



# Updates on the Rapid Cycling Synchrotrons studies

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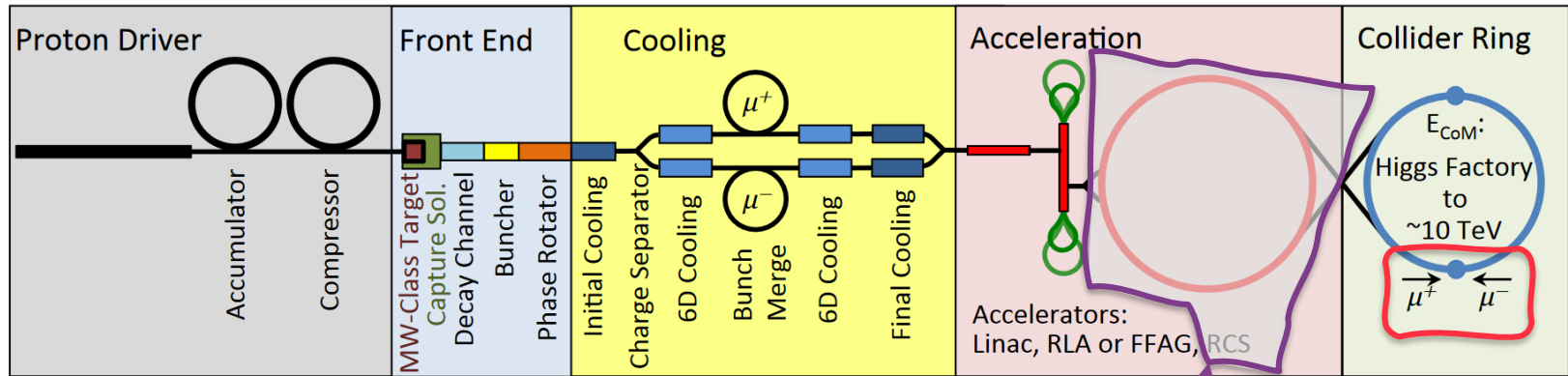
# Outline

- **Design aspects of the Rapid Cycling Synchrotrons**
- **Magnets and RF layout**
- **Synchrotron tune mitigation**
- **Nonlinear ramping functions**
- **Summary and outlook on longitudinal tracking studies**



# Reminder on design baselines

- Base for the work is the US Muon Accelerator Program (MAP)
- High energy complex consist of a chain of rapid cycling synchrotrons (RCS)



1 bunch per beam

Part of interest for us

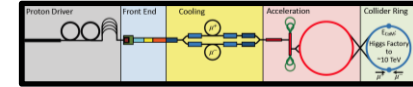
# Reminder on design baselines

- Design oriented on reaching the performance parameter [[webpage](#)]
- The relevant target parameters are: [[presentation](#) by D. Schulte]

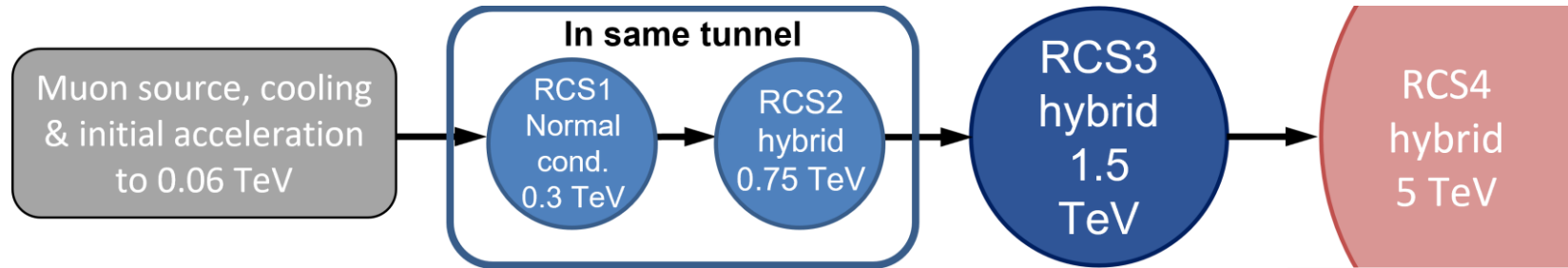
Parameter	Unit	3 TeV	10 TeV
L	$10^{34} \text{ cm}^{-2}\text{s}^{-1}$	1.8	20
N	$10^{12}$	2.2	1.8
$f_r$	Hz	5	5
$\langle B \rangle$ (average)	T	7	10.5
$\epsilon_L$ (norm, $1\sigma_z\sigma_E$ )	MeV m	7.5	7.5
$\sigma_E / E$	%	0.1	0.1
$\sigma_z$	mm	5	1.5

Repetition rate of 5 Hz  
→ RCS

# The high-energy complex



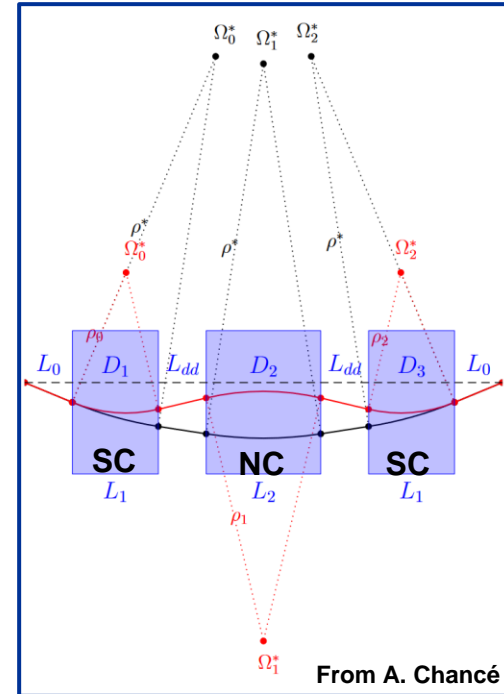
- Chain of rapid cycling synchrotrons, counter-rotating  $\mu^+/\mu^-$  beams  
 $\rightarrow 60 \text{ GeV} \rightarrow 314 \text{ GeV} \rightarrow 750 \text{ GeV} \rightarrow 1.5 \text{ TeV} \rightarrow 5 \text{ TeV}$



- Hybrid RCSs have interleaved normal conducting (NC) and superconducting (SC) magnets, see also talks by A. Chancé [[talk1](#), [talk2](#)]
- This would be the first hybrid RCSs in the world!

# Hybrid RCS magnet layout

- SC magnets provide high average  $B$ , but not fast ramping  $\rightarrow$  fixed-field,  $B_{sc} = 10$  T
- NC magnets require fast ramping within  $B_{nc} = \pm 1.8$  T
- Adopted achieved parameters: below saturation of both technologies
- Beam orbit not constant during acceleration
  - $\rightarrow$  Orbit length and  $f_{rev} \neq \text{const.}$
  - $\rightarrow$   $f_{RF}$  tuning for cavities



# Parameters and tools: General parameter

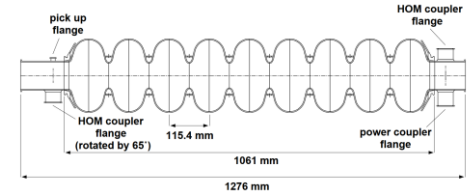
Details on RF requirements in presentation by H. Damerau

Detailed parameter table: [\[link\]](#)

	RCS1→314 GeV	RCS2→750GeV	RCS3→1.5TeV
Circumference, $2\pi R$ [m]	5990	5590	10700
Energy factor, $E_{ej}/E_{inj}$	5.0	2.4	2.0
Repetition rate, $f_{rep}$ [Hz]	5 (asym.)	5 (asym.)	5 (asym.)
Number of bunches	$1\mu^+$ , $1\mu^-$	$1\mu^+$ , $1\mu^-$	$1\mu^+$ , $1\mu^-$
Bunch population	$>2.5E12$	$>2.3E12$	$2.2E12$
Survival rate per ring	90%	90%	90%
Acceleration time, $t_{acc}$ [ms]	0.34	1.04	2.37
Number of turns	17	55	66
Energy gain per turn, $\Delta E$ [GeV]	14.8	7.9	11.4
Acc. gradient for survival [MV/m]	2.4	1.3	1.1
Acc. field in RF cavity [MV/m]	30 (45 optimistically)	30	30

	Symbol	Unit	Stage 1	Stage 2	Stage 3
			Value	Value	Value
Basic data					
Particles	$\mu$				
Costs	-	MC			
Type	-		RCS	hybrid RCS	hybrid RCS
Dynamics					
Acceleration time	$T_{acc}$	[ms]	0.34	1.09704995	2.37
Injection energy	$E_{inj}$	[MeV]	63000	313830	750000
Ejection energy	$E_{ej}$	[MeV]	313830 (defined by $\mu$ )	750000	1500000
Energy ratio	$E_{ej}/E_{inj}$		4.96	2.39	2.30
Momentum at $s$	$p[s]$	MeV/c	63106	313935	750106
Momentum at $e$	$p[e]$	MeV/c	213935	750106	1501066
Number of turns	$N_{turn}$		17	66	66
Planned Survival rate	$N_s/N_{inj}$		0.9	0.9	0.9
Total survival rate	$N_s/N_{inj}$		0.9	0.81	0.729
Accel. Gradient, linear for survival	$G_s$	[MV/m]	2.44	1.33	1.06
Required energy gain per turn	$\Delta E$	[MeV]	14755	7930	11364
Transition gamma	$\gamma_t$		20.41	20.41	-30
Injection relativistic mass factor	$\gamma_{inj}$		597	2971	7099
Ejection relativistic mass factor	$\gamma_{ej}$		2971	7099	14198
Injection v/c	$\beta_{inj}$	%	0.99999996	0.999999942	0.999999921
Ejection v/c	$\beta_{ej}$	%	0.999999943	0.999999901	0.999999975
Parameter Classical RCS					
Radius	$R$	[m]	953.3	953.3	1703.0
Circumference	$2\pi R$	[m]	5990	5990	10700
Circumference Ratio	$R_2/R_1$		1	1	1.79
Pack fraction	$\eta$		0.61	0.61	0.628
Bend radius	$\rho_b$	m	581.8	581.8	1070.2
Total straight section length	$L_{str}$	[m]	2338.7	2338.7	3975.7
Injection bending field (average)	$B_{inj}$	[T]	0.36	1.80	2.34
RF					
Systems	-		TESLA	TESLA	TESLA
Main RF frequency	$f_{RF}$	[MHz]	1300	1300	1300
Harmonic number	$h$		29667	29667	46367
Revolution frequency $\omega$	$f_{rev}$	[kHz]	50.08	50.08	28.64
Revolution period	$T_{rev}$	[ns]	20.0	20.0	35.7
Max RF voltage	$V_{RF}$	[kV]	20.87	11.22	16.07
Max RF power	$P_{RF}$	[MW]	-	-	-
RF Filling factor	-		0.4	0.4	0.45
Number RF stations	-		Around 50	Around 50	Around 50
Cavities	-		9-cell	9-cell	9-cell
Number of cavities	$N_c$		888	374	536
Peak impedance	$Z_{peak}$	[ $\Omega$ ]	-	-	-
Gradient in cavity	$\Delta V/L$	[MV/m]	30	30	30
Average energy gain per total straight	$\Delta E/L$	[MeV/m]	6.3	3.4	2.9
Accelerating field per total straight	$\Delta V/L$	[MeV/m]	6.9	4.8	4.0
Accelerating field gradient, with FF	$\Delta V/L$	[MeV/m]	22.3	12.0	9.0
Stable phase	$\phi_s$	[ $^\circ$ ]	45	45	45
Conversion factor mm mrad -eVs	$k$	$\mu\text{m mrad}^{-1} \text{eVs}^{-1}$	69.40	166.86	331.72
Longitudinal emittance ( $\sigma_x^2 + \sigma_z^2$ )	$\epsilon_{L,0}$	[eVs]	0.02575 5eV m	0.025	0.025
Longitudinal emittance (phase space area)	$\epsilon_{L,0}$	[eVs]	0.079	0.079	0.079
Injection bucket area	$A_{inj}$	[eVs]	0.62	1.01	1.40
Ejection bucket area	$A_{ej}$	[eVs]	1.37	1.56	1.97
Bucket area reduction factor	$A_{inj}/A_{ej}$		0.172	0.172	0.172
Horizontal betatron tune	$Q_x$		-	-	-
Vertical betatron tune	$Q_y$		-	-	-
Average horizontal Twiss beta	$\beta_x$	[m]	10	10	10
Average vertical Twiss beta	$\beta_y$	[m]	10	10	10
Injection synchrotron frequency	$f_{s,inj}$	[kHz]	76.83	25.07	14.53
Ejection synchrotron frequency	$f_{s,ej}$	[kHz]	34.20	18.22	10.27
Injection synchrotron tune $Q_s$	$Q_{s,inj}$		1.52	0.50	0.52
Ejection synchrotron tune $Q_s$	$Q_{s,ej}$		0.68	0.32	0.37

# The cavity assumption: TESLA



(From [design report](#))

- High  $G_{acc}$  and strong beam loading  $\rightarrow$  SRF
- $\rightarrow$  1.3 GHz TESLA-like cavity as assumption for muon collider RCS [\[Phys. Rev. ST Accel. Beams 3, 092001, 2000\]](#)
- Cavity parameter (9 cells,  $L=1.06$  m):
  - Harmonic number  $h = 25957$  to 46367
  - $R/Q = 518 \Omega$ , total  $R_s = 306 G\Omega$
  - Gradient of structure 30 MV/m
  - $Q_L = 2.2e6$  (for beam loading compensation with  $\Delta f = 320$  Hz, [\[ref\]](#)), probably value to high and to be corrected

Table 2: TTF cavity design parameters.<sup>a</sup>

type of accelerating structure	standing wave
accelerating mode	TM <sub>010</sub> , $\pi$ mode
fundamental frequency	1300 MHz
design gradient $E_{acc}$	25 MV/m
quality factor $Q_0$	$> 5 \cdot 10^6$
active length $L$	1.038 m
number of cells	9
cell-to-cell coupling	1.87 %
iris diameter	70 mm
geometry factor	270 $\Omega$
$R/Q$	518 $\Omega$
$E_{peak}/E_{acc}$	2.0
$B_{peak}/E_{acc}$	4.26 mT/(MV/m)
tuning range	$\pm 300$ kHz
$\Delta f/\Delta L$	315 kHz/mm
Lorentz force detuning at 25 MV/m	$\approx 600$ Hz
$Q_{ext}$ of input coupler	$3 \cdot 10^6$
cavity bandwidth at $Q_{ext} = 3 \cdot 10^6$	430 Hz
RF pulse duration	1330 $\mu$ s
repetition rate	5 Hz
fill time	530 $\mu$ s
beam acceleration time	800 $\mu$ s
RF power peak/average	208 kW/1.4 kW
number of HOM couplers	2
cavity longitudinal loss factor $k_l$ for $\sigma_z = 0.7$ mm	10.2 V/pC
cavity transversal loss factor $k_t$ for $\sigma_x = 0.7$ mm	15.1 V/pC/m
parasitic modes with the highest impedance :	type
$\pi/9$ $(R/Q)/$ frequency	TM <sub>011</sub>
$2\pi/9$ $(R/Q)/$ frequency	80 $\Omega$ /2454 MHz
	67 $\Omega$ /2443 MHz
bellows longitudinal loss factor $k_l$ for $\sigma_z = 0.7$ mm	1.54 V/pC
bellows transversal loss factor $k_t$ for $\sigma_x = 0.7$ mm	1.97 V/pC/m



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Details on RF requirements in presentation by H. Damerau

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- High  $\Delta E = V_{RF} \cdot \cos(\phi_s) \rightarrow$  Unique RF requirements such as high synchrotron tune

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Max RF power	$P_{RF}$	[MW]	-	-	-
RF Filling factor	-	-	0.4	0.4	0.45
Number RF stations	-	-	Around 50	Around 50	Around 50
Cavities	-	-	-	9-cell	9-cell
Number of cavities	$N_c$	-	88	374	536
Peak impedance	$Z_{peak}$	[ $\Omega$ ]	-	-	-
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Ejection bucket area	$A_{ej}$	[eVs]	1.37	1.56	1.97
Bucket area reduction factor	$A_{inj}/A_{ej}$	-	0.172	0.172	0.172
Horizontal betatron tune	$Q_x$	-	-	-	-
Vertical betatron tune	$Q_y$	-	-	-	-
Average horizontal Twiss beta	$\beta_x$	[m]	10	10	10
Average vertical Twiss beta	$\beta_y$	[m]	10	10	10
Injection synchrotron frequency	$f_{syn}$	[kHz]	76.83	25.07	16.13
Ejection synchrotron frequency	$f_{syn}$	[kHz]	34.20	16.22	10.27
Injection synchrotron tune $Q_s$	$Q_s$	-	1.52	0.50	0.52
Ejection synchrotron tune $Q_s$	$Q_s$	-	0.68	0.32	0.37

# Synchrotron tune and number of RF stations

- Number of synchrotron oscillations per turn proportional to  $\sqrt{V_{\text{RF}}}$ :

$$Q_s = \frac{\omega_s}{\omega_0} = \sqrt{-\frac{h\eta e V_{\text{RF}} \cos \phi_s}{2\pi E \beta^2}} \propto \sqrt{V_{\text{RF}} \cos \phi_s}$$

LHC:  $Q_s=0.005$

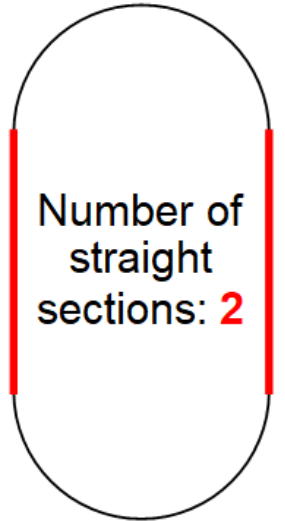
- Stable synchrotron oscillations and phase focusing only for  $Q_s \ll 1/\pi$  (T. Suzuki, [KEK Report 96-10](#))

→ RCSs would exceed this limit:  $0.3 < Q_s < 1.5$

→ Several longitudinal kicks per turn for small  $Q_s$  between stations, i.e., small  $Q_s/n_{\text{RF}}$

→ Distribute RF system over  $n_{\text{RF}}$  sections

→  $n_{\text{RF}}$  is an important quantity to determine!



From: H. Damerau



# Synchrotron tune and number of RF stations

International  
UON Collider  
Collaboration

*Why not choosing a high  $n_{RF}$  to fulfil  $Q_s \ll 1/\pi$  ?*

- High  $n_{RF} \rightarrow$  smaller quadrupole-like oscillations caused by discrete energy steps and resulting mismatching
- **BUT:** higher  $n_{RF}$  results in higher construction / cooling / cryogenics and powering costs, even though the number of cavities is constant and defined by  $\Delta E$  per turn, plus lattice restrictions

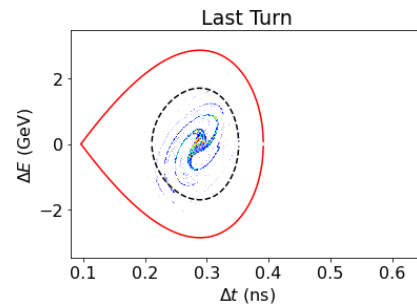
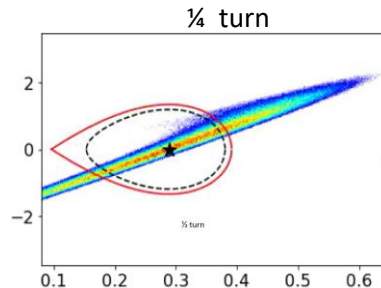
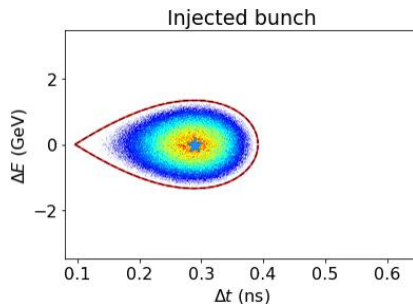
$\rightarrow$  Determine emittance growth, also as a function of  $n_{RF}$

**Examples:**  $n_{RF} = 4$

RCS1, no  $V_{ind}$ ,  $\mathcal{E}_L = 0.31$  eVs

$$\frac{Q_s}{n_{RF}} = \frac{1.5}{4} = 0.38$$

High  $Q_s$  destroys bunch!





# Synchrotron tune and number of RF stations

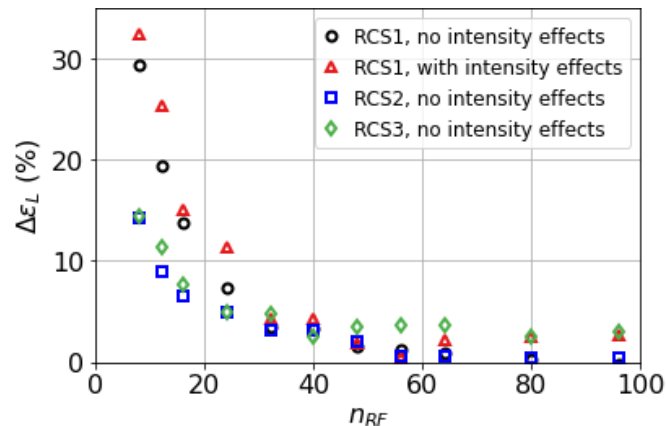
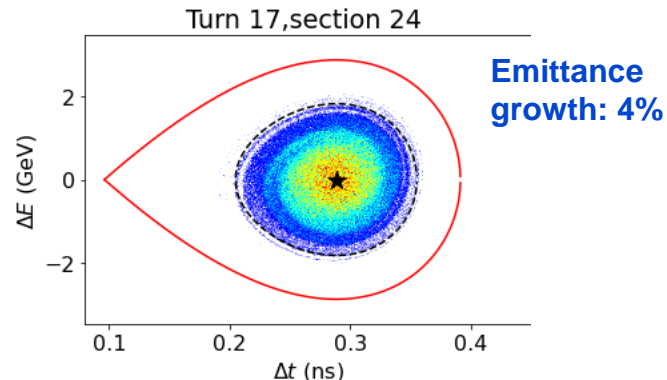
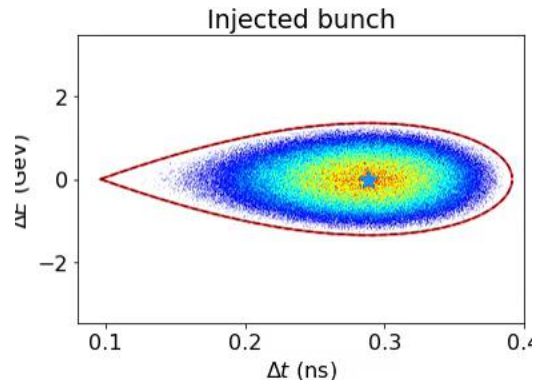
International  
UON Collider  
Collaboration

**Examples:**  $n_{RF} = 32$

RCS1, no  $V_{ind}$ ,  $\varepsilon_L = 0.31$  eVs

$$\frac{Q_s}{n_{RF}} = 0.05$$

- For  $n_{RF} > 48$ ,  $\Delta\varepsilon_L = 3\%$ , no further improvement
- For RCS1,  $n_{RF} \approx 32$  best candidate
- For RCS2 and 3,  $n_{RF} \approx 24$  could be enough



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Number of turns			
Energy gain per turn, $\Delta E$ [GeV]			
Acc. gradient for survival [MV/m]			
Acc. field in RF cavity [MV/m]			
Ramp rate, $\dot{B}_{nc}$ [kT/s]	4199	3281	1518

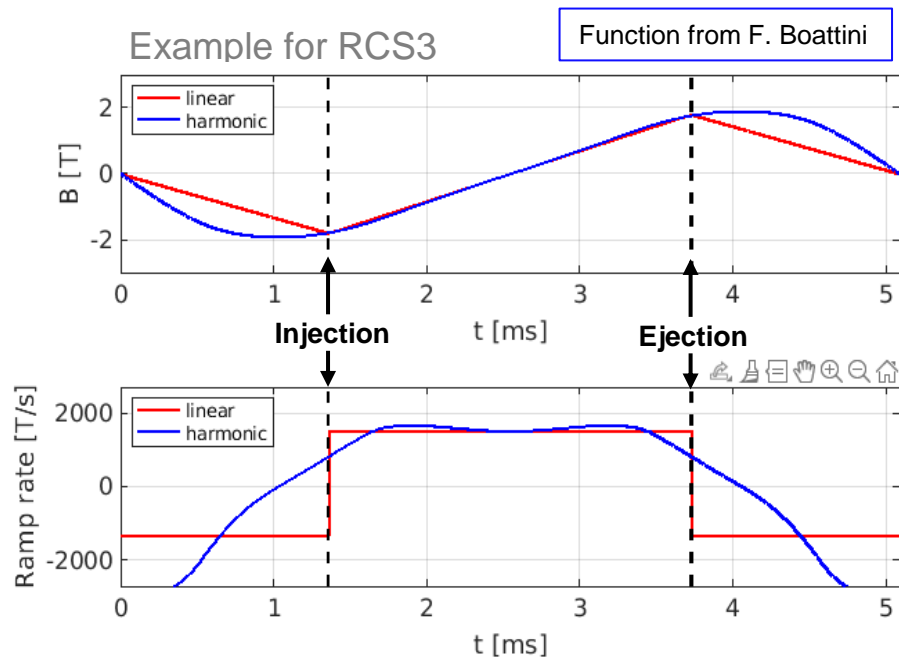
Fast ramping within  $B_{nc} = \pm 1.8 T$

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38	Pack fraction	$\eta$	0.61	0.61	0.628
39	Bend radius	$\rho_b$ m	581.8	581.8	1070.2
40	Total straight section length	$L_{str}$ [m]	2338.7	2338.7	3975.7
41	Injection bending field (average)	$B_{inj}$ [T]	0.36	1.80	2.34
42	RF				
43	Systems		TESLA	TESLA	TESLA
44	Main RF frequency	$f_{RF}$ [MHz]	1300	1300	1300
45	Harmonic number	$h$	2967	2967	4637
46	Revolution frequency $\omega$	$f_{rev}$ [kHz]	50.08	50.08	28.84
47	Revolution period	$T_{rev}$ [ns]	20.0	20.0	35.7
48	Max RF voltage	$V_{RF}$ [kV]	20.87	11.22	16.87
49	Max RF power	$P_{RF}$ [MW]	0.4	0.4	0.45
50	RF Filling factor		Around 50	Around 50	Around 50
51	Number RF stations				
52	Cavities		9-cell	9-cell	9-cell
53	Number of cavities		81	81	81
54	Peak impedance	$Z_{peak}$ [Ω]	30	30	30
55	Gradient in cavity	$\Delta V/L$ [MeV/m]	6.3	6.3	6.3
56	Average energy gain per total straight	$\Delta E/L$ [MeV/m]	6.9	4.8	4.0
57	Accelerating field per total straight	$\Delta V/L$ [MeV/m]	6.9	4.8	4.0
58	Accelerating field gradient, with FF	$\Delta V/L$ [MeV/m]	22.3	12.0	9.0
59	Stable phase	$\phi_s$ [°]	45	45	45
60	Conversion factor mm mrad - eVs	$k_{conv}$ m/rp	69.40	166.86	331.72
61	Longitudinal emittance ( $\sigma_E^2 + \Delta z^2$ )	$\epsilon_{long}$ [eVs]	0.02575 MeV m	0.025	0.025
62	Longitudinal emittance (phase space area)	$\epsilon_{long}$ [eVs]	0.079	0.079	0.079
63	Injection bucket area	$A_{inj}$ [eVs]	1.01	1.01	1.40
64	Ejection bucket area	$A_{ej}$ [eVs]	1.37	1.56	1.97
65	Bucket area reduction factor	$A_{inj}/A_{ej}$	0.172	0.172	0.172
66	Horizontal betatron tune	$Q_x$	10	10	10
67	Vertical betatron tune	$Q_y$	10	10	10
68	Average horizontal Twiss beta	$\beta_x$ [m]	10	10	10
69	Average vertical Twiss beta	$\beta_y$ [m]	10	10	10
70	Injection synchrotron frequency	$f_{inj}$ [kHz]	34.20	16.22	10.27
71	Ejection synchrotron frequency	$f_{ej}$ [kHz]	1.52	0.50	0.52
72	Injection synchrotron tune $Q_s$	$Q_{s,inj}$	0.68	0.32	0.37
73	Ejection synchrotron tune $Q_s$	$Q_{s,ej}$			



# Fast ramping considerations

- **Optimization problem** between linearizing magnet ramps and installed voltage, or muon loss
  - **Linear ramping**  $\rightarrow$  constant  $V_{RF}$   
 $\rightarrow$  simplest RF solution, best for  $\mu$
  - **Non-linear ramping**  $\rightarrow$  decrease peak power  
 $\triangleq$  magnet powering costs significantly  
(see [talk](#) by F. Boattini)
  - **Sinusoidal ramp function**  $\rightarrow$  performance decrease of 50%
- $\rightarrow$  **Study quasi-linear ramping** by e.g. natural resonant discharge of e.g. two harmonics

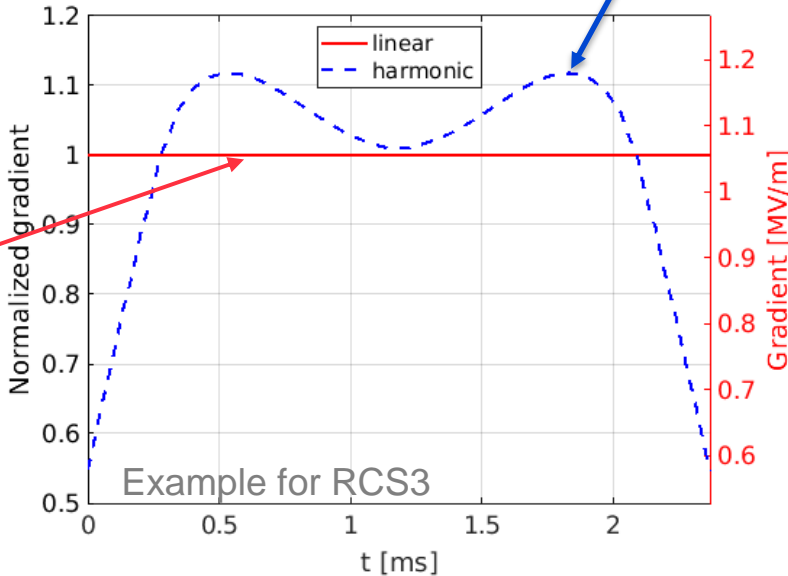
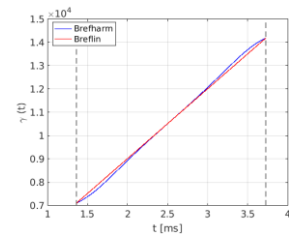
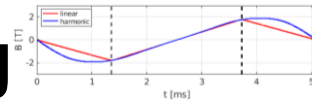




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# RF requirements with non-linear ramping

- $V_{acc}$  and  $G_{acc}$  must be increased by 12% to achieve the same  $\tau_{acc}$   
 $\Leftrightarrow \neq 200\%$  as for a sine-like ramp!



Constant  
gradient for  
linear ramp

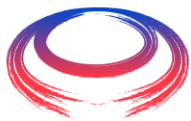
Average gradient over ring  
for 90% survival

$B \propto E$  defines all dynamics!

$$G_{harm}(t) = \frac{(\gamma_{ej} - \gamma_{inj})}{2} \cdot \frac{m_{\mu}}{c} \left( \frac{\dot{B}_{harm}(t)}{B_{ej}} \right)$$





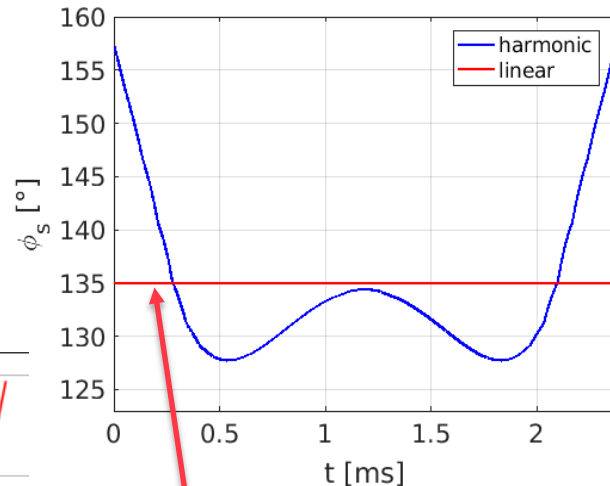
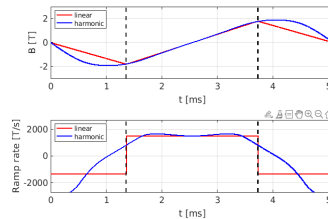
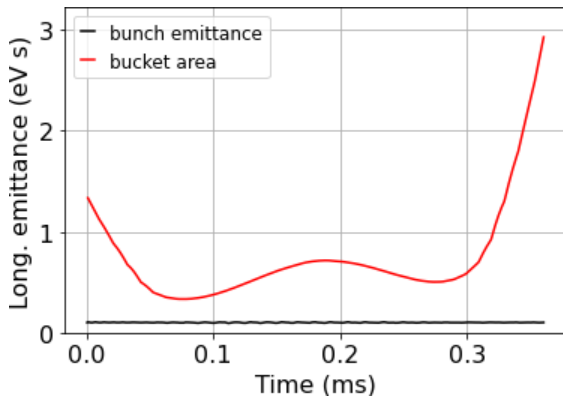
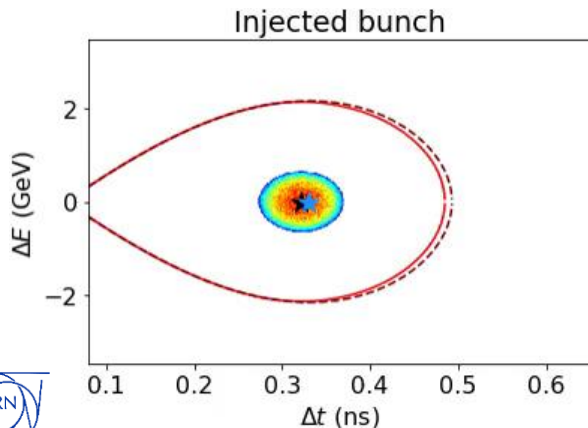


# Acc. gradient with non-linear ramping

→ Adjust the voltage by sweeping the synchrotron phase  $V(t) = V_{RF} \cdot \sin(\phi_s(t))$

and 
$$\phi_s(t) = \arcsin\left(\frac{\dot{B}_{harm}(t)}{\dot{B}_{lin}(t)} \cdot \sin \phi_{s,0}\right)$$

- Example for RCS3, no intensity effects:



$\phi_{s,0} = \pi - 45^\circ$  for linear, above transition



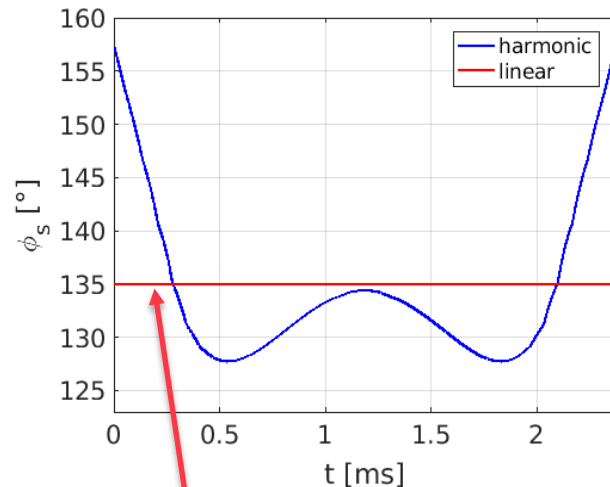
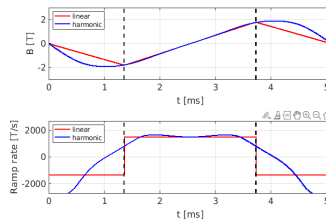
# Acc. gradient with non-linear ramping

→ Adjust the voltage by sweeping the synchrotron phase  
 $V(t) = V_{RF} \cdot \sin(\phi_s(t))$

and  $\phi_s(t) = \arcsin\left(\frac{\dot{B}_{harm}(t)}{\dot{B}_{lin}(t)} \cdot \sin \phi_{s,0}\right)$

- Example for RCS3, no intensity effects

→ Powering and ramping function optimization ongoing, combined with synchronous phase and RF voltage optimization (see [next](#) talk and [talk](#) by F. Boattini)



$\phi_{s,0} = \pi - 45^\circ$  for linear, above transition

# Summary

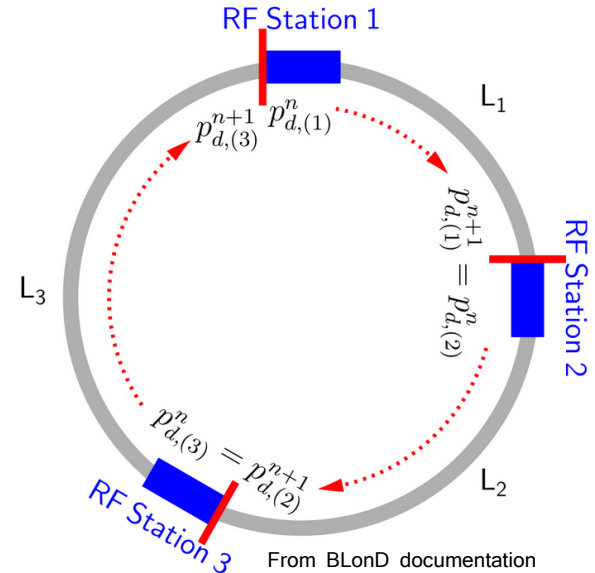
- **The muon decay brings unique challenges: fast acceleration, large voltages, high intensities, high synchronous tune, small number of turns**
- **Fast ramping asks for RCS, high energies for hybrid magnet structure**
- **Developing an integrated design with respect to magnets + RF and ramping function, but also lattice (see talks by A. Chancé)**
- **Large, number of RF stations, e.g. 30, to mitigate extreme synchrotron tune**
- **Magnet ramping and RF voltages require optimization of acceleration parameters (RF voltages, synchronous phase, acceleration time = decay)**



# The BLonD code

(Beam Longitudinal Dynamics code)

- **BLonD**: macro-particle tracking code, developed at CERN since 2014
- **Links**: [documentation](#) and [github](#)
- **MuC-specific to multiple RF stations & muon decay**
- **Studies of today with only one bunch, 2<sup>nd</sup> to follow**



# Studies with BLoND

(Beam Longitudinal Dynamics code)

- Using the **BLoND** code to observe effects of
  - Synchrotron tune  $Q_s$
  - Choice of synchronous phase
  - Short-range wakefields
  - Beam loading at fundamental frequency
  - Induced HOM powers
  - Nonlinear ramping functions (RF and magnets)