Standard Model PDFs for High-Energy Lepton Colliders

David Marzocca

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LePDF

Francesco Garosi, D.M., Sokratis Trifinopoulos [**2303.16964**]

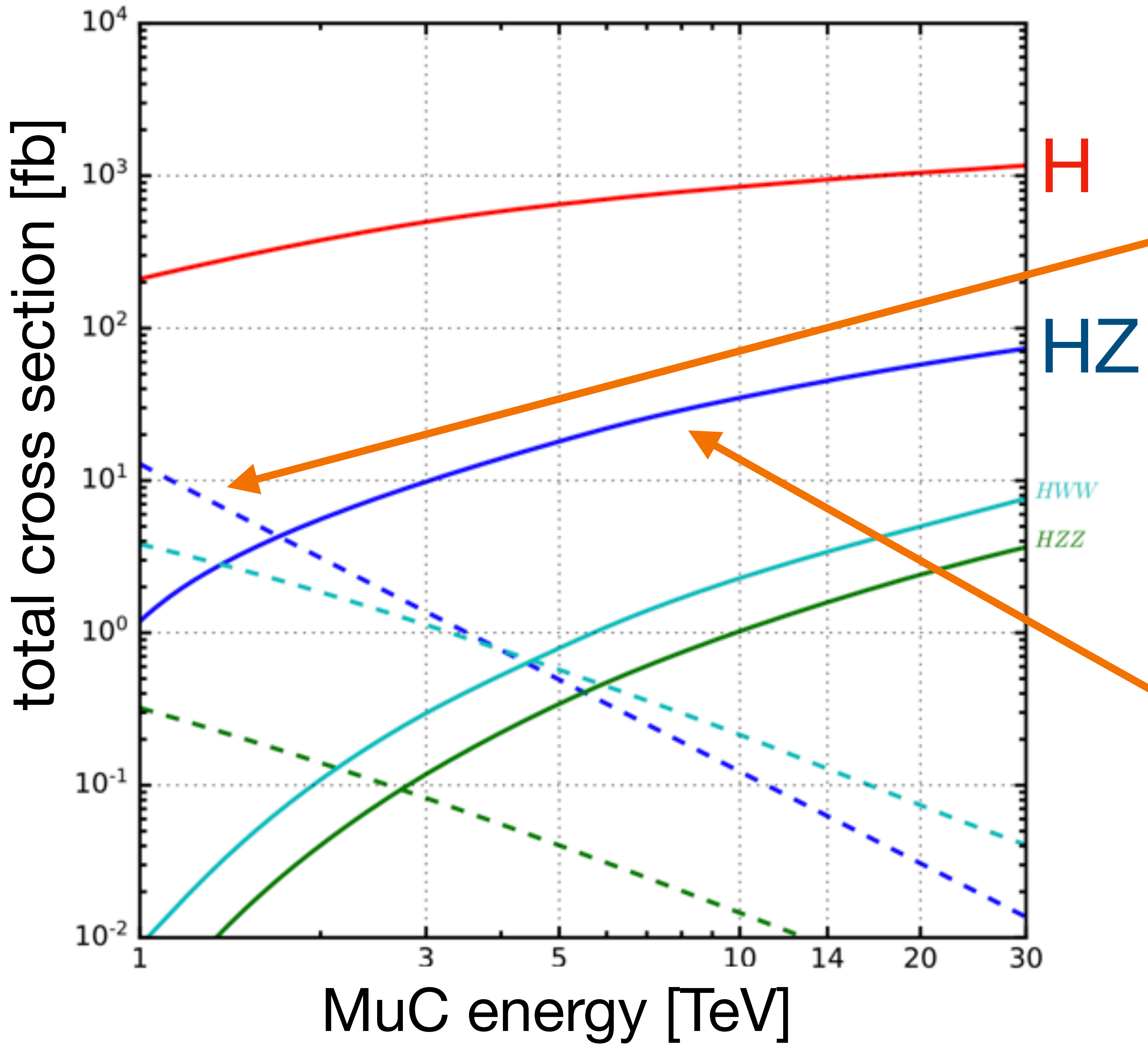
Available form:
- https://github.com/DavidMarzocca/LePDF
- https://lepdf.hepforge.org/ (still empty)

Based on several previous works, most notably:
P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047],
Bauer, Webber [1703.08562, 1808.08831],
Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2007.14300, **2103.09844**]
Azatov, Garosi, Greljo, DM, Salko, Trifinopoulos [2205.13552]

IMCC Annual Meeting 2023 - 20/06/2023

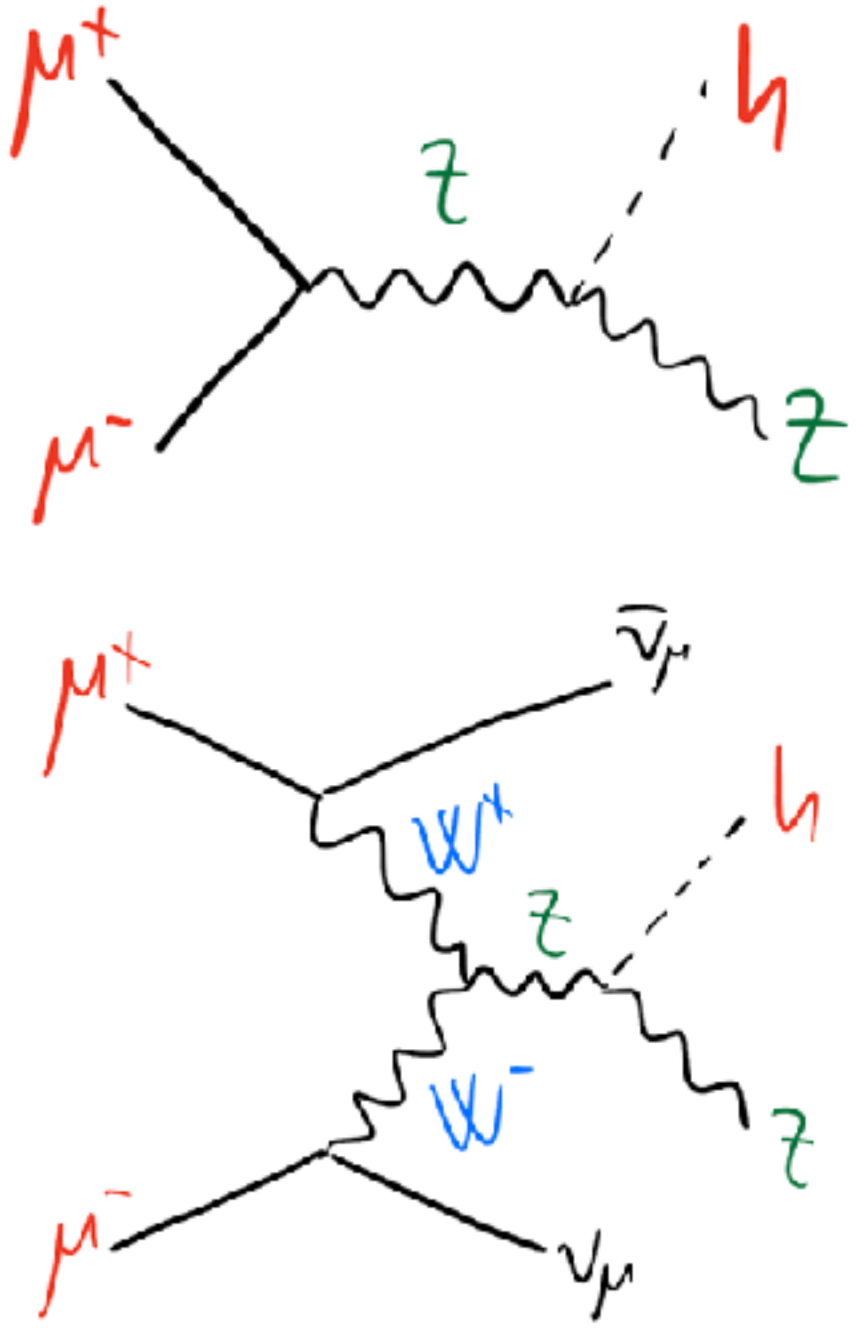
MuC is a VBC

[Costantini et al. 2005.10289]



dashed: annihilation

solid: VBF



$$\sigma_{ann} \propto \frac{\alpha^2}{s}$$

$$\frac{d\sigma_{VBF}}{dm_h^2} \propto \frac{\alpha^2}{m_{hZ}^2}\,\alpha^2 \log^2\frac{m_{hZ}^2}{m_W^2}\,\log\frac{s}{m_{hZ}^2}$$

[see also Costantini et al. 2005.10289]

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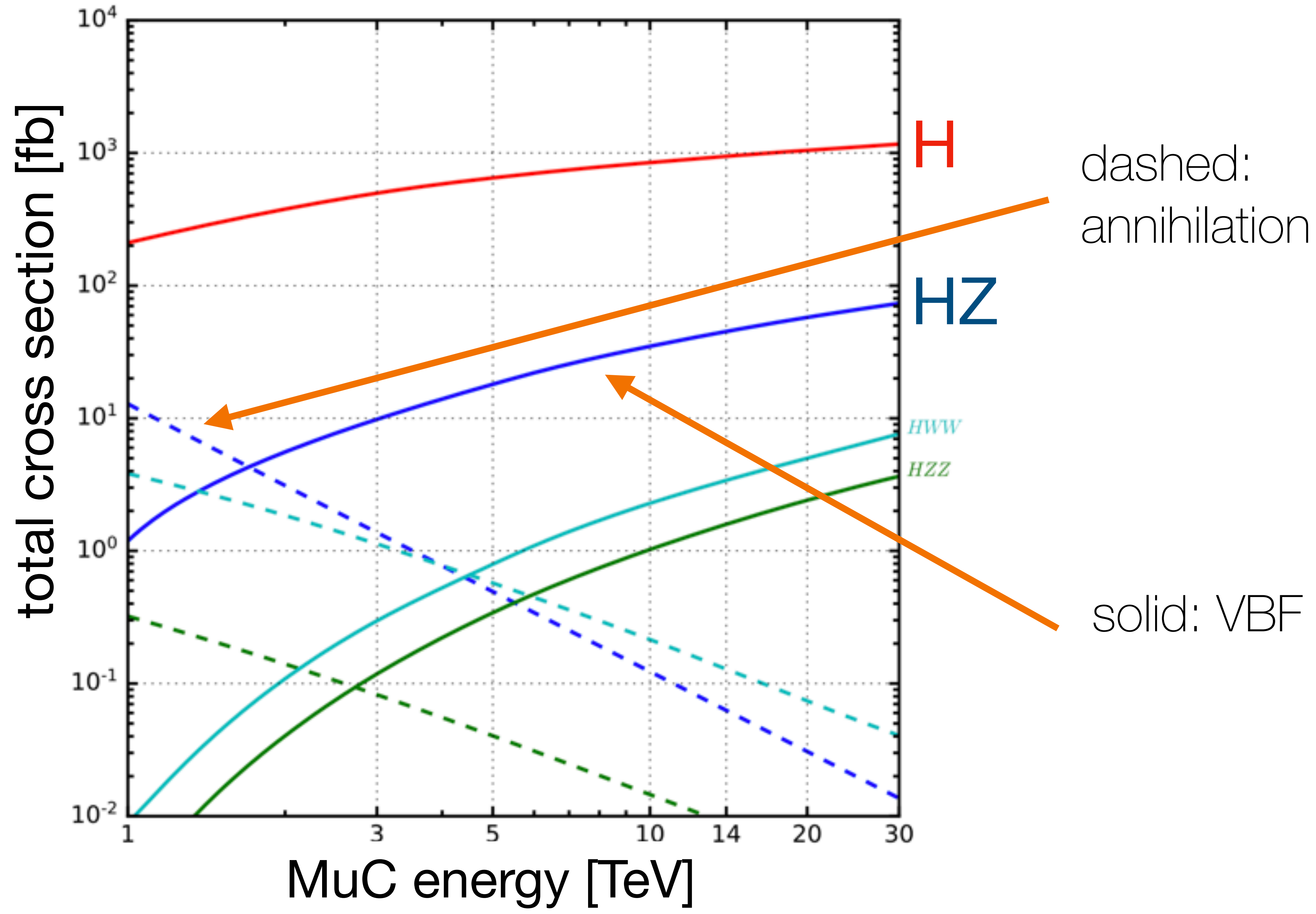

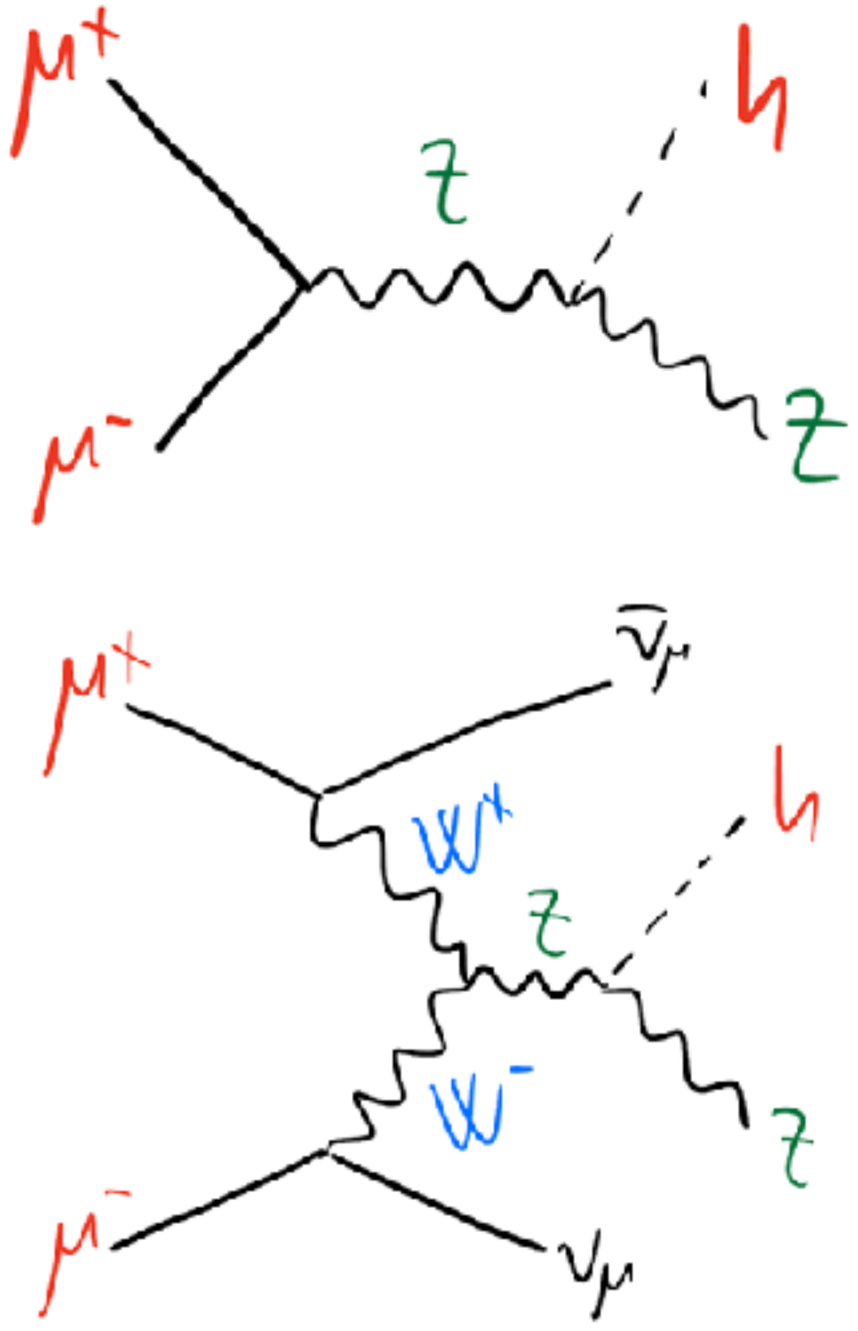

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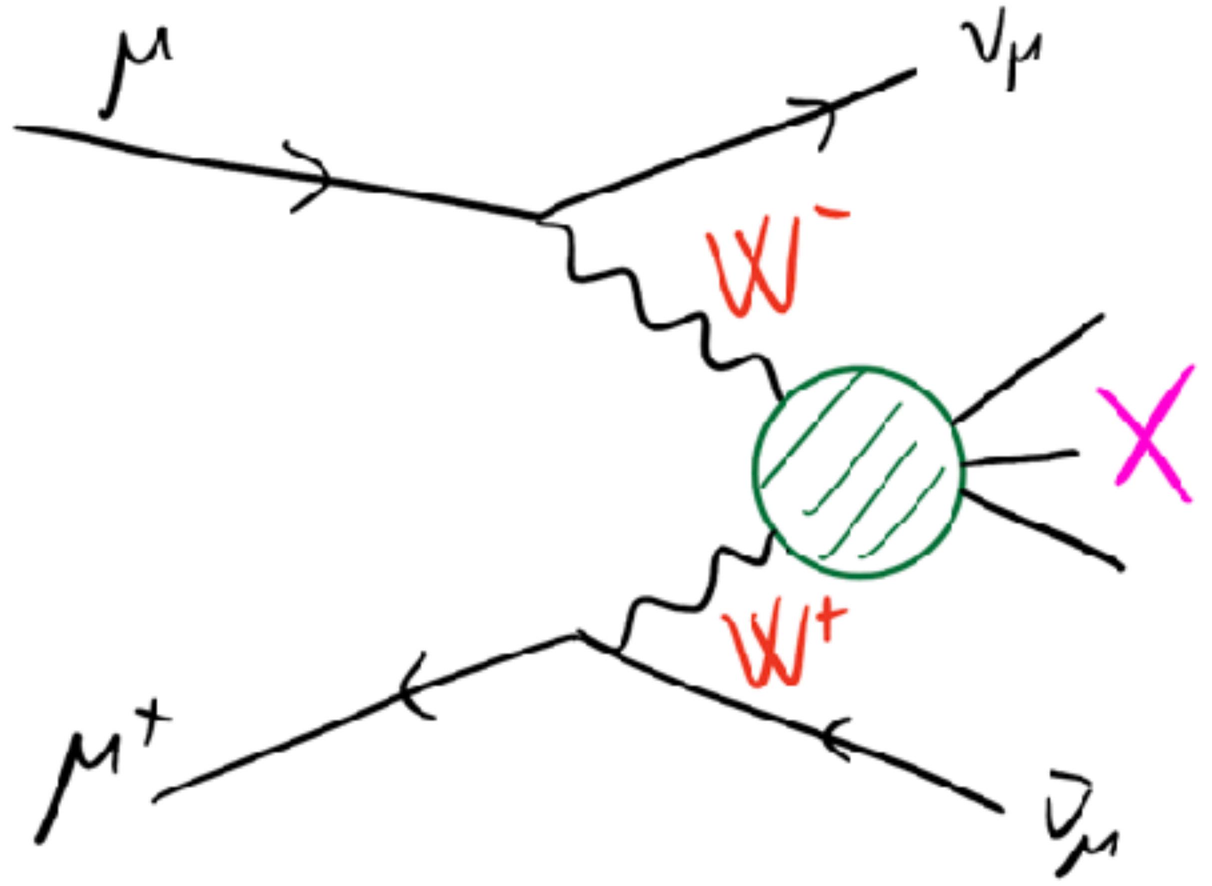
[see also Costantini et al. 2005.10289]

At muon colliders above ~1 - 5 TeV,
the VBF process is always dominating over annihilation,
mostly collinear emission.

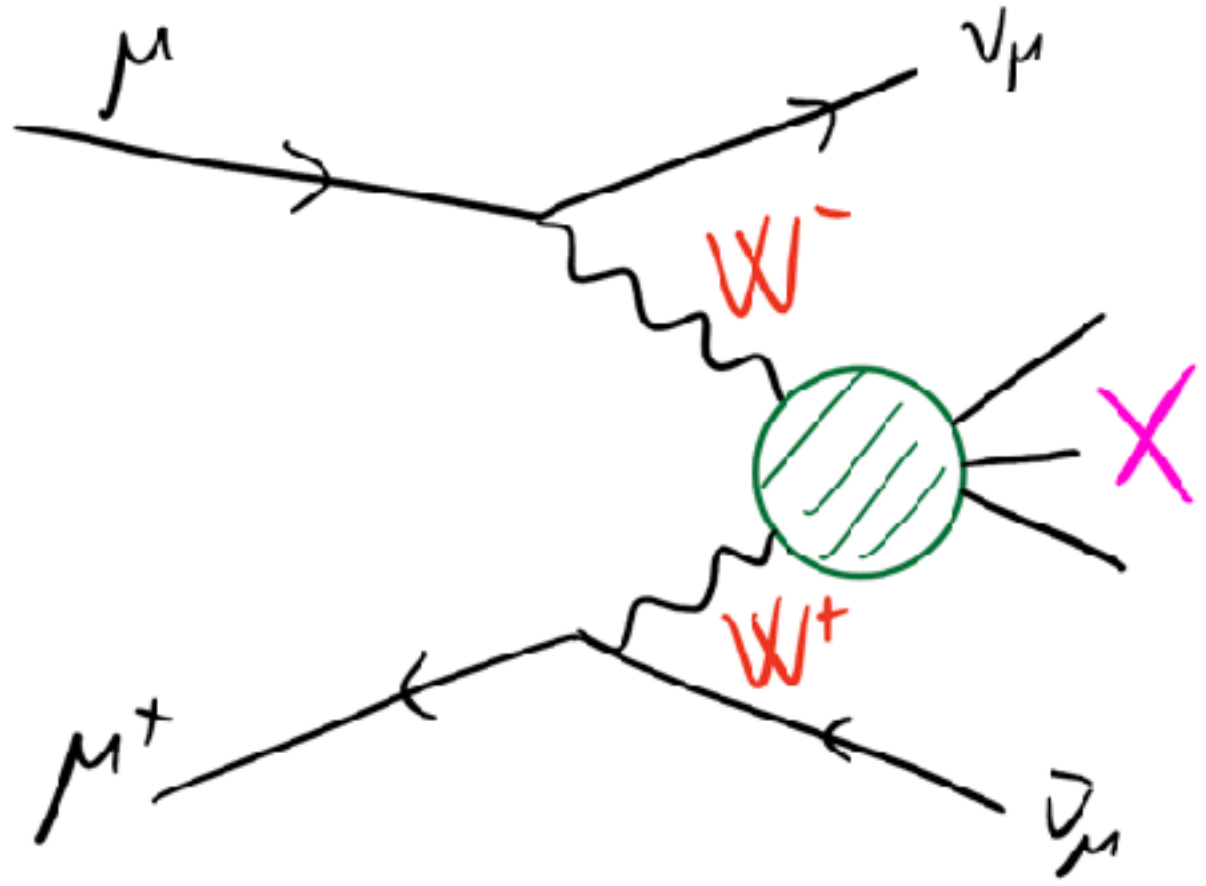
A high-energy Muon Collider is a Vector Boson Collider!

This is what allows a MuC to reach exquisite precision in Higgs and EW physics.



The emission of **collinear radiation** (photon, W, Z, etc..) **off a muon** can be factorised from the hard scattering. [Cuomo, Vecchi, Wulzer 1911.12366]

$$p_T,\, m_W \ll E$$



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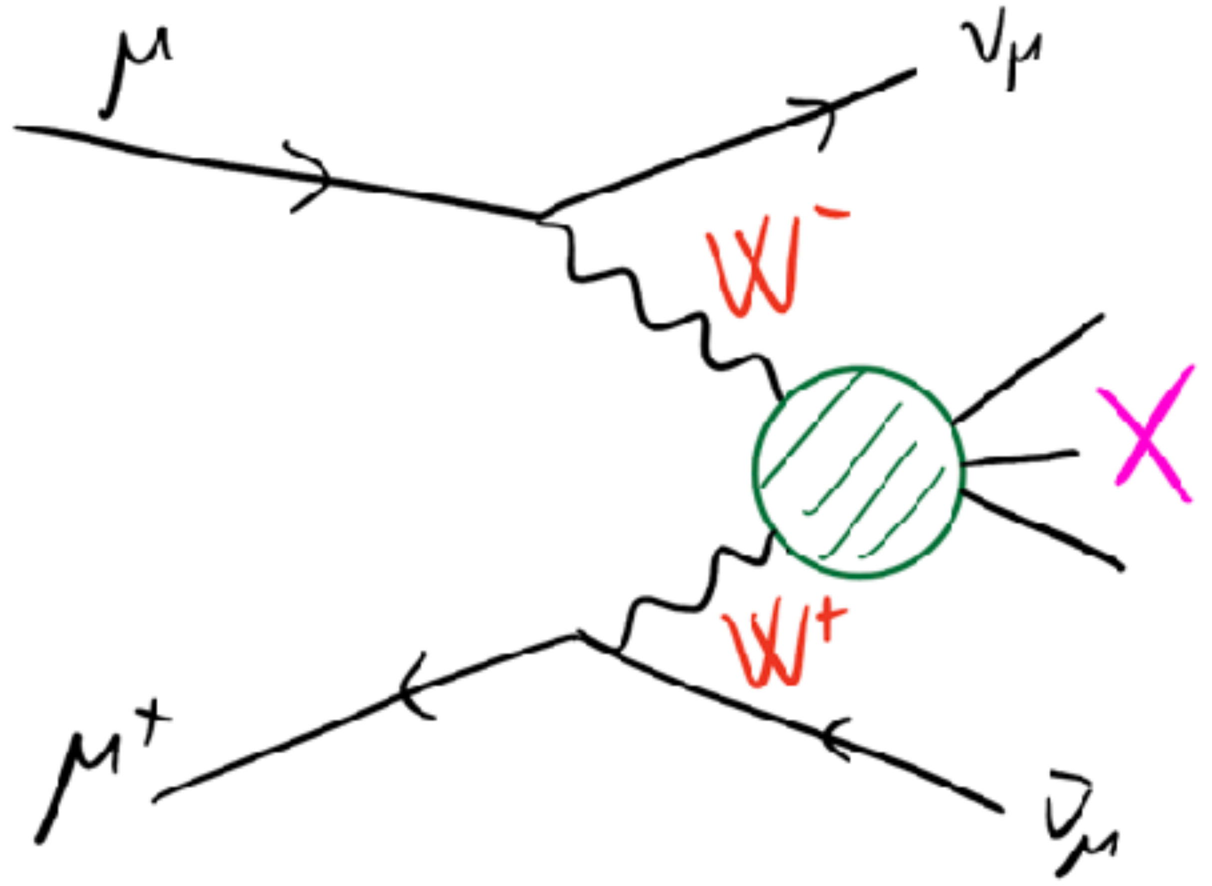
This can be described in terms of
generalised Parton Distribution Functions, like for proton colliders:

$$\sigma(\mu\bar{\mu} \rightarrow C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1, Q)\, f_j(x_2, Q)\, \hat{\sigma}(i\, j \rightarrow C)(\hat{s})$$



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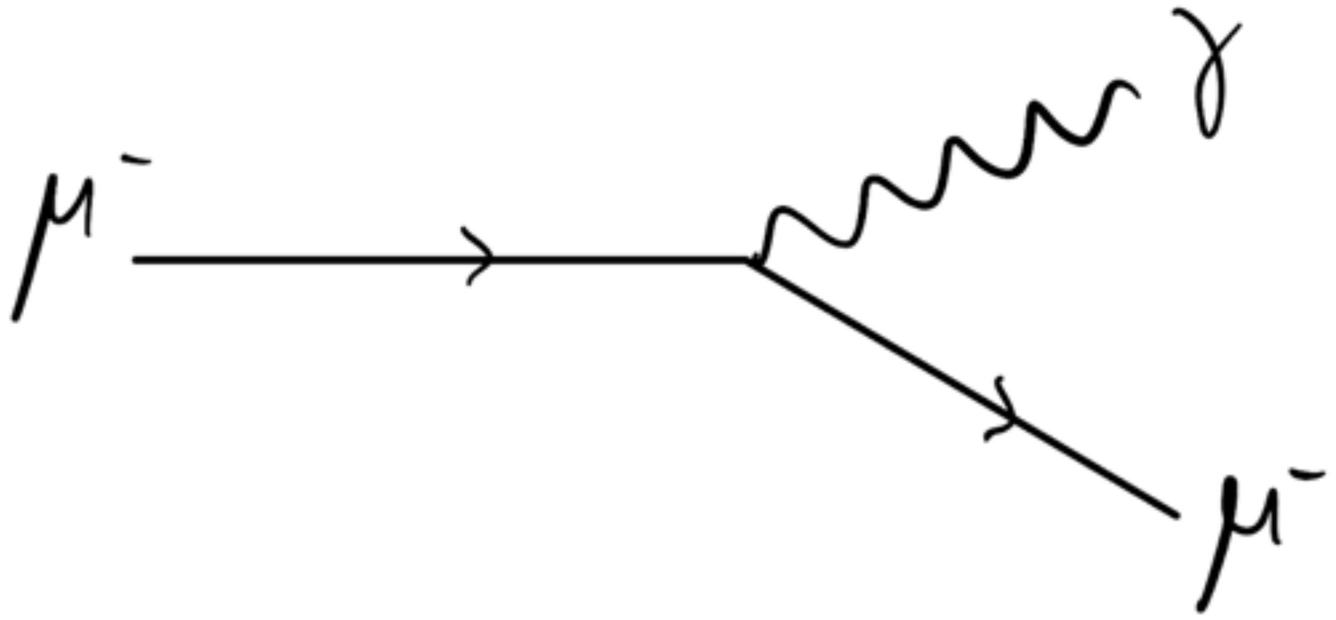
The case of collinear photon emission from an electron gives the ***Equivalent Photon Approximation***



$$f_\gamma^{\mathrm{EPA}}(x) = \frac{\alpha}{2\pi} P_{\gamma e}(x) \log \frac{E^2}{m_e^2}$$

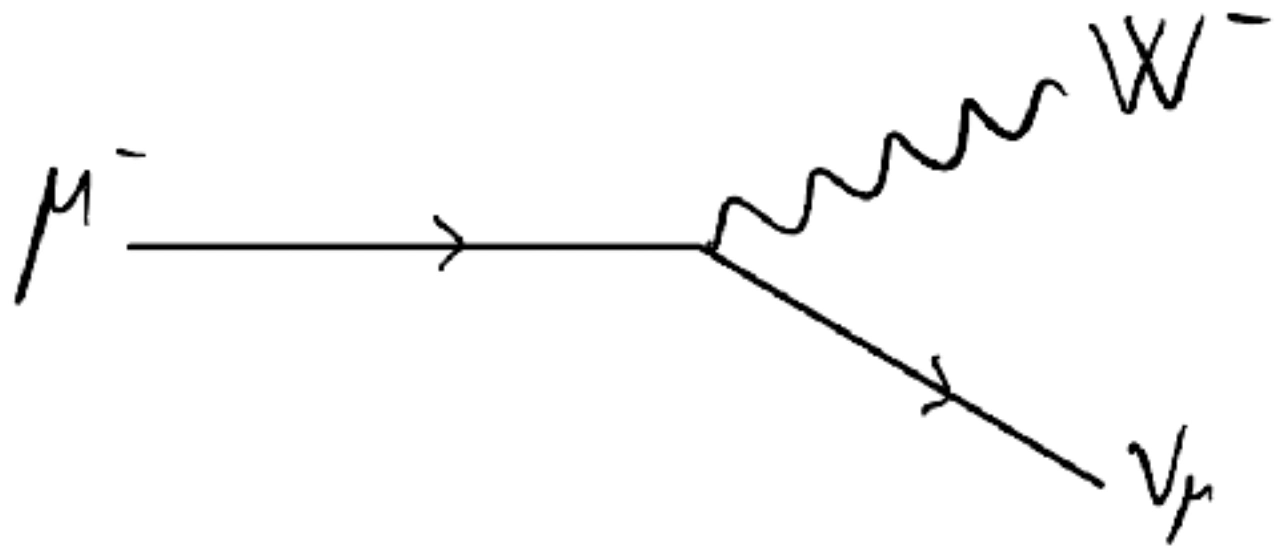
Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34)

LO Splitting function:

$$P_{\gamma e}(x) = \frac{1+(1-x)^2}{x}$$

At energies **above the EW scale**, similarly one can obtain the **Effective Vector Approximation** for EW gauge boson PDFs

Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84,
See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…



$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \left(\log \frac{Q^2 + (1-x) m_W^2}{m_\mu^2 + (1-x) m_W^2} - \frac{Q^2}{Q^2 + (1-x) m_W^2} \right)$$

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This one is now implemented in `MadGraph5_aMC@NLO`
[Ruiz, Costantini, Maltoni, Mattelaer 2111.02442]

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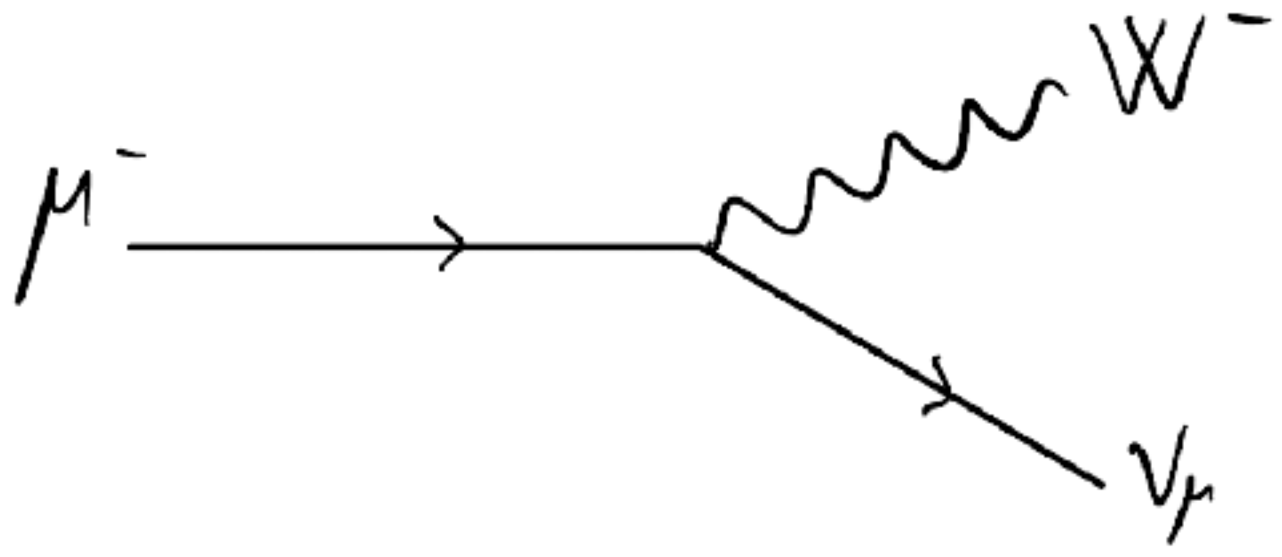

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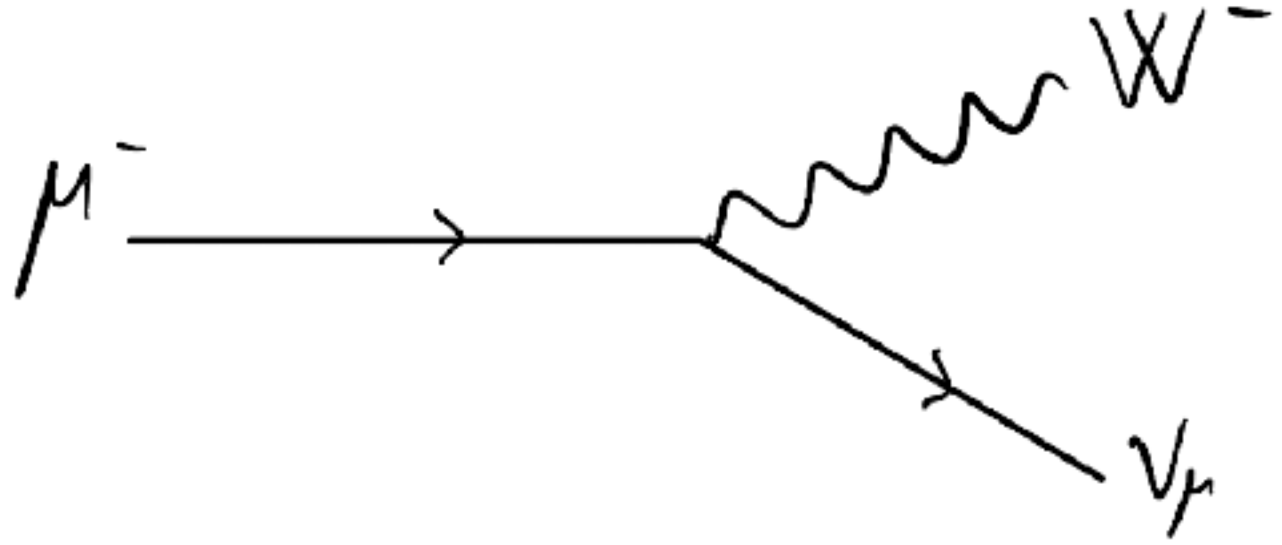

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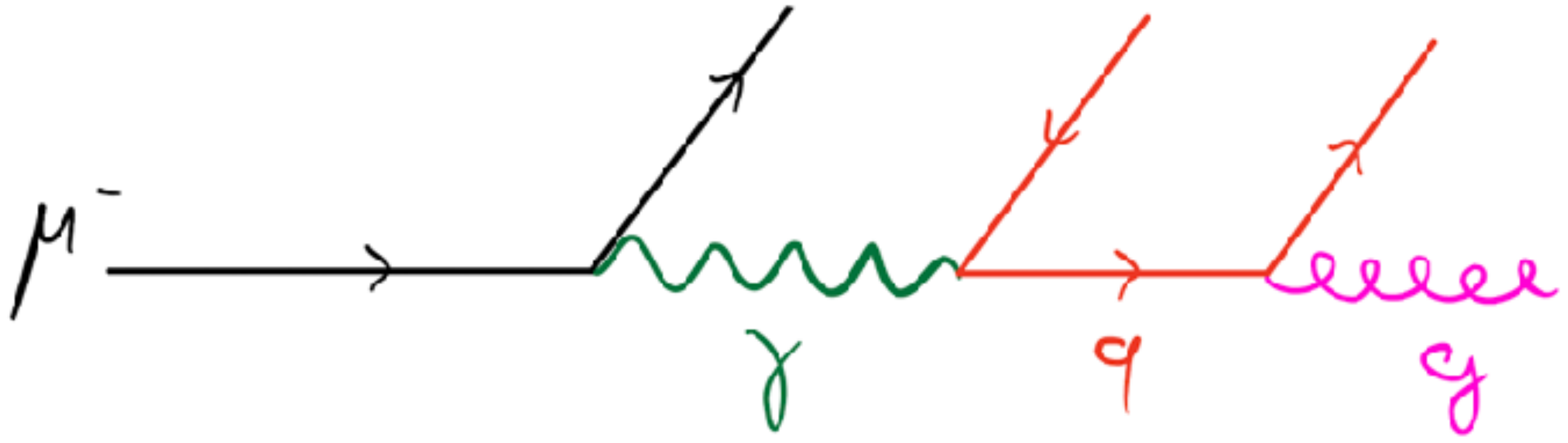
★ The expected relative corrections to the LO EVA result are proportional to (Sudakov double logs)

$$\alpha_2 \left(\log \frac{Q^2}{m_W^2} \right)^2 \sim 1 \quad \text{for } Q \sim 1.5\ \text{TeV}.$$

For precise vector boson PDFs at the TeV scale it is important to re-sum the EW double logs.

Furthermore, if we are also interested in the **QCD (gluon and quarks) PDFs**, then **resummation is required** since α_s is large at small scales.



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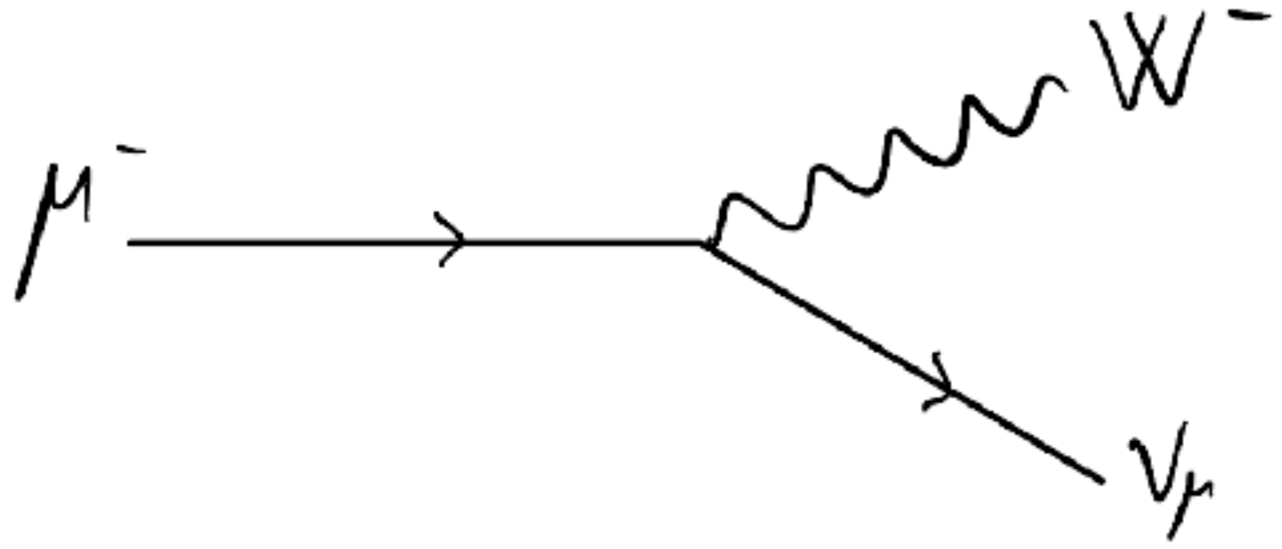

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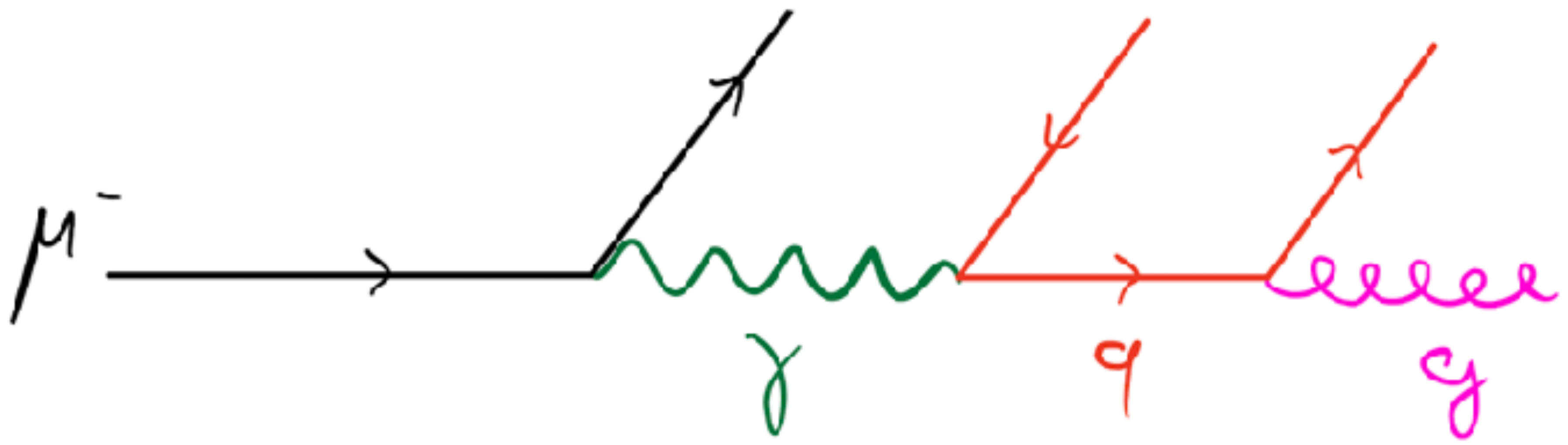
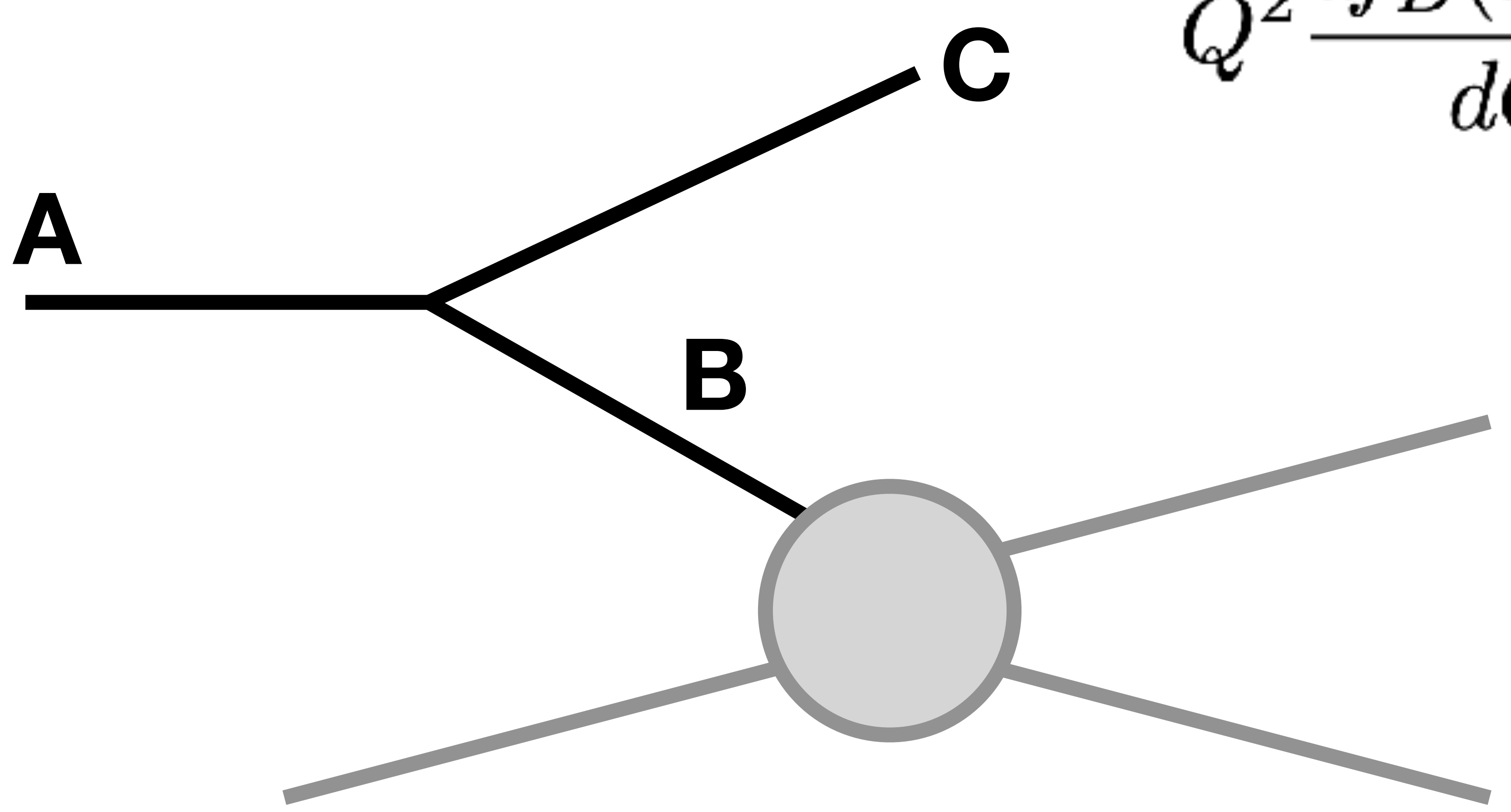


★ Mass effects remain relevant up to several TeV of energy. The $Q \gg m_W$ approx. is not sufficient for good precision.

Collinear Radiation and PDFs

Strongly ordered emission from multiple splittings can be resummed by solving the **DGLAP equations**



$$Q^2 \frac{d f_B(x, Q^2)}{dQ^2} = P_B^v \, f_B(x, Q^2) + \sum_{A,C} \frac{\alpha_{ABC}(Q)}{2\pi} \int_x^1 \frac{dz}{z} P_{BA}^C(z) \, f_A\left(\frac{x}{z}, Q^2\right)$$

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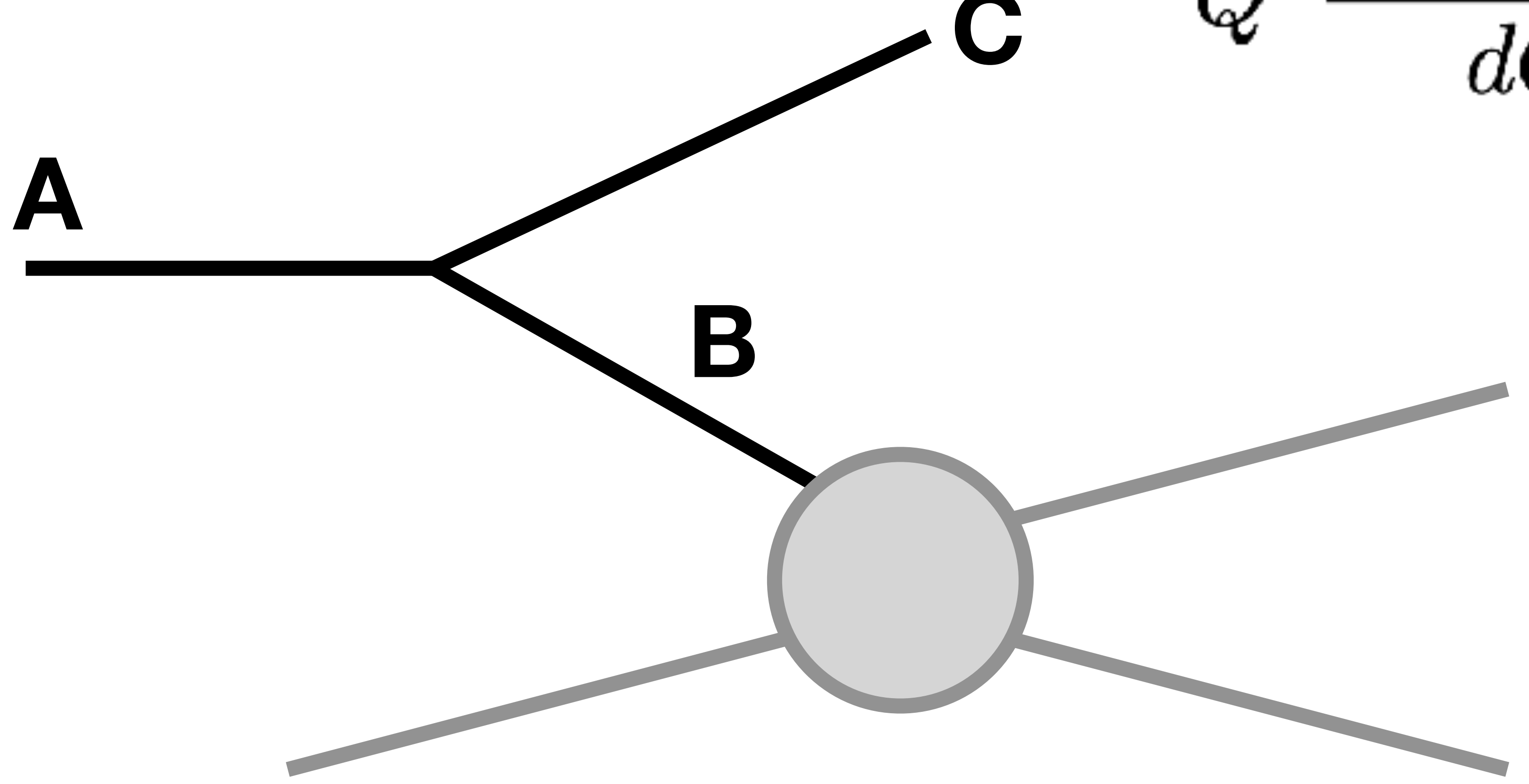

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Real emission

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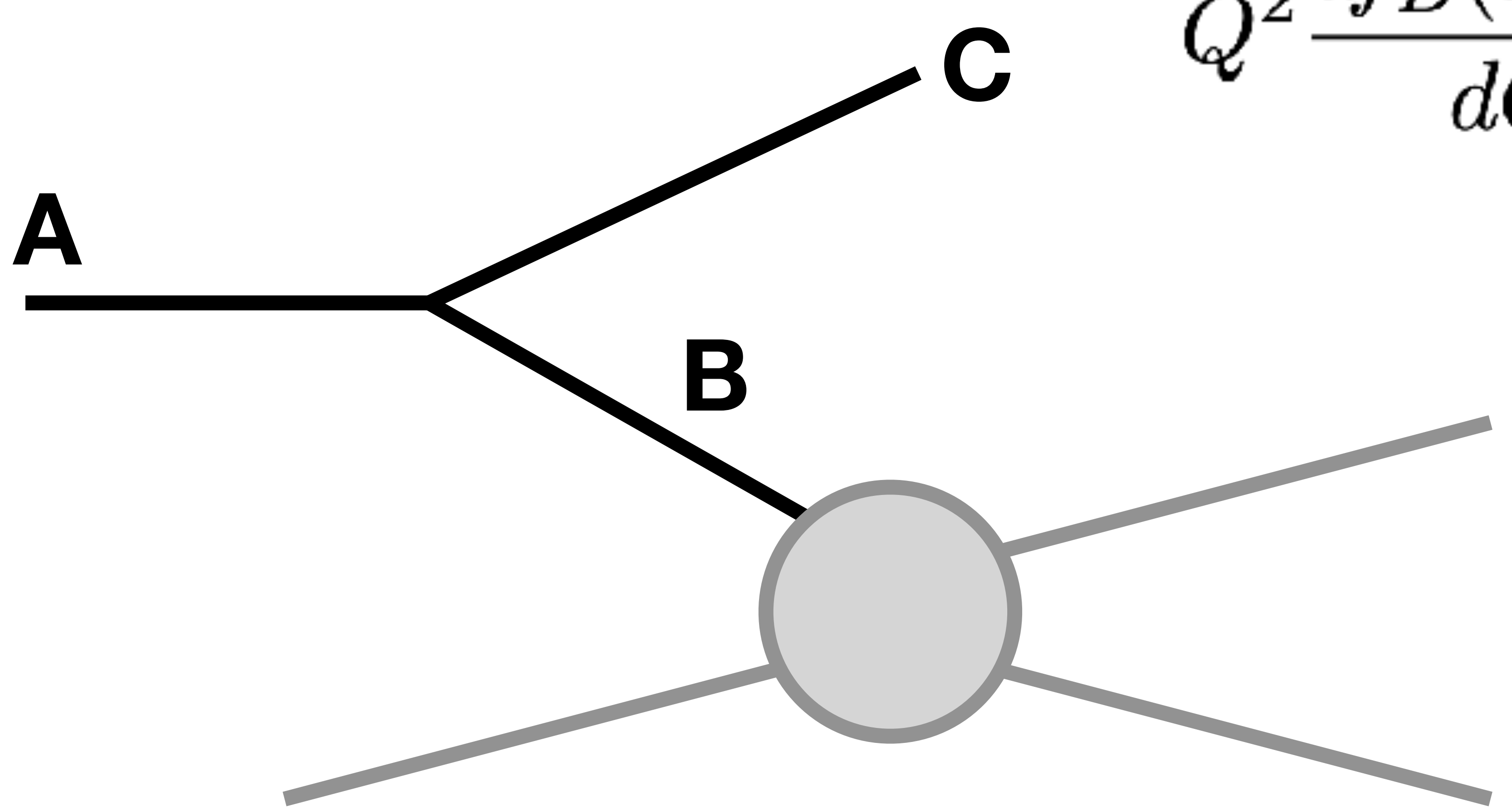

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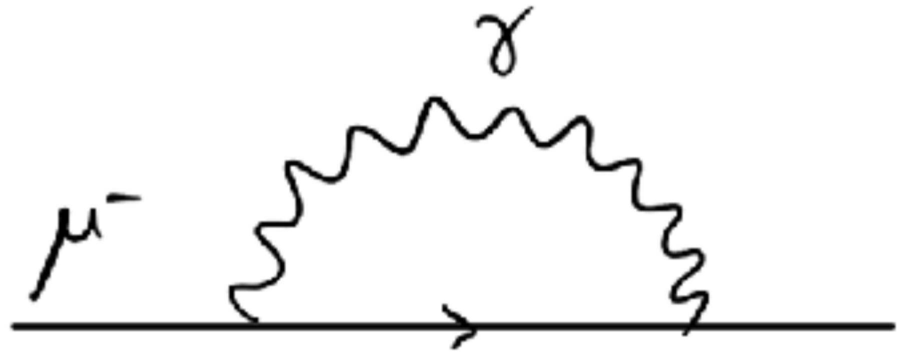
Virtual corrections

Real emission



They cancel the IR divergence ($z \to 1$)
of real soft emissions.

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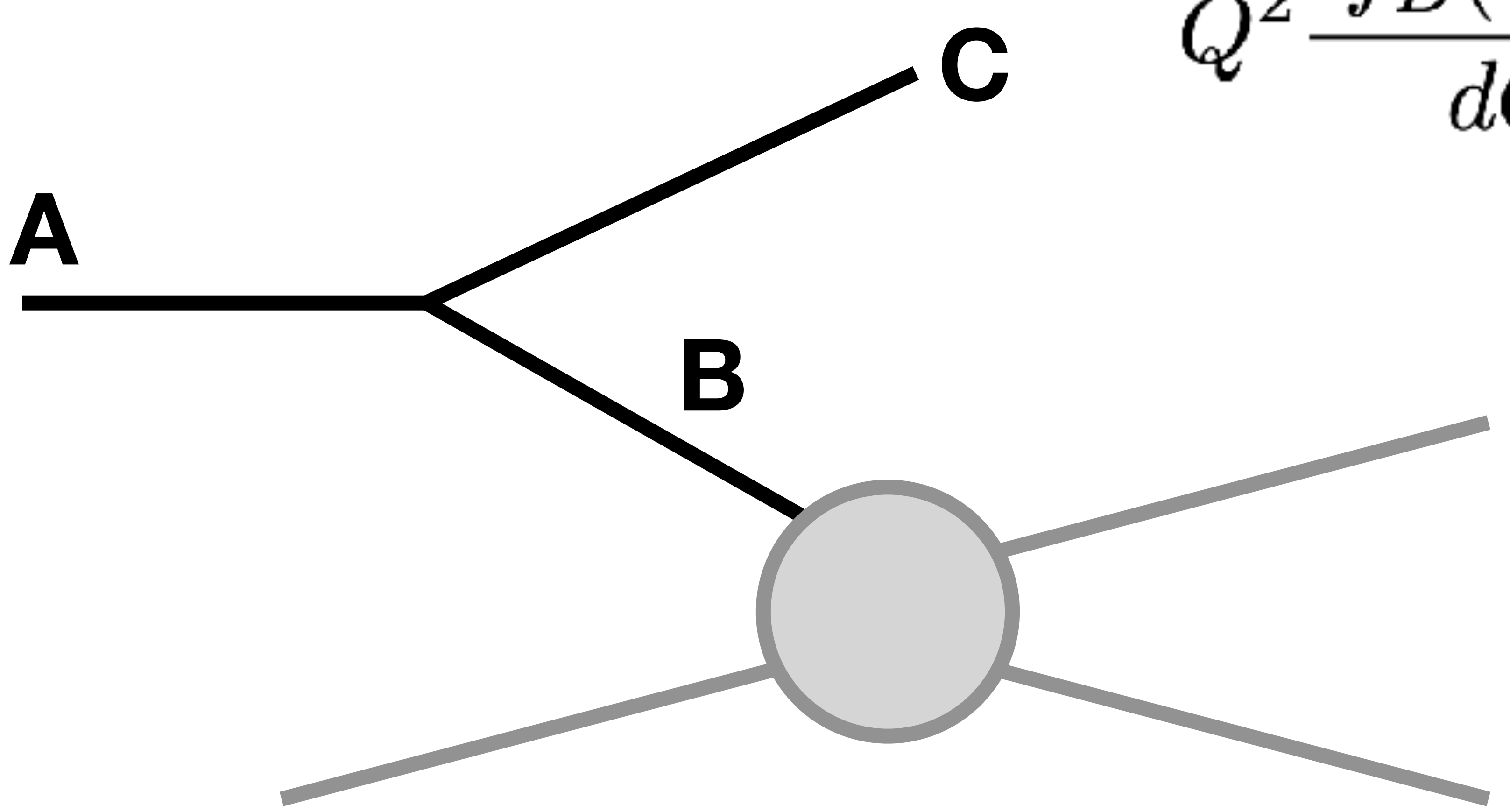

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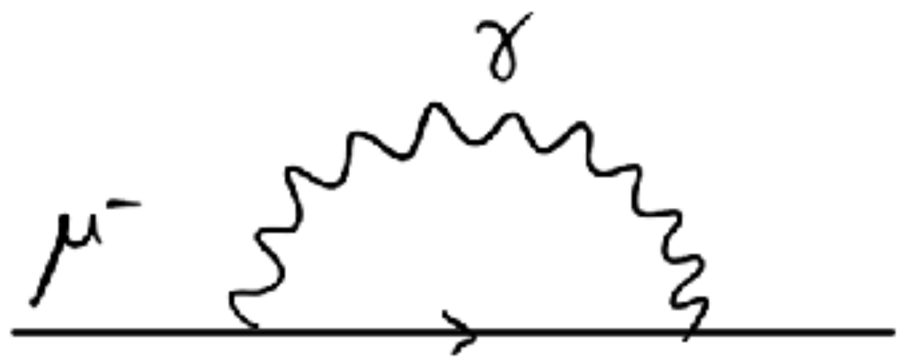


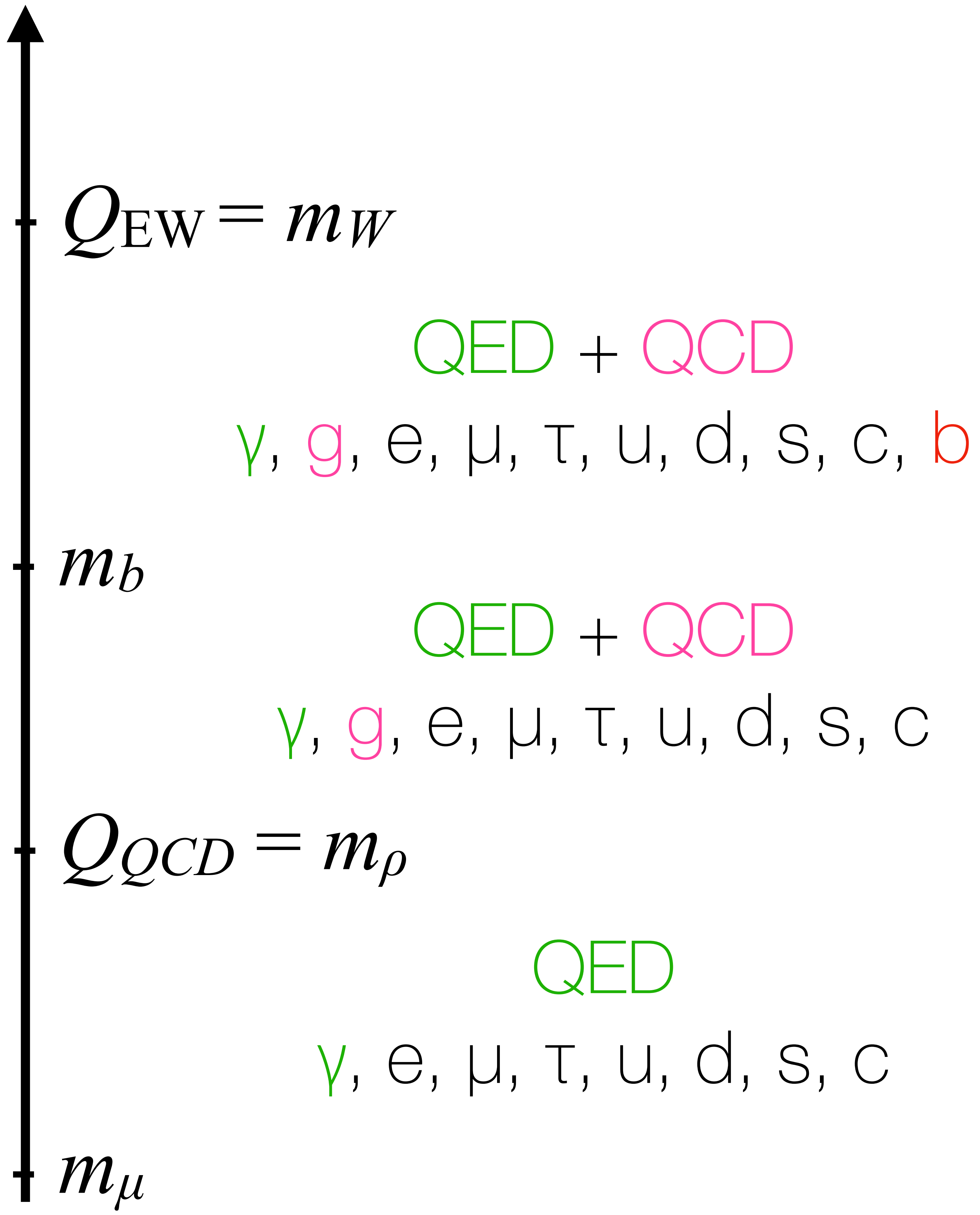
They cancel the IR divergence ($z \to 1$)
of real soft emissions.

Unlike for protons, since the muon is elementary this can be done **from first principles**.

The **boundary condition** is set by $f_\mu(x, m_\mu) = \delta(1\text{-}x) + O(\alpha), \quad f_{i\neq\mu}(x, m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]

DGLAP equations below EW scale



Below the EW scale only **QED** + **QCD** interactions are relevant.

QCD is introduced at the rho meson scale (we vary it for uncertainty), and we add the threshold for the b quark.

Drees, Godbole [hep-ph/9403229], Schuler and Sjostrand [hep-ph/9503384,9601282]

DGLAP equations below EW scale

$Q_{\rm EW} = m_W$

QED + QCD

γ, g, e, μ, τ, u, d, s, c, b

m_b

QED + QCD

γ, g, e, μ, τ, u, d, s, c

$Q_{QCD} = m_\rho$

QED

γ, e, μ, τ, u, d, s, c

m_μ

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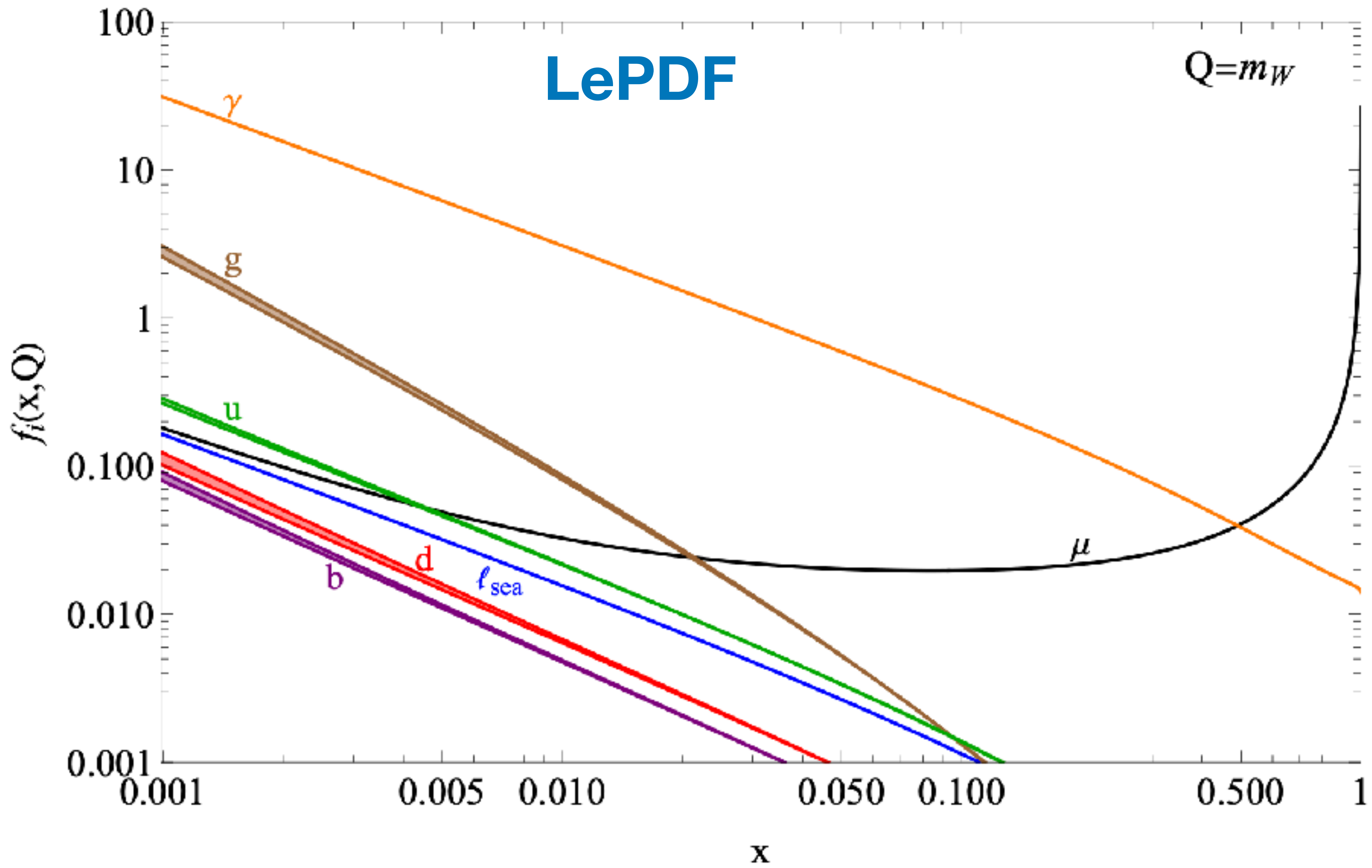
PDFs of a muon at the EW scale:



$$f_{\ell_{\rm sea}} = f_e = f_\tau = f_{\bar e} = f_{\bar\mu} = f_{\bar\tau}$$
$$f_{q^u} = f_u = f_{\bar u} = f_c = f_{\bar c} \,,$$
$$f_{q^d} = f_d = f_{\bar d} = f_s = f_{\bar s} \,,$$
$$f_b = f_{\bar b} \,.$$

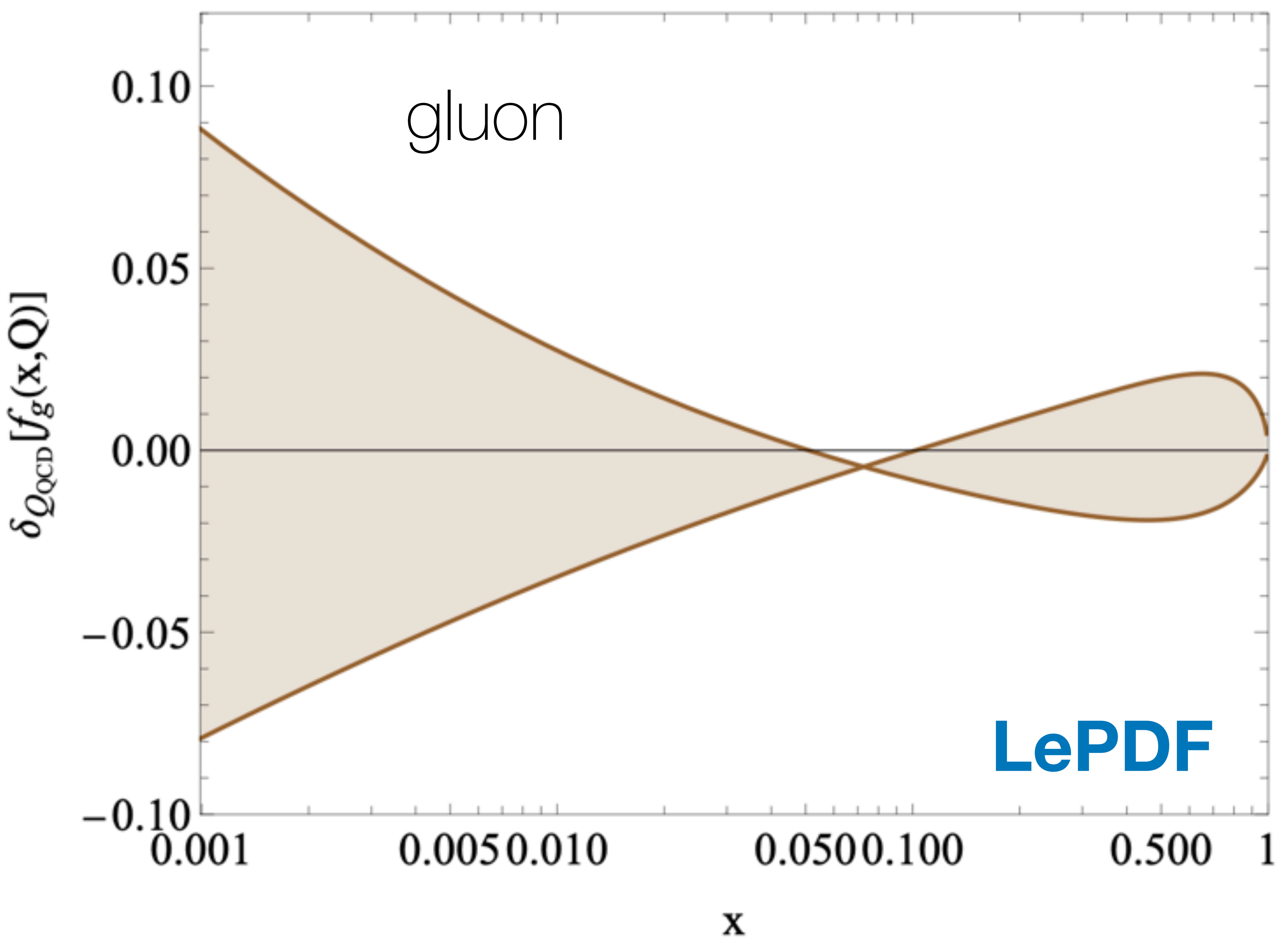
The band width represents uncertainty due to choice of $Q_{\rm QCD}$

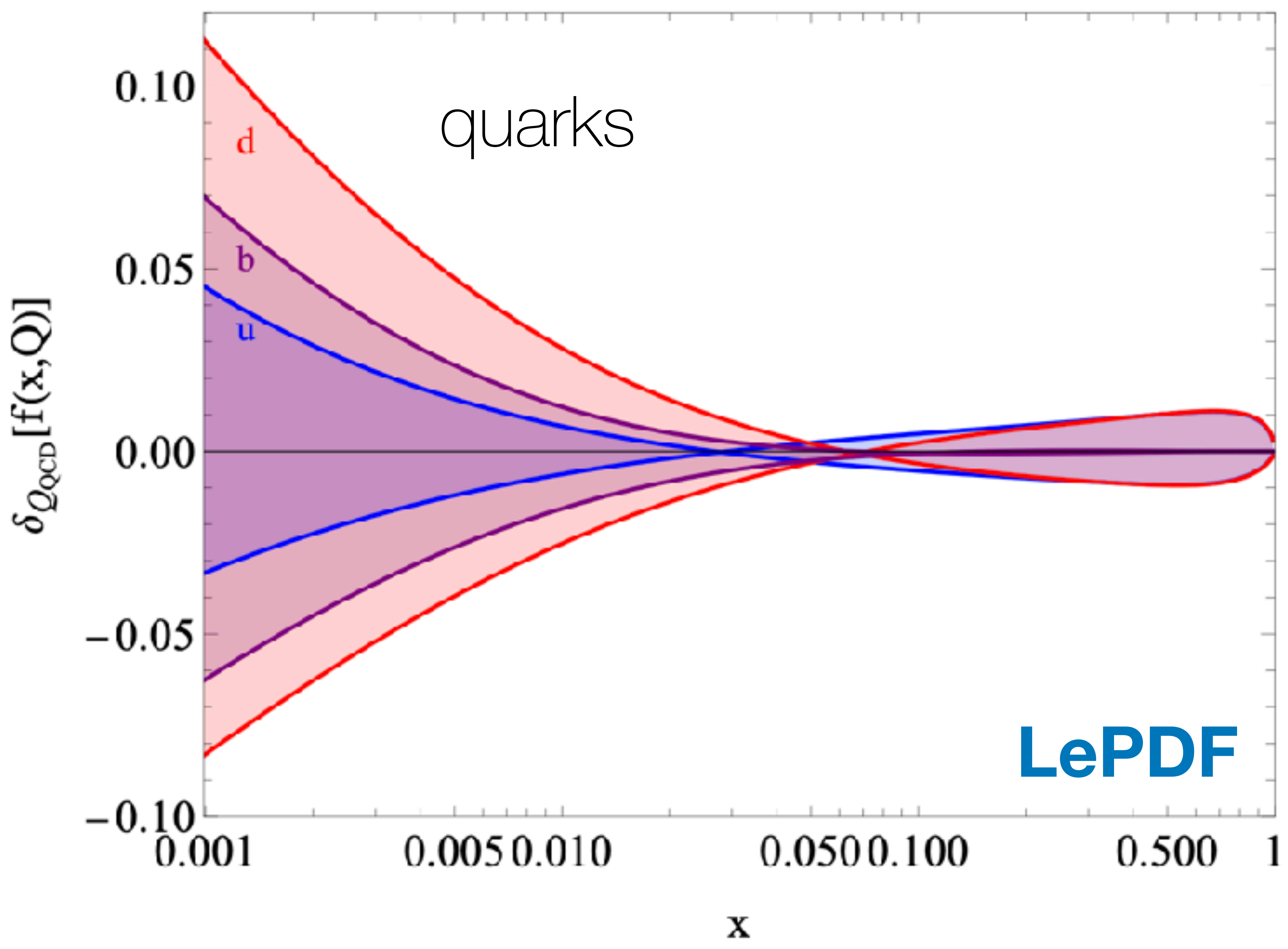
We checked for agreement with Han, Ma, Xie [2103.09844]

Uncertainty due to choice of Q_{QCD}

Changing the scale in the interval $Q_{QCD} =$ [0.5 - 1] GeV

Relative variation in the PDFs, evaluated at the $\boldsymbol{m_W}$ **scale**.





For **leptons** and the **photon**, relative variations are **smaller than 10^{-5}**.

Above the EW scale

All SM interactions and fields must be considered and

several new effects must be taken into account:

- **PDFs become polarised**, since EW interactions are chiral. Bauer, Webber [1808.08831]
- At high energies **EW Sudakov double logarithms** are generated. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047], Bauer, Webber [1703.08562, 1808.08831], Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2103.09844], F. Garosi, D.M., S. Trifinopoulos [2303.16964]
- Neutral bosons interfere with each other: **Z/γ and h/Z_L PDFs mix**. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047] Chen, Han, Tweedie [1611.00788]
- **Mass effects** of partons with EW masses (W, Z, h, t) become relevant and remain so even at multi-TeV scale.
- EW symmetry is broken. Another set of splitting functions, proportional to v^2 instead of p_T^2, arise: **ultra-collinear splitting functions**. Chen, Han, Tweedie [1611.00788]

Implementation

We work in the **mass eigenstate basis**,
same numerical method used below the EW scale.

After identifying PDFs which are identical because of flavour symmetry, we remain with **42 independent PDFs**:

$$f_{e_L} = f_{\tau_L}\,, \quad f_{\bar{\ell}_L} = f_{\bar{e}_L} = f_{\bar{\mu}_L} = f_{\bar{\tau}_L}\,,$$
$$f_{e_R} = f_{\tau_R}\,, \quad f_{\bar{\ell}_R} = f_{\bar{e}_R} = f_{\bar{\mu}_R} = f_{\bar{\tau}_R}\,,$$
$$f_{\nu_e} = f_{\nu_\tau}\,, \quad f_{\bar{\nu}_\ell} = f_{\bar{\nu}_e} = f_{\bar{\nu}_\mu} = f_{\bar{\nu}_\tau}\,,$$
$$f_{u_L} = f_{c_L}\,, \quad f_{\bar{u}_L} = f_{\bar{c}_L}\,, \quad f_{u_R} = f_{c_R}\,, \quad f_{\bar{u}_R} = f_{\bar{c}_R}\,,$$
$$f_{d_L} = f_{s_L}\,, \quad f_{\bar{d}_L} = f_{\bar{s}_L}\,, \quad f_{d_R} = f_{s_R}\,, \quad f_{\bar{d}_R} = f_{\bar{s}_R}\,.$$

Leptons	μ_L	μ_R	e_L	e_R	ν_μ	ν_e	$\bar{\ell}_L$	$\bar{\ell}_R$	$\bar{\nu}_\ell$
Quarks	u_L	d_L	u_R	d_R	t_L	t_R	b_L	b_R	+ h.c.
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z\gamma_\pm$	$W^\pm_\pm$	$G_\pm$				
Scalars	h	Z_L	hZ_L	$W^\pm_L$					

Starting from $Q_{\text{EW}} = m_W$, heavy states are added at the corresponding mass threshold.

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DGLAP equations:

$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P^v_B\, f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}^C_{BA} \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}^C_{BA} \otimes f_A$$

$$\widetilde{P}^C_{BA}(z, p_T^2) = \left(\frac{p_T^2}{\widetilde{p}_T^2}\right)^2 P^C_{BA}(z)$$

$$\widetilde{p}_T^2 \equiv \bar z(m_B^2 - p_B^2) = p_T^2 + z m_C^2 + \bar z m_B^2 - z\bar z m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

Ultra-collinear splittings

Polarisation

Since EW interactions are chiral,
PDFs become polarised. Bauer, Webber [1808.08831]

E.g. in case of $\boldsymbol{W^-}$ PDF, coupled to $\boldsymbol{\mu_L}$,
the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's,
since $P_{V+f_L}(z) = (1\text{-}z)/z$ while $P_{V-f_L}(z) = 1/z$.

Splitting functions depend on the helicity of the states, e.g. :

$$P_{V_+ f_L}(z) = P_{V_- f_R}(z) = \frac{\bar{z}^2}{z} \,,$$
$$P_{V_- f_L}(z) = P_{V_+ f_R}(z) = \frac{1}{z} \,,$$
$$P_{f_L V_+}(z) = P_{f_R V_-}(z) = \bar{z}^2 \,,$$
$$P_{f_L V_-}(z) = P_{f_R V_+}(z) = z^2 \,,$$

$\bar{z} = 1 - z$

Polarisation

Since EW interactions are chiral,
PDFs become polarised. Bauer, Webber [1808.08831]

E.g. in case of $\boldsymbol{W^-}$ PDF, coupled to $\boldsymbol{\mu_L}$,
the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's,
since $P_{V+f_L}(z) = (1-z)/z$ while $P_{V-f_L}(z) = 1/z$.

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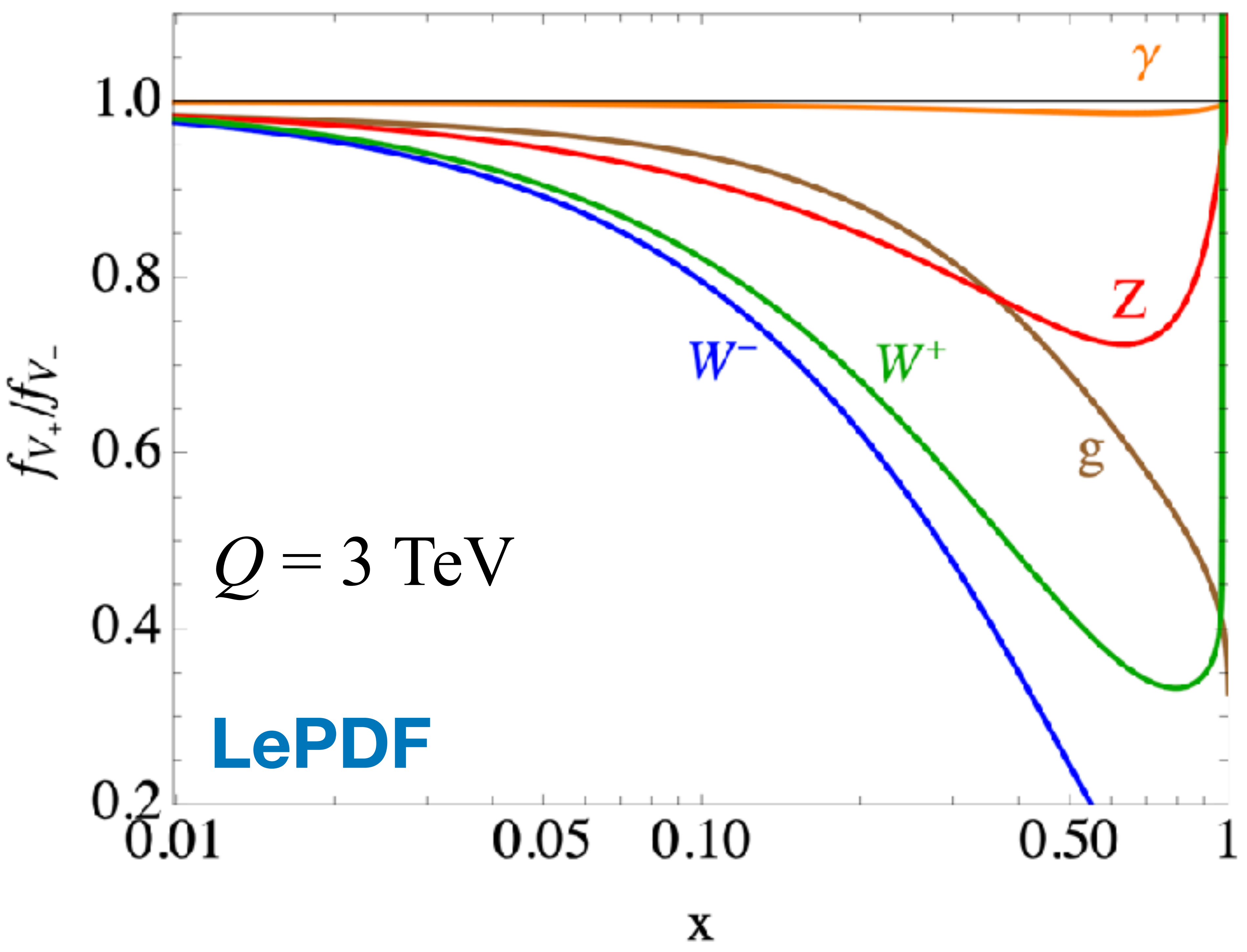
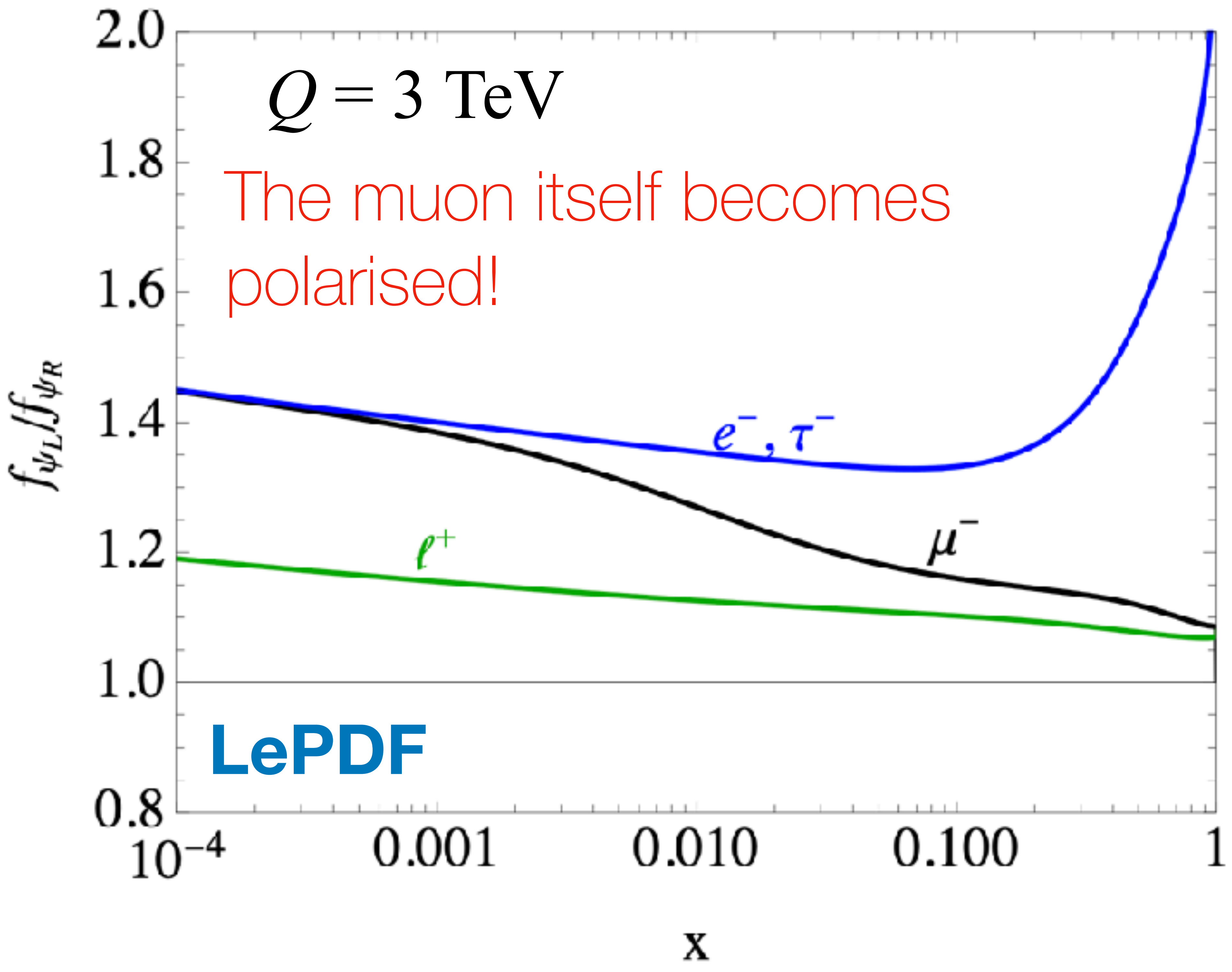
Vectors polarisation: V_+ / V_-



Fermions polarisation: ψ_L / ψ_R



EW Sudakov double logs from ISR

The Bloch-Nordsieck theorem is violated for non-abelian gauge theories
→ IR divergencies are not cancelled in inclusive processes, since the initial state is EW non-singlet
→ We are often interested in exclusive processes, since we measure the SU(2) charge (W vs Z, t vs b, etc…)

The **EW Sudakov double logs** arises as a non-cancellation of the IR soft divergences $(z \to 1)$ between real emission and virtual corrections.

P. Ciafaloni, Comelli [hep-ph/9809321], Fadin et al. [hep-ph/9910338], M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
see also Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077], Manohar [1409.1918], Pagani, Zaro [2110.03714], …
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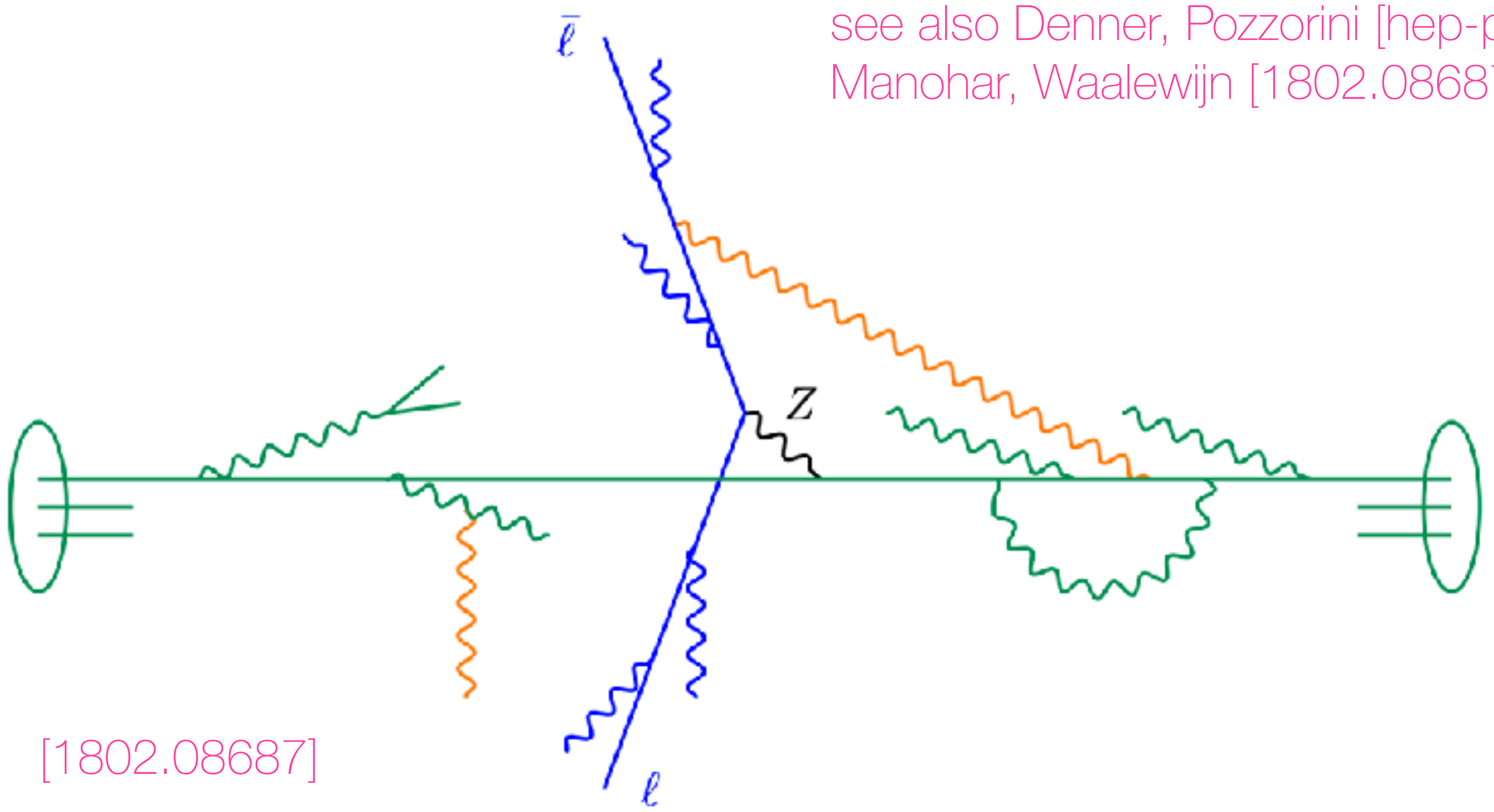

[1802.08687]

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At the leading-log level we can neglect soft radiation

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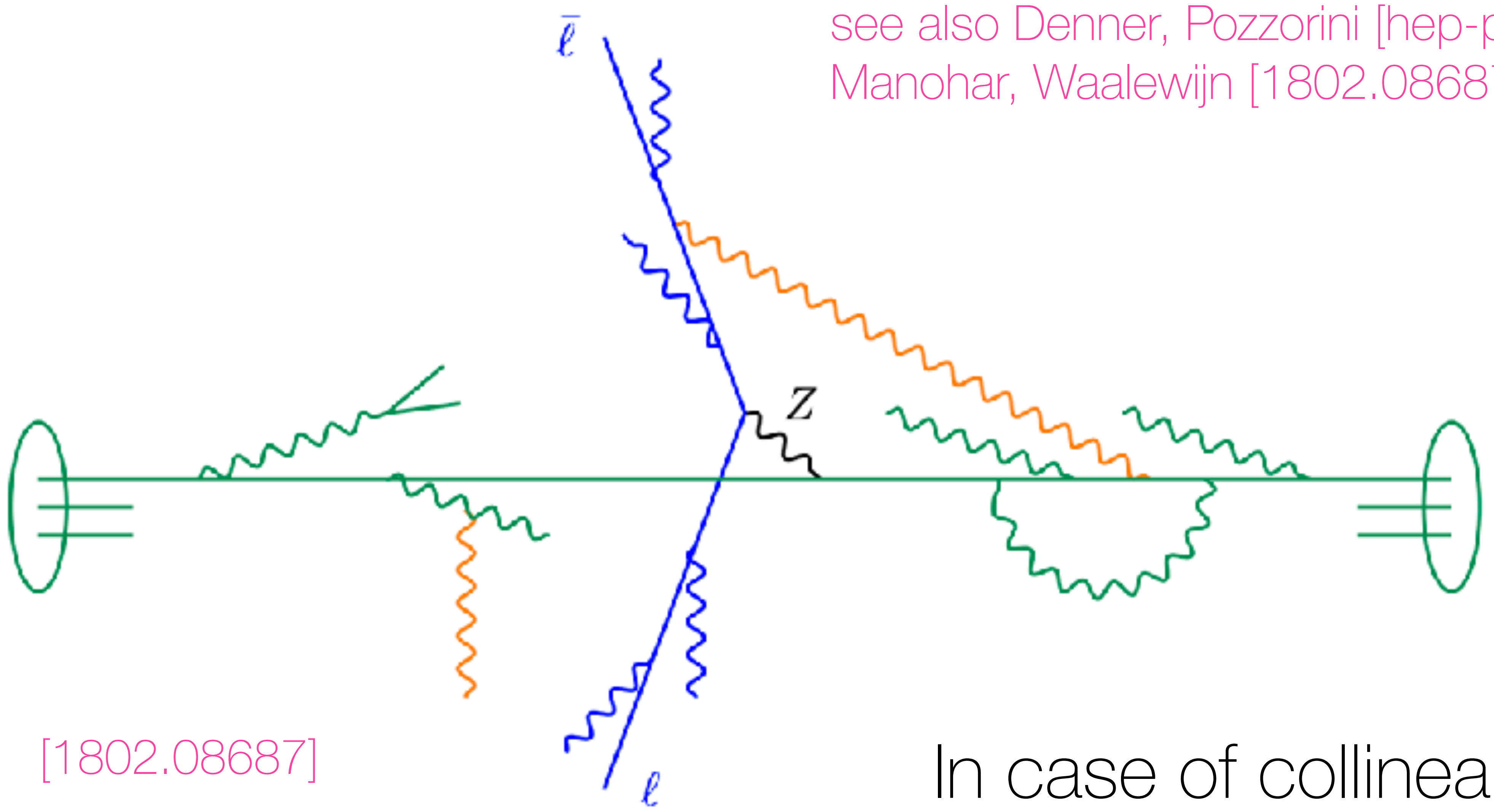

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Manohar, Waalewijn [1802.08687]

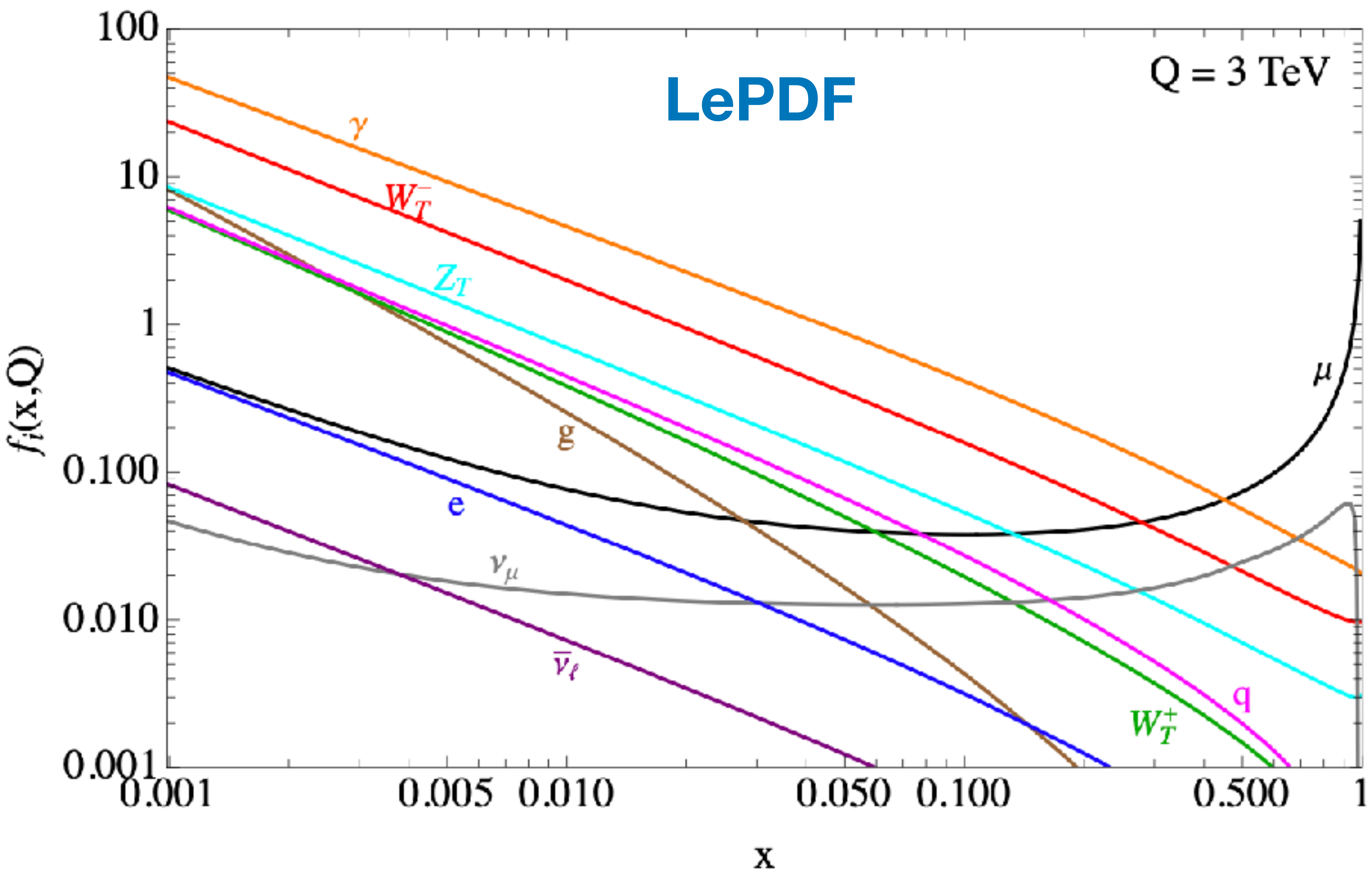
In case of collinear W emission they can be implemented (and resummed) at the **Leading Log** level by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]

LePDF

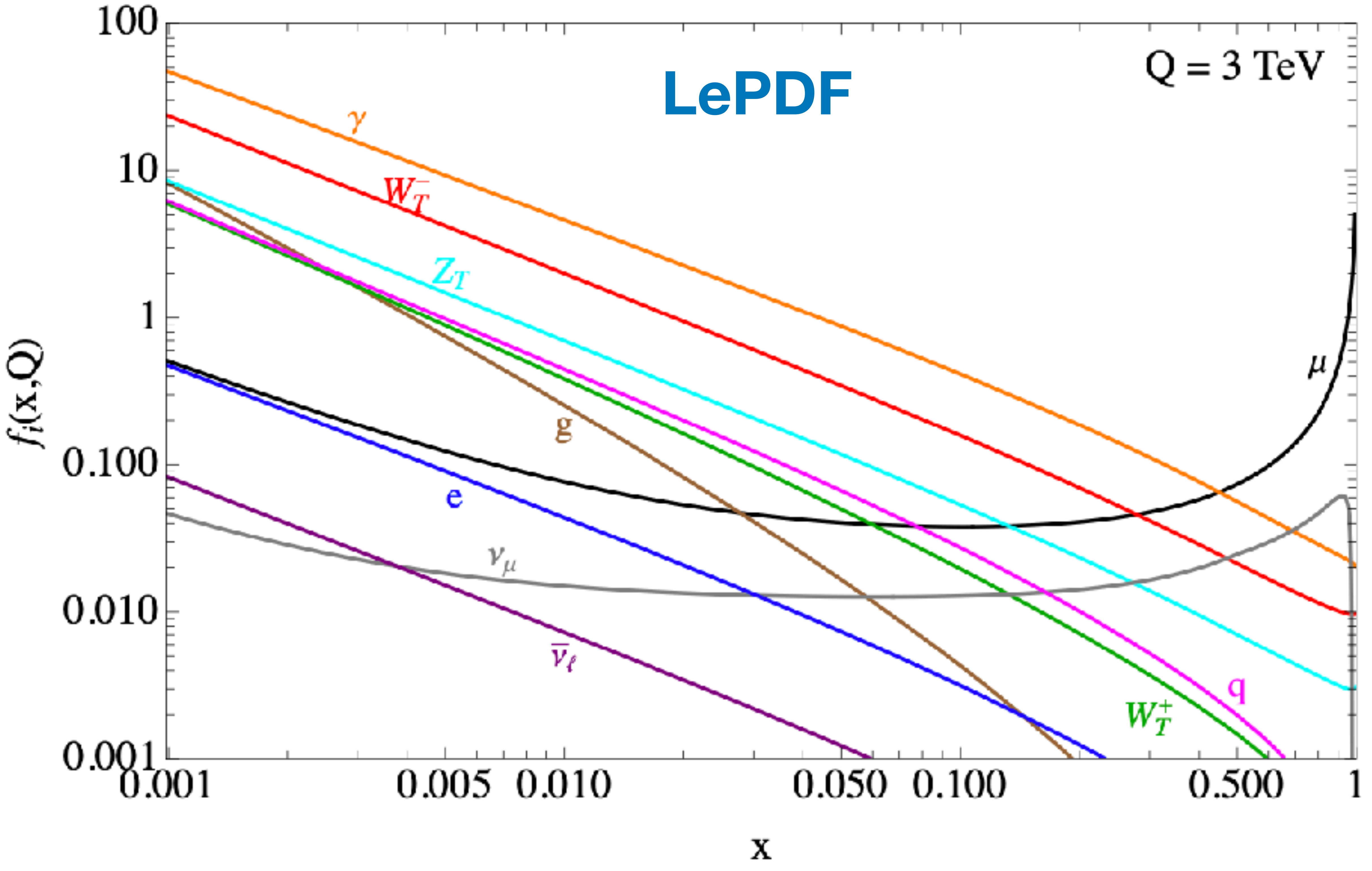
Results

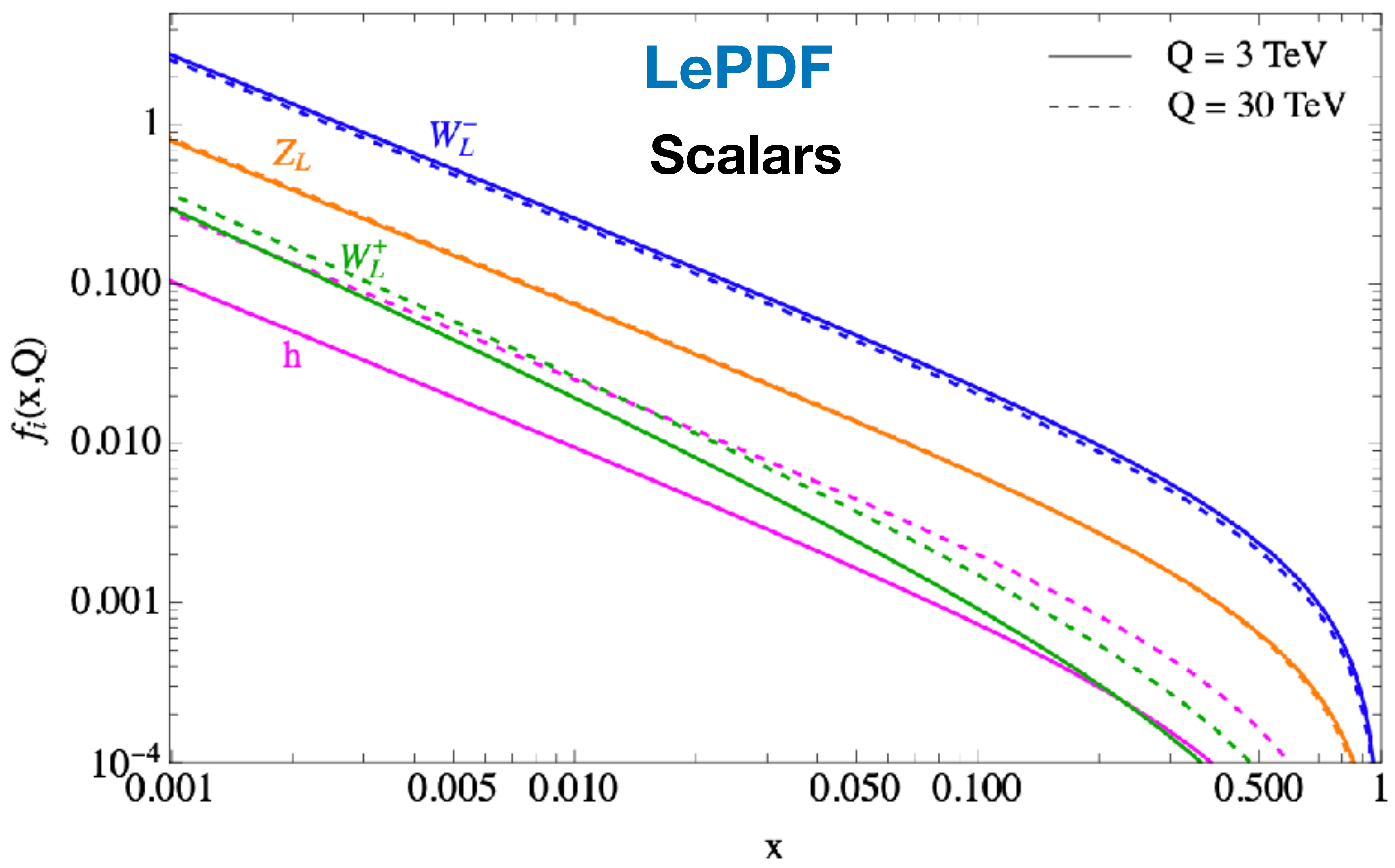
PDFs of a muon



At high scales and **well above threshold** (small x)
a muon has large EW boson PDFs,
but also **gluons** and **quarks**.

PDFs of a muon





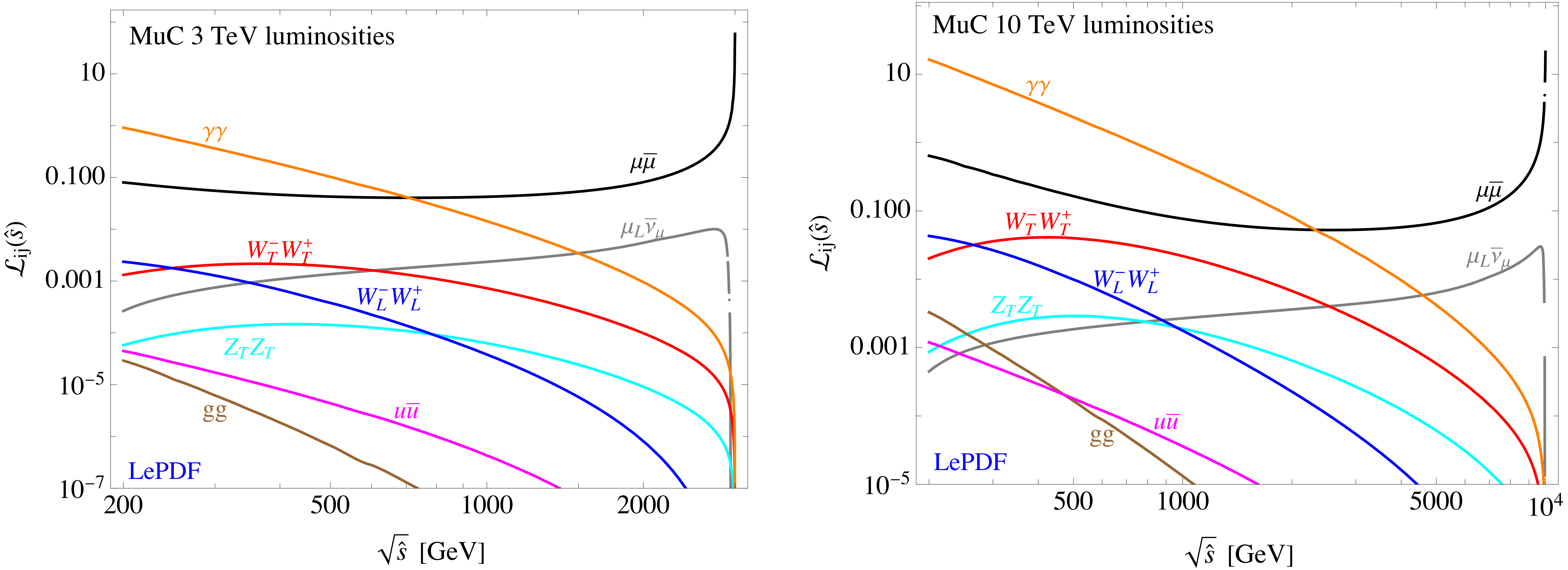
At high scales and **well above threshold** (small x) **a muon has large EW boson PDFs**, but also **gluons** and **quarks**.

Longitudinal gauge bosons PDFs are dominated by **ultra-collinear contributions** from the muon (and muon neutrino, for the W^+), which **do not scale**.

The **Higgs** instead has **no coupling to massless fermions**, so its PDF has no large u.c. contributions.

Some examples of **parton luminosities** for muon colliders.

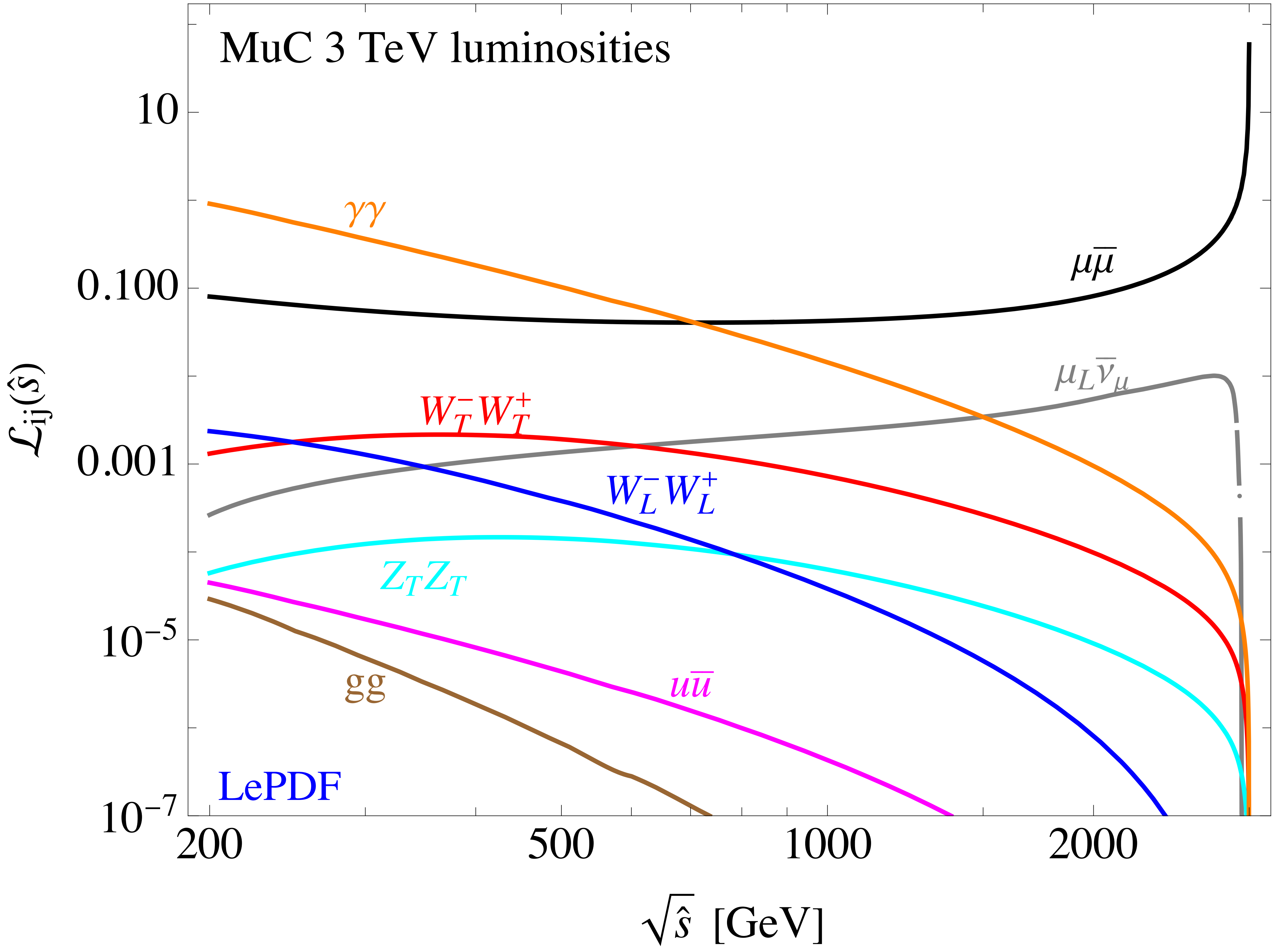
$$\mathcal{L}_{ij}(\hat{s}) = \int_{\hat{s}/s_0}^{1} dx \frac{1}{x} f_i^{(\mu)}\left(x, \frac{\sqrt{\hat{s}}}{2}\right) f_j^{(\bar{\mu})}\left(\frac{\hat{s}}{x s_0}, \frac{\sqrt{\hat{s}}}{2}\right)$$



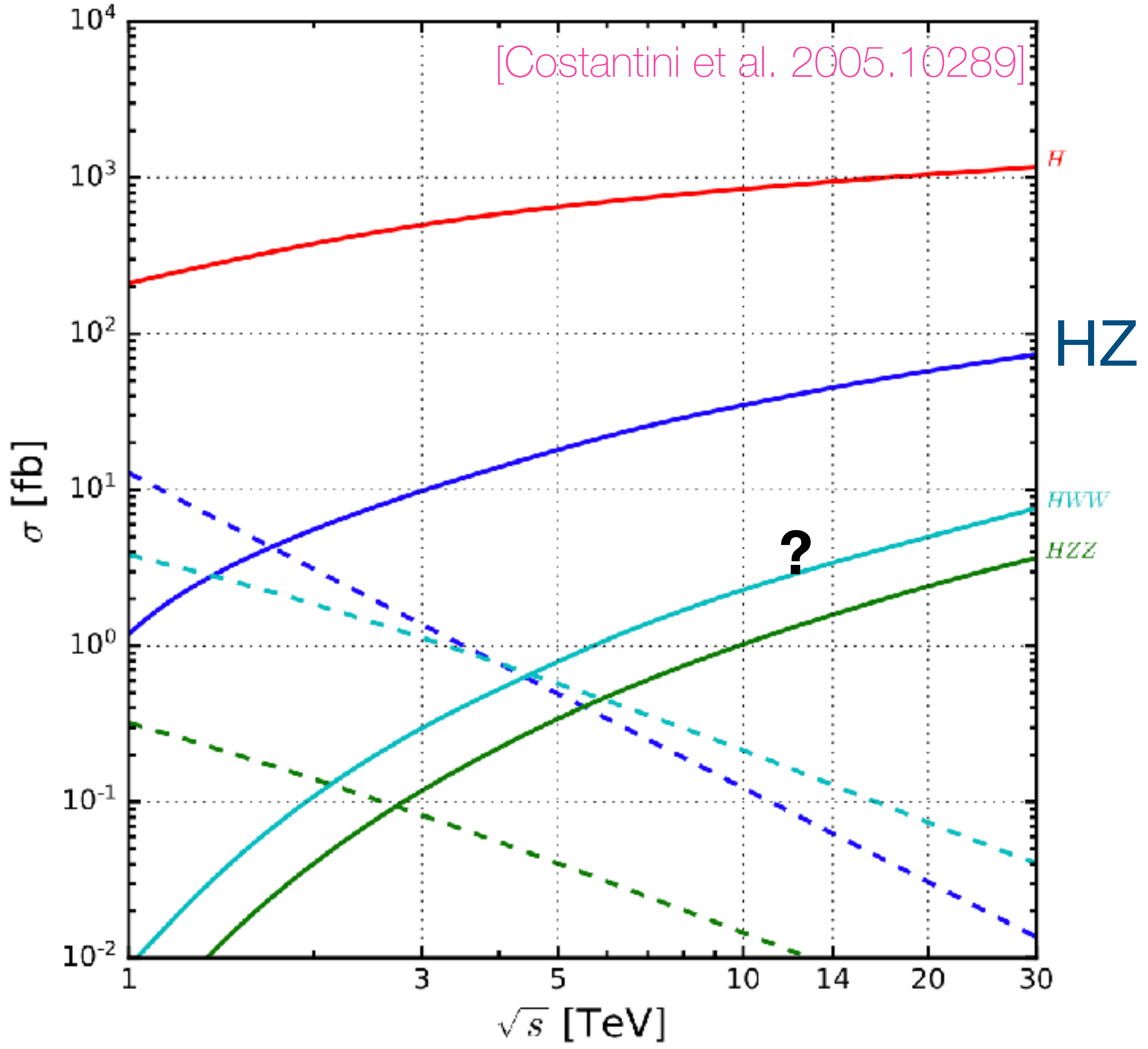
Some comments:

- **large $\mu\bar{\mu}$ and $\mu_L\bar{\nu}_\mu$ lumi** at **small $\sqrt{\hat{s}}$**: possible sizeable **impact on VBF studies** from annihilation channel?
- The **very large $\gamma\gamma$ lumi** could dominate over Z contributions.
- **gluon and quark luminosities are very small**: small impact from QCD-induced backgrounds.

large $\mu\bar{\mu}$ and $\mu_L\bar{\nu}_\mu$ lumi at **small $\sqrt{\hat{s}}$**: possible sizeable **impact on VBF studies** from annihilation channel?



$$\sigma = \sum_{ij} \int d\hat{s}/s_0 \, \mathcal{L}_{ij}(\hat{s}) \, \hat{\sigma}(\hat{s})$$



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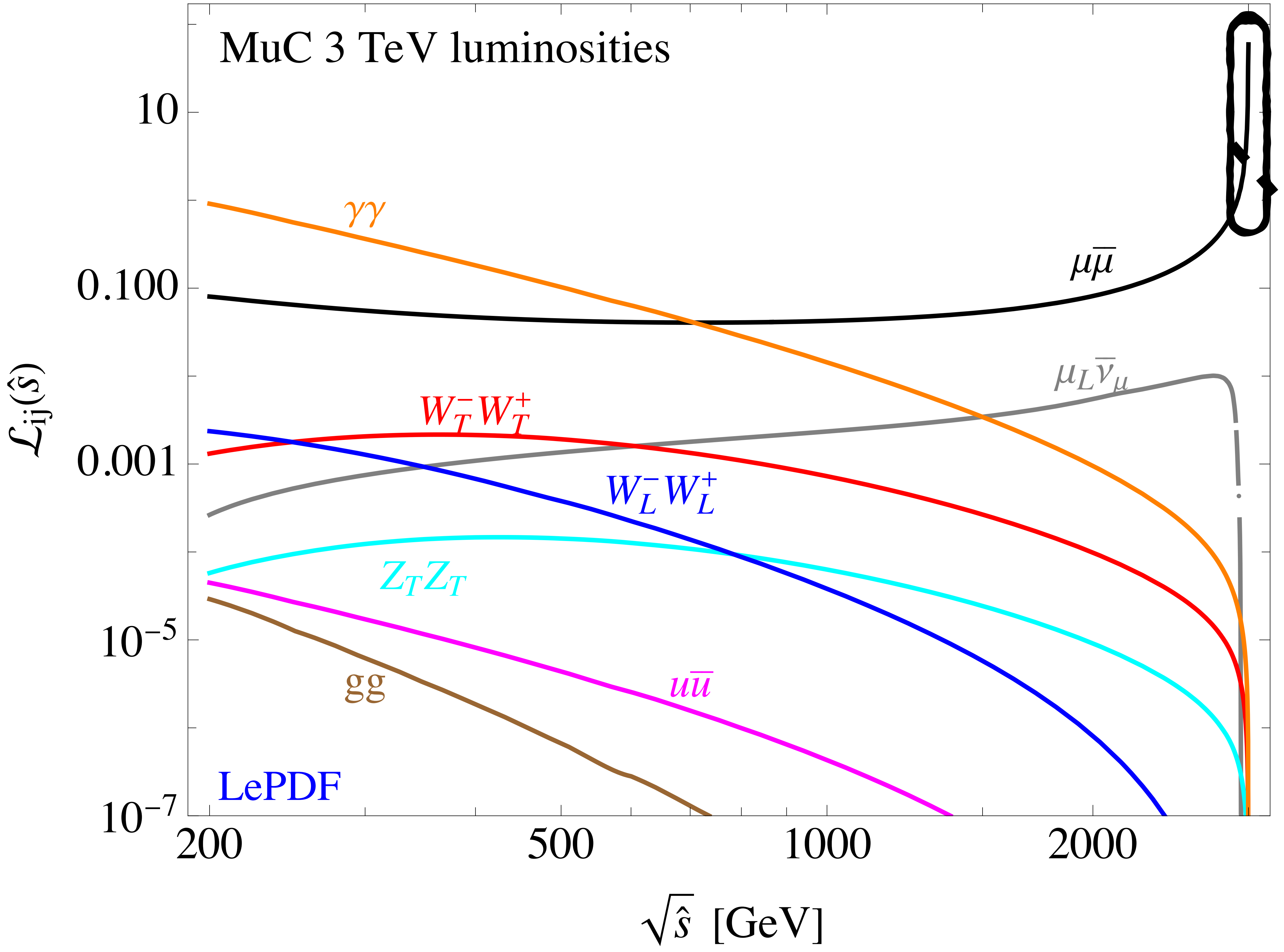

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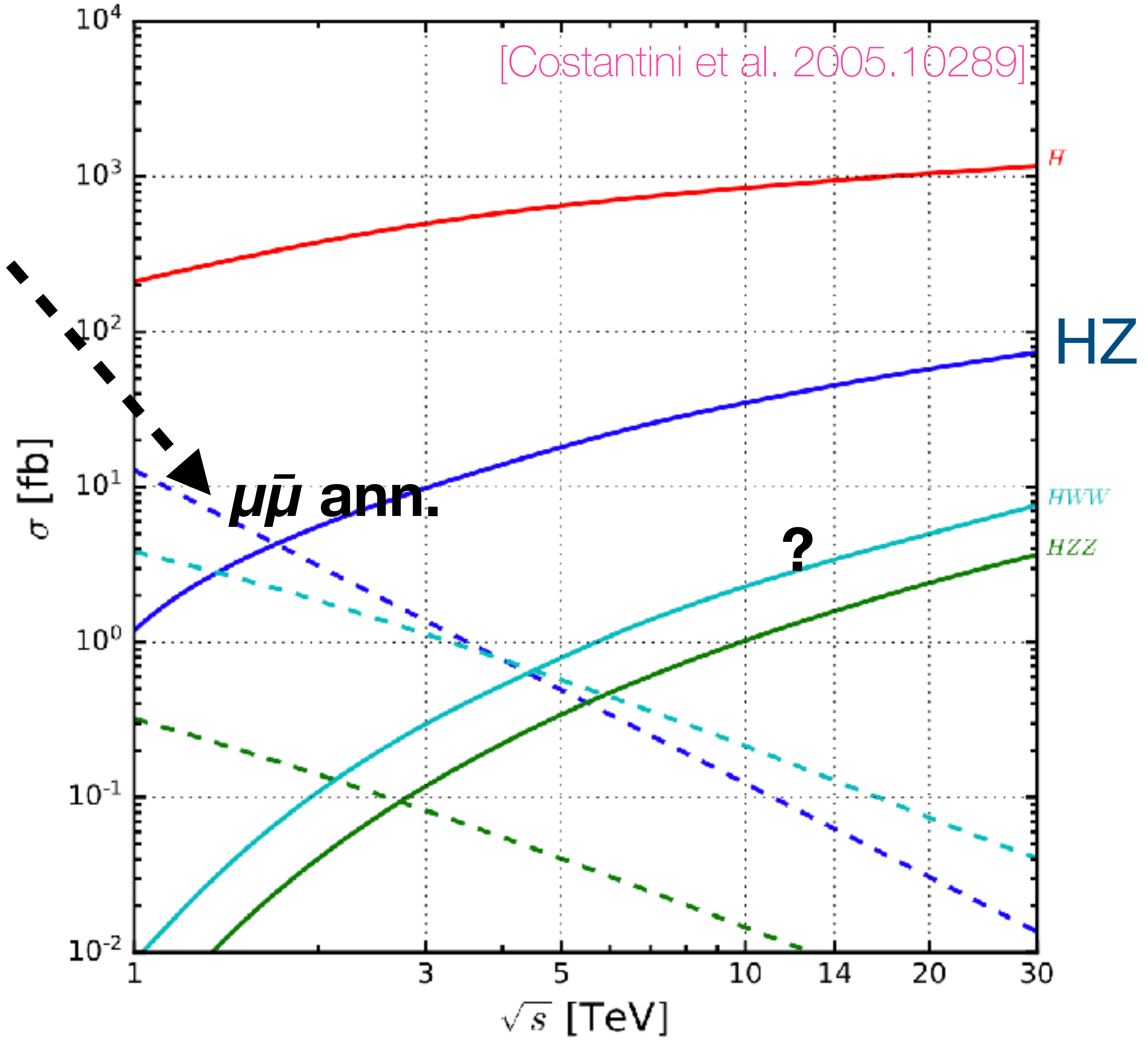

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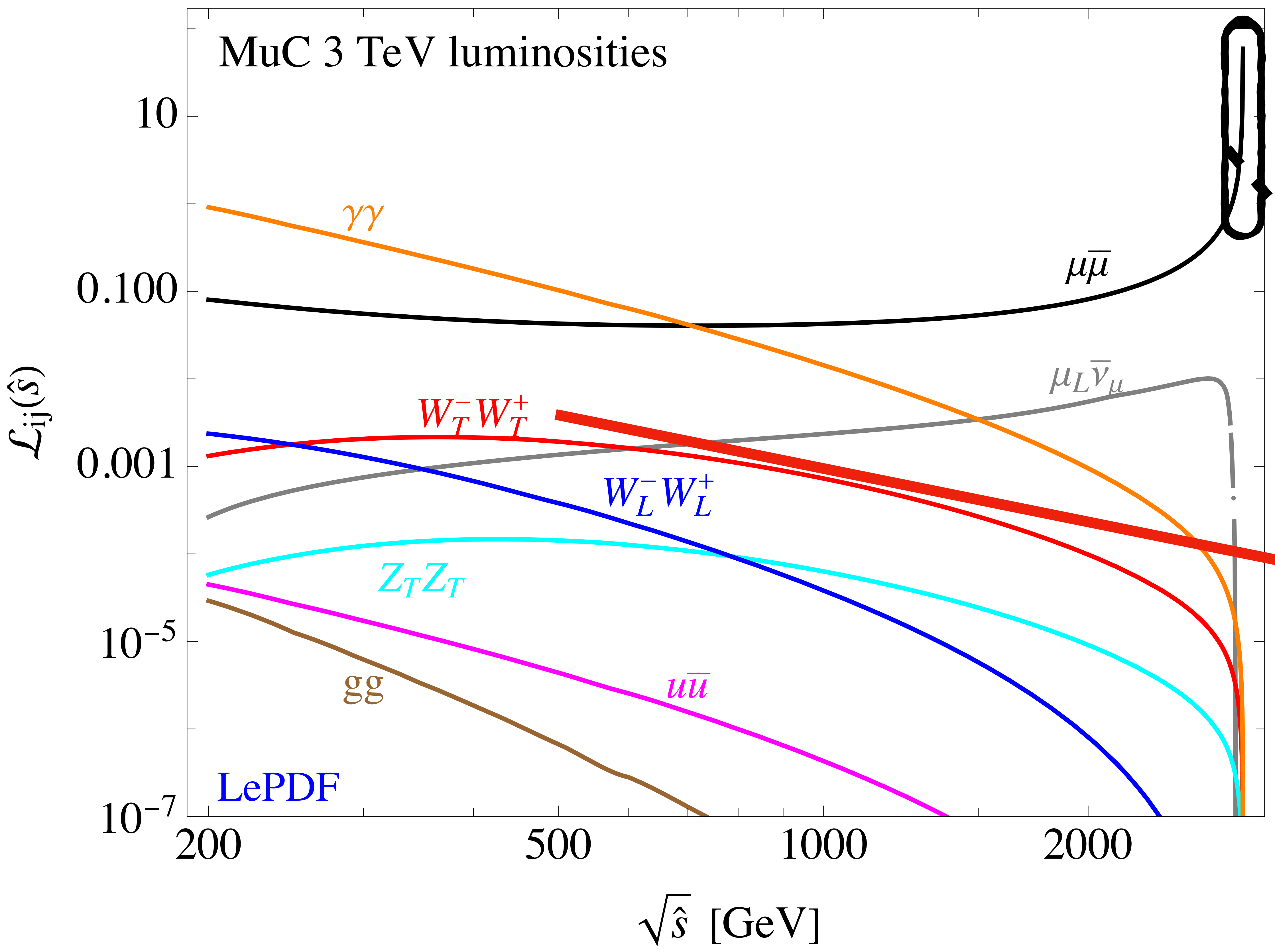

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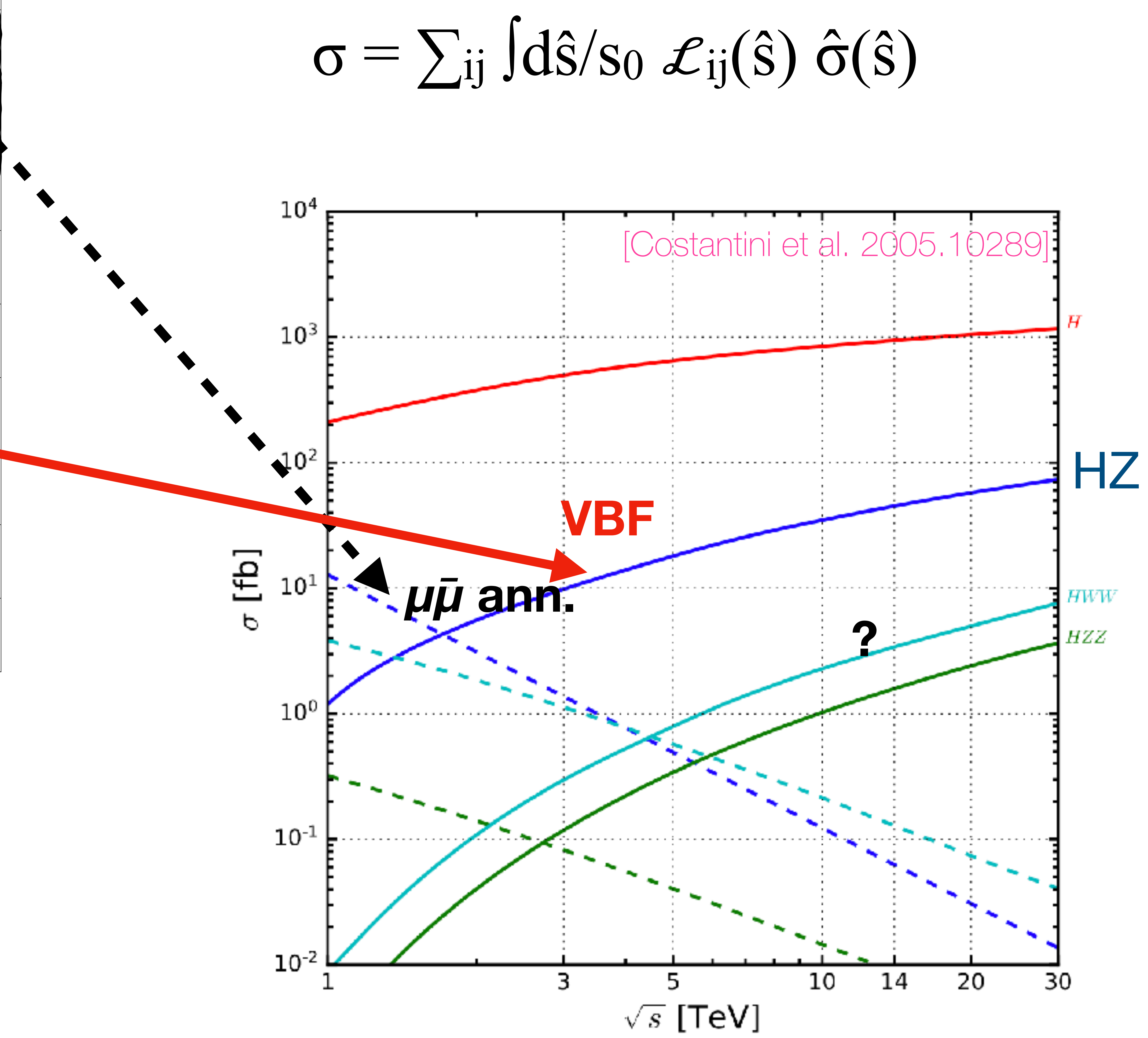

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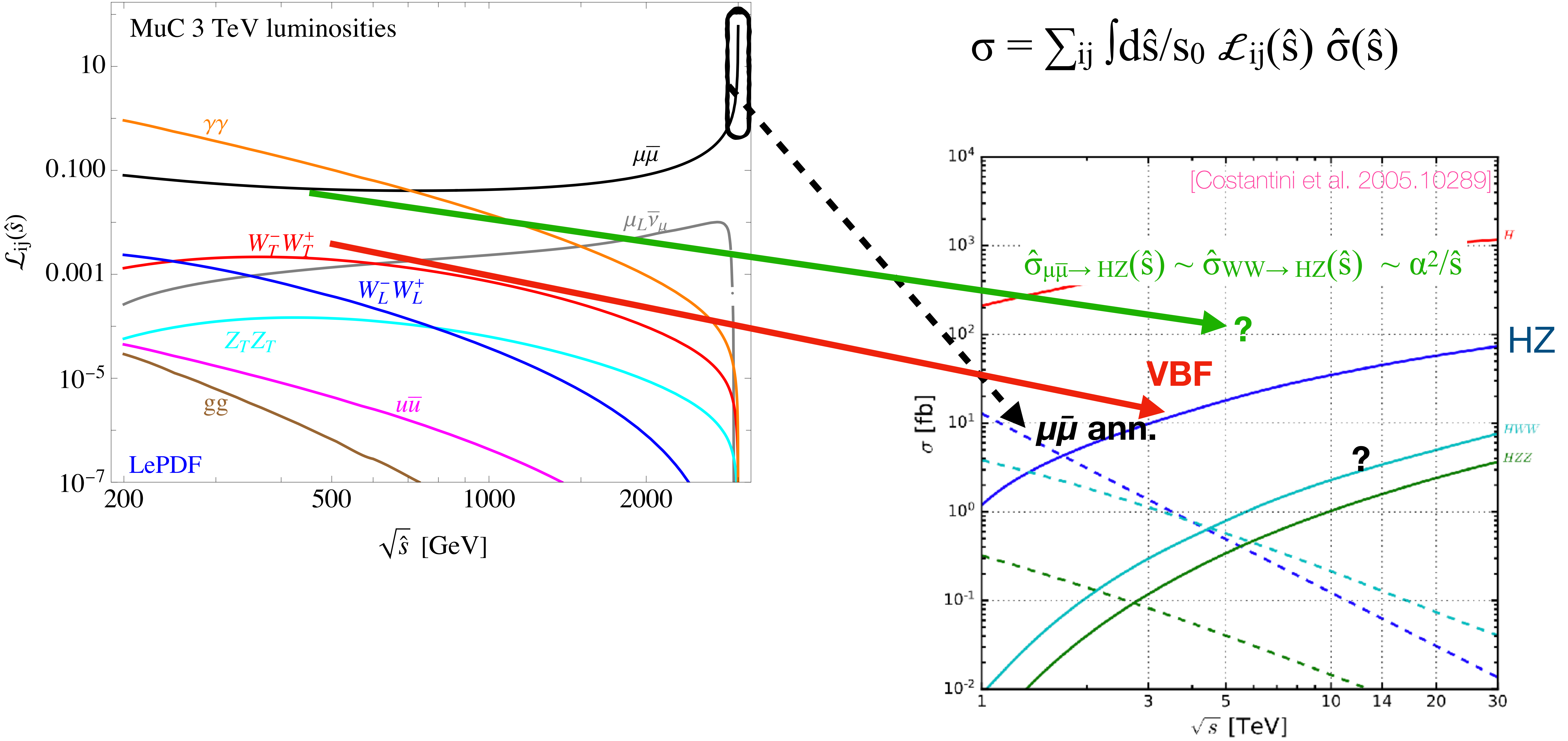

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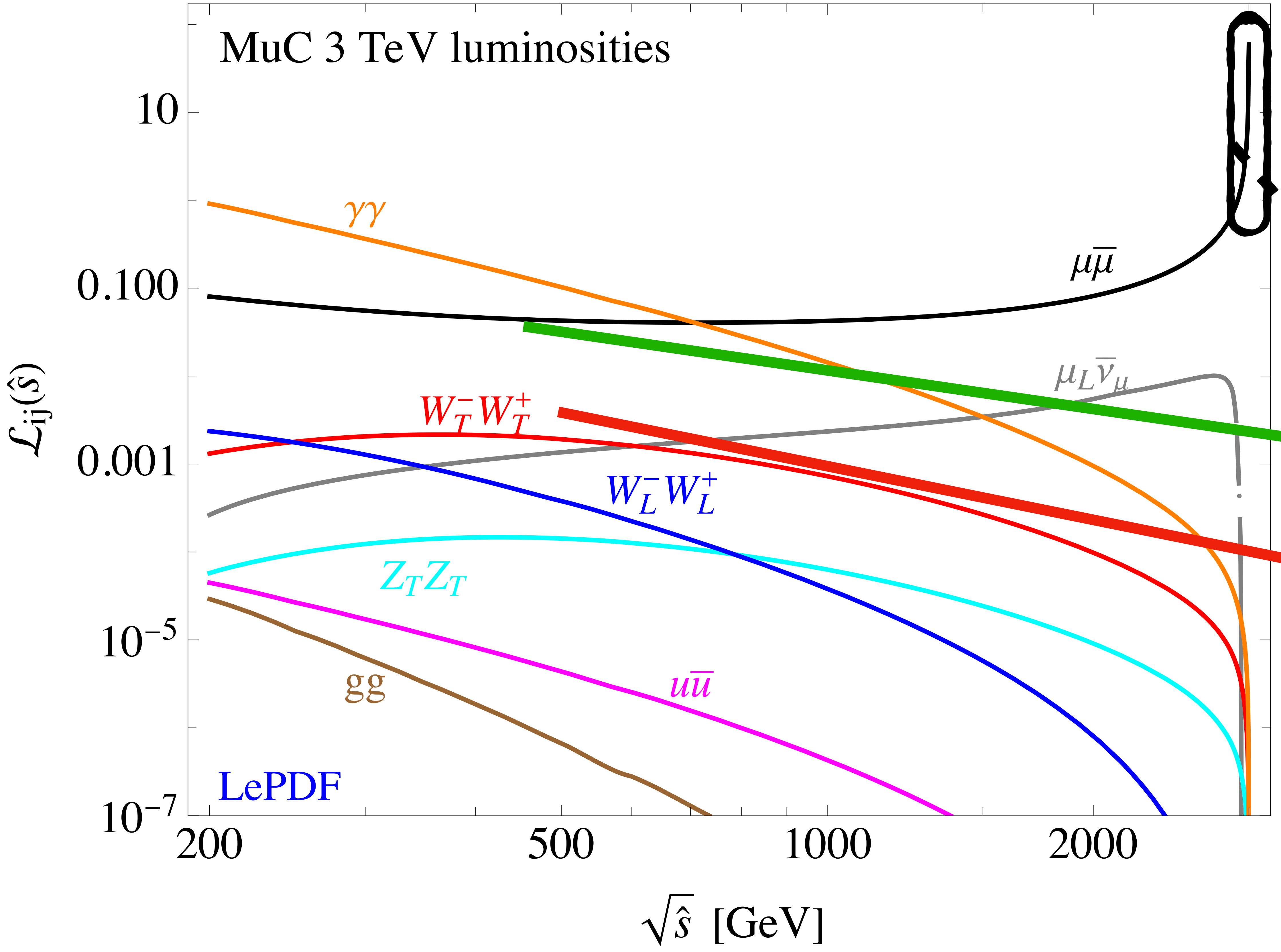

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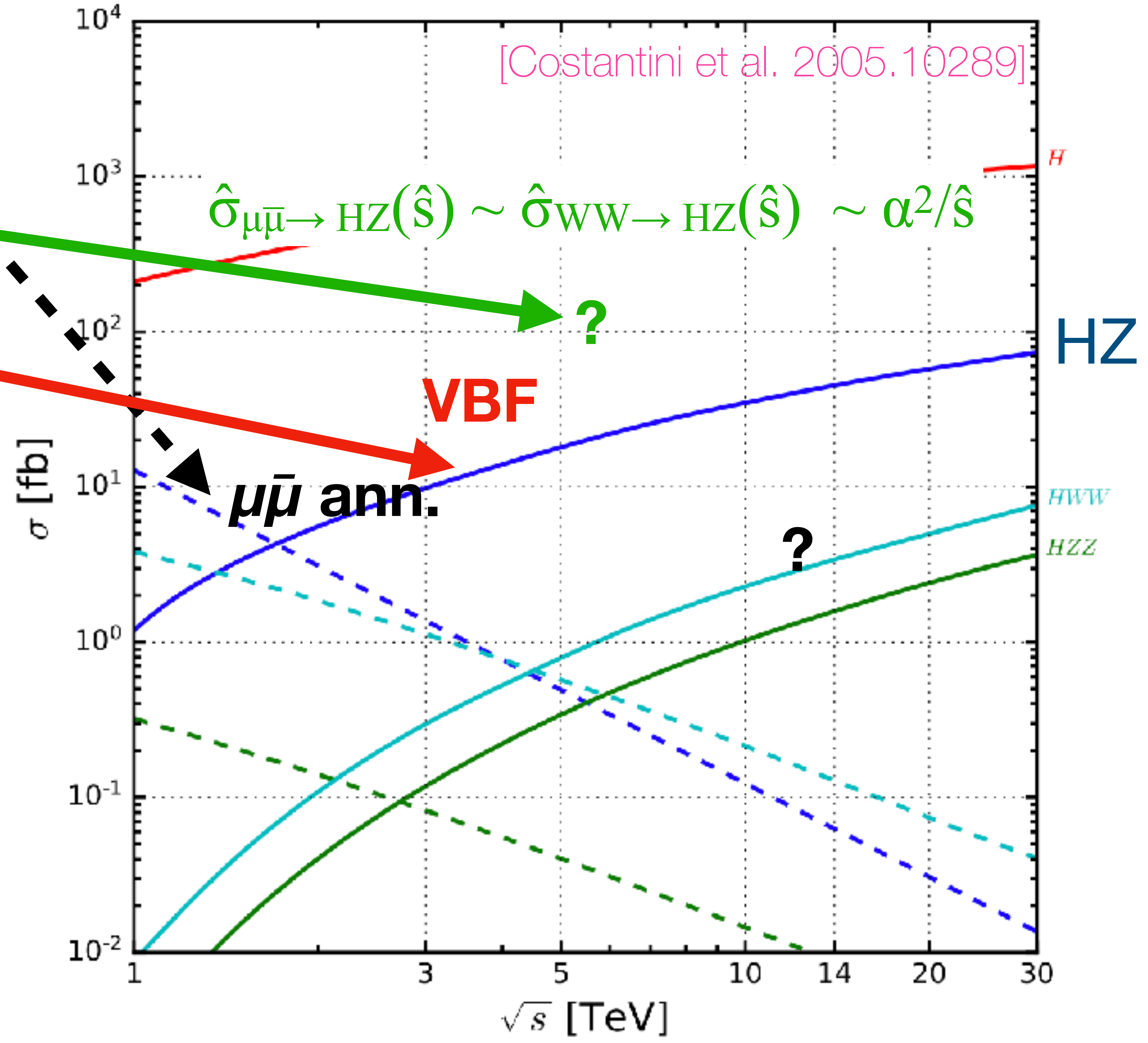

The **effect of realistic rapidity cuts** on the final state should be considered, the $\mu\bar{\mu}$ channel is clearly unbalanced in rapidity (PDF larger at x~1 and x≪1): very forward final state.

LePDF vs. EVA

EVA$_{\mathbf{LO}}$: $f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x) \left(\log \frac{Q^2 + (1-x)m_W^2}{m_\mu^2 + (1-x)m_W^2} - \frac{Q^2}{Q^2 + (1-x)m_W^2} \right)$ $\qquad f^{(\alpha)}_{W^-_L}(x,Q^2) = \frac{\alpha_2}{4\pi} \frac{1-x}{x} \frac{Q^2}{Q^2 + (1-x)m_W^2}$

We can expect large deviations from EVA, since $\alpha_2 \left(\log \frac{Q^2}{m_W^2} \right)^2 \sim 1$ for $Q \sim 1.5$ TeV.

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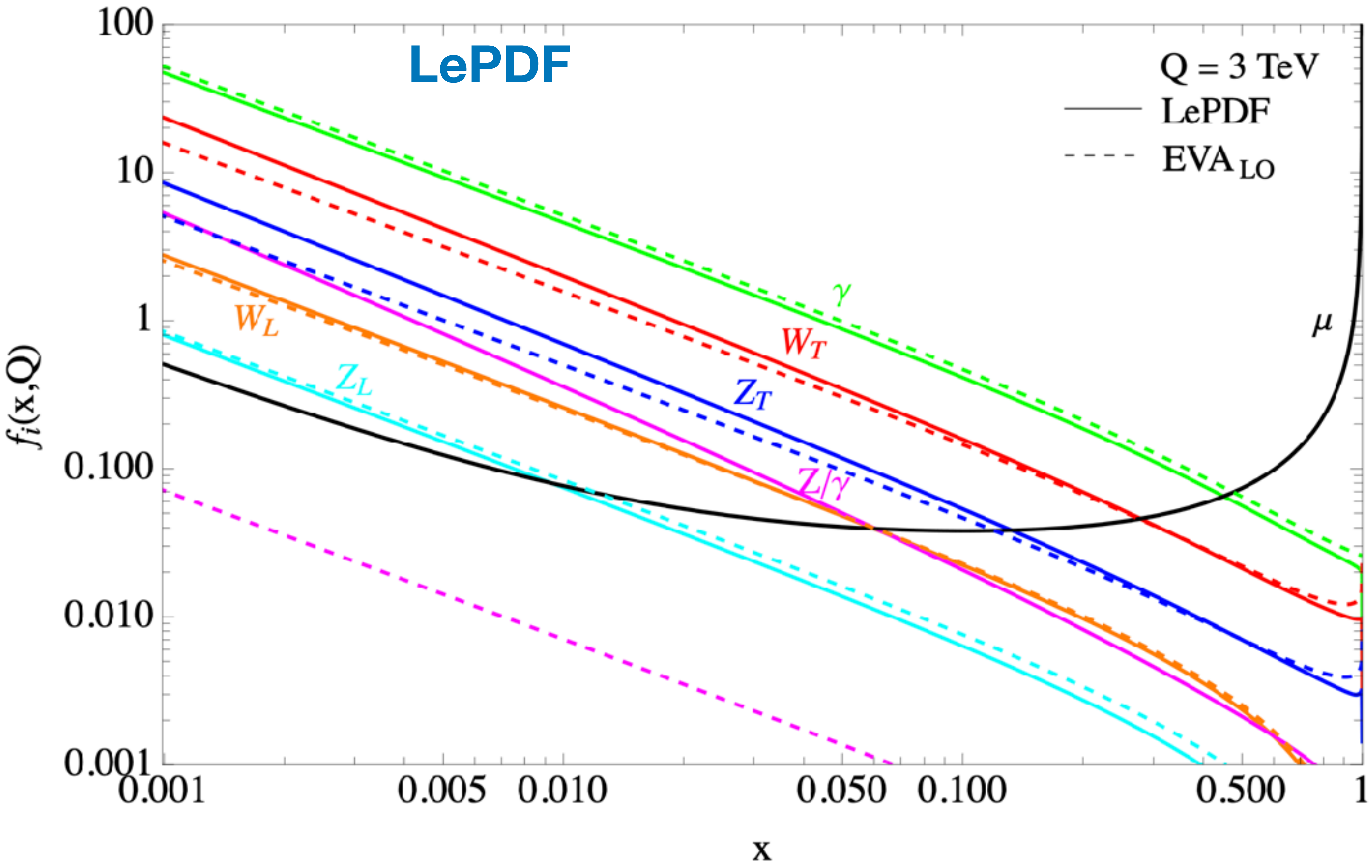


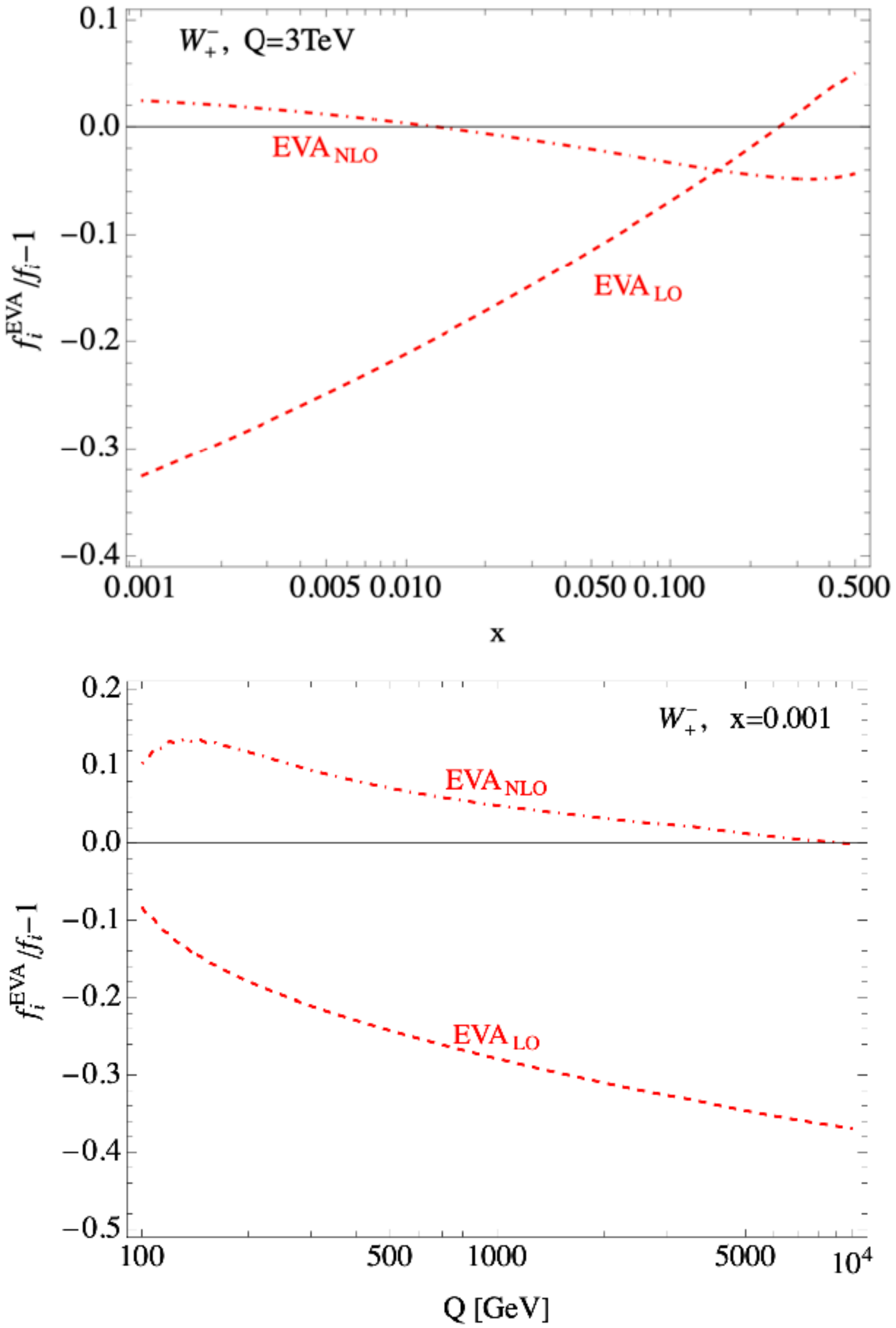
- The **EVA Z/γ PDF is off by ~10²**, due to the fact that in EVA the muon is taken unpolarised and

$$Q_{\mu_L}^Z + Q_{\mu_R}^Z = -\frac{1}{2} + 2s_W^2 \ll 1$$

Instead, the muon gains a O(1) polarisation, so the Z/γ PDF is much larger.

- We can also see a **sizeable deviation** (in this log-log plot) for the **W_T** and **Z_T** PDF.

LePDF vs. EVA



The **deviation becomes larger at small x and at large scales** (Sudakov double logs are absent in EVA).

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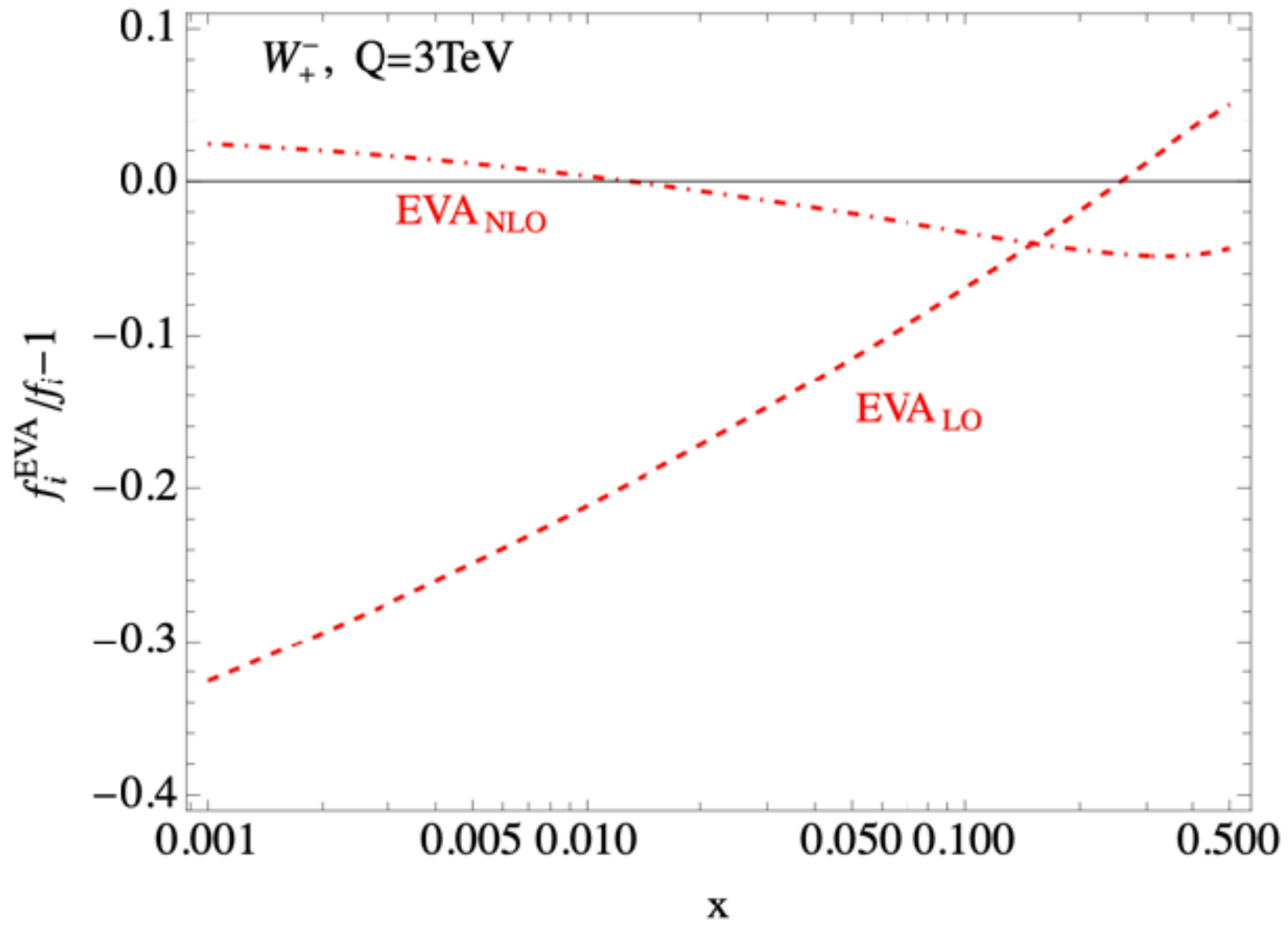
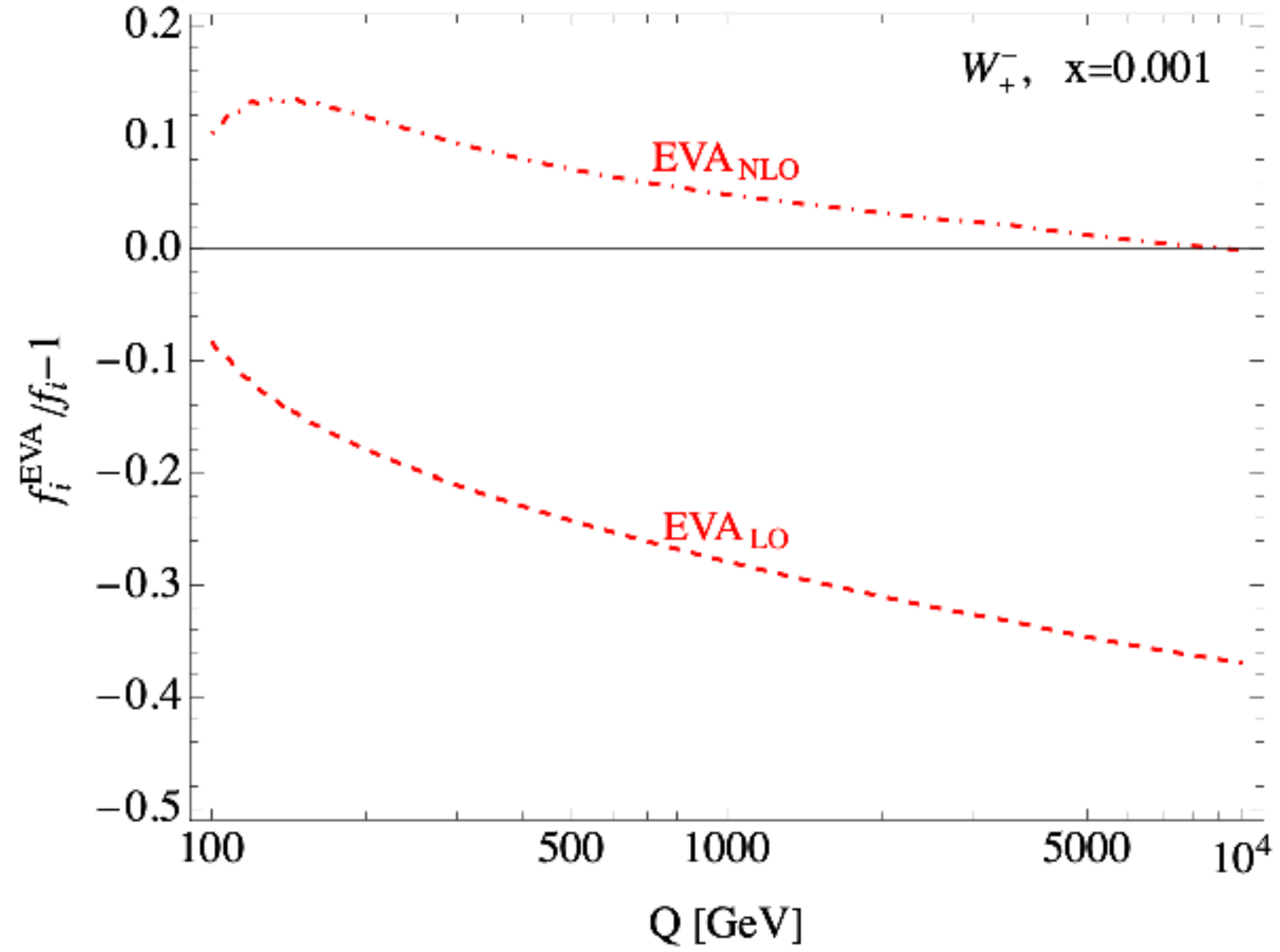
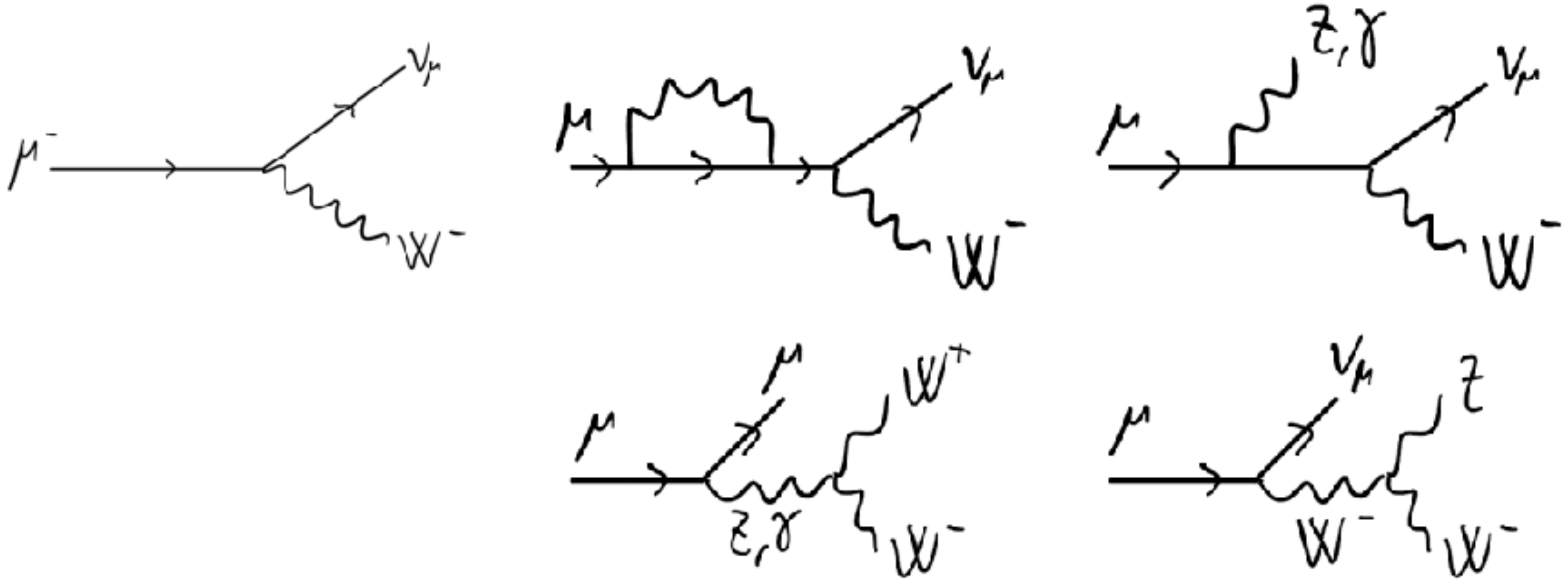




The **deviation becomes larger at small x and at large scales** (Sudakov double logs are absent in EVA).

We improve EVA by computing iteratively the W^-_+ **PDF at O(α^2)**. *



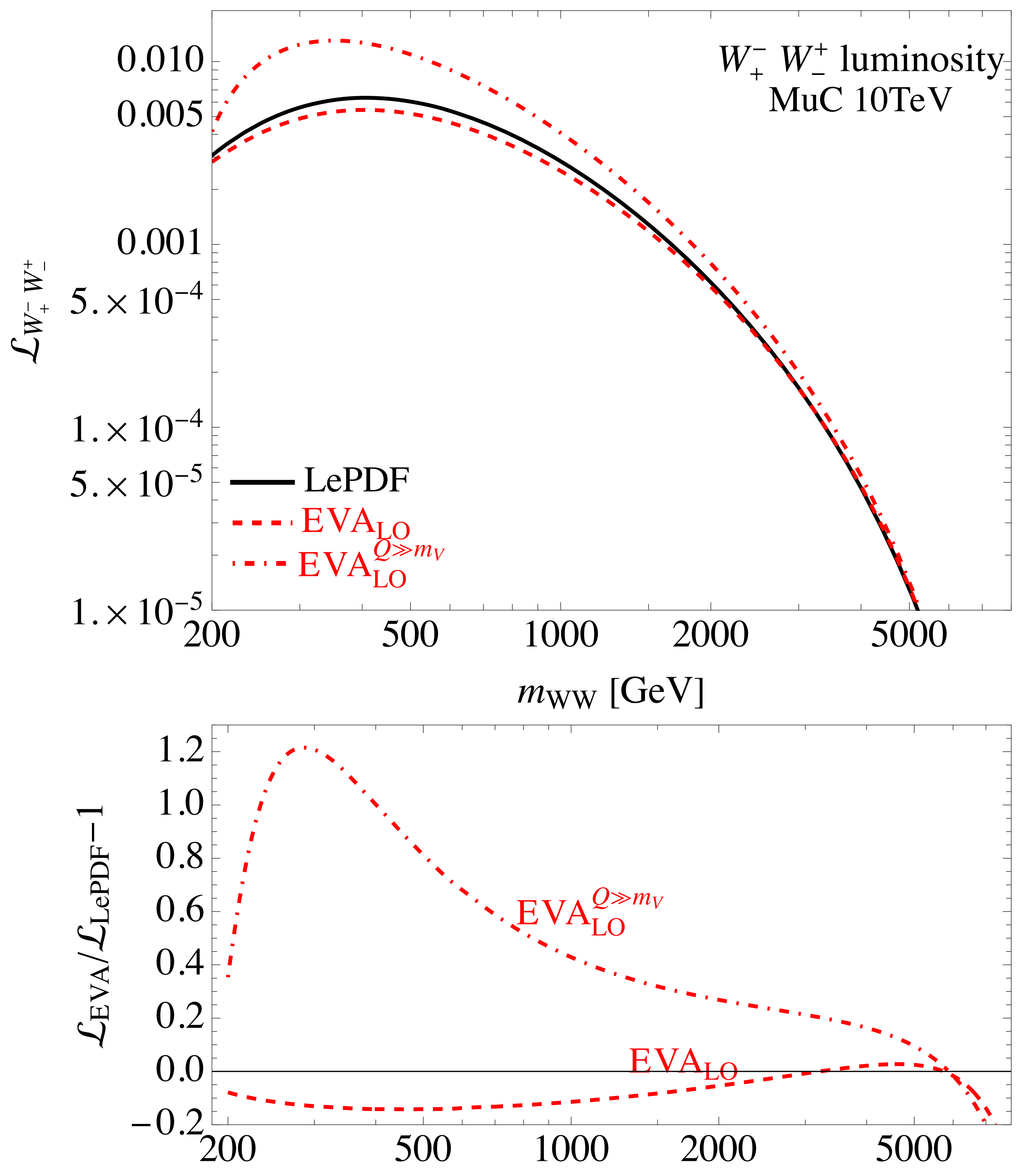
$$f^{(\alpha)}_{\mu_L}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(\frac{1}{2} P^v_{\mu_L}(t')\delta(1-x) + \frac{\alpha_\gamma}{4\pi} P^V_{ff}(x) + \frac{\alpha_2}{4\pi c_W^2} (Q^Z_{\mu_L})^2 P^V_{ff}(x) \right) ,$$

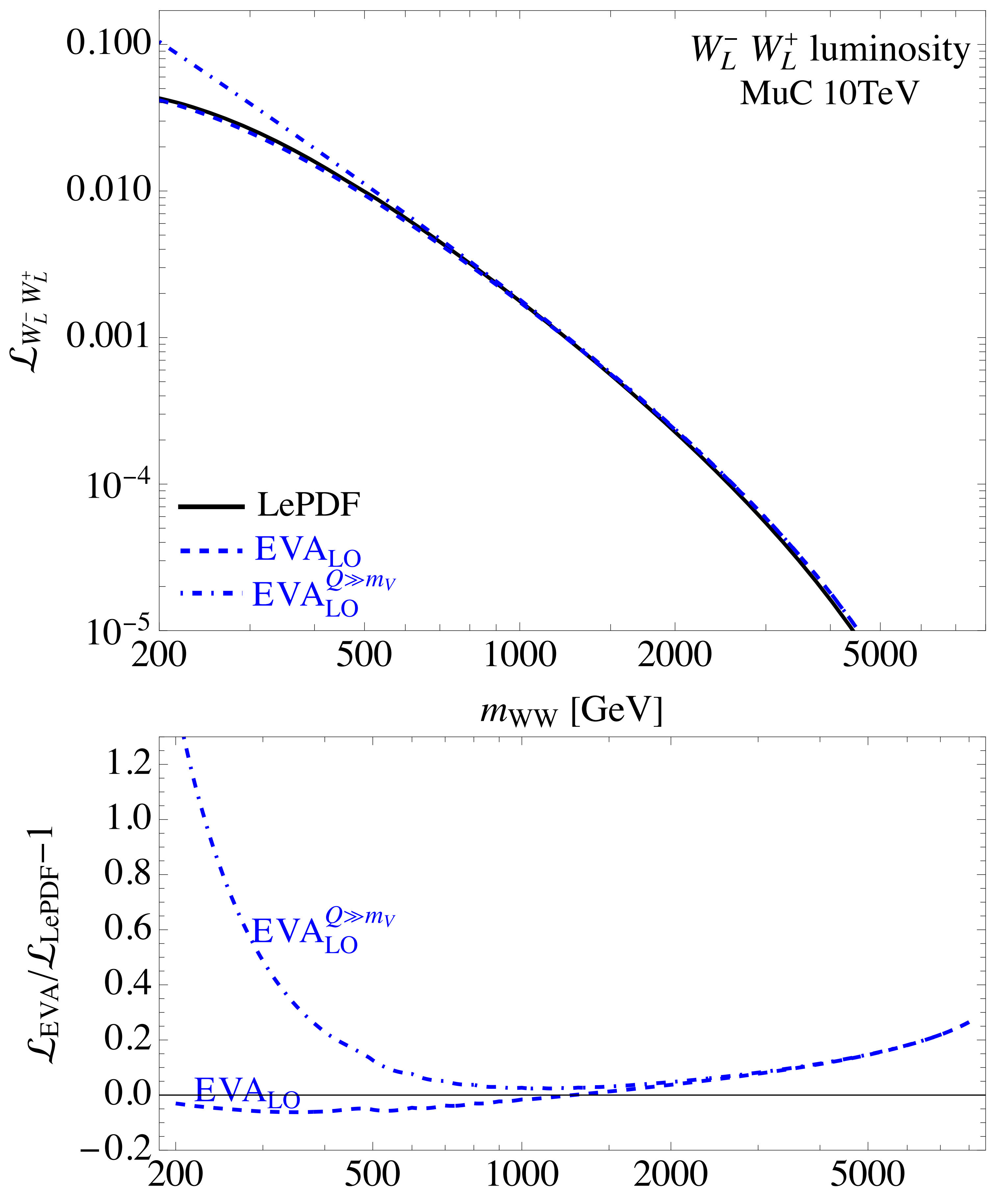
$$f^{(\alpha^2)}_{W^-_+}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(P^v_{W^-_+} f^{(\alpha)}_{W^-_+} + \frac{\alpha_2}{4\pi} P^f_{V_+ f_L} \otimes f^{(\alpha)}_{\mu_L} + \frac{\alpha_2}{2\pi} c_W^2 P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{Z_s}) + \right.$$
$$\left. + \frac{\alpha_\gamma}{2\pi} P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{\gamma_s}) + \frac{\sqrt{\alpha_\gamma \alpha_2}}{2\pi} c_W P_{V_+ V_s} \otimes f^{(\alpha)}_{Z/\gamma_s} \right) .$$

* for simplicity, in the NLO part we take the Q ≫ m_W and x ≪ 1 limit in the LO EVA expression.

Several **double logs appear at this order**, we find a **much improved agreement** with the LePDF resummation.

LePDF vs. EVA: WW Luminosity





At the level of **parton luminosity**:

- for **$W_T W_T$**: $\mathrm{EVA_{LO}}$ is accurate to **~15%**
- for **$W_L W_L$**: $\mathrm{EVA_{LO}}$ is accurate to **~5%**
- The **$Q \gg m_V$** approximation does not reproduce well the complete result, with **O(1) differences** up to large scales (particularly for transverse modes).

$\mathrm{EVA_{LO}}$ $$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

$\mathrm{EVA_{LO}}^{m_V\to 0}$ $$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

Implemented in `MadGraph5_aMC@NLO`
Ruiz, Costantini, Maltoni, Mattelaer [2111.02442]

Conclusions

We derived resummed **SM PDFs for lepton colliders** at the leading-log level: **LePDF**.

The results are made public in a **LHAPDF6**-*type* format: **extended to include helicity dependence**.

https://github.com/DavidMarzocca/LePDF

Some work is required to allow an interface with MadGraph5, *anybody interested*?

The **large muon PDF at small x** (an $O(\alpha^2)$ effect) could be **relevant for all VBF studies**.

We show that the implementation of EVA with the Q≫mW approximation is not sufficient, even at TeV scales.

When mass terms are included, **EVA @ LO deviates by**:

- **up to O(30-40%) for Z_T and W_T** at small x and large Q (few TeV) ,
- **~10^2 for the Z/γ** PDF.

At the level of $W_T W_T$ luminosities the deviation is ~15% for MuC10.

This should be much improved by using EVA @ NLO (ongoing work).

We hope this tool can allow **more comprehensive studies of physics potential at Muon Colliders**!

Thank you!

Backup

Evolution below the EW scale

DGLAP equations below the EW scale:

$$\frac{df_l}{dt} = \frac{\alpha_\gamma(t)}{2\pi}\left[\left(P_f^v + P_{ff,G}\right)\otimes f_l + P_{fV}\otimes f_\gamma\right] ,$$

$$\frac{df_{q^u}}{dt} = \frac{\alpha_\gamma(t)}{2\pi}Q_u^2\left[\left(P_f^v + P_{ff,G}\right)\otimes f_{q^u} + N_c P_{fV}\otimes f_\gamma\right]$$
$$+ \frac{\alpha_3(t)}{2\pi}\left[C_F\left(P_f^v + P_{ff,G}\right)\otimes f_{q^u} + T_F P_{fV}\otimes f_g\right] ,$$

$$\frac{df_{q^d,b}}{dt} = \frac{\alpha_\gamma(t)}{2\pi}Q_d^2\left[\left(P_f^v + P_{ff,G}\right)\otimes f_{q^d,b} + N_c P_{fV}\otimes f_\gamma\right]$$
$$+ \frac{\alpha_3(t)}{2\pi}\left[C_F\left(P_f^v + P_{ff,G}\right)\otimes f_{q^d,b} + T_F P_{fV}\otimes f_g\right] ,$$

$$\frac{df_\gamma}{dt} = \frac{\alpha_\gamma(t)}{2\pi}\left[P_\gamma^v f_\gamma + \sum_f Q_f^2 P_{Vf}\otimes\left(f_f + f_{\bar{f}}\right)\right] ,$$

$$\frac{df_g}{dt} = \frac{\alpha_3(t)}{2\pi}\left[C_A\left(P_g^v + P_{VV}\right)\otimes f_g + C_F P_{Vf}\otimes\sum_q\left(f_q + f_{\bar{q}}\right)\right] ,$$

$t \equiv \log\left(\mu^2/m_\mu^2\right)$

Thanks to flavour symmetry and C and P invariance of QED+QCD we can identify:

$$f_{\ell_{\rm sea}} = f_e = f_\tau = f_{\bar{e}} = f_{\bar{\mu}} = f_{\bar{\tau}}$$
$$f_{q^u} = f_u = f_{\bar{u}} = f_c = f_{\bar{c}} ,$$
$$f_{q^d} = f_d = f_{\bar{d}} = f_s = f_{\bar{s}} ,$$
$$f_b = f_{\bar{b}} .$$

IR poles in z=1 are regulated using the +-distribution

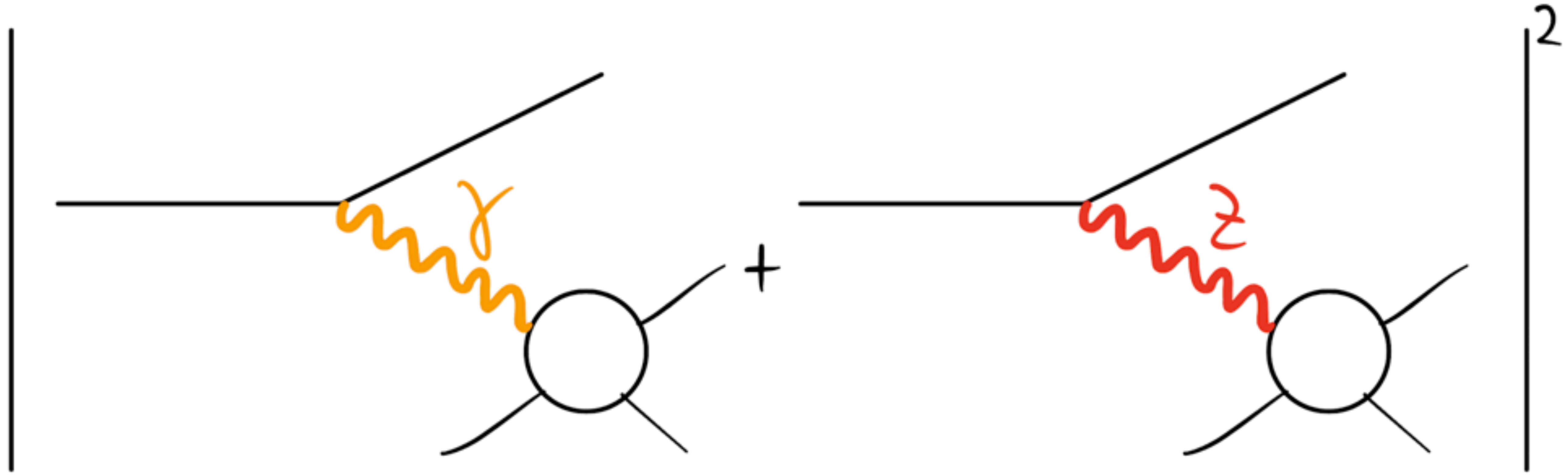
$$\int_x^1 dz\frac{f(z)}{(1-z)_+} = \int_x^1 dz\frac{f(z)-f(1)}{1-z} - f(1)\int_0^x \frac{dz}{1-z}$$

Numerical procedure:

- Discretize the equations in a grid in “x”
- Perform the integrals with the rectangles method
- Solve numerically the coupled set of differential equations using the Runge-Kutta algorithm (we implemented both in c++ and Mathematica, for cross-check).
- Momentum conservation is imposed at each step.

Photon - Z mixing

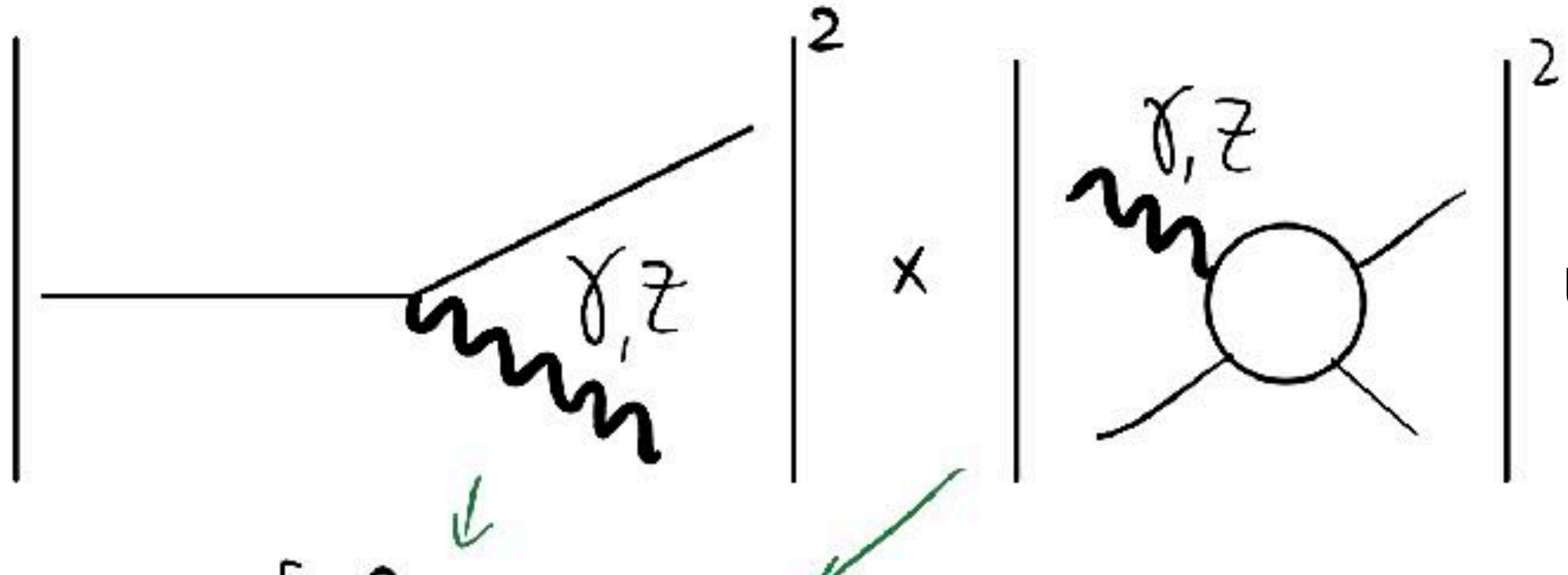
Photon and Z bosons can interfere



In the collinear limit this can be described by a **mixed Z/γ PDF**.
Similarly for Z_L and H.

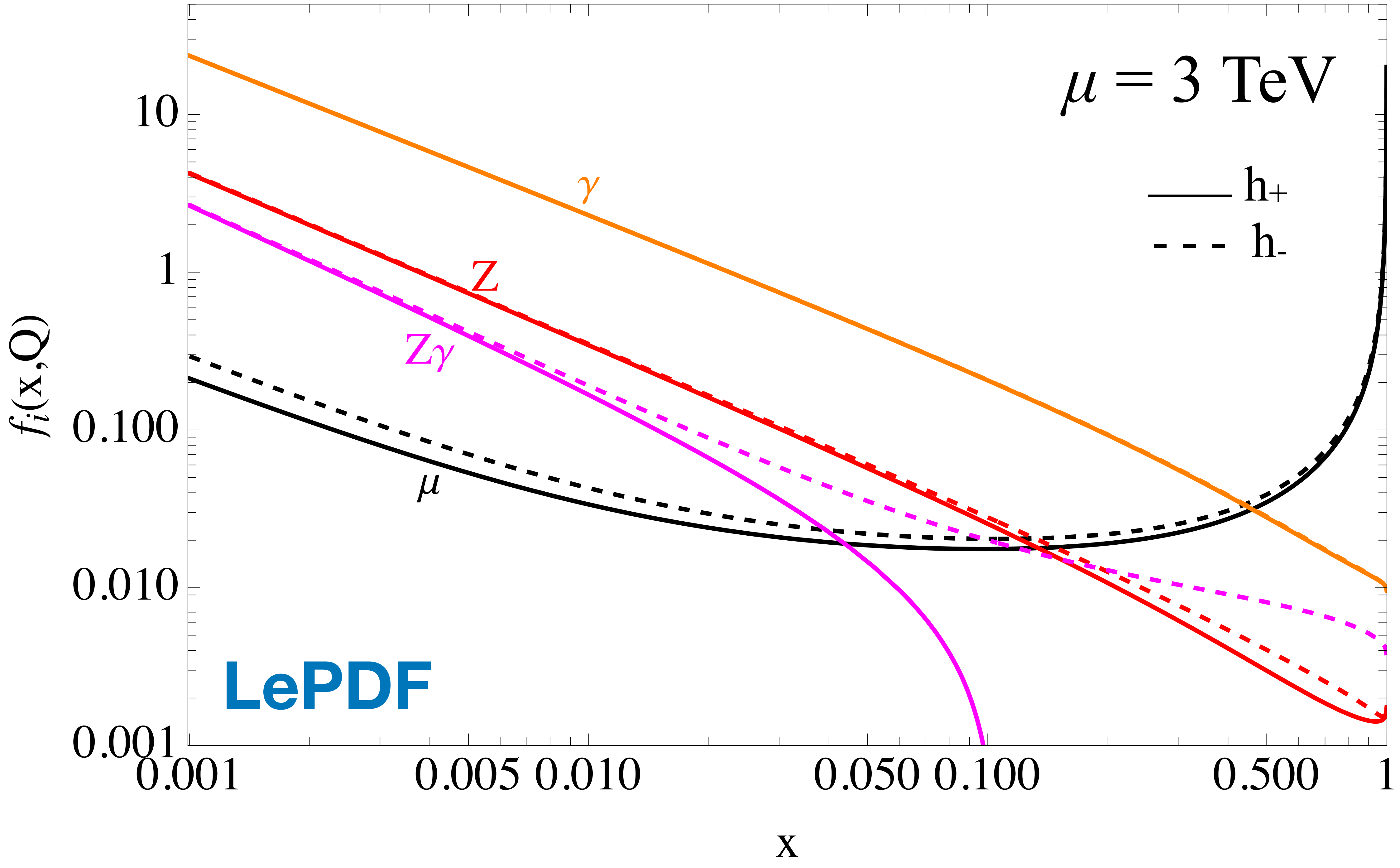
P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]

The splitting function must be generalised to a splitting matrix.
The rate is computed by tracing against the matrix of the hard scattering process



up to $O(k_T^2/E^2, m^2/E^2)$

$$\sigma \propto \left[\frac{dP_{A\to BC}}{dz\, dk_T^2}\right]_{ij} \hat{\sigma}_{ij} \, , \quad i,j = \gamma, Z$$



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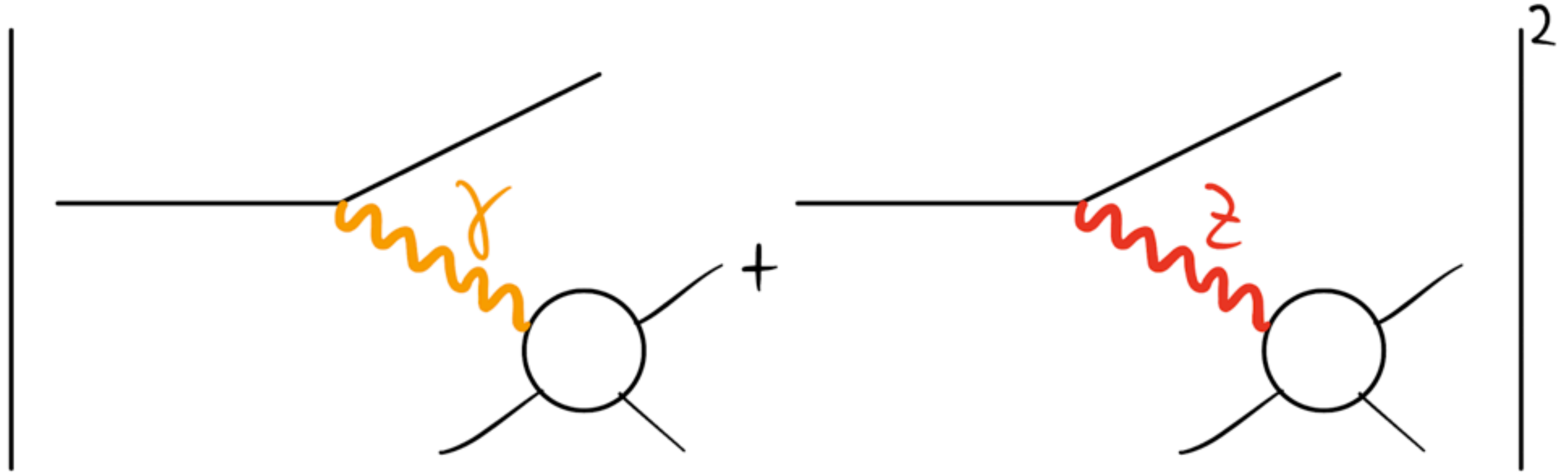


Photon and Z bosons can interfere.
The interference term is described by a **mixed Z/γ PDF**.
Similarly for Z_L and H.

P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]

Matrix splitting function:

$$\left[\frac{d\mathcal{P}_{A\to B+C}}{dz\, dk_T^2}\right]_{ij} \simeq \frac{1}{16\pi^2}\,\frac{1}{z\bar{z}}\,\mathcal{M}_k^{(\mathrm{split})*}\mathcal{D}_{ki}^*\mathcal{D}_{jl}\mathcal{M}_l^{(\mathrm{split})}$$

The propagators are diagonal in the mass basis:

$$\mathcal{D}_{\gamma\gamma} = \frac{i}{q^2}\,, \quad \mathcal{D}_{ZZ} = \frac{i}{q^2 - m_Z^2}\,, \quad \mathcal{D}_{\gamma Z} = \mathcal{D}_{Z\gamma} = 0$$

$$\mathcal{D}_{hh} = \frac{i}{q^2 - m_h^2}\,, \quad \mathcal{D}_{Z_L Z_L} = \frac{i}{q^2 - m_Z^2}\,, \quad \mathcal{D}_{hZ_L} = \mathcal{D}_{Z_L h} = 0.$$

One can go from the mass to the gauge basis via a Weinberg angle rotation, e.g.:

$$\begin{pmatrix} B \\ W_3 \\ BW_3 \end{pmatrix} = \begin{pmatrix} \cos^2\theta_W & \sin^2\theta_W & -\cos\theta_W\sin\theta_W \\ \sin^2\theta_W & \cos^2\theta_W & \cos\theta_W\sin\theta_W \\ 2\cos\theta_W\sin\theta_W & -2\cos\theta_W\sin\theta_W & \cos^2\theta_W - \sin^2\theta_W \end{pmatrix} \begin{pmatrix} \gamma \\ Z \\ Z\gamma \end{pmatrix}$$

EW Sudakov double logs from ISR

In case of collinear W emission they can be implemented (and resummed) at he **Double Log** level equations by putting an **explicit IR cutoff $z_{max} = 1 - Q_{EW}/Q$** $(Q_{EW} = m_W)$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]
see Manohar, Waalewijn [1802.08687] for a different approach

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P^C_{BA}(z) f_A\left(\frac{x}{z},Q^2\right) \rightarrow \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{z^{ABC}_{\max}(Q)} \frac{dz}{z} P^C_{BA}(z) f_A\left(\frac{x}{z},Q^2\right)$$

This modifies also the **virtual corrections** as:

$$P^v_A(Q) \supset -\sum_{B,C} \frac{\alpha_{ABC}(Q)}{2\pi}\int_0^{z^{ABC}_{\max}(Q)} dz\, z\, P^C_{BA}(z)$$

The non-cancellation of the z_{max} dependence between emission and virtual corrections generates the double logs.

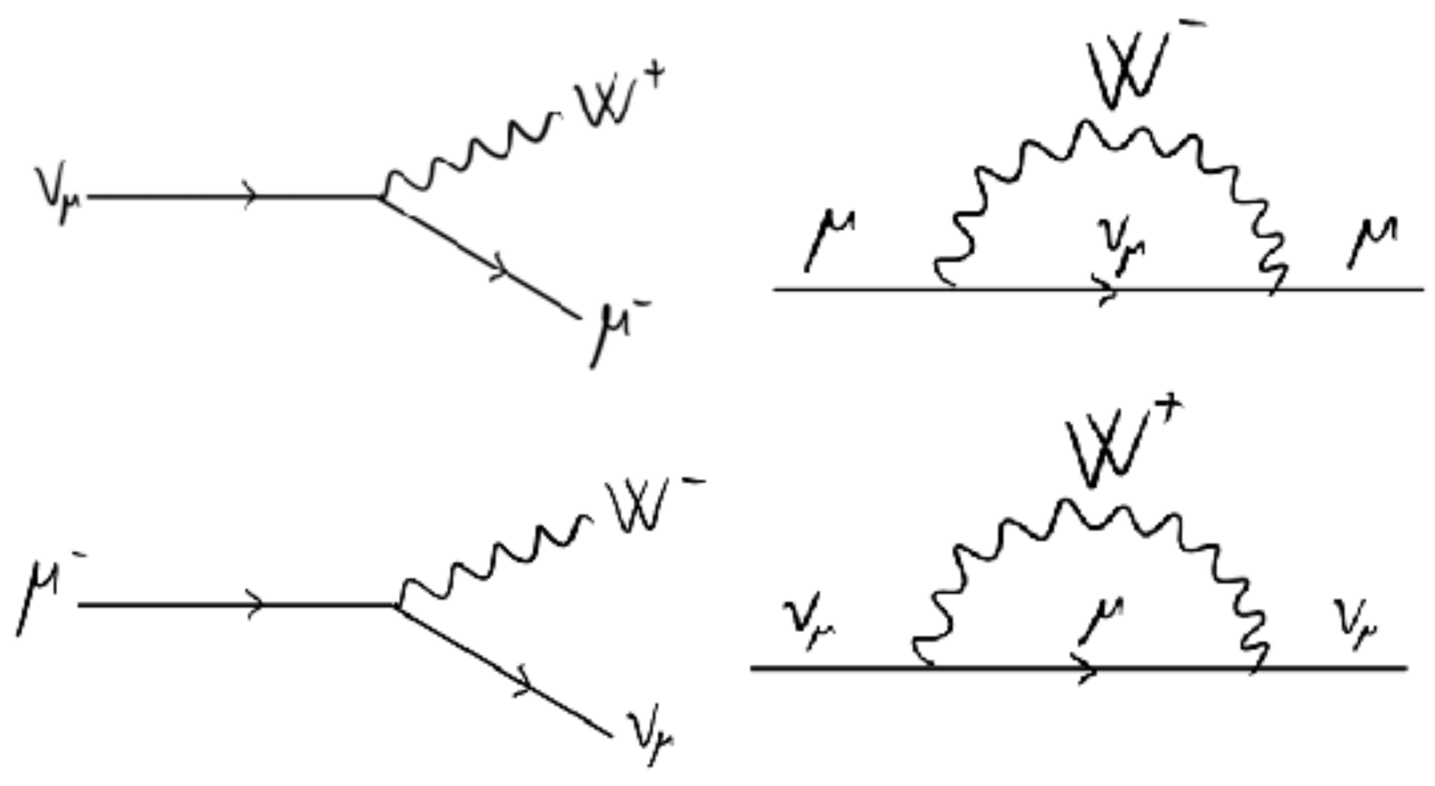
This happens if $P^C_{BA},\ U^C_{BA} \propto \frac{1}{1-z}$ and $A \neq B$ otherwise we set $z_{max}=1$ and use the +-distribution.

EW Sudakov double logs from ISR

For <u>illustration</u>, let us consider the **μ_L and ν_μ DGLAP equations** and only **interactions with transverse $W^\pm$**

$$\frac{df_{\mu_L}}{d\log Q^2} = \frac{\alpha_2}{2\pi}\frac{1}{2}\int_0^{z_{\max}(Q)} dz P^V_{ff}(z)\left(\frac{1}{z}f_{\nu_\mu}\left(\frac{x}{z},Q^2\right) - z f_{\mu_L}(x,Q^2)\right) + \text{ IR-finite terms}$$

$$\frac{df_{\nu_\mu}}{d\log Q^2} = \frac{\alpha_2}{2\pi}\frac{1}{2}\int_0^{z_{\max}(Q)} dz P^V_{ff}(z)\left(\frac{1}{z}f_{\mu_L}\left(\frac{x}{z},Q^2\right) - z f_{\nu_\mu}(x,Q^2)\right) + \text{ IR-finite terms}$$



We are interested in the **IR divergent terms**, take $z \to 1$ for all regular terms inside the integrand:

$$\frac{df_{\mu_L}}{d\log Q^2} \approx -\frac{\alpha_2}{4\pi}\Delta f_{L_2}(x)\int_0^{z_{\max}(Q)} dz\frac{2}{1-z} + \ldots \approx -\frac{\alpha_2}{4\pi}\log\frac{Q^2}{Q^2_{\rm EW}}\Delta f_{L_2}(x) + \ldots\,,$$

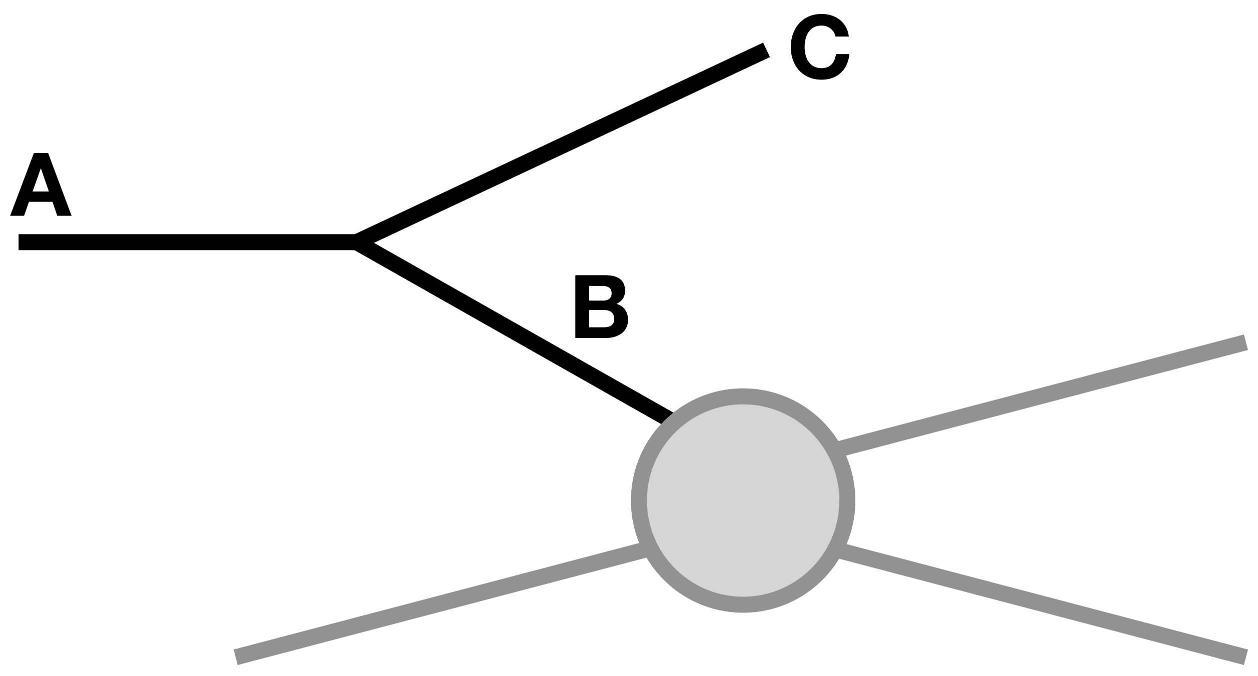
$$\frac{df_{\nu_\mu}}{d\log Q^2} \approx \frac{\alpha_2}{4\pi}\Delta f_{L_2}(x)\int_0^{z_{\max}(Q)} dz\frac{2}{1-z} + \ldots \approx \frac{\alpha_2}{4\pi}\log\frac{Q^2}{Q^2_{\rm EW}}\Delta f_{L_2}(x) + \ldots\,,$$

$$\Delta f_{L_2}(x) \equiv f_{\mu_L}(x) - f_{\nu_\mu}(x)$$

Upon integration in $\log Q^2$ one gets the **double log**: it is **negative for μ_L** and **positive for ν_μ**, tends to restore $SU(2)_L$ invariance at high scales and vanishes when the two become equal.

It is **not present for Z and γ** interactions with fermions, since in the RHS the same fermion PDF enters.

Mass effects



1) Kinematical effects of emitted real radiation

The particle C is emitted on-shell: its energy is bounded to be $E_C = (z\text{-}x)\, E > m_C$ $\quad z \geq x + \frac{m_C}{E}$

In the limit where collinear factorisation is valid, $E \gg p_T, m$, **we can neglect this effect**.

2) Propagator effects

The mass modifies the propagator of the off-shell parton which then enters the hard scattering:

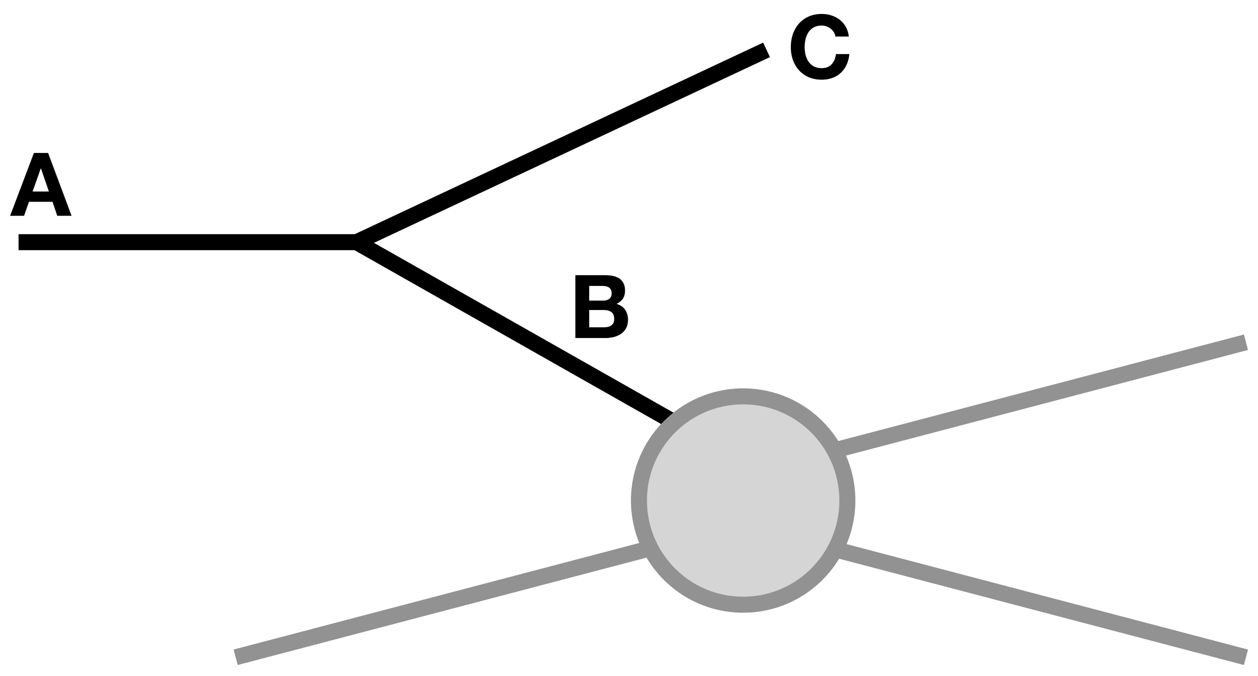
$$\tilde{p}_T^2 \equiv \bar{z}(m_B^2 - q^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

This can be implemented by a rescaling of the massless splitting functions:

$$P_{BA}^C(z) \quad \to \quad \tilde{P}_{BA}^C(z, p_T^2) = \left(\frac{p_T^2}{\tilde{p}_T^2}\right)^2 P_{BA}^C(z)$$

Chen, Han, Tweedie [1611.00788]

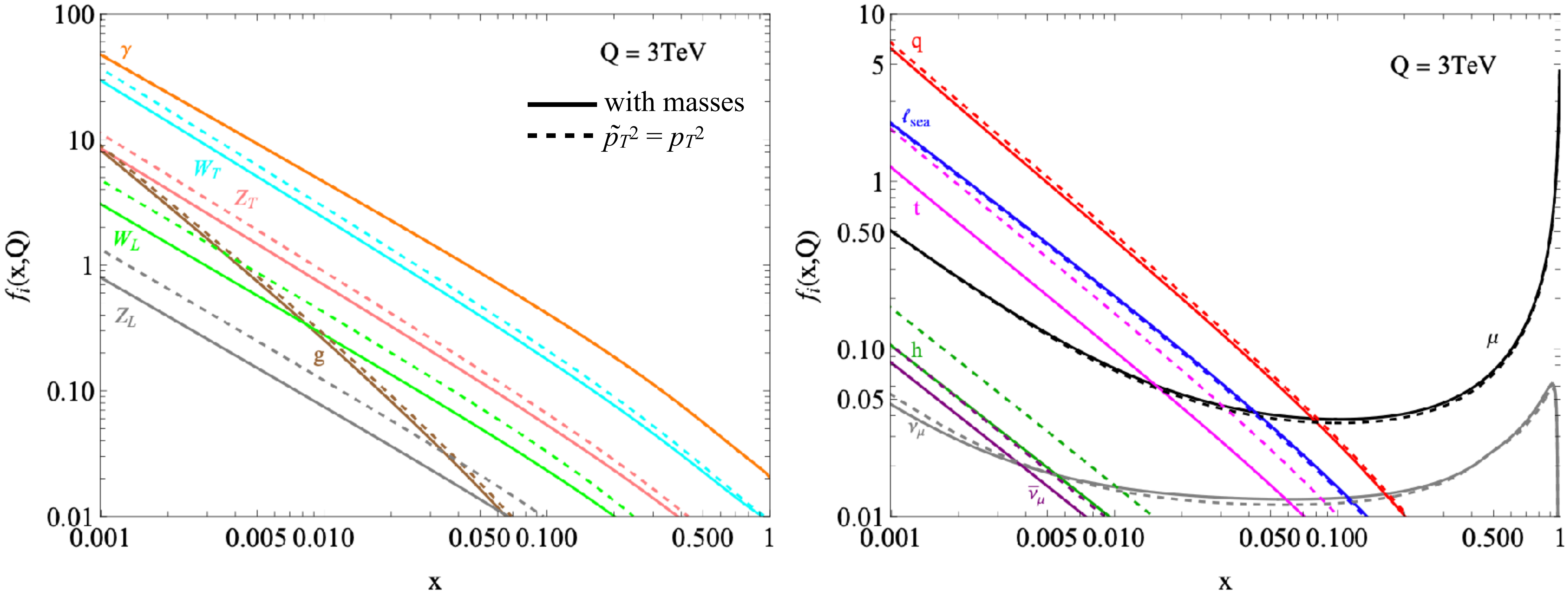
Mass effects



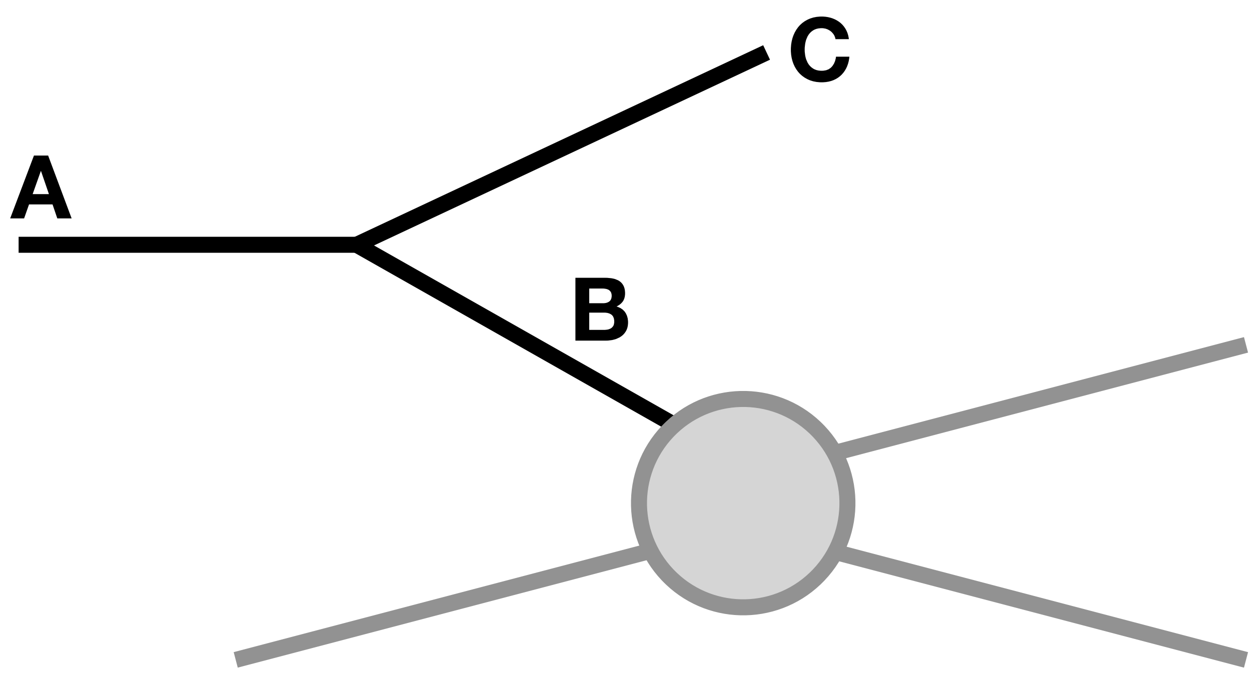
2) Propagator effects

$$\widetilde{P}^{C}_{BA}(z, p_T^2) = \left(\frac{p_T^2}{\widetilde{p}_T^2}\right)^2 P^C_{BA}(z)$$

$$\widetilde{p}_T^2 \equiv \bar{z}(m_B^2 - p_B^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$



Ultracollinear splittings



In the unbroken phase, splitting matrix elements are proportional to p_T^2

$$|\mathcal{M}(A \to B + C)|^2 \equiv 8\pi\alpha_{ABC}\frac{p_T^2}{z\bar{z}}P_{BA}^C(z)$$

Ultra-collinear splitting function Chen, Han, Tweedie [1611.00788]

Upon EWSB, further splittings proportional to v^2 are generated.
They generalise the EWA splitting $f \to W_L f'$

$$|\mathcal{M}_{A\to B+C}|^2 \equiv \frac{v^2}{z\bar{z}}P_{BA,C}^{u.c.}(z)$$

For example: $$P^{u.c.}_{f_L^{(2)} f_L^{(1)}, W_L}(z) = \left(y_{f_1}^2 z\bar{z} - y_{f_2}^2 \bar{z} - g_2^2 z\right)^2 \frac{1}{2\bar{z}_+}$$

coupling of massless fermions to W_L, with no chirality flip
(via coupling to remainder gauge field W_n in GEG)

The missing p_T^2 factor removes the log enhancement at high scales, making the **u.c. terms approach a constant value**.

The DGLAP equations are generalised as:

$$Q^2\frac{df_B(x,Q^2)}{dQ^2} = P_B^v f_B(x,Q^2) + \sum_{A,C}\frac{\alpha_{ABC}}{2\pi}\widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2}\sum_{A,C}\widetilde{U}_{BA}^C \otimes f_A$$

Effective W approximation

Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84,

Solving the DGLAP equations iteratively at LO we recover the EWA:

$$f^{\rm LO}_{W_T^-}(x,\mu^2) = \int_{m_\mu^2}^{\mu^2} dp_T^2 \frac{1}{2}\frac{d\mathcal{P}_{\psi\to W_T\psi}}{dp_T^2}(x,p_T^2) = \int_{m_\mu^2}^{\mu^2} dp_T^2\, \frac{g^2}{32\pi^2}\frac{p_T^2}{(p_T^2+(1-x)m_W^2)^2}\frac{1+(1-x)^2}{x} =$$

$$\approx \frac{g^2}{32\pi^2}\frac{1+(1-x)^2}{x}\left(\log\frac{\mu^2}{m_W^2} - \log(1-x) - 1\right) + \mathcal{O}\left(\frac{m_W^2}{\mu^2}\right)$$

$$f^{\rm LO}_{W_L^-}(x,\mu^2) = \int_{0}^{\mu^2} dp_T^2 \frac{1}{2}\frac{d\mathcal{P}_{\psi\to W_L\psi}}{dp_T^2}(x,p_T^2) = \int_{0}^{\mu^2} dp_T^2\, \frac{g^2}{16\pi^2}\frac{m_W^2}{(p_T^2+(1-x)m_W^2)^2}\frac{(1-x)^2}{x}$$

$$= \frac{g^2}{16\pi^2}\frac{1-x}{x}\frac{\mu^2}{\mu^2+(1-x)m_W^2} \approx \frac{g^2}{16\pi^2}\frac{1-x}{x} + \mathcal{O}\left(\frac{m_W^2}{\mu^2}\right).$$

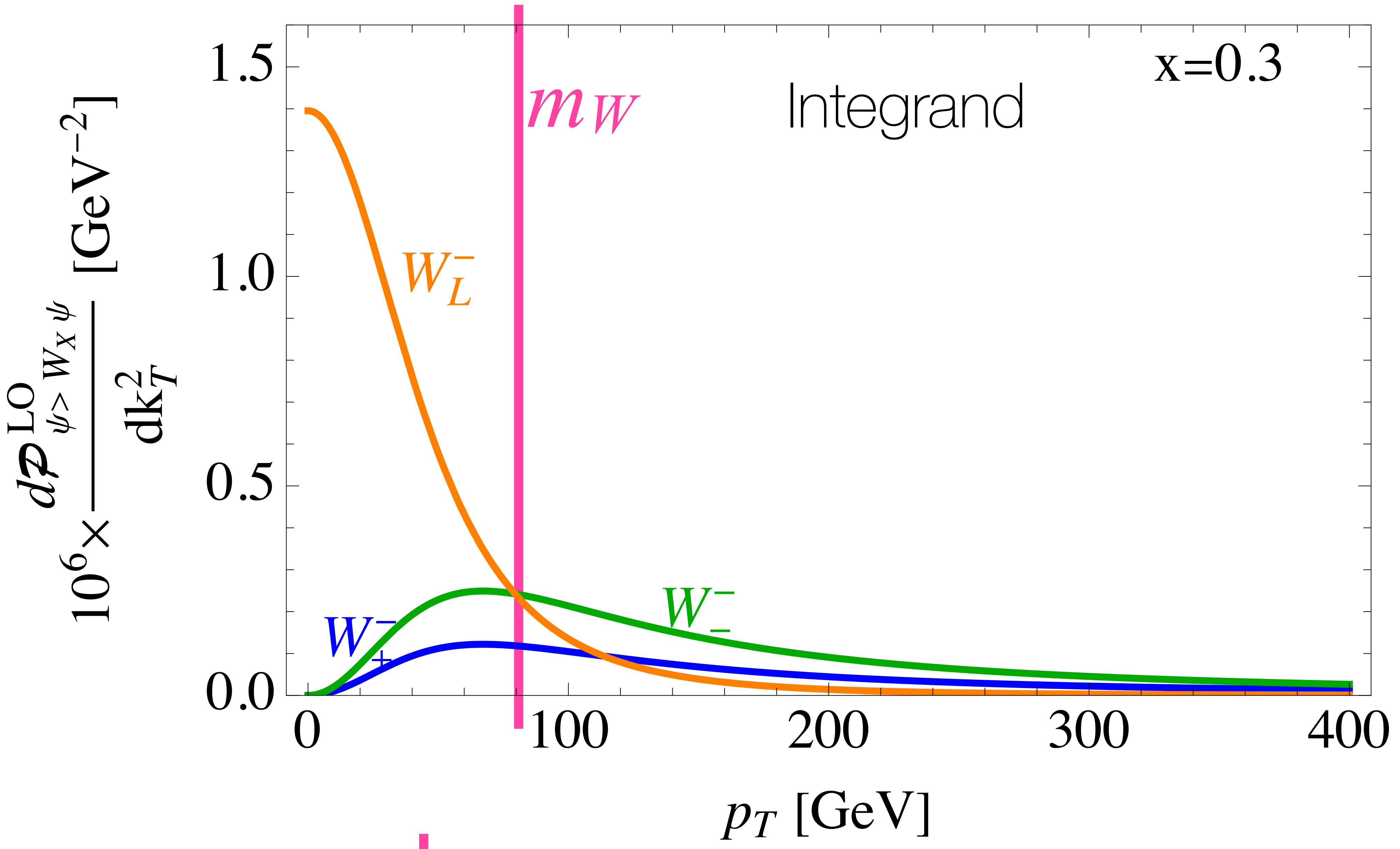
ultra-collinear term, vanishes for v → 0

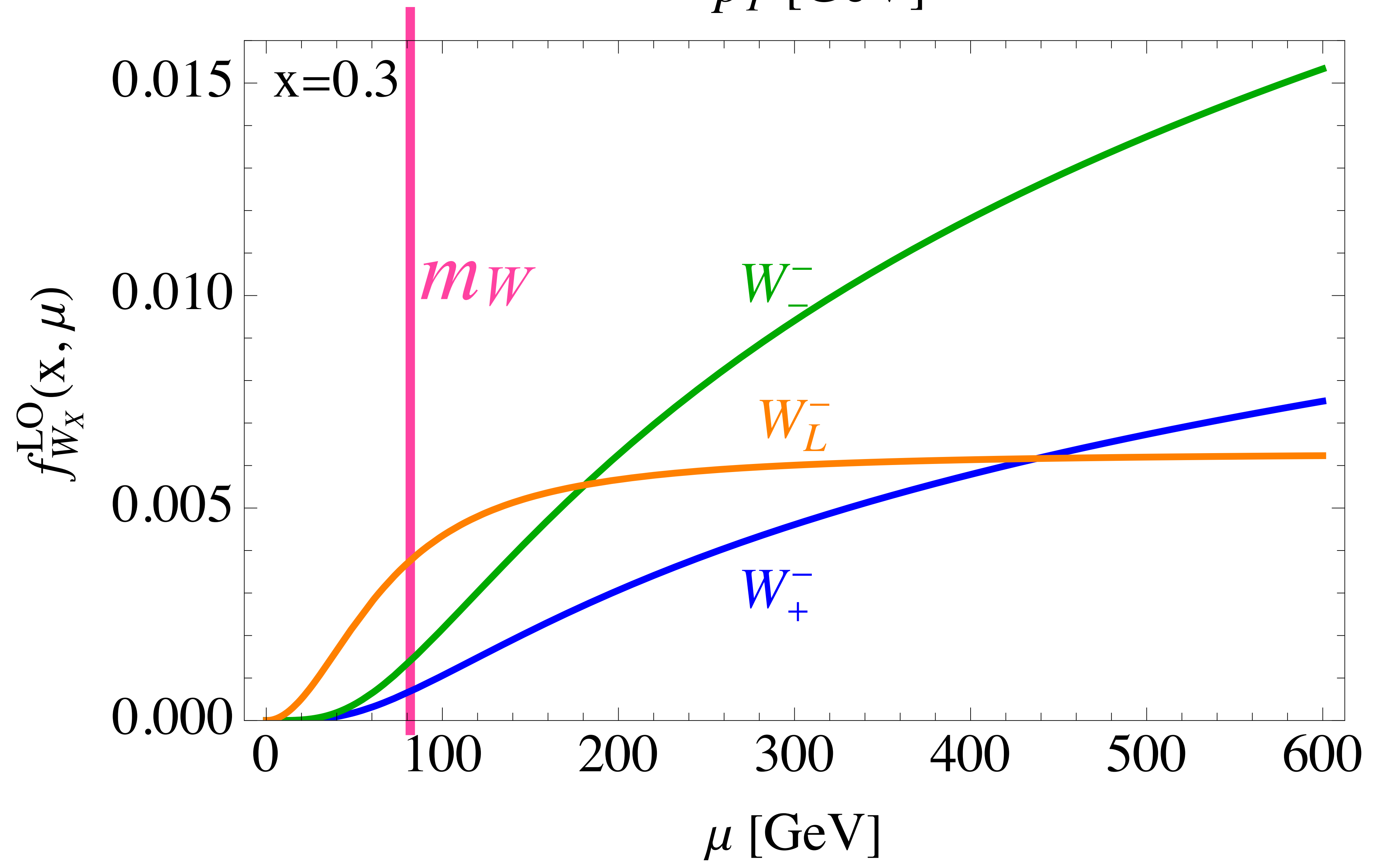
See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…

They receive a substantial contribution from scales $\mu < m_W$, **particularly the longitudinal polarization**.

To recover this, at the EW matching scale we use the EWA as boundary condition for the EW gauge bosons:

$$f_{W^-_{L,\pm},Z_{L,\pm}}(x,\mu^2_{\rm EW}) \equiv f^{\rm LO}_{W^-_{L,\pm},Z_{L,\pm}}(x,\mu^2_{\rm EW})$$

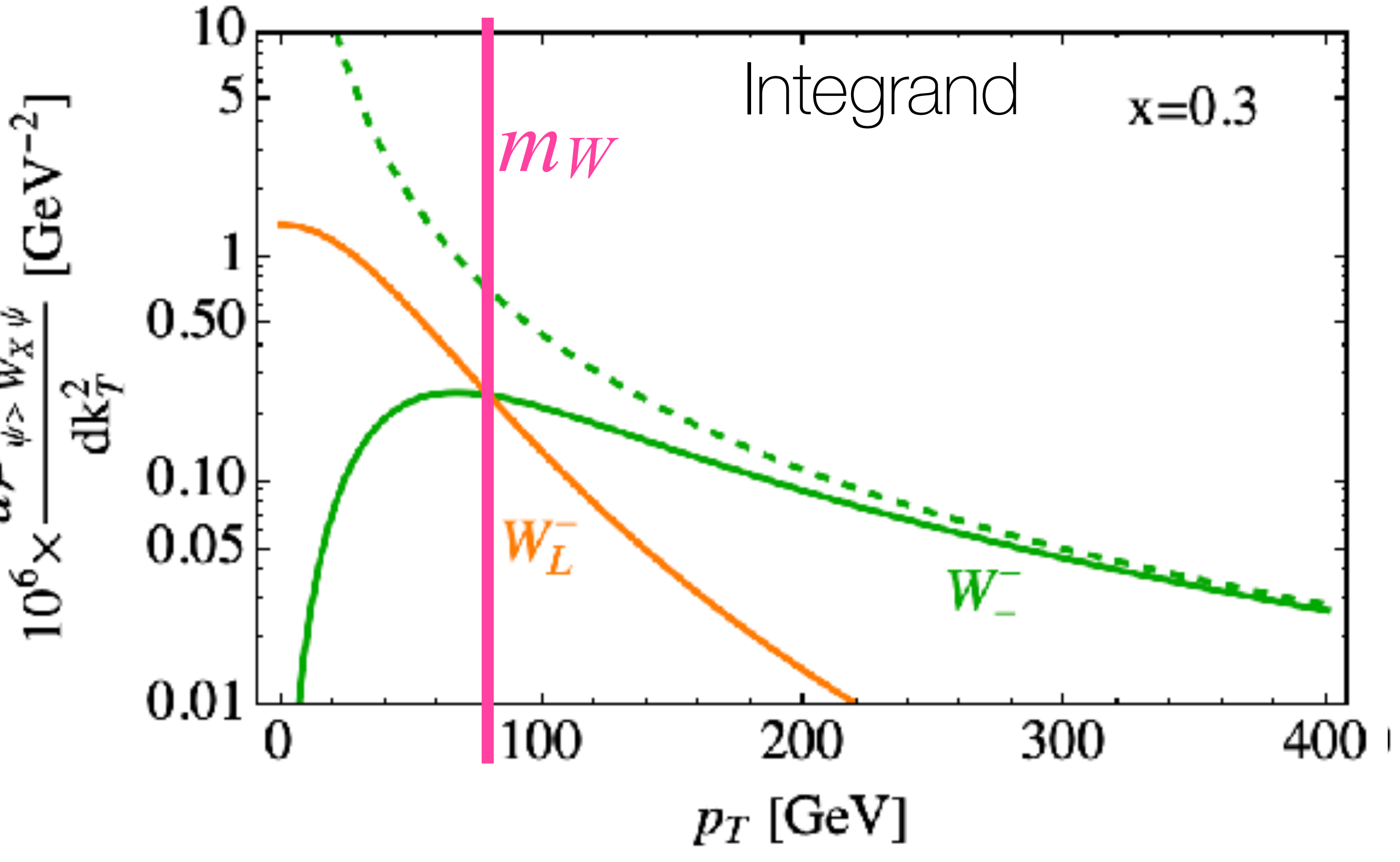


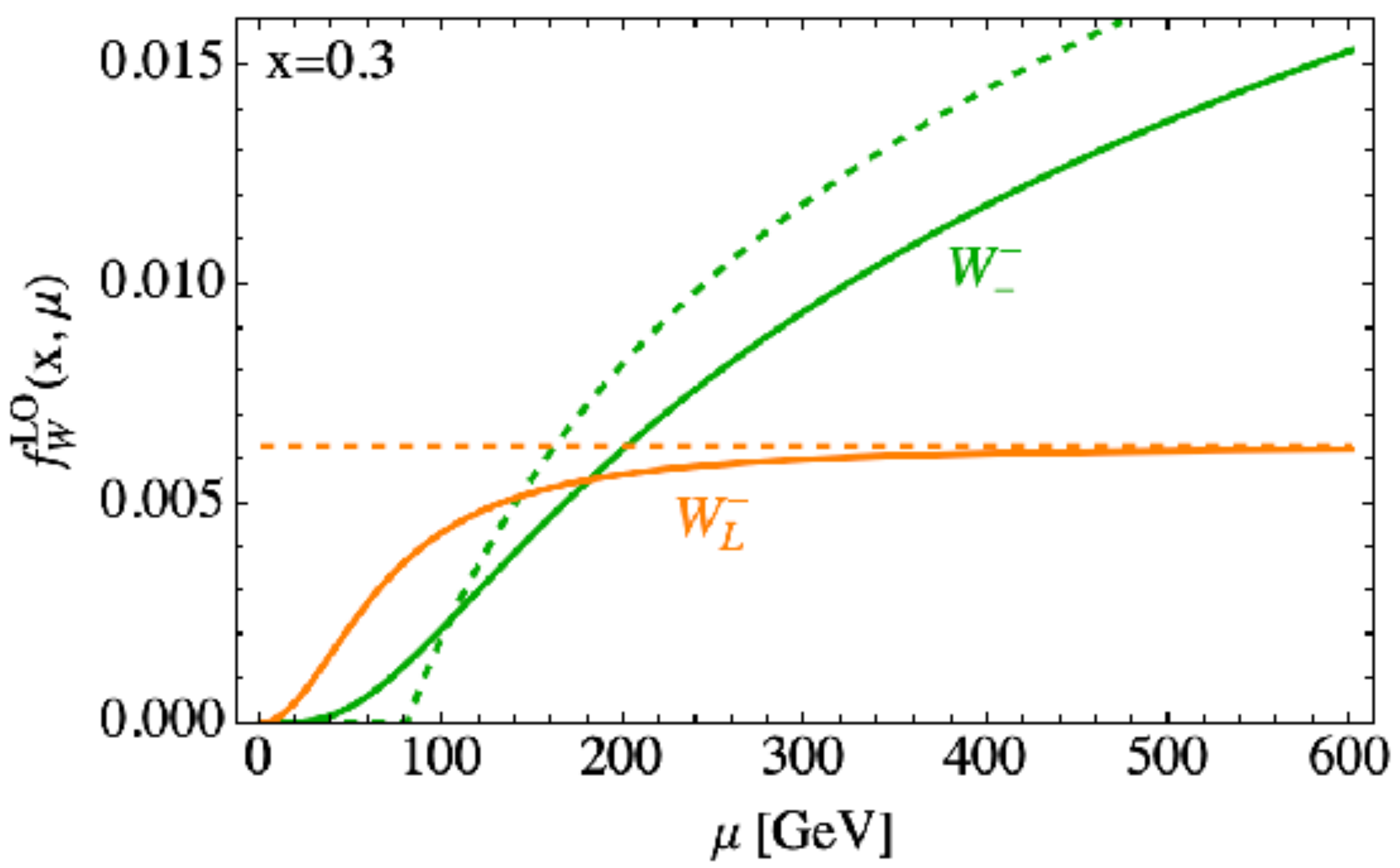


Effective W approximation

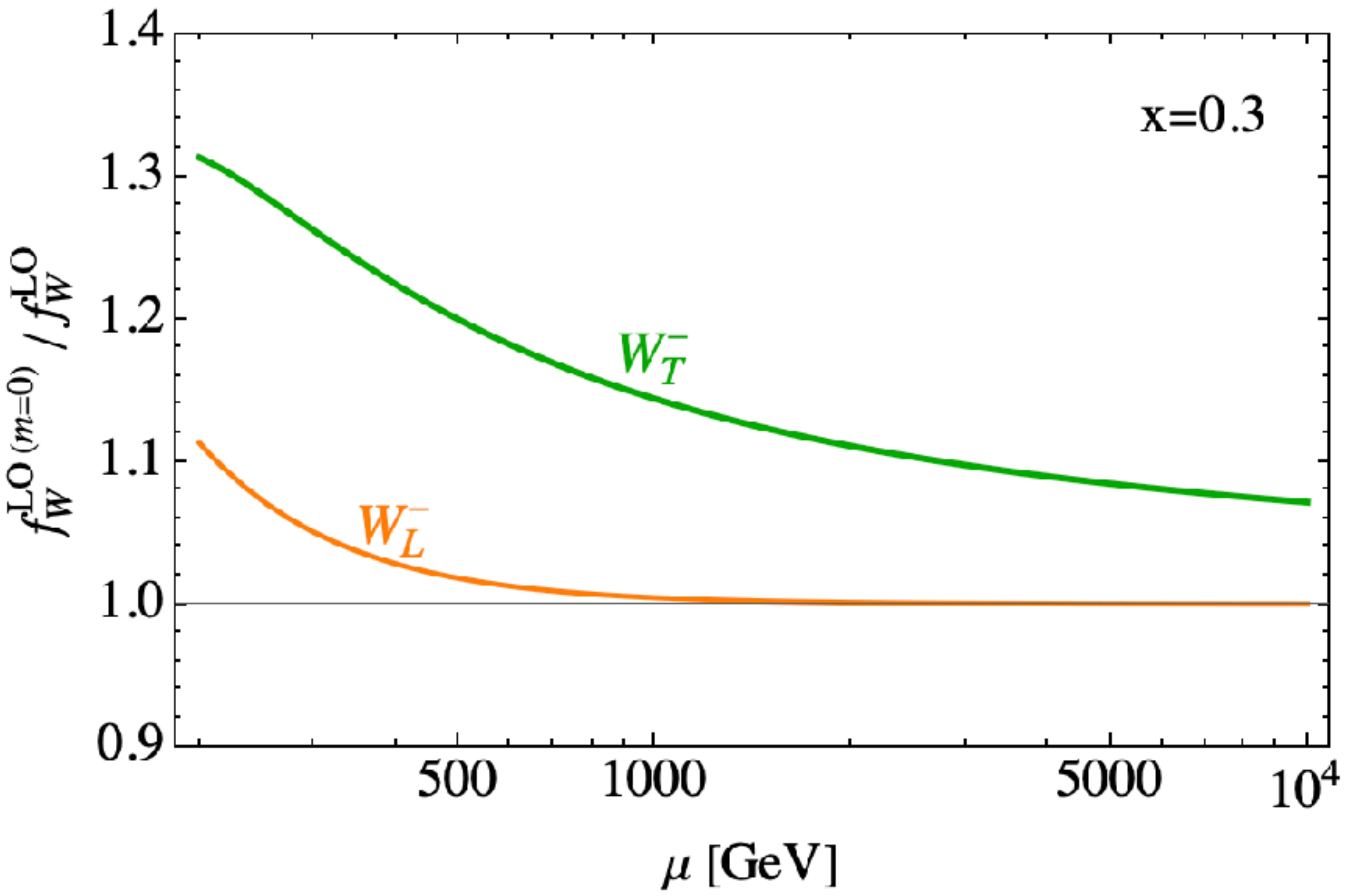
dashed: massless

Looking at the effect of the mass in the propagator.



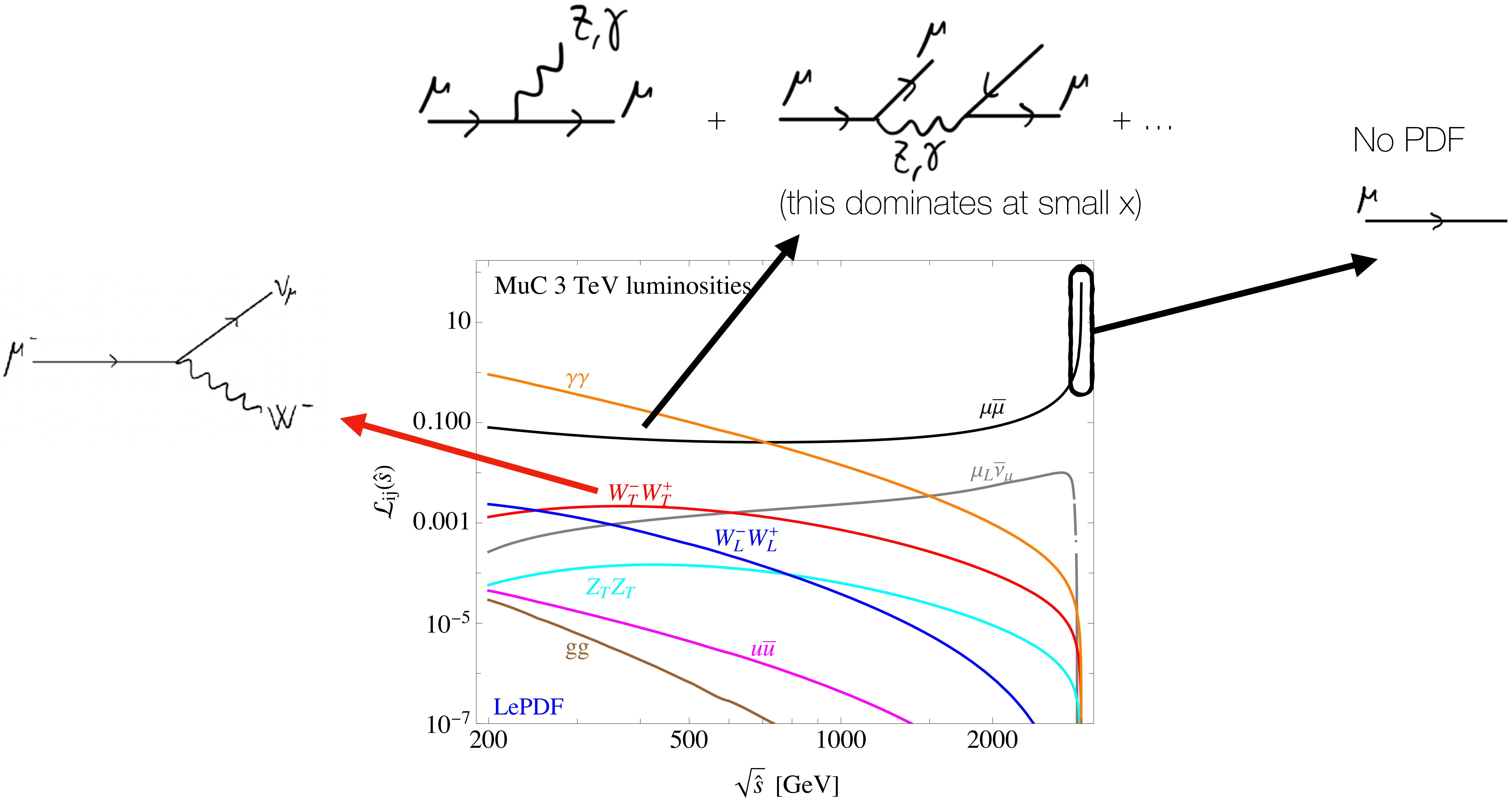


For the W_T a correct treatment of the mass is important also at very high scales.



In case of W_L , the effect is purely an EWSB one, so we can take the massless limit only at the end of the computation.
In case of WT I do the integral with massless propagator from m_W up to μ.

large $\mu\bar{\mu}$ and $\mu_L\bar{\nu}_\mu$ lumi at **small $\sqrt{\hat{s}}$**: possible sizeable **impact on VBF studies** from annihilation channel?



Top quark PDF

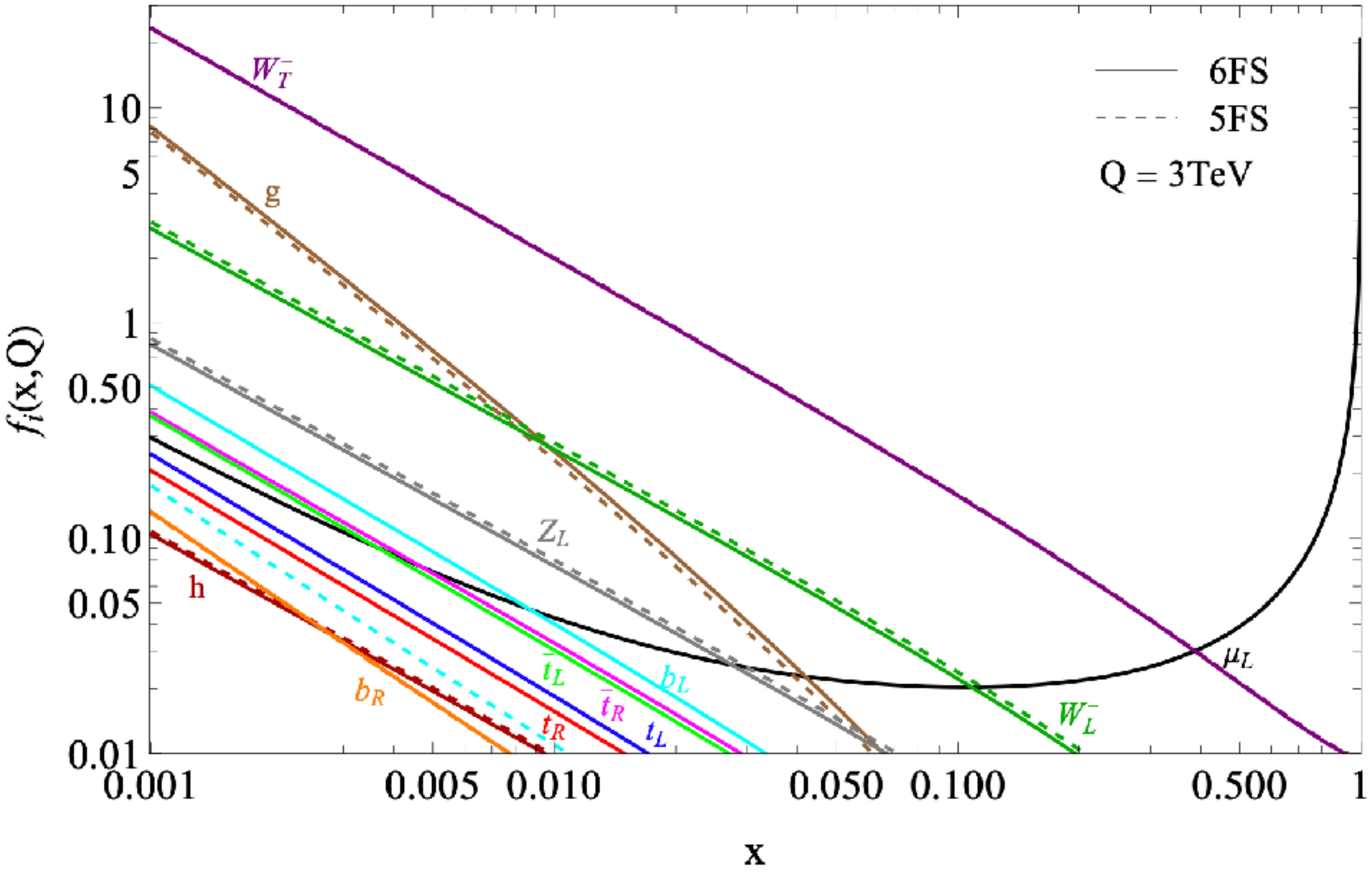
For hard scattering energies $E \gg m_t$, terms with $\log E/m_t$ due to collinear emission of top quarks can arise.

These can be resummed by including the **top quark PDF** within the DGLAP evolution, in a **6FS**.

Barnett, Haber, Soper '88; Olness, Tung '88

Whether or not this is useful depends on the process under consideration.

Dawson, Ismail, Low [1405.6211]
Han, Sayre, Westhoff [1411.2588]



We provide two version of the codes: **5FS** and **6FS**.
In the 6FS we keep **finite top quark mass** effects, like we do for other heavy SM states.

LePDF - LHAPDF6

LHAPDF6: Buckley et al. [1412.7420]

Extended to include helicity & Zγ and Z_Lh PDFs

LePDF_mu_6FS.info

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SetDesc: "PDFs of muon obtained in the 6FS."
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Reference: 2303.16964
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DataVersion: 1
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3   4   4   5   5   6   6   -1  -1  -2  -2  -3  -3  -4  -4  -5  -5      -6  -6  21  21  22  22  23  23
23  2223    2223    24  24  24  -24 -24 -24 25  2523]
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+   -   +   -   0   +   -   0   0   0]
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QMax: 5.7222760e+004
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MZ: 91.1876
MHiggs: 125.25
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MStrange: 0
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(x Grid)

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(Q Grid)

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0    +    -    +    -    0    +    -    0    0    0
```

$x\, f_i(x,Q)$ Grid:

$$
\begin{array}{cccc}
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x f_{e_L}(x_0,Q_1) & \dots & \dots & x f_{h/Z_L}(x_0,Q_1) \\
\vdots & \vdots & \vdots & \vdots \\
x f_{e_L}(x_0,Q_{N_Q}) & \dots & \dots & x f_{h/Z_L}(x_0,Q_{N_Q}) \\
x f_{e_L}(x_1,Q_0) & \dots & \dots & x f_{h/Z_L}(x_1,Q_0) \\
x f_{e_L}(x_1,Q_1) & \dots & \dots & x f_{h/Z_L}(x_1,Q_1) \\
\vdots & \vdots & \vdots & \vdots \\
x f_{e_L}(x_1,Q_{N_Q}) & \dots & \dots & x f_{h/Z_L}(x_1,Q_{N_Q}) \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
x f_{e_L}(x_{N_x},Q_0) & \dots & \dots & x f_{h/Z_L}(x_1,Q_0) \\
x f_{e_L}(x_{N_x},Q_1) & \dots & \dots & x f_{h/Z_L}(x_1,Q_1) \\
\vdots & \vdots & \vdots & \vdots \\
x f_{e_L}(x_{N_x},Q_{N_Q}) & \dots & \dots & x f_{h/Z_L}(x_{N_x},Q_{N_Q})
\end{array}
$$

LePDF: Numerical Implementation

We solve the DGLAP numerically in x space. Due to the sharp behaviour of the muon PDF near x=1, the typical interpolation techniques used for PDFs of proton do not work.

We discretise x interval $[x_{min}=10^{-6},1]$ in N_x small intervals, denser for $x \approx 1$:

$$x_\alpha = 10^{-6((N_x-\alpha)/N_x)^{2.5}}$$
$$\alpha = 0, 1, \ldots, N_x$$

For the splitting functions divergent in $z \to 1$ we us the "+" distribution

$$\int_x^1 dz \frac{f(z)}{(1-z)_+} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} - f(1)\int_0^x \frac{dz}{1-z} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} + f(1)\log(1-x)$$

The differential evolution is done in $t = \log Q^2/m_\mu^2$ with 4th order Runge-Kutta.

At x=1 we fix
$$f_{iN_x}(t) = \begin{cases} \frac{L(t)}{\delta x_{N_x}} & i = \mu \\ 0 & i \neq \mu \end{cases}$$

where L(t) is fixed imposing momentum conservation:

Han, Ma, Xie [2103.09844]

$$L(t) = 1 - \sum_{i=1}^{N_f} \sum_{\alpha=1}^{N_x-1} \delta x_\alpha x_\alpha f_{i\alpha}(t)$$

The uncertainties due to x and t discretisation are estimated to be of ~1% and ~0.1%, respectively, for $N_x=1000$.

Leptoquarks: S_3

$S_3 = (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$ $\quad \mathcal{L}^{\text{int}}_{S_3} = \lambda_{i\mu}\, \overline{Q^{i\,c}_L}\, \epsilon\, \sigma^I L^2_L S^I_3 + \text{h.c.}$

***s*-channel LQ:**
quarks inside the PDF of muon