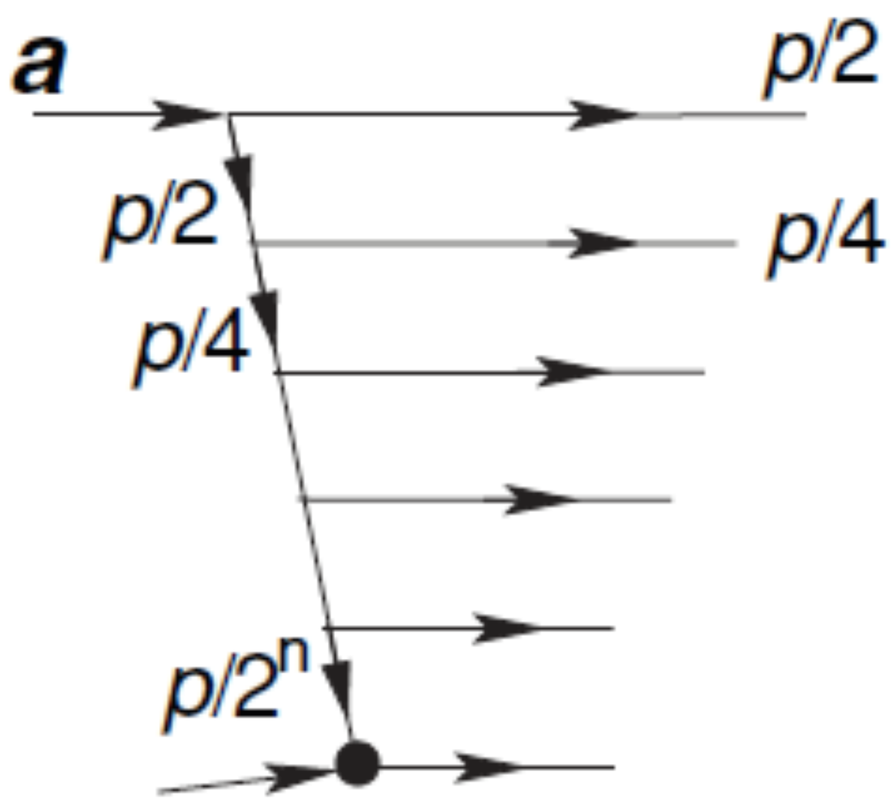
lifetime of a parton xp

$k_\perp = \mathcal{O}(\mu)$

its transverse velocity

The drift that the system accumulates after **n(s)** decays amounts to $\rho_0(s) = \left|\sum_{i=1}^{n(s)} \Delta\rho_i\right|$

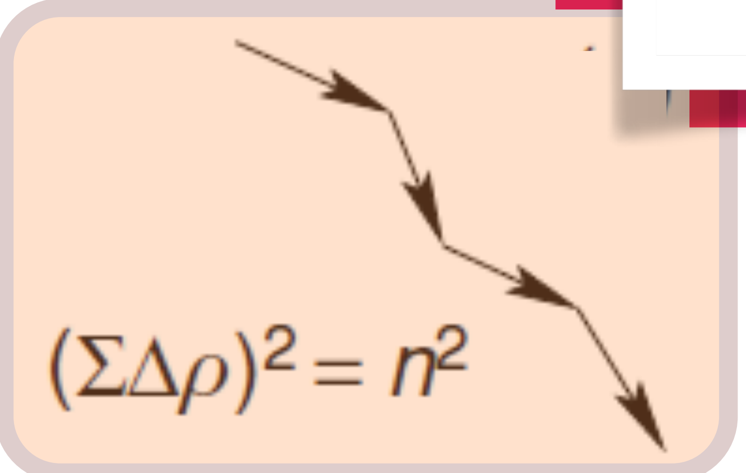
The number of steps necessary to degrade the offspring energy down to the hadron mass scale can be estimated as



$$p_n \sim \frac{p}{2^n}, \qquad n \simeq \frac{\ln(p/\mu)}{\ln 2}$$

$$\rho_0(s) \sim n(s) \cdot |\Delta\rho| \sim \frac{1}{\mu} \ln\frac{s}{\mu^2}$$

The *fastest possible growth* of the effective radius of the projectile hadron could be achieved if successive parton decays were ***aligned***:

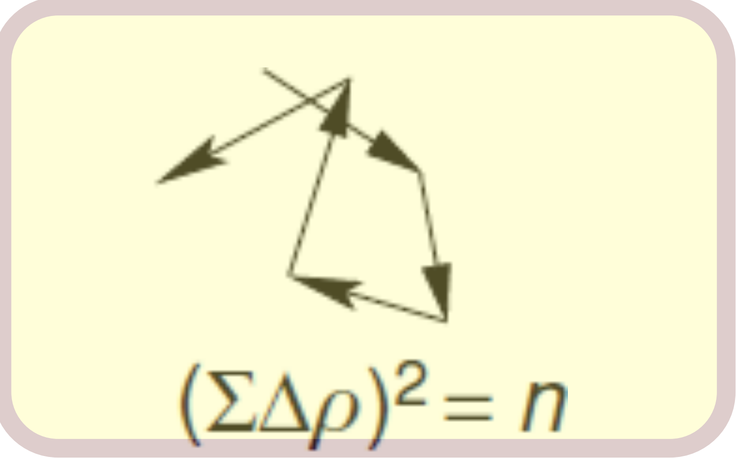


Froissart !

Gribov

Alternatively, for ***random walk*** in the impact parameter space,

(Brownian motion of a drunk sailor)



$$\rho_0(s) \sim \sqrt{n(s) \cdot |\Delta\rho|^2} \sim \frac{1}{\mu}\sqrt{\ln\frac{s}{\mu^2}}$$

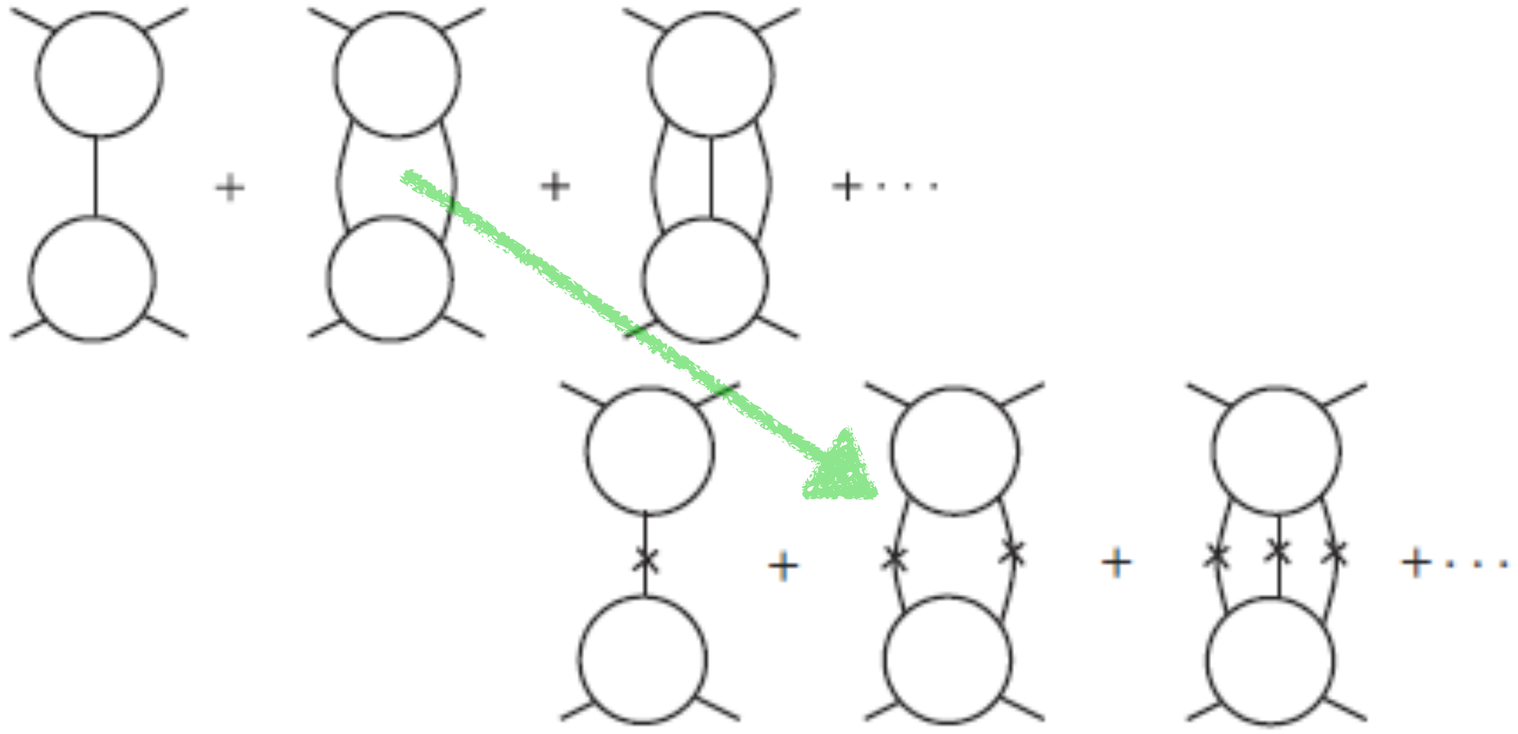
The 'comb' picture of a multi-particle state of a relativistic hadron leads us to the idea of an interaction radius growing with **s**.

Any scientific ground under our *Fingerspitzengefühl* ? Actually, our much praised basic principles ***forbid*** hadron radii to stay ***constant***!

This was the first serious prediction concerning high energy hadron physics after *Froissart* and *Pomeranchuk* theorems $\left(\lim_{s\to\infty} \sigma_{pp}^{\rm tot} / \sigma_{p\bar{p}}^{\rm tot} = 1\right)$

Until now we have been exploiting analyticity and unitarity in the s-channel.

We saw how the s-channel unitarity restricted the dynamics in the impact parameter plane and gave rise to the *Froissart* limit.



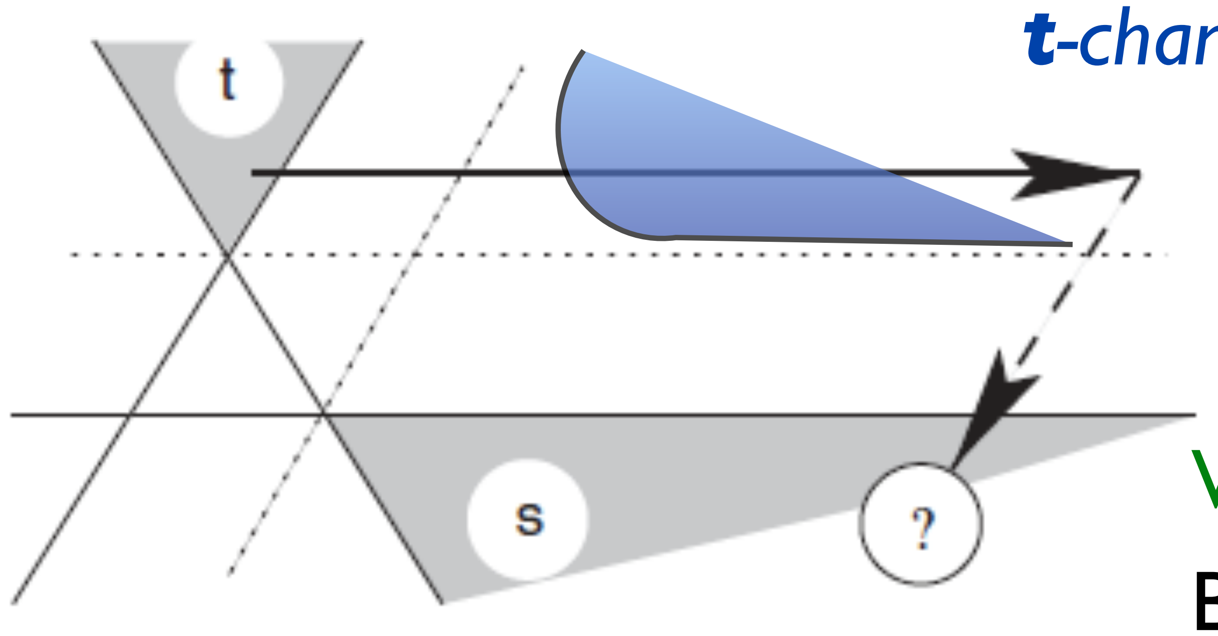
Analyticity, as we learned, follows from *causality*.

Unitarity : the *sum of probabilities* of all possible channels of particle ***creation*** equals one.

There must be one more condition that is not so easy to formulate : the probability that colliding particles ***exchange*** something also cannot be bigger than one.

It is real (on-mass-shell) particles that one can measure and '***count***'.

We could make exchange particles real ('countable') if we chose positive ***t*** above thresholds :



t*-channel unitarity* operates at positive ***t*** and ***s<0***, while we are interested in ***t<0*** and positive (large) ***s*** ...

Need to ***analytically continue*** the ***t***-channel unitarity relation to large unphysical angles, z>>1, to find ourselves ***not too far*** from the physical region of the ***s***-channel scattering!

What is wrong with $\rho_0 = \text{const}$? This regime is viable only if $\sigma_{\rm tot}(s) < \dfrac{\text{const}}{\ln s}, \quad s \to \infty$

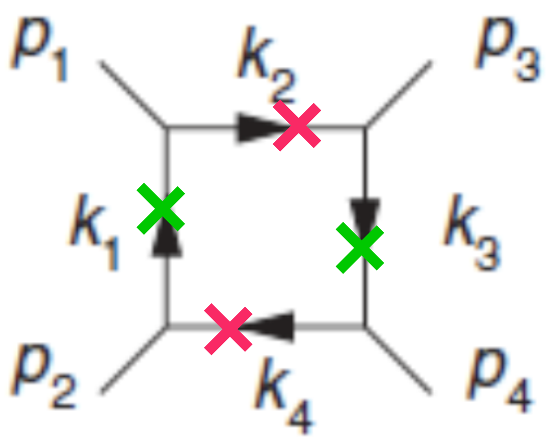
But we know that the total strong cross sections *do not fall with energy*. (They slowly *increase* instead.)

$\rho_0 = \text{const}$ clashes with the hidden knowledge bordered by the *secret hyperbola* - the *Karplus curve*.

Muddling through analytic continuation of ***t***-unitarity would imply changing the theme from ***thriller*** to ***horror***.

I will give you but the sketch of the path

$$(s - 4m^2)(t - 4m^2) = 4m^4$$

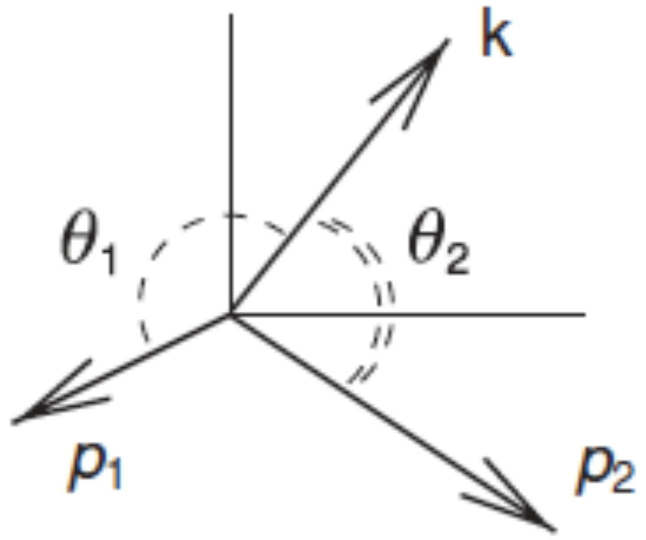


Go to the physical region of the **t**-channel where the energy **t** is just above the first threshold where 2-particle unitarity applies

$$\mathrm{Im}_t\, A(t,s) = \frac{1}{2i}\left[A(t+i\epsilon,s) - A(t-i\epsilon,s)\right] \equiv A_3(t,z) = \frac{1}{2}\cdot$$

$$= \tau \int \frac{d\Omega}{4\pi} A(\mathbf{p}_1,\mathbf{k})A^*(\mathbf{k},\mathbf{p}_2)$$

$$z = \cos\Theta_{12} = 1 + \frac{2\,s}{t-4\mu^2}$$



Integration over *polar* and *azimuthal* angles of the intermediate state momentum **k** can be traded for *two cosines*:

$$d\Omega = d(\cos\Theta_1)\cdot d\phi = dz_1\cdot dz_2 \times \frac{2}{\sqrt{-K}} \qquad K = z^2 + z_1^2 + z_2^2 - 2zz_1z_2 - 1$$

: *simple trigonometry*

What is *not so simple* is to determine where the following integral develops singularity in **z** (*read:* **s**).

$$A_3(t,z) = \frac{\tau}{2\pi}\iint \frac{dz_1\,dz_2}{\sqrt{-K(z,z_1,z_2)}} A(t,z_1)A^*(t,z_2) \qquad -1 \le z_1,\, z_2 \le 1$$

To get singularity in **z** it is not enough to hit the point K=0. The *integration paths* can be deformed as to avoid vanishing of the denominator. The singularity appears when such deformation is no longer possible. Namely, when $z_1,\ z_2$ hit *unitary cuts* of their respective amplitudes!

Evaluating $\mathrm{Im}_s A_3$, we express the *double spectral function Im Im* via *Im* parts as

$$\mathrm{Im}_s A_3 = \mathrm{Im}_s \mathrm{Im}_t A \equiv \rho_{st}(s,t) = \frac{\tau}{\pi}\iint \frac{dz_1\,dz_2}{\sqrt{K(z,z_1,z_2)}}\left[A_1(t,z_1)A_1^*(t,z_2) + A_2(t,z_1)A_2^*(t,z_2)\right]$$

with integration running over the region $z_1z_2 + \sqrt{(z_1^2-1)(z_2^2-1)} \le z\,;\quad z_1 > 1,\, z_2 > 1.$

The *double spectral function* - a specifically relativistic object - bears information about unitarity in both channels, and allow to "*counting exchanges*".
Knowing it, we can restore the amplitude itself using the Double Dispersion (*Mandelstam*) Representation

$$A(s,t) = \frac{1}{\pi^2}\iint \frac{\rho_{st}(s',t')\,ds'\,dt'}{(s'-s)(t'-t)} + [s\to u] + [t\to u]$$

$\rho_0 = \mathrm{const}$ implies *factorisation* of **s**- and **t**- dependence:

$$A_1(s,t) \simeq s\cdot h(s)\cdot F_1(t), \quad \rho_{st} \simeq s\cdot h(s)\cdot \mathrm{Im}\,F_1(t)$$

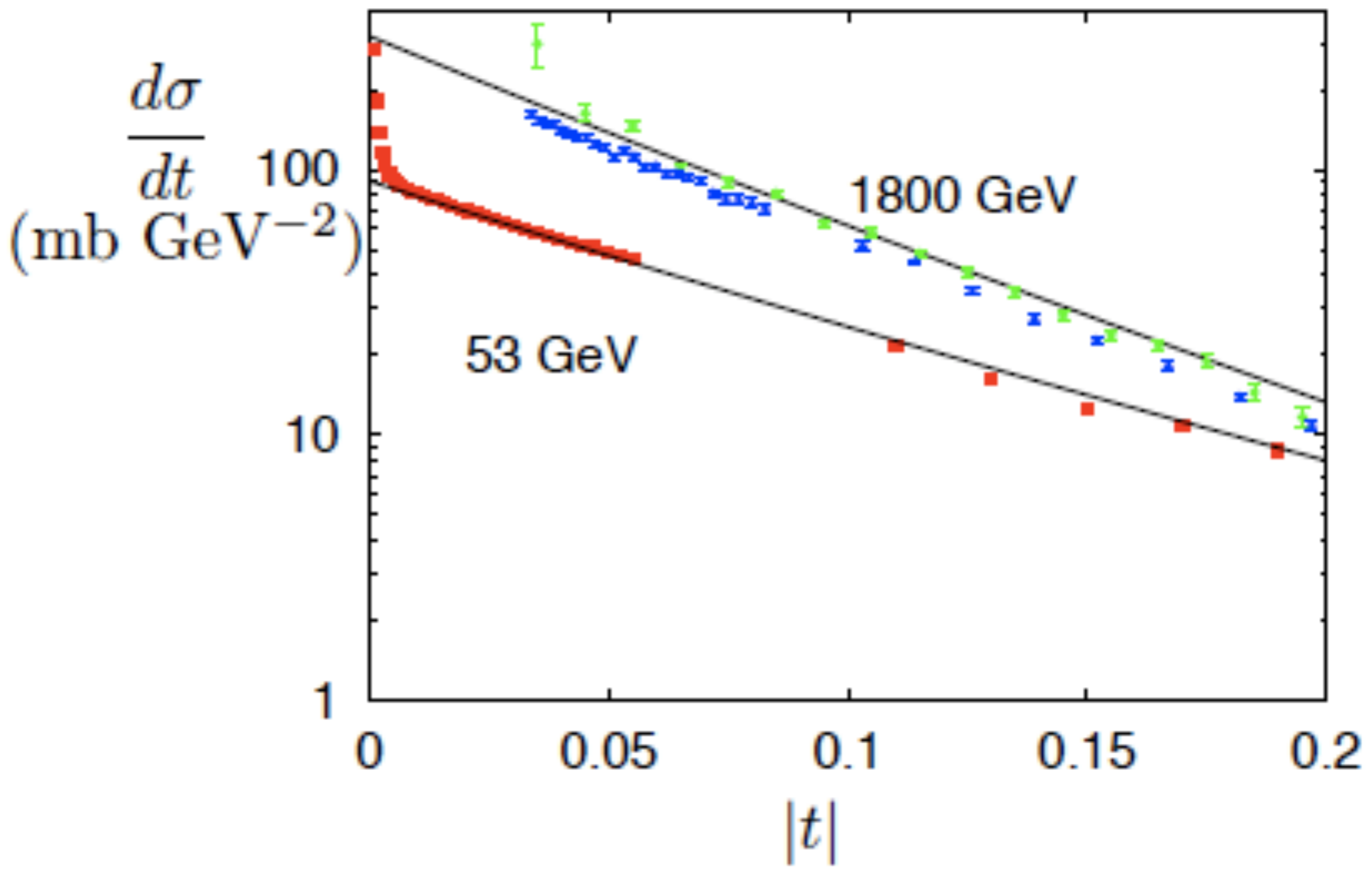
Substituting this ansatz into DDR results in the r.h.s. coming out $\ln s$ times **larger** than the l.h.s.!

$\rho_0 = \mathrm{const}$ hypothesis (*prejudice*) did not survive the Unitarity test!

proton swelling

How about experiment (*The last judgement*)?

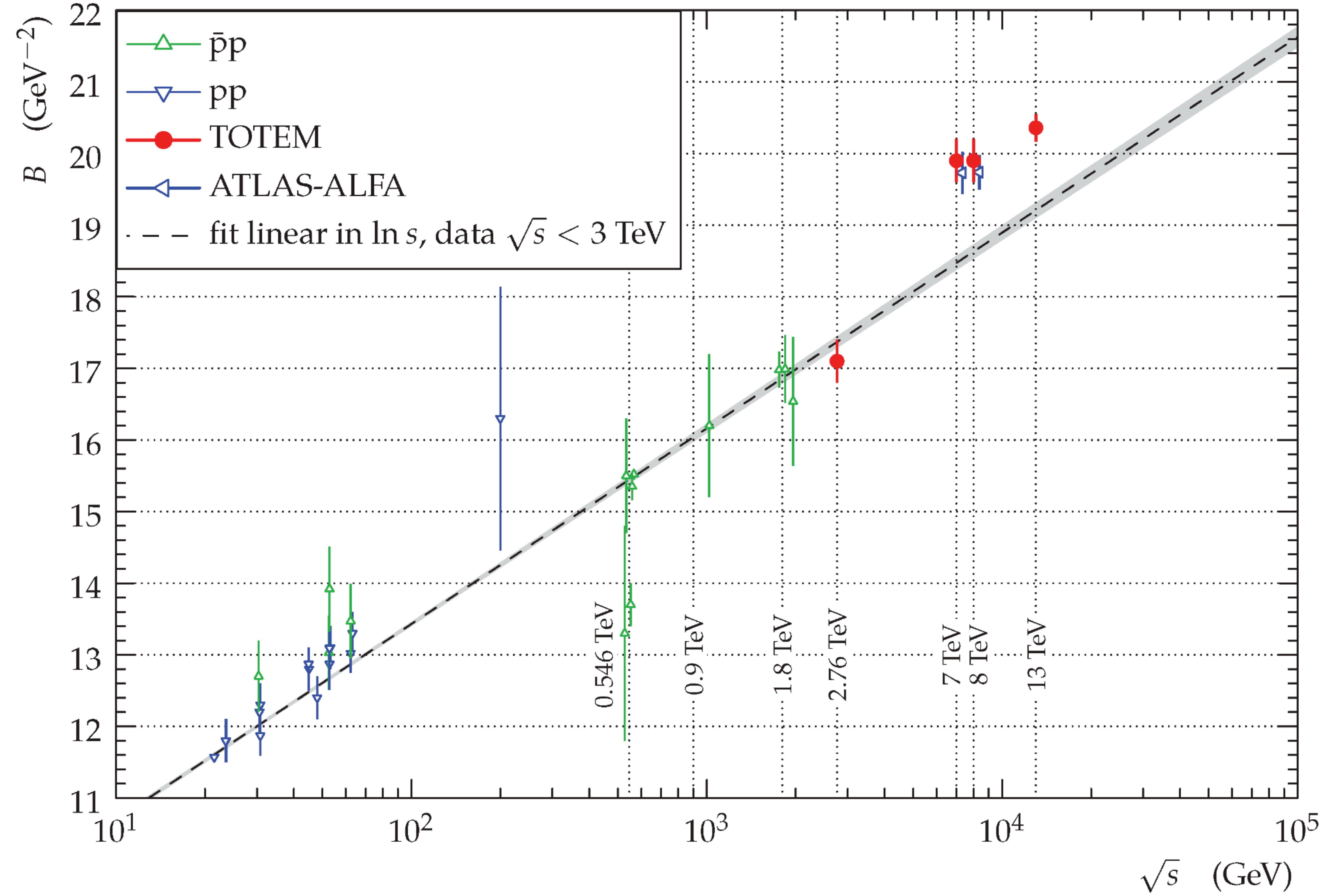
Proton radius can be measured by examining the shape of the forward **pp** elastic peak.



The differential spectrum falls exponentially with increasing momentum transfer,

$$\frac{d\sigma_{\rm el}}{dq^2} \simeq \sigma_0 e^{-{\rm B_{el}}(s)|q^2|}$$

whose falloff parameter translates into proton radius $\quad {\rm B_{el}} = R^2_{\rm proton} \equiv \rho_0^2(s)$



Experimental data from moderate energies all the way up to the LHC show a steady growth of proton radius as $\sqrt{\ln s}$.

... or maybe a little faster (*?*)

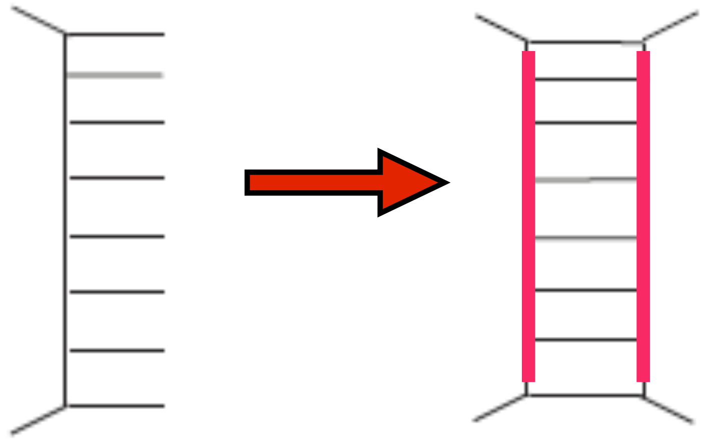
Similar conclusions follow also from $p\bar{p}, \pi p, K\pi$ reactions.

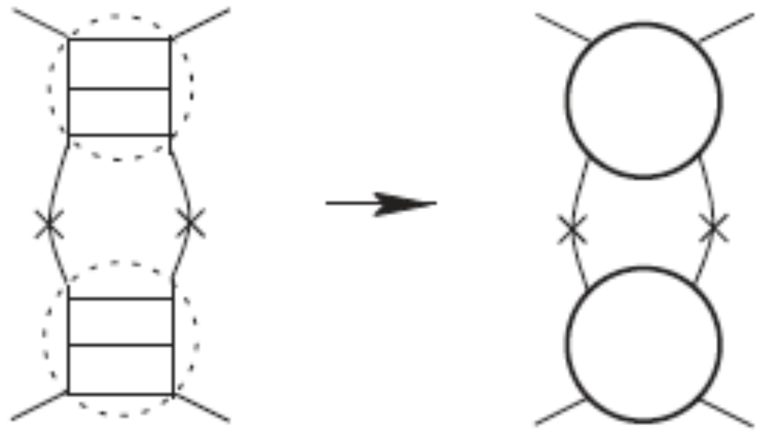
Squaring the "*comb*" amplitude we get a ***ladder** graph* for the cross section.

A remarkable thing about ***ladders***: they satisfy the **t**-channel unitarity!

There is one more hint:

*from **t**-channel point of view,* ***ladder rungs*** make one think of *potential interaction* between 2 particles.





(recall: *Repetition*= *Unitarity*)

This makes one wonder about possible role of **t**-*channel* ***resonances*** in the energy behaviour of **s**-*channel* amplitudes.

This expectation turns out to be true and constitutes the main discovery of the ***Complex Angular Momenta*** theory (***TCAM***).

Write down the amplitude expansion in terms of **t**-*channel partial waves* f(s) rather than f(s) that we employed while deriving the *Froissart bound*:

$$A(s,t) = \sum_{n=0}^{\infty}(2n+1)f_n(t)P_n(z)$$

For a finite **t** a limited number of partial waves contribute.

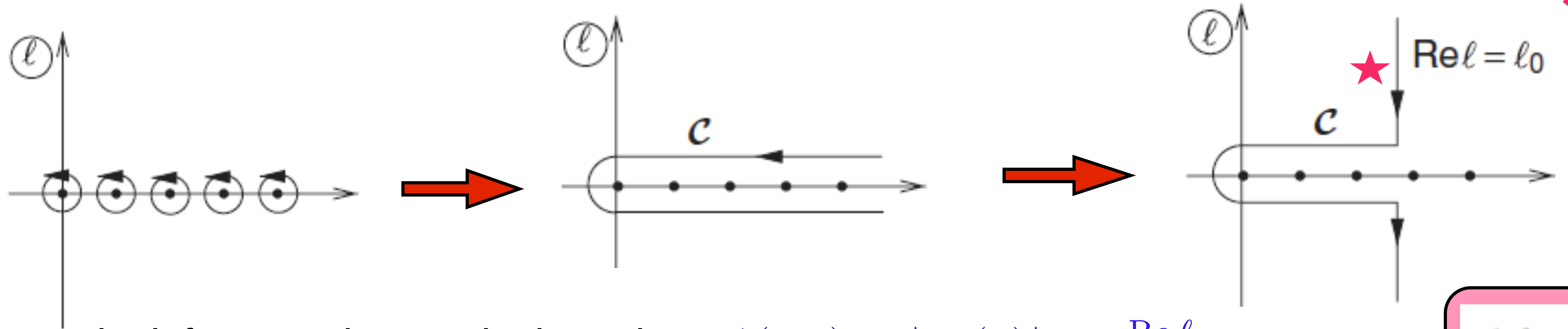
Let boldly truncate the sum and look at behaviour at large (*unphysical*) $z \longrightarrow \infty$.

$$\left[z \equiv z_t = 1 + \frac{s}{t-4m^2}\right] \qquad = \sum_{n=0}^{n_0(t)}(2n+1)f_n(t)P_n(z) + \sum_{n_0(t)}^{\infty}\cdots \quad \sim z^{n_0(t)} \propto s^{n_0(t)}.$$

This is by no means proof. But a good hint at a link worth exploring: characteristic **t**-channel angular momentum as exponent of the **s**-behaviour!

Suppose we knew how to construct a function $f_\ell(t)$ that would be ***analytic*** in ℓ and would coincide with the partial waves in integer points, $f_\ell(t)|_{\ell=n} = f_n(t), \quad n = 0,1,2,\ldots,\infty.$ Employing the Cauchy integral along a circle $\mathcal{C}_n$ surrounding an integer point $\ell = n$, one replaces the sum by a contour in the complex ℓ plane, running along the imaginary axis (*Sommerfeld–Watson integral*)

$$\frac{1}{2i}\int_{\mathcal{C}_n}\frac{d\ell\, P_\ell(-z)}{\sin \pi\ell}f_\ell = f_n$$



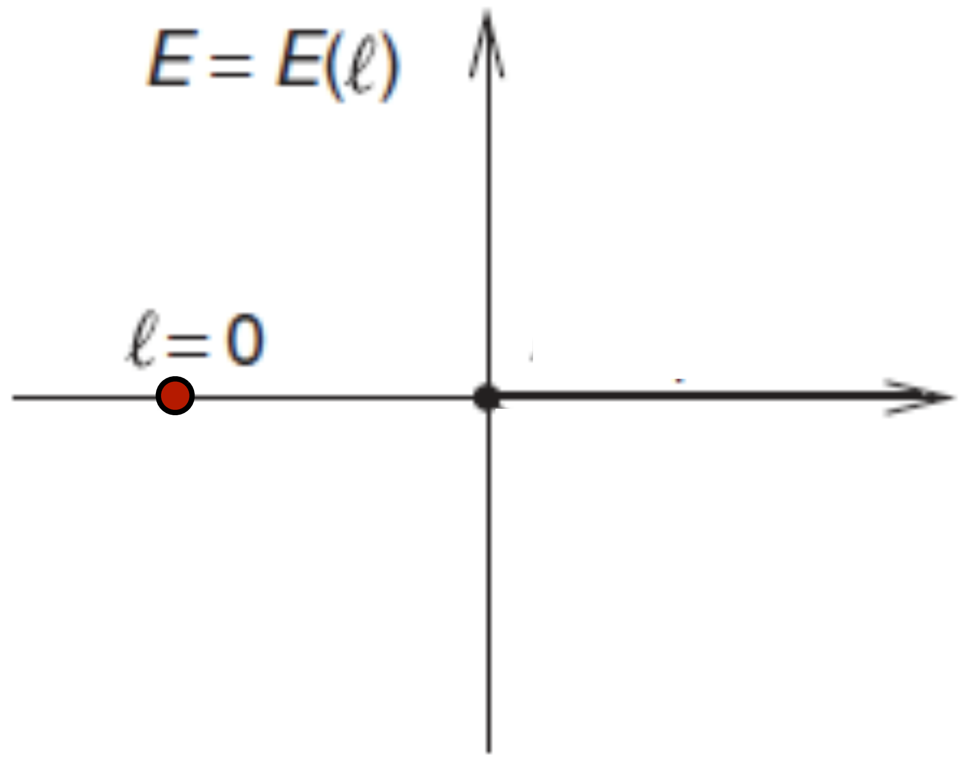
We can move the contour to the left, strengthening the boundary $A(s,t) \propto |P_\ell(z)| \leq z^{\mathrm{Re}\,\ell}$ until a ***singularity*** of the p.w. $f_\ell(t)$ is hit at some $\ell = \alpha(t)$

$$A(s,t) \sim z^{\alpha(t)} \propto s^{\alpha(t)}, \quad s \longrightarrow \infty.$$

Partial wave amplitude as an analytic function of angular momentum was first considered in NQM by T. Regge in 1959.

In the attractive potential, ***bound states*** and ***resonances*** in different partial waves can exist.

Imagine we took a shallow potential such that there is only ℓ=0 bound state.



Let increase ℓ continuously. The repulsive centrifugal potential grows, the energy of the level goes up.

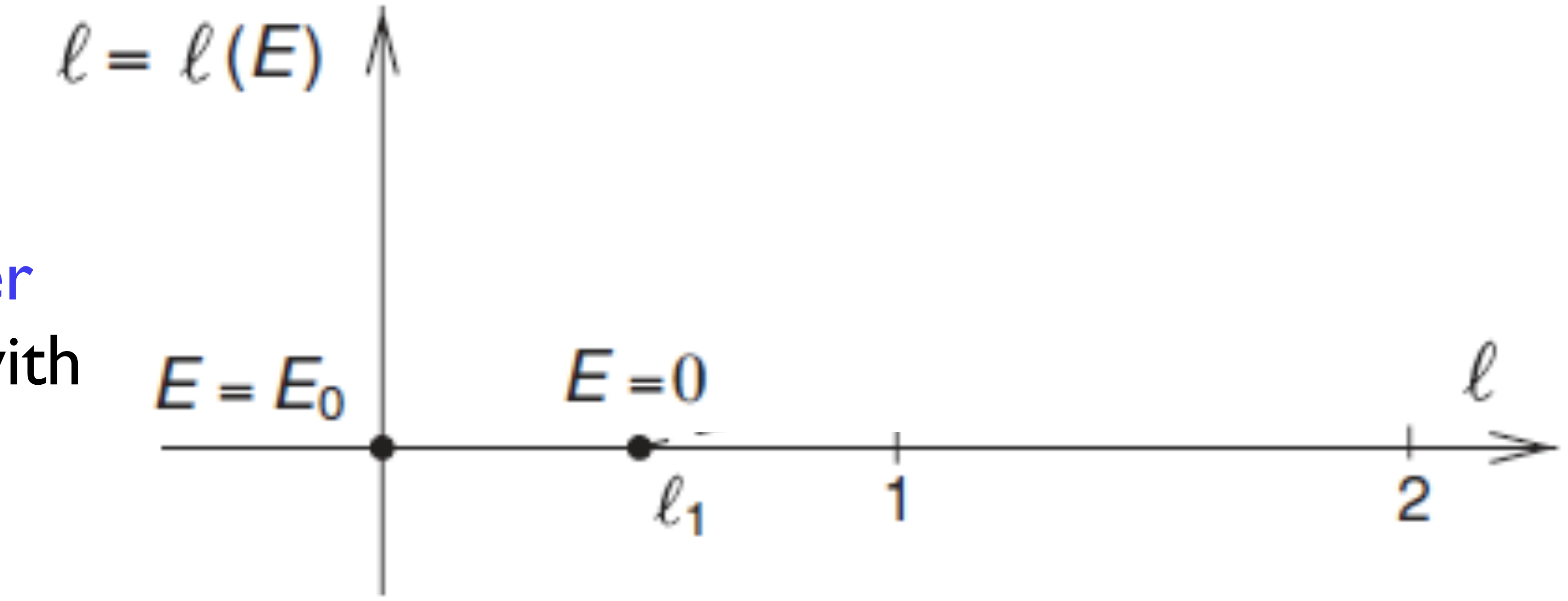
It hits **E=0** at some angular momentum $\ell = \ell_1$. E becomes *complex* and the level dives onto unphysical sheet .

The pole (at integer value ℓ=n) is nothing but a ***resonance*** with spin n

Now reverse the picture : by increasing energy from E_0, we will determine at which ℓ the energy level occurs.

When we kept ℓ real, above E=0 the energy of the level became complex. If we want now to keep E real, it is ℓ to becomes complex above $\ell = \ell_1$.

This curve -"Regge trajectory" - quantifies the characteristic angular momentum that determines asymptotics of the scattering amplitude when $\cos\Theta \to \infty$.



This tells us what is the maximum value of angular momentum at which the attraction is still stronger than the centrifugal repulsion, and the wave function is concentrated at small distances so that f_n with $n < \alpha(t)$ are large and contribute significantly to the partial-wave expansion.

For NQM this was no more than a mathematical curiosity.

In Relativistic Theory the problem of analytic continuation to complex ℓ is solved by two equations for the absorptive ("Im") part of the amplitude

$$A^{\pm}_{\text{abs}}(s,t) = \frac{1}{4i}\int_C d\ell(2\ell+1)f^{(\pm)}_{\ell}(t)P_{\ell}(z)$$

$$f^{(\pm)}_{\ell}(t) = \frac{2}{\pi}\int_{z_0}^{\infty} dz\, Q_{\ell}(z)A^{\pm}_{\text{abs}}(z,t)$$

"Gribov-Froissart projection"

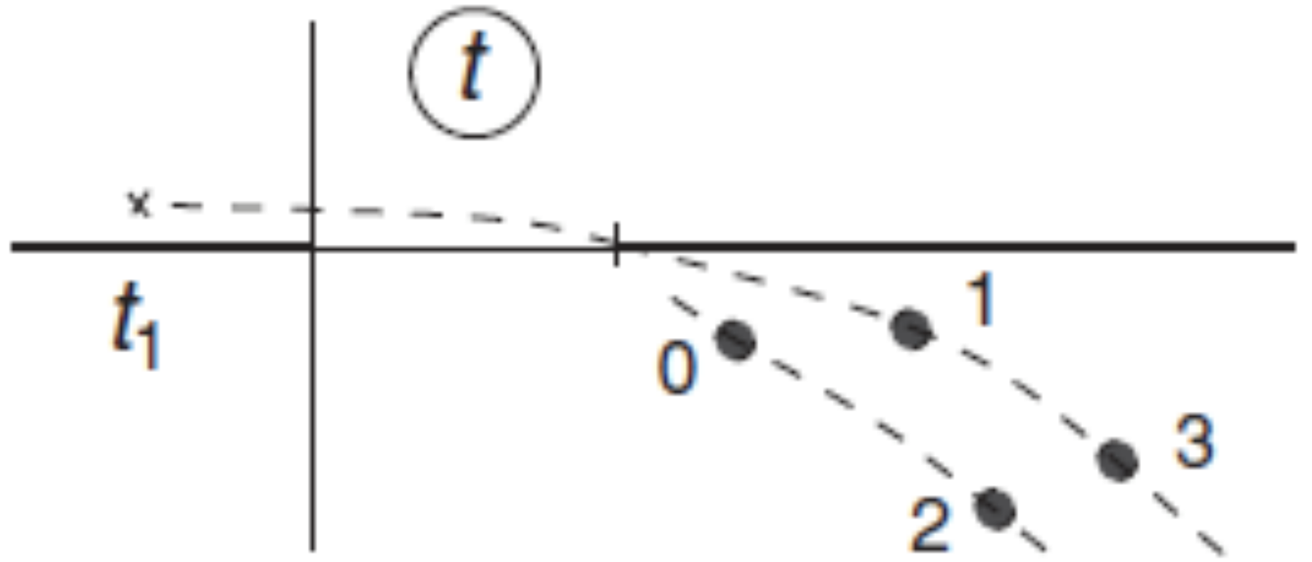
There is a subtlety *: in RT one has to continue even and odd angular momenta independently.*
The reason - presence of the left cut, bringing in a non-continuable factor $(-1)^{\ell}$.
The cure - treat separately $s \leftrightarrow u$ *symmetric and antisymmetric combinations* $A^{\pm} = A(s) \pm A(u)$
This property is called "signature" (positive, negative).

Singularities, as before, arise when analysing the unitarity relation of the t-channel

Reggeon

$$\frac{1}{2i}[\phi_n(t+i\epsilon)-\phi_n(t-i\epsilon)] = C_n\phi_n(t+i\epsilon)\phi_n(t-i\epsilon)$$

which relation *holds for arbitrary complex* ℓ.



Without much ado, we dive under the unitary cut to get a Regge pole

$$\phi_\ell^{(\pm)}(+) = \frac{\phi_\ell^{(\pm)}(-)}{1-2iC_\ell\,\phi_\ell^{(\pm)}(-)} \qquad \phi_{\ell_0}^{(\pm)}(t) = \frac{1}{2iC_{\ell_0}(t)} \;\longrightarrow\; \ell_0 = \ell^{\pm}(t).$$

A truly remarkable picture! We knew that resonances with different spins **n** live on the unphysical sheet. Now not only have we got the statement about the large-s behaviour, but also about the resonances themselves : *they are analytically linked to each other!*

Regge trajectory is characterised by its quantum numbers (I, B, S, ...), plus "signature". *Reggeon akin particle with varying spin* **J(t)**

Two (generally speaking, different) analytic curves $\ell^{\pm}(t)$ *that combine together* ***t****-channel* ***resonances*** *with even and odd spins at the same time determine* ***asymptotic behaviour*** *of the symmetric and anti-symmetric parts of the* ***s****-channel amplitude.*

Understanding this cross-channel relation constitutes the main achievement of the theory of complex angular momenta.

Substituting the pole $f_\ell^{\pm}(t) = \dfrac{r^{\pm}(t)}{\ell-\alpha^{\pm}(t)}$ into the Sommerfeld–Watson integral results in

$$A^{\pm}(s,t) = -\frac{\pi}{2}r\frac{2\alpha+1}{\sin\pi\alpha}[P_\alpha(-z)\pm P_\alpha(z)]$$

$$z = 1+\frac{2s}{t-4\mu^2}$$

- **I** Near an integer point (of proper signature) the reggeon amplitude reproduces particle exchange
- **II** The residue *factorises*, $r = g_a g_b$ as it did before in the case of resonances - follows from ***unitarity*** !
- **III** The Reggeon propagator differs essentially from the particle propagator: it contains a non-trivial *complexity*

$$D^{\pm}(s,t) = -\frac{(-s)^{\alpha^{\pm}(t)}\pm s^{\alpha^{\pm}(t)}}{\sin\pi\alpha^{\pm}(t)} = s^{\alpha^{\pm}(t)}\cdot\xi_\alpha^{\pm}$$

signature factor $\xi_\alpha^{\pm} = -\dfrac{e^{-i\pi\alpha^{\pm}(t)}\pm 1}{\sin\pi\alpha^{\pm}(t)}$

$$\xi^{+}_{\alpha(t)} = i-\cot\frac{\pi\alpha(t)}{2}$$
$$\xi^{-}_{\alpha(t)} = i+\tan\frac{\pi\alpha(t)}{2}$$

Near those integers where ξ_α *has poles* ($\alpha^{+}=2n$, $\alpha^{-}=2n+1$) *it is almost real* (*as it should be for particle exchange*).
In physical points of *'alien signature'*, ($\alpha^{+}=2n+1$, $\alpha^{-}=2n$), *the amplitude is purely imaginary,* $\xi_\alpha = i$ (*classical diffraction off a black disc*).

Take $\pi N \to \pi N$ scattering as example.

The two-pion system in the physical region of the **t**-channel has ***resonances***.

Look at the famous vector meson ρ (J=1) and its known "*excitations*" with spins J=3, 5, ...

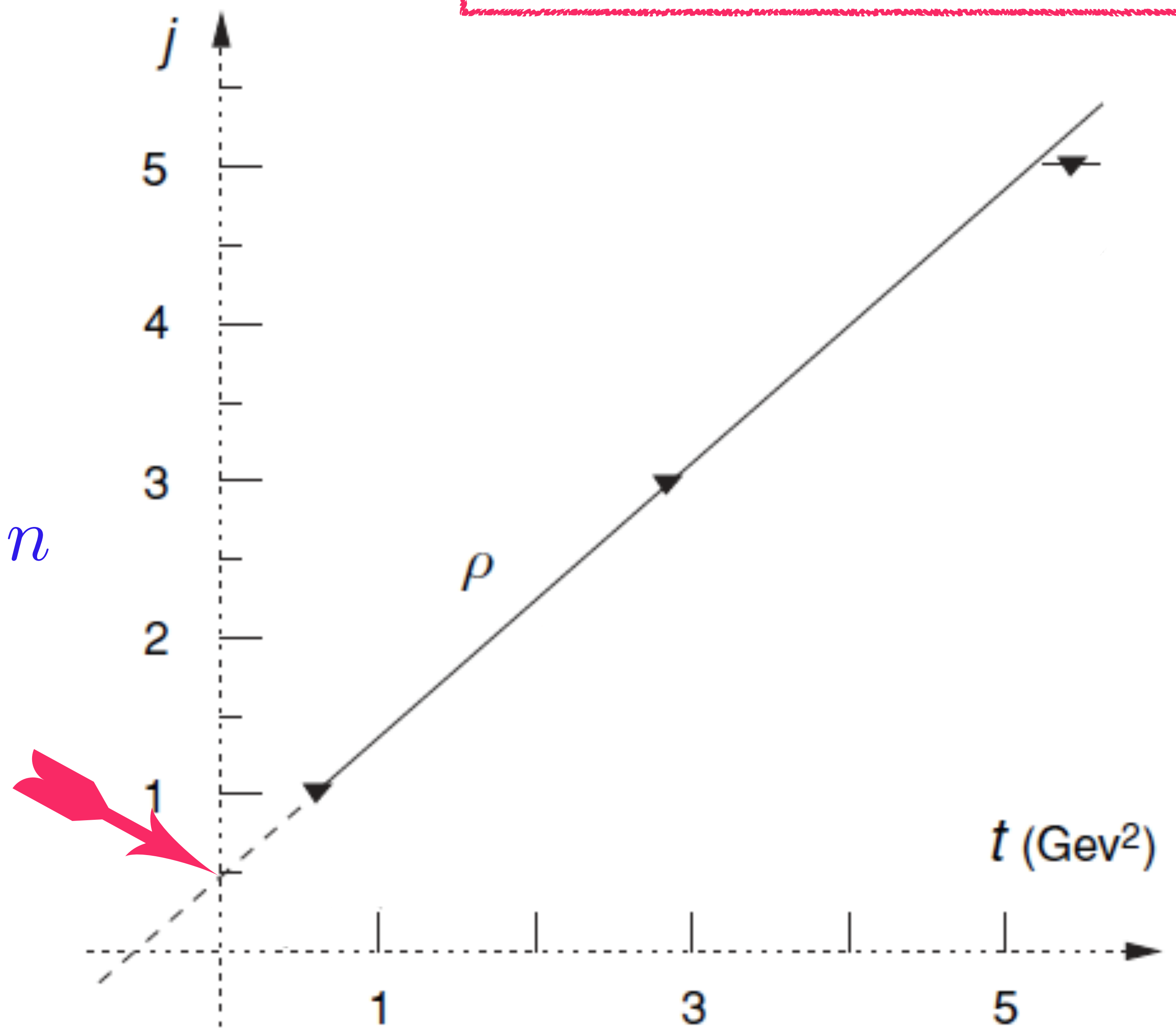
Plot their spin vs squared masses (**t**). This is the Chew-Frautschi diagram.

Since the Regge trajectory looks *well linear*, continue drawing to **t**=0 and further into **t**<0.

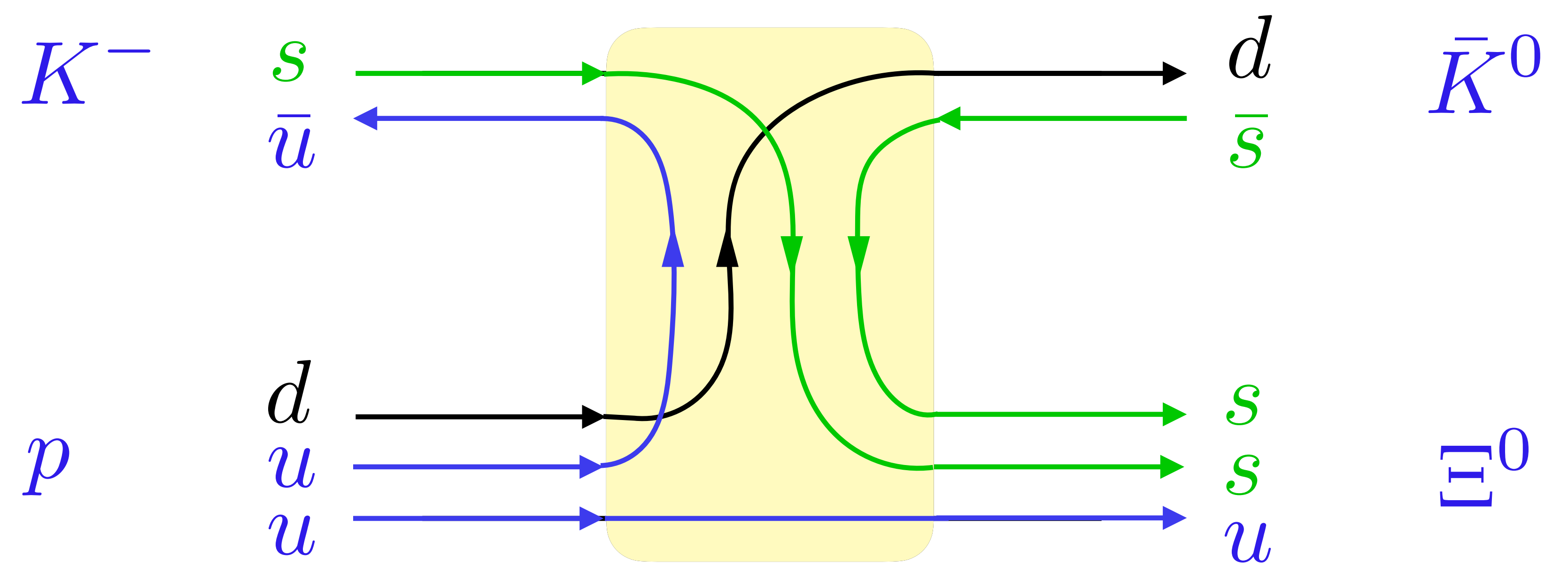
We expect that the differential cross section of the change-exchange reaction $\pi^+ p \to \pi^0 n$ in the *near-to-forward* direction should fall with energy. As it does indeed.

$$\frac{d\sigma_{\pi^+ N \to \pi^0 n}}{dt} \simeq \frac{1}{4\pi} \left| \frac{A(s,t)}{s} \right|^2 \Longrightarrow s^{2(\alpha_\rho(t)-1)} g_\rho^{\pi\pi}(t) g_\rho^{NN}(t) \Big|_{t\simeq 0} \sim s^{-1}$$

Chew-Frautschi plot



Another example or Regge phenomenology - of the *opposite kind*



Looks a perfectly legitimate reaction :

electric charge	Q	-1+1 = 0+0
baryon charge	B	0+1 = 0+1
strangeness	S	-1+0 = 1-2

to better see strangeness balance, here is the quark content of participants

Quantum numbers in the t-channel here are B=0, S=2.

So, we would need to exchange a double-strange meson that cannot be assembled from a quark and an antiquark (as all known mesons are).

We stumble upon an "exotic" state - ***tetra-quark ...*** As long as such animal (*and its Regge intercept*) is not found, we can expect the corresponding cross section to fall extremely fast with energy. Which is does.

Not only mesons but also baryons fall onto Regge trajectories.

All trajectories happen to be *approximately linear* (?)
This phenomenological finding has ignited the development of the **String theory** (quantum spectrum of a rotating stick - linear in J).

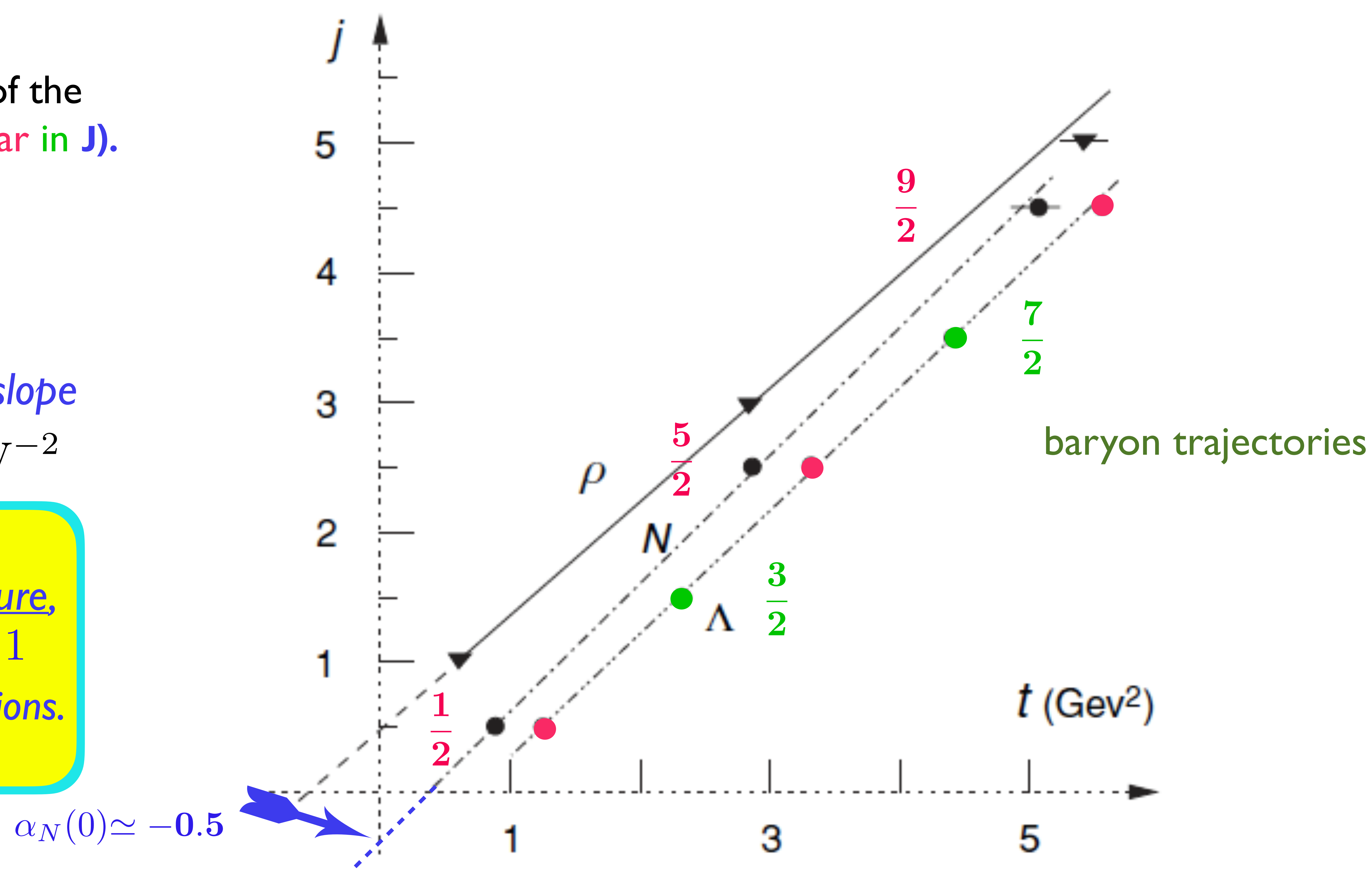
Two *Lambda*-trajectories display *degeneracy in signature*.

(The same degeneracy manifests itself in the K^* family.)

All *Reggeons* (*but one, t.b.c.*) have approximately the same *slope*

$$\alpha_{\rm R}(t) \simeq \alpha_{\rm R}(0) + \alpha'_{\rm R}\cdot t \qquad \alpha'_R \simeq 0.9 - 1.0\ {\rm GeV}^{-2}$$

NB: The trajectory ***apart*** bears the name "**Pomeron**".
It has *quantum numbers of the vacuum* and *positive signature*,
and has to have the maximally allowed *intercept* $\alpha_{\rm P}(0) = 1$
Pomeron is responsible for the behaviour of *total cross sections*.
Its **t**- *slope* is much smaller: $\alpha'_P \simeq 0.25\ {\rm GeV}^{-2}$



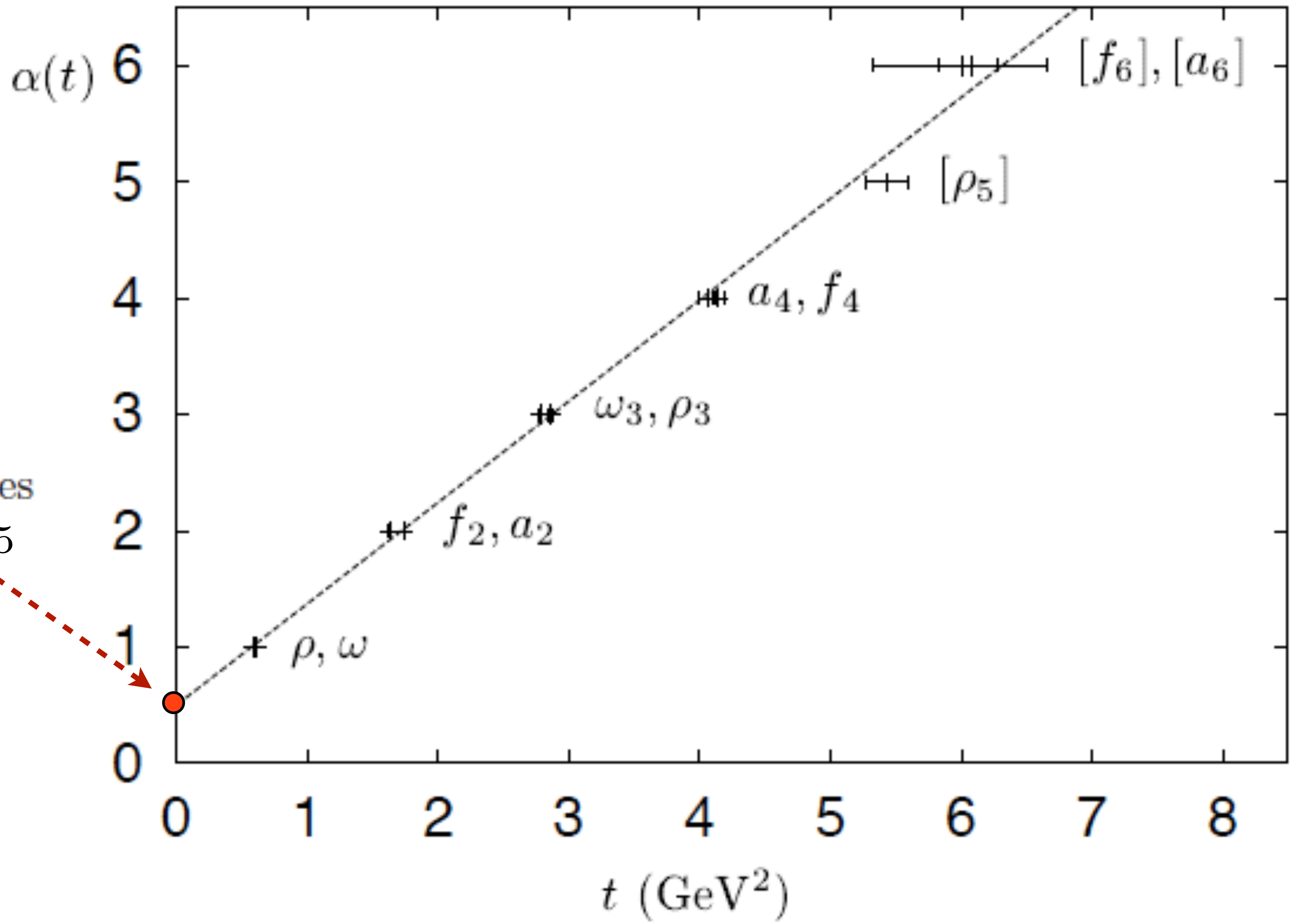
By continuing the N trajectory down to **t=0**, we conclude that the differential Xsection of $\pi N \to \pi N$ scattering in the ***backward*** direction $-t \simeq s \to \infty, |u| = {\rm const}$ should falls with **s** much faster:

$$\frac{d\sigma_{\pi^+ N\to n\pi^0}}{du} \simeq \frac{1}{4\pi}\left|\frac{A(s,t)}{s}\right|^2 \propto s^{2(\alpha_N(u)-1)}\left[g_N^{\pi N}(u)\right]^2\Big|_{u\simeq 0} \sim s^{-3}$$

The whole bunch of *"natural parity"* trajectories happen to be *degenerate* :

degeneracy

rho($I^G{}_{sgn}=1^-{}_-$)   omega($0^+{}_-$) f_2($0^+{}_+$) a_2($1^-{}_+$)



for ρ, ω, f_2, a_2 trajectories

$\mathrm{Im}\ A(s, t=0) \ \propto\ s^{0.55}$

what is "natural parity"?

$$P_r = P \cdot (-1)^\ell$$

scalar, vector, tensor mesons

P_r = +1 (*"natural"*)

pseudoscalar, axial vector,..

P_r = -1 (*"unnatural"*)

"unnatural parity" trajectories $(\pi,\ a_1)$ *are poorly known;* $\alpha(0) \simeq 0$

Which singularity is the rightmost ?

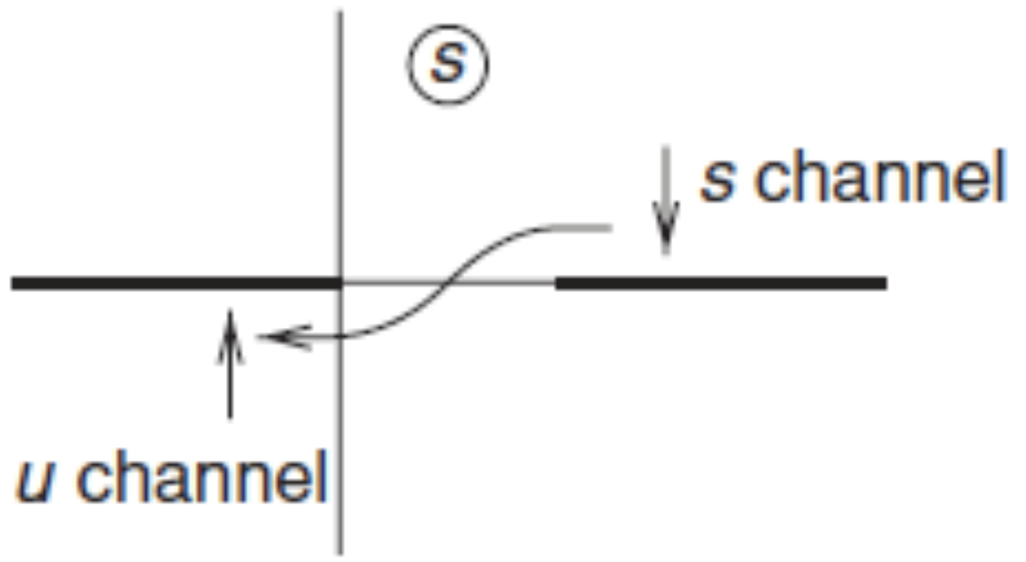
It is straightforward to prove that

> *In the interval $0 \leq t \leq 4\mu^2$ (of which $t = 0$ is the point of the most interest to us) the rightmost pole in the j-plane has to have positive signature, $Pj = +1$, and all the quantum numbers = those of the vacuum.*

To assure *non-falling* σ_{tot}, it has to have the *maximal intercept* $\alpha_P(0) = 1$

*Moreover, from **s**-channel unitarity follows that it has to have positive signature*

$$\sigma_{ab} \propto \mathrm{Im}\, A_{ab}(s,t) = g_a g_b s^{\alpha} \cdot \mathrm{Im}\, \xi_{\alpha} > 0$$



Otherwise, *the cross section in the **u** channel* would turn negative.

A similar story with other quantum numbers like *isospin* (any irreducible tensor has a negative diagonal element ...)

Hypothesis of the leading pole = "*Pomeron*" (Gell-Mann's name for Gribov's *vacuum pole*)

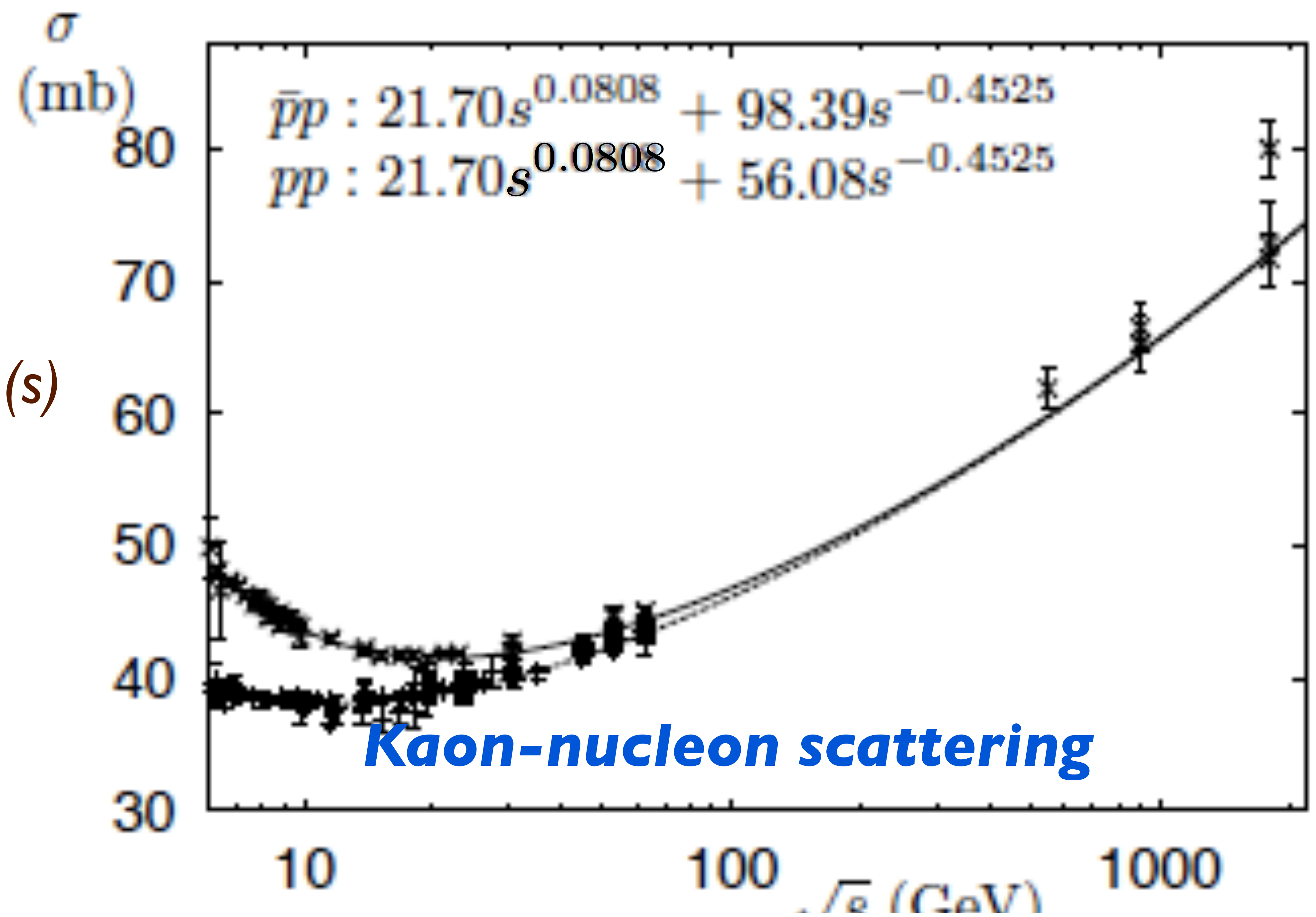
A weird thing about the Pomeron trajectory: *no known mesons* are ascribed to! (all info comes from $t<0$)

How does this work it practice ?

Total cross sections :

Pheno

nucleon-nucleon scattering



A few things to notice :

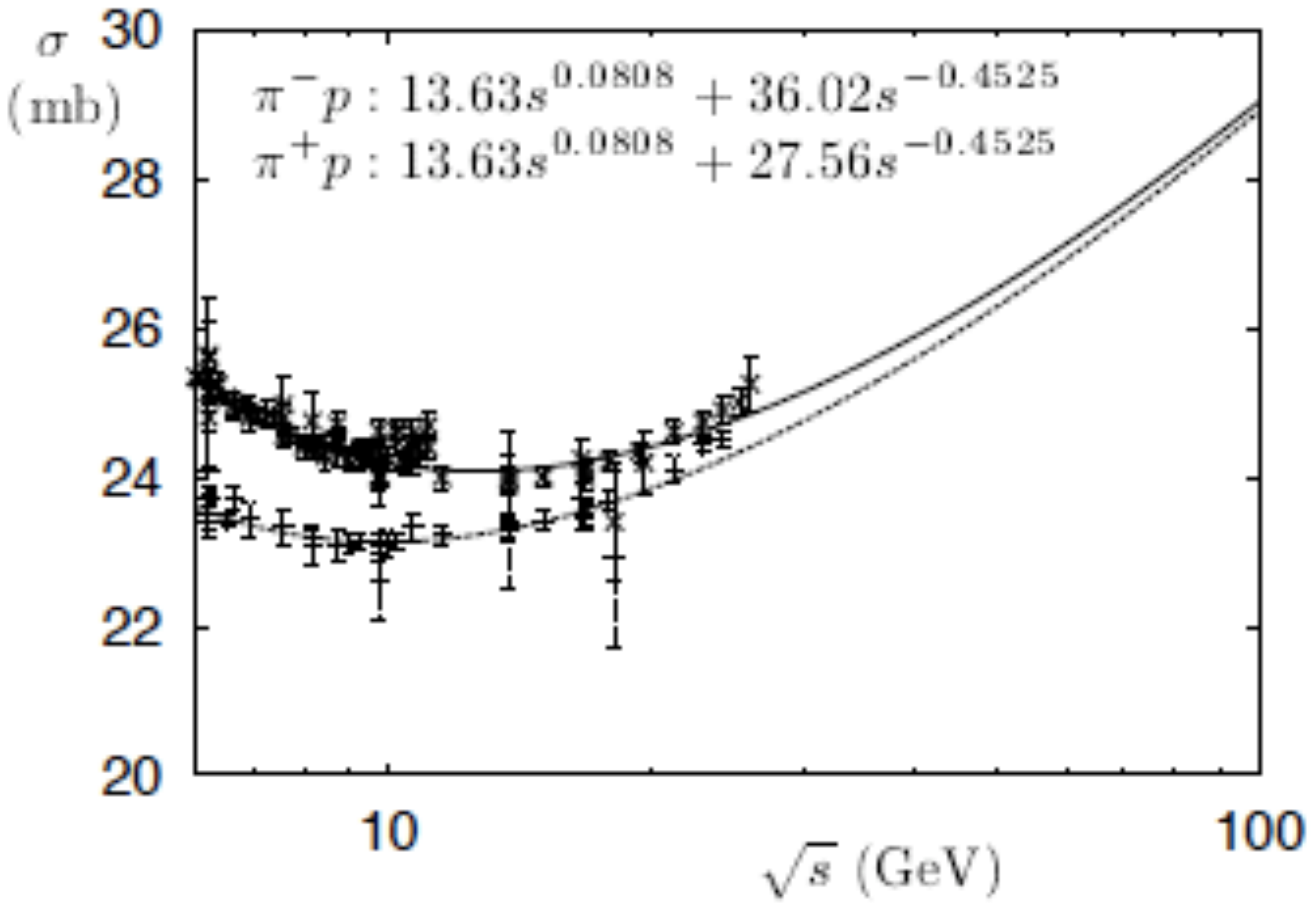
Donnachie & Landshoff fit

hardly distinguishable from *log(s) + log²(s)*

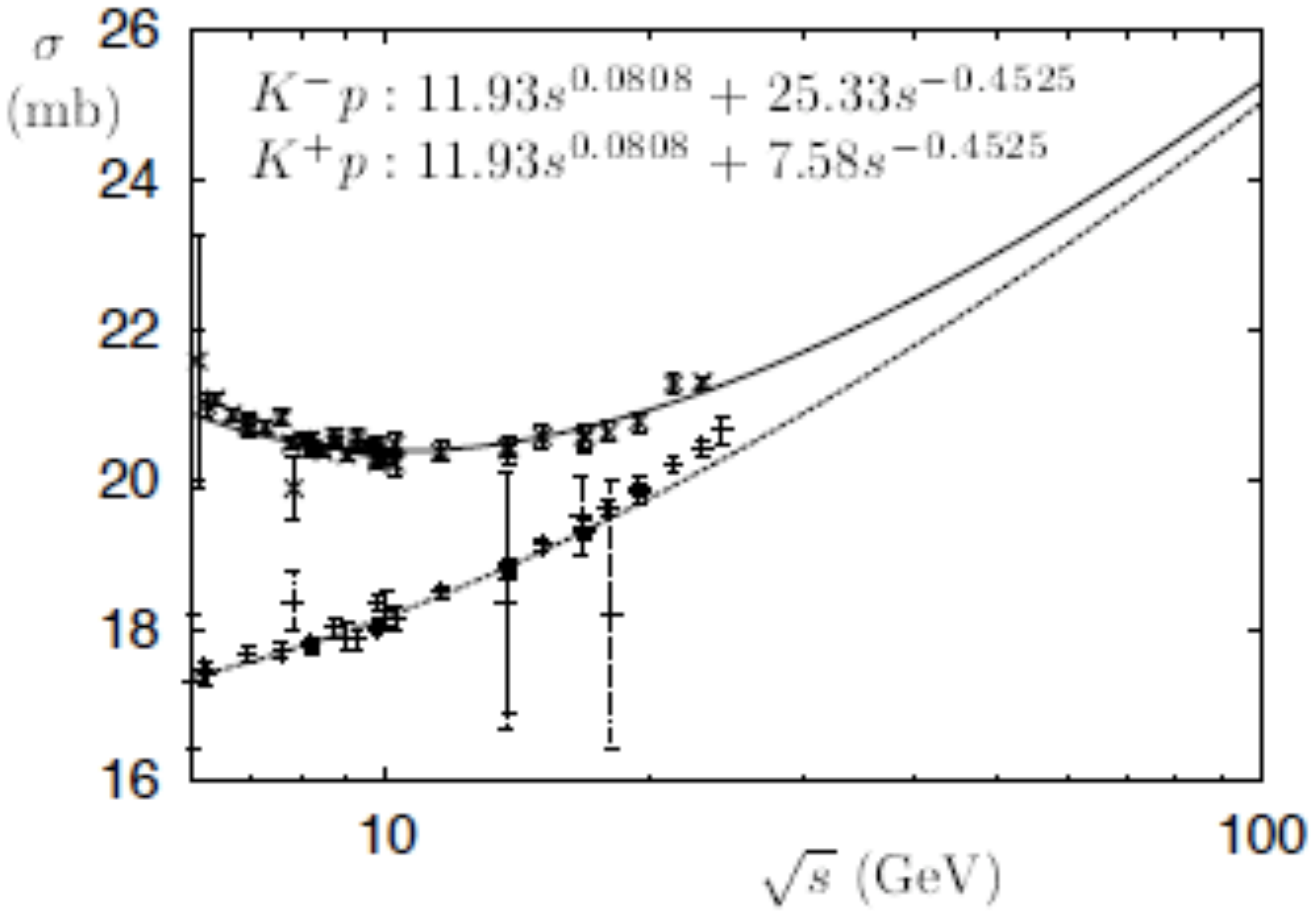
interplay between positive (f_2, a_2) and negative signature (ρ, ω) reggeons

pion-nucleon scattering

$\pi : p \simeq 2 : 3$ *quark counting !*



Kaon-nucleon scattering



Differential cross sections

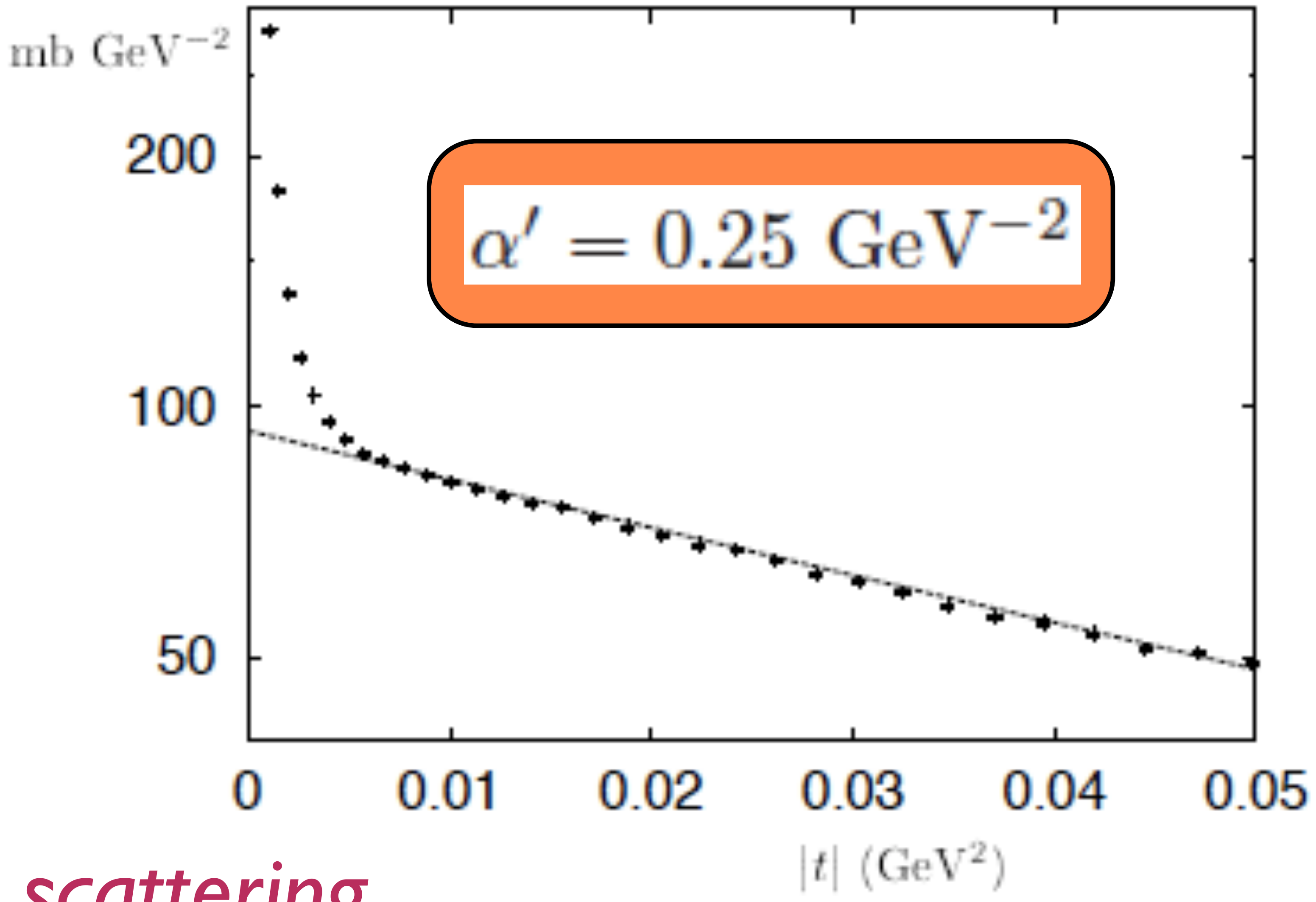
Pheno

A miraculous simplicity : suppose that ***P*** couples to hadrons as *the photon* does and that its coupling is proportional to the number of *valence quarks*.

Then for the cross section of elastic *nucleon-nucleon* scattering we expect

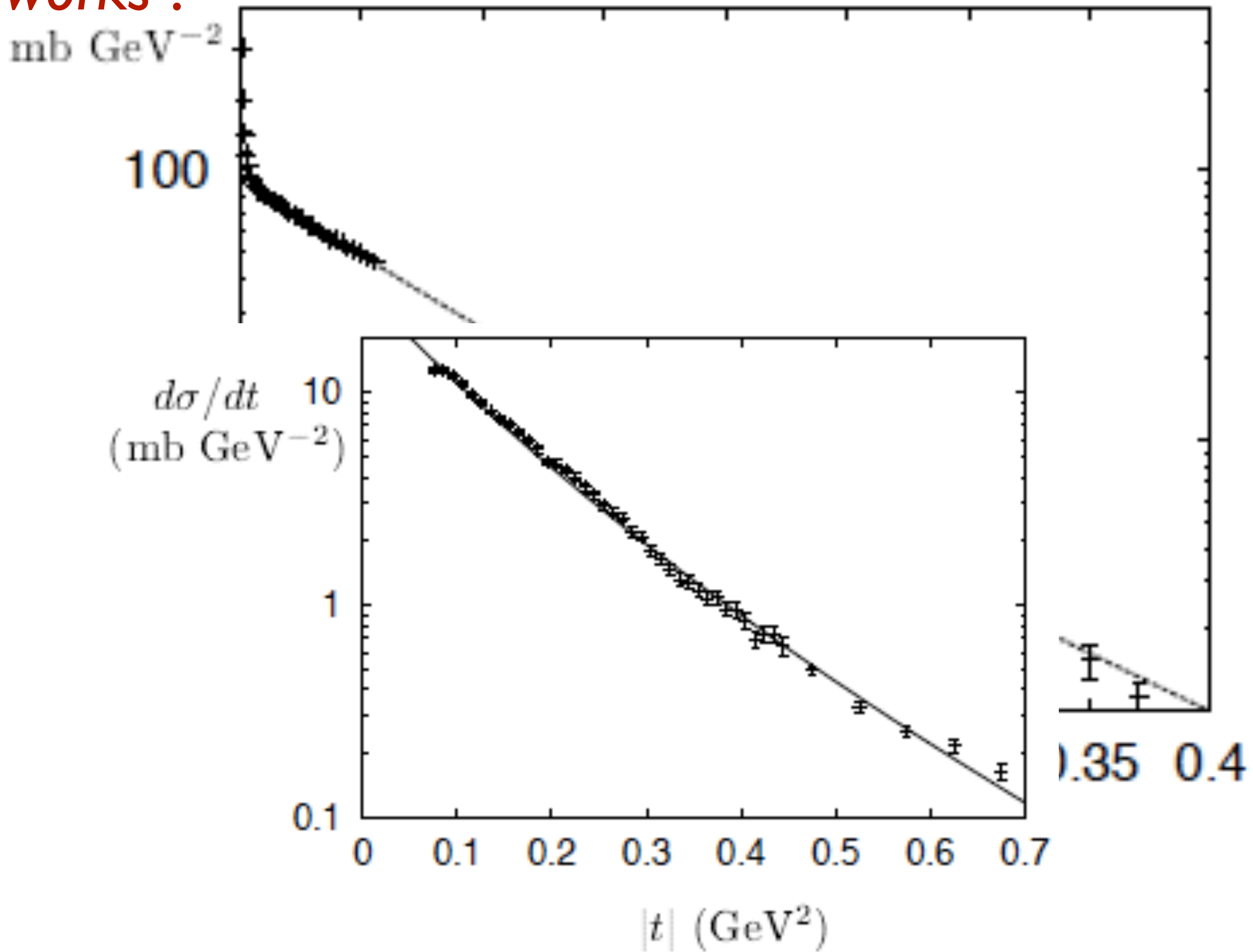
$$\frac{d\sigma}{dt} = \frac{[3\beta_{\mathbb{P}} F_1(t)]^4}{4\pi} (\alpha'_{\mathbb{P}} s)^{2(\epsilon_{\mathbb{P}} + \alpha'_{\mathbb{P}} t)}$$

(F_1 *the e.m. Dirac proton form factor*)



Stepping away from the *Coulomb peak* (smallest **t**) we determine the slope of the ***P*** trajectory

And it works !



pion-nucleon scattering

measure e.m. pion form factor

replace

$$[3F_1(t)]^4 \longrightarrow [3F_1(t)]^2 \; [2F_\pi(t)]^2$$

and obtain a parameter-free prediction for

$$\frac{d\sigma^{\pi p}(s,t)}{dt}$$

The *goods* and *bads* of the Regge pole picture.

Universal high energy behaviour of total scattering cross sections. (Pomeron, P)
Energy growth of the hadron interaction radii.
Fragmentation scaling and universality of the Feynman plateau.
High energy factorisation of cross sections of various processes.
Power energy decrease of two-particle "charge exchange" cross sections. (Reggeons, R)
Steep falloff of cross sections with "exotic" quantum numbers in the t *channel.*

Apparent linearity of Regge trajectories remains unexplained (if not mysterious).
Absence of predicted parity degeneracy among baryons.
Elastic scattering angular distribution at (relatively) large $|t|$ *is poorly described.*

The latter "difficulty" is not accidental.

As we will see, Pomeron pole is not alone on the angular momentum plane.
As particles/resonances were creating threshold cuts, in the very same manner a Regge pole gives rise to branch singularities.

P *is special in this respect : on the a.m. plane Pomeron branchings are not separated from the pole but are sitting "on top of it" ...*

From the s*-channel point of view, Pomeron branchings are intimately related with the pattern of multiplicity fluctuations in multi-particle production.*
They are also responsible for important phenomenon of "shadowing", typical for strong interaction dynamics

s-channel structure of the Pomeron exchange & physics of Reggeon branchings

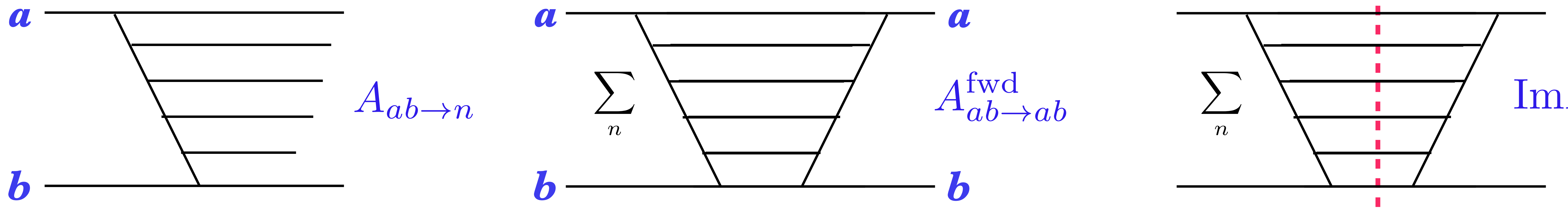
The Reggeon exchange amplitude $\dfrac{P_{\alpha(t)}(-z) \pm P_{\alpha(t)}(z)}{\sin \pi\alpha(t)}$ *replaces that of usual particle exchange* $\dfrac{P_\sigma(z)}{m_\sigma^2 - t}$ $\left[z \equiv \cos\Theta_t = 1 + \dfrac{2s}{t-4m^2} \right]$

Regge pole contribution bears essential complexity : $A^{\pm}_{\rm regg}(s,q^2) \propto \xi_\alpha s^\alpha$ $\xi_\alpha = \dfrac{e^{-i\pi\alpha} \pm 1}{-\sin\pi\alpha}$ *- the signature factor*

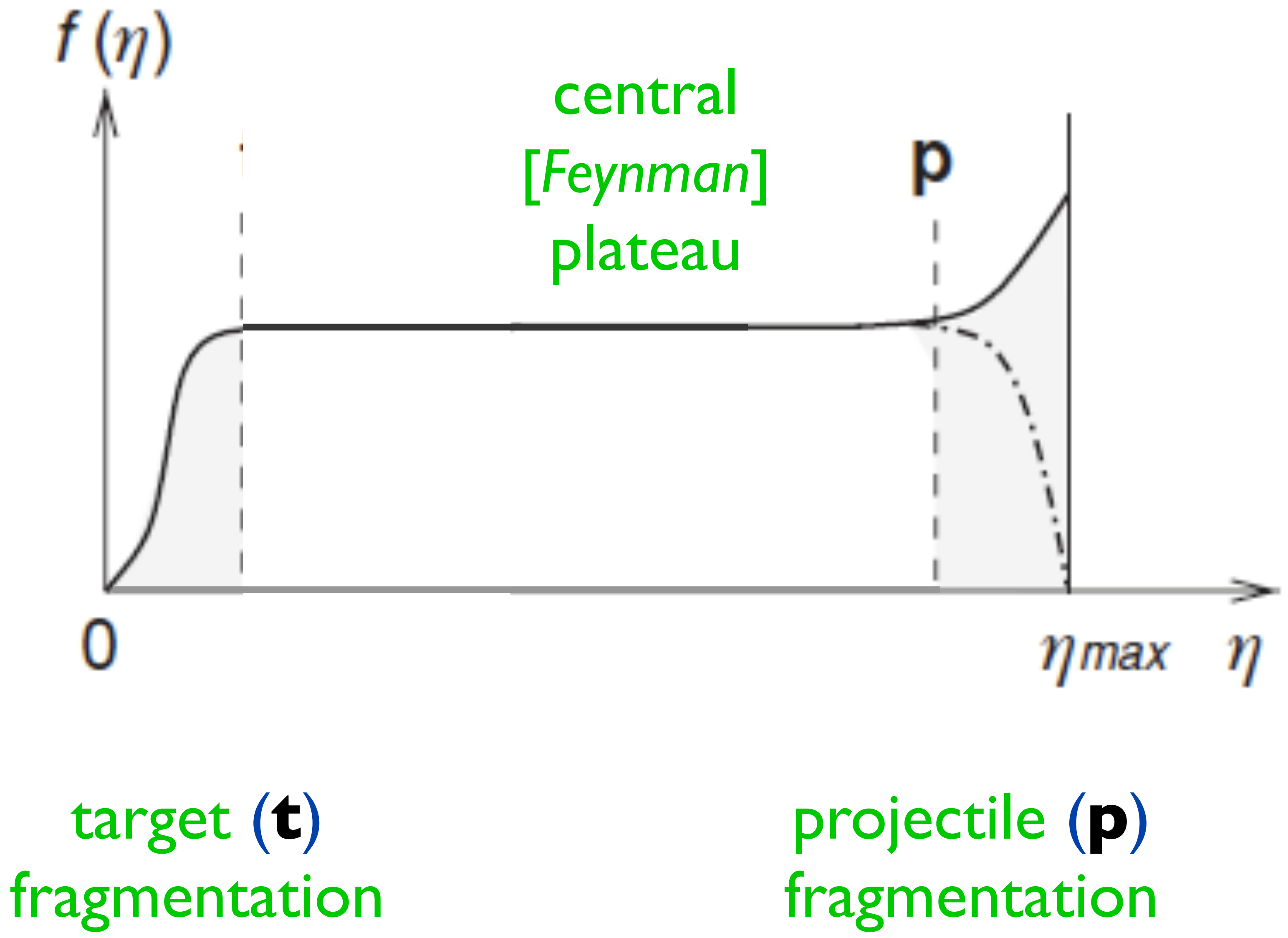
We observe that the **Im** part is large: $\mathrm{Im}\, A^{\pm}_{\rm regg} \propto \mathrm{Im}\left[i - \dfrac{\cos\pi\alpha \pm 1}{\sin\pi\alpha} \right] s^\alpha = s^\alpha$

Moreover, if $\alpha_P(0) = 1$ *, (Pomeron) the forward amplitude is purely imaginary (cf. black disk)*

Imaginary Pomeron exchange means that the elastic amplitude "contains inside" lot of inelastic processes. These are our ladders (squared "combs").



$$\mathrm{Im} A^{\rm fwd}_{ab\to ab} = \sum_n |A_{2\to n}|^2 = s\,\sigma^{tot}_{ab}$$



Hadrons in the "comb" have limited transverse momenta and distributed uniformly in rapidity

*Nice thing about this variable is its friendliness towards Lorentz boosts in the direction of collision (**z**).*

$$\eta = \frac{1}{2}\ln\frac{k_0 + k_z}{k_0 - k_z} = \ln\frac{k_0 + k_z}{m_\perp}$$

The absolute value of rapidity depends on the frame, but $\Delta\eta = \eta_1 - \eta_2$ *stays boost invariant.*

$\eta_t = 0,\ \eta_p = \ln s$ *in the lab. turns into*

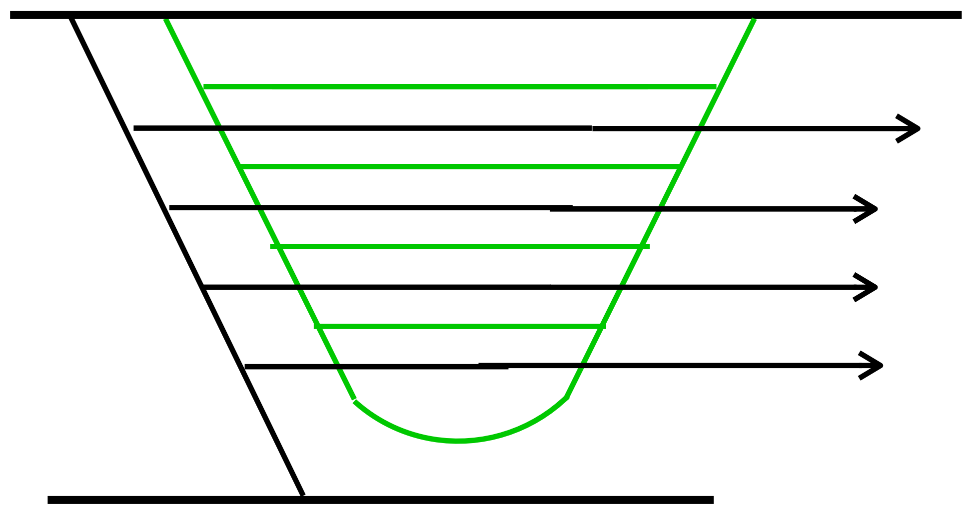
$\eta_t = -\dfrac{1}{2}\ln s,\ \eta_p = +\dfrac{1}{2}\ln s$ *in c.m.s., etc*

$$d\Gamma(k) = \frac{d^3\mathrm{k}}{k_0} = d^2\mathrm{k}_\perp \frac{dk_z}{k_0} = d^2\mathrm{k}_\perp\, d\eta$$

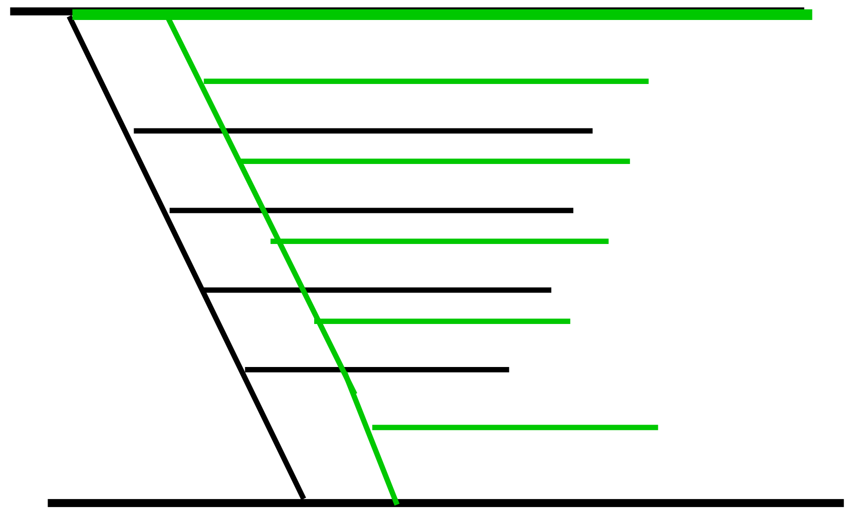
*With **s** increasing, the plateau widens while the fragmentation regions stay intact while parting.*

We established a link between long living "comb" fluctuation and the Pomeron.

What if our projectile fluctuates more than once?

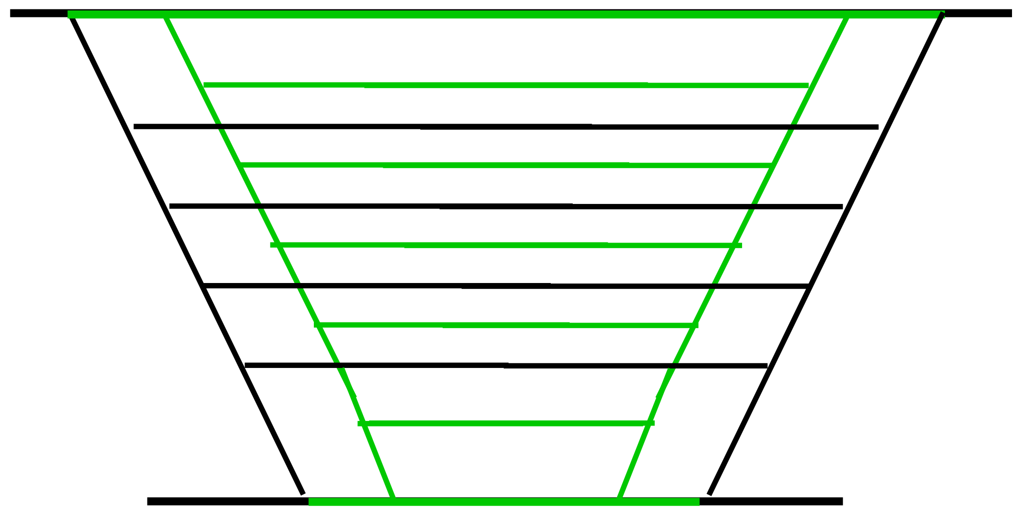


If only one parton hits the target, the adjacent comb - the virtual fluctuation that did not loose coherence - collapses back without affecting the interaction.

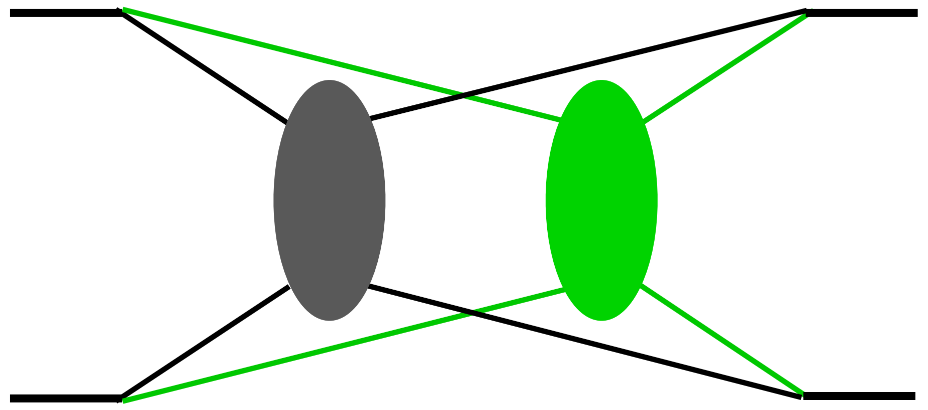


Meanwhile, if both branches interact with the target, we get an amplitude with double parton content.

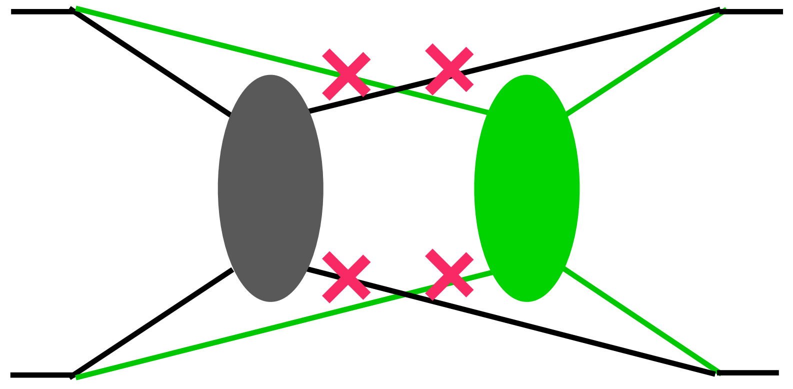
Squaring this amplitude one arrives at the picture of two parallel ladders:



$\equiv$

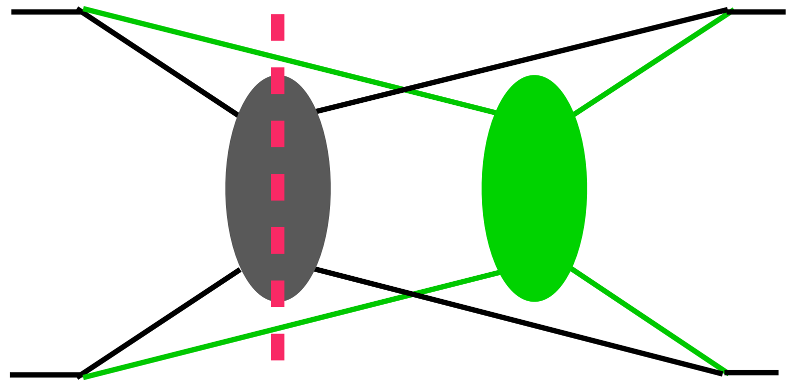


What sort of final states this amplitude contains? *The double-Reggeon amplitude can be cut in three different ways:*



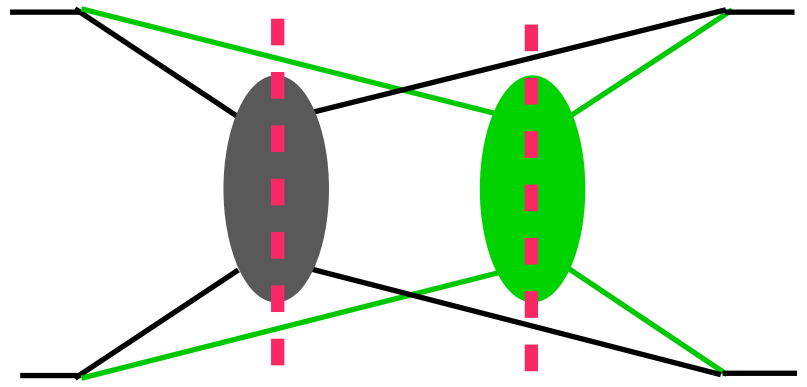
$2\times|f|^2$

quasi-elastic process



$2\times\,[2\mathrm{Im}_s f]\cdot[if+(if)^*]$

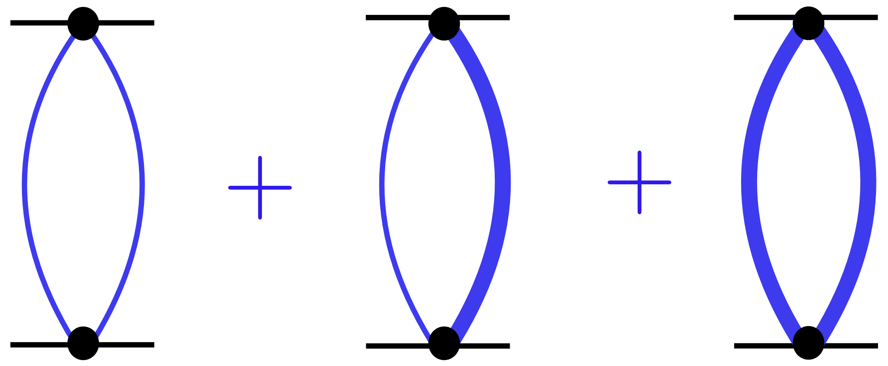
correction to plateau



$(2\mathrm{Im}_s f)^2$

double particle density

$\equiv$

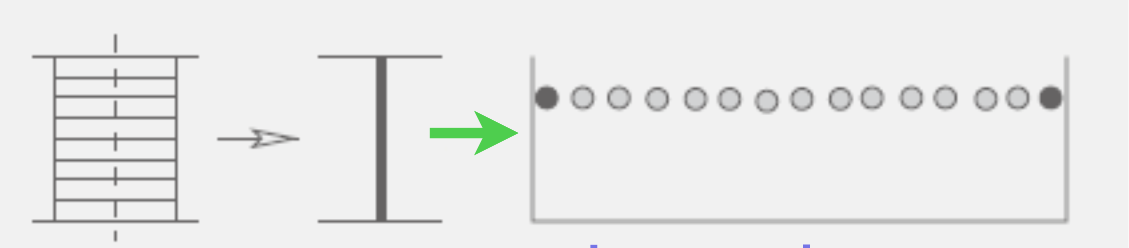


$\mathbf{2} \quad -\mathbf{8} \quad +\mathbf{4} = -\mathbf{2}$

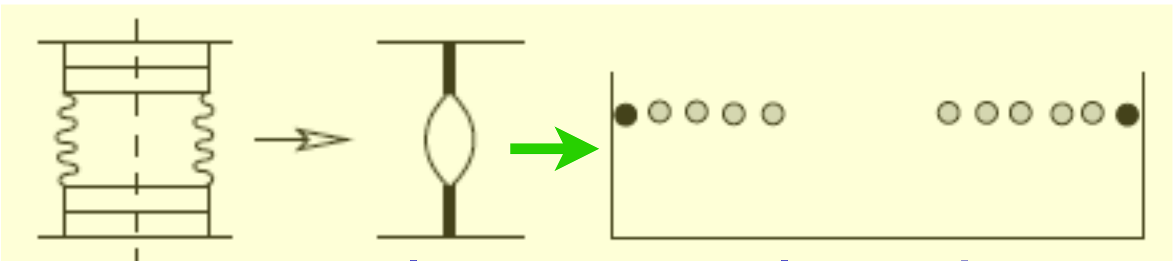
Small and large density fluctuations plus correction to average events combine into ***suppression*** of the total cross section - shadowing effect.

General formula has been derived (AGK cutting rules) that determines relative weights of various particle density fluctuations due to an arbitrary number of Pomeron exchanges.

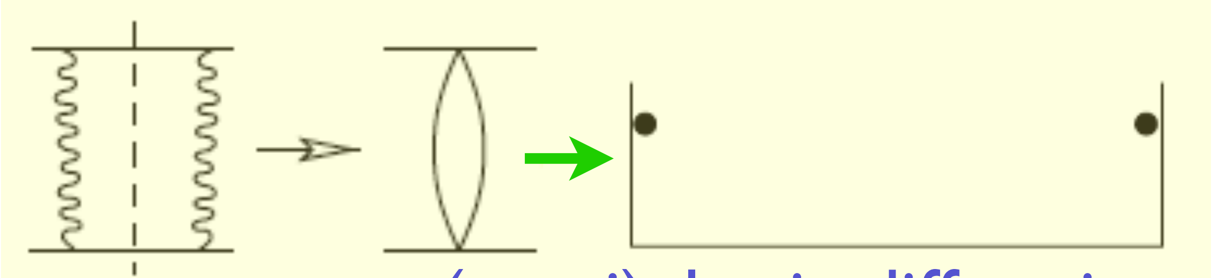
Exchanging multiple Pomerons we get various fluctuations in high energy multi-particle production pattern.



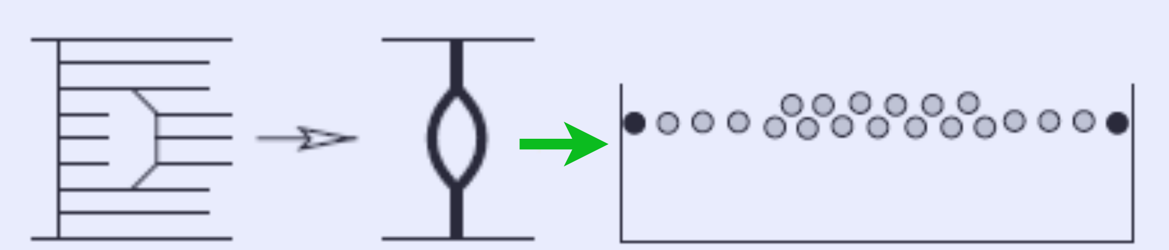
basic plateau



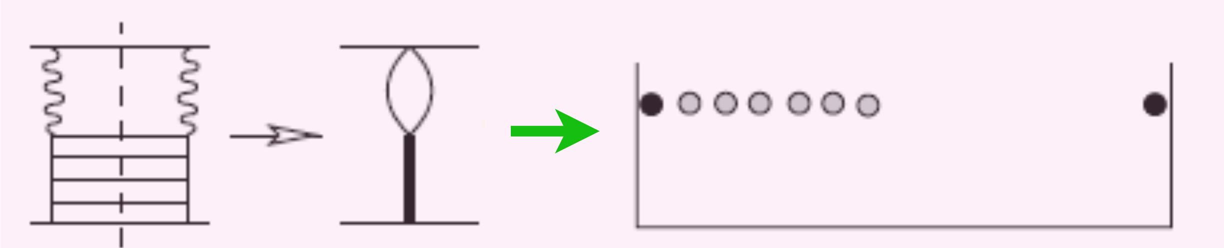
large central rapidity gap



(quasi)elastic diffraction



double central density



high-mass inelastic diffraction

This one in particular (*known as 3-Pomeron limit*) proved catastrophic: $\frac{\sigma_{3\mathbf{P}}}{\sigma_{\mathbf{P}}} \propto \ln\ln s \to \infty$

The study of small multiplicities $\mathbf{n} \ll \overline{\mathbf{n}}(\mathbf{s})$ *provides a crucial test of the Pomeron pole hypothesis.*

TCAM would have been self-consistent if multiplicity fluctuations were rare and the Pomeron 'resistant' to the ***s****-channel unitarity.*

In reality fluctuations happen to be omnipresent and contribute 'too much' to σ_{tot} *.*

Position in the ***j*** *plane of the* ***n****-Reggeon exchange branch cut singularity reads (Mandelstam)*

$$j_n(t) = n\alpha\left(\frac{t}{n^2}\right) - n + 1$$

Specifically for the Pomeron case, at the point $t = 0$ *responsible for* σ_{tot} *one obtains*

$$j_n(0) = 1 + n(\alpha(0) - 1) = 1 = \alpha_P(0)$$

Pomeron collides with all its branching and is no longer an isolated leading singularity...

P *shares an honourable role of the rightmost singularity in the* ***j*** *plane with branch cuts due to multiple* ***P*** *exchange.*

Moreover, at $t < 0$ *branchings dominate over the pole!* *(Hence problems predicting diff. distribution of elastic scattering.)*

Attempts at solving the problem of 'interacting Pomerons' triggered a breakthrough in theory of second order phase transitions.

(similar physics of systems with untameable long-range fluctuations, near the "boiling point")

Looking at high energy behaviour of total, inelastic, diffractive hadron cross sections we are facing an intrinsically unstable dynamics difficult to handle.

QCD has only added insult to injury : another infrared instability - that of colour confinement.

wrap-up

Talking about physics of hadrons we practically did not mention quarks neither referred to QCD.

I did this on purpose.

I have tried to convince you that when it comes to high-energy processes, the knowledge of microscopic dynamics is not obligatory.
It goes without saying that having that knowledge would be helpful.

However, we are still hurting: QCD as we know it does not apply to the physics of soft hadron interactions.
All the beauty of the quark-gluon dynamics fades away when it comes to processes characterised by large cross sections.

Meantime, it is worth keeping in mind that the concept of quarks + gluons + colour confinement goes along.

The basic features of high-energy hadron phenomena vs. QCD :

cross sections not falling with energy	vector gluon as interaction mediator
vanishing cross sections with exotic quantum number exchange	quark model
uniform rapidity distribution (Feynman plateau)	accompanying gluon radiation (bremsstrahglung)
rare appearance of large $k_\perp$	dimensionless coupling + asymptotic freedom
hadrons belong to Regge families	quarks and gluons reggeize too
the vacuum channel	**t**-channel two-gluon exchange - a perfect sketch for Pomeron

So, there is well harmony on a qualitative level.

On a quantitative level*, it's more about deception than progress.*

To give an example, a pQCD shot at Pomeron has earned widespread fame but lacks relevance. Perturbative approach is inapplicable here.

As far as general principles are concerned, in 50 years there has been no understanding or tools developed
*that allow unitarity (and even causality) to be formulated in the language of INFO (**I**dentified but **N**on-**F**lying **O**bjects).*

Remind you of the announced SBC options [Scared (and/or) Bored (and/or) Curious]

How complete the failure was is for you to judge.