Initial Evaluations of the Cooling Solenoids for the Rectilinear 6D Cooling Channel

Siara Fabbri, Jonathan Pavan

DISCLAIMER:

These are tentative results based on the geometries and parameters from the US MAP original design [1]

[1] Stratakis, Diktys, and Robert B. Palmer. "Rectilinear six-dimensional ionization cooling channel for a muon collider: A theoretical and numerical study." *Physical Review Special Topics-Accelerators and Beams* 18.3 (2015): 031003.

Outline

Goal: simulate and characterize the cooling solenoid magnets based on geometries and initial parameters from the US MAP study.

- *Overview*
- *Simulation study*
  - Approach and validation
  - COMSOL setup
  - Analytic formulas
- *Results*
  - Characterization of all stages
- *Summary & Next Steps*

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Overview: Cooling for a Muon Collider 1/2

- Rectilinear 6D cooling scheme to reduce emittance of muon beam by several orders of magnitude

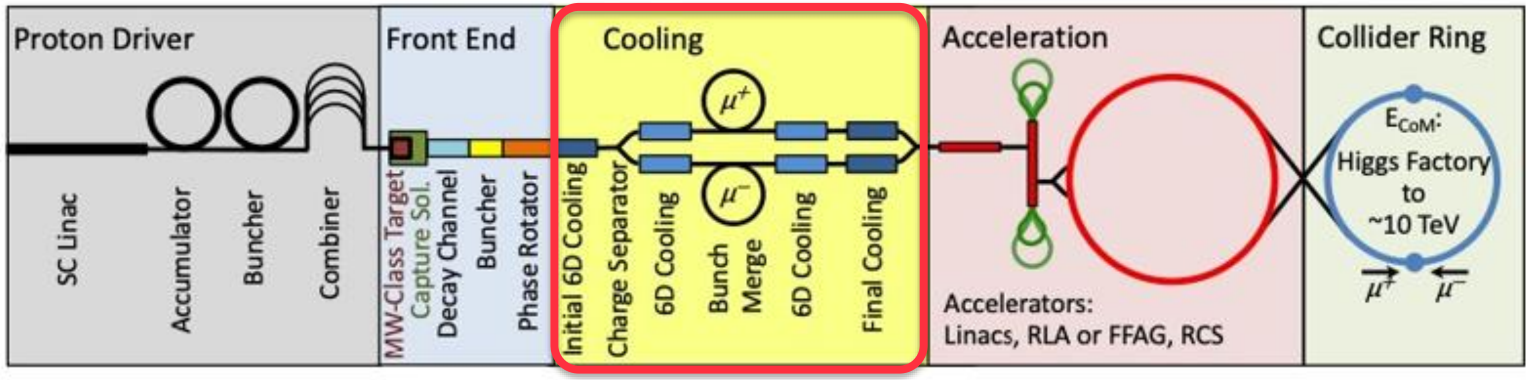


Figure from MAP study, ref [1]

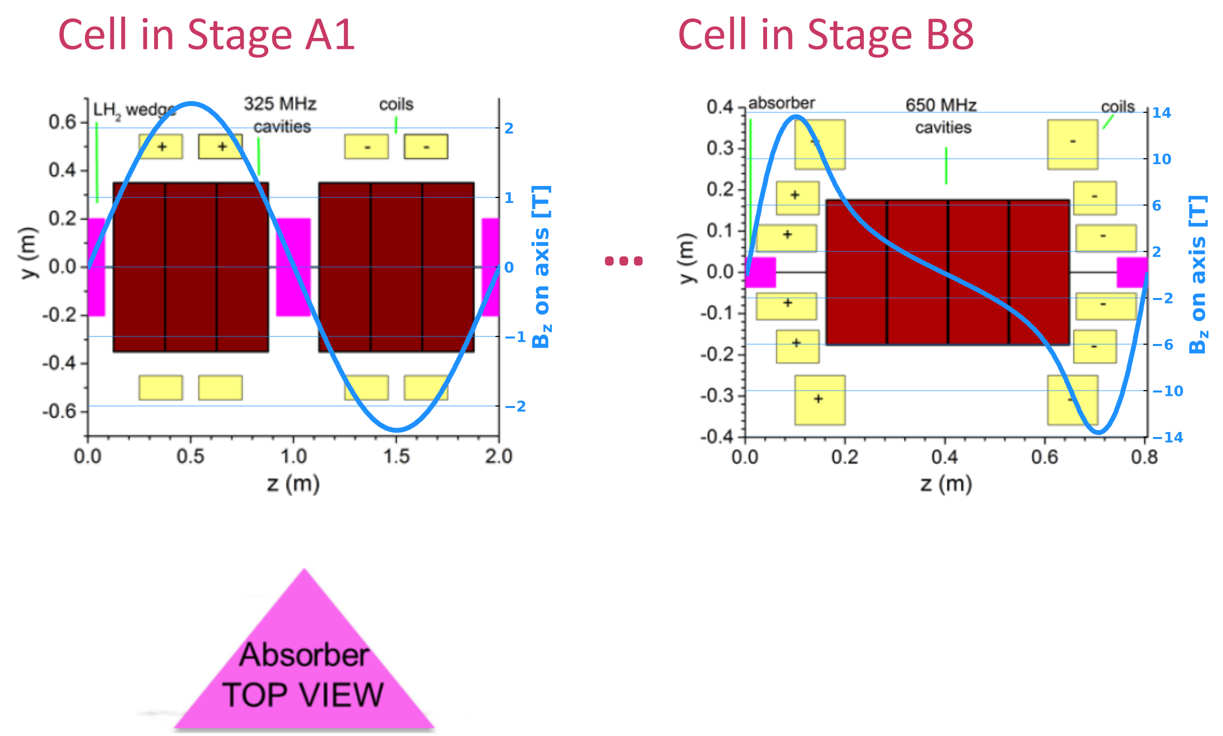
- 12 unique stages:
  - 4 cooling stages <u>*before*</u> bunch recombination (A1-A4)
  - 8 cooling stages <u>*after*</u> bunch recombination (B1-B8)
- *Each stage has a repeating series of a cell type*

Overview: Cooling for a Muon Collider 2/2

- High field, very compact solenoids
- Each cell has symmetric solenoids of opposite polarity
- Dipole component separated out!

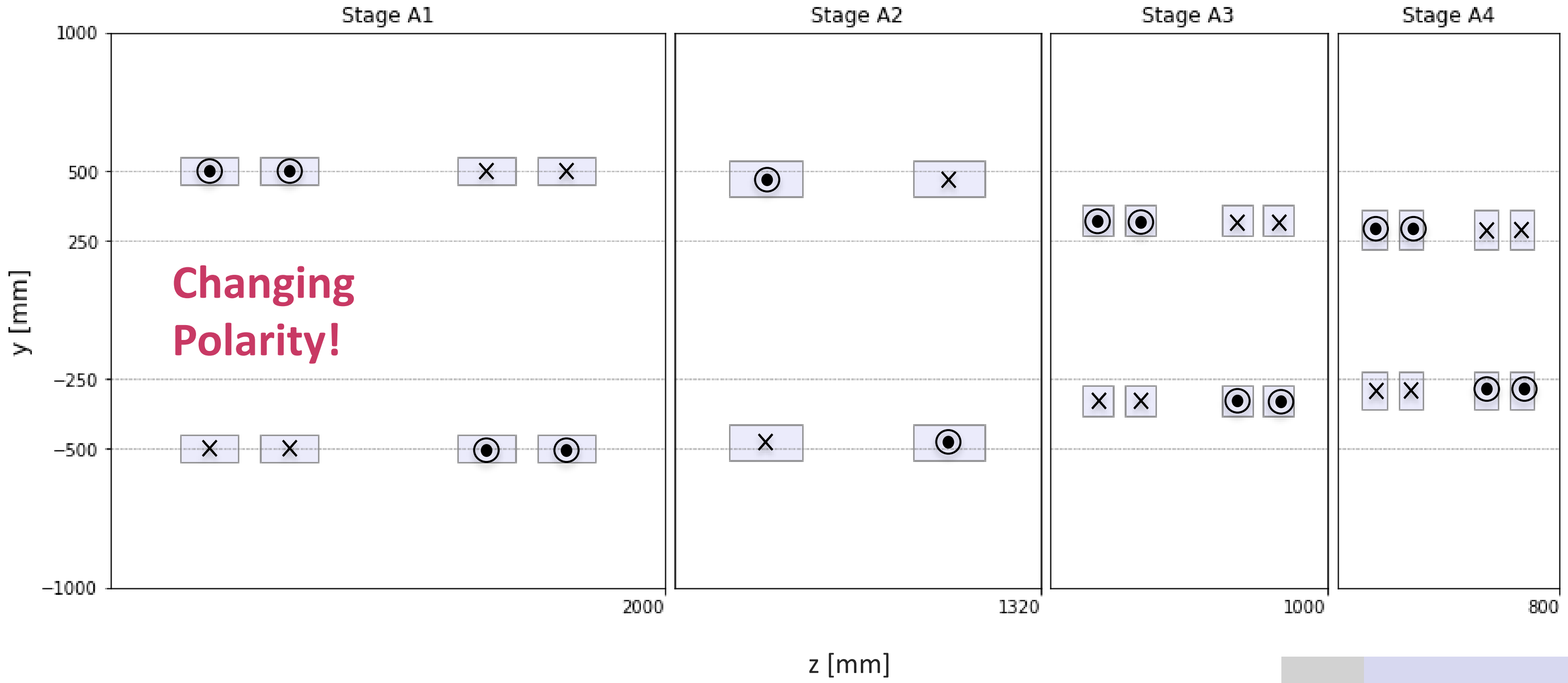
Some stats:

- Fields on axis: 2 to 14 T
- Cell Lengths: 0.8 to 2.7 m
- Total length of all Stages: ~ **1 km**
- Total number of solenoids: 2432



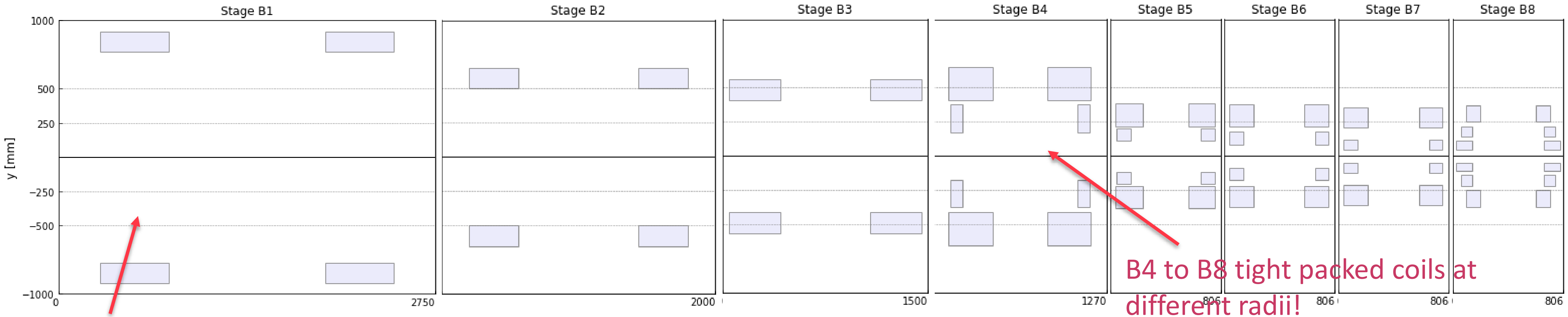
Images taken from ref [1] Stratakis, Diktys and corresponding presentations

Overview: Input Parameters and Geometry 1/2



			A1	A2	A3	A4
			01	01	01	01
Input Parameters	Bore Radius	mm	450	410	270	220
	Magnet Thickness	mm	100	130	110	140
	Magnet Length	mm	210	260	110	90
	Current Density	A/mm^2	63.25	126.6	165	195
	Cell Length	m	2	1.32	1	0.8

Overview: Input Parameters and Geometry 2/2



B4 to B8 tight packed coils at different radii!

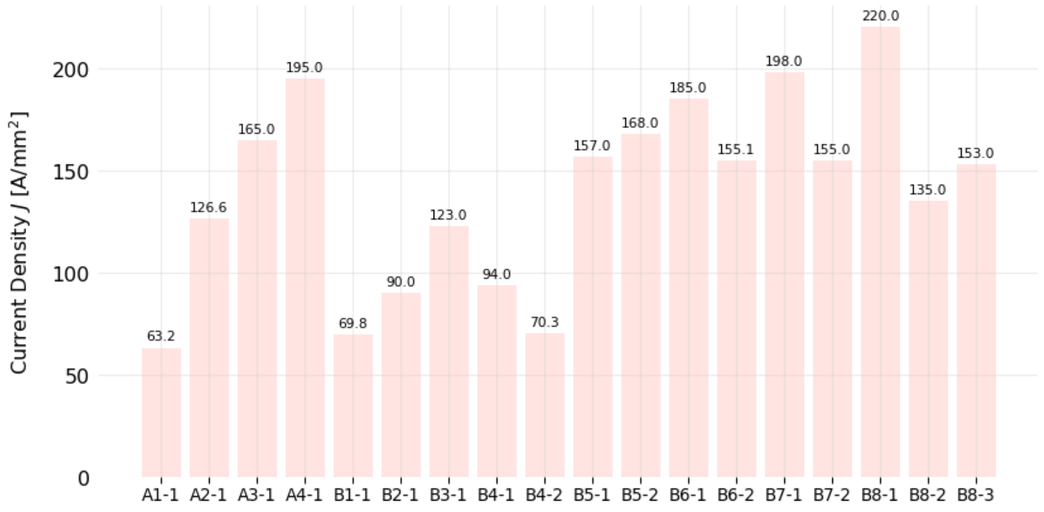
Large bore!

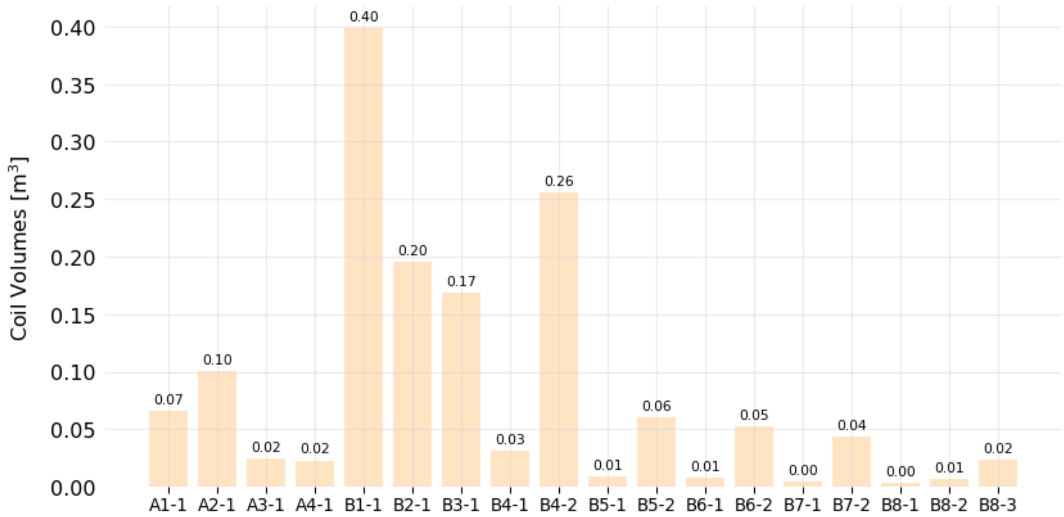
Input Parameters		B1	B2	B3	B4		B5		B6		B7		B8		
		01	01	01	01	02	01	02	01	02	01	02	01	02	03
Bore Radius	mm	770	500	410	175	410	113	217	84	215	50	210	45	140	250
Magnet Thickness	mm	150	150	150	200	240	88	165	92	160	74	145	65	80	120
Magnet Length	mm	500	360	370	92	320	100	196	100	177	100	170	120	80	100
Current Density	A/mm^2	69.8	90	123	94	70.3	157	168	185	155.1	198	155	220	135	153
Cell Length	m	2.75	2	1.5	1.27		0.806		0.806		0.806		0.806		

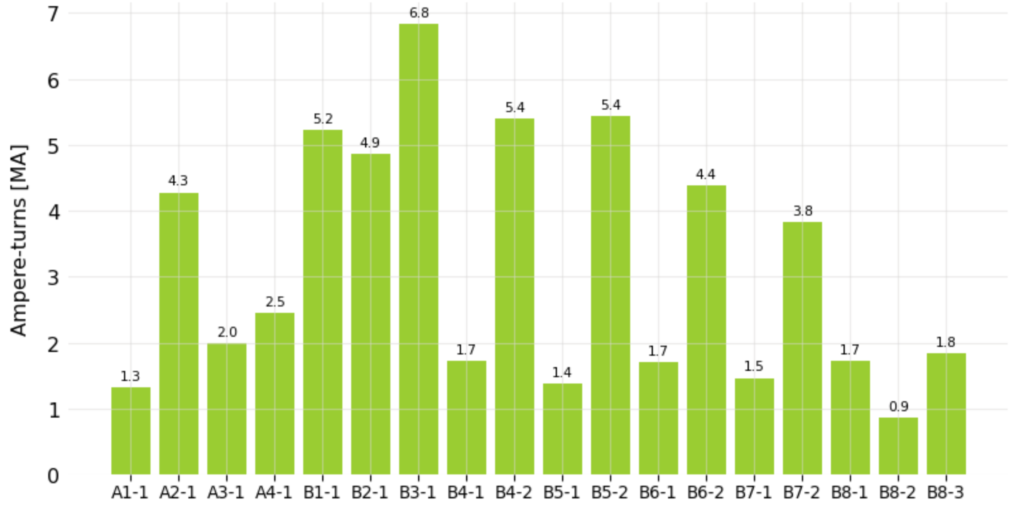
Overview: Coil Properties

18 unique coil types

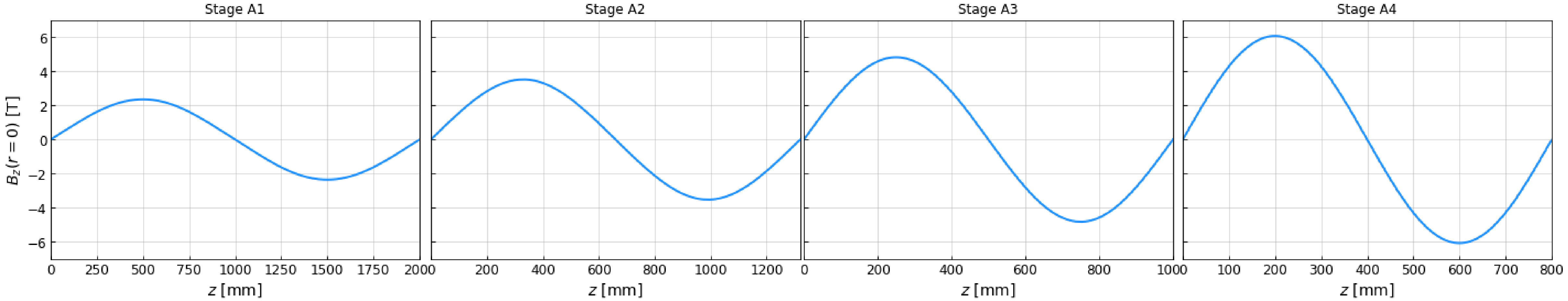
- 2 to 6 coils per cell
- Inner bore diameter from 90 mm to 1540 mm
- Lengths from 80 mm to 210 mm
- Current densities from 63 to 220 $\mathrm{A/mm^2}$



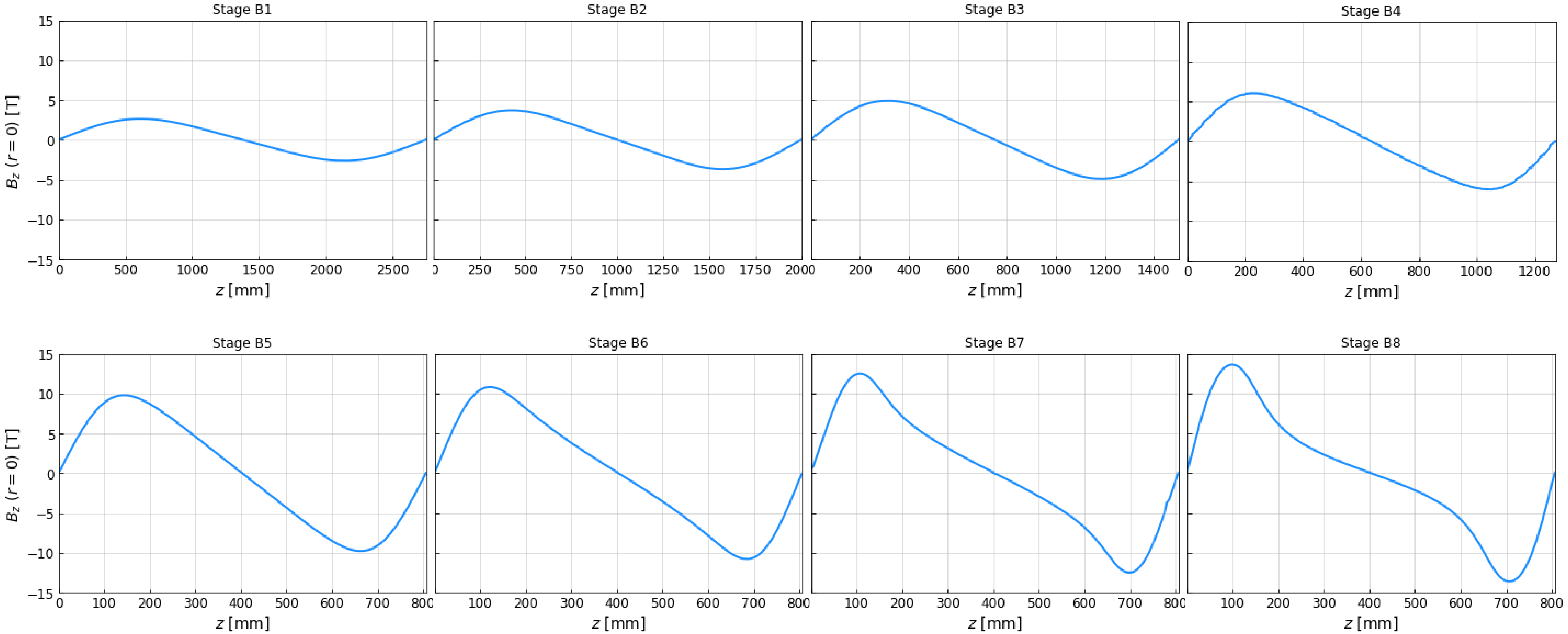




Overview: On-Axis Magnetic Field (B_z) in A1 to A4



Overview: On-Axis Magnetic Field (B_z) in B1 to B8



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- *Overview*
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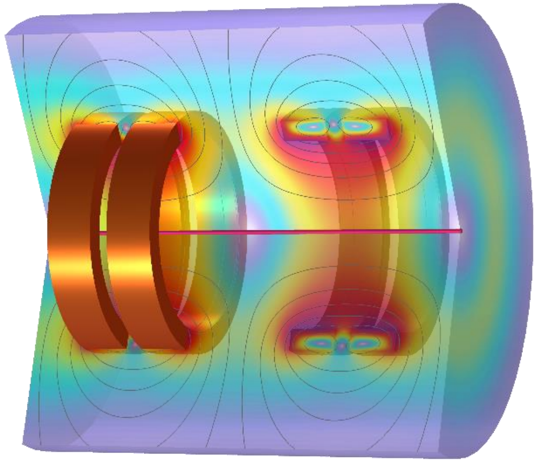
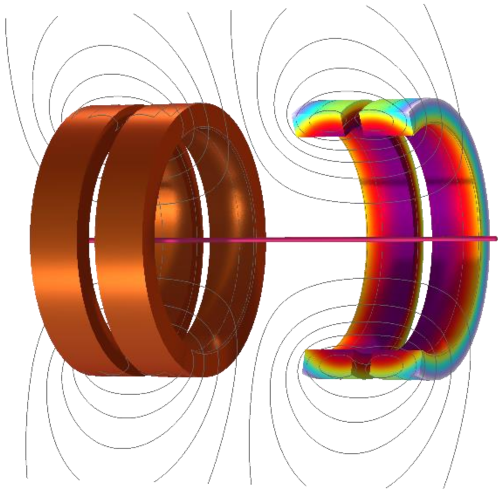
Simulation Study: Our Focus

1. Magnetic Properties

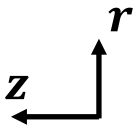


- B_z , B_r , $|B|$
- B_{max} in coils
- Stored magnetic energy
- Inductance
- Stray fields

2. Mechanical Properties



- *Stresses*
- *Force densities*
- *Coil parameters*



Simulation Study: Approach

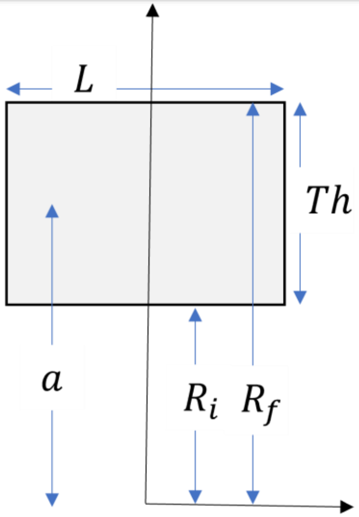
Simulations done in COMSOL but continuously validated against analytical formulas and supplied G4beamline fieldmaps.

1. COMSOL

2. Analytic Formulas

	Parameter	Equation	References /comments
1	Self Inductance L	$L = \frac{aN^2}{\mu_0}\left[\log\left(\frac{8a}{R}\right)\left(1+\frac{3R^2}{16a^2}\right)-\left(2+\frac{R^2}{16a^2}\right)\right]$ $R = 0.2235(a+L)$	[1]
2	Radial Stress σ_r	$\sigma_r = \frac{JBz(R_i,0)R_i}{\alpha-1}\left[\frac{2+\nu}{3}(\alpha-\kappa)\left(\frac{\alpha^2+\alpha+1-\frac{\alpha^2}{\rho^2}}{\alpha+1}-\rho\right)-\frac{3+\nu}{8}(1-\kappa)\left(\alpha^2+1-\frac{\alpha^2}{\rho^2}-\rho^2\right)\right]$	[2] and [3]
3	Hoop Stress σ_θ	$\sigma_\theta = \frac{JBz(R_i,0)R_i}{\alpha-1}\left\{(\alpha-\kappa)\left[\frac{2+\nu}{3}\left(\frac{\alpha^2+\alpha+1+\frac{\alpha^2}{\rho^2}}{\alpha+1}\right)-\frac{1+2\nu}{3}\rho\right]-(1-\kappa)\left[\frac{3+\nu}{8}\left(\alpha^2+1+\frac{\alpha^2}{\rho^2}\right)-\frac{1+3\nu}{8}\rho^2\right]\right\}$	[2] and [3]
4	Magnetic Energy	$W = \frac{1}{2}LI^2$	
5	Peak field at $(r=0, z=0)$ of a single *ideal* coil	$B_0 = \mu_0 J R_i \beta \ln\left[\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right]$	[3]
6	Peak field at inner radius of a single *ideal* coil	$B = B_0\left[1-\frac{1}{2}E_2(\alpha,\beta)\left(\frac{r}{R_i}\right)^2+\frac{3}{8}E_4(\alpha,\beta)\left(\frac{r}{R_i}\right)^4-\frac{5}{16}E_6(\alpha,\beta)\left(\frac{r}{R_i}\right)^6+\ ...\right]$	[3]
7	Mutual Inductance	$M = \mu_0\sqrt{a_1 a_2}\left[\left(\frac{2}{k}-k\right)F-\frac{2}{k}E\right]+corrections$	Eqs. 1, 29 and 33 from [1]

Simulation Study: Analytic Formulas for a Single Coil



$$\alpha = \frac{R_f}{R_i}, \beta = \frac{L}{2R_i}$$

	Parameter	Equation	References /comments
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2	Radial Stress σ_r	$\sigma_r = \frac{JBz(R_i,0)R_i}{\alpha-1}\left[\frac{2+\nu}{3}(\alpha-\kappa)\left(\frac{\alpha^2+\alpha+1-\frac{\alpha^2}{\rho^2}}{\alpha+1}-\rho\right)-\frac{3+\nu}{8}(1-\kappa)\left(\alpha^2+1-\frac{\alpha^2}{\rho^2}-\rho^2\right)\right]$	[2] and [3]
3	Hoop Stress σ_θ	$\sigma_\theta = \frac{JBz(R_i,0)R_i}{\alpha-1}\left\{(\alpha-\kappa)\left[\frac{2+\nu}{3}\left(\frac{\alpha^2+\alpha+1+\frac{\alpha^2}{\rho^2}}{\alpha+1}\right)-\frac{1+2\nu}{3}\rho\right]-(1-\kappa)\left[\frac{3+\nu}{8}\left(\alpha^2+1+\frac{\alpha^2}{\rho^2}\right)-\frac{1+3\nu}{8}\rho^2\right]\right\}$	[2] and [3]
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[1] Rosa, Edward Bennett, and Frederick Warren Grover. *Formulas and tables for the calculation of mutual and self-inductance*. No. 169. US Government Printing Office, 1948.

[2] Wilson, Martin N. "Superconducting magnets." (1983).

[3] Iwasa, Yukikazu. *Case studies in superconducting magnets: design and operational issues*. Springer science & business media, 2009.

Simulation Study: Analytic Formulas → Coil Calculator

COIL: from Cell A1

Description: all coils in cell A1 are the same.

	Parameter	Unit	Value	Value in SI un	SI unit
Input	Length	mm	210	0.21	m
	Inner Radius (Ri)	mm	450	0.45	m
	Outer Radius (Rf)	mm	550	0.55	m
	Current Density	A/mm2	63.25	63250000	A/m
	Tape Length	mm	12	0.012	m
	Tape width	mm	0.11	0.00011	m
	Bz(Ri, L/2) -> For stress calc	T	3.9	3.9	T
	Bz(Rf, L/2) -> For stress calc	T	-2	-2	T
Other deduced parameters	Cross Section Area	mm2	2.10E+04	0.021	m2
	Number of turns (rounded)		15909	15909	
	Thickness	mm	100	0.10	m
	Ratio Rf/Ri (alpha)		1.22222	1.22	
	Ratio L/(2Ri) (beta)		0.23333	0.23333333	
	Distance to center of coil (a)	mm	500	0.50	m
	Geometric mean distance (R)	mm	69.285	0.069285	m

Calculated Parameters

[1] Peak Field on axis [T]: 1.638406392

[2] Approximate Peak Field at Ri [T]: 3.835081091

Expansion Term	Value
H0	1303802.651
FE2	-0.051773348
FE4	0.046289401
FE6	-0.036231531
FE8	0.025046476

[3] Self Inductance [H]: 329.062102

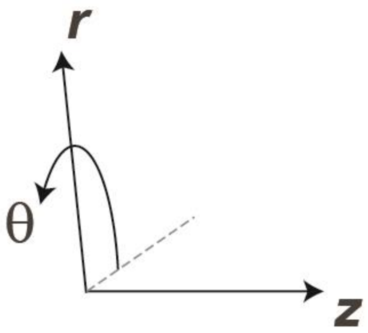
[4] Stresses [Mpa]

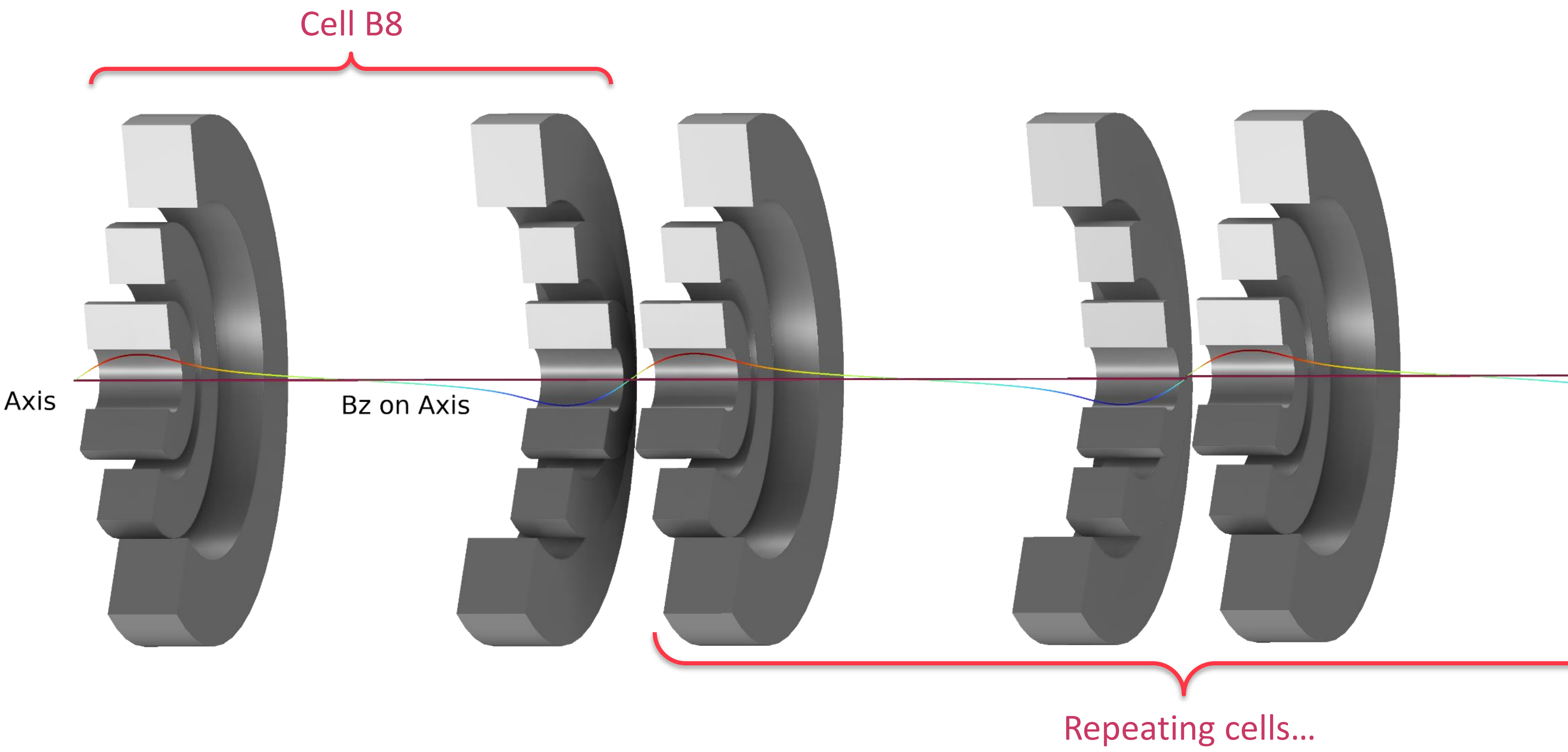
r/Ri	Hoop Stress	Radial Stress
1	32.25149125	
1.01709402	31.09633419	
1.04639805	29.31217634	
1.06105006	28.51527155	
1.07570208	27.78314768	
1.09035409	27.11672009	
1.10500611	26.51684387	
1.11965812	25.98431854	
1.13431013	25.51989229	
1.14896215	25.12426593	
1.16361416	24.79809643	
1.17826618	24.54200016	
1.19291819	24.35655587	
1.20757021	24.24230741	
1.22222222	24.19976625	

Coil Current [A]	Magnetic Energy [kJ
83.49	1146.88

Calculates basics parameters for a given single coil geometry, current density, and tape size.

Simulation Study: Geometry and Definitions





Stresses

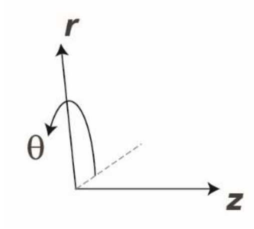
Hoop: σ_θ

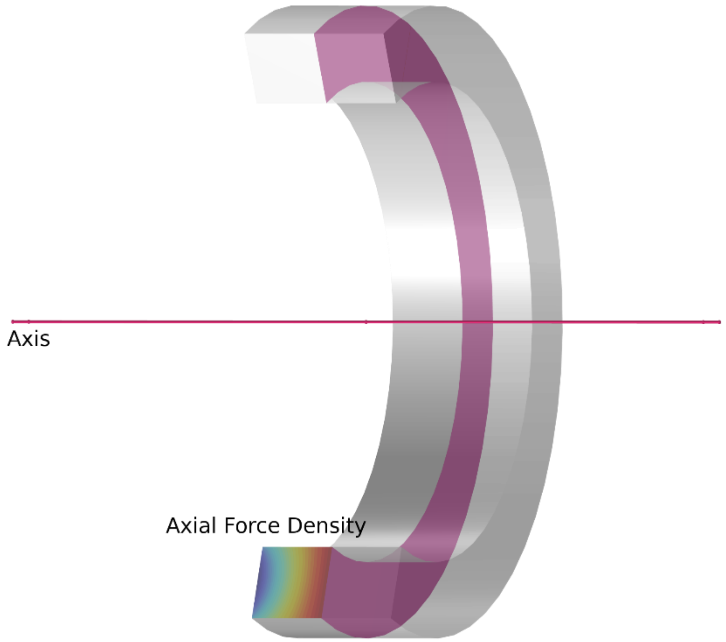
Radial: σ_r

Longitudinal: σ_z

Simulation Study: Mechanics Calculations

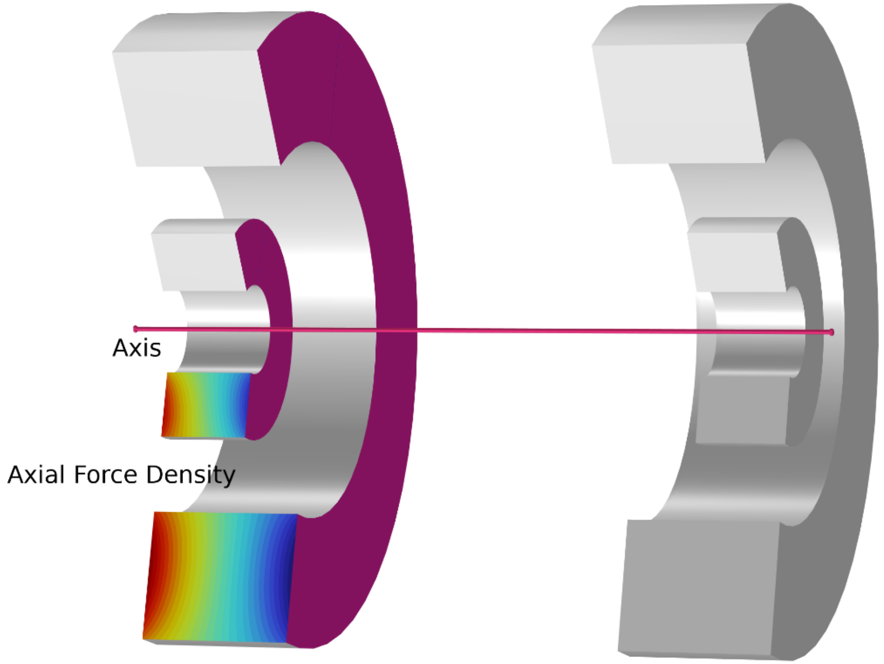
1. Single Coil





A single coil is split exploiting the symmetry of the system. In the middle of the coil a **roller** boundary (shown in purple) is placed.

2. Single Cell / Periodic Lattice



The stresses are evaluated on just 2 of the 4 coils in the shown cell, exploiting symmetry. **Rollers** are placed such that they oppose the net axial force on a coil.

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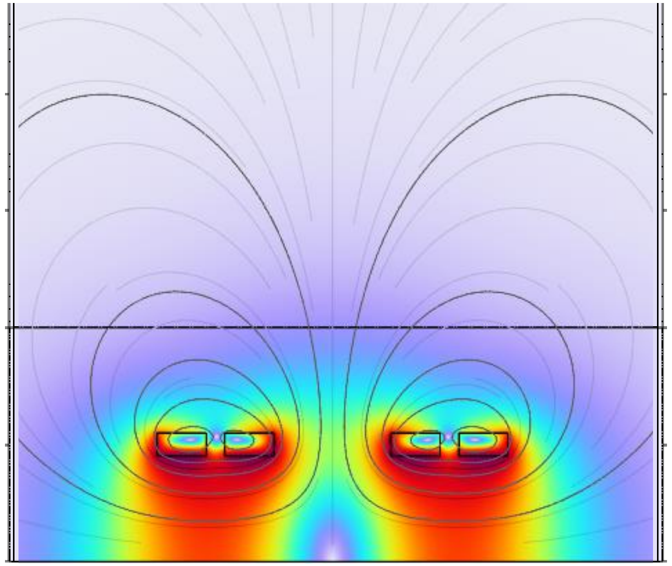
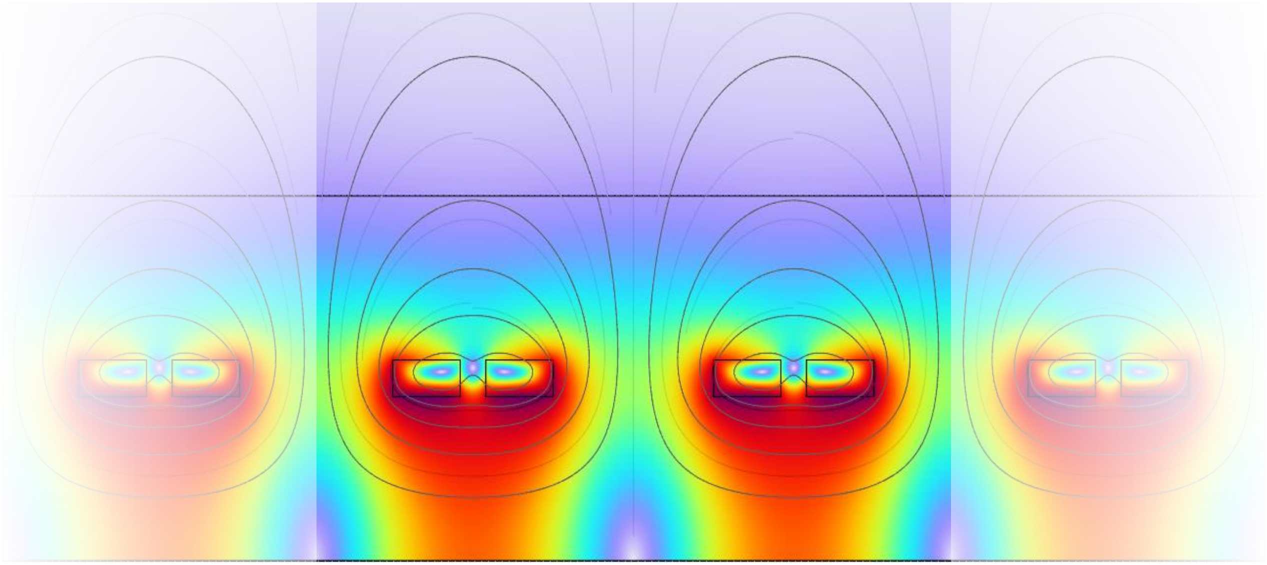
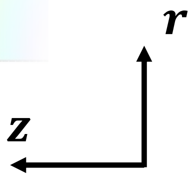
Results Part 3: Characterisation of all Stages

1. **Single Cell**

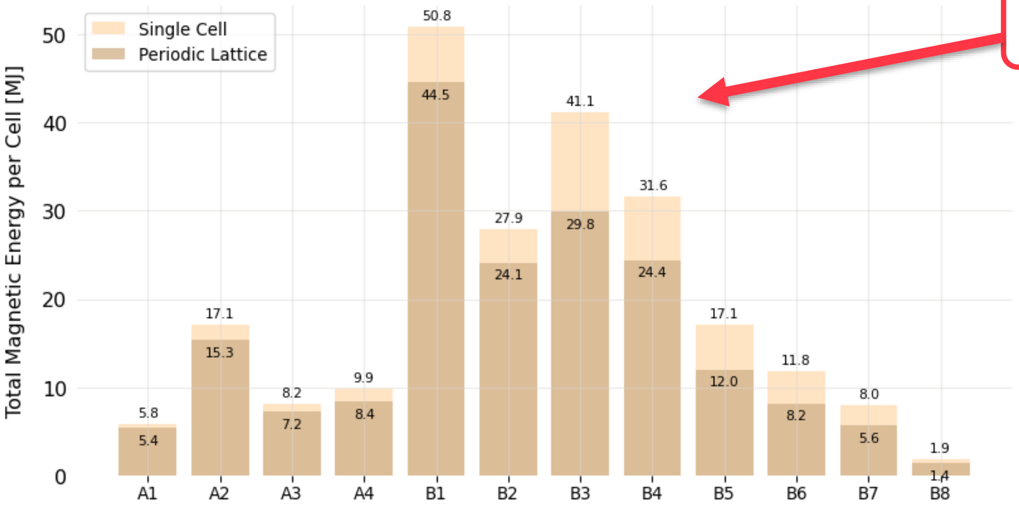


2. **Periodic Lattice**



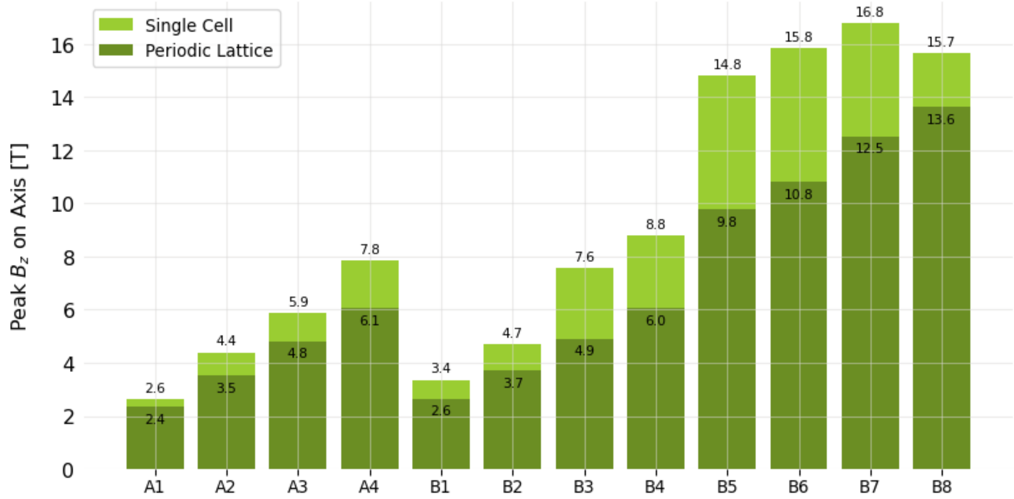


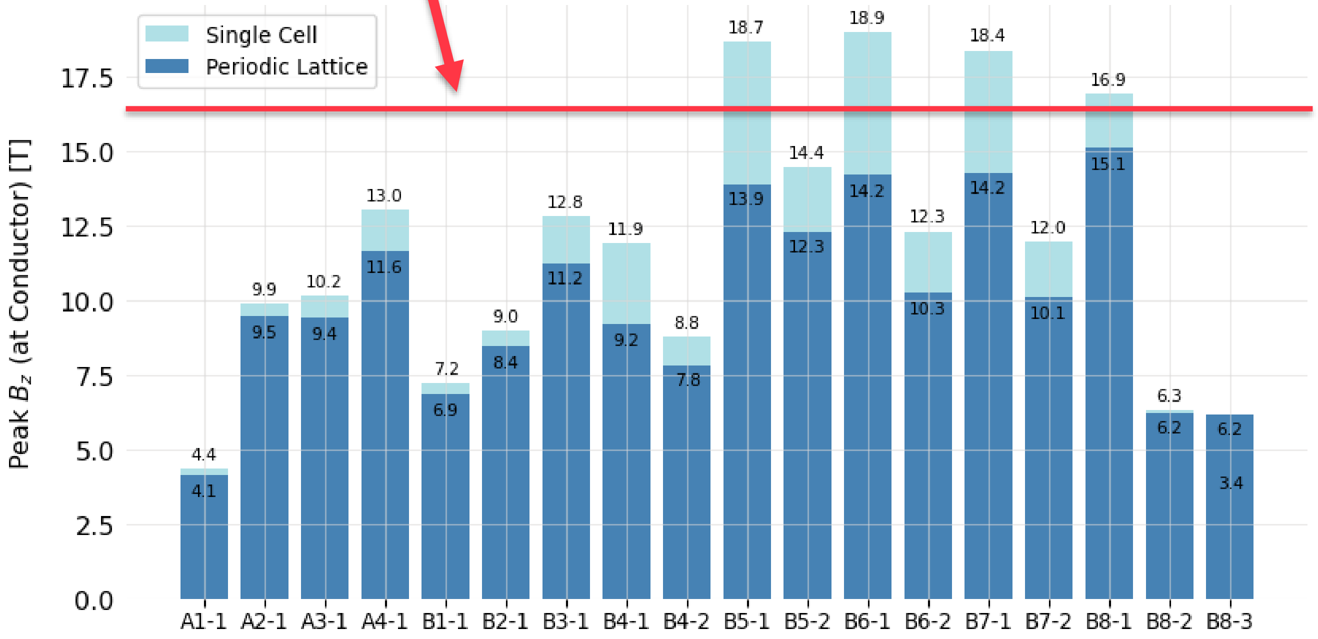
Results Part 3: Characterization of all Stages, 1/4
Magnetic Properties – Cells and Lattices



For comparison purposes an LHC dipole stores ~7 MJ

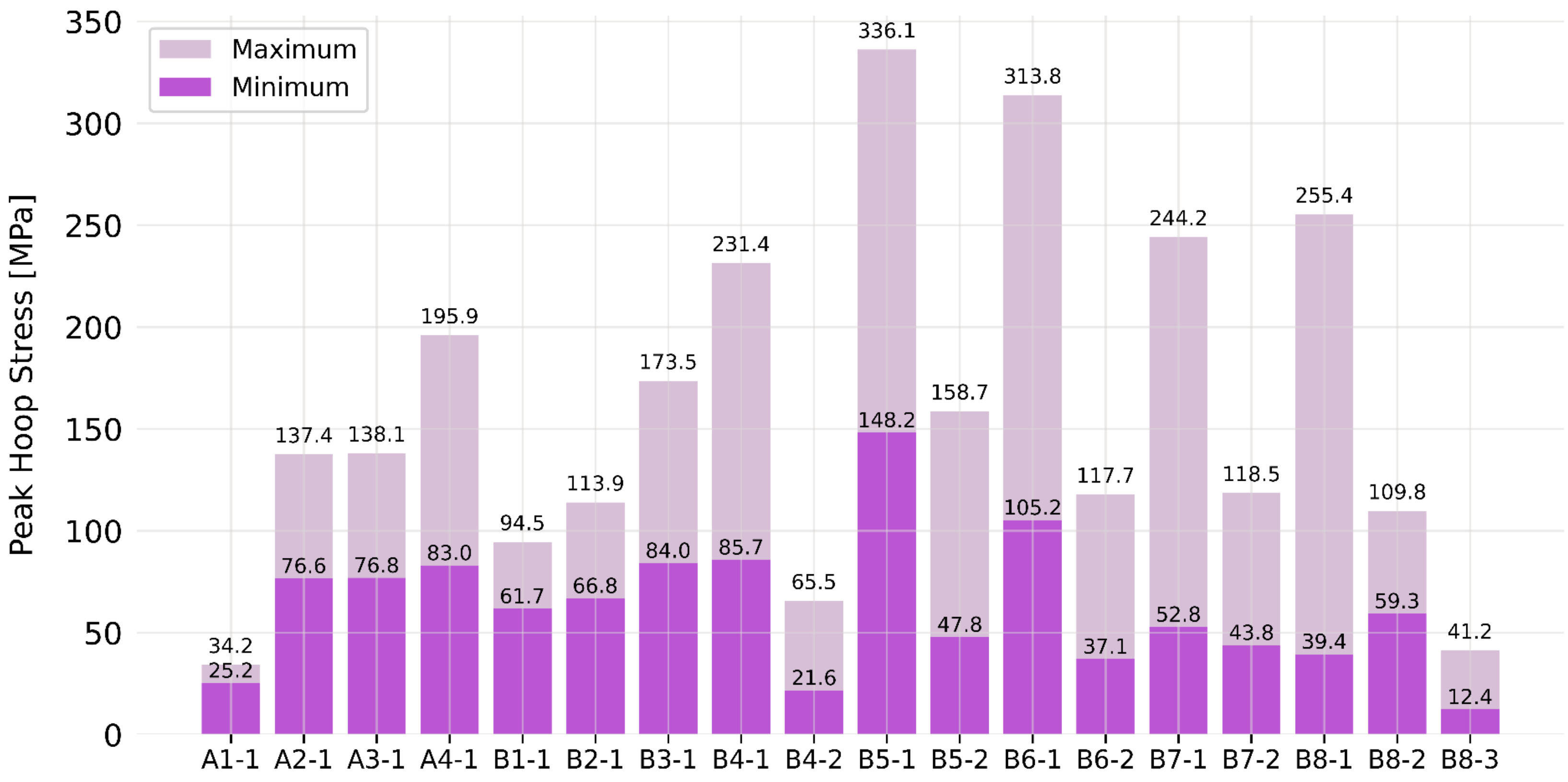
Above 17 T (B5 to B8), highest performance required & difficult for Nb_3Sn





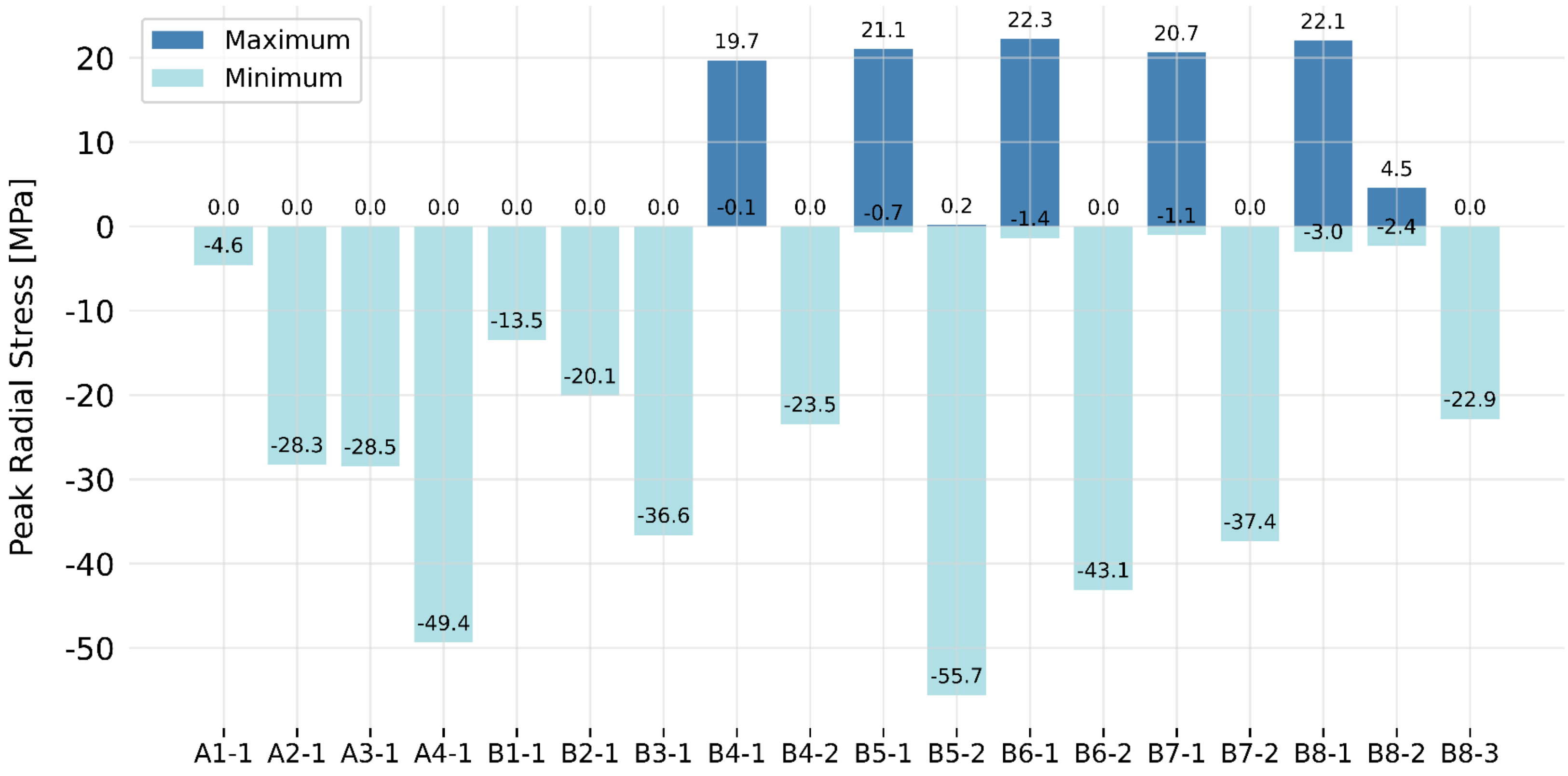
Results Part 3: Characterization of all Stages, 2/4
Mechanical Calculations in Lattices

Maximum and minimum values of **hoop stress σ_θ**



- **Coils with >150MPa will probably require reinforcement**
- Multi-radius-coil cells (B4-1, B5-1, B6-1, B7-1, B8-1, B8-2): the hoop stress decreases monotonically radially
- For all other coils (A1 to B3, B4-2, B5-2, B6-2, B7-2, B8-3): there is a local minimum in hoop stress

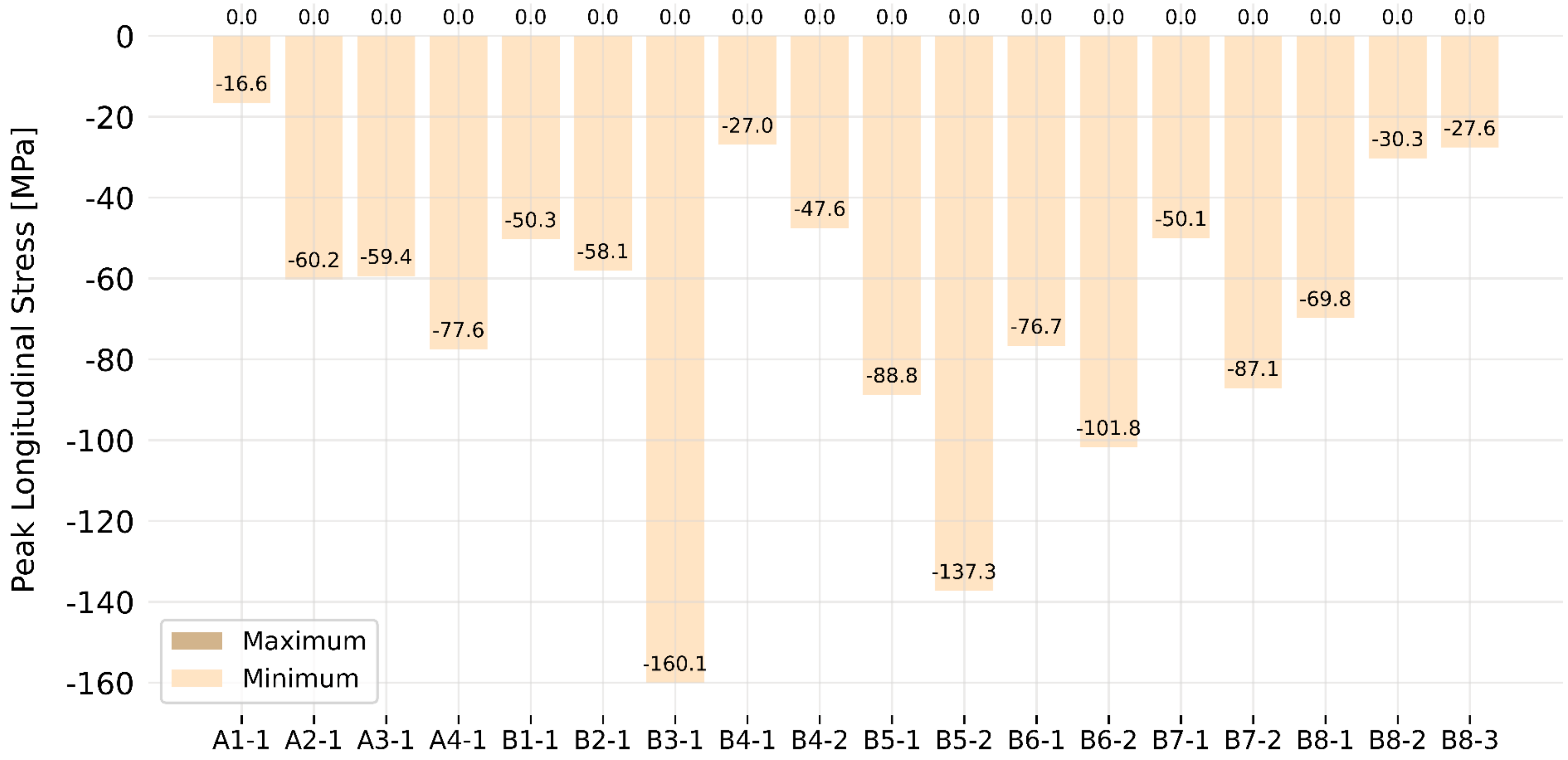
Maximum and minimum values of **radial stress σ_r**



- *Multi-radius-coil cells (B4-1, B5-1, B6-1, B7-1, B8-1, B8-2)*: **tensile** saddle-like profile on inner coil. **To go towards a compressive stress profile → these coils would need to be wound in independent layers!**
- *For all other coils (A1 to B3, B4-2, B5-2, B6-2, B7-2, B8-3):* compressive 'parabolic-like' profile, with a minimum stress in the center

Results Part 3: Characterization of all Stages, 2/4
Mechanical Calculations in Lattices – Longitudinal Stress

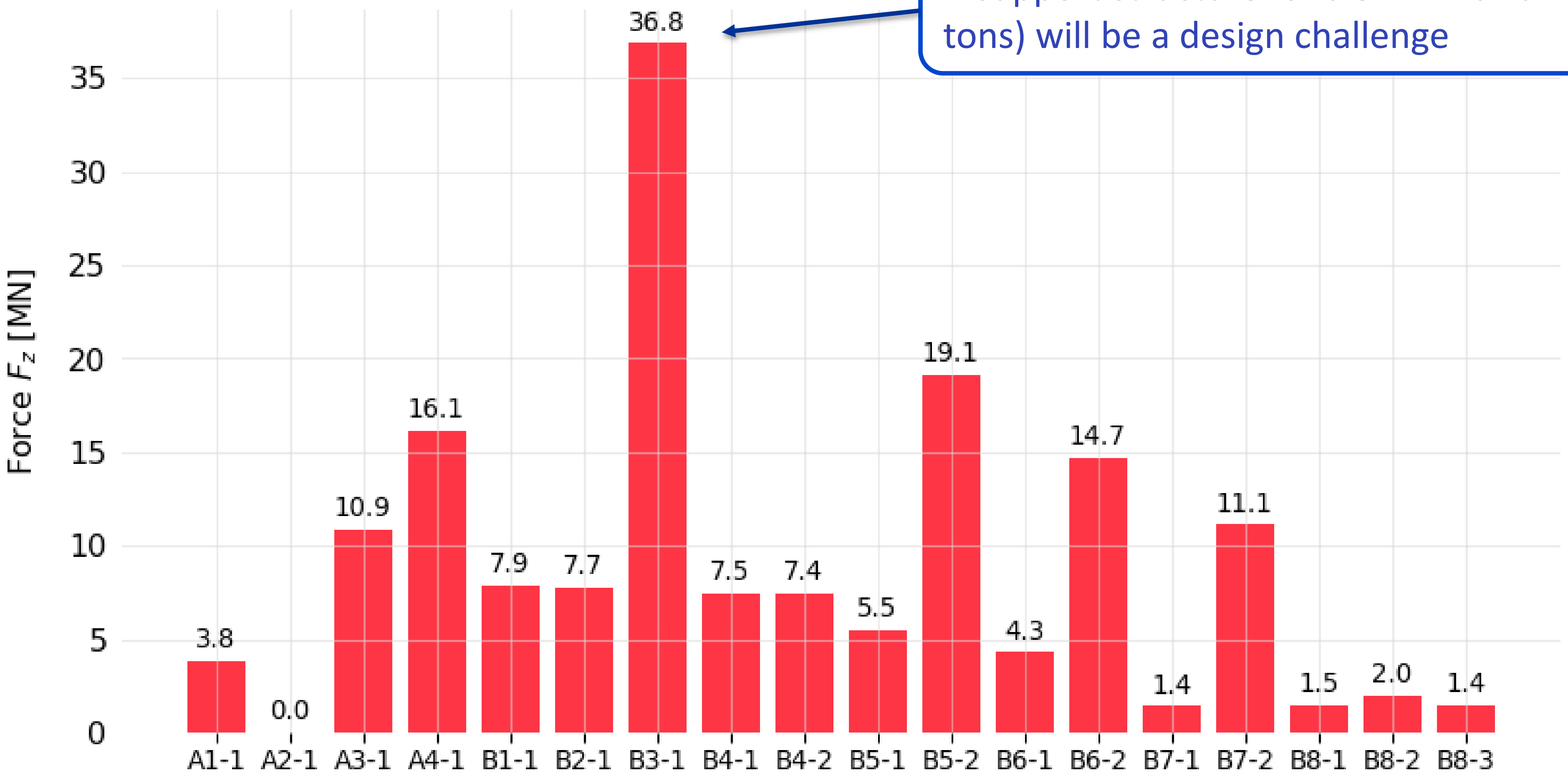
Maximum and minimum values of **longitudinal stress σ_z**



- As expected, the stresses are all compressive based on the roller positioning

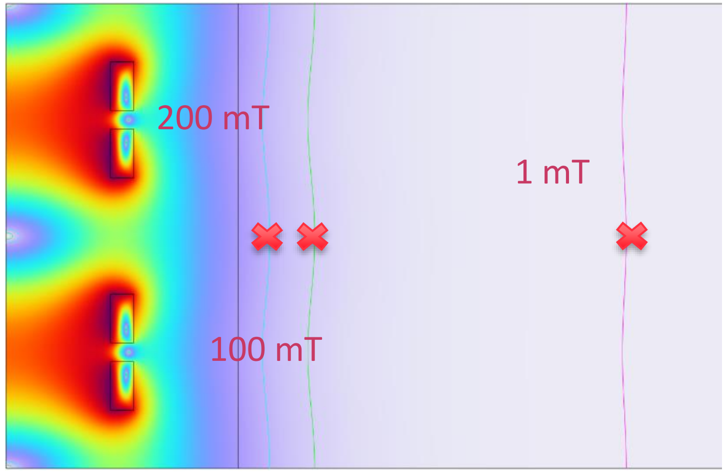
Results Part 3: Characterization of all Stages, 2/4
Mechanical Calculations in Lattices - Forces

A support structure for a 37 MN axial force (3700 tons) will be a design challenge

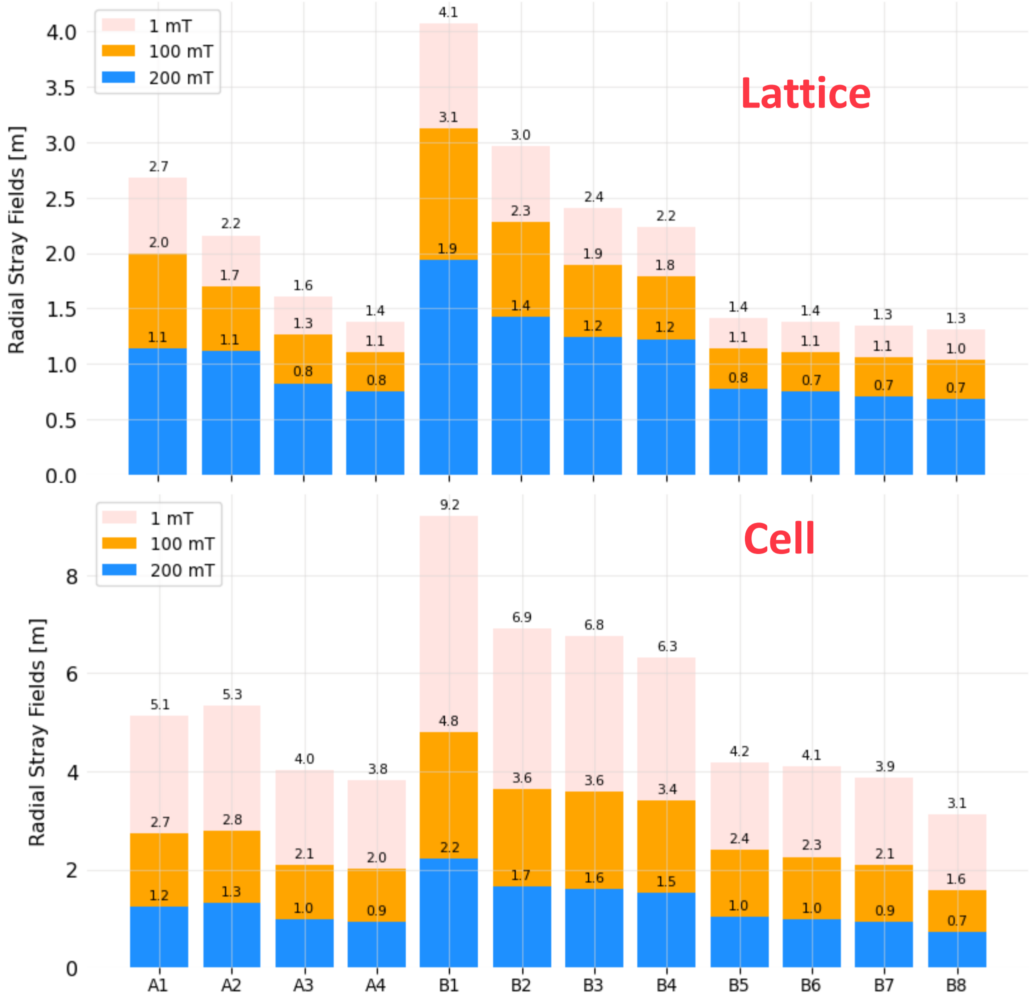


Results Part 3: Characterization of all Stages 4/4

Stray Fields



*Turbopumps can operate at 5 to 10 mT



Technology Options – Table of Stages and Coils

Stage	[1]		[2]		Coil	[1] Input geometries from US MAP Study				[2] Calculated Parameters using COMSOL (cell in a lattice unless otherwise stated)							Technology Options			
	Cell Length [m]	Solenoids/Cell	Peak Bz field on axis [T]	Stored Magnetic Energy [MJ]		Length [mm]	Radius [mm]	Thickness [mm]	Current Density [A/mm2]	Peak Bz Field in Coil (lattice configuration) [T]	Peak Bz Field in Coil (Single Cell) [T]	Maximum (+)Peak Hoop stress (see 1) [MPa]	Minimum (-) Longitudinal Stress [MPa]	Maximum (+) Radial Stress [MPa]	Minimum (-) Radial Stress [MPa]	Peak Longitudinal Force [MN]	NbTi (4 K)	Nb3Sn (4 K)	HTS (4 K)	HTS (20 K)
A1	2	4	2.4	5.38	A1-1	210	450	100	63.25	4.1	4.4	34.2	-16.6	0.0	-4.6	3.8	X	X	X	X
A2	1.32	2	3.5	15.35	A2-1	260	410	130	126.6	9.5	9.9	137.4	-60.2	0.0	-28.3	0.0		X	X	X
A3	1	4	4.8	7.23	A3-1	110	270	110	165	9.4	10.2	138.1	-59.4	0.0	-28.5	10.9		X	X	X
A4	0.8	4	6.1	8.39	A4-1	90	220	140	195	11.6	13.0	195.9	-77.6	0.0	-49.4	16.1		X	X	X
B1	2.75	2	2.6	44.54	B1-1	500	770	150	69.8	6.9	7.2	94.5	-50.3	0.0	-13.5	7.9	X	X	X	X
B2	2	2	3.7	24.1	B2-1	360	500	150	90	8.4	9.0	113.9	-58.1	0.0	-20.1	7.7		X	X	X
B3	1.5	2	4.9	29.83	B3-1	370	410	150	123	11.2	12.8	173.5	-160.1	0.0	-36.6	36.8		X	X	X
B4	1.27	4	6	24.4	B4-1	92	175	200	94	9.2	11.9	231.4	-27.0	19.7	-0.1	7.5		X	X	X
					B4-2	320	410	240	70.3	7.8	8.8	65.5	-47.6	0.0	-23.5	7.4		X	X	X
B5	0.806	4	9.8	12.03	B5-1	100	113	88	157	13.9	18.7	336.1	-88.8	21.1	-0.7	5.5			X	X
					B5-2	196	217	165	168	12.3	14.4	158.7	-137.3	0.2	-55.7	19.1		X	X	X
B6	0.806	4	10.8	8.19	B6-1	100	84	92	185	14.2	18.9	313.8	-76.7	22.3	-1.4	4.3			X	X
					B6-2	177	215	160	155.1	10.3	12.3	117.8	-101.8	0.0	-43.1	14.7		X	X	X
B7	0.806	4	12.5	5.65	B7-1	100	50	74	198	14.3	18.4	244.2	-50.1	20.7	-1.1	1.4			X	X
					B7-2	170	210	145	155	10.1	12.0	118.5	-87.1	0.0	-37.4	11.1		X	X	X
B8	0.806	6	13.6	1.42	B8-1	120	45	65	220	15.1	16.9	118.5	-69.9	22.1	-3.0	1.5			X	X
					B8-2	80	140	80	135	6.2	6.3	109.8	-30.3	4.5	-2.4	2.0	X	X	X	X
					B8-3	100	250	120	153	6.2	3.4	41.2	-27.6	0.0	-22.9	1.4	X	X	X	X

Outline

Goal: simulate and characterize the cooling solenoid magnets based on geometries and initial parameters from the US MAP study.

- *Overview*
- *Simulation study*
  - Approach and Validation
  - COMSOL Setup
  - Analytic formulas
- *Results*
  - Characterization of all coil types
  - Case Study: Stage A1
  - Characterization of all stages
- ***Summary & Next Steps***

Summary of Initial Evaluations

This study has presented a first look at the cooling stages and their solenoids based on input parameters from the US MAP study. From these results, it is obvious that the magnet parameters will need to be optimised from an energy/cost and engineering perspective

Some key notes:

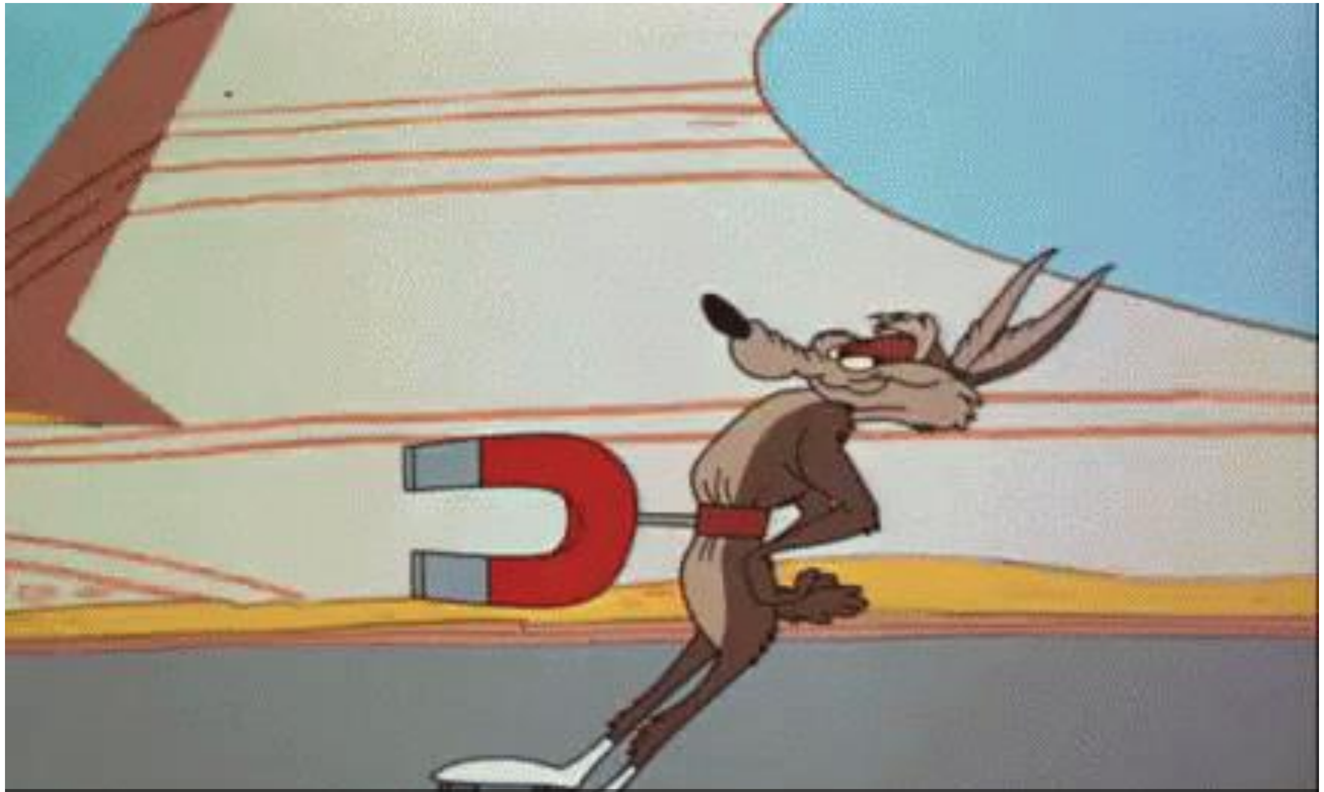
- Potentially large self inductance and large stored magnetic energy
- Hoop stresses > 150 MPa
- Tensile radial stresses
- Longitudinal forces (F_Z) on coils up to 37 MN
- Large stray fields

Questions going forward / some next steps:

- Beam dynamics vs. field quality and magnet alignment (as the magnet configurations are iterated on)
- Options to change towards optimized magnet configurations: higher current density (practical limits), number of magnets, magnet size (radius), etc.

What this study did not include:

- A more complete mechanical structure
- **Matching sections** between stages
- Deeper engineering considerations
  - Iron
  - Realistic space requirements (e.g. B8 is super tight)
  - …
- Dipole magnets



Thank you for listening!

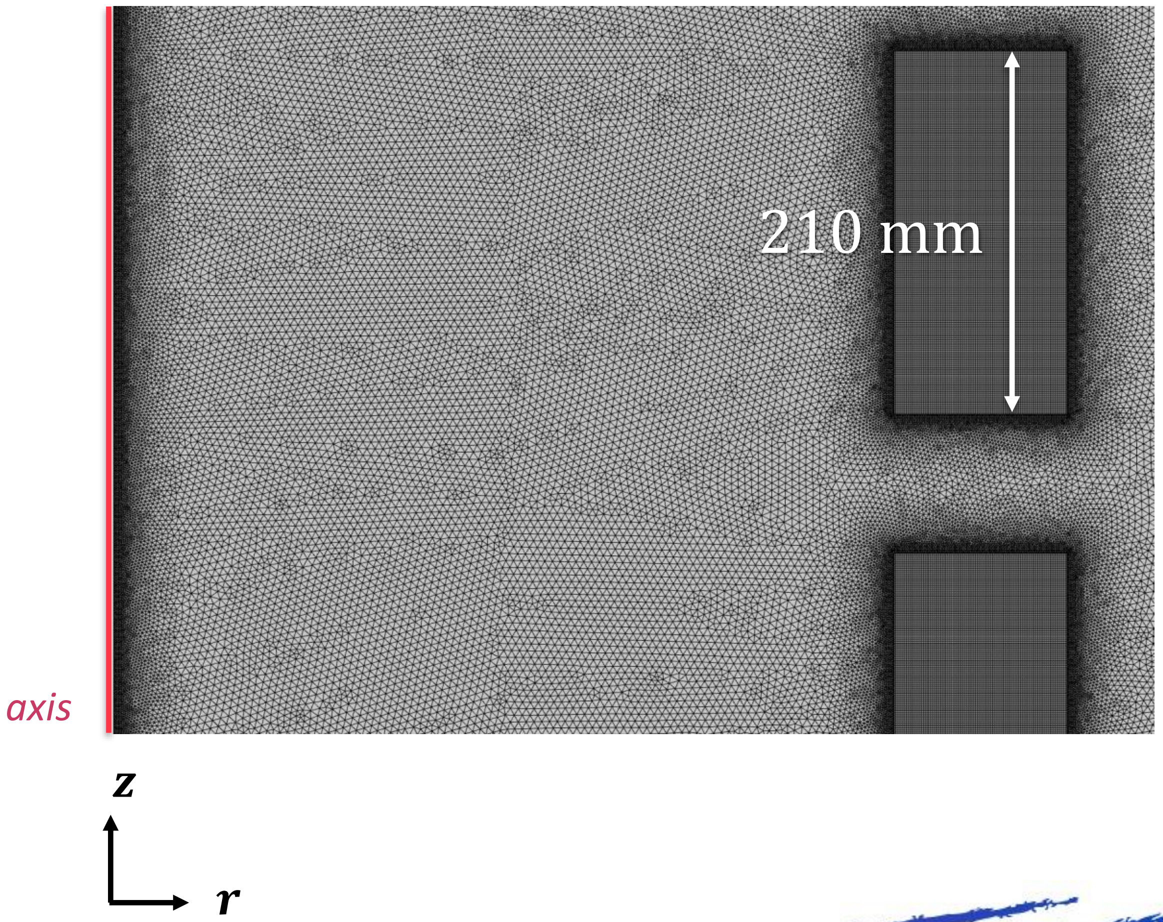
Questions?

Additional Slides

Simulation Study: COMSOL Setup

- *2D Axisymmetric* → stationary
- Physics modules
  - Magnetic
  - Solid Mechanics
- Very fine mesh (Fig. 1)
  - *Max element size* in air region: 5 mm
  - *Max element size* in coil region: 1 mm
- Relative tolerance: < 1e-5
- Use of *infinite domain* for cell and periodic cells:
  - Magnetic insulation at boundaries: $\boldsymbol{n} \; x \; \boldsymbol{A} = 0$
- Use of *periodic boundary condition* for cell-cell interface:
  - Periodicity on vector potential: $\boldsymbol{A}_{left} = \boldsymbol{A}_{right}$
- Mechanics interface
  - *Linear elastic materials*
  - Lorentz force on coil: $\sim J \; x \; B$
  - Rollers on specific faces
  - *Homogeneous copper*: $(E = 120 \text{ GPa}, \nu = 0.34)$

Fig. 1



Outline

Goal: simulate and characterize the cooling solenoid magnets based on geometries and initial parameters from the US MAP study.

- *Overview*
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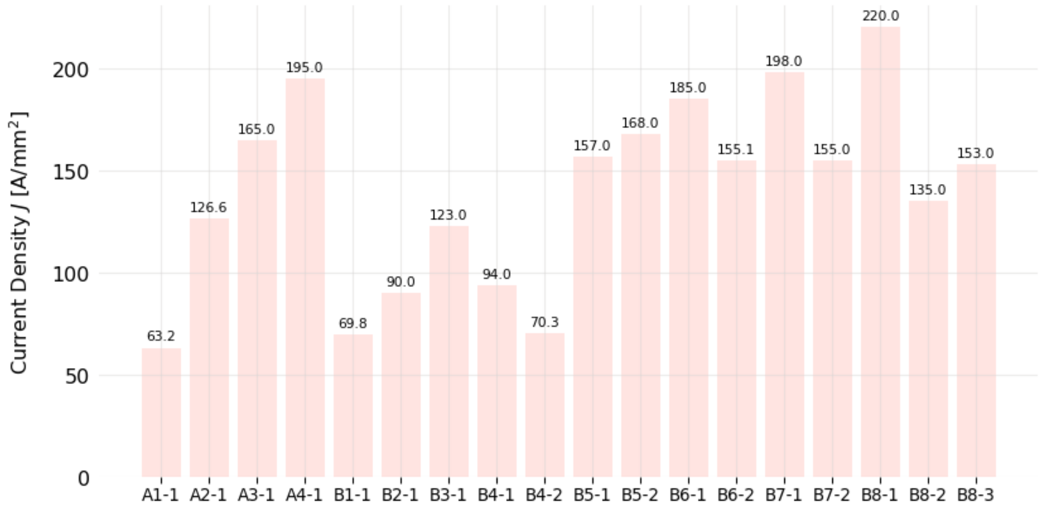
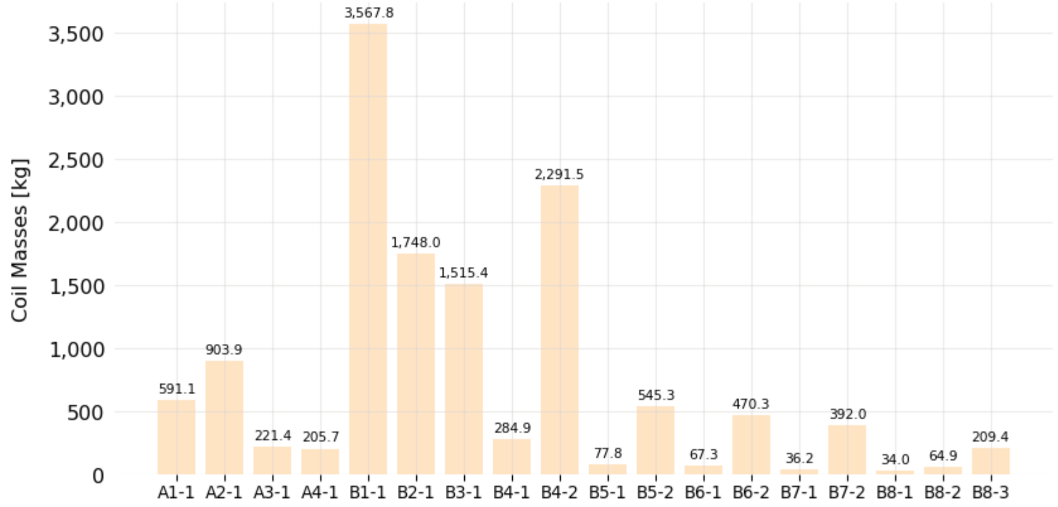
Results Part 1: Characterization of all Coil Types 1/4

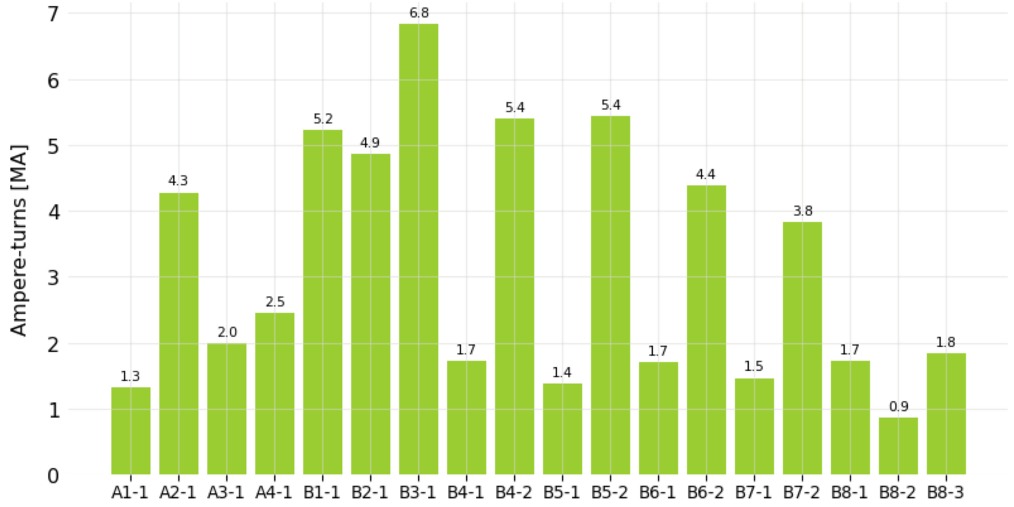
Coil Properties

18 unique coil types

- 2 to 6 coils per cell
- Inner bore diameter from 90 mm to 1540 mm
- Lengths from 80 mm to 210 mm
- Current densities from 63 to 220 $\mathrm{A/mm^2}$

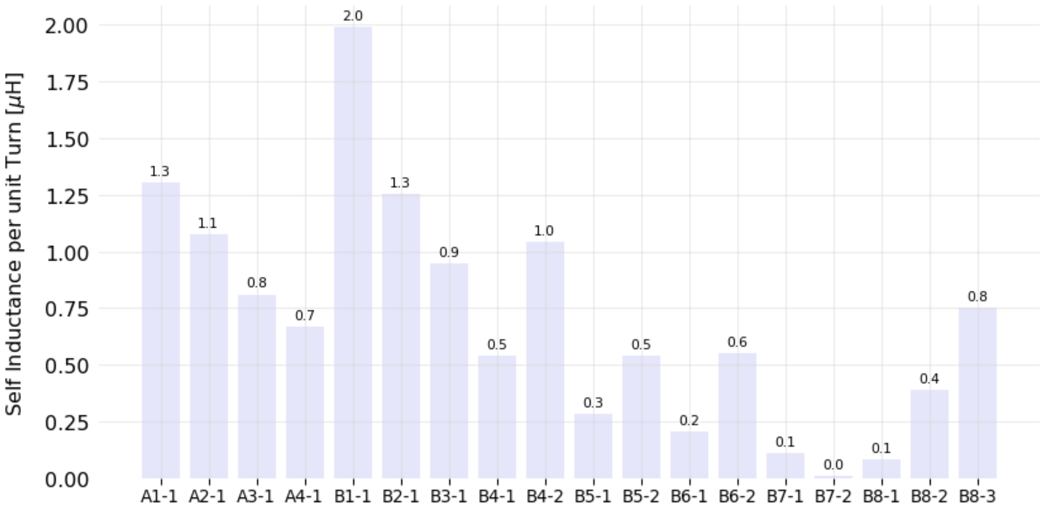


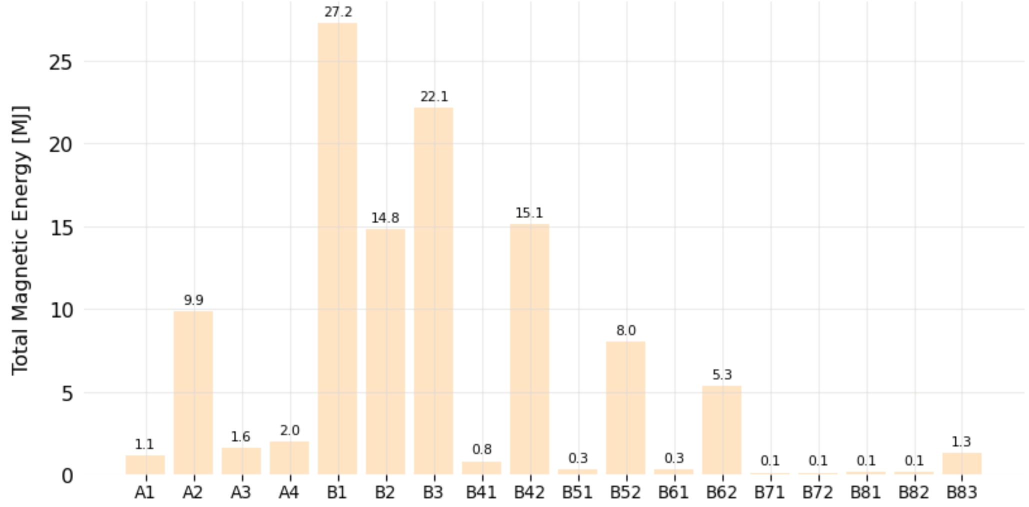


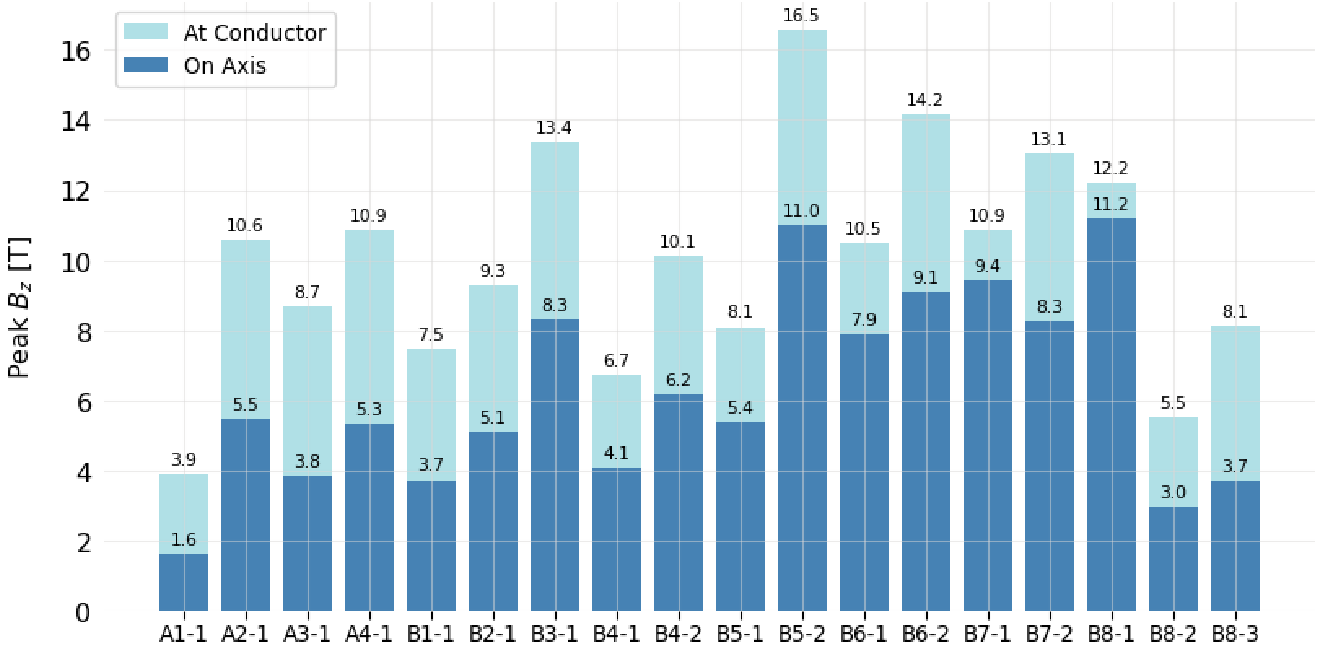


Results Part 1: Characterization of all Coil Types 2/4

Magnetic Properties

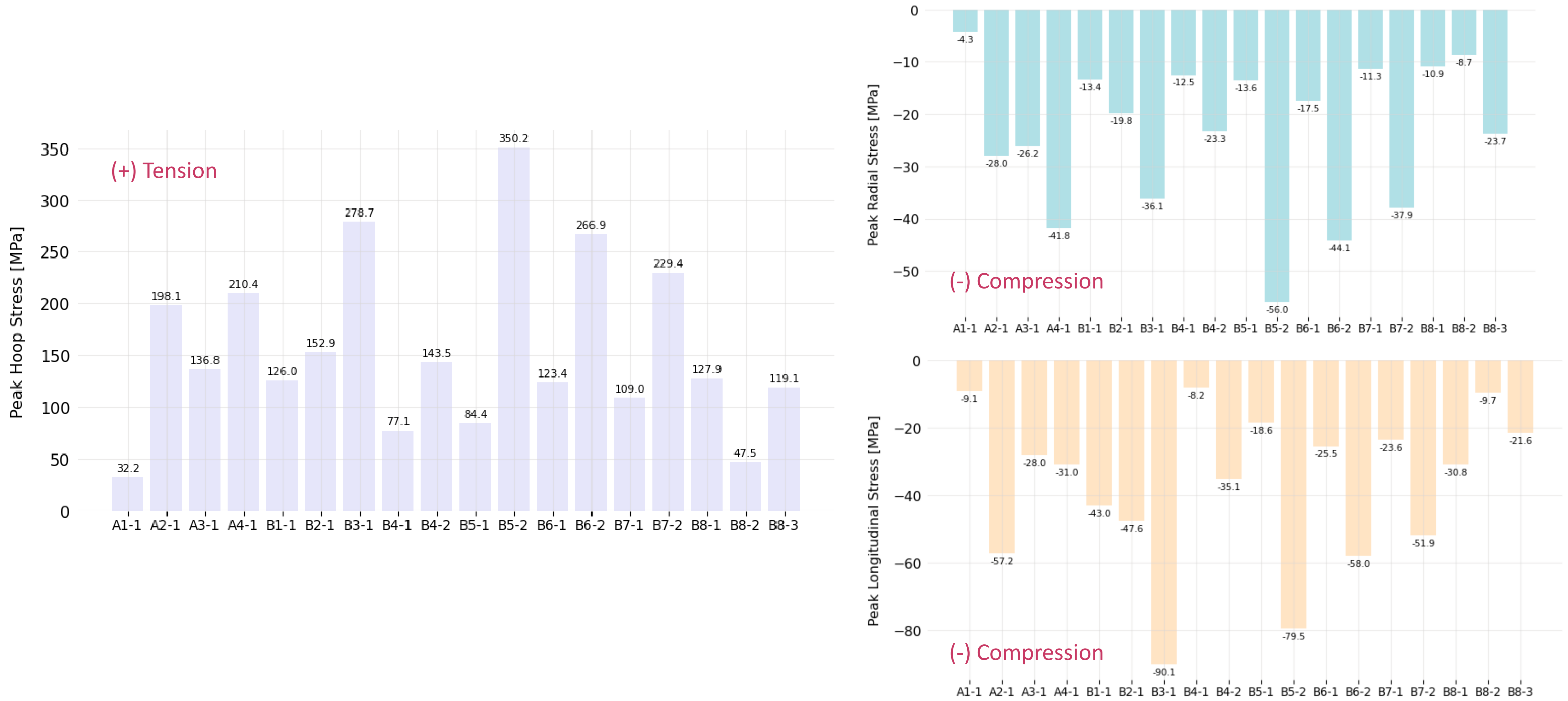






Results Part 1: Characterization of all Coil Types 3/4

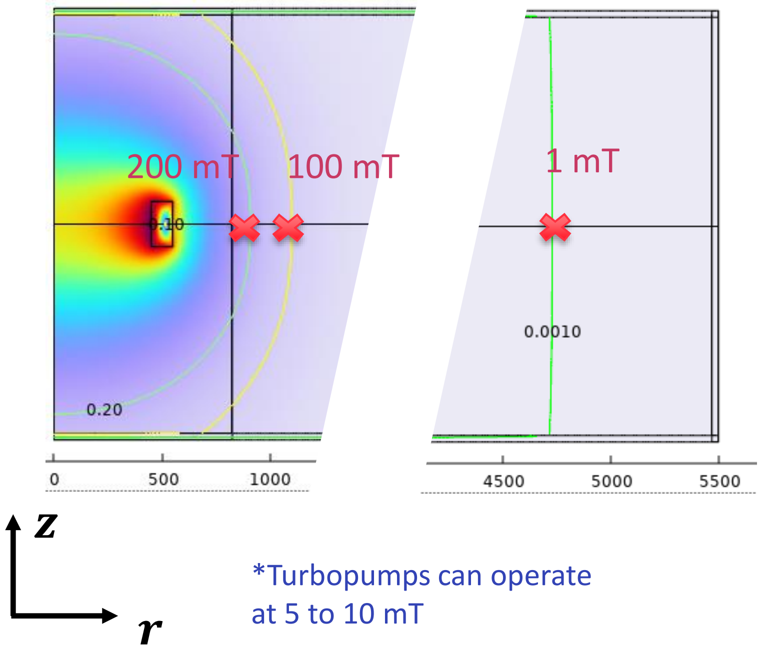
Mechanical Properties



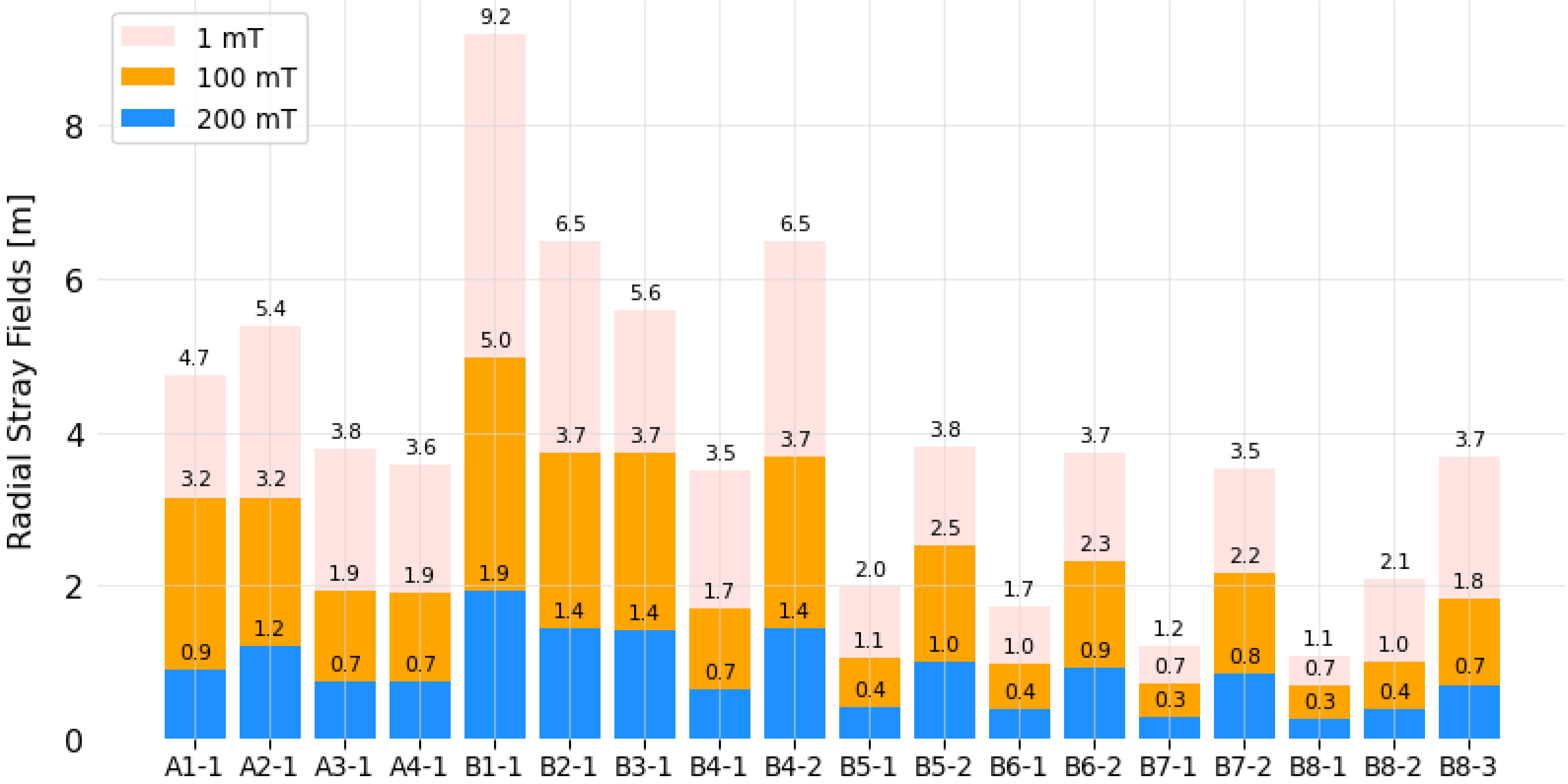
Results Part 1: Characterization of all Coil Types 4/4

Stray Fields

Example



*Turbopumps can operate at 5 to 10 mT

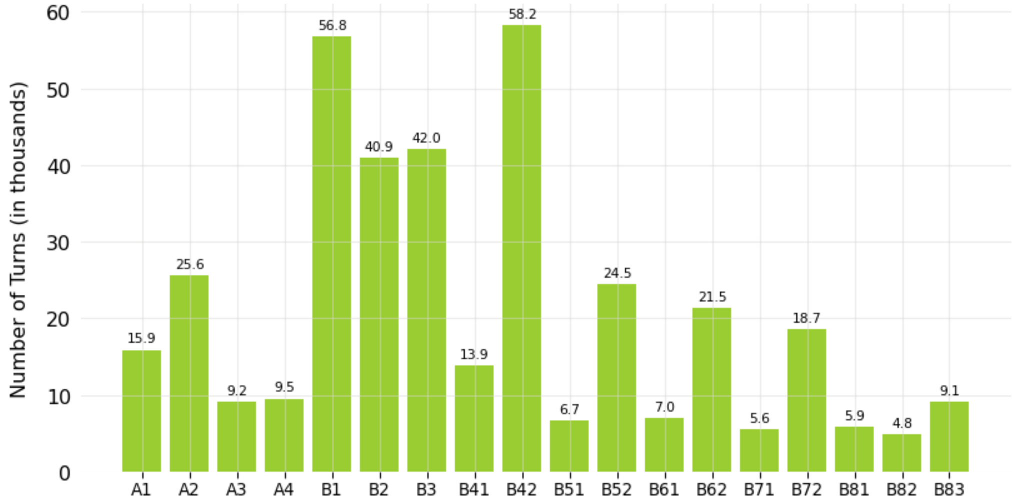


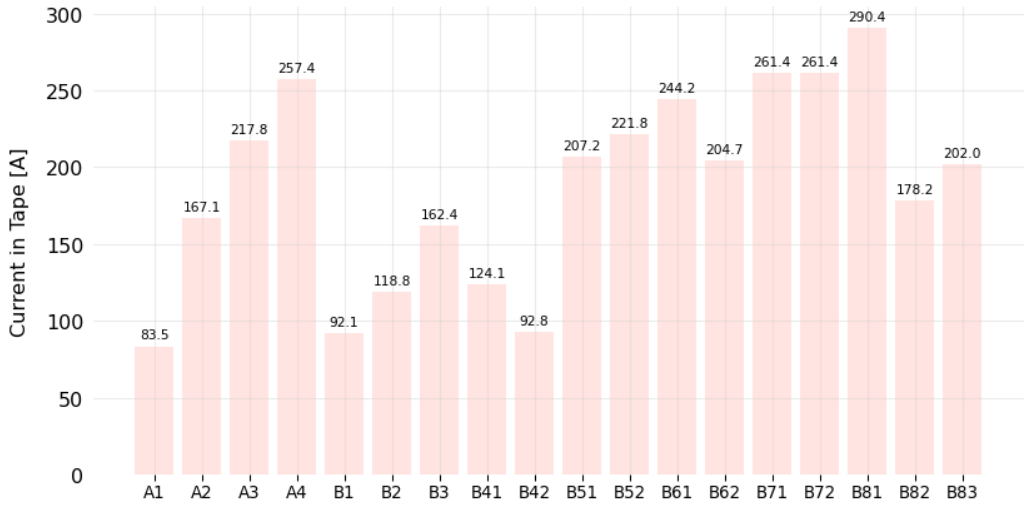
Results Part 1: Characterization of all Coil Types 1/4

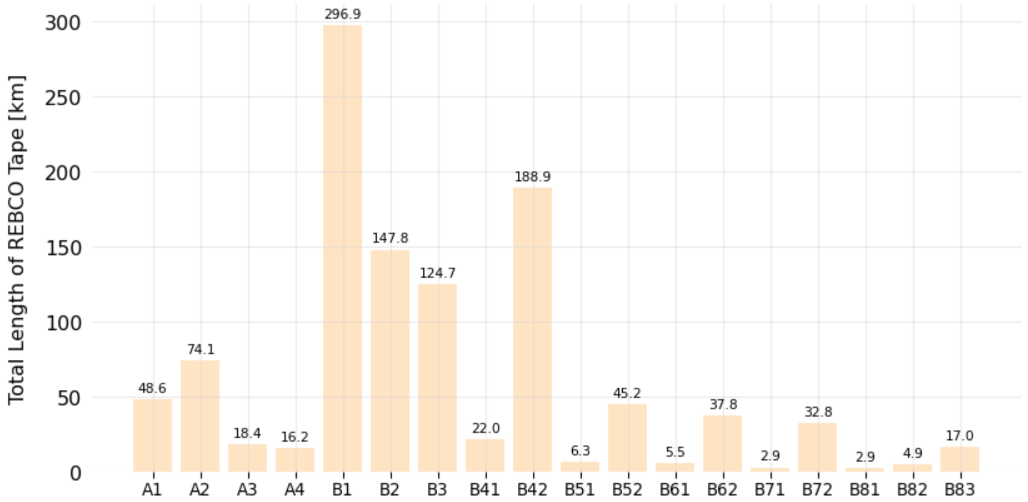
Tape Properties

Assuming ReBCO 12 mm $\times$ 0.11 mm!

- 18 unique coil types
- 2 to 6 coils per cell
- Inner bore diameter from 90 mm to 1540 mm
- Lengths from 80 mm to 210 mm
- Current densities from 63 to 220 $\mathrm{A/mm^2}$







Results Part 1: Characterization of all Coil Types 2/4

Magnetic Properties

Assuming ReBCO 12 mm $\times$ 0.11 mm!