Sudakov Logs at the Muon Collider

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IMCC Annual Meeting 2023
20/06/2023

Based on:
Learning from radiation at a very high energy lepton collider
Chen, Glioti, Rattazzi, Ricci, Wulzer
2202.10509

Introduction

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

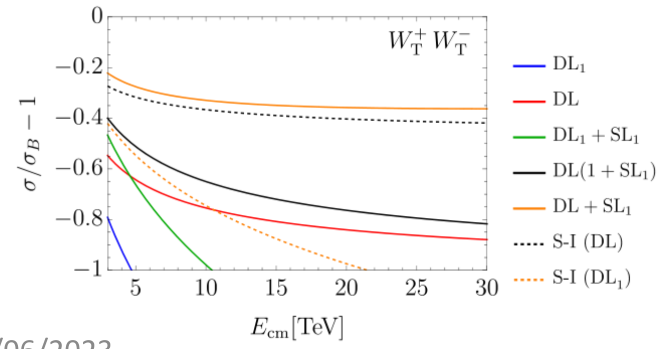
Double (Sudakov) Logs appear in cross-sections involving **soft** and **collinear EW bosons**

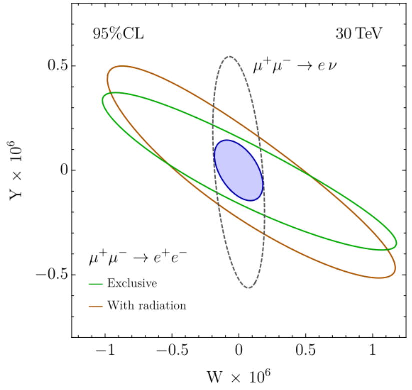
$$\frac{\alpha_w}{4\pi}\log^2 E^2/m_W^2$$

This talk has **two** objectives

Show why understanding Double (and Single) Electroweak Logs is fundamental for a future Muon Collider

Show how these effects can be exploited in order to probe new physics much more effectively





EW Double Logs

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

Why are Electroweak Double Logs **special**?

- The EW group is **non-abelian** and **spontaneously broken**:
  - Double Logs do **not cancel out** even for inclusive observables
  - Initial/final states are **not** EW **singlets**
- EW theory is **weakly coupled**
- IR cutoff (m_W) is **physical**
- Double Logs are **extremely large** at a Muon Collider

BN Theorem violation
Ciafaloni, Ciafaloni, Comelli (2000)

$$E \gg m_w \quad \Longrightarrow \quad \frac{\alpha_w}{4\pi} \log^2 E^2/m_W^2 \sim O(1)$$

$10\ \text{TeV} \implies \sim 0.25$

+ group theory factors…

These effects must be **resummed** in order to reach % level precision!

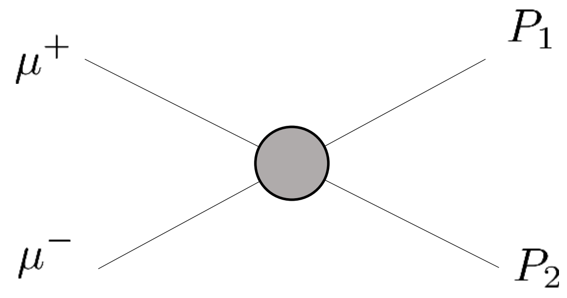
Part 1 – Resumming Double Logs

Which observables?

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

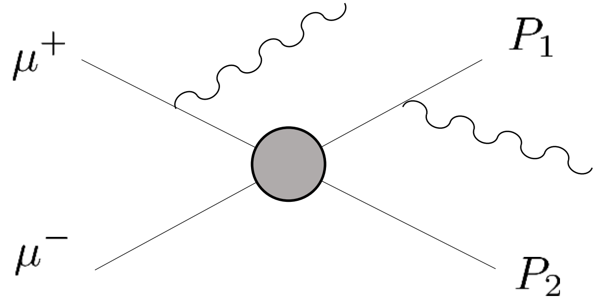
We consider **two representative observables** for which we know how to perform the DL resummation

Exclusive



- **Two hard** final particles with **definite** EW **color**
- **Veto** on soft/collinear EW **radiation**
- Inclusive on soft photons/gluons

Semi-Inclusive



- **Two hard** final particles with **definite** EW **color**
- All **radiation** is **allowed** (up to some hardness threshold)
- "**Semi**" since we don't **sum** over external legs colors

IREE strategy

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

Our resummation strategy is based on an **InfraRed Evolution Equation** (IREE)

Fadin, Lipatov, Martin, Melles, 1999

$\lambda = E^2$ Born obs.

↓ IREE

$\lambda = m_W^2$ Resummed obs.

- Introduce an unphysical **IR cutoff** λ

$$\lambda < \min \left| \frac{(k_i \cdot q)(k_j \cdot q)}{(k_i \cdot k_j)} \right|$$

- Compute derivative of the observable wrt λ through **diagrammatic** techniques

$$\frac{d}{d\lambda} \mathcal{O}^\lambda = \mathcal{K} \cdot \mathcal{O}^\lambda$$

- Solve the IREE with the **boundary condition**

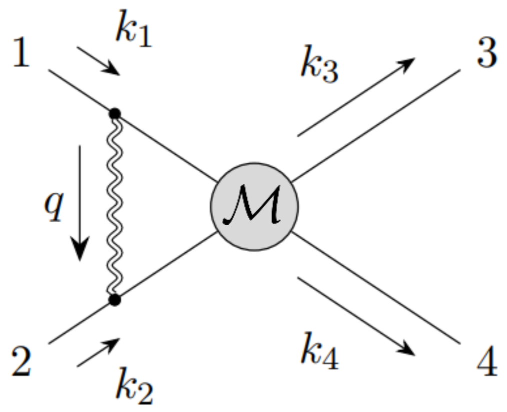
$$\mathcal{O}^{\lambda = E^2} \equiv \mathcal{O}^{\text{Born}}$$

Exclusive observables

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

Exclusive on EW radiation, but inclusive on soft photons

$$\mu^+\mu^- \to P_1 P_2 + n\gamma$$



IREE defined at the level of **Amplitudes**

Kernel is **color-diagonal**

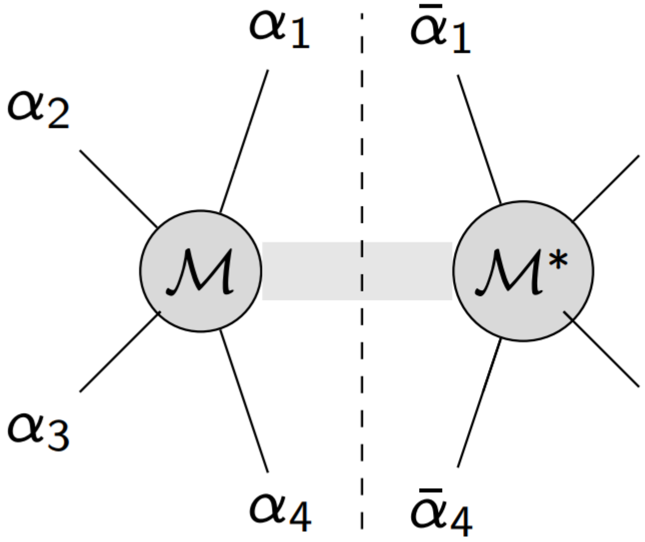
$$\frac{d\mathcal{M}_\lambda^\alpha}{d\log^2(E^2/\lambda)} = -\frac{1}{2}\mathcal{K}\,\mathcal{M}_\lambda^\alpha\,, \qquad \mathcal{K} = \frac{1}{16\pi^2}\sum_i \left[g^2 c_i + g'^2 y_i^2\right]$$

Semi-inclusive observables

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

$$\mu^+\mu^- \to P_1 P_2 + n(W,\, Z,\, \gamma)$$

P1 and P2 carry ~90% of COM energy



$$\equiv \mathcal{D}_\lambda^{\alpha\bar{\alpha}}$$

IREE defined at the level of **Density Matrix**

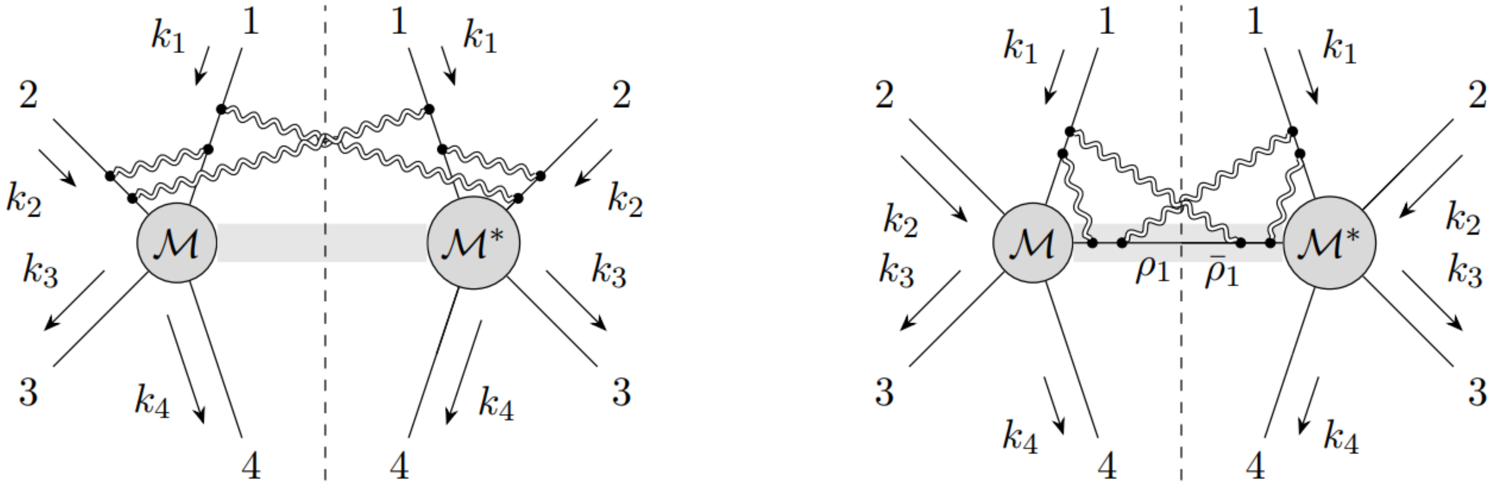
$$\mathcal{D}_\lambda^{\alpha\bar{\alpha}} \equiv \mathcal{M}_\lambda^{\alpha}(\mathcal{M}_\lambda^{\bar{\alpha}})^* + \sum_{N=1}^{\infty} \int \mathrm{dPh}_{N,\lambda}^{\mathcal{H}} \sum_{\rho_1 \dots \rho_N} \mathcal{M}_\lambda^{\alpha;\rho}(\mathcal{M}_\lambda^{\bar{\alpha};\rho})^*$$

Semi-inclusive observables

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

$$\mu^+\mu^- \to P_1 P_2 + n(W,\, Z,\, \gamma)$$

P1 and P2 carry ~90% of COM energy



These do **not** contribute at DL level

$$\frac{d\mathcal{D}_\lambda^{\alpha\bar{\alpha}}}{d\log^2(E^2/\lambda)} = -\mathcal{K}^{\alpha\bar{\alpha}}_{\beta\bar{\beta}}\,\mathcal{D}_\lambda^{\beta\bar{\beta}}$$

Kernel is **off-diagonal**

All **Born hard processes mix** with each other

Summary of Results

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

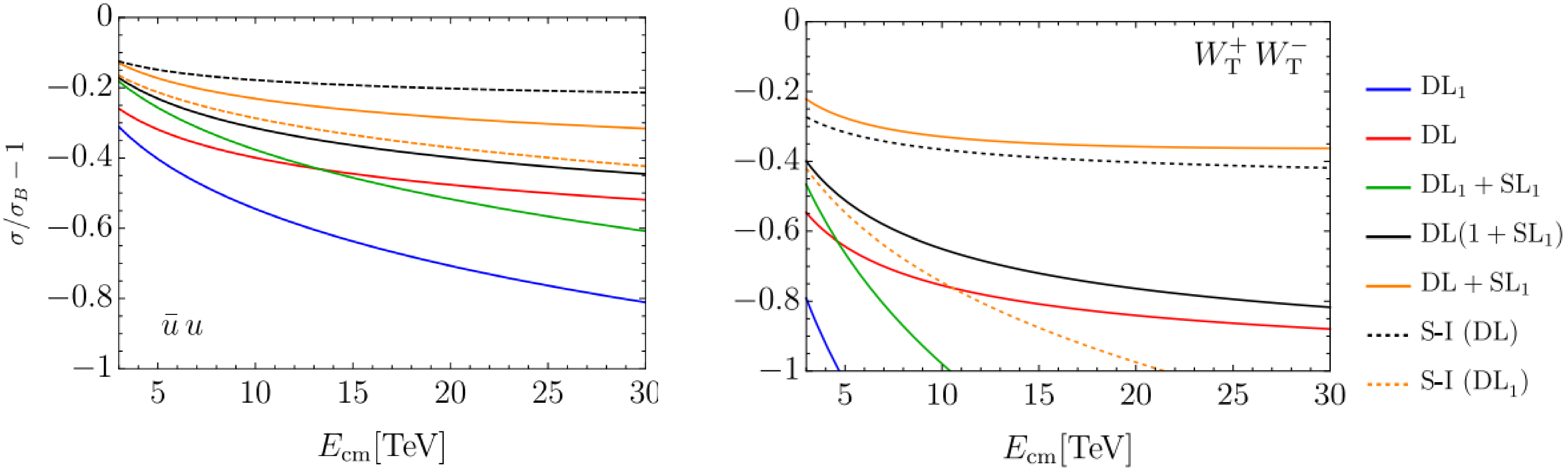
- In both cases the Kernel and double logs simply **exponentiate**
- **Exclusive** Kernel is **color-diagonal**. Gives an exponential **suppression** factor for each external leg **proportional** to its EW **Casimir**
- **Semi-inclusive** Kernel is **off-diagonal**. All allowed Born amplitude are **mixed** together
- SI-Kernel has a **leg-by-leg structure** that vanishes when summed over leg color (similar to KLN theorem, but stronger result)

$$\mathcal{K}^{\alpha\bar{\alpha}}_{\beta\bar{\beta}} = \frac{g^2}{16\pi^2}\sum_i \left[c_i\, \delta^{\alpha_i}_{\beta_i}\delta^{\bar{\alpha}_i}_{\bar{\beta}_i} + \sum_{A=1,2,3} (T^A_i)^{\alpha_i}_{\beta_i}(T^A_{i^c})^{\bar{\alpha}_i}_{\bar{\beta}_i}\right]\left[\prod_{j\neq i}\delta^{\alpha_j}_{\beta_j}\delta^{\bar{\alpha}_j}_{\bar{\beta}_j}\right]$$

- Abelian **hypercharge** term **cancels out** in semi-inclusive observables as expected from BN theorem

Effect of Double and Single Logs

Chen, AG, Rattazzi, Ricci, Wulzer (2022)



Single-Logs (virtual only) added at fixed-order from Denner, Pozzorini (2000)

Effect of Double and Single Logs

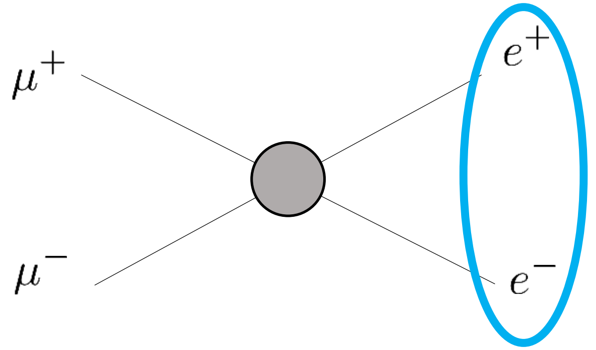
	10 TeV		
	DL	$e^{\mathrm{DL}}-1$	$\mathrm{SL}(\frac{\pi}{2})$
$\ell_L \to \ell'_L$	-0.82	-0.56	0.33
$\ell_L \to q_L$	-0.78	-0.54	0.34
$\ell_L \to e_R$	-0.56	-0.43	0.17
$\ell_L \to u_R$	-0.48	-0.38	0.15
$\ell_L \to d_R$	-0.43	-0.35	0.13
$\ell_R \to \ell'_L$	-0.56	-0.43	0.17
$\ell_R \to q_L$	-0.53	-0.41	0.16
$\ell_R \to \ell'_R$	-0.30	-0.26	0.09
$\ell_R \to u_R$	-0.22	-0.20	0.07
$\ell_R \to d_R$	-0.17	-0.16	0.05

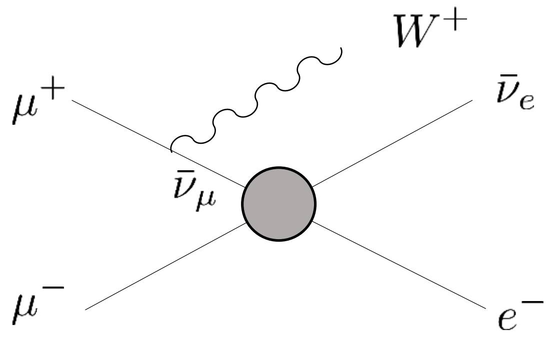
Part 2 – Probing New Physics

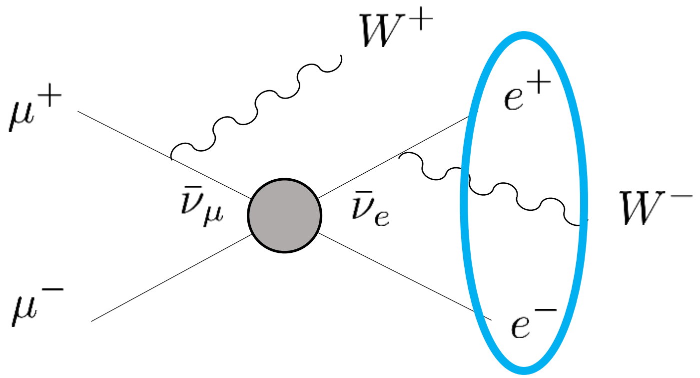
Radiation for BSM

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

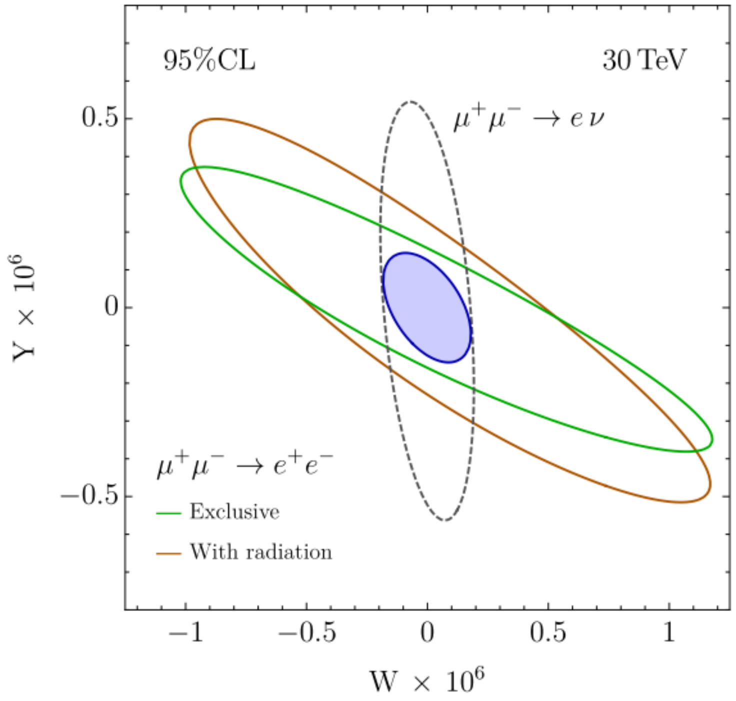
Thanks to the large double **logs processes with soft emission** become **as likely as** processes allowed at **tree-level**







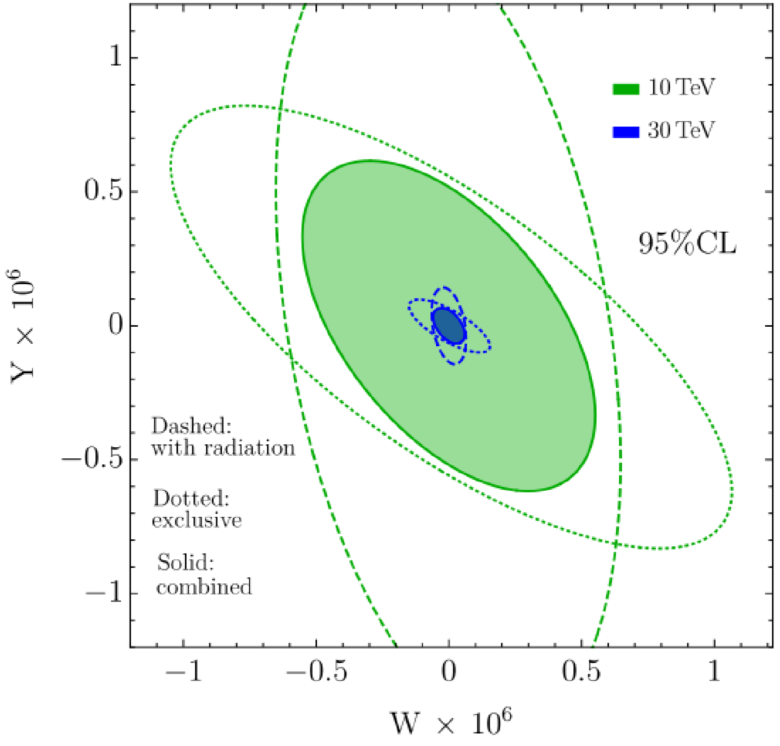
Hard processes sensitive to **different combinations** of new physics parameters



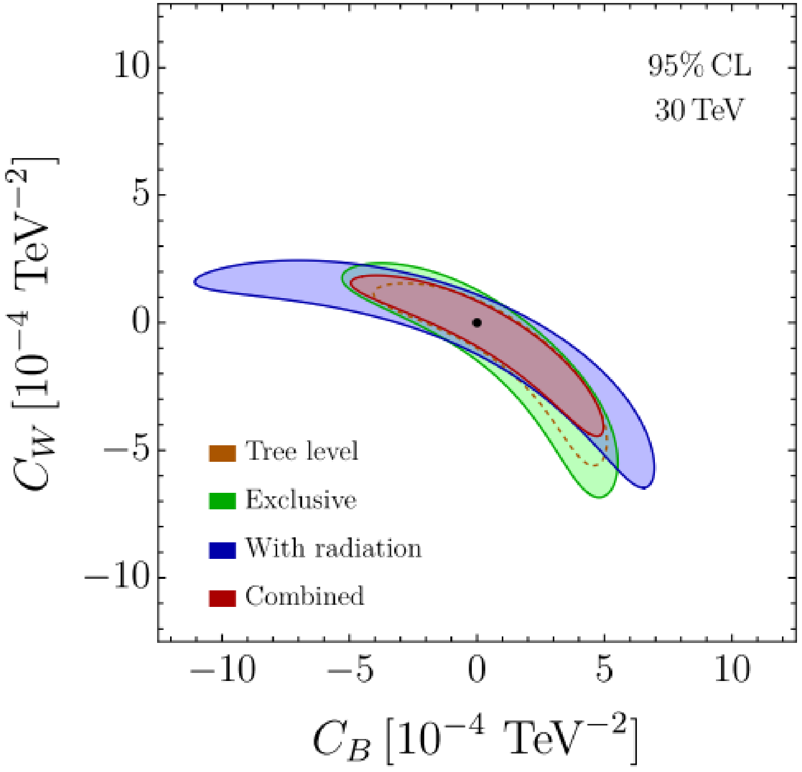
Results on effective operators

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

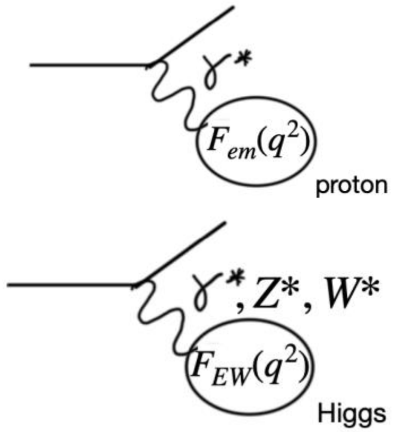
Difermion production



Diboson production



$$O_{2W} = (D_\mu W^{\mu\nu,a})^2 \sim W$$
$$O_{2B} = (\partial_\mu B^{\mu\nu})^2 \sim Y$$
$$O_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}$$
$$O_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}$$

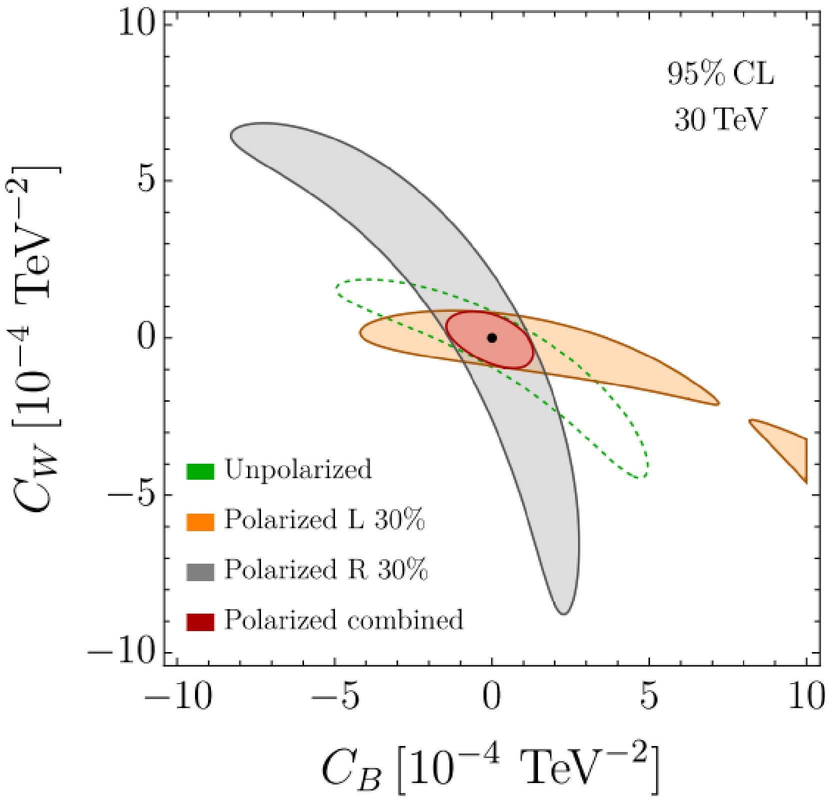


Avoiding flat direction

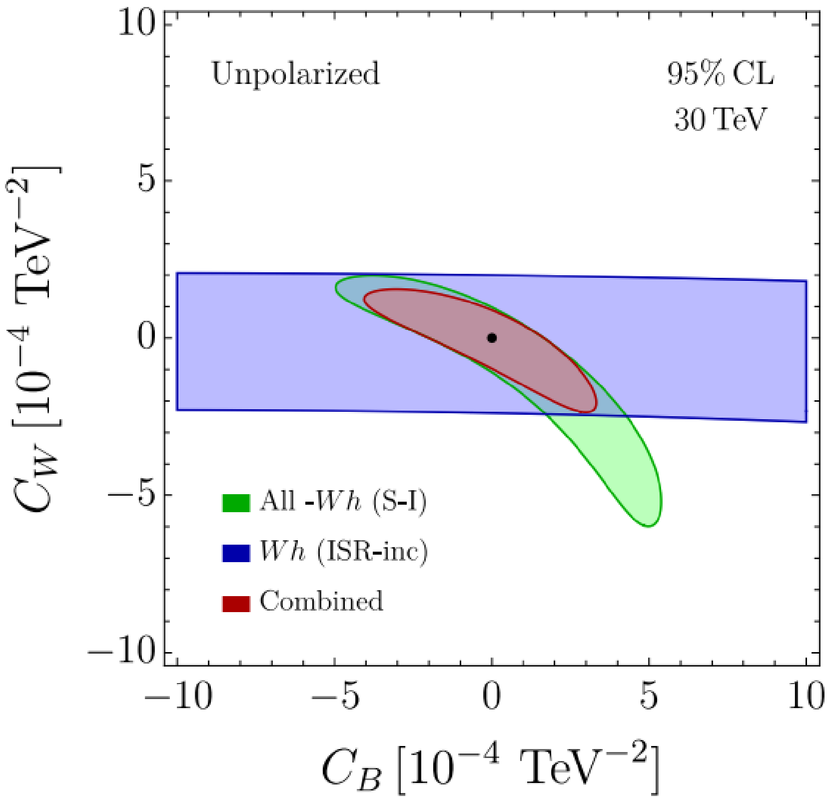
Chen, AG, Rattazzi, Ricci, Wulzer (2022)

Beam polarization reduces the effect of the flat direction

Beam polarization



ISR-inclusive

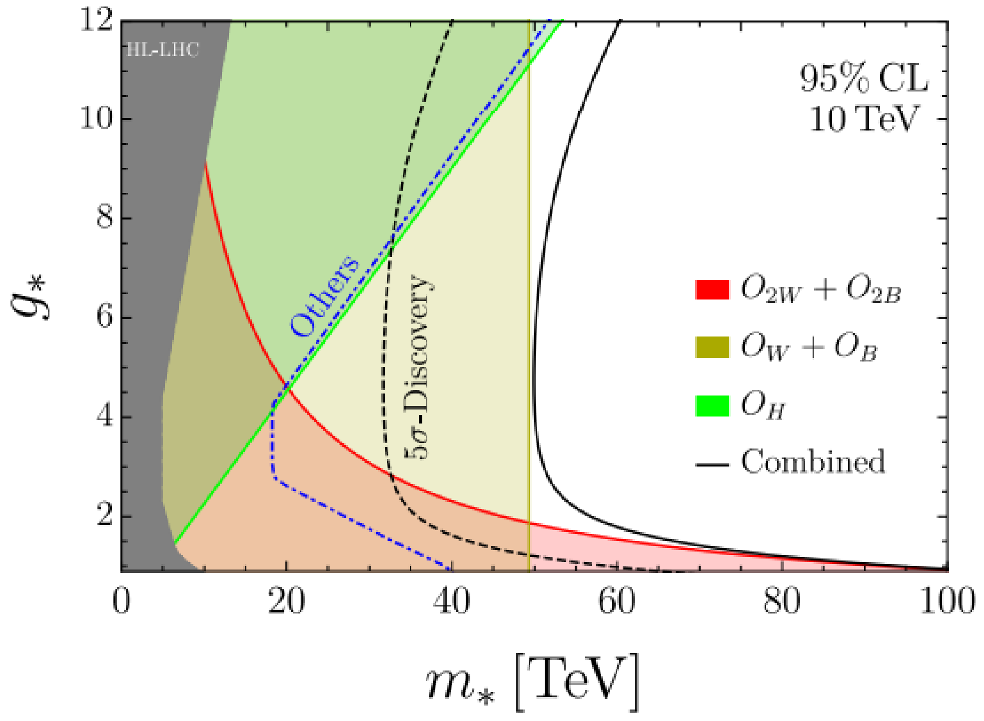


Wh exclusive on the final state but ISR inclusive effectively left-polarizes the beam

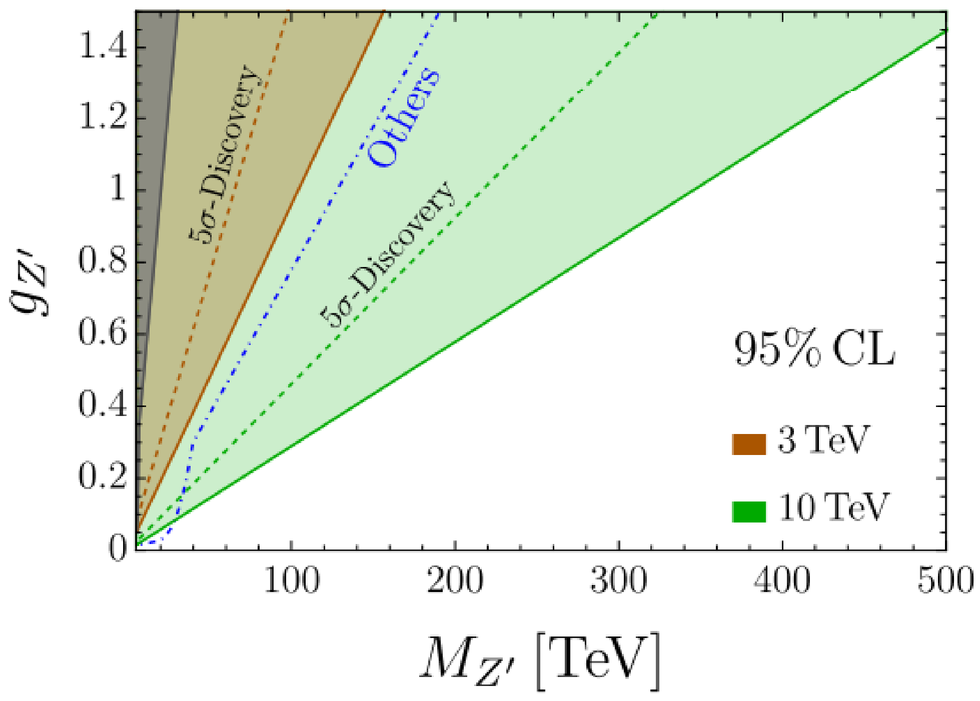
Reach on BSM models

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

CH universal reach



Z' model

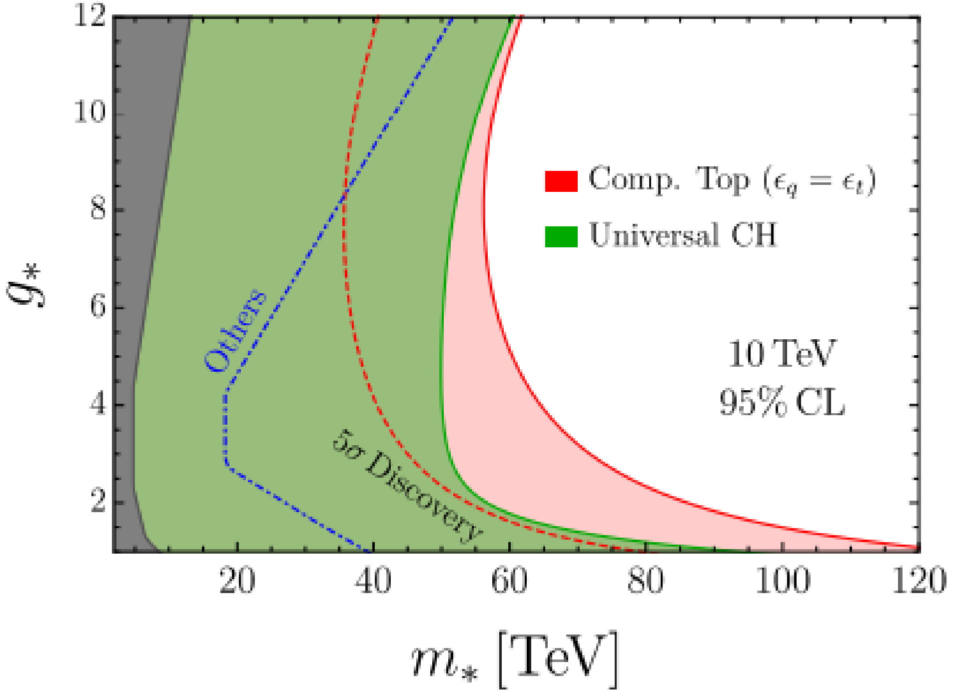


Tree-level prediction for OH from Buttazzo, Franceschini, Wulzer, 2020

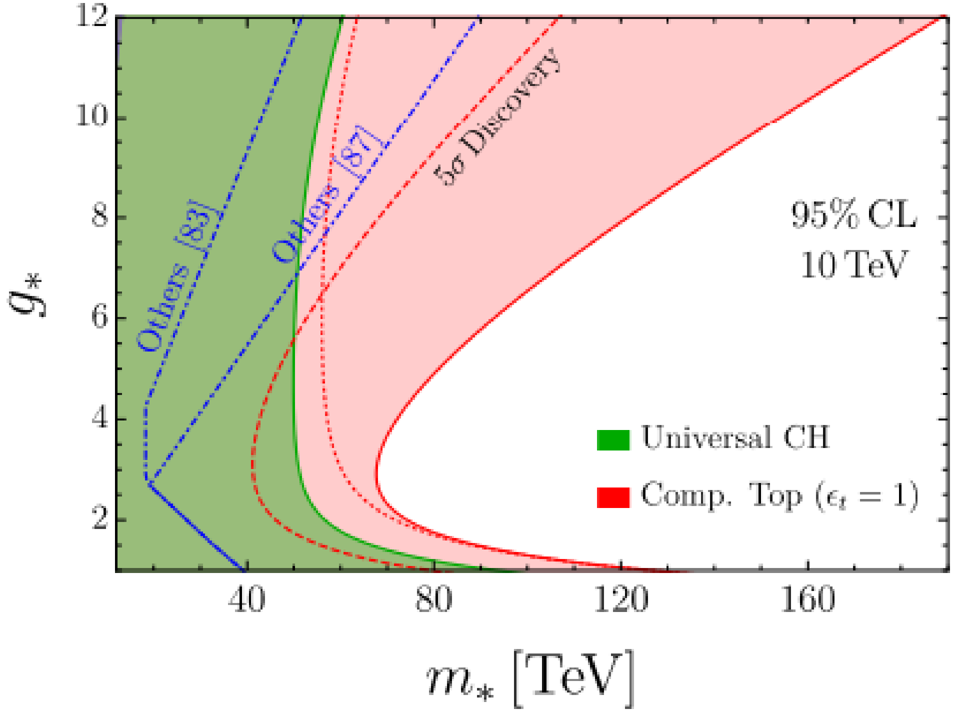
Composite top

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

Equally composite top



Right composite top



Conclusions

Chen, AG, Rattazzi, Ricci, Wulzer (2022)

- The effects **of EW radiation** are extremely **large** and must be **resumed** to reach % level of precision at a Muon Collider

- A **better understanding of EW radiation is needed**: how to compute more general observables? (eg. ISR inclusive) how to include single-logs?

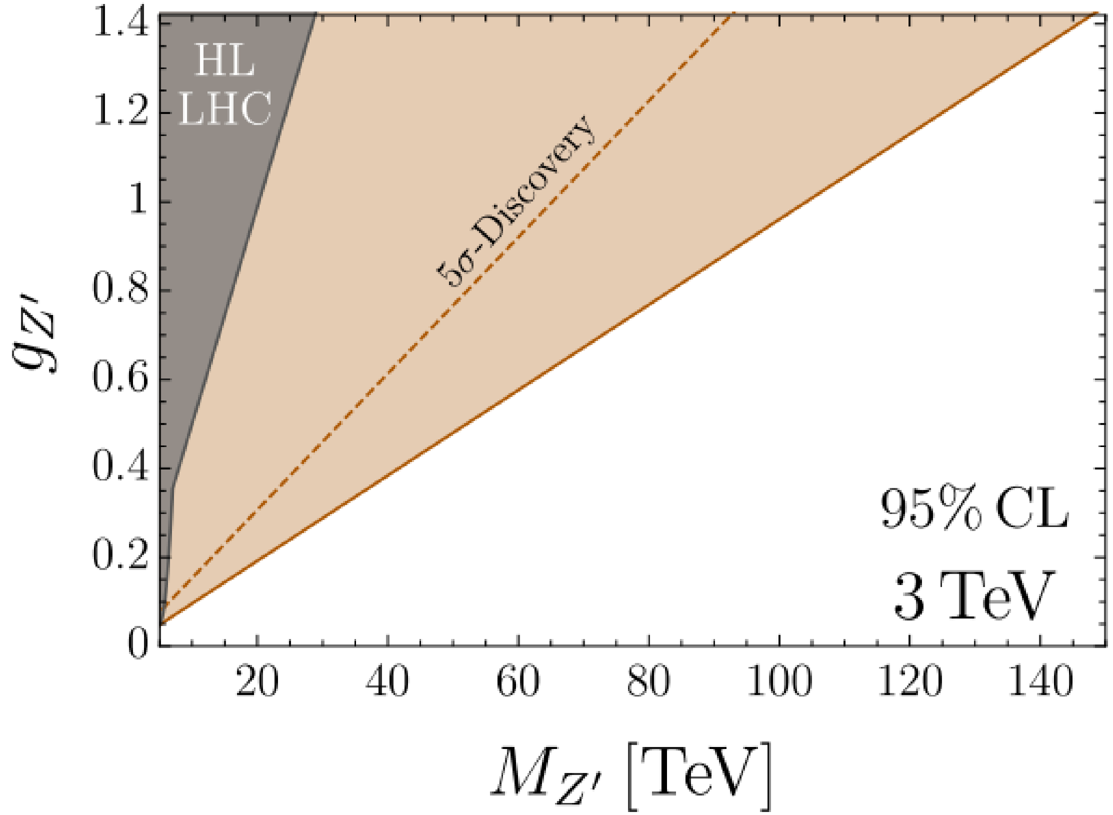
- Large radiation effect allows to **probe New Physics** in a **richer way** compared to tree-level prediction

- A **10 TeV Muon Collider** can indirectly probe New Physics up to **100+ TeV**

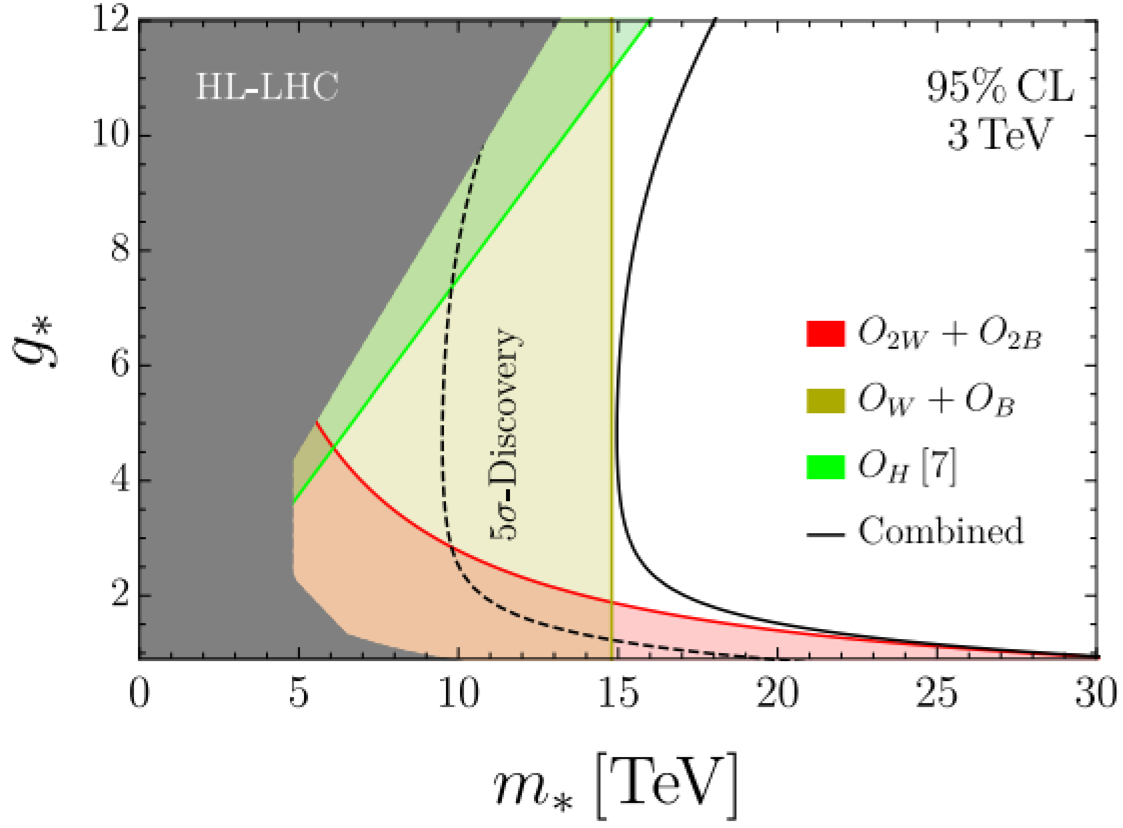
- **WIP**: a user-friendly implementation of these computations for these processes + production of heavy EW multiplets

BACKUP

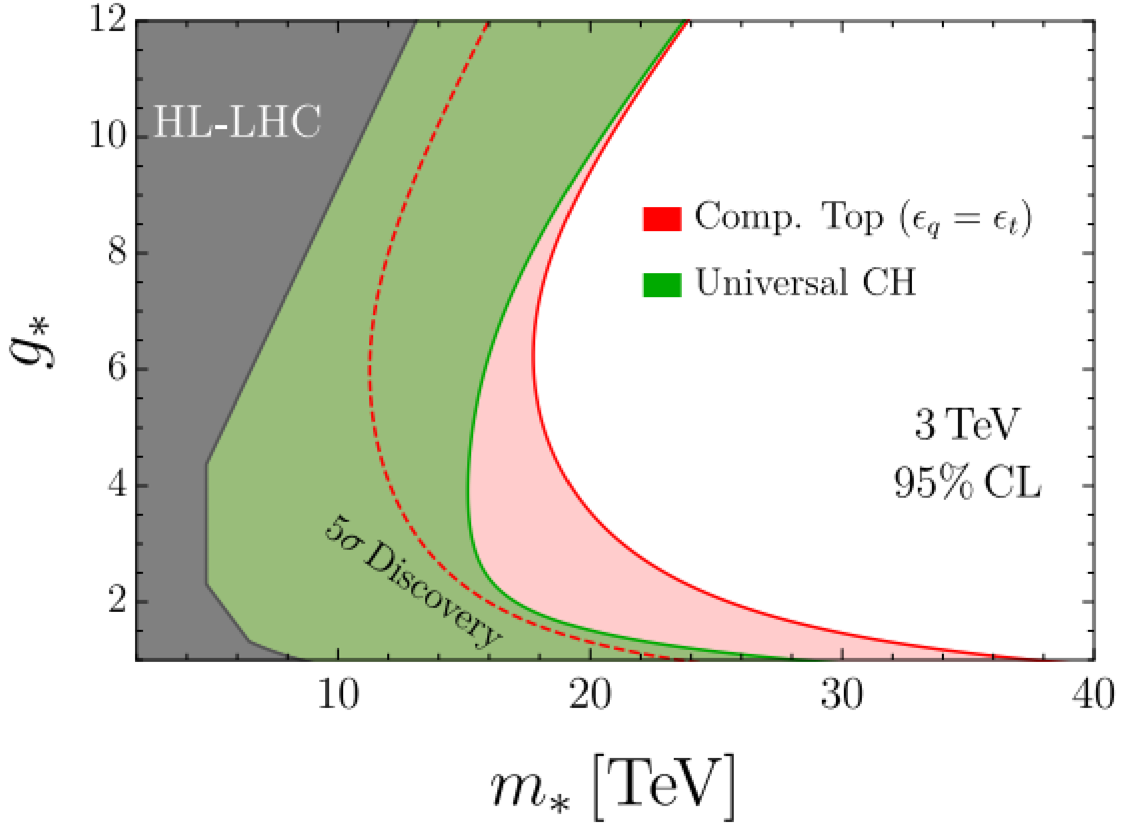
	3 TeV			10 TeV		
	DL	$e^{\mathrm{DL}}-1$	$\mathrm{SL}(\frac{\pi}{2})$	DL	$e^{\mathrm{DL}}-1$	$\mathrm{SL}(\frac{\pi}{2})$
$\ell_L \to \ell'_L$	-0.46	-0.37	0.25	-0.82	-0.56	0.33
$\ell_L \to q_L$	-0.44	-0.36	0.25	-0.78	-0.54	0.34
$\ell_L \to e_R$	-0.32	-0.27	0.13	-0.56	-0.43	0.17
$\ell_L \to u_R$	-0.27	-0.24	0.11	-0.48	-0.38	0.15
$\ell_L \to d_R$	-0.24	-0.21	0.10	-0.43	-0.35	0.13
$\ell_R \to \ell'_L$	-0.32	-0.27	0.13	-0.56	-0.43	0.17
$\ell_R \to q_L$	-0.30	-0.26	0.12	-0.53	-0.41	0.16
$\ell_R \to \ell'_R$	-0.17	-0.16	0.07	-0.30	-0.26	0.09
$\ell_R \to u_R$	-0.12	-0.12	0.05	-0.22	-0.20	0.07
$\ell_R \to d_R$	-0.09	-0.09	0.04	-0.17	-0.16	0.05



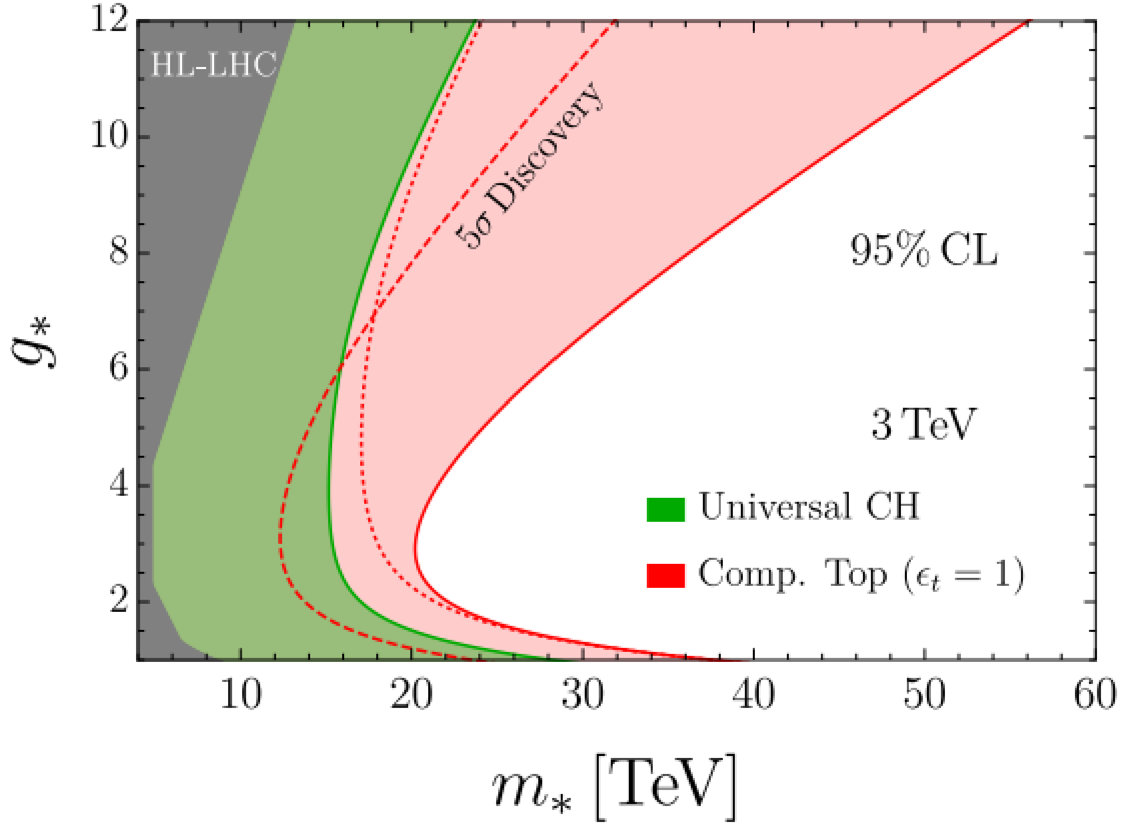
HL LHC
5σ-Discovery
95% CL
3 TeV
$g_{Z'}$
$M_{Z'}$ [TeV]



HL-LHC
95% CL
3 TeV
5σ-Discovery
$O_{2W} + O_{2B}$
$O_W + O_B$
O_H [7]
Combined
g_*
m_* [TeV]



HL-LHC
Comp. Top ($\epsilon_q = \epsilon_t$)
Universal CH
3 TeV
95% CL
5σ Discovery
g_*
m_* [TeV]



HL-LHC
5σ Discovery
95% CL
3 TeV
Universal CH
Comp. Top ($\epsilon_t = 1$)
g_*
m_* [TeV]