IWHSS 2023
International Workshop on Hadron Structure and Spectroscopy 2023

Fragmentation Functions from e^+e^- annihilation experiments

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University of Ferrara

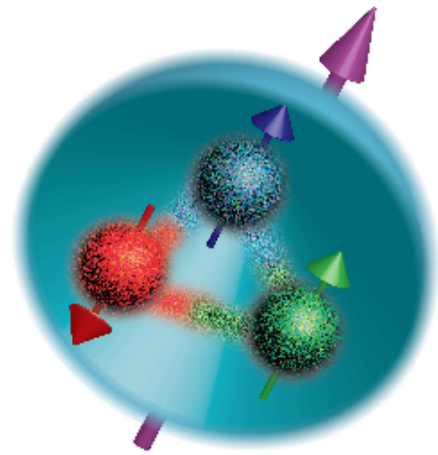
INFN Istituto Nazionale di Fisica Nucleare

STRONG 2020

Introduction

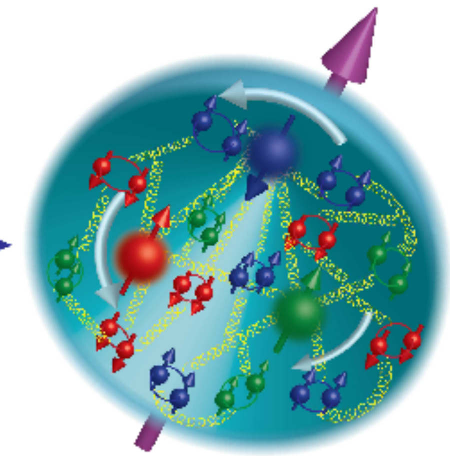
QCD picture of the nucleon

Naive picture



three non-relativistic quarks

Realistic picture



infinite number of relativistic quarks and gluons

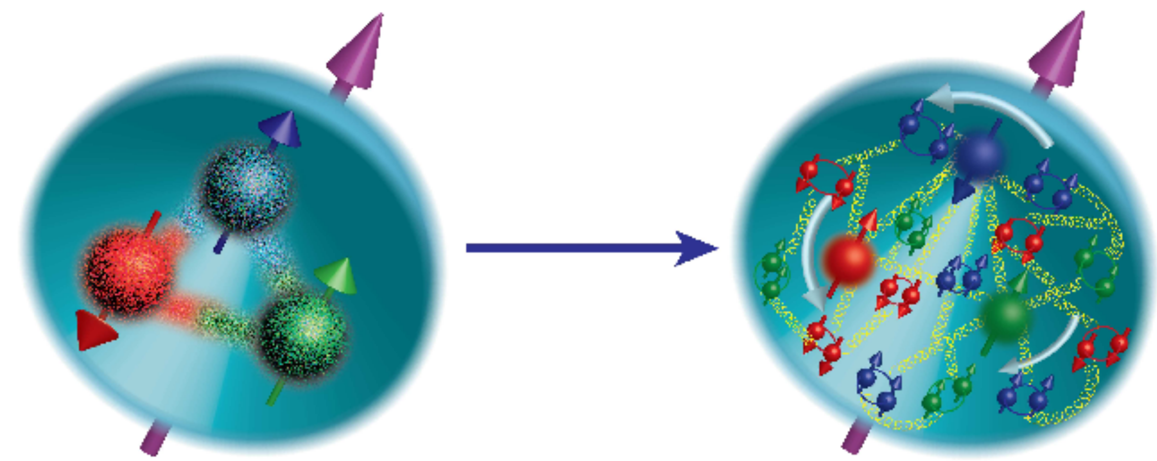
Parton Distribution Functions (PDFs)

https://www.annualreviews.org/doi/pdf/10.1146/annurev-nucl-011720-042725

Introduction

Naive picture | Realistic picture

QCD picture of the nucleon



Parton Distribution Functions (PDFs)

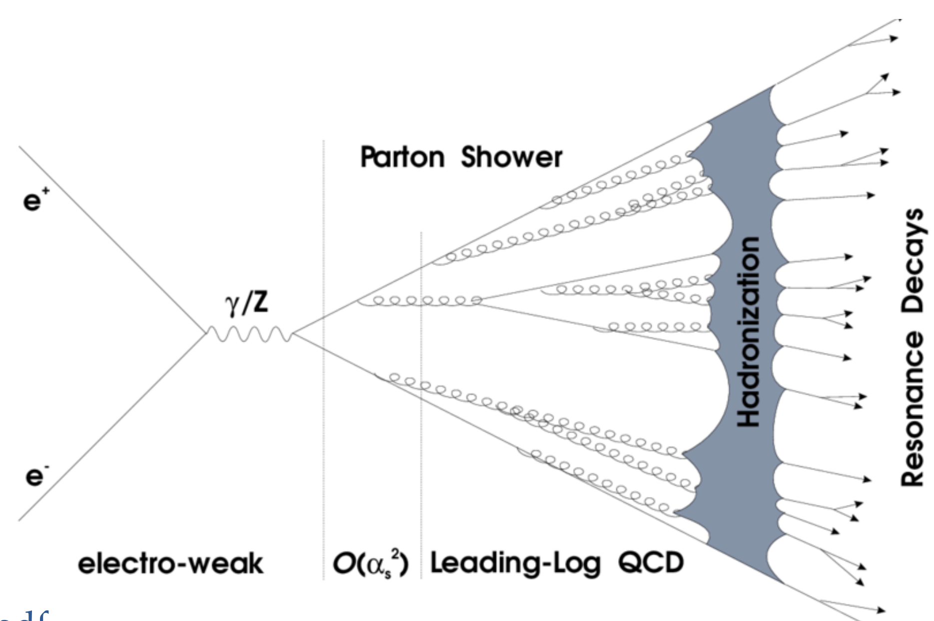
three non-relativistic quarks

infinite number of relativistic quarks and gluons

https://www.annualreviews.org/doi/pdf/10.1146/annurev-nucl-011720-042725

Hadron formation

- How many particles and how many jets created?
- What fraction of the initial parton momenta do they carry?

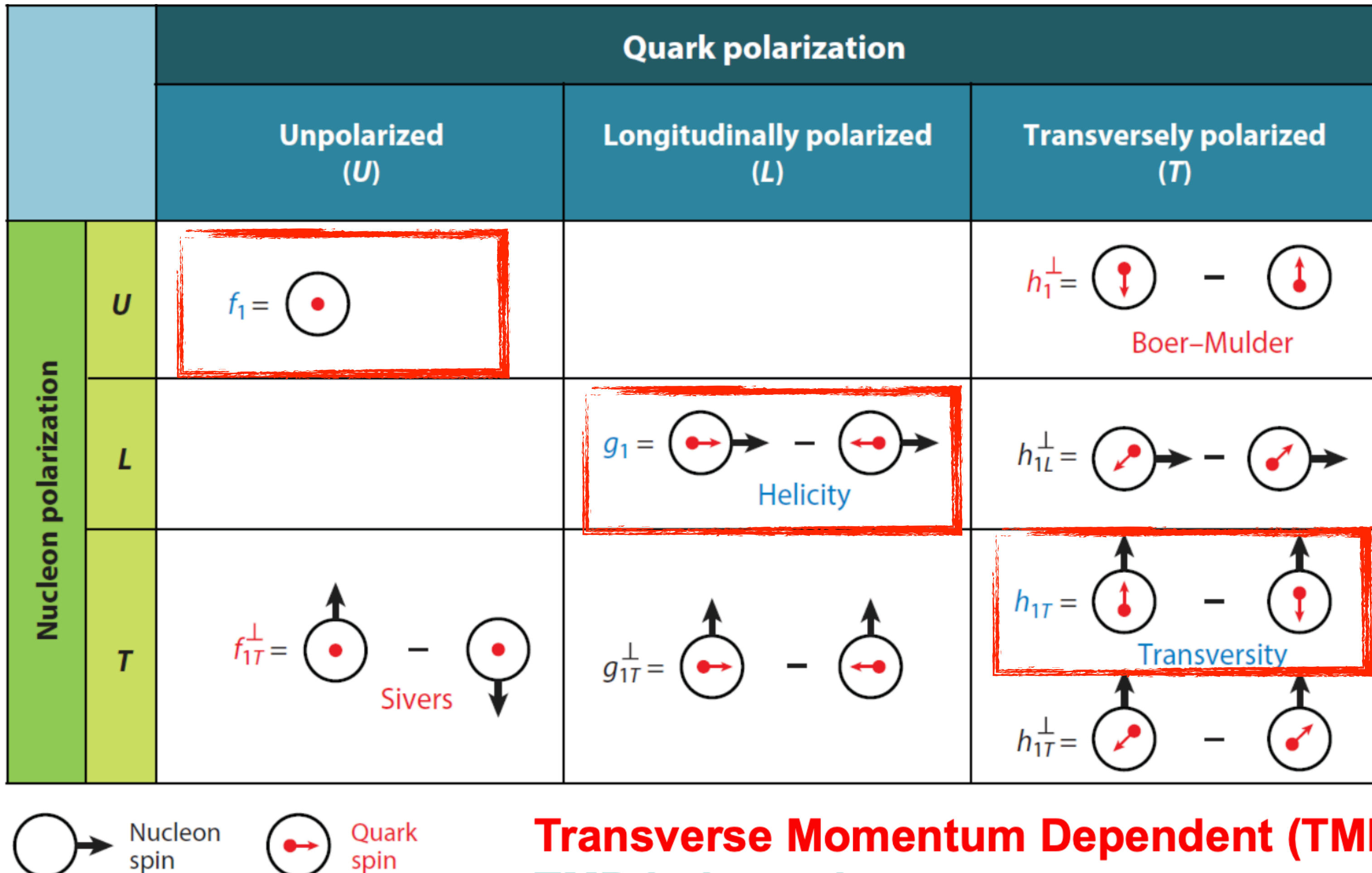


Fragmentation Functions (FFs)

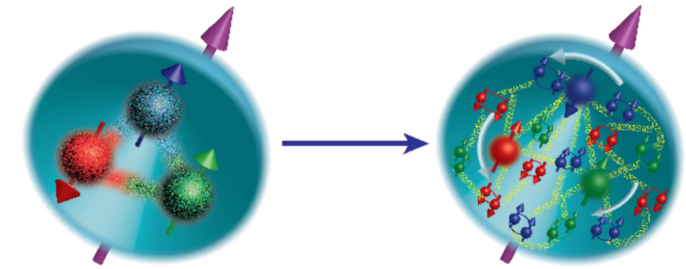
https://pdg.lbl.gov/2019/reviews/rpp2019-rev-frag-functions.pdf

Universal *and* ***non-perturbative*** *objects*

Parton Distribution Functions



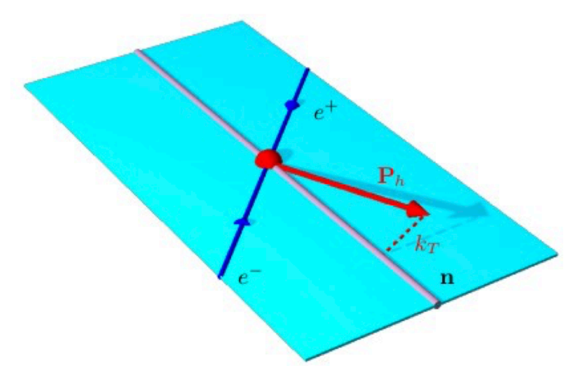
Transverse Momentum Dependent (TMD)
TMD independent



f_1, g_1, and h_1 : three leading-twist PDFs
- probability interpretation;
- complete description of the nucleon structure at leading-twist level;
- provide 2-dim (collinear) picture of the nucleon structure
- TMD: 3-D picture

h1: Transversity PDF

- Describe the distribution of quark's transverse spin in a transversely polarized nucleon
- Chiral-odd



Fragmentation Functions

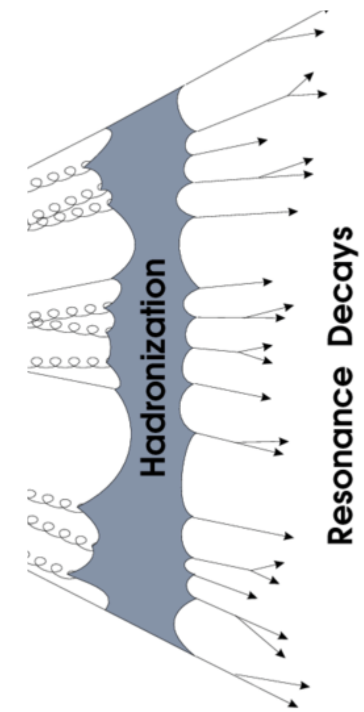
Formation of colourless hadrons starting from a coloured partonic initial state

Nucleon polarization	Quark polarization: Unpolarized (U)	Quark polarization: Longitudinally polarized (L)	Quark polarization: Transversely polarized (T)
U	D_1 = Unpol.		$H_1^\perp$ = − Collins
L		G_1 = −	$H_{1L}^\perp$ = −
T	$D_{1T}^\perp$ = − Polarized FF	$G_{1T}^\perp$ = −	H_1 = − $H_{1T}^\perp$ = −

Nucleon spin   Quark spin

Transverse Momentum Dependent (TMD)
TMD independent

- probability that a parton i fragments into an hadron h carrying away a fraction z of the parton's momentum



$H_1^\perp$: Collins FF

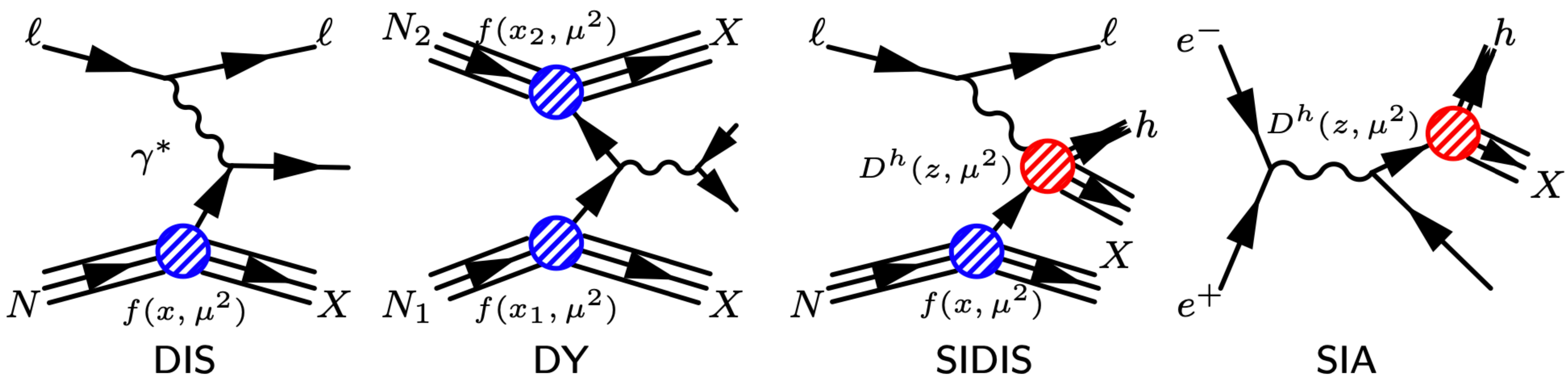
- Chiral-odd function

Transverse Momentum Dependent (TMD) FFs ⇒ to study the spin-dependent observables

- tools to investigate the 3D-structure of nucleons
- when only spinless hadrons (π, K) are considered, we have only D_1 and $H_1^\perp$
- TMDs evolution

How to access PDFs and FFs

- Physical observables are written as a convolutions of coefficient functions and PDFs/FFs
- *QCD factorization theorem*: separate the cross sections into a perturbatively calculable scattering contributions and non-perturbative one encoded in PDFs/FFs



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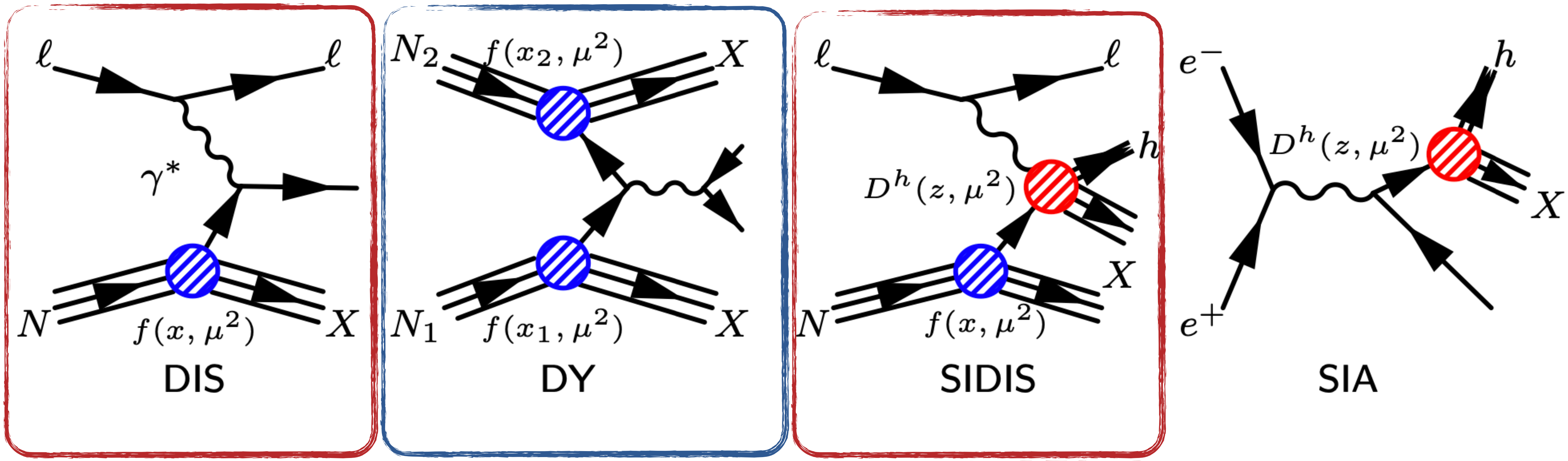

Flavour/charge separation
SIDIS: needed input on FFs;
Limited access to gluons

Direct access to PDFs; Sensitive to gluon FFs; large z behaviour;
Large theoretical uncertainties; no sensitive to heavy quarks

How to access PDFs and FFs

- Physical observables are written as a convolutions of coefficient functions and PDFs/FFs
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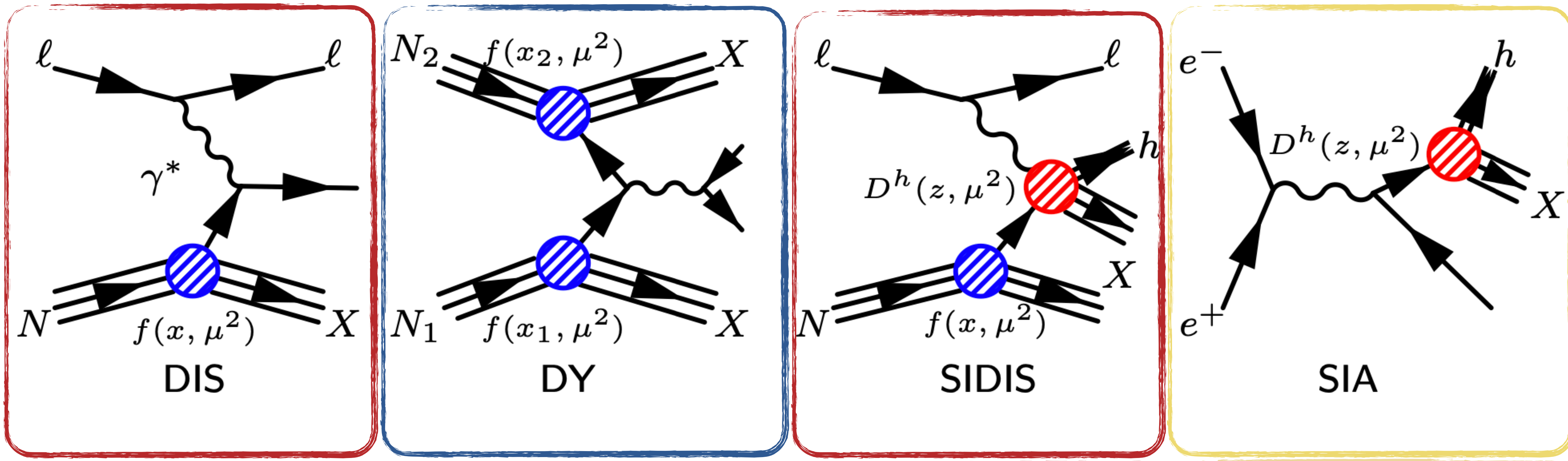


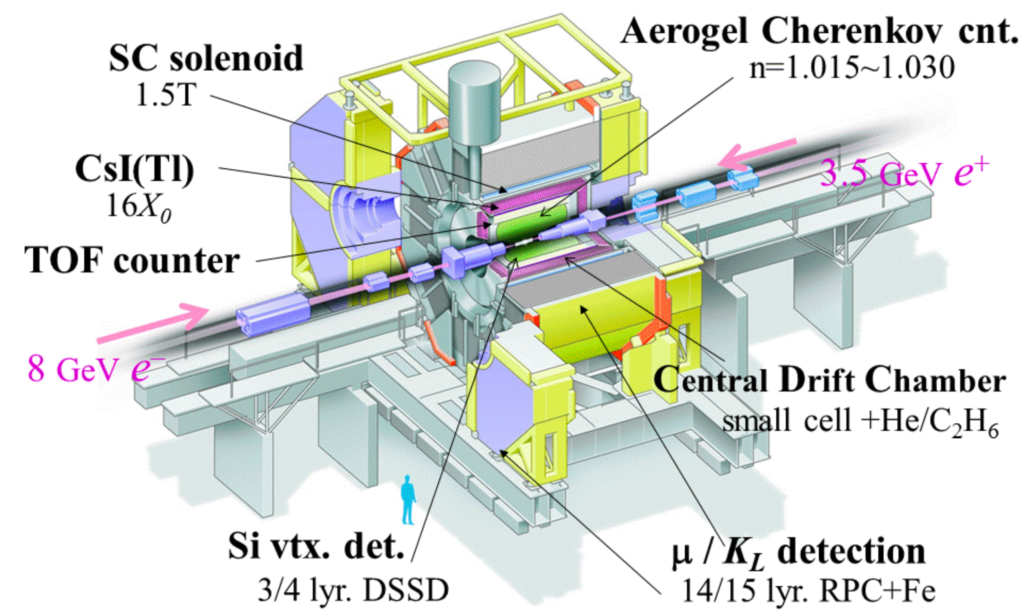
Flavour/charge separation
SIDIS: needed input on FFs;
Limited access to gluons

Direct access to PDFs; Sensitive to gluon FFs; large z behaviour;
Large theoretical uncertainties; no sensitive to heavy quarks

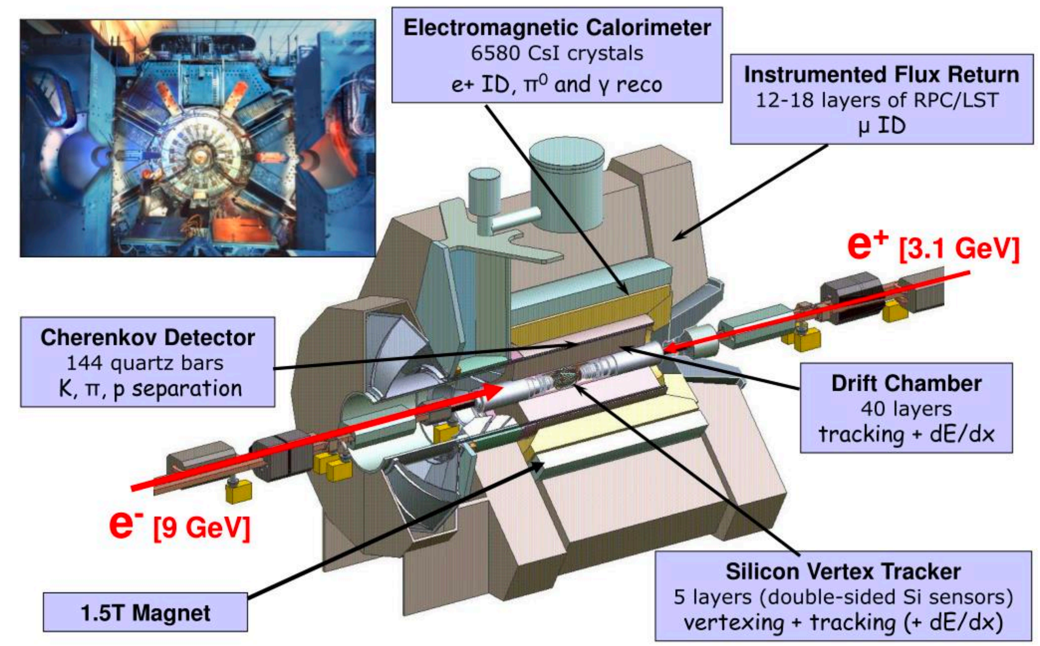
Clean process, high statistic;
Sensitivity to heavy quarks;
Limited in flavour separation; Limited access to gluon FFs

Belle, BaBar, BESIII detectors

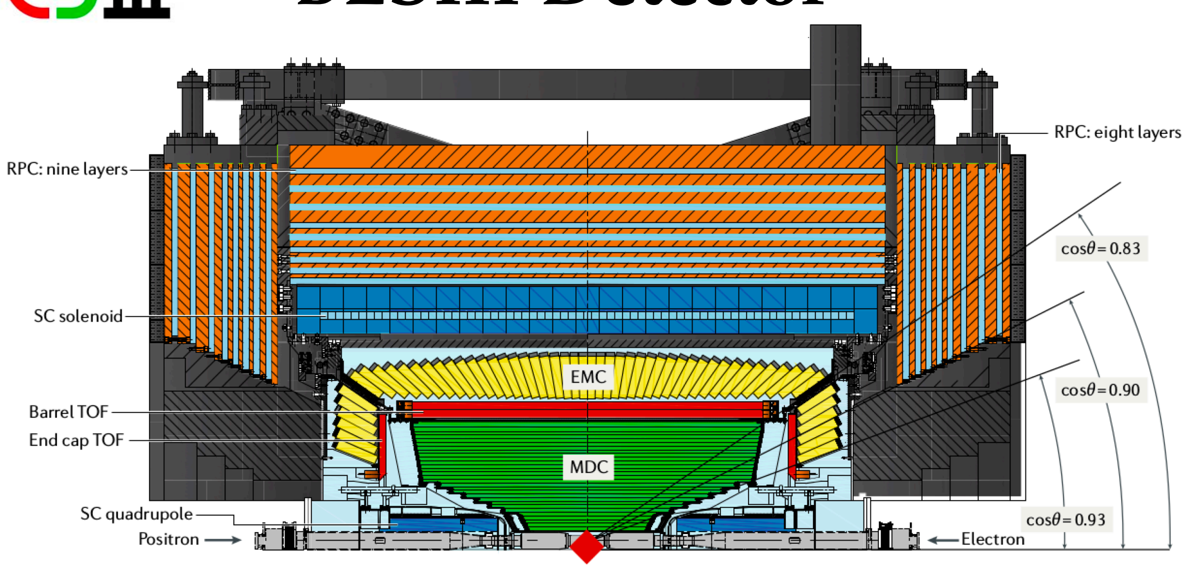
Belle Detector

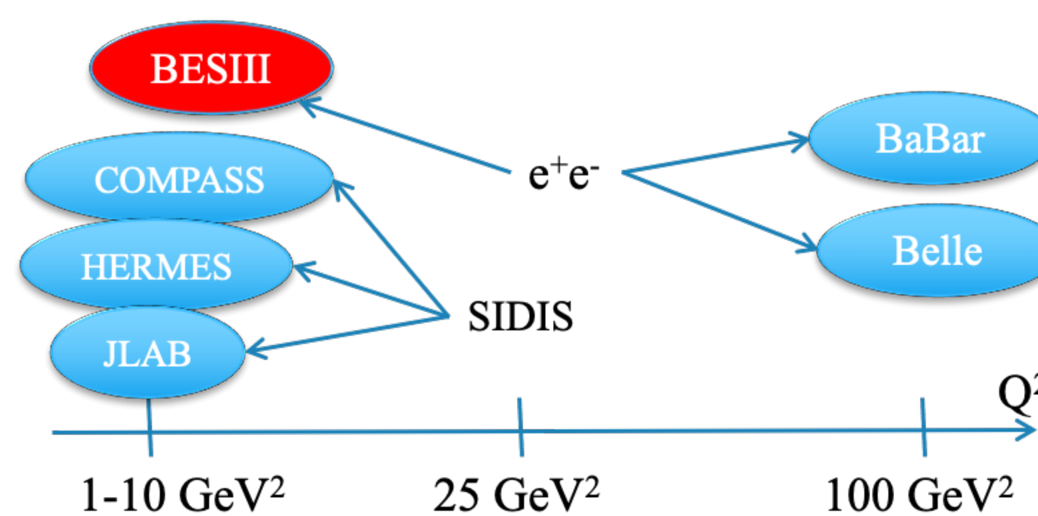


BaBar Detector

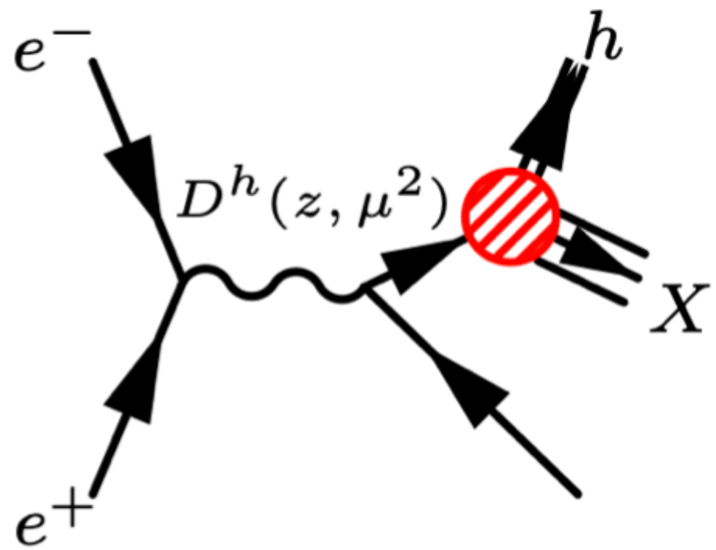


BESIII Detector



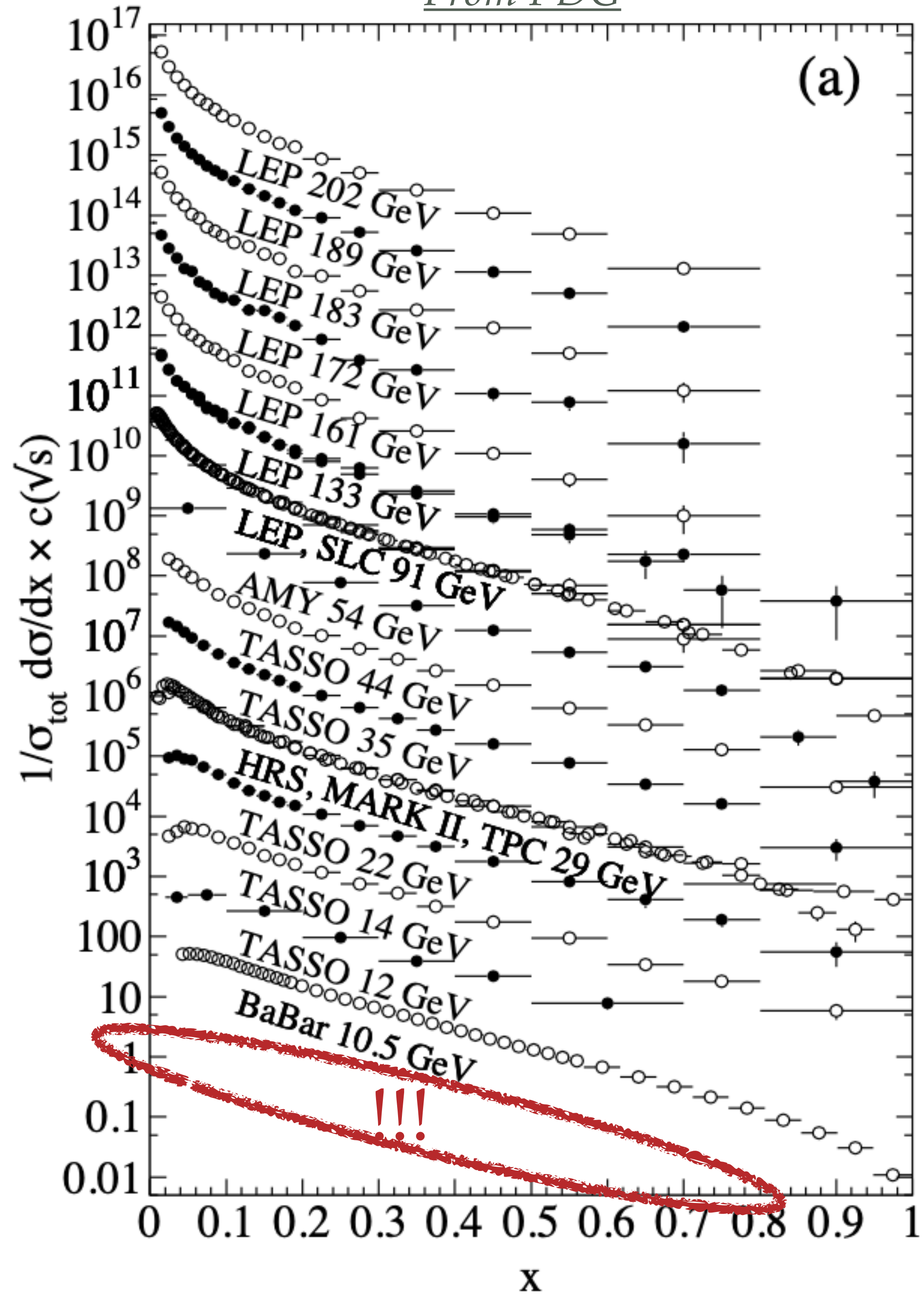


Unpolarised FFs

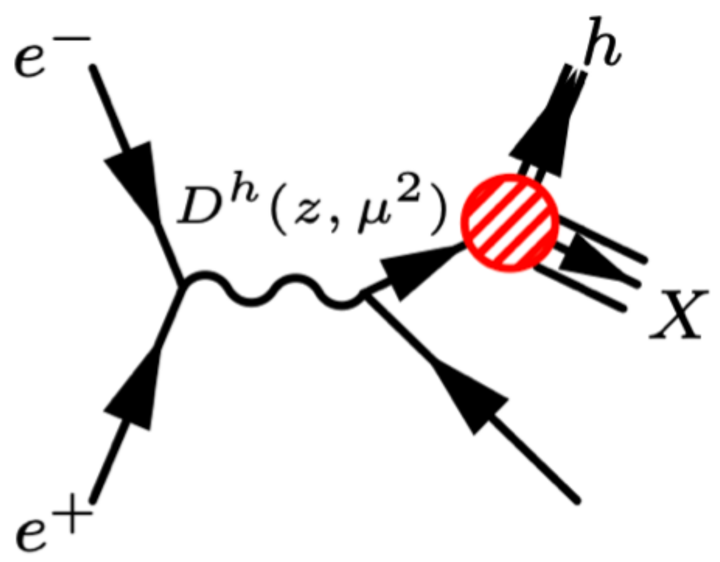


$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})} \frac{d\sigma(e^+e^- \to h + X)}{dp_h},$$

From PDG



Unpolarised FFs



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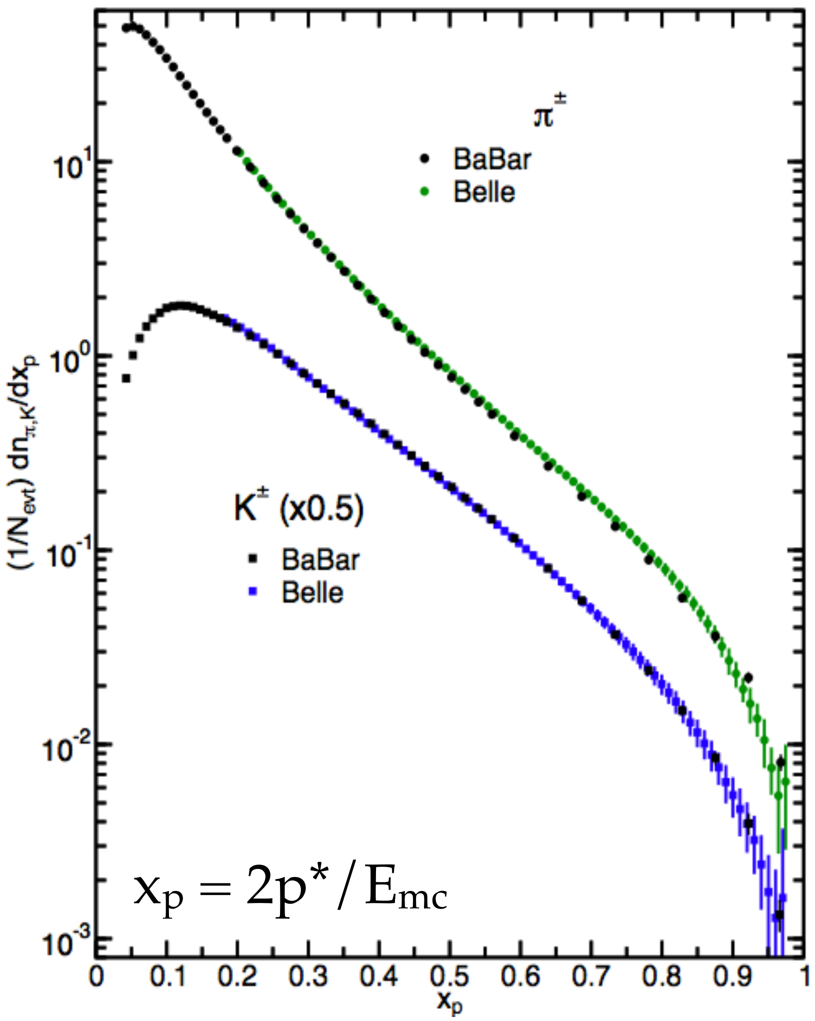
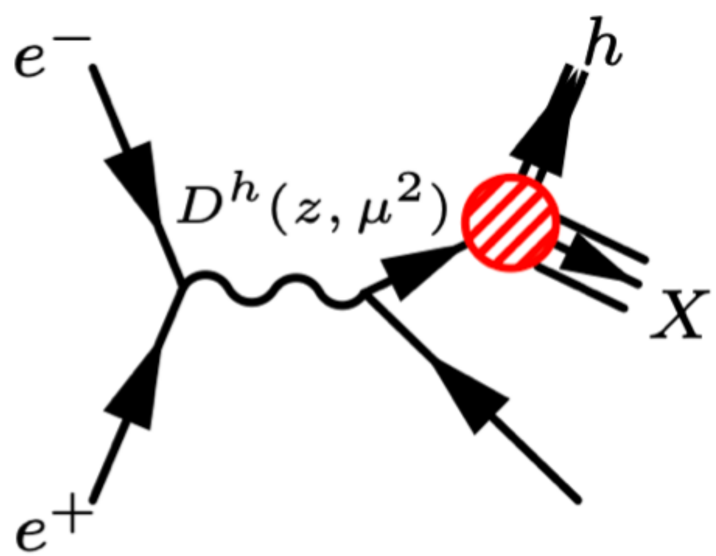


BaBar: PhysRevD.88.032011

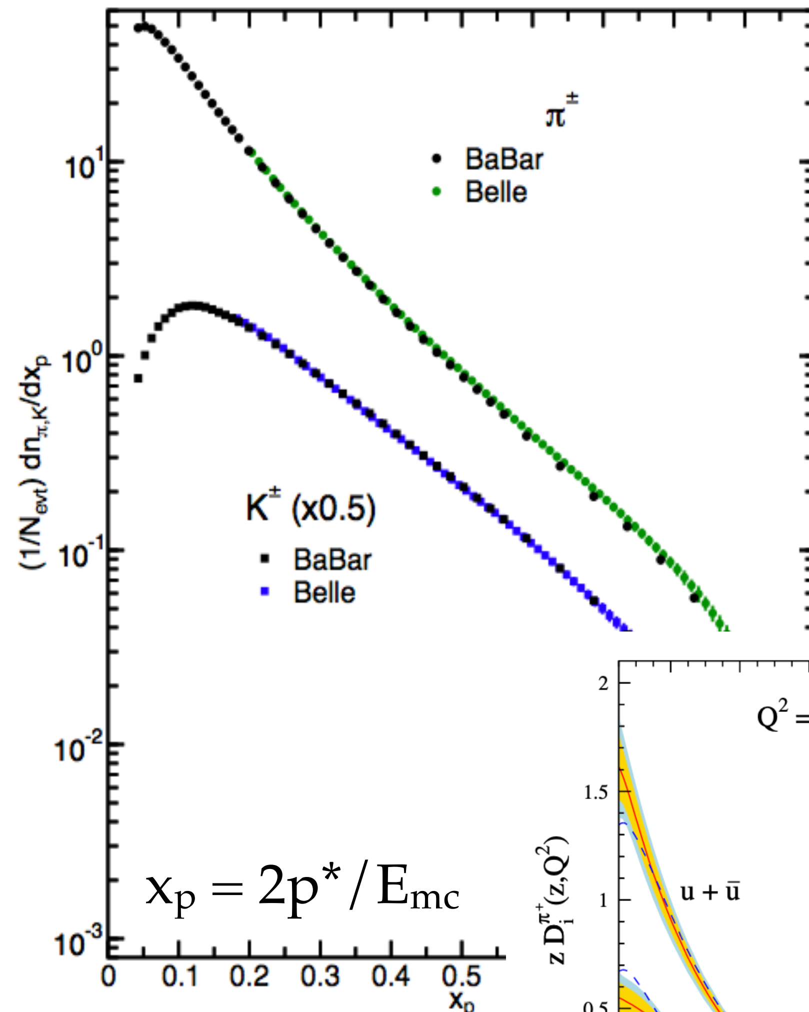
Belle: PhysRevLett.111.062002

**arbitrary normalisation in order to compare the shape*

Unpolarised FFs

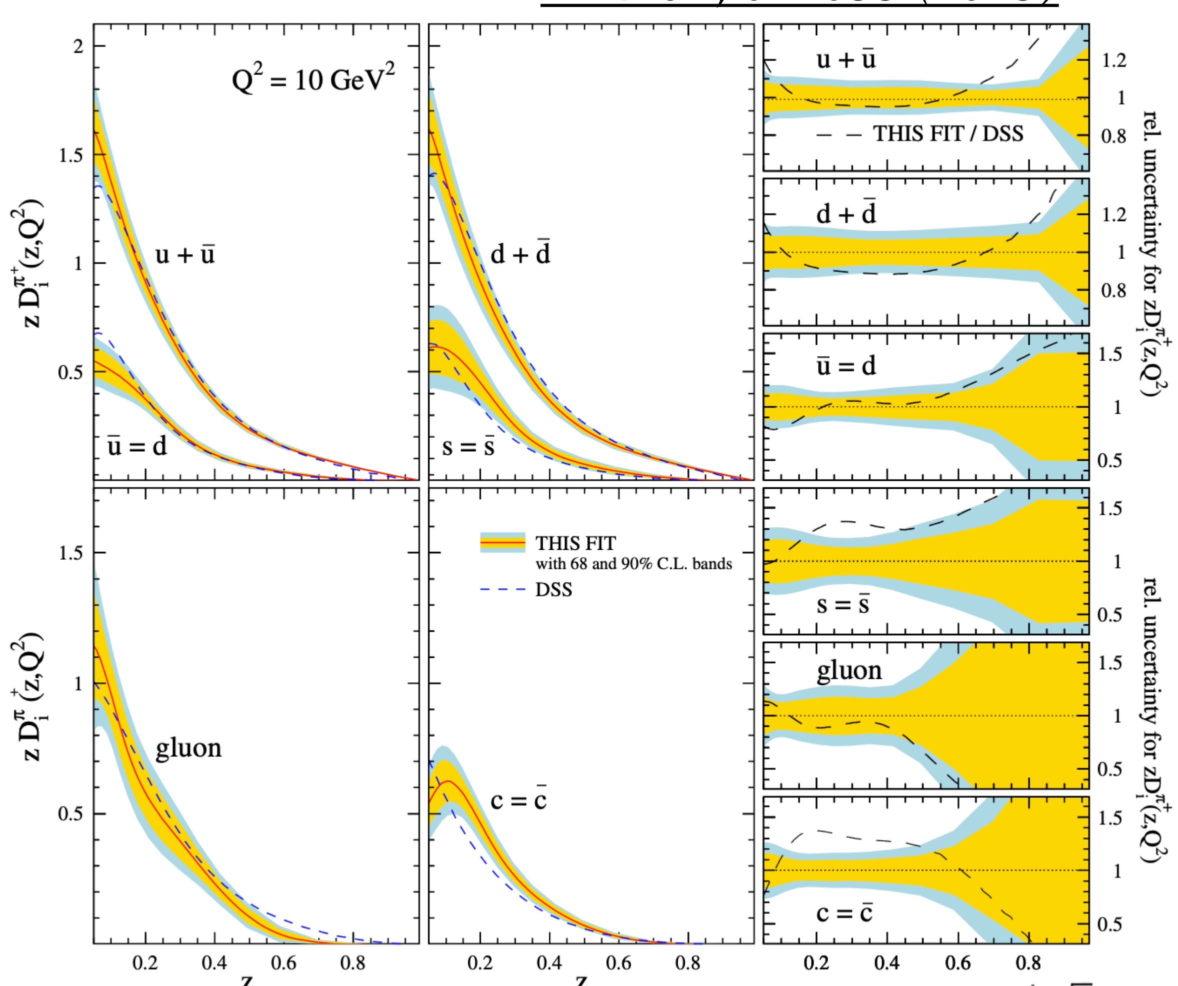


$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})}\frac{d\sigma(e^+e^- \to h + X)}{dp_h},$$



BaBar: PhysRevD.88.032011
Belle: PhysRevLett.111.062002
**arbitrary normalisation in order to compare the shape*

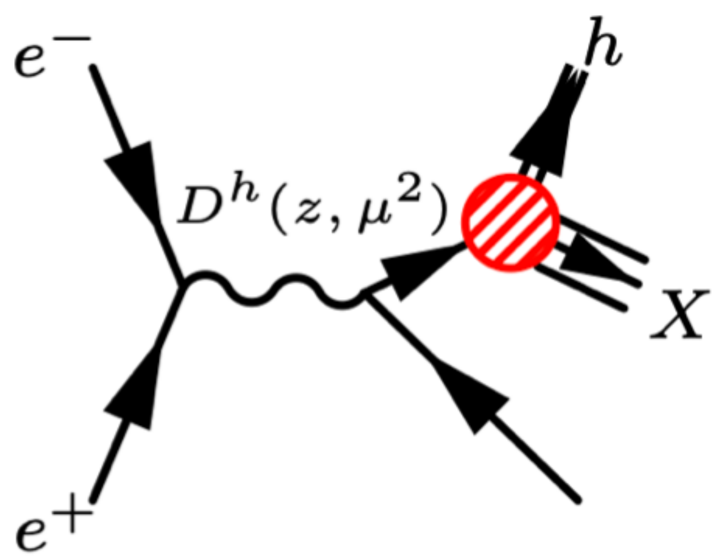
PRD 91, 014035 (2015)



$z = 2E_h/\sqrt{s}$,

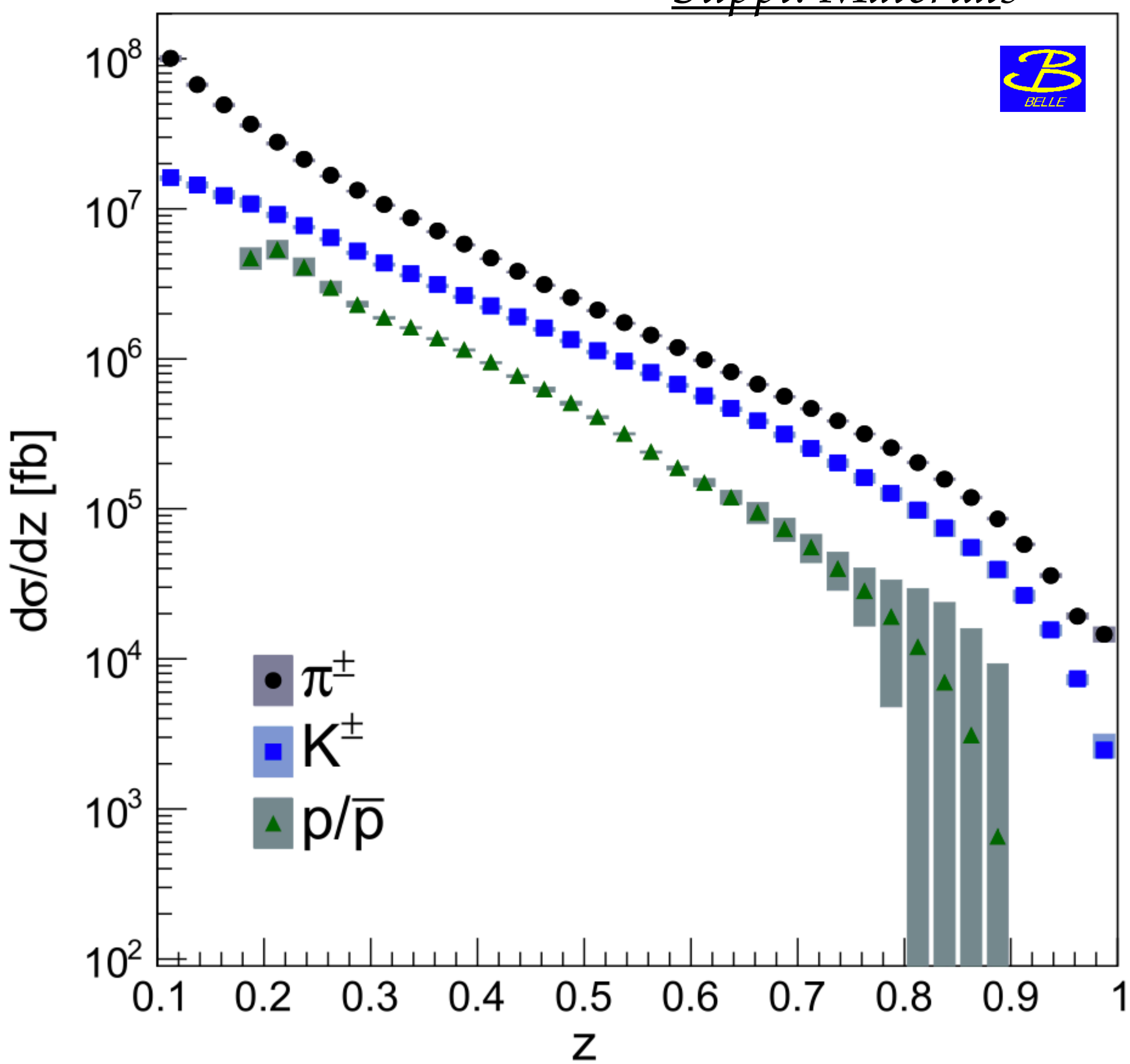
- Global fit analysis (e+e-, SIDIS and pp data) performed in order to extract the FFs for gluons and different quark flavours
  - DSS fit: PRD 91, 014035 (it includes Belle and BaBar data)

Inclusive light hadrons productions



$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})} \frac{d\sigma(e^+e^- \to h + X)}{dp_h},$$

Belle Update: PRD 101, 092004 (2020)
Suppl. Materials



- Data set used: 558 fb^{-1} @ 10.58 GeV
- Update version of ISR correction ==> direct applicable in global fit analyses
  - correction obtained by calculating the ratios of MC cross sections with and without ISR
  - overall systematic uncertainty ~10% (dependence with *s* of fragmentation models; shift in the energy fraction)

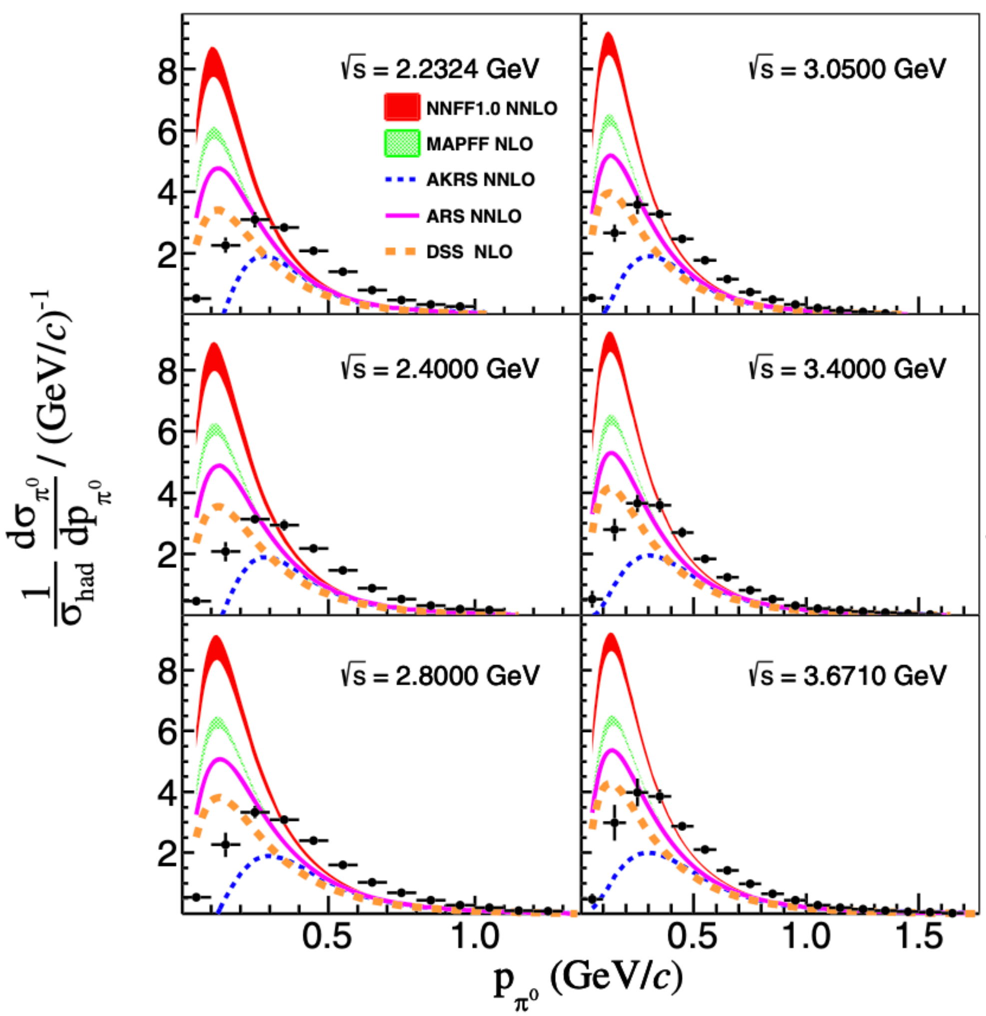
Inclusive π^0 and K^0_S production @ BESIII

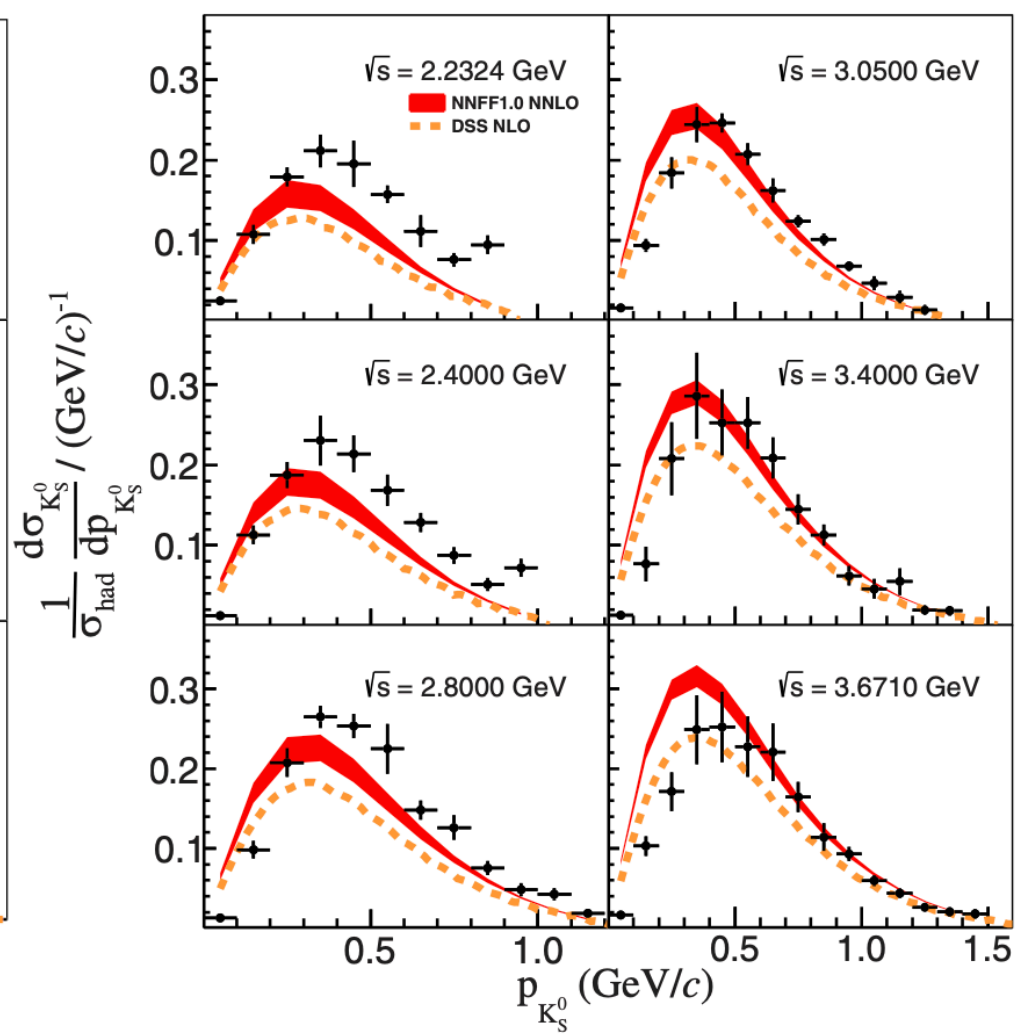
BESIII: PRL 130, 231901 (2023)
Suppl. Material

$e^+e^- \to \pi^0/K^0_S + X$ studied at six c.m. energies from 2.2324 to 3.6710 GeV

- $M(\gamma\gamma)$ and $M(\pi^+\pi^-)$ spectra divided into $\Delta p_{\pi/K}$=0.1 GeV/c intervals in order to extract the corresponding number of signal events
- Normalized differential cross section:

$$\frac{N_h^{\rm obs}}{N_{\rm had}^{\rm obs}} \frac{1}{\Delta p_h} f_h$$





NNFF1.0, ARS, AKRS: inclusive e+e- data at NNLO accuracy

MAPFF: + Lepton-proton fixed target

DSS: + Lepton-proton fixed target and proton-proton collision

Disagreement observed to depend on both c.m energy and hadron momentum

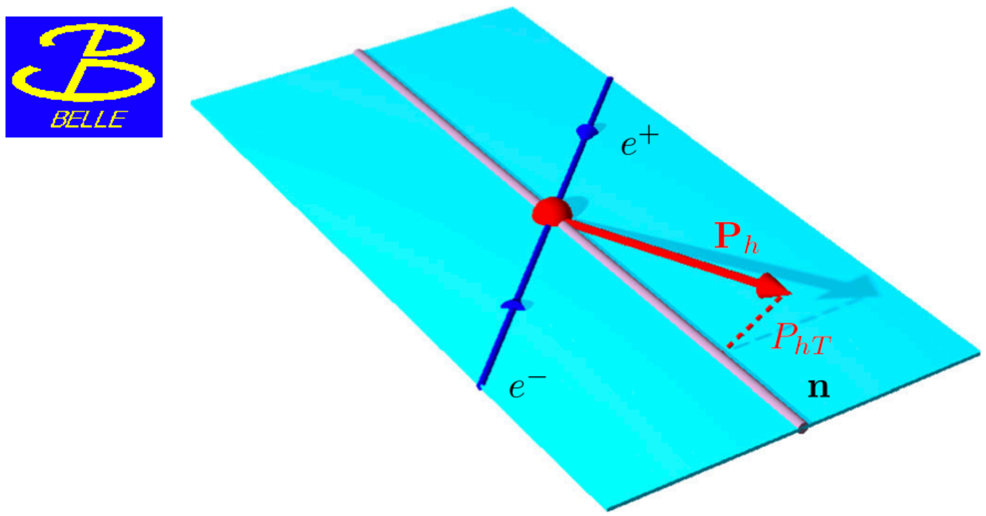
Leading twist calculation not sufficient at BESIII energy scale? Consider quark mass and hadron mass correction effects? small-z resummption effects? problem in the extrapolation of FFs from high energy data to low-energy scale?

TMD single-hadron production cross sections @ Belle

Belle: PRD 99, 112006 (2019)

First direct measurement of unpolarised cross section as a function of z, p_T and thrust

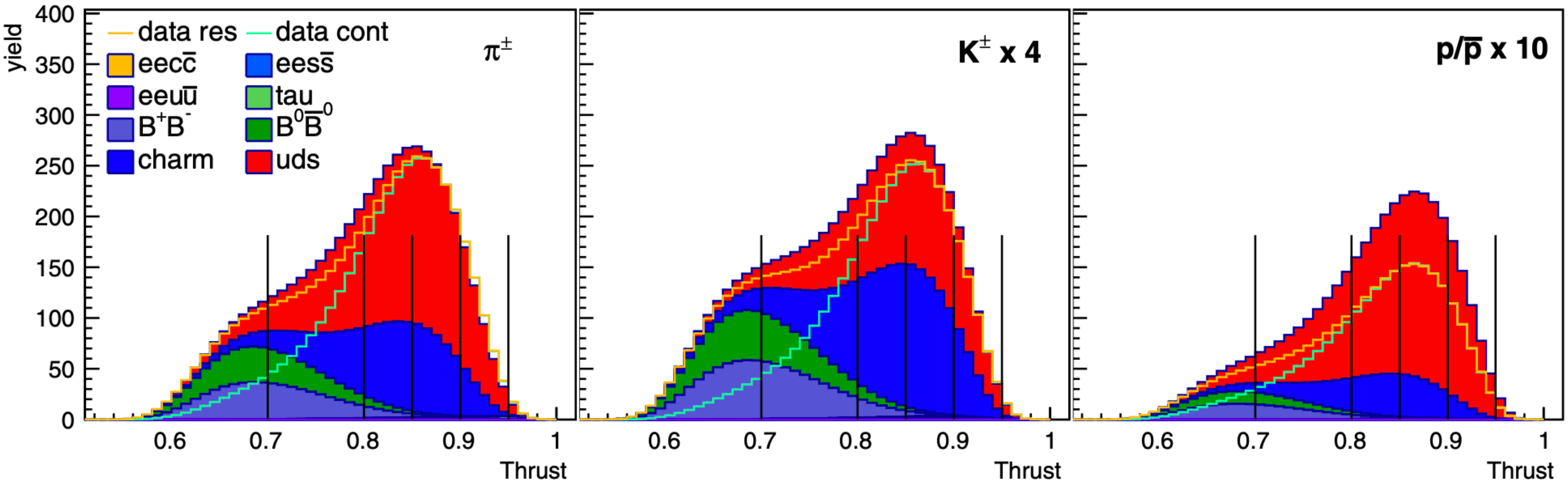
$$D_1^h(z, \mu^2, k_T)$$



Definition of thrust event shape variable:

$$T \stackrel{\max}{=} \frac{\sum_h |\mathbf{P}_h^{\text{CMS}} \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h^{\text{CMS}}|}$$

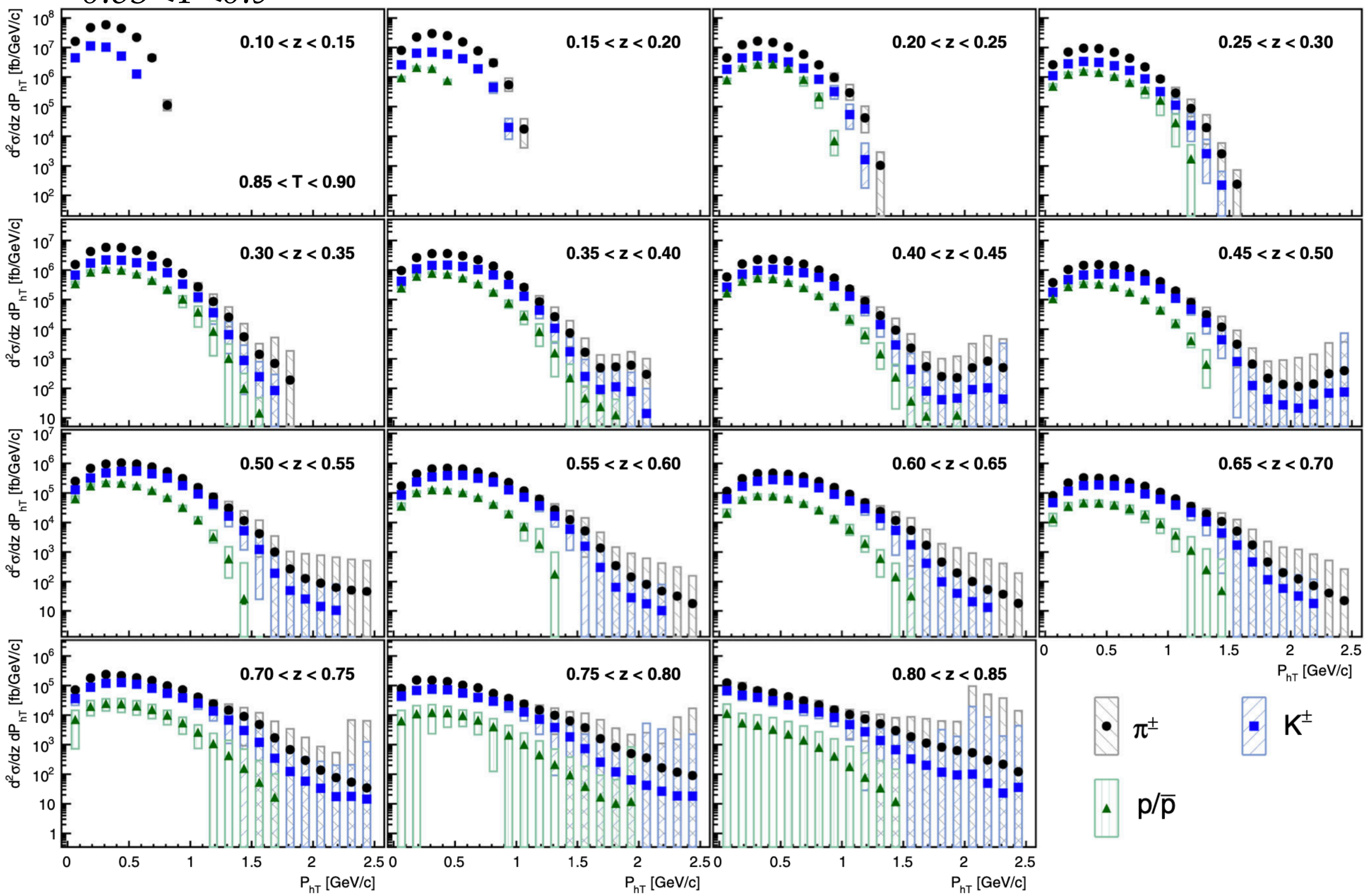
$$z = 2E_h/\sqrt{s}, \qquad \mathbf{p}_T = -z\mathbf{k_T}$$



TMD single-hadron production cross sections @ Belle

Belle: PRD 99, 112006 (2019)

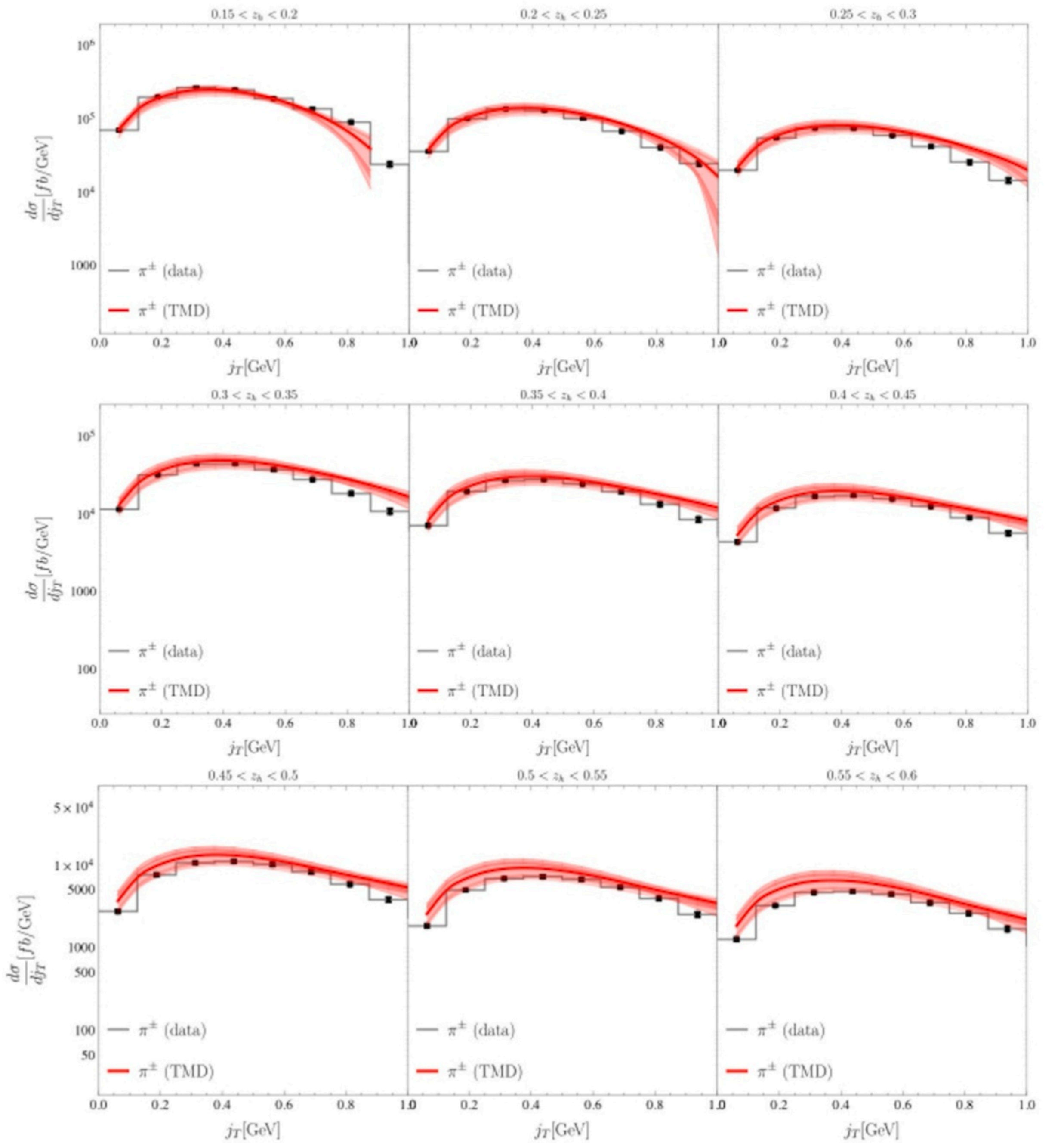
0.85<T<0.9



- Similar behaviour between hadron types
- At lower p_T, the behaviour resembles a Gaussian (which is assumed for TMD FFs); the gaussian widths vary with z and thrust

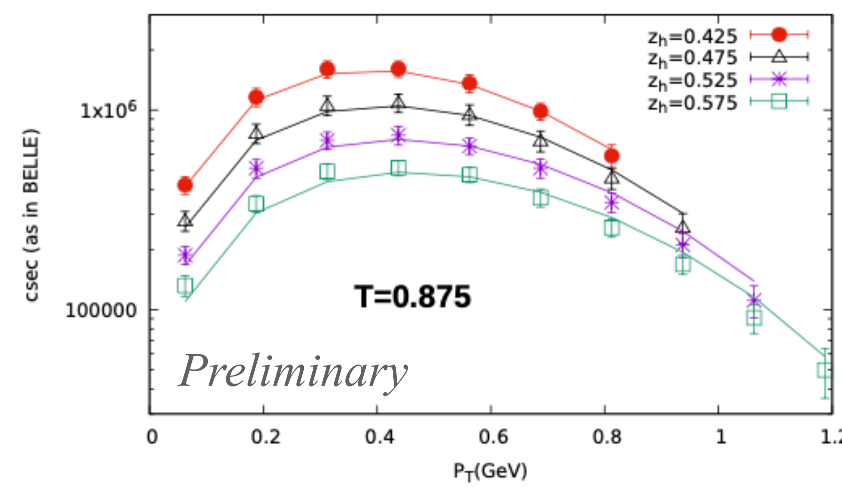
Extraction of TMD single-hadron production cross sections

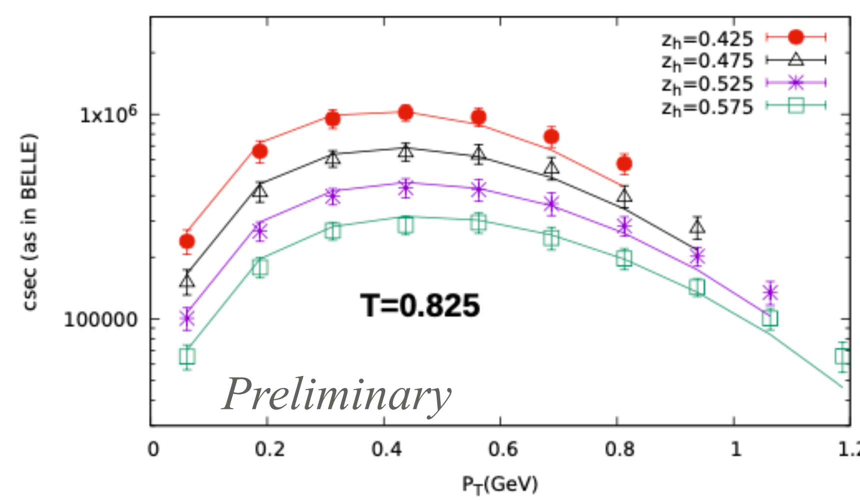
Zhang-Bo Kang et al. JHEP12(2020)127



TMD factorization formalism developed: good description of p_T distribution for z<0.65, threshold (~ ln(1−z_h)) resummation required for z>0.65

M. Boglione et al. SPPP8, 139(2022) JHEP02(2021)076

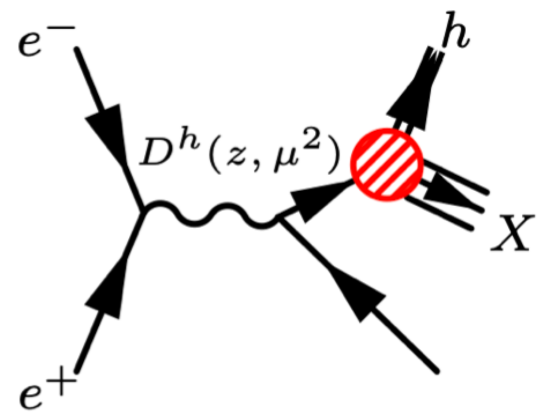




The factorized $e^+e^- \to hX$ cross section is differential in three variables: z, p_T, T

Thrust dependence of the $e^+e^- \to hX$ cross section described for the first time

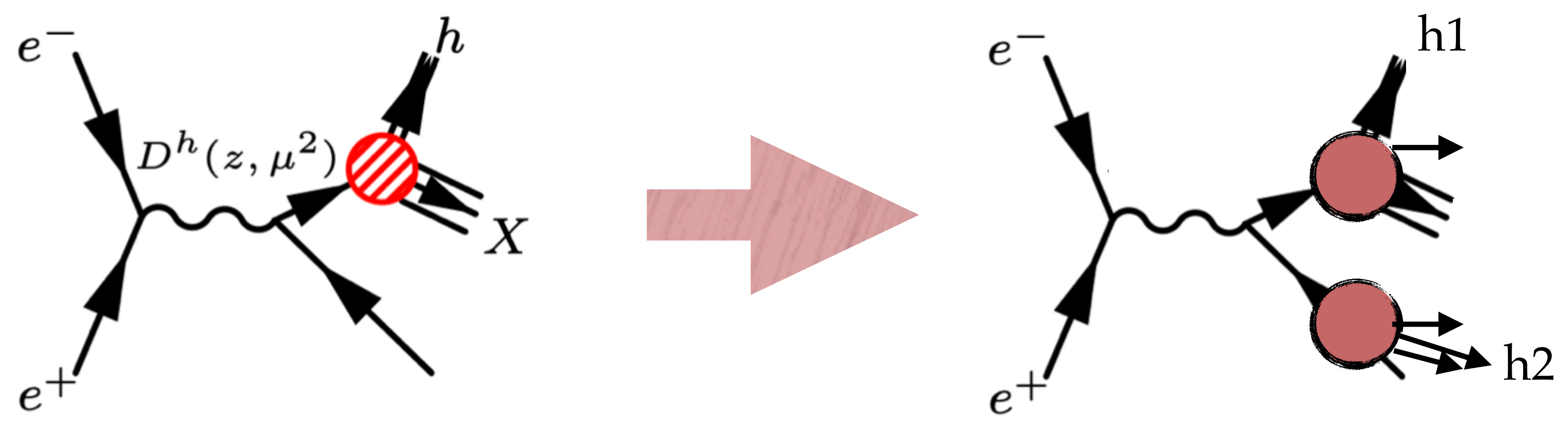
Polarised FF: the Collins effect



$H_1^\perp$?

chiral-odd function

Polarised FF: the Collins effect



q↑→hX: $$D_1^{q\uparrow}(z,\mathbf{P}_\perp;s_q) = D_1^q(z,P_\perp) + \frac{P_\perp}{zM_h} H_1^{\perp q}(z,P_\perp)\,\mathbf{s}_q\cdot(\mathbf{k}_q\times\mathbf{P}_\perp)$$

Unpolarized FF

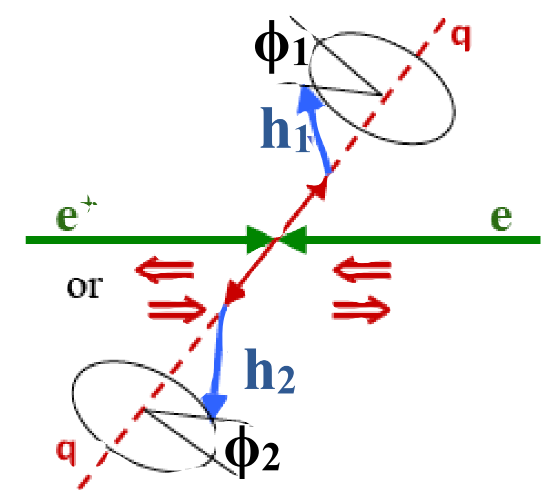
Collins FF [NP**B 396**, 161 (1993)]: related to the probability that a transversely polarized quark (q↑) fragments into a spinless hadron

- Evolution of TMD objects
- Global analysis (PRD 78,032011 (2007); PRD 87,094019 (2013), PRD 91,014034 (2015)):
  - combines Semi Inclusive Deep Inelastic Scattering (SIDIS) and e^+e^- data
  - extraction of $H^\perp_1$ and transversity parton distributions h_1 for the "u" and "d" quarks

Collins effect in e^+e^- annihilation

In $e^+e^- \to q\bar{q}$, spins unknown, but $s_q \parallel s_{\bar{q}}$

- exploit this correlation by using hadrons in opposite jets
- define **favored** ($u\to\pi^+$, $d\to\pi^-$) and **disfavored** ($d\to\pi^+$, $u\to\pi^-$, $s(\bar{s})\to\pi^\pm$) FFs

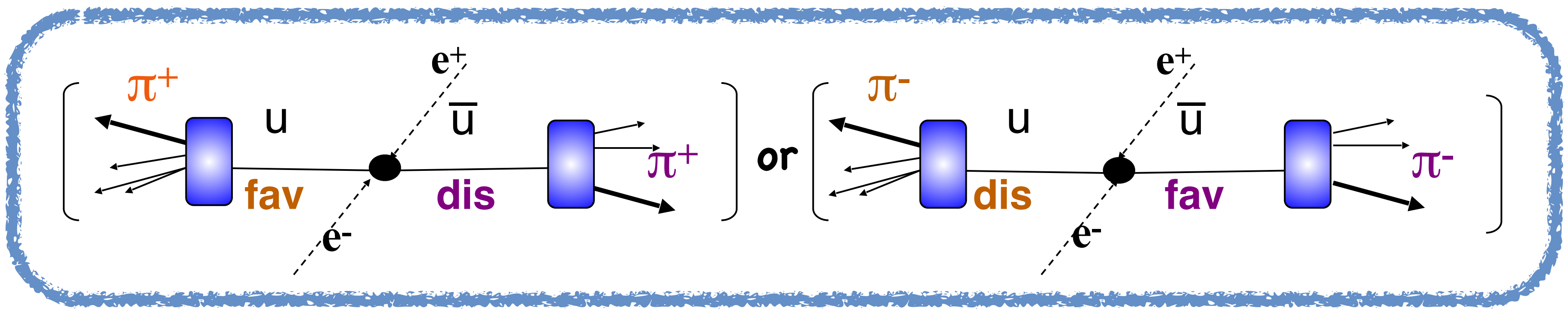


$e^+e^- \to q\bar{q} \to h_1h_2X$ (q=u,d,s) $\Rightarrow \sigma \propto \cos(\phi_1+\phi_2) H_1^{\perp(h1)} \times H_1^{\perp(h2)}$

Azimuthal modulation wrt the quark spin direction:
Collins effect (or Collins asymmetry)

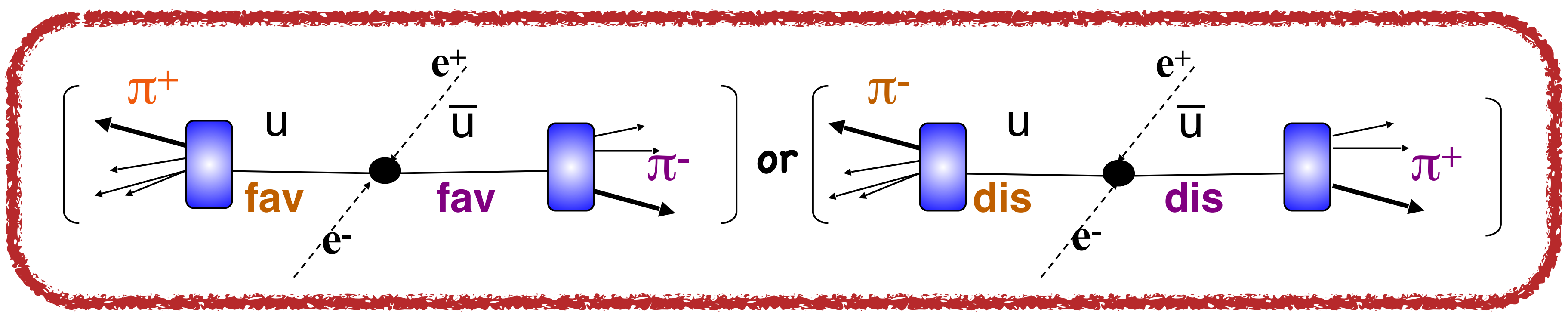
Example: Like $\pi\pi$ pairs (L)

Collins asymmetry for $\pi\pi$



Example: Unlike $\pi\pi$ pairs (U)

Collins asymmetry for $\pi\pi$



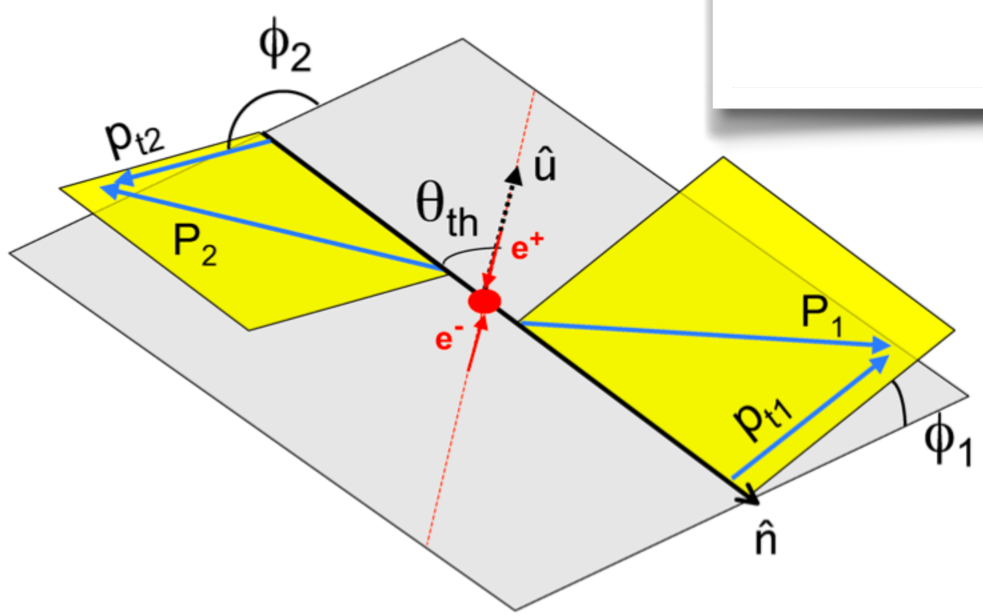
Reference Frames

See D. Boer, NPB 806, 23 (2009)

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d\phi_1 d\phi_2} = \sum_{q,\bar{q}} \frac{3\alpha^2}{Q^2}\frac{e_q^2}{4} z_1^2 z_2^2 \Big[(1+cos^2\theta) D_1^{q,(0)}(z_1)\bar{D}_1^{q,(0)}(z_2) + sin^2(\theta) cos(\phi_1+\phi_2) H_1^{\perp,(1),q}(z_1)\bar{H}_1^{\perp,(1),q}(z_2)\Big]$$

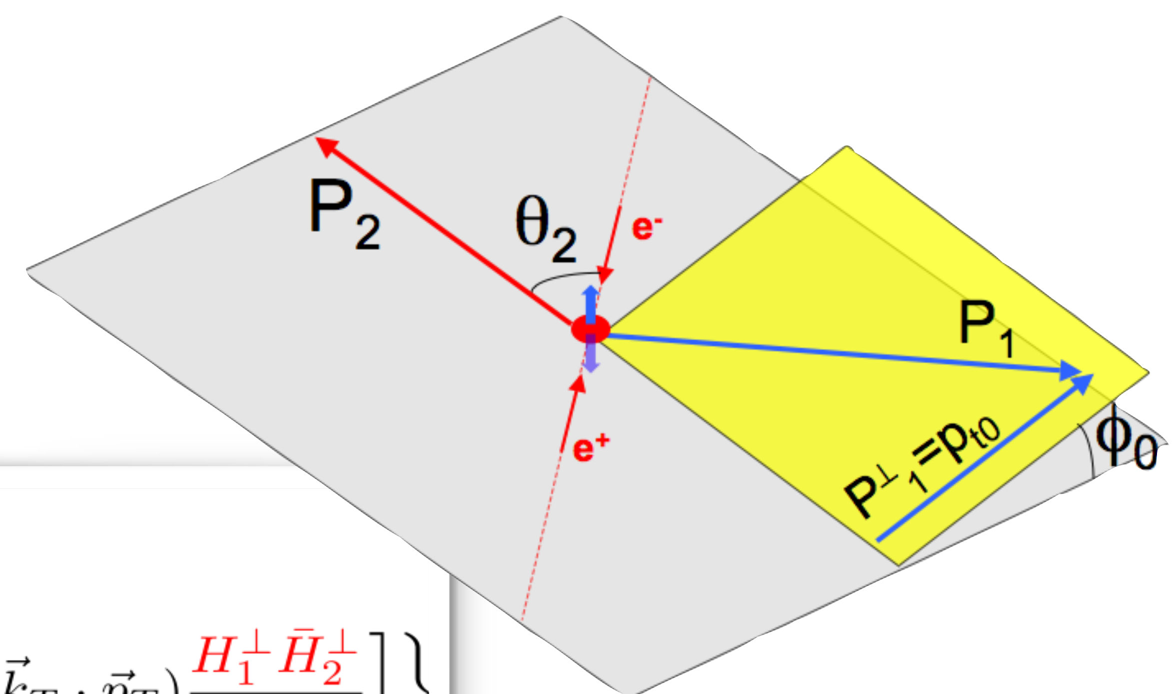
All quantities in e+e- center of mass

RF12



θ: angle between the e^+e^- axis and the thrust axis;
$\varphi_{1,2}$: azimuthal angles between $P_{h1(h2)}$ and the scattering plane

RF0



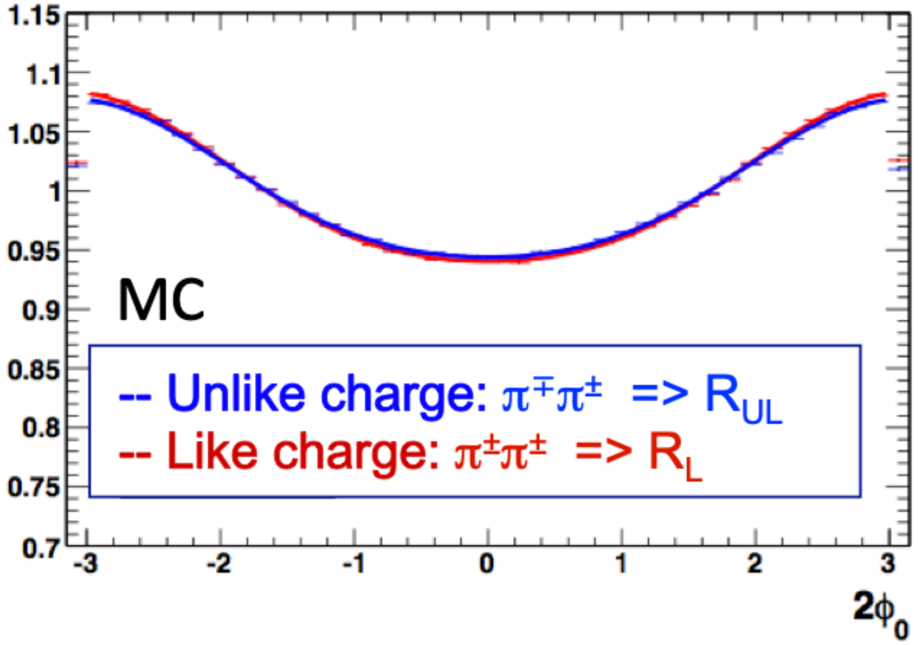
θ_2: angle between the e^+e^- axis and P_{h2};
φ_0: angle between the plane spanned by P_{h2} and the e^+e^- axis, and the direction of P_{h1} perpendicular to P_{h2}.

All quantities in e+e- center of mass

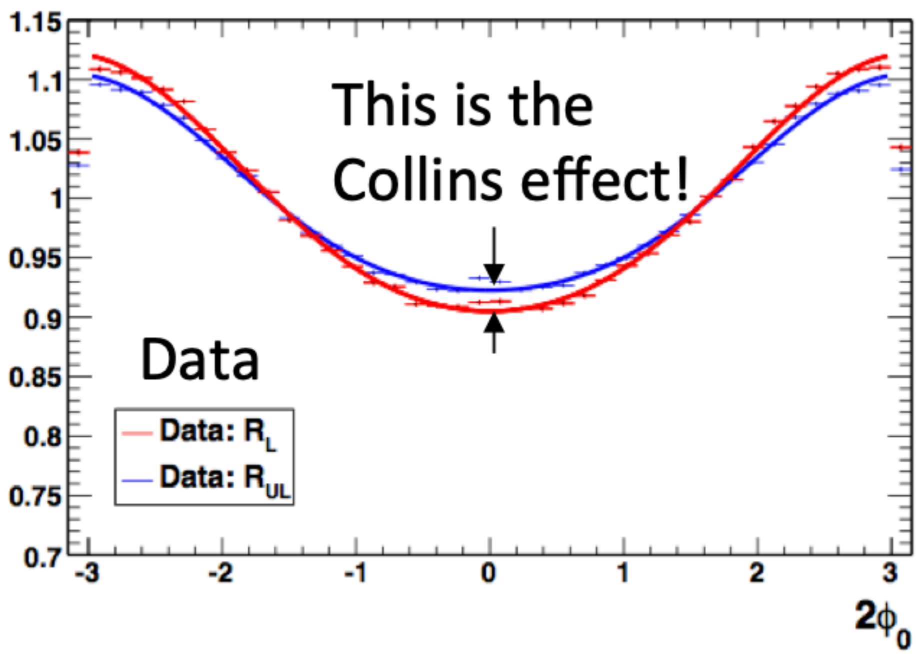
$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega dz_1 dz_2 d^2\vec{q}_T} = \frac{3\alpha^2}{Q^2} z_1^2 z_2^2 \Big\{A(y)\mathcal{F}[D_1\bar{D}_2] + B(y) cos(2\phi_0)\mathcal{F}\Big[(2\hat{h}\cdot\vec{k}_T\hat{h}\cdot\vec{p}_T - \vec{k}_T\cdot\vec{p}_T)\frac{H_1^\perp \bar{H}_2^\perp}{M_1 M_2}\Big]\Big\}$$

Asymmetry Measurements from Data

$N^{(U,L)}(\phi)/<N^{(U,L)}(\phi)>$ in MC sample



$N^{(U,L)}(\phi)/<N^{(U,L)}(\phi)>$ in data sample



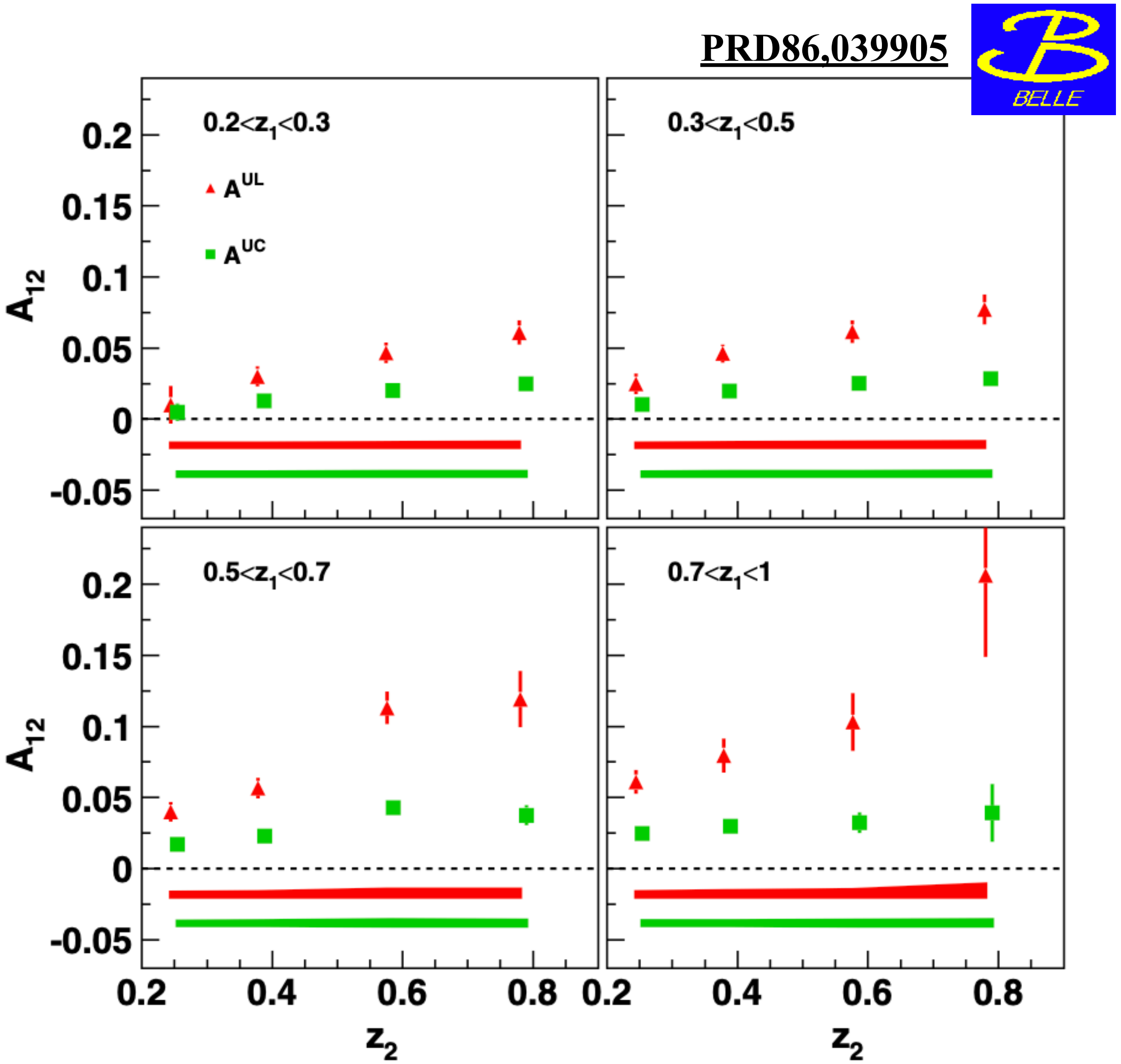
- Collins asymmetries extracted from fit to the normalised azimuthal distribution

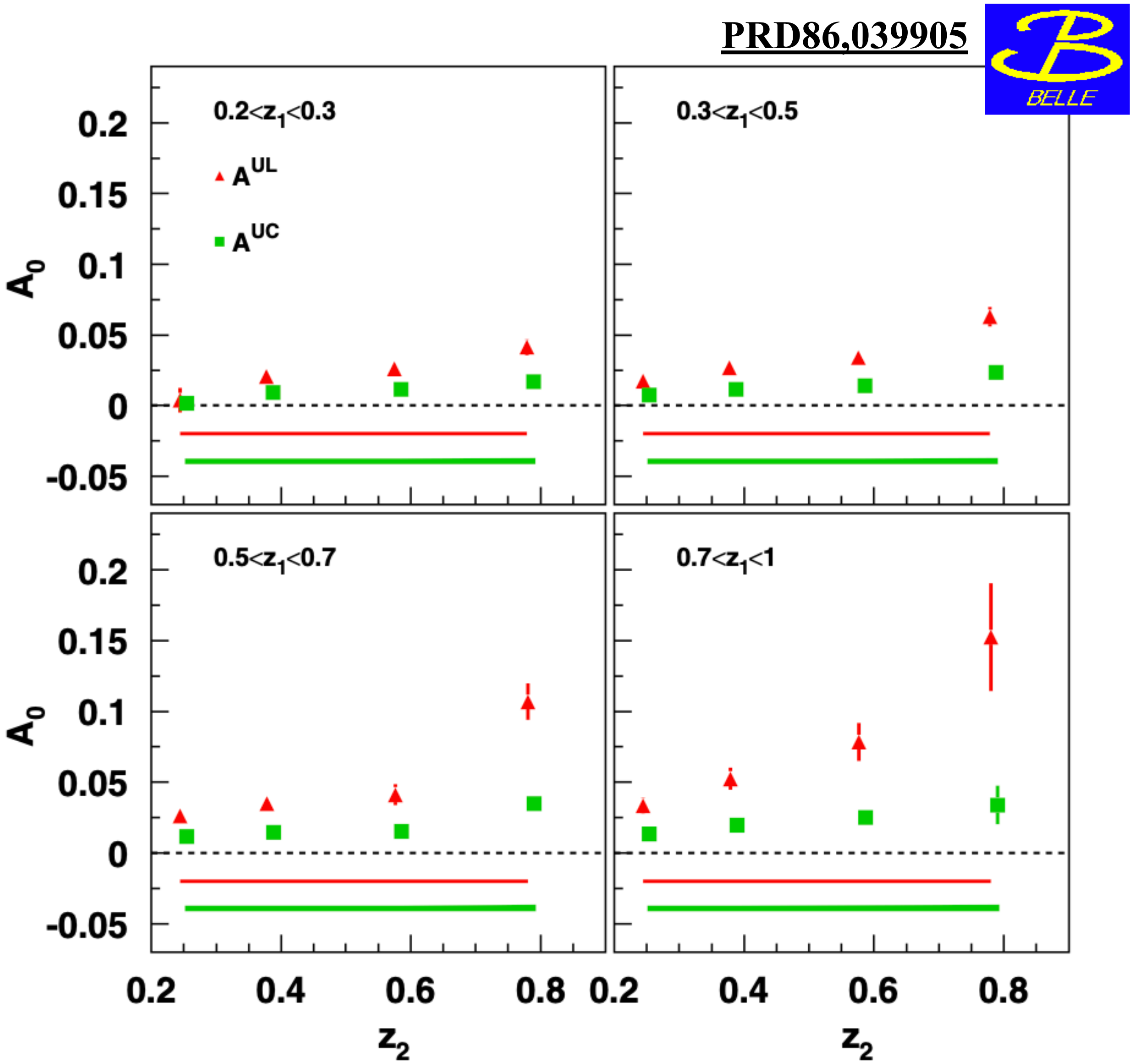
$$R_\alpha = \frac{N(\phi_\alpha)}{< N_\alpha >} = a + b \cdot \cos(\phi_\alpha)$$

- MC generator does not include polarised FFs as the Collins FF
  - modulation observed in MC sample produced by detector acceptance
  - correction of these effects would bring too large systematic uncertainties
- Ratio of U and L distributions
  - Collins effect is not sensitive to the electric charge
  - U and L sigli different in data due to the different contribution of favoured and disfavoured FFs

Collins FFs Results

First measurement of Collins effects in e+e− annihilation (PRL 96,232002)



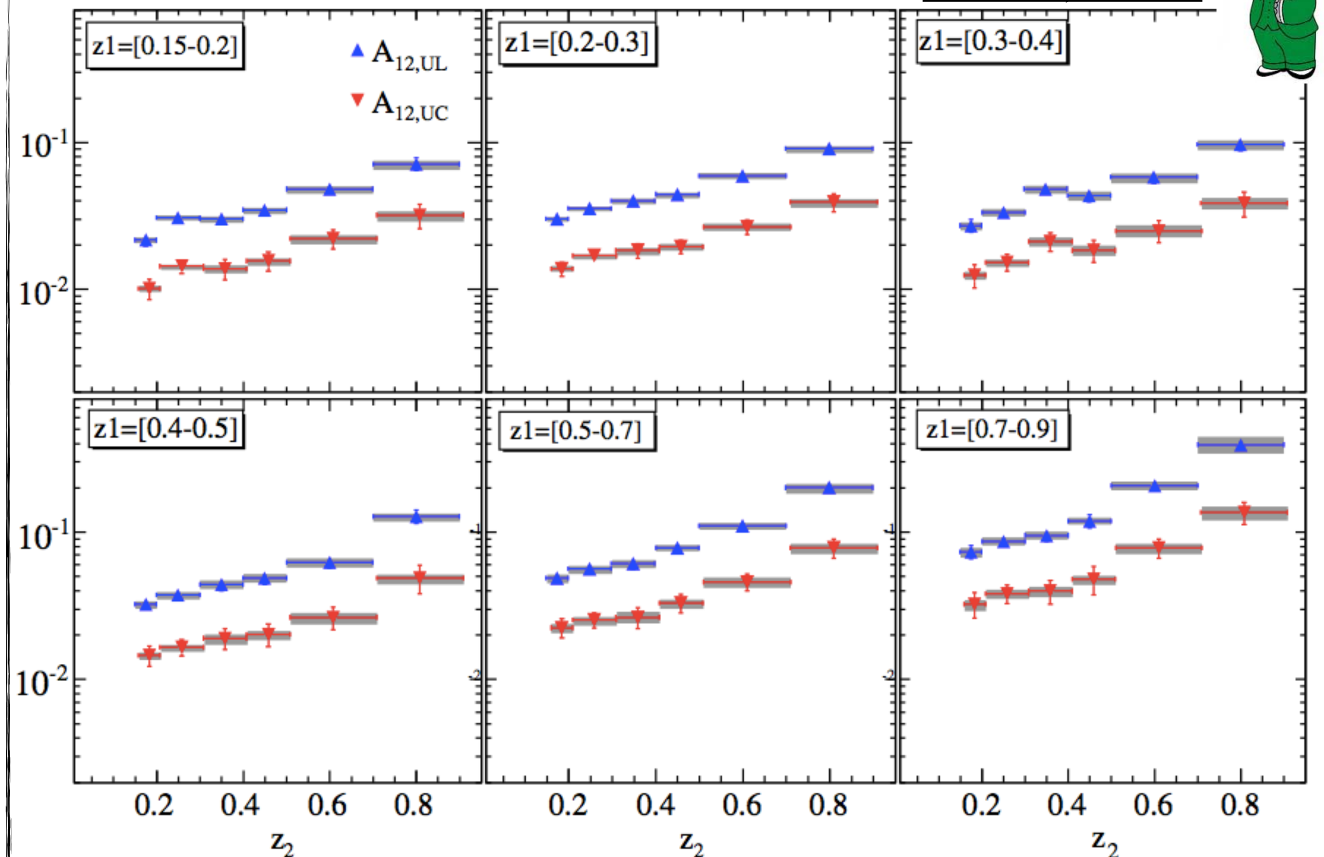


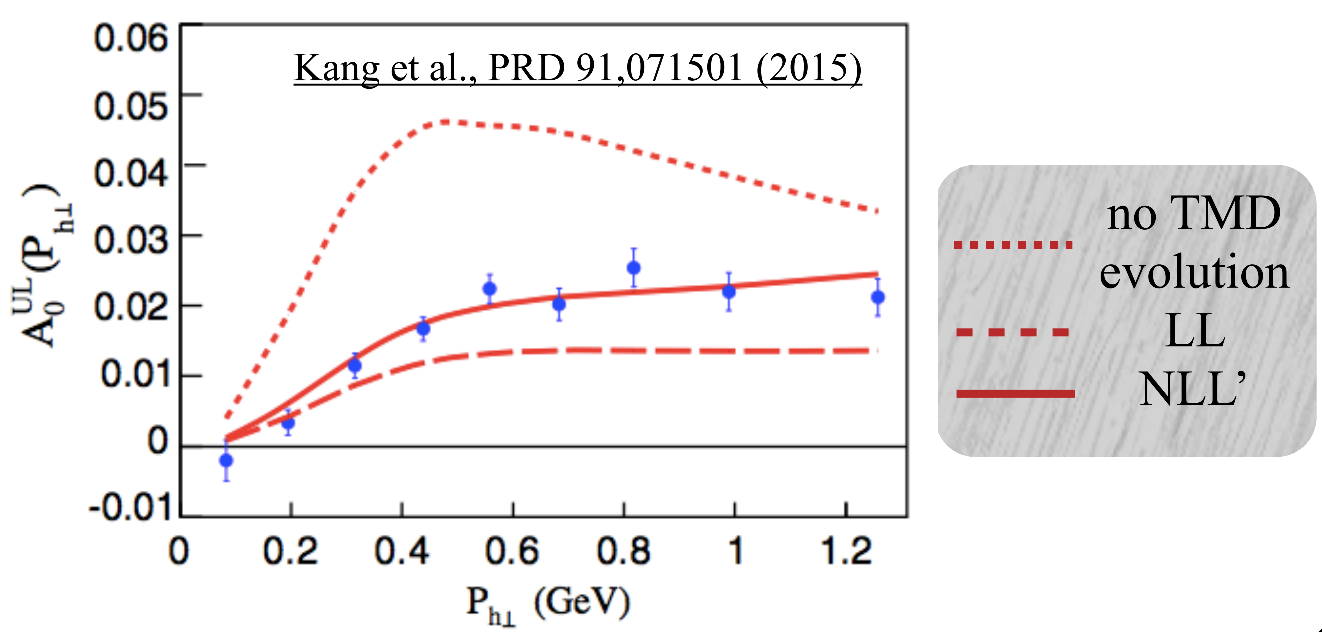
- $\mathcal{L}$ ~ 547 fb^{-1} at ~10.58 GeV
- Non-zero asymmetries increasing with z

Collins FFs Results

$\mathcal{L}$ ~ 468 fb^{-1} at ~10.58 GeV

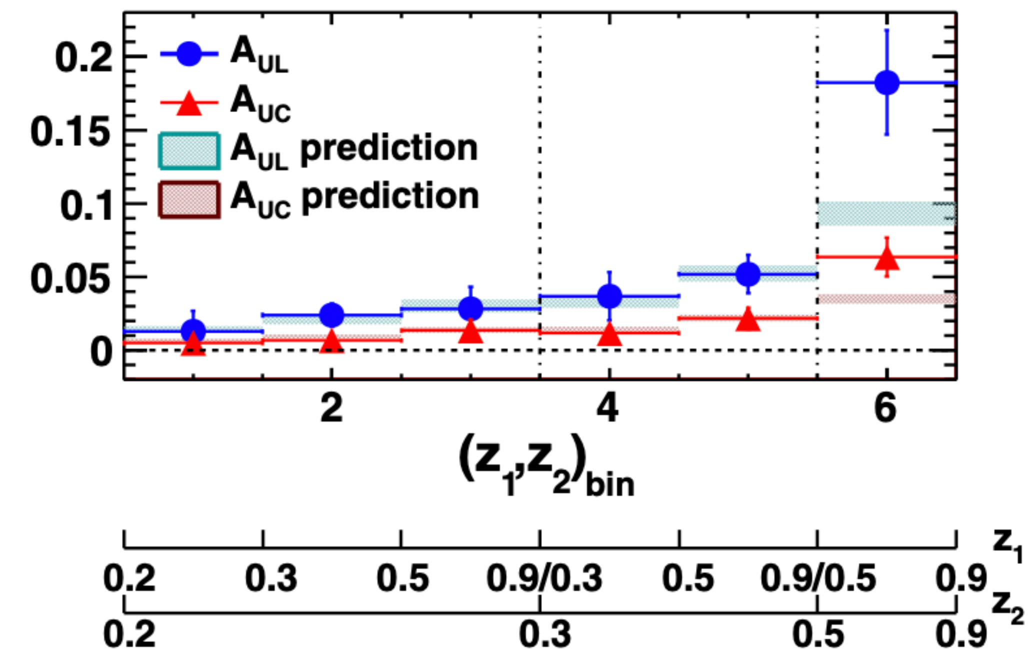
PRD 90,052003

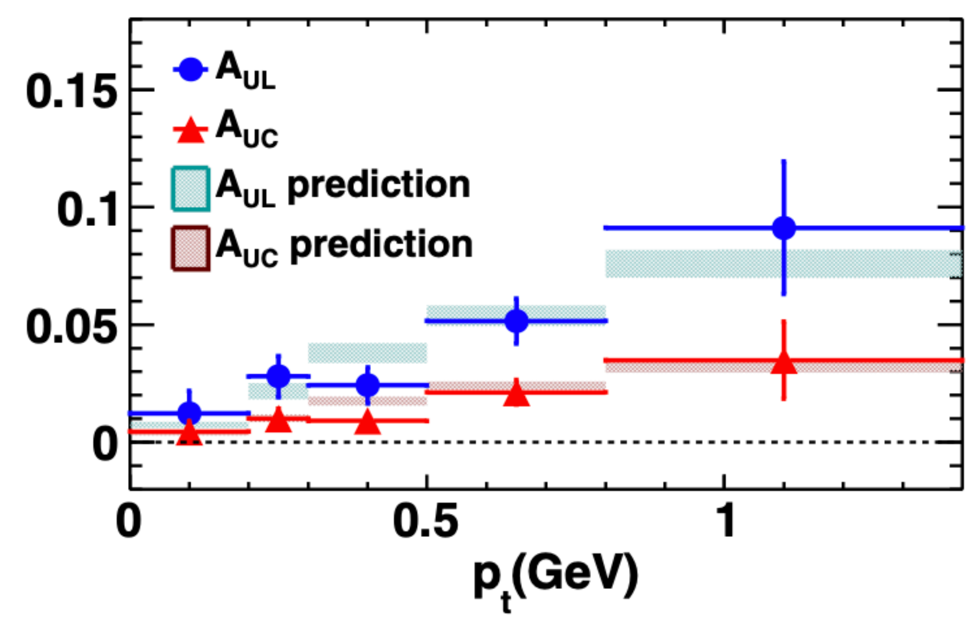




$\mathcal{L}$ ~ 62 pb^{-1} at ~3.65 GeV

PRL116, 042001





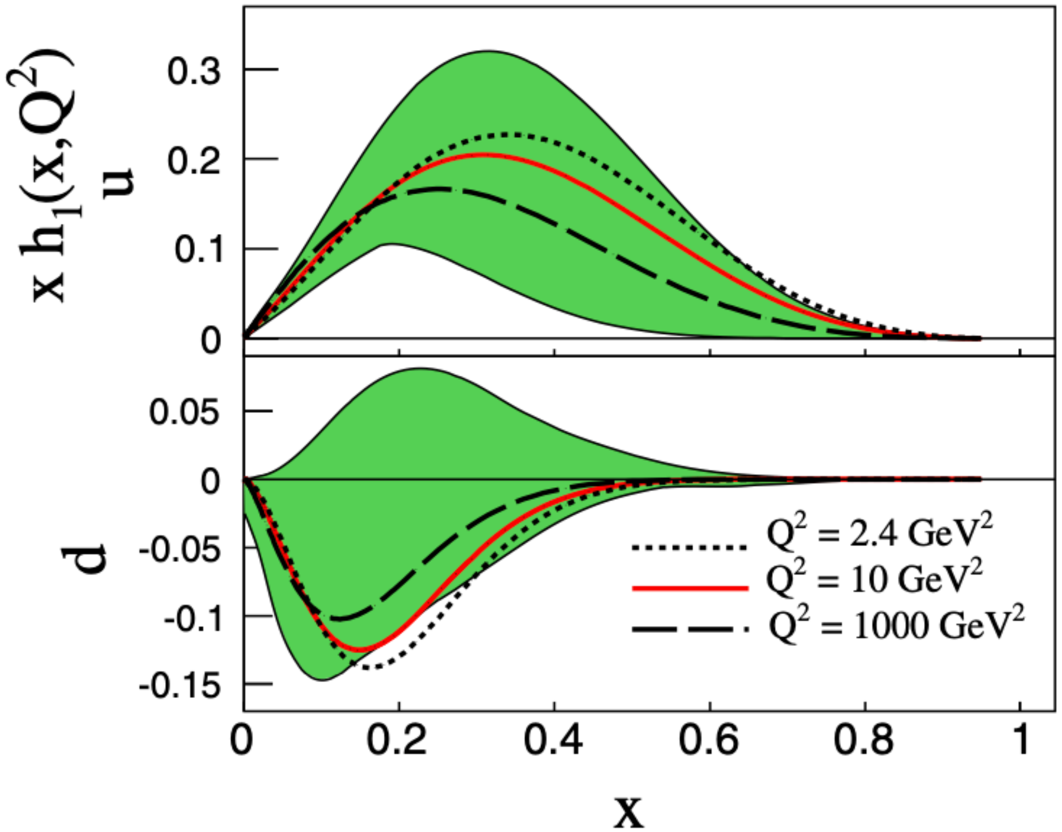
Extraction of h_1 and Collins FF

PRD 93,014009 (2016)

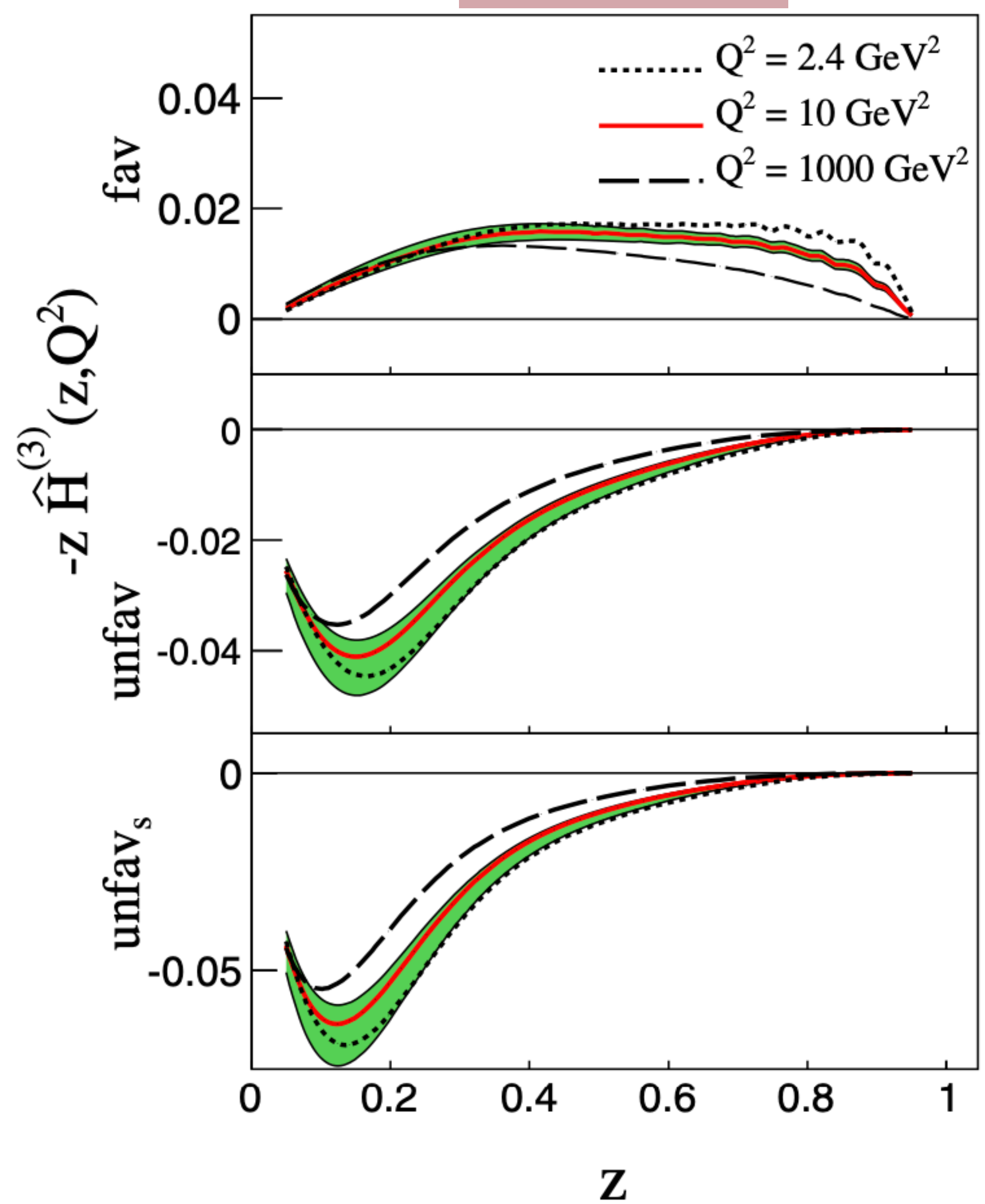
- First extraction: Anselmino et al. PRD 75,054032
- Global analysis with SIDIS (HERMES (PRL**103**,152002), COMPASS (PLB**744**,250; PLB**673**,127)) + BaBar RF0 data (PRD**90**,052003) and Belle RF0 (PRD**78**,032011) data

Full QCD dynamics taken into account, including TMD evolution effects at the NLL' order and perturbative QCD corrections at the NLO

Transversity PDFs

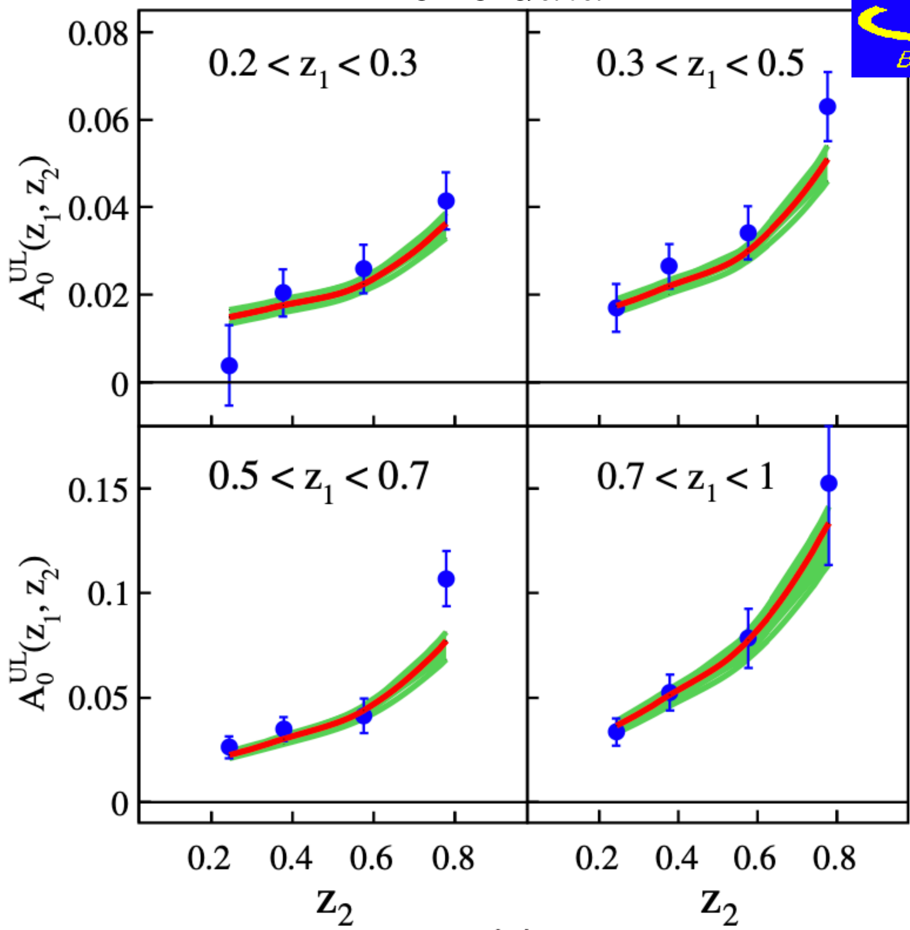


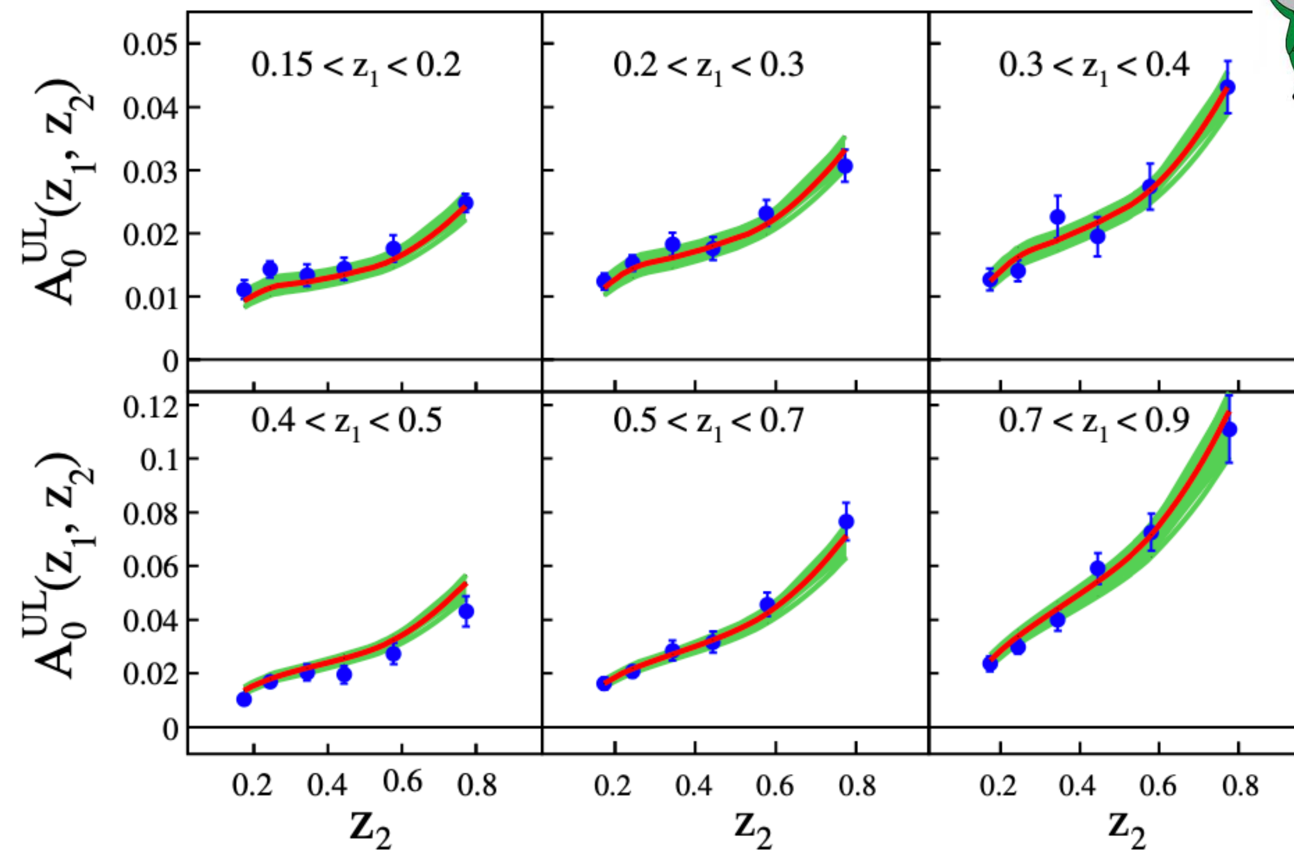
Collins FFs



Extraction of Collins FF: data description

PRD 93,014009 (2016)



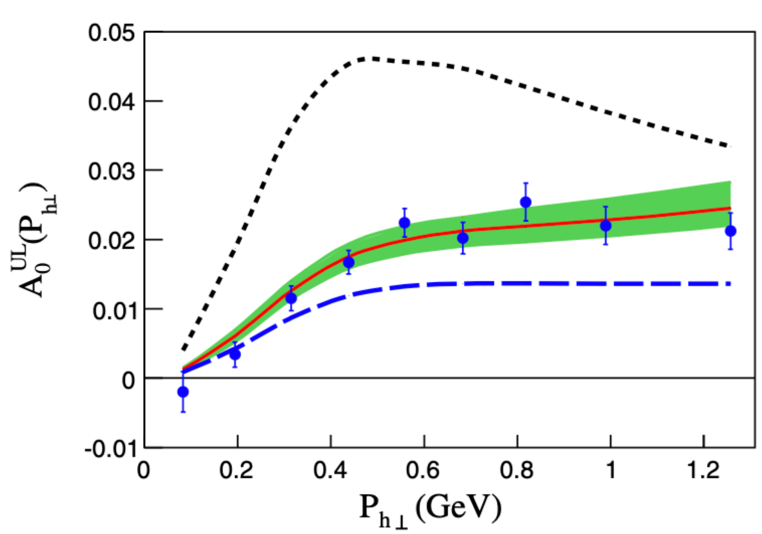


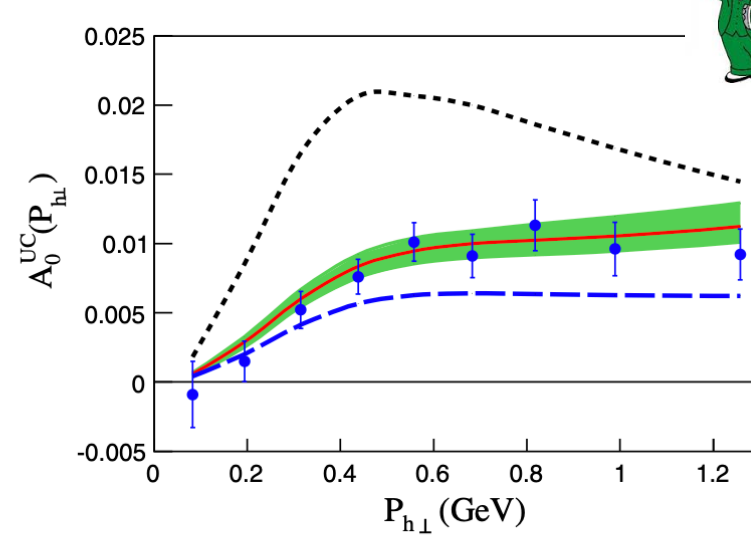
NLL' calculations

LL calculation

no TMD evolution

e+e− data are very sensitive to the inclusion of higher order corrections



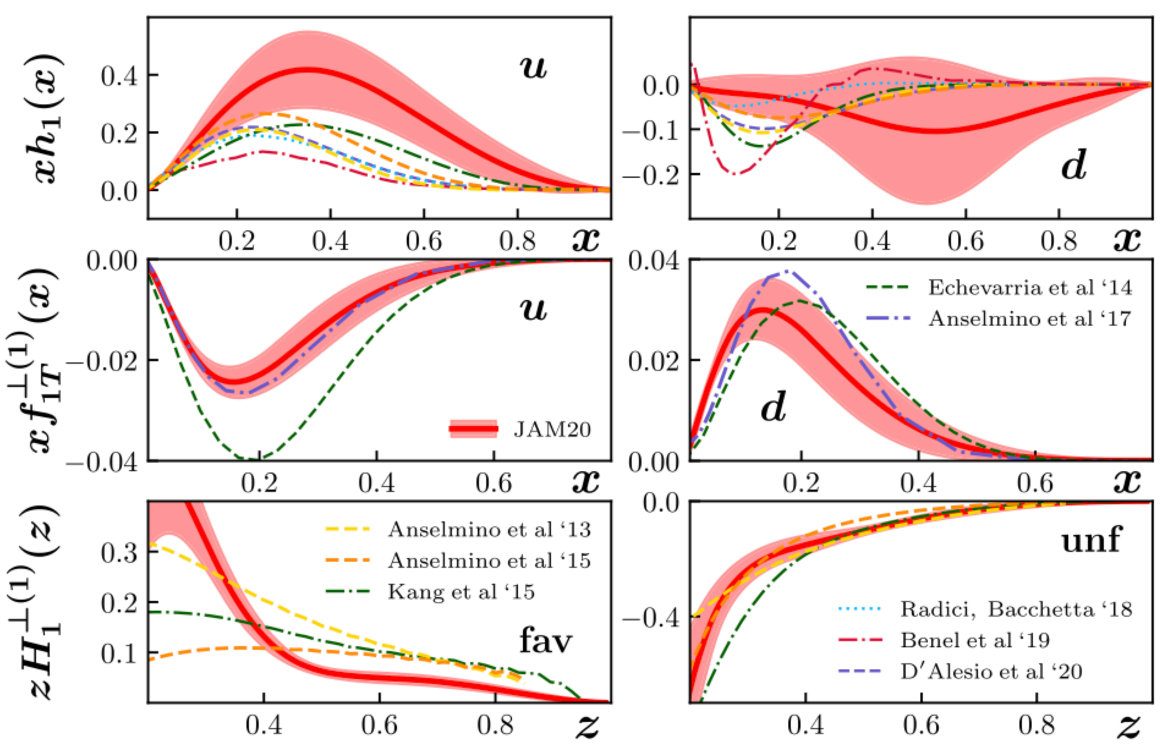


First Global Analysis

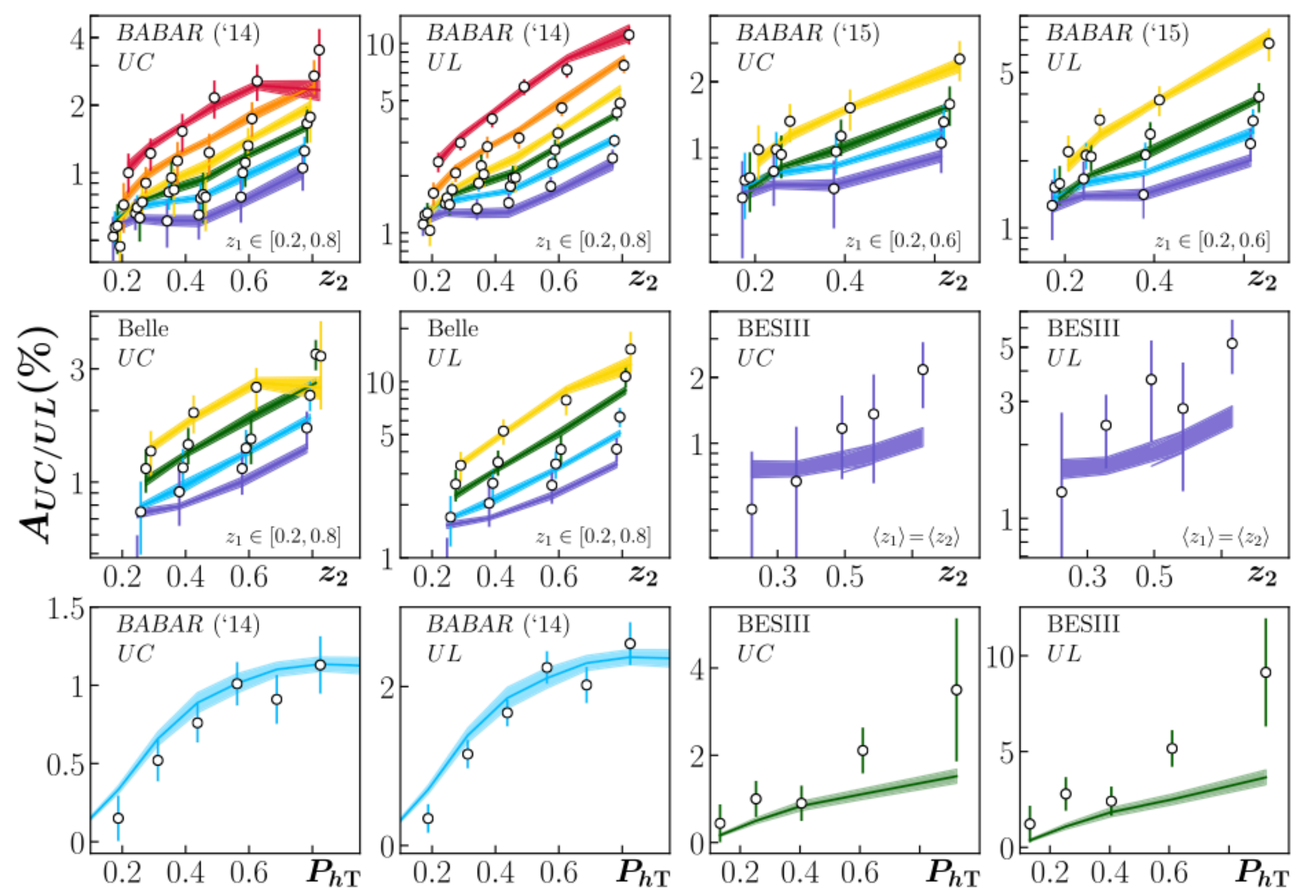
PRD 102, 054002 (2020)

First simultaneous QCD global analysis with SIDIS, e+e- annihilation, DY and proton-proton collisions

- Test of universality
- Indication that transverse-spin asymmetries in high-energy collisions have a common origin
- Extracted quark tensor charges are in excellent agreement with lattice QCD



Anselmino'13, Anselmino'15, Anselmino'17, Benel'19, D'Alessio'20, Echevarria'14, Kang'15, Radici-Bacchetta'18

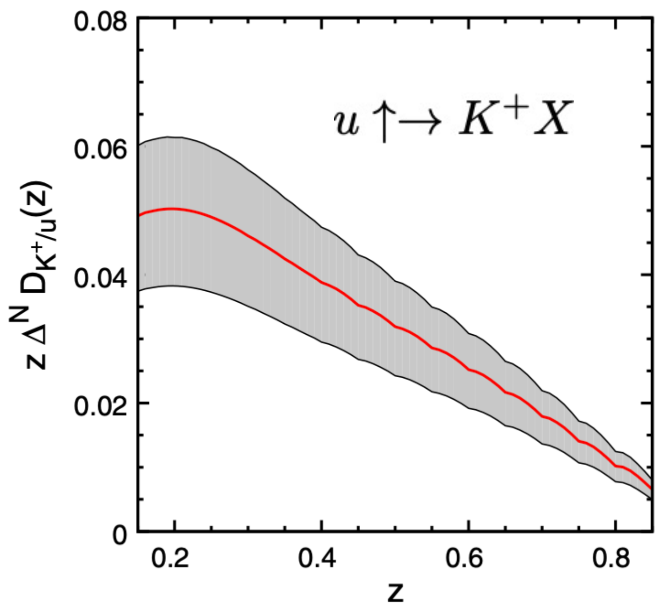


Comparison between theory and data
(BaBar, Belle, BESIII)

Extraction of Collins FF for Kaons

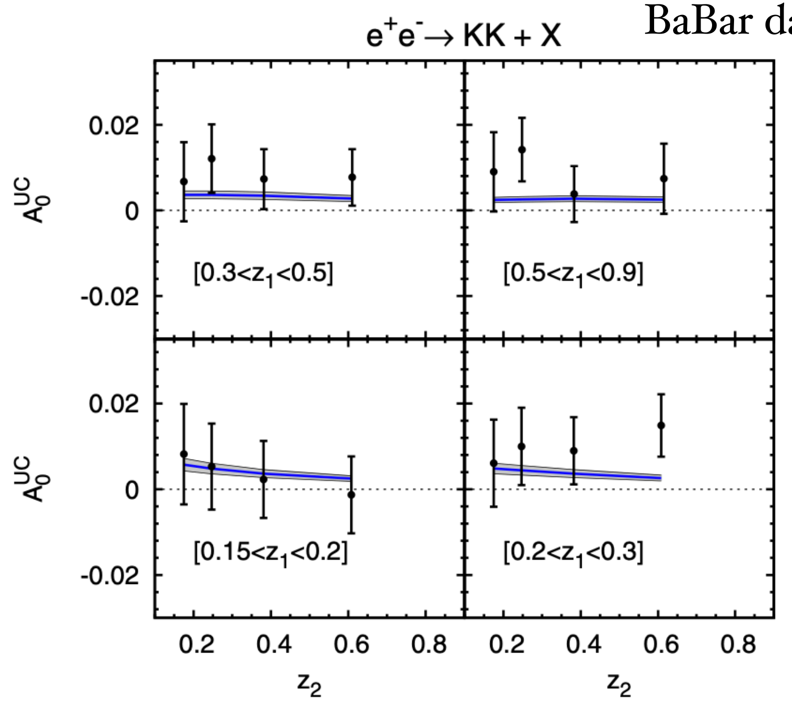
PRD 93, 034025 (2016)

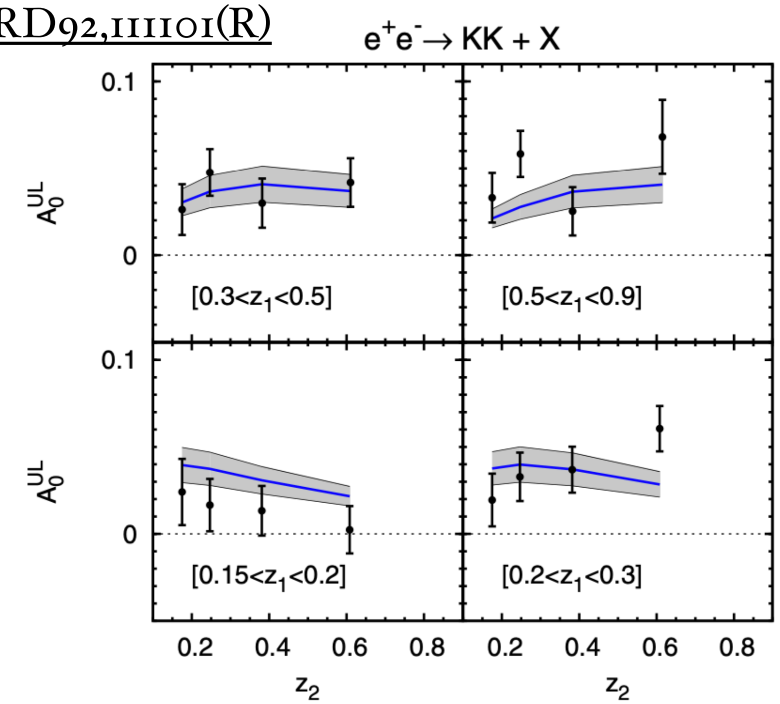
- Pions FF from previous analysis
- Similar parametrisation for kaon favoured and disfavoured FFs as used for pions

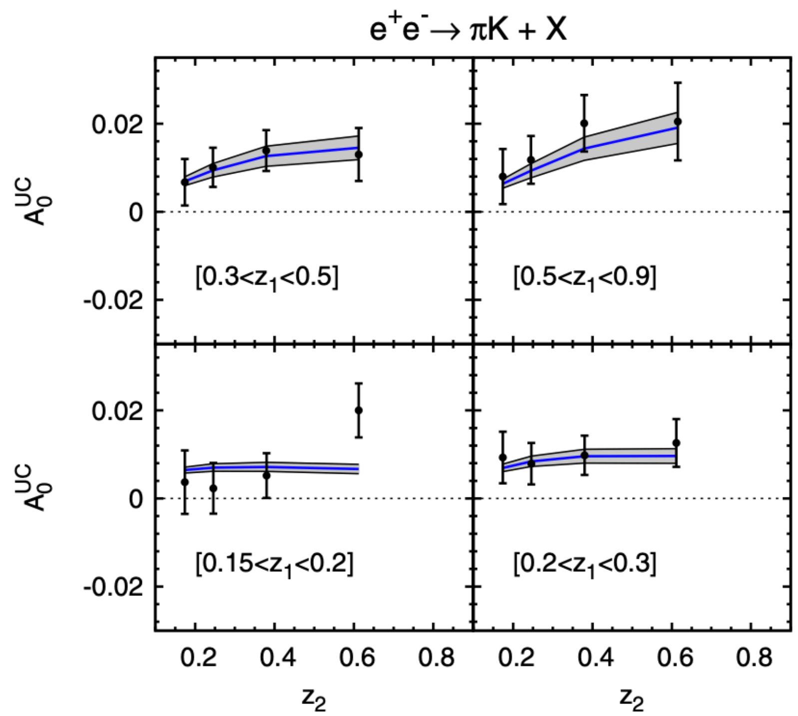


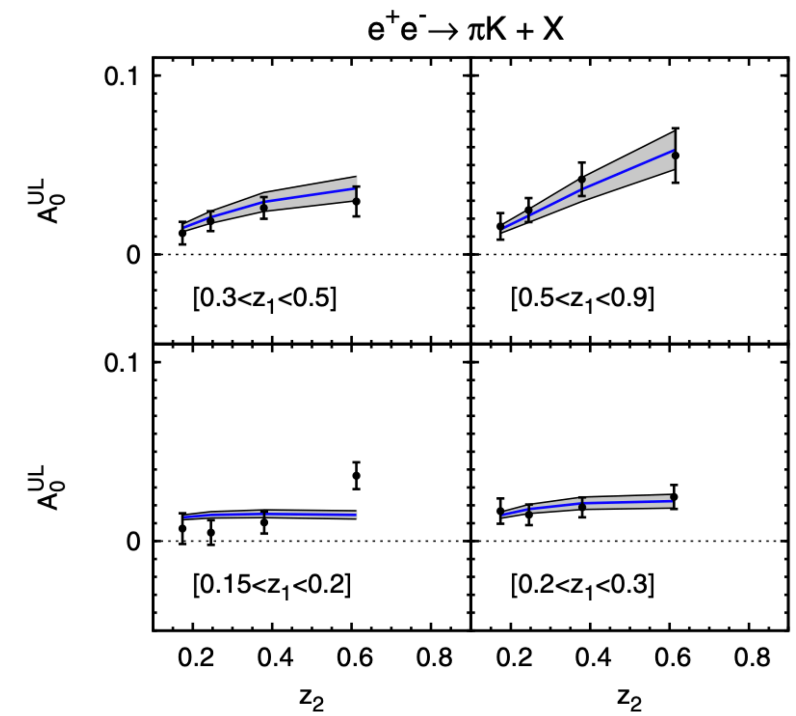
- Positive favored Collins function and a disfavoured contribution compatible with zero
- Large uncertainty
- No definitive conclusions can be extracted for strange Collins FF nor on the disfavoured ones

BaBar data: PRD92,111101(R)



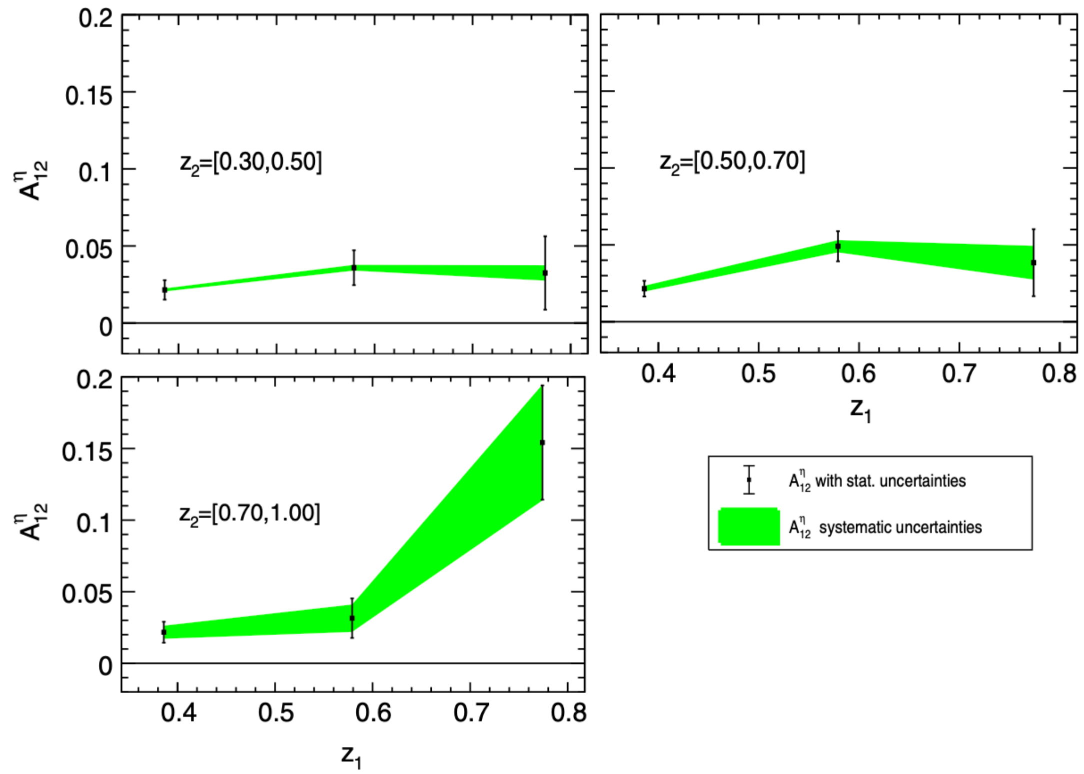


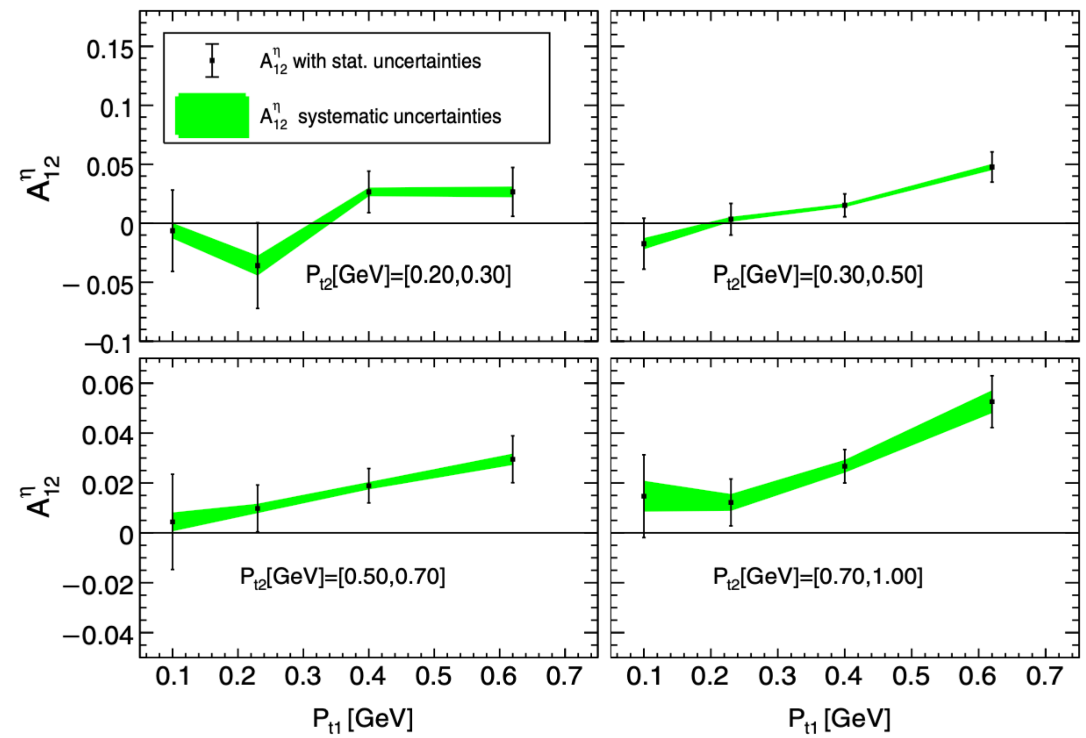




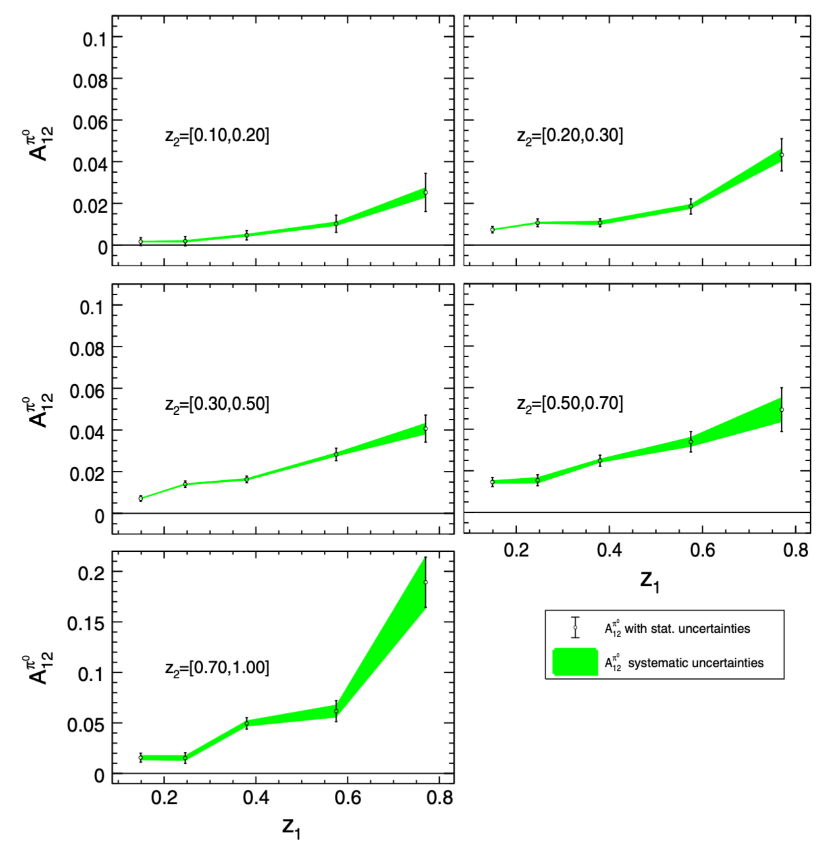
Azimuthal asymmetries of back-to-back $\pi^{\pm}\eta/\pi^{\pm}\pi^{0}$ from Belle

PRD 100, 092008 (2019)





- L = 980.4 fb^{-1}; RF12 (thrust reference frame) only
- π^0:
  - Significant asymmetries that rise with z
  - Continuous rise with P_{t1} consistent with a linear behaviour
- η:
  - Larger uncertainties
  - The rise with z is much less pronounced
  - Rise of the asymmetry with increasing pt
  - Charm contribution ~ 20-30% larger than neutral pions sample
  - for those bins with similar charm contributions η and π^0 asymmetries are fully consistent



$\Lambda/\overline{\Lambda}$ hyperon polarization at Belle

PRD 122, 042001 (2019)

- $e^+e^- \rightarrow \Lambda(\overline{\Lambda})X$ and $e^+e^- \rightarrow \Lambda(\overline{\Lambda})h^\pm X$ using 800.4 fb^{-1} collected by Belle at or near $\sqrt{s}$=10.58 GeV

$$D_{1T}^{\perp\Lambda/q}(z, p_\perp^2)$$

==> production of transversely polarized Λ hyperons from unpolarised quark
==> chiral-even and T-odd function

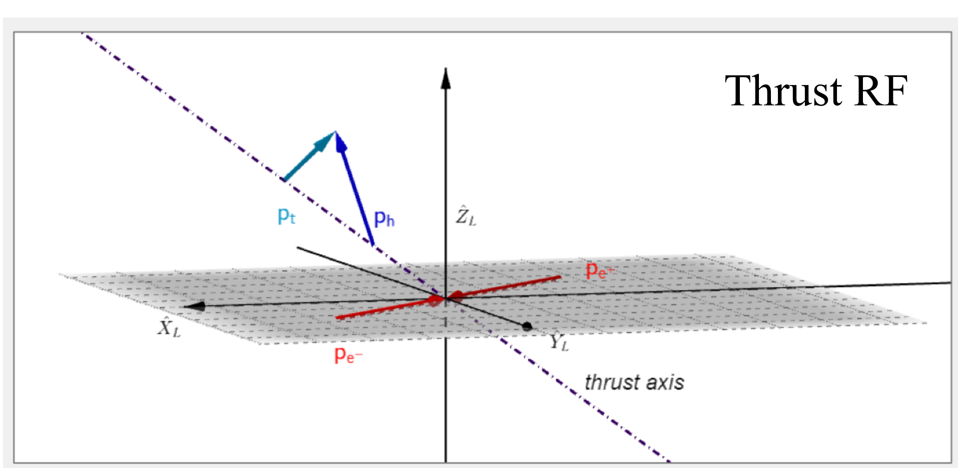
Distribution of protons from Λ decays with a transverse polarisation P:

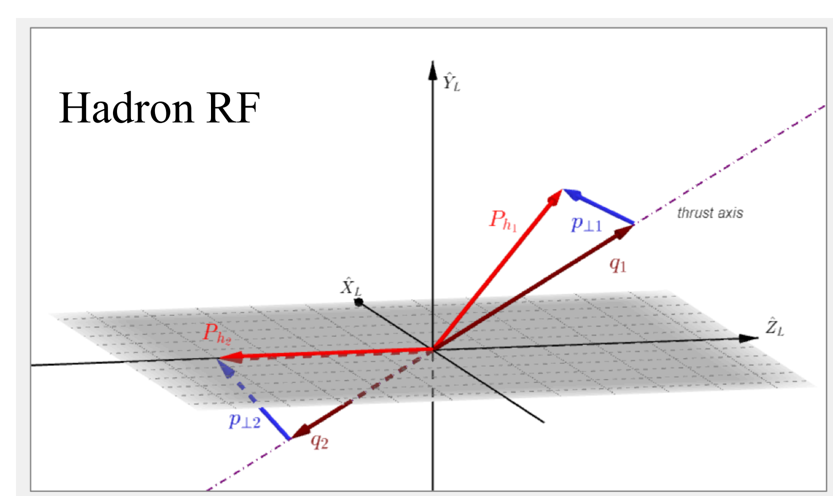
$$\frac{1}{N}\frac{dN}{d\cos\theta} = 1 + \alpha P\cos\theta,$$

$$P_\Lambda(z, p_\perp) = \frac{\sum_q e_q^2 \frac{4zp_\perp\sqrt{s}}{z^2 s + 4p_\perp^2}\Delta^N D_{\Lambda^\uparrow/q}(z, p_\perp)}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_\perp)} \times \frac{\int d(\cos\theta)\sin(2\theta)}{\int d(\cos\theta)(1+\cos^2\theta)}$$

PRD **100**,014029

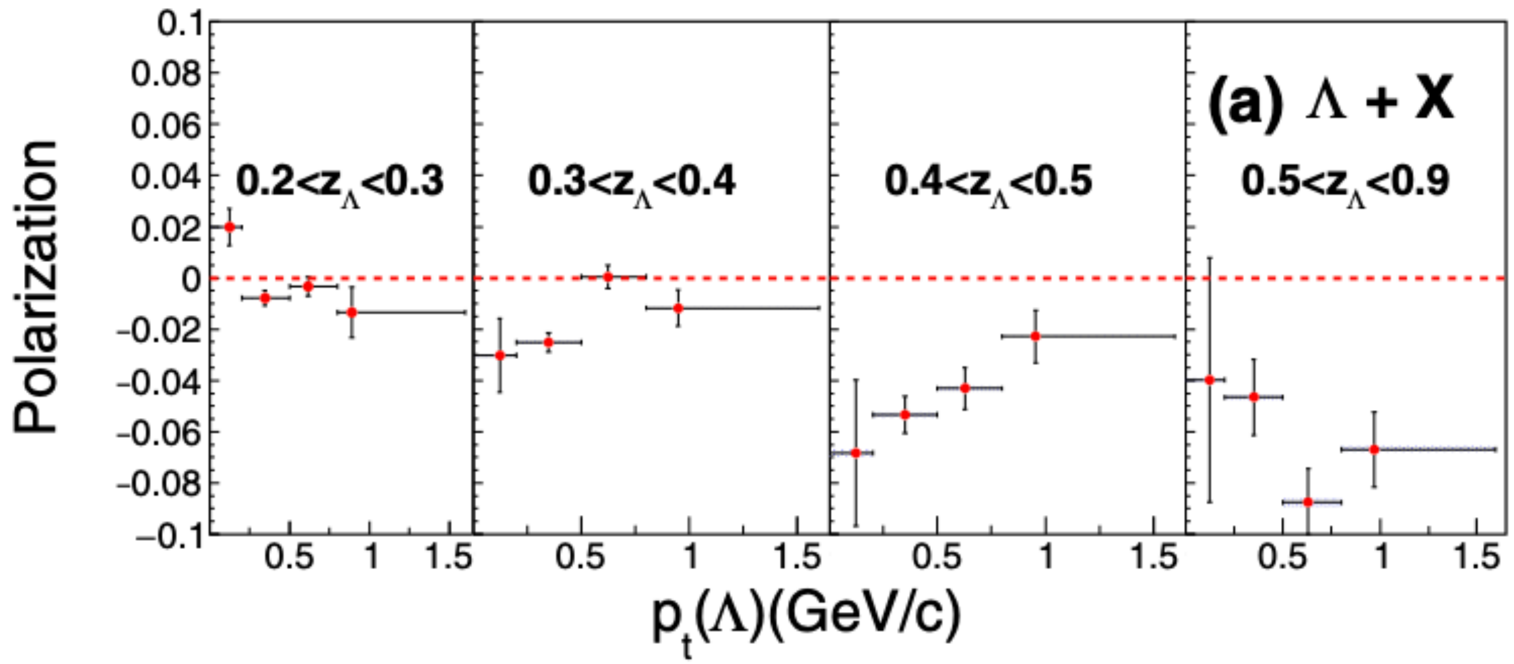
θ: angle between the axis $\hat{n} \propto \hat{T} \times \hat{p}_\Lambda$ and the proton momentum in the Λ rest frame; α is the world average value of the parity-violating decay asymmetry for the Λ (from PDG)
==> Significant transverse polarization is observed

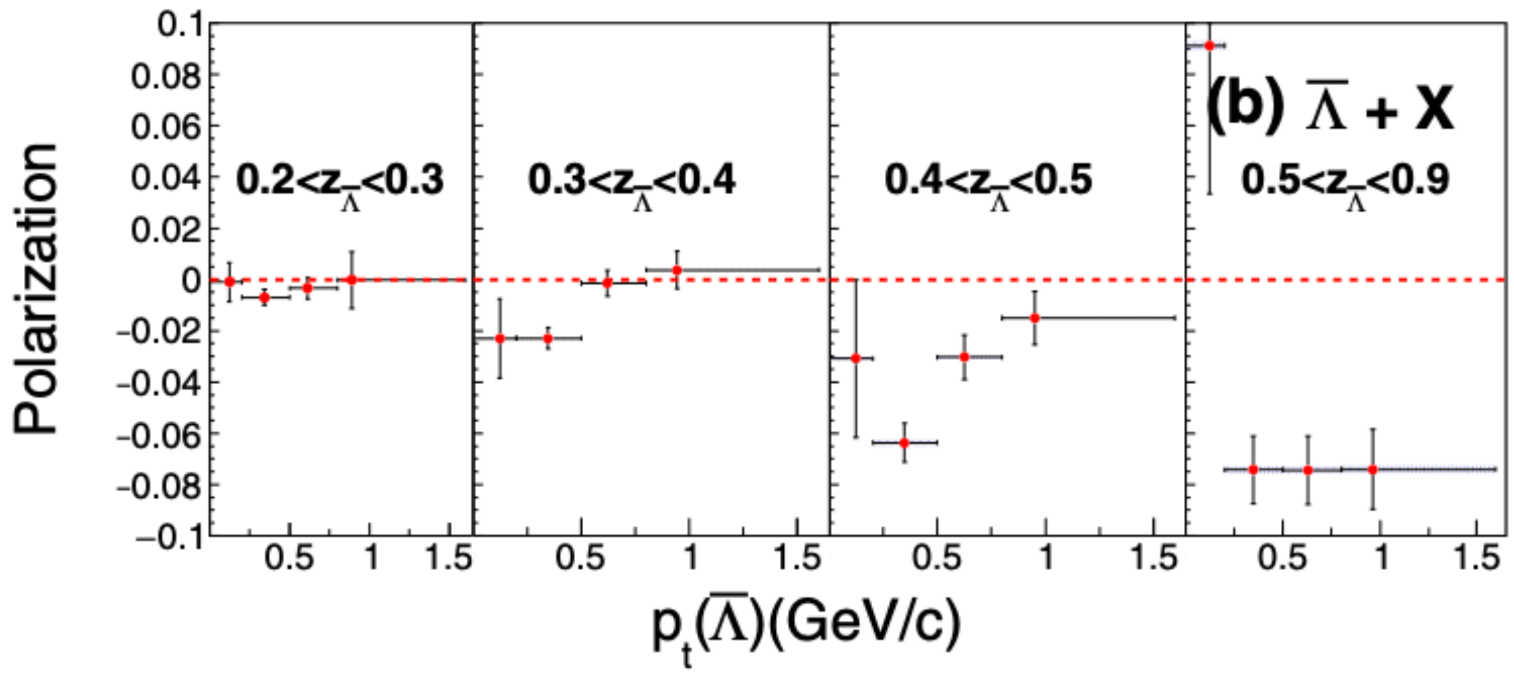




$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle: results

PRD 122, 042001 (2019)



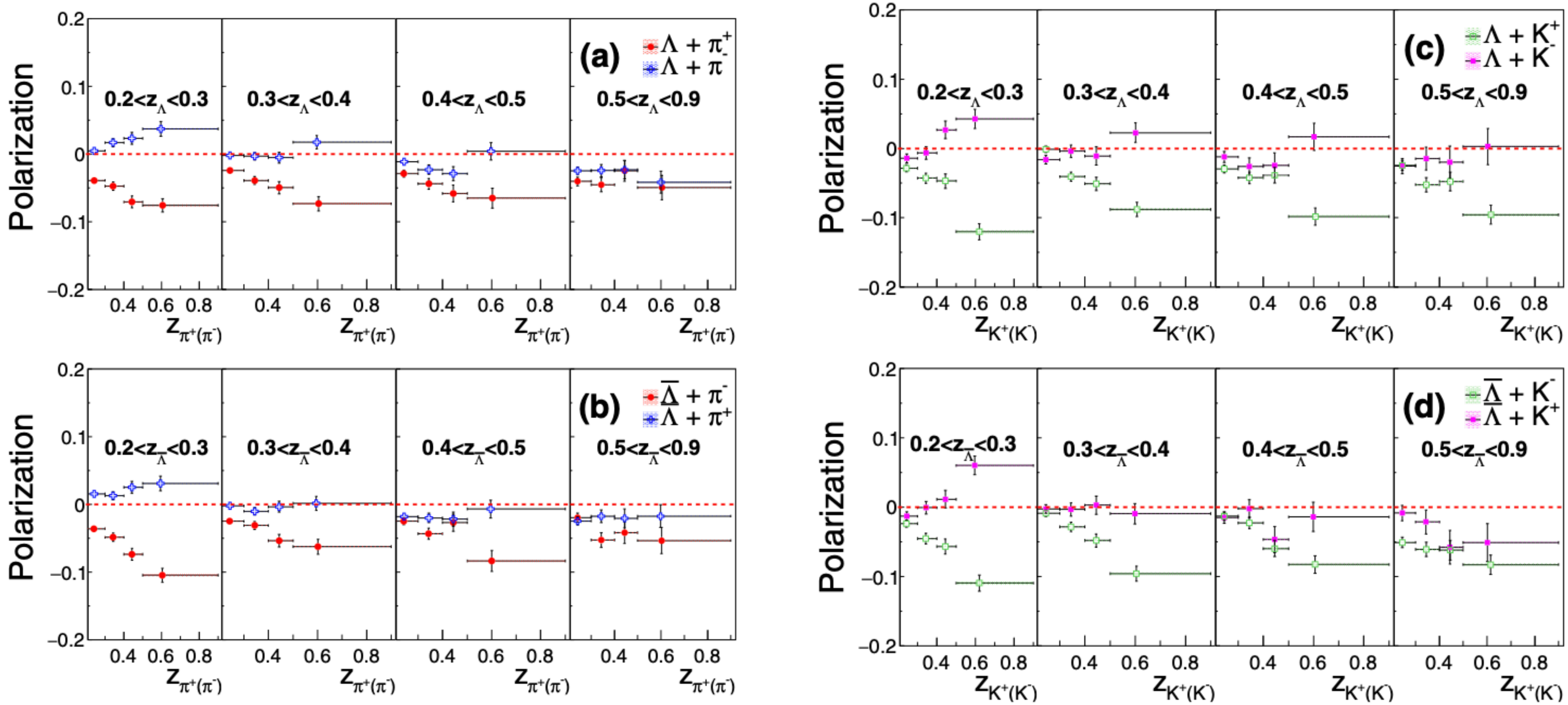


Complex p_t dependence

- $z_\Lambda>0.5$: increasing asymmetry with pt; opposite behaviour for intermediate z_Λ
  - different quark flavour contributions in different kinematic regions (s quark contribution is dominant in the highest z_Λ bin; u quark and charm contribution in the intermediate regions)
  - large charm contribution in intermediate z-bins

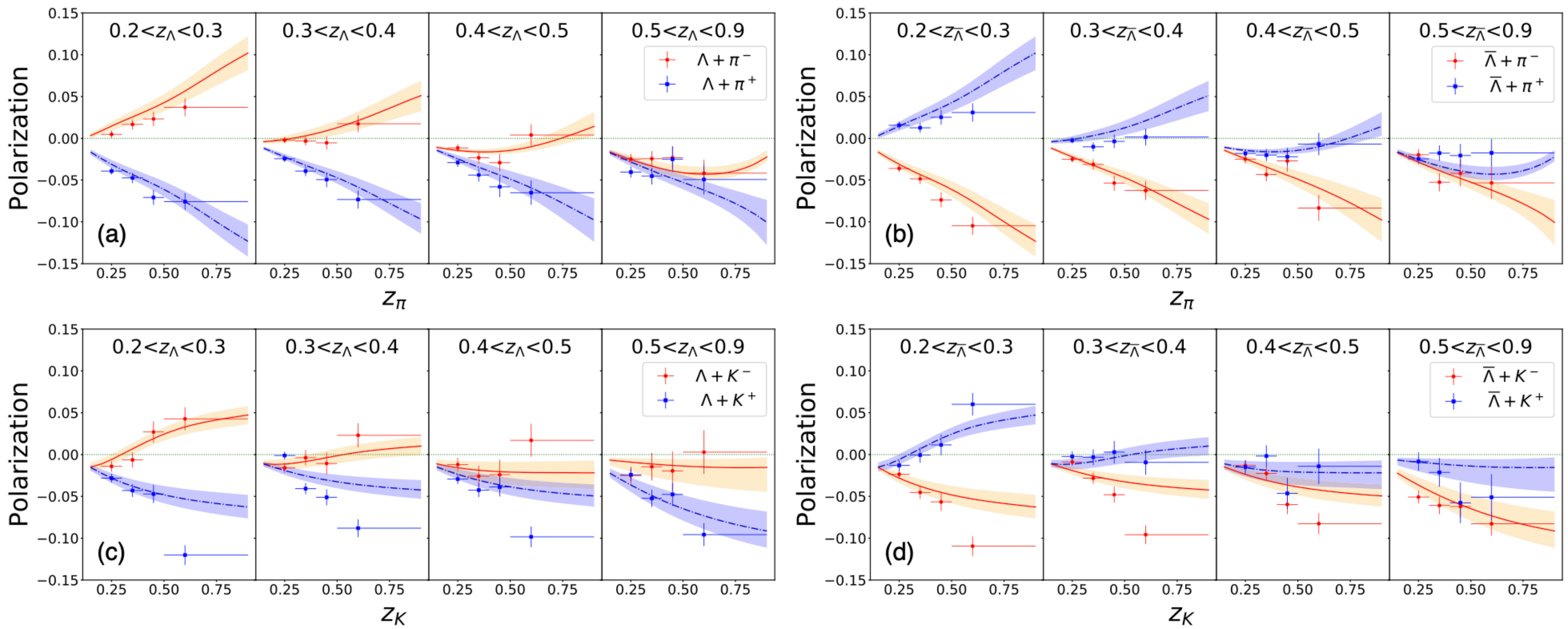
$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle: results

PRD 122, 042001 (2019)



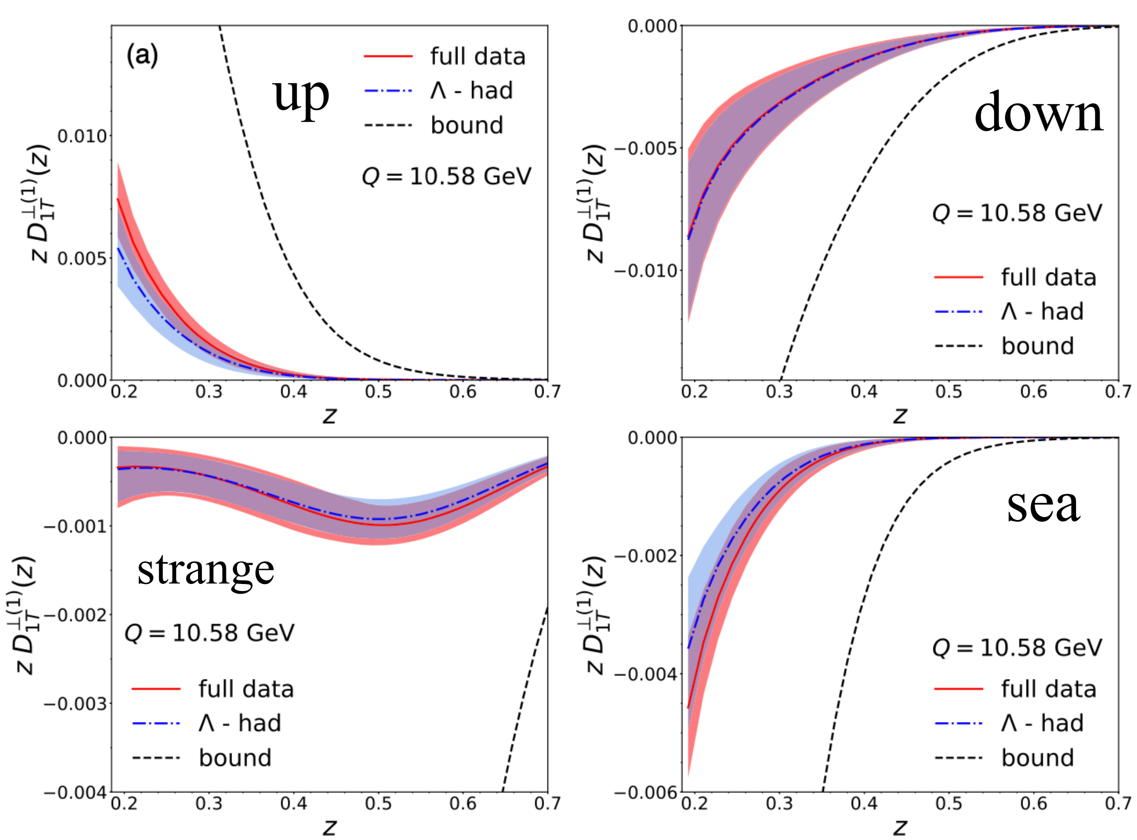
- low z_Λ: opposite site and increasing magnitude
- Sensitivity to the flavour dependence by selecting a light hadron in the opposite hemisphere:
  - Λ polarization from s quark is negative: negative asymmetry observed in ΛK^+X at high z_Λ, where s to Λ fragmentation dominates
  - In $\Lambda\pi^-X$ and ΛK^-X $u\to\Lambda$ fragmentation dominates at low z_Λ: u fragmentation to Λ is positive

First extraction of Λ polarised FF from Belle data



First extraction from Belle data within a TMD approach: D'Alessio, Murgia, Zaccheddu

PRD 102, 054001 (2020)

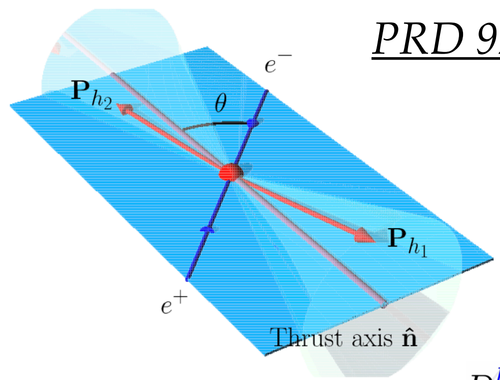


- Clear separation in flavour achieved
- The needed of sea-quark polarised FF is well supported
- Extracted the first pt dependence

Higher statistic and complementary studies in other processes needed for a deeper understanding of this TMD-FF and solving the long-standing puzzle of the observed spontaneous transverse hyperon polarisation

Di-hadron Fragmentation Functions @ Belle

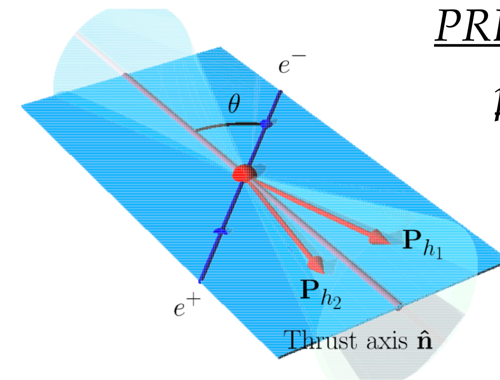
PRD 92, 092007 (2015)



$D^h_{1,q}(z_1, Q^2) D^h_{1,q}(z_2, Q^2)$

more likely hadrons emerge from different parton
==> single hadron FF for each hadron

PRD 92, 092007 (2015)
pol. counterpart: IFF



$D^{h_1 h_2}_{1,q}(z, m, Q^2)$

more likely hadrons emerge from the same parton ==> di-hadron FF

Give access to favoured and disfavoured fragmentation processes: the ratio of inclusive cross sections for charged di-hadrons in various topologies and for different hadron type combinations are calculated

Several definition of fractional energies: Belle: PRD 101, 092004 (2020)

$$z = 2E_h/\sqrt{s}$$

$$z_1 = \frac{2P_1 \cdot q}{q^2} \qquad z_2 = u = \frac{P_1 \cdot P_2}{P_1 \cdot q}$$

AEMP
Nucl. Phys. B160, 301 (1979)

facilitate the interpretation of cross section in term of single-hadron FFs

$$z_1 = \left(P_1 \cdot P_2 - \frac{M_{h1}^2 M_{h2}^2}{P_1 \cdot P_2}\right) \frac{1}{P_2 \cdot q - M_{h2}^2 \frac{P_1 \cdot q}{P_1 \cdot P_2}}$$

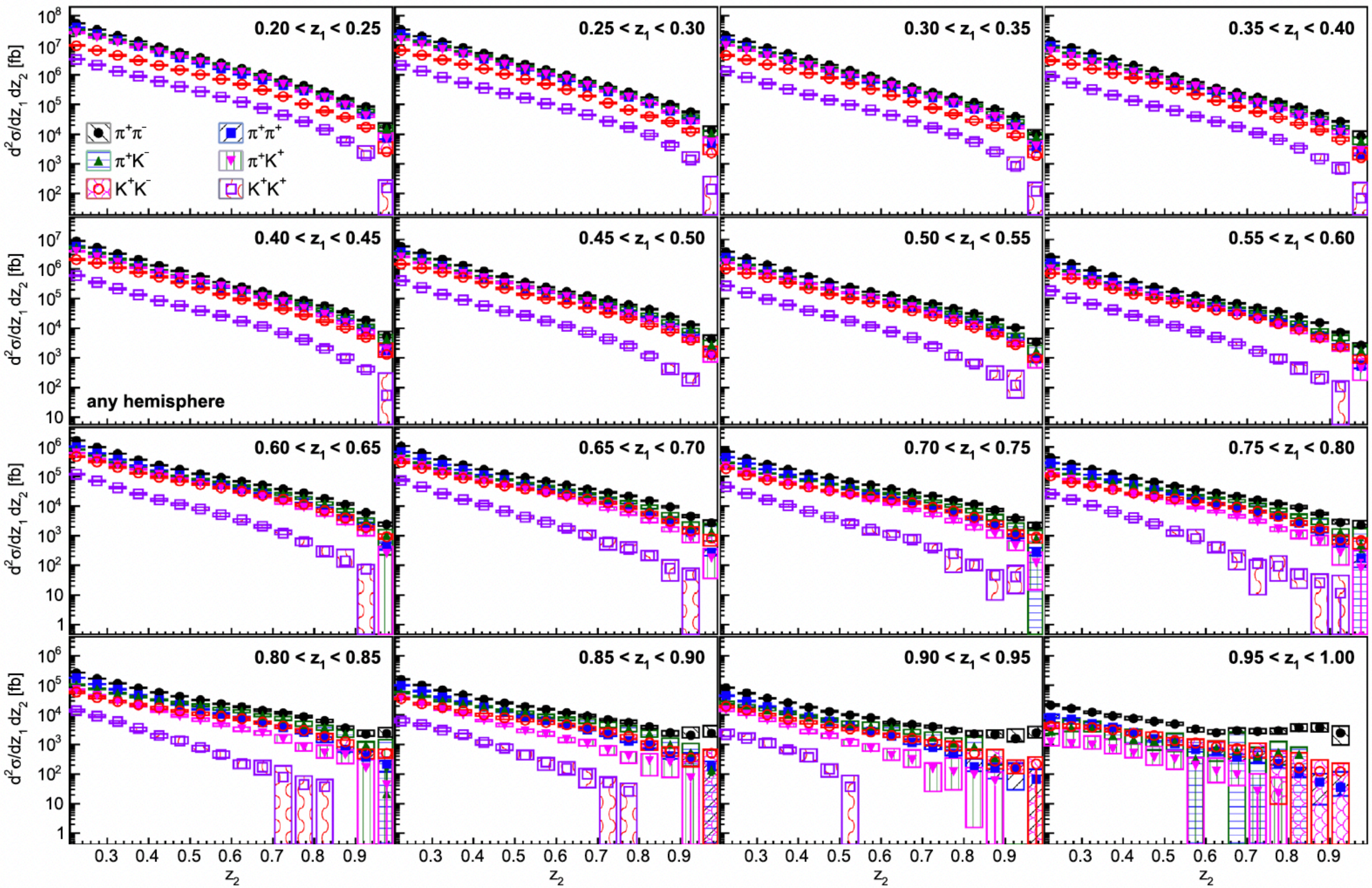
$$1 \leftrightarrow 2$$

MVH:
PRD 100, 034011 (2019)

highlight the transverse momentum produced in the fragmentation process

Di-hadron Fragmentation Functions @ Belle

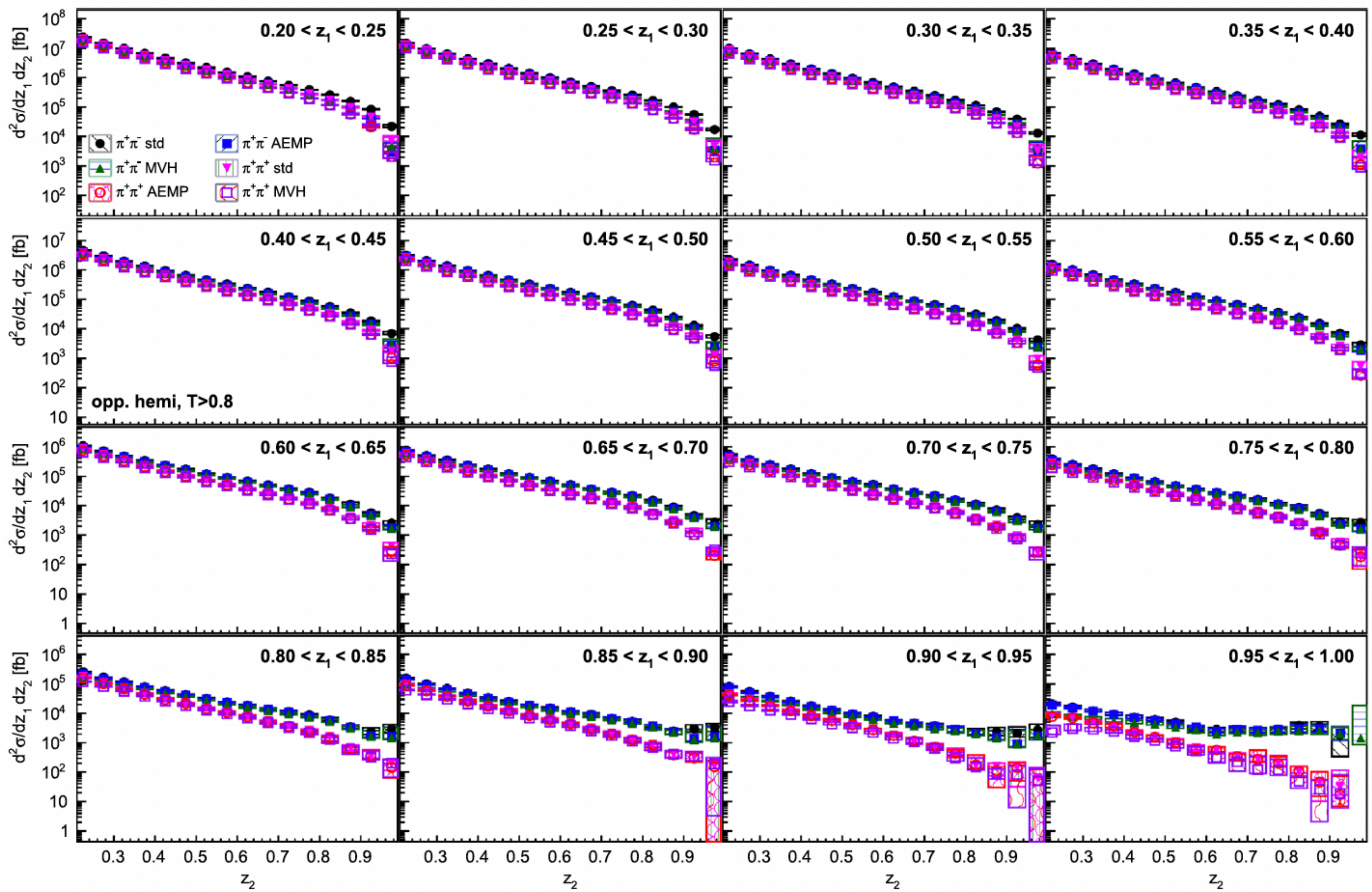
Belle: PRD 101, 092004 (2020)



Di-Hadron in any hemispheres as a functions of z_1 and z_2 using conventional definition

Di-hadron Fragmentation Functions @ Belle

Belle: PRD 101, 092004 (2020)



Di-Hadron in opposite hemispheres as a functions of z_1 and z_2 for same sign and opposite sign pion pairs, and comparison between conventional, MVH, and AEMP fractional energy definition

MVH and AEMP suppressed at small fractional energies; close to unity at high z

Summary and Conclusions

- e^+e^- annihilation experiment is an excellent laboratory to study fragmentation processes

- Crucial information for global analysis
  - combined with SIDIS and pp measurements to access PDFs

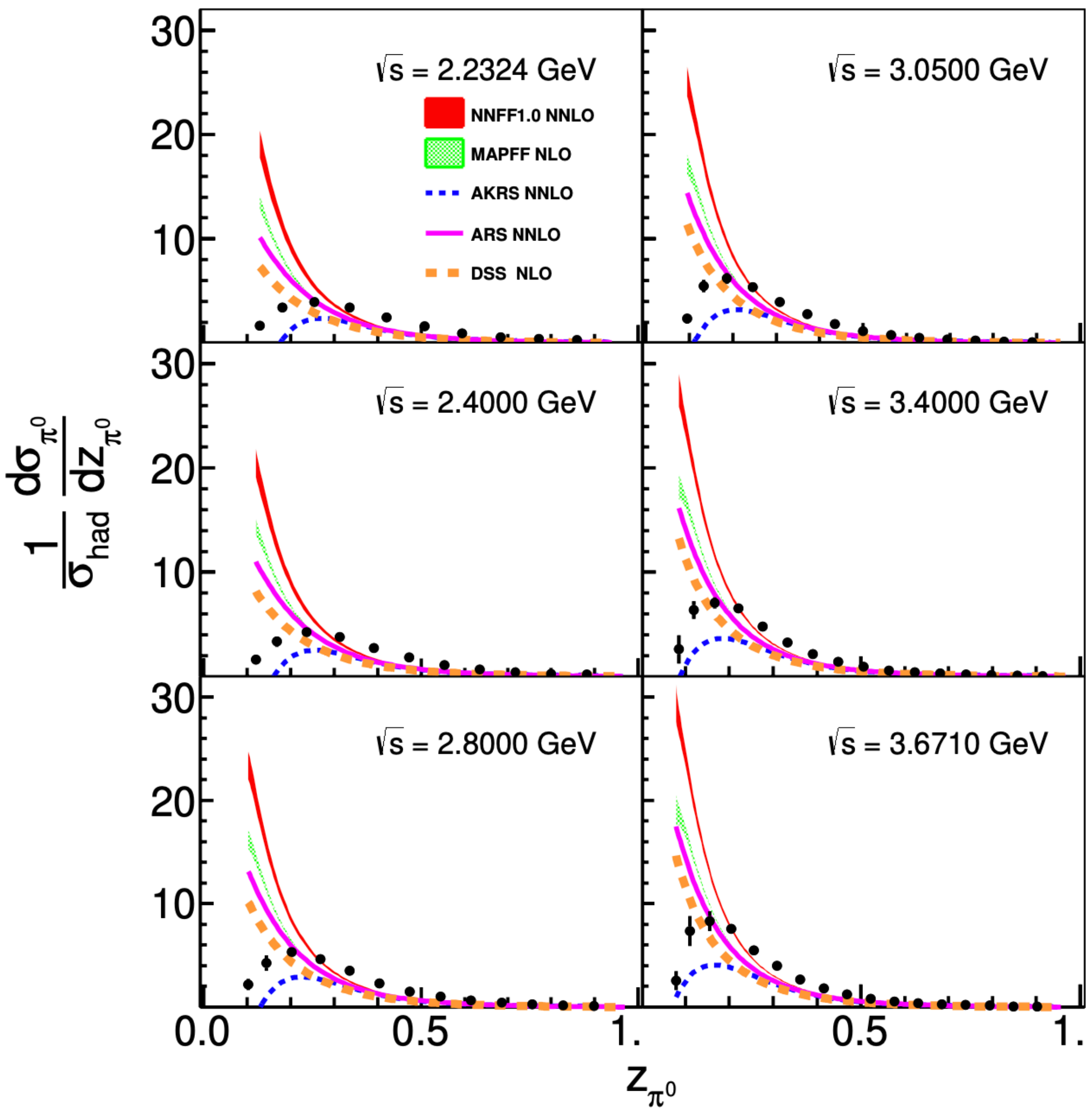
- Sensitivity not only to collinear FFs but also TMD FFs:
  - explicitly p_T dependent unpolarised FFs
  - Collins azimuthal asymmetries
  - Polarized Λ fragmentation
  - Di-hadron fragmentation function measurements

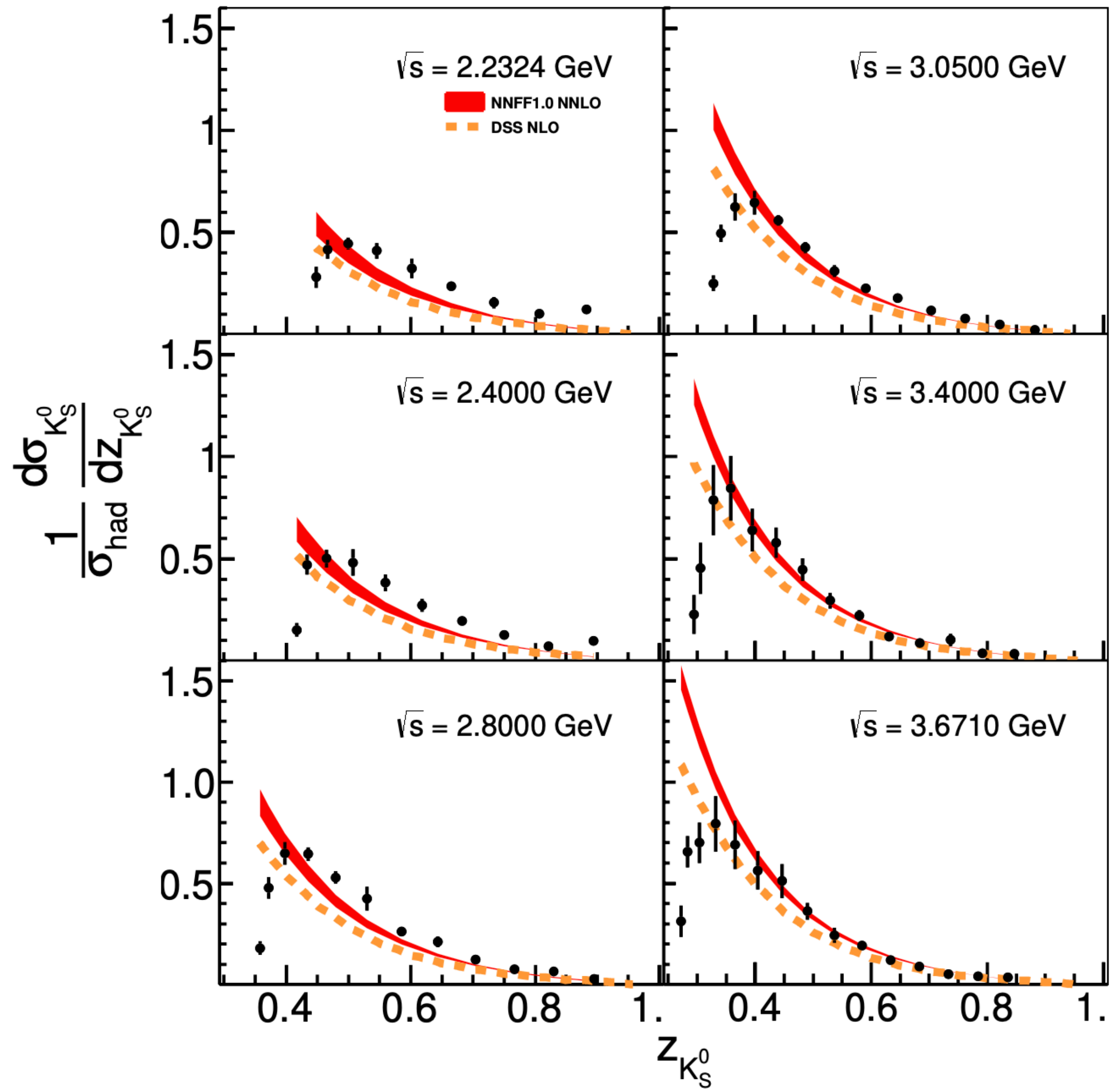
- Many other new analyses and studies in ongoing

Backup

Inclusive π^0 and K^0_S production @ BESIII

$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})}\frac{d\sigma(e^+e^- \to h+X)}{dz_h} = \frac{\sqrt{s}}{2}\sqrt{1+\frac{M_h^2c^2}{p_h^2}}\frac{1}{\sigma(e^+e^- \to \text{hadrons})}\frac{d\sigma(e^+e^- \to h+X)}{dp_h}$$

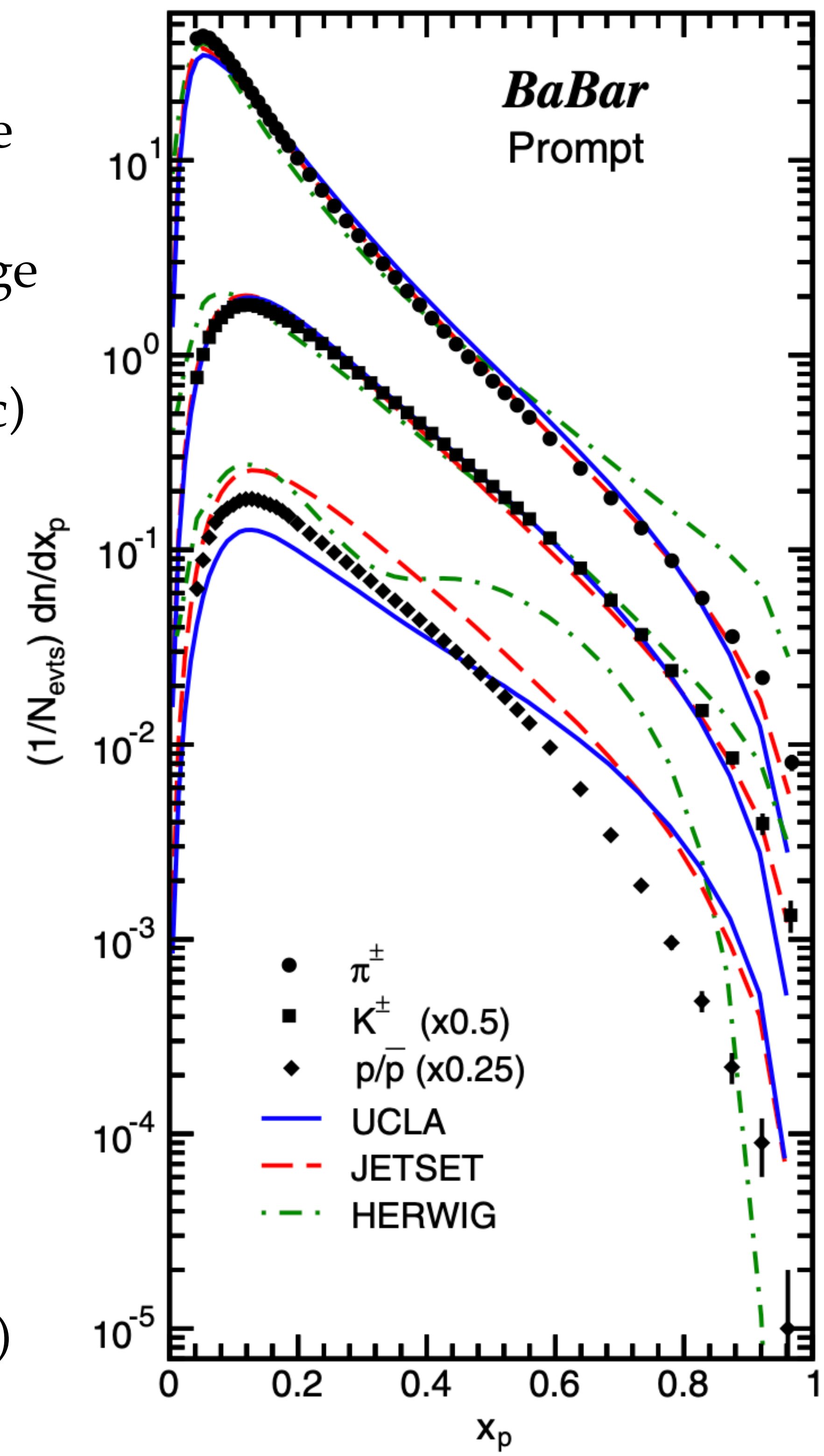




Inclusive light hadrons productions

- Data set used: 0.9 fb^{-1} @ $\Upsilon(4S)$ and 3.6 fb^{-1} at 10.54 GeV
- measured both *prompt* and *conventional* hadron cross section
  - *prompt*: primary hadrons or products of a decay chain where all particles have a lifetime shorter than 10^{-11} s
  - *conventional*: includes weak decay products of KS and strange baryons
- Scaled momentum distribution: $x_p = 2p^*/E_{mc}$ (0.2<p<5.27 GeV/c)

PRD88,032011



JETSET model:

- represent the color field between the parton by a "string", and according to an iterative algorithm breaks the string into several pieces, each corresponding to a primary hadron
- **large number of free parameters** (models many hadron species)

HERWIG model:

- splits the gluons produced into $q\bar{q}$ pairs, combines these quark and antiquark locally to form colorless "clusters", and decay these "clusters" into primary hadrons
- **few free parameters**

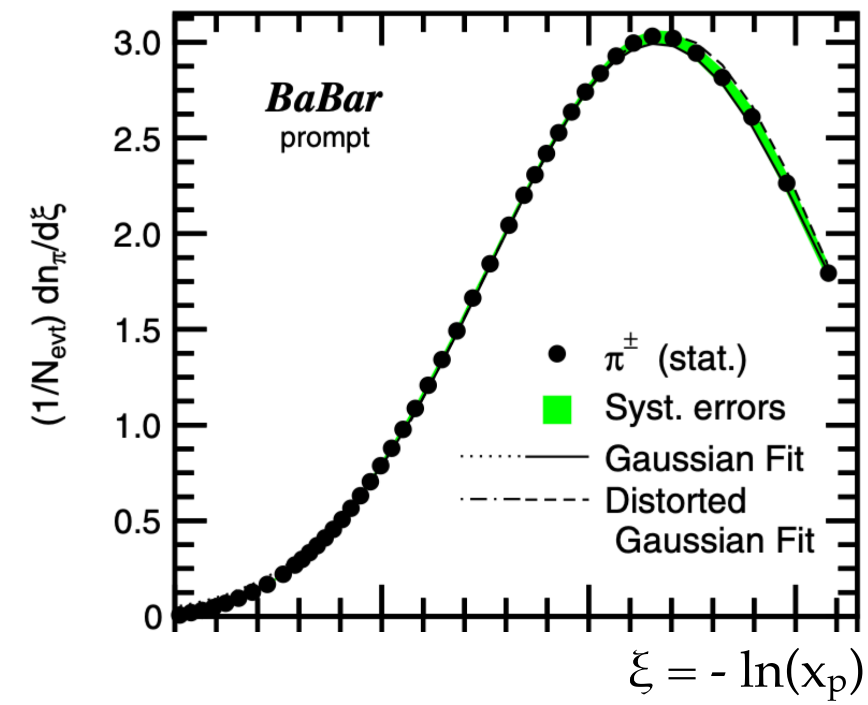
UCLA model:

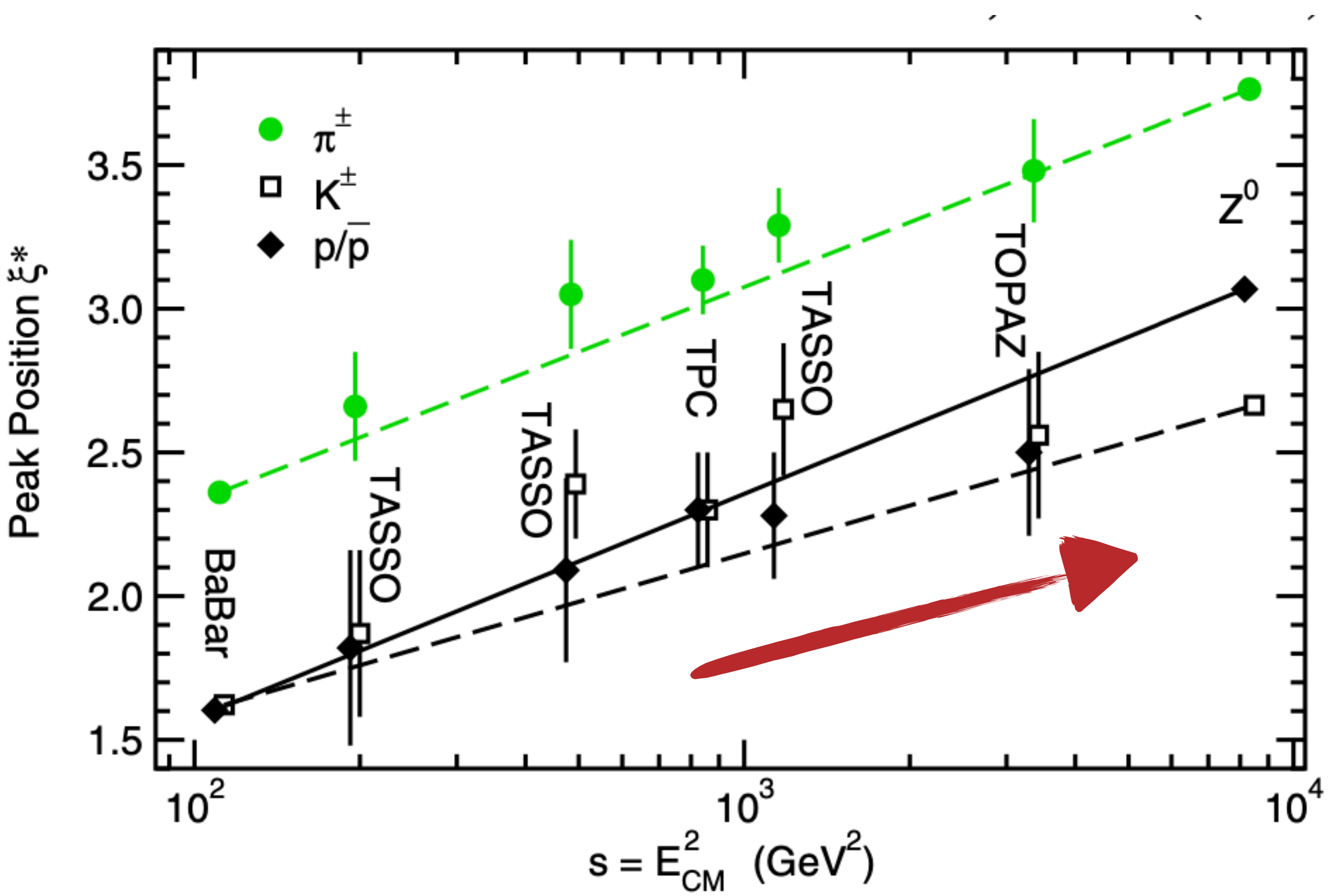
- generates whole events according to weights derived from phase space and Clebsch-Gordan coefficients
- **few free parameters**

- Good qualitative description of the bulk spectra (no the details)

Test of MLLA+LHPD QCD predictions

Modified Leading Logarithmic Approximation (MLLA) with Local Parton-Hadron Duality (LHPD) ansatz (Z.Phys.C27,65):
1) the multiplicity distributions vs $\xi = -\ln(x_p)$ should be Gaussian near the peak





2) the peak position should decrease exponentially with increasing hadron mass at a given E_{cm}

- observed that $\xi_K \approx \xi_p$

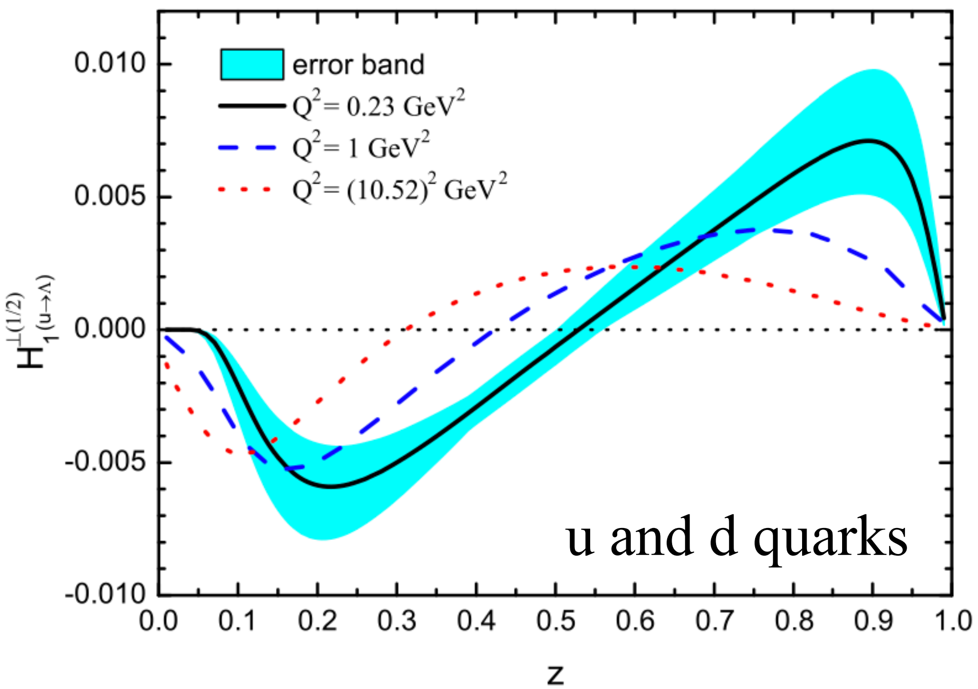
3) ξ^* should increase logarithmically with E_{mc} for a given hadron type

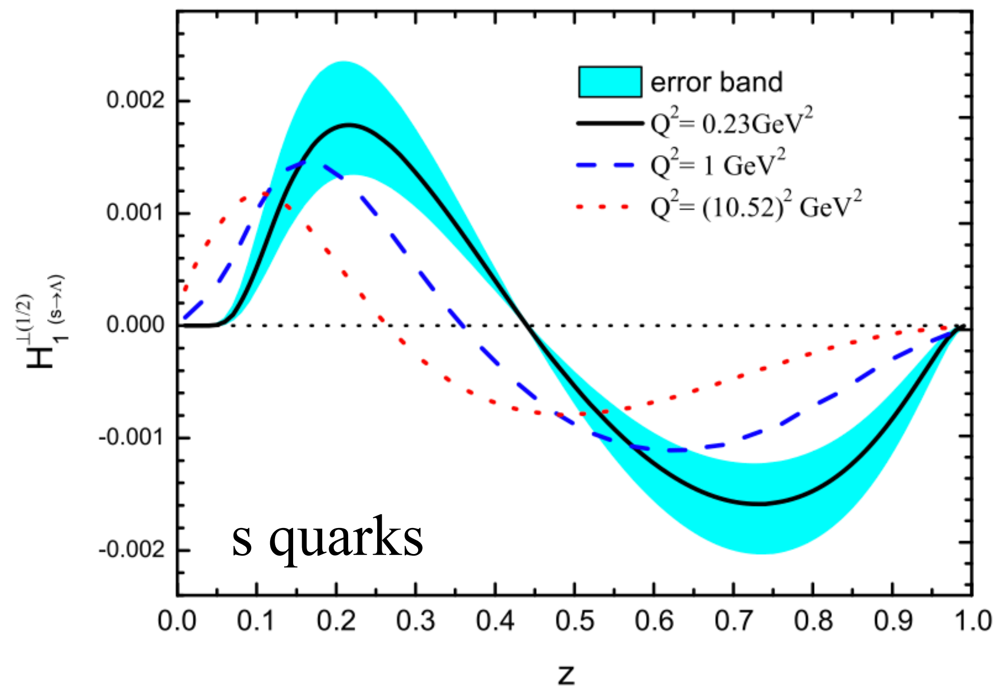
- similar slope for pions and protons, different for kaons
- possibly due to flavour composition changing with E_{cm}

Λ hyperon Collins FFs prediction

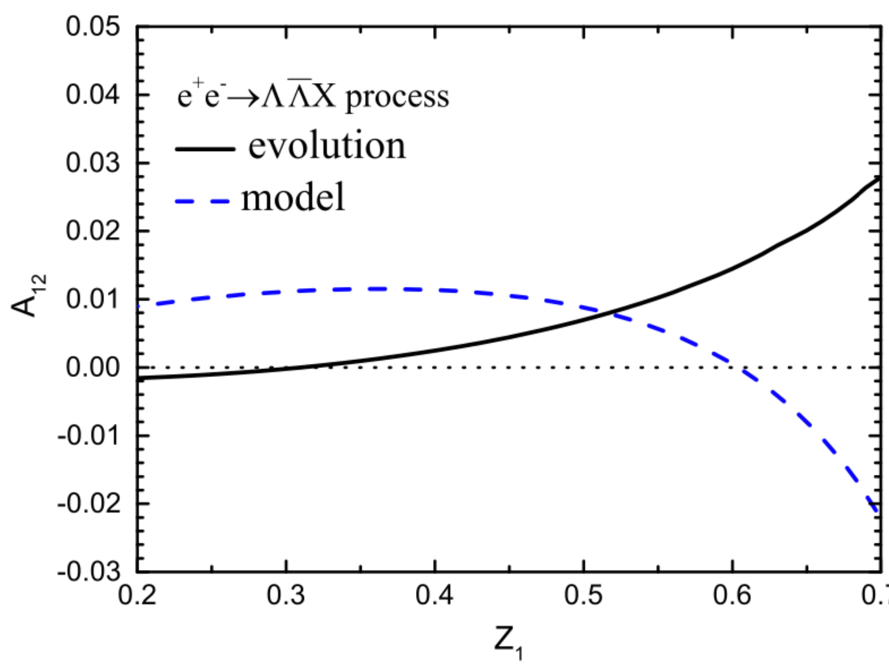
PRD 97, 114015 (2018)

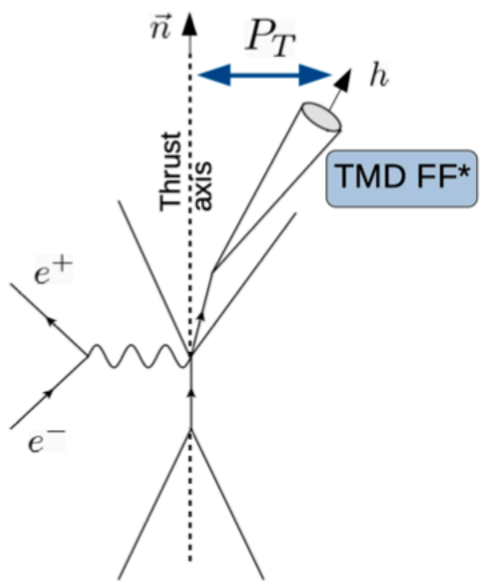
- Theoretical expectation from Collins FFs for Λ hyperon: $e^+e^- \to \Lambda\bar{\Lambda}X$
  - Process accessible in e^+e^- facilities
  - Collins FF: complementary information to $D_{1T}^{\perp}$

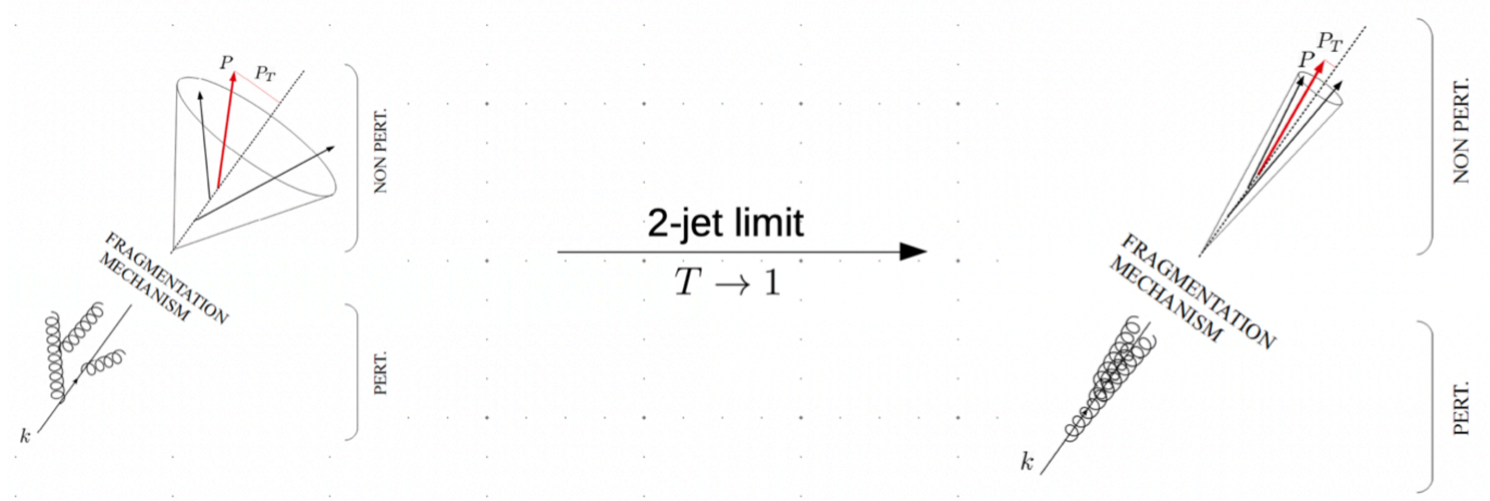


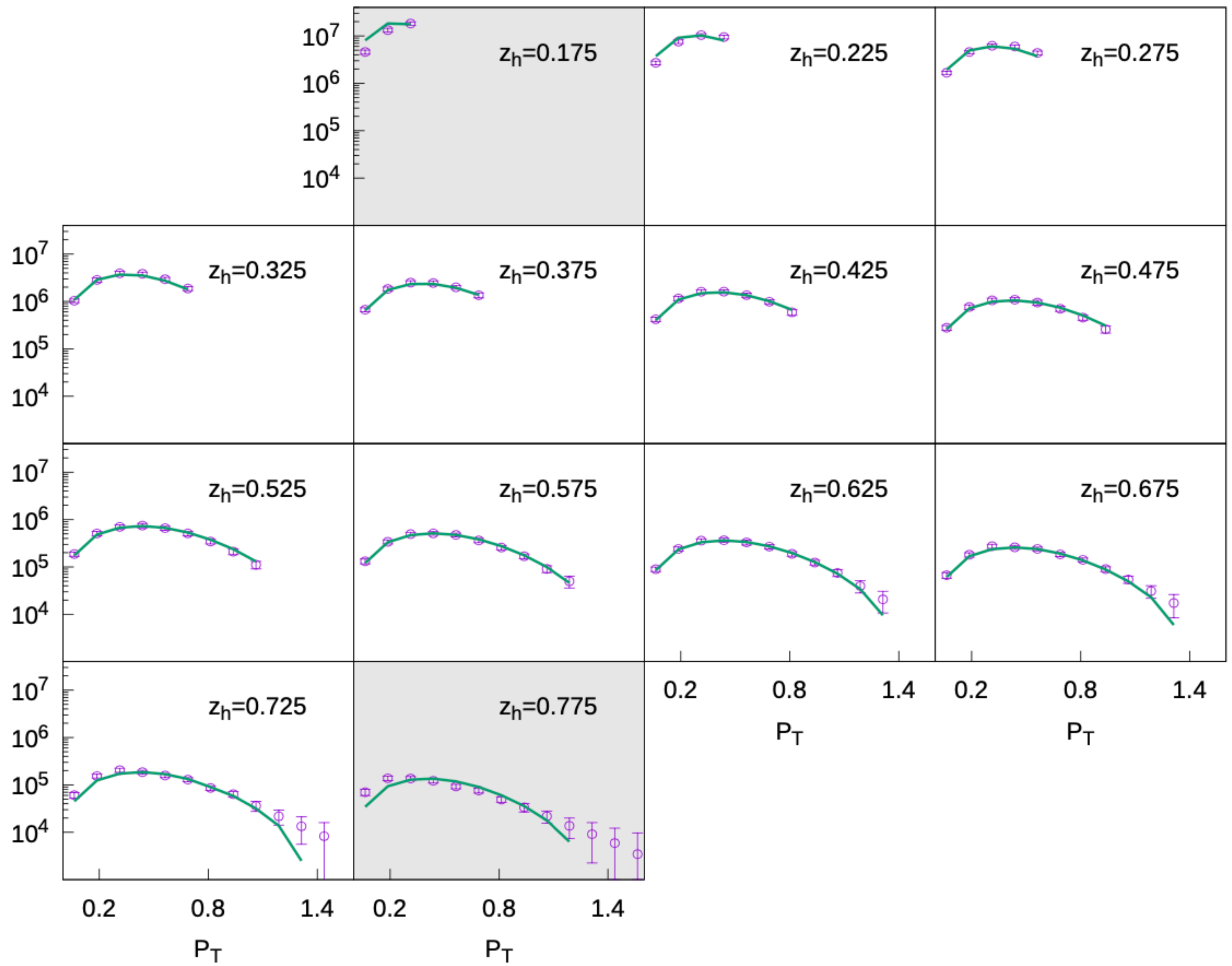


- Azimuthal asymmetry prediction in RF12 frame for $0.2<z_1<0.7$ (integrated over z_2)
- Increasing asymmetry with z, of the order of several percent



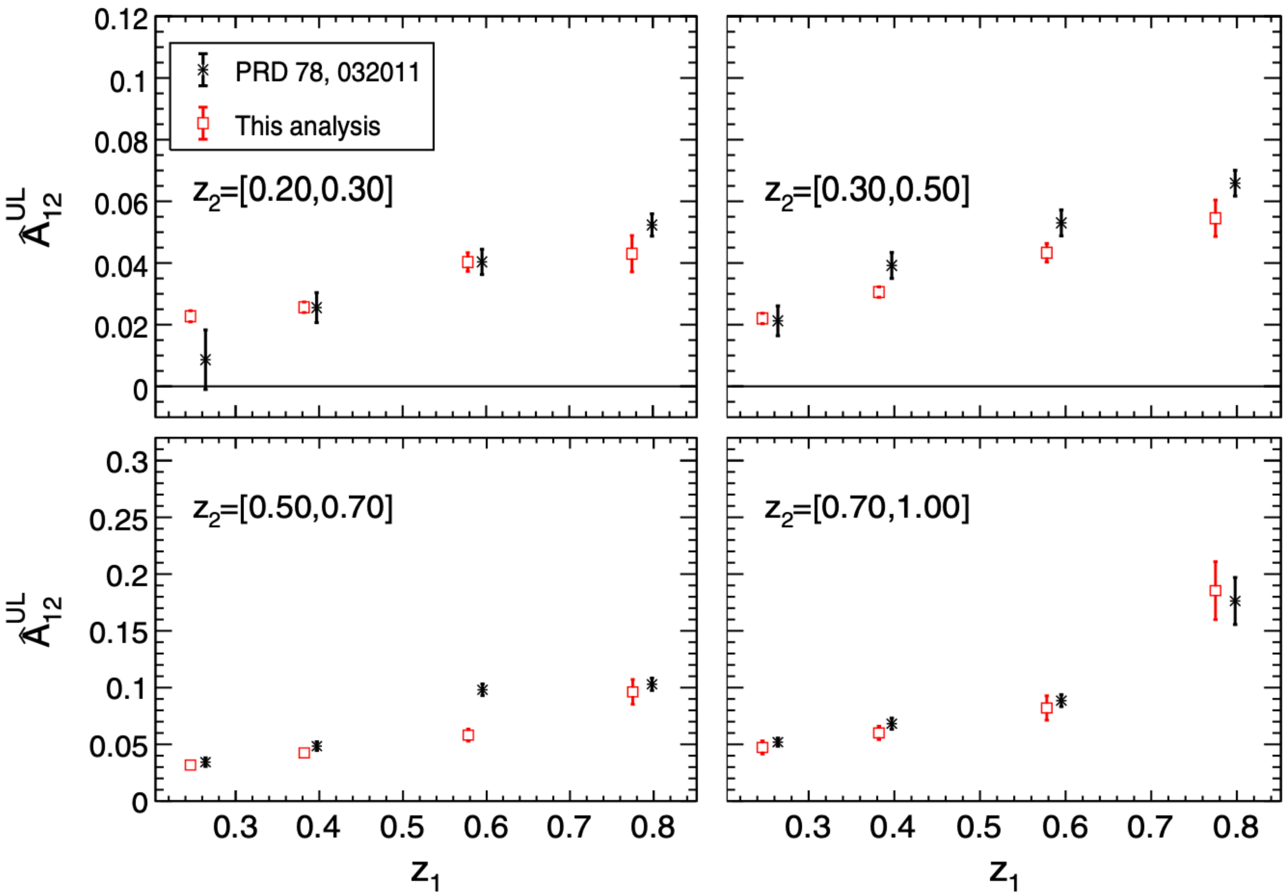

$\vec{n}$
P_T
h
Thrust axis
TMD FF*
e^+
e^-


2-jet limit
$T \to 1$
FRAGMENTATION MECHANISM
NON PERT.
PERT.
FRAGMENTATION MECHANISM
NON PERT.
PERT.


z_h=0.175
z_h=0.225
z_h=0.275
z_h=0.325
z_h=0.375
z_h=0.425
z_h=0.475
z_h=0.525
z_h=0.575
z_h=0.625
z_h=0.675
z_h=0.725
z_h=0.775
10^7
10^6
10^5
10^4
0.2
0.8
1.4
P_T

Azimuthal asymmetries of back-to-back $\pi^\pm$ - (π^0,η, $\pi^\pm$) from Belle

PRD100,092008

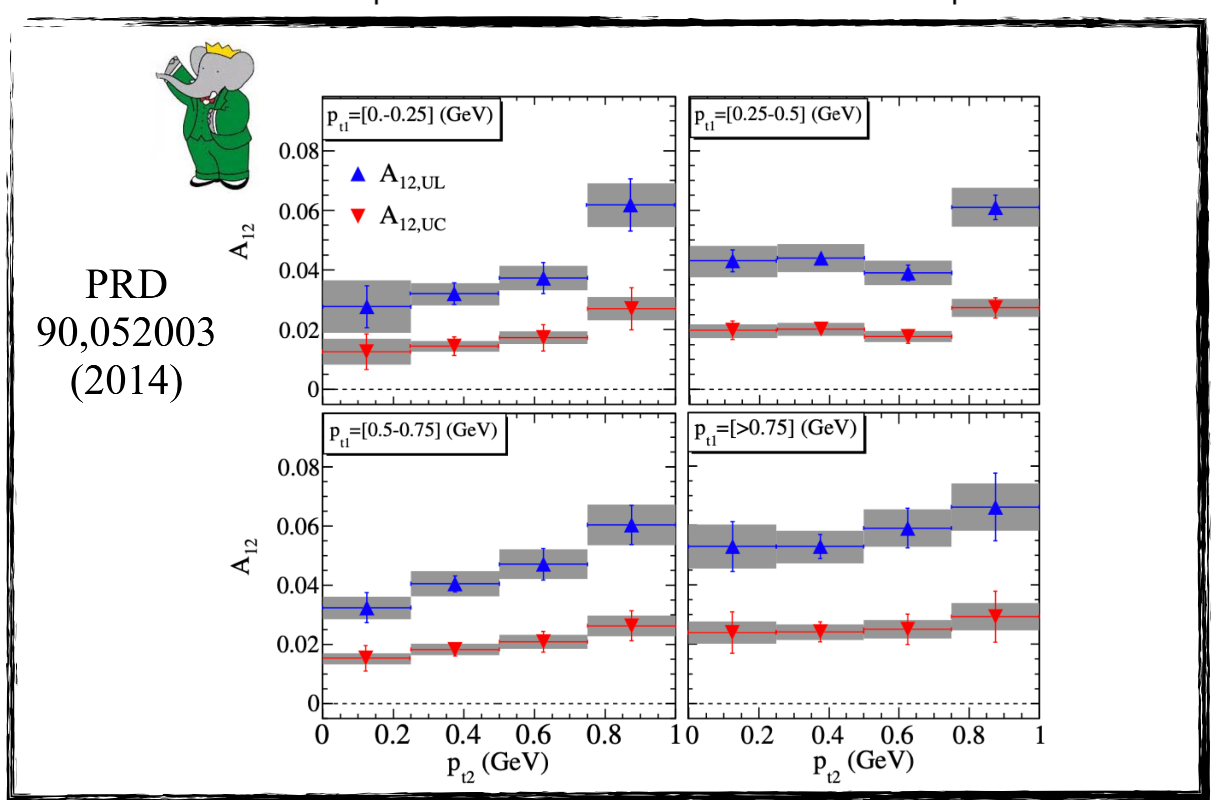


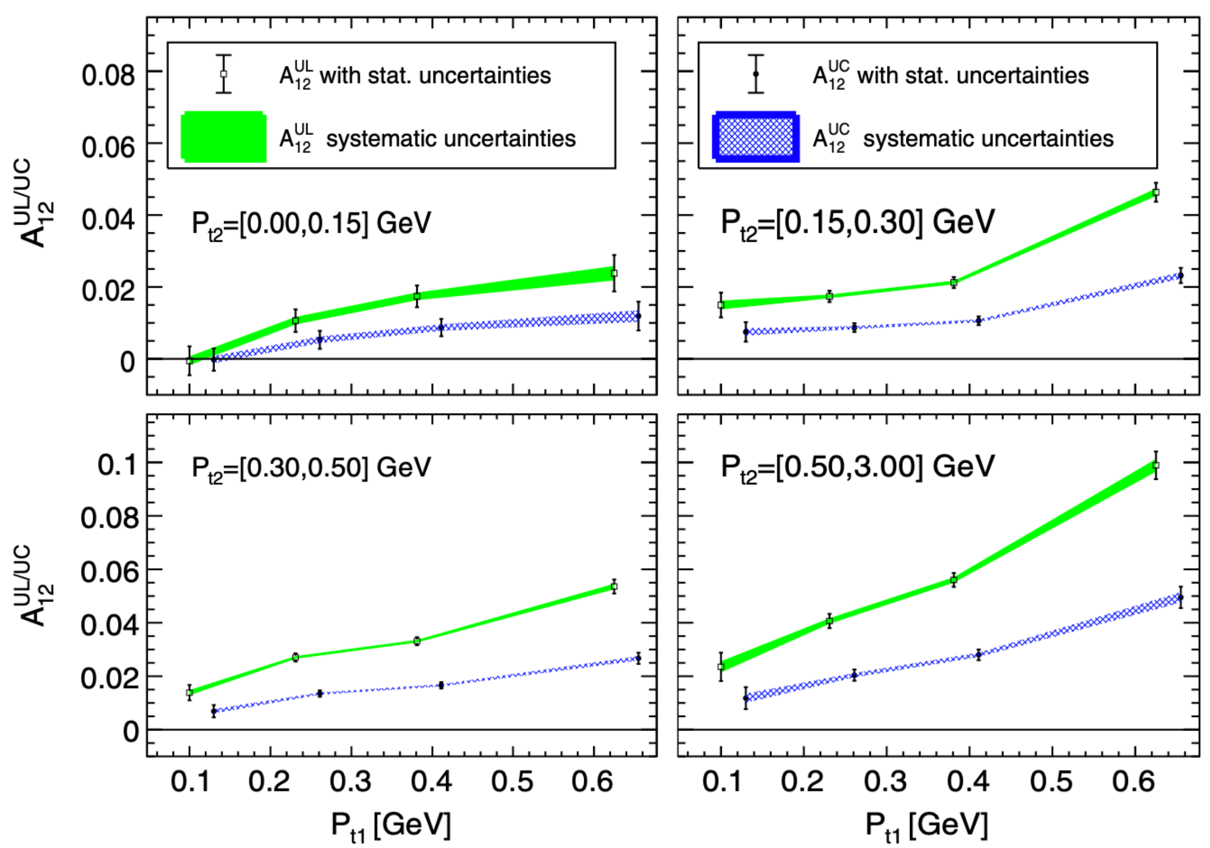
New Belle analysis: RF12 only

- No charm correction: only the estimated charm fractions are provided
- bin-by-bin generated-thrust axis correction (not the qqbar axis)
- Pt smering correction
- MC asymmetry subtraction

Only for results comparison:

- Vanishing charm asymmetry
- No thrust axis correction
- Kinematic factors taken into account





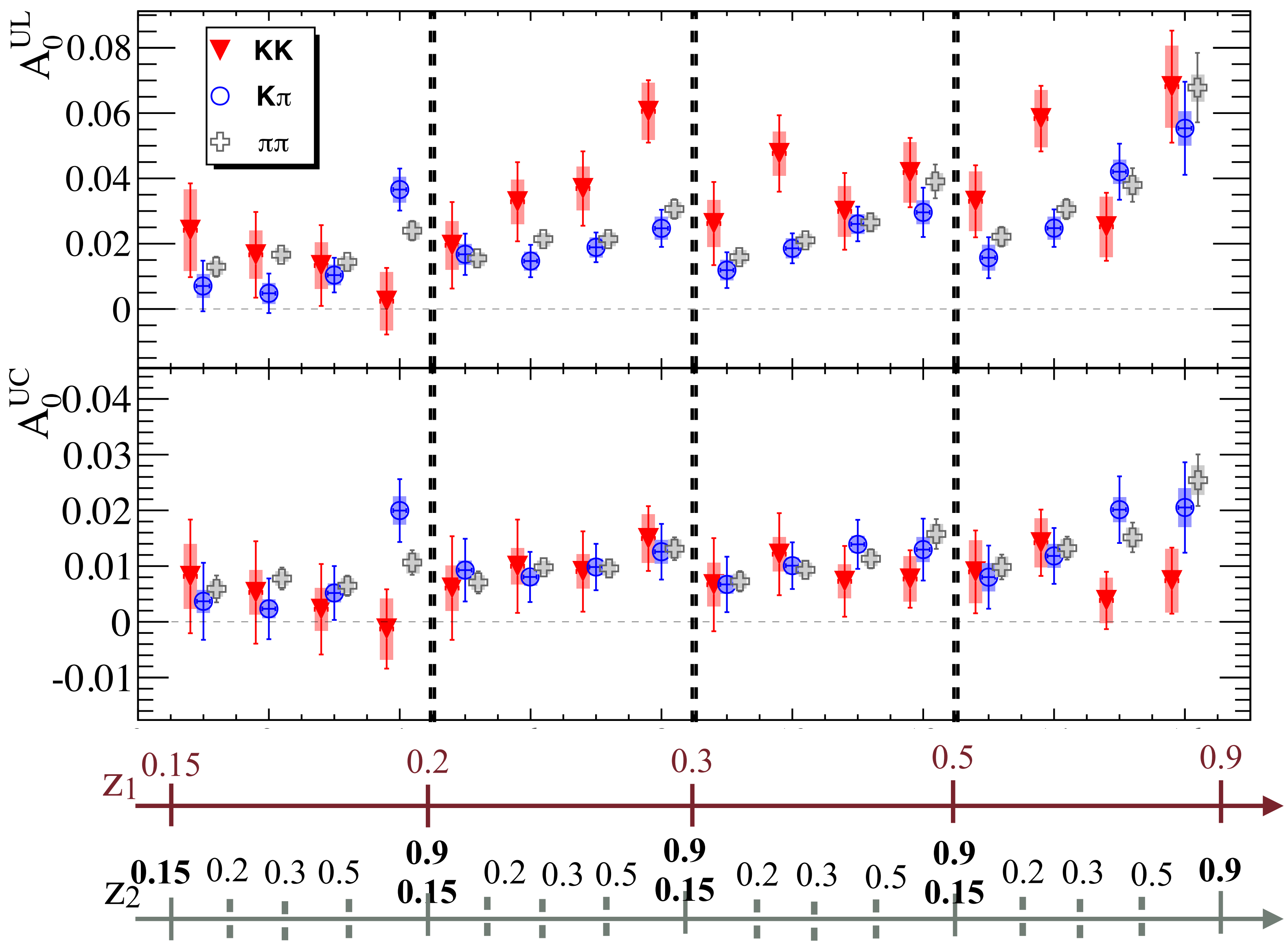
Results: RF0

PRD92,111101(R)

Simultaneous measurement of KK, Kπ and ππ Collins asymmetries

- all corrections are applied

- Rising of the asymmetry as a function of *z*:
  - more pronounced for U/L
- A^{UL} KK asymmetry slightly higher than pion asymmetry for high z
- KK asymmetry consistent with zero at lower z



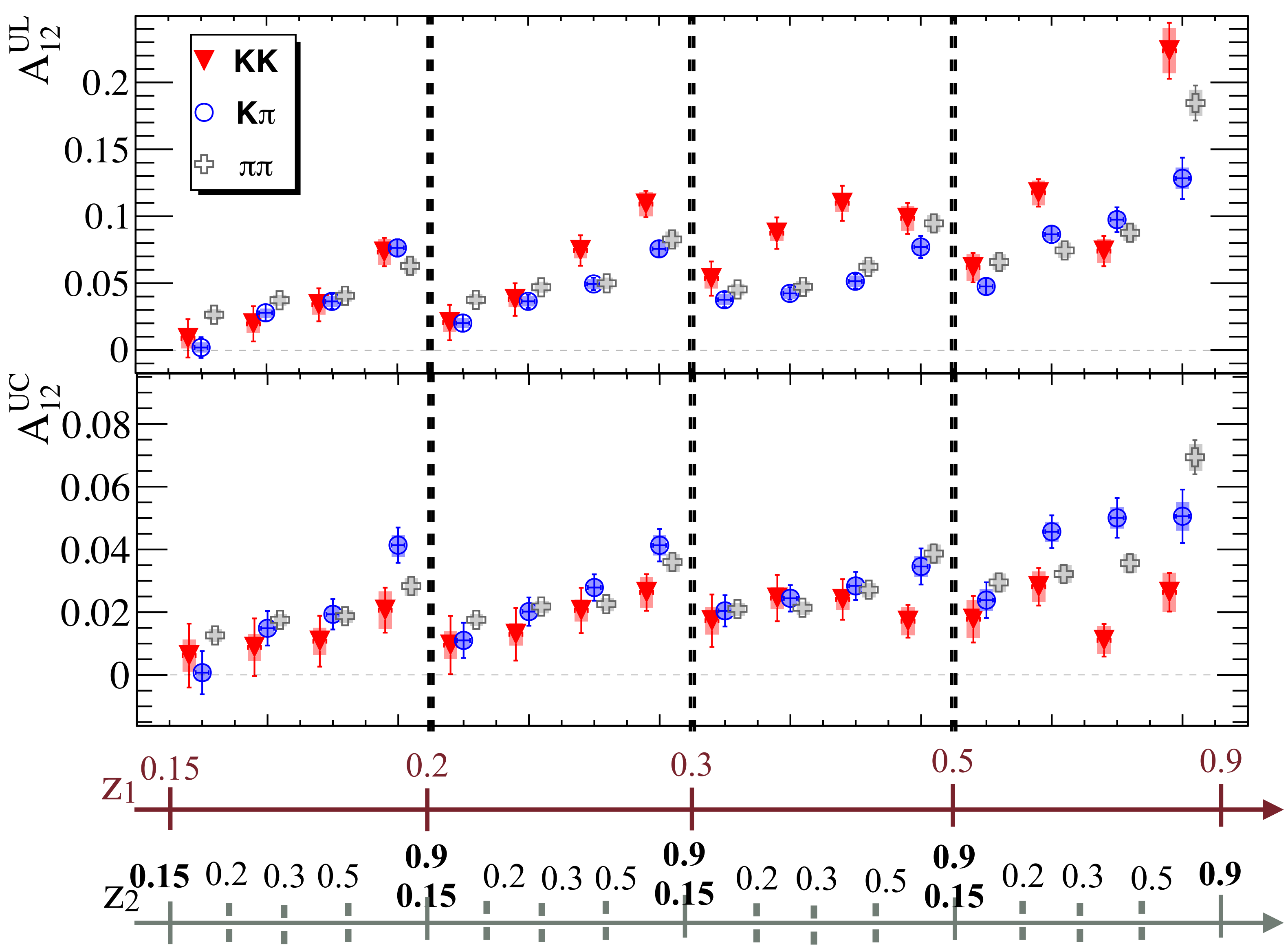
Note that A^{UL} and A^{UC} asymmetries are obtained using the same data sample, and are strongly correlated

Results: RF12

PRD92,111101(R)

Simultaneous measurement of KK, Kπ and ππ Collins asymmetries

- all corrections are applied



- Rising of the asymmetry as a function of *z*:
  - more pronounced for U/L
- A^{UL} KK asymmetry slightly higher than pion asymmetry for high z
- KK asymmetry consistent with zero at lower z

Note that A^{UL} and A^{UC} asymmetries are obtained using the same data sample, and are strongly correlated

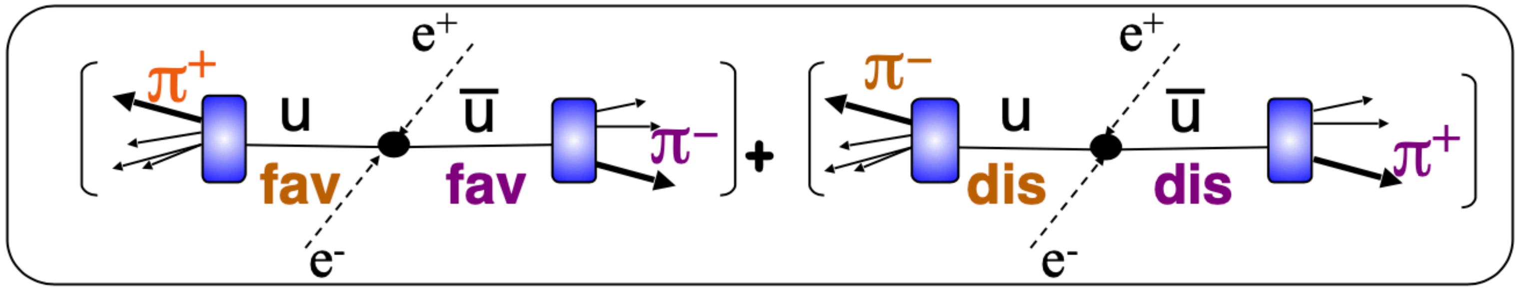
Favored and Disfavored processes

Different combinations of charged pions ⇒ sensitivity to **favored** or **disfavored** FFs

- **favored** process: fragmentation of a quark of flavor q into a hadron with a valence quark of the same flavor: i.e.: $u \rightarrow \pi^+$, $d \rightarrow \pi^-$
- **disfavored** for $d \rightarrow \pi^+$, $u \rightarrow \pi^-$, and $s \rightarrow \pi^\pm$

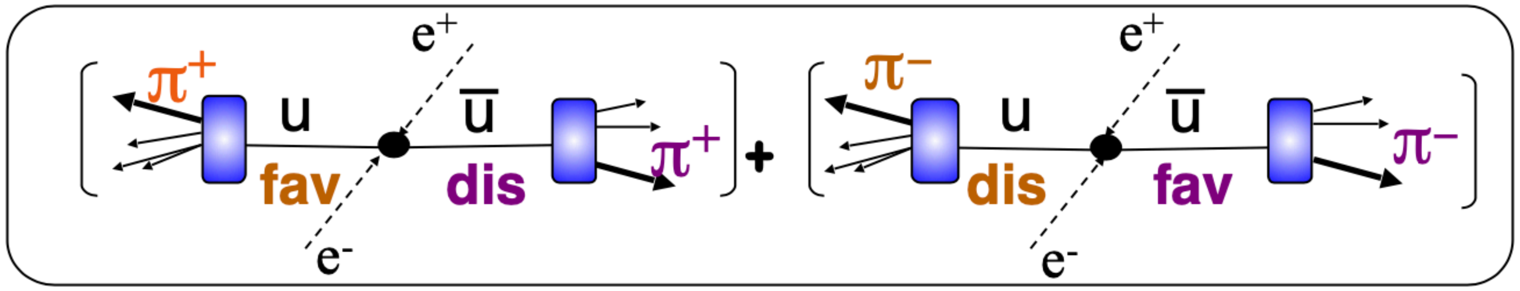
Unlike-sign pion pair = **U:**

$\pi^\mp\pi^\pm$**: (fav x fav)+(dis x dis)**



Like-sign pion pair = **L:**

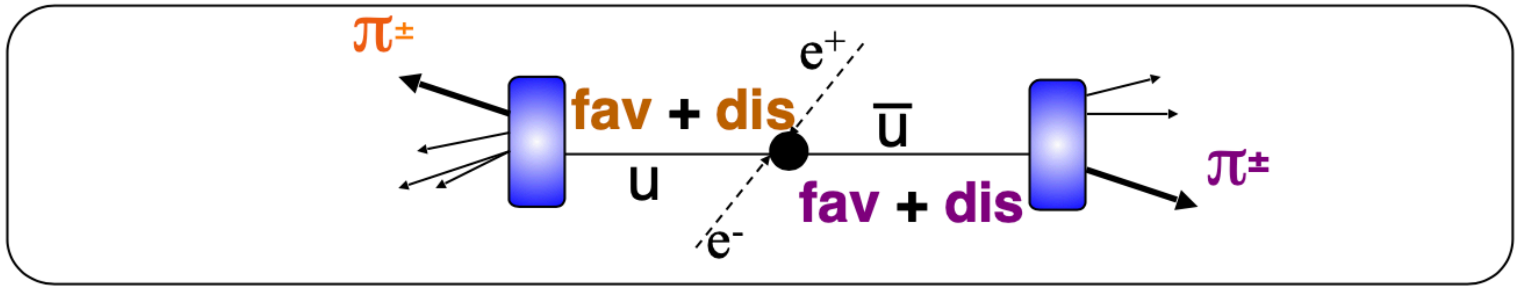
$\pi^\pm\pi^\pm$**: (fav x dis)+(dis x fav)**



Charged pion pair = **C (U+L):**

$\pi\pi$**: (fav + dis)x(fav + dis)**

$\pi = \pi^\pm$



Backgrounds: contributions and corrections

- In each bin, the data sample includes pairs from
  - signal uds events
  - $B\overline{B}$ events (small, mostly at low z)
  - $c\overline{c}$ events (important at low/medium z)
  - $\tau^+\tau^-$ events (important at high z)

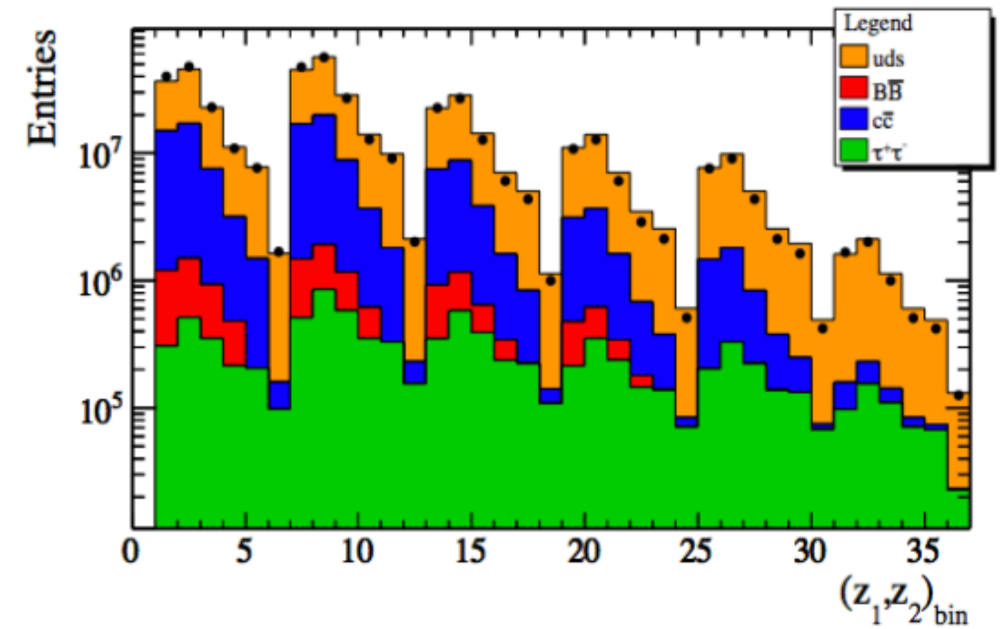
Fraction of ππ due to the i[th] bkg process

True asymmetry

$$A_\alpha^{meas} = (1 - \sum_i F_i) \cdot A_\alpha + \sum_i F_i \cdot A_\alpha^i$$

Bkg asymmetry

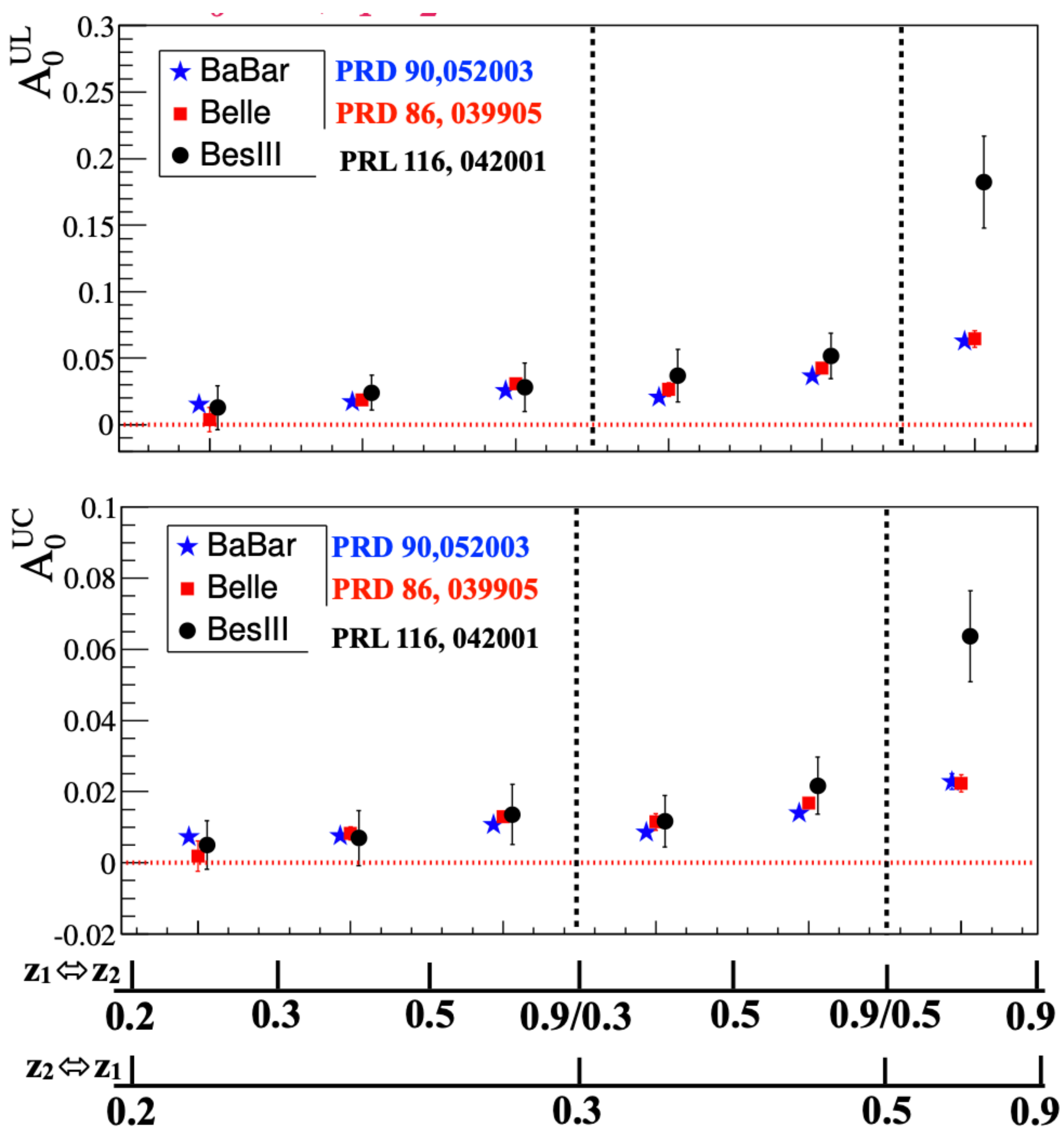
- We must calculate these quantities:
  - F_i using MC sample; we assign MC-data difference in each bin as systematic error
  - $A^{B\overline{B}}$ must be zero; we set $A^{B\overline{B}} = 0$
  - A^τ small in simulation; checked in data; we set $A^\tau = 0$

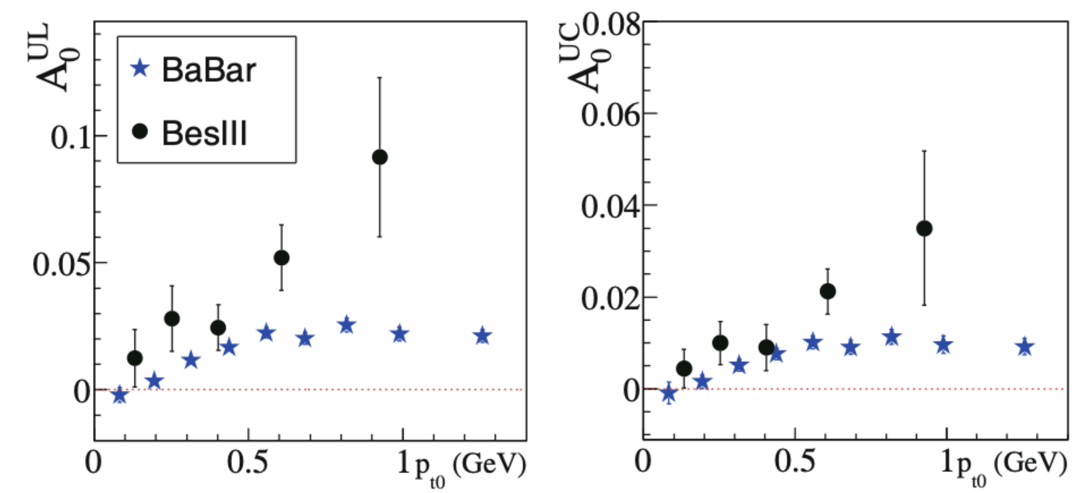


- **Charm** background contribution is about 30% on average
  - Both fragmentation processes and weak decays can introduce azimuthal asymmetries
  - We used a **D*±-enhanced control sample** to estimate its effect

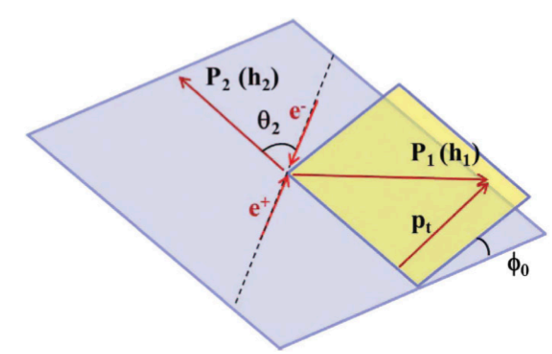
$$\begin{cases} A_\alpha^{meas} = (1 - F_c - F_B - F_\tau) \cdot A_\alpha + F_c \cdot A_\alpha^{ch} \\ A_\alpha^{D^*} = f_c \cdot A_\alpha^{ch} + (1 - f_c - f_B) \cdot A_\alpha \end{cases}$$

Collins FFs Results Comparison

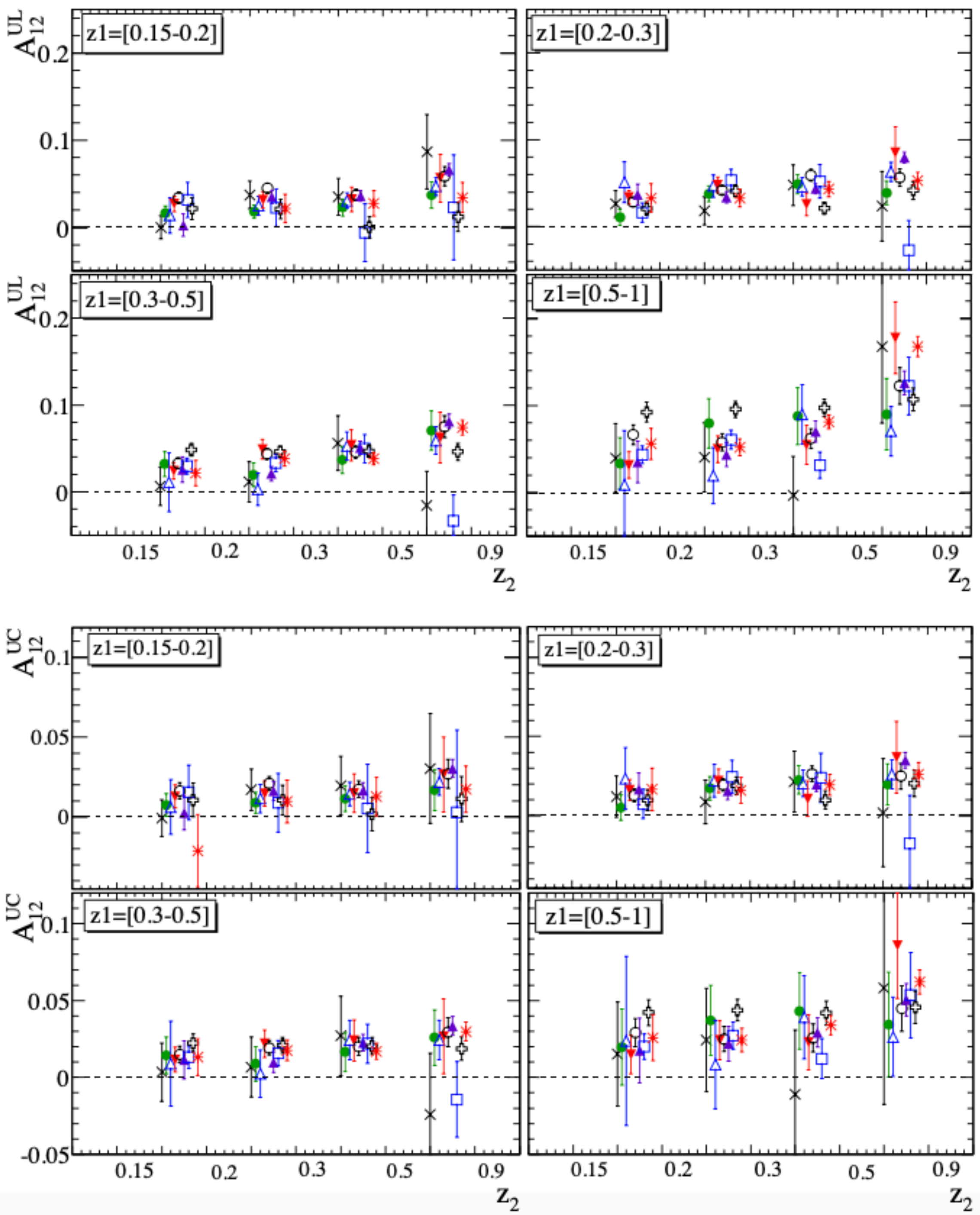




- Larger asymmetry from BESIII data, in agreement with the prediction (PRD93,014009)



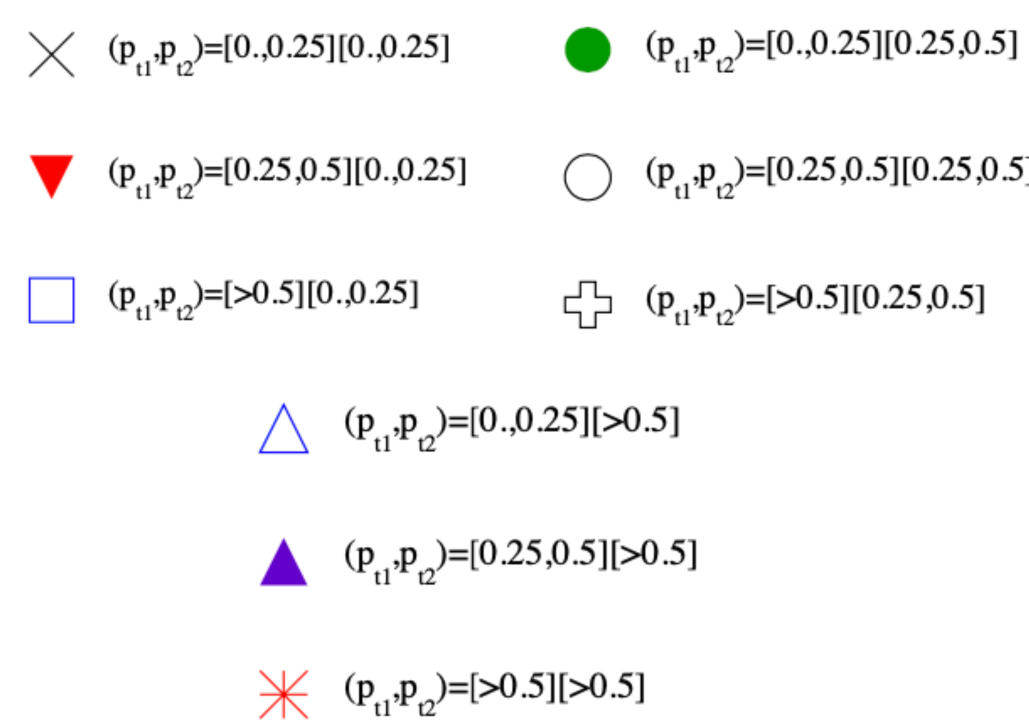
4-D: asymmetry vs. $(z_1, z_2) \times (p_{t1}, p_{t2})$



We study the asymmetries in the RF12 frame in a four-dimensional space:

$(z_1, z_2, p_{t1}, p_{p2})$

- We use 4 z_i and 3 p_t intervals
- Test to probe the factorization of the Collins fragmentation functions
- Powerful tools to access p_t - z correlation



Azimuthal asymmetries of back-to-back $\pi^{\pm}$ - (π^0,η, $\pi^{\pm}$) from Belle

PRD100,092008

$$\mathcal{R}_{12}^{UL} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp,\mathrm{fav}} \otimes H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}} \otimes H_1^{\perp,\mathrm{dis}}) + 2H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{5(D_1^{\mathrm{fav}} \otimes D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}} \otimes D_1^{\mathrm{dis}}) + 2D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} - \frac{10H_1^{\perp,\mathrm{fav}} \otimes H_1^{\perp,\mathrm{dis}} + 2H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{10D_1^{\mathrm{fav}} \otimes D_1^{\mathrm{dis}} + 2D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} \right\},$$

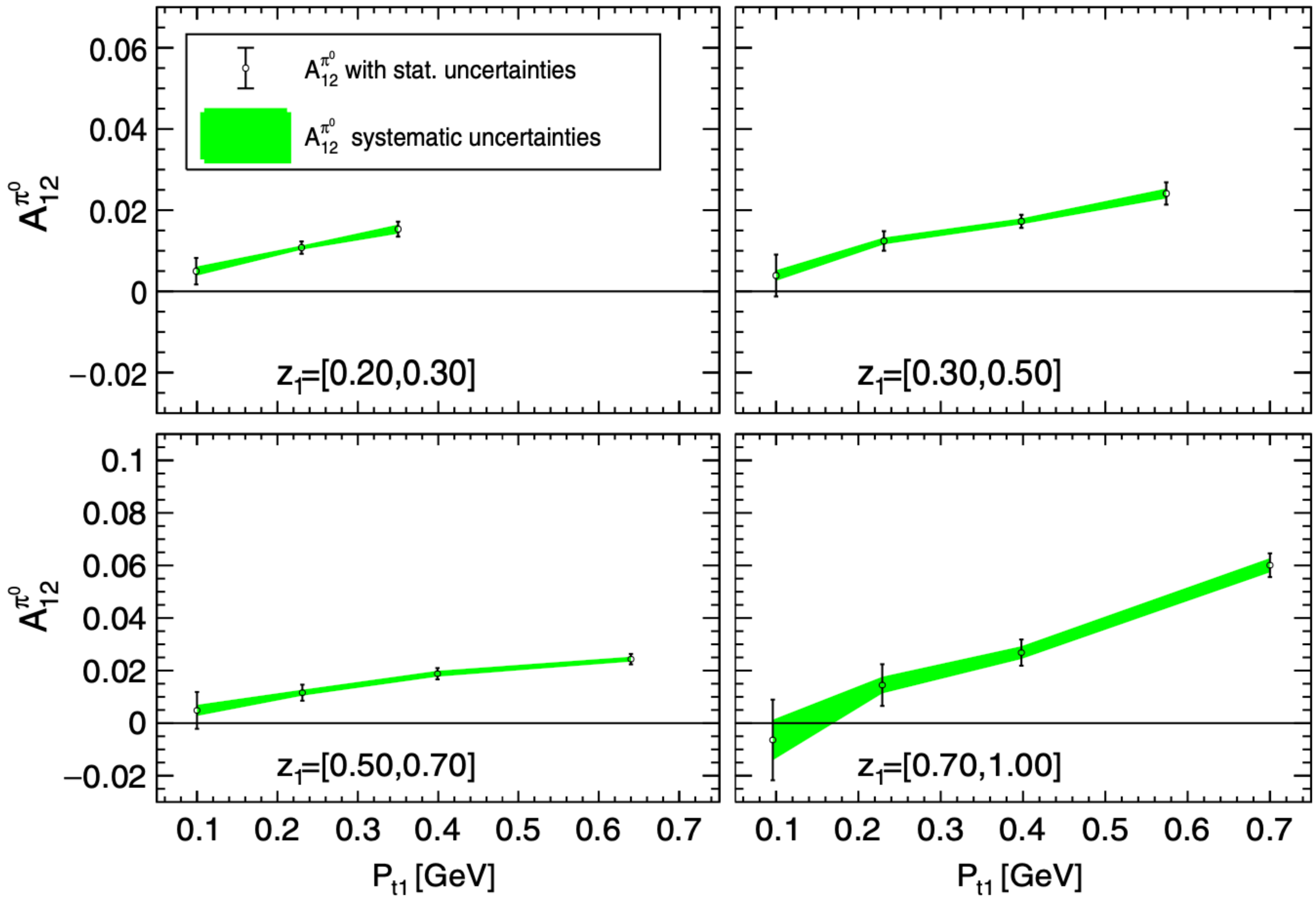
$$\mathcal{R}_{12}^{UC} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp,\mathrm{fav}} \otimes H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}} \otimes H_1^{\perp,\mathrm{dis}}) + 2H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{5(D_1^{\mathrm{fav}} \otimes D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}} \otimes D_1^{\mathrm{dis}}) + 2D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} - \frac{5(H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}}) \otimes (H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}}) + 4H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{5(D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}}) \otimes (D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}}) + 4D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} \right\},$$

$$\mathcal{R}_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}}) \otimes (H_1^{\perp,\mathrm{fav}} + H_1^{\perp,\mathrm{dis}}) + 4H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{5(D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}}) \otimes (D_1^{\mathrm{fav}} + D_1^{\mathrm{dis}}) + 4D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}})} - \frac{10H_1^{\perp,\mathrm{fav}} \otimes H_1^{\perp,\mathrm{dis}} + 2H_{1,s\to\pi}^{\perp,\mathrm{dis}} H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{10D_1^{\mathrm{fav}} \otimes D_1^{\mathrm{dis}} + 2D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} \right\}.$$

$$\mathcal{R}_{12}^{\eta} = \frac{R_{12}^{\eta\pm}}{R_{12}^{L}} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp,\mathrm{fav}_\eta} + H_1^{\perp,\mathrm{dis}_\eta}) \otimes (H_1^{\perp,\mathrm{dis}} + H_1^{\perp,\mathrm{fav}}) + 4H_{1,s\to\eta}^{\perp} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{5(D_1^{\perp,\mathrm{fav}_\eta} + D_1^{\perp,\mathrm{dis}_\eta}) \otimes (D_1^{\perp,\mathrm{dis}} + D_1^{\perp,\mathrm{fav}}) + 4D_{1,s\to\eta} \otimes D_{1,s\to\pi}^{\mathrm{dis}})} - \frac{10H_1^{\perp,\mathrm{fav}} \otimes H_1^{\perp,\mathrm{dis}} + 2H_{1,s\to\pi}^{\perp,\mathrm{dis}} \otimes H_{1,s\to\pi}^{\perp,\mathrm{dis}}}{10D_1^{\perp,\mathrm{fav}} \otimes D_1^{\perp,\mathrm{dis}} + 2D_{1,s\to\pi}^{\mathrm{dis}} \otimes D_{1,s\to\pi}^{\mathrm{dis}}} \right\}.$$

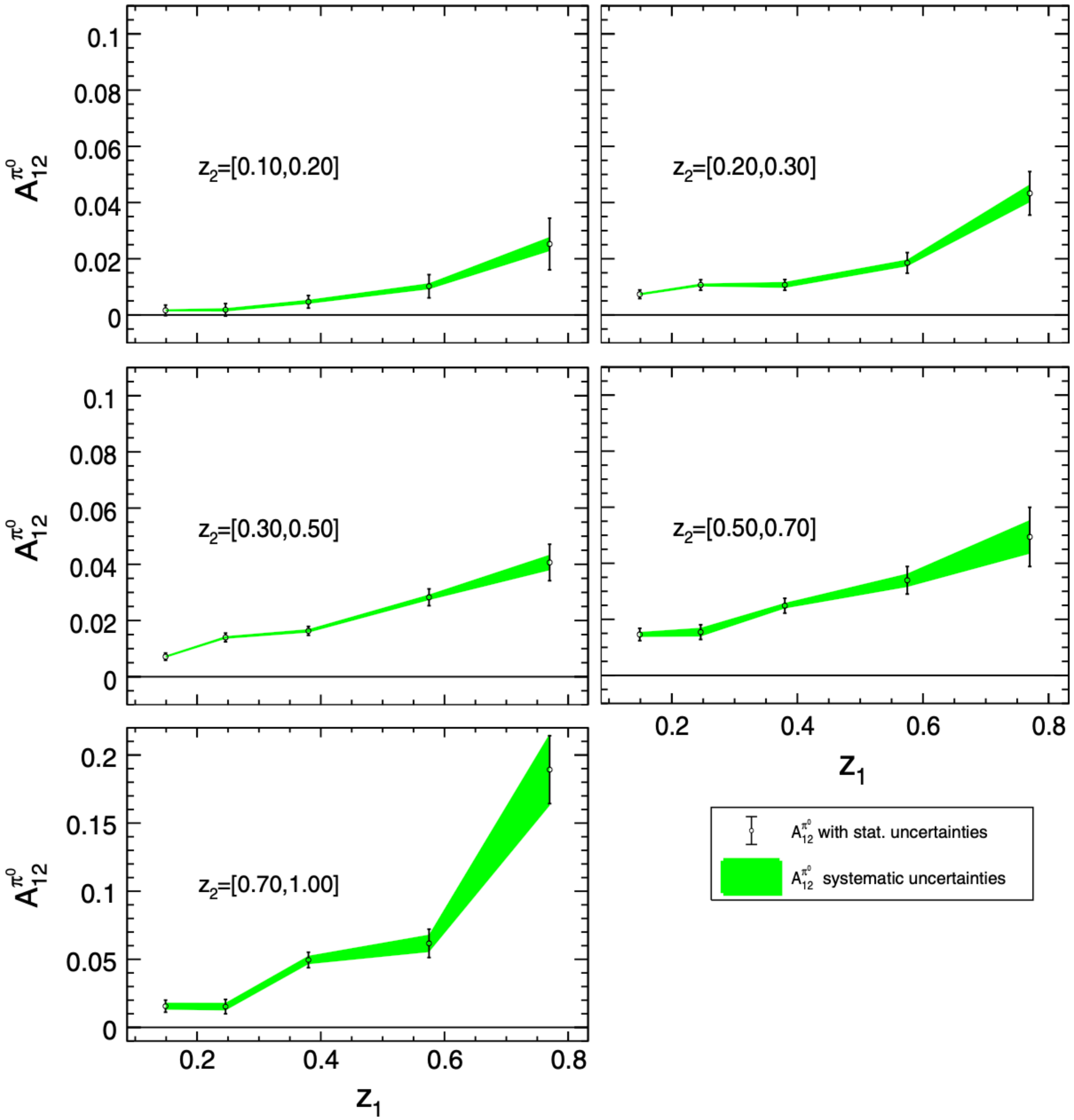
Azimuthal asymmetries of back-to-back $\pi^{\pm}$ - (π^0,η, $\pi^{\pm}$) from Belle

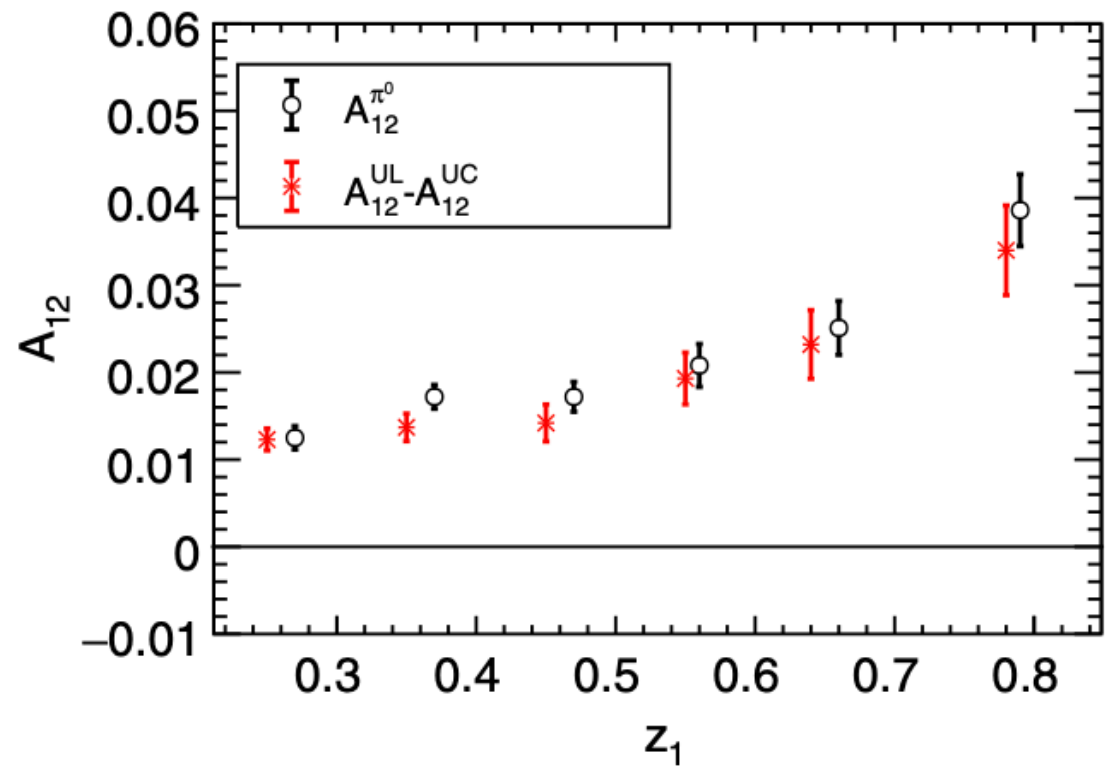
PRD100,092008



Azimuthal asymmetries of back-to-back $\pi^{\pm}\pi^{0}$ from Belle

PRD100,092008

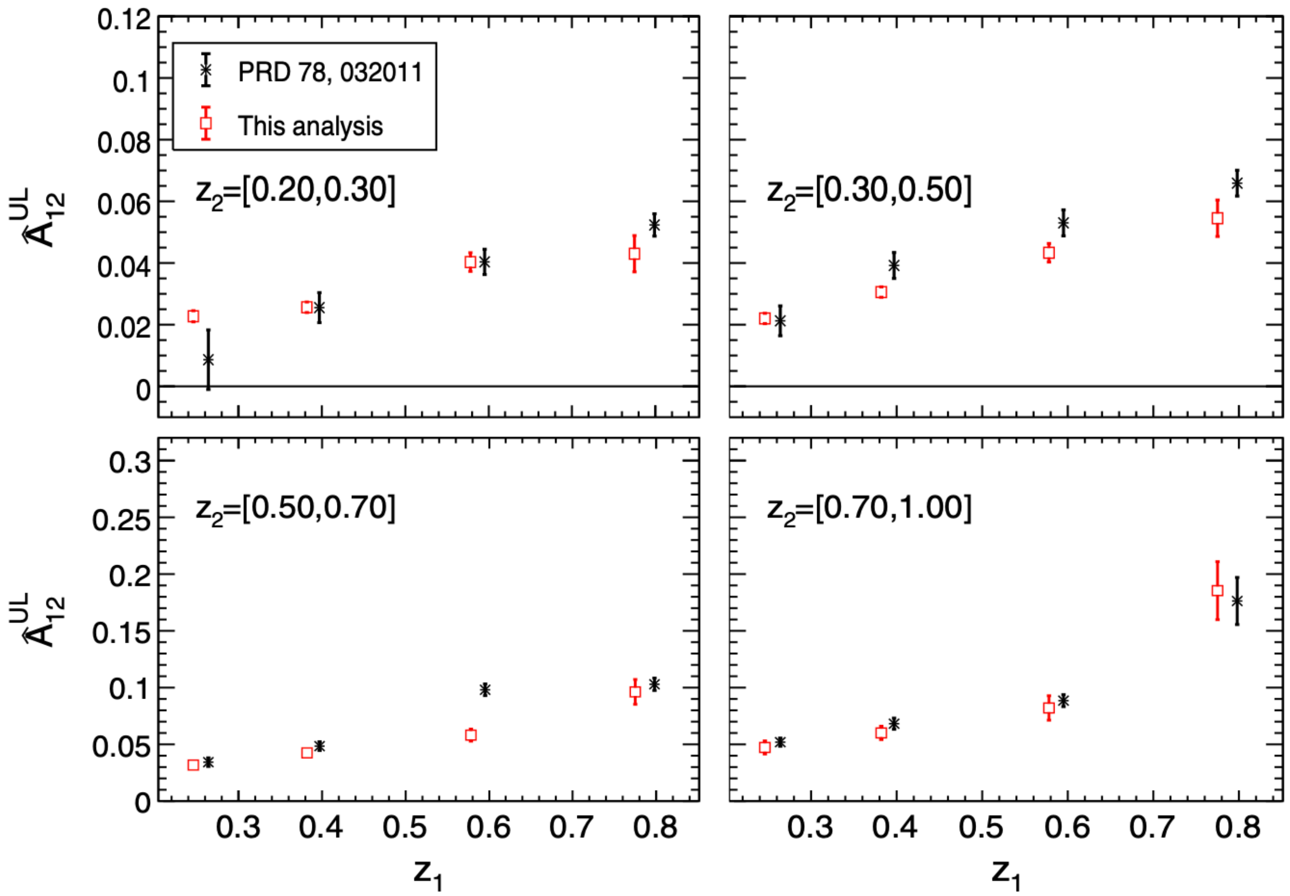




- Good agreement between A^{UL}-A^{UC} and $A^{\pi 0}$ as expected from isospin relation (taking into account statistical and systematic uncertainties)

Azimuthal asymmetries of back-to-back $\pi^\pm$ - (π^0,η, $\pi^\pm$) from Belle

PRD100,092008

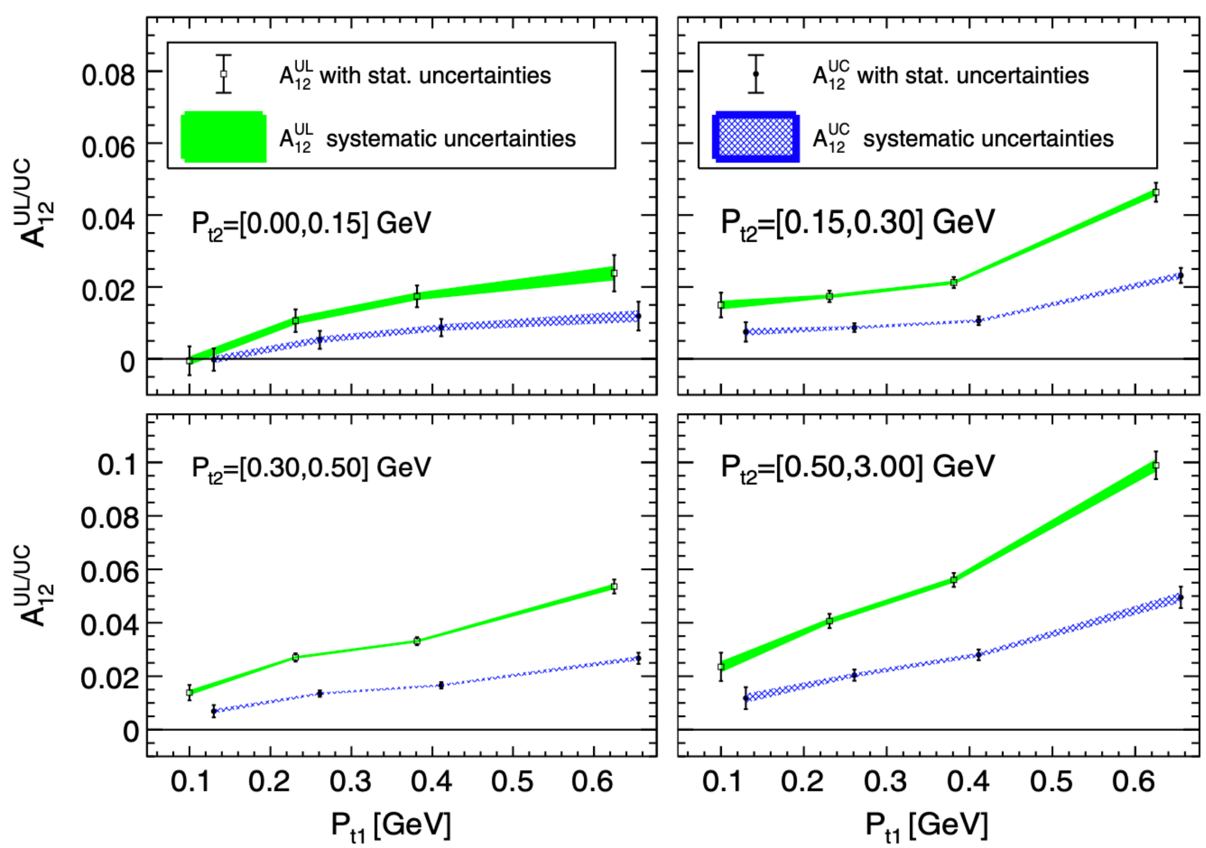


New Belle analysis: RF12 only

- No charm correction: only the estimated charm fractions are provided
- bin-by-bin generated-thrust axis correction (not the qqbar axis)
- Pt smering correction
- MC asymmetry subtraction

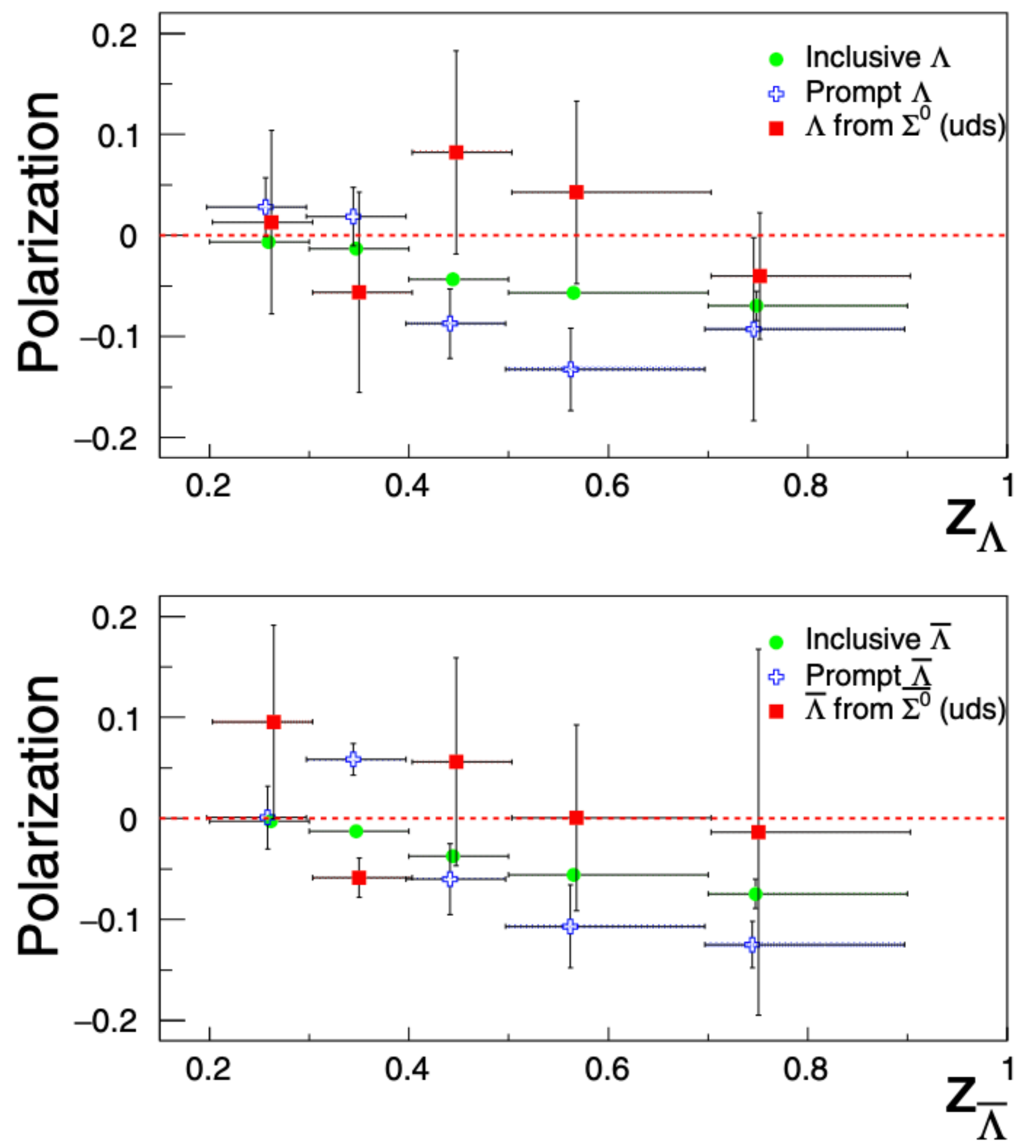
Only for results comparison:

- Vanishing charm asymmetry
- No thrust axis correction
- Kinematic factors taken into account



$\Lambda/\bar{\Lambda}$ hyperon polarization at Belle

PRL122,042001



- Charm corrected unfolded transverse polarisation also obtained ==> large statistical uncertainties

$\bar{\Lambda}/\Lambda$ hyperon polarization

PRD **100**,014029

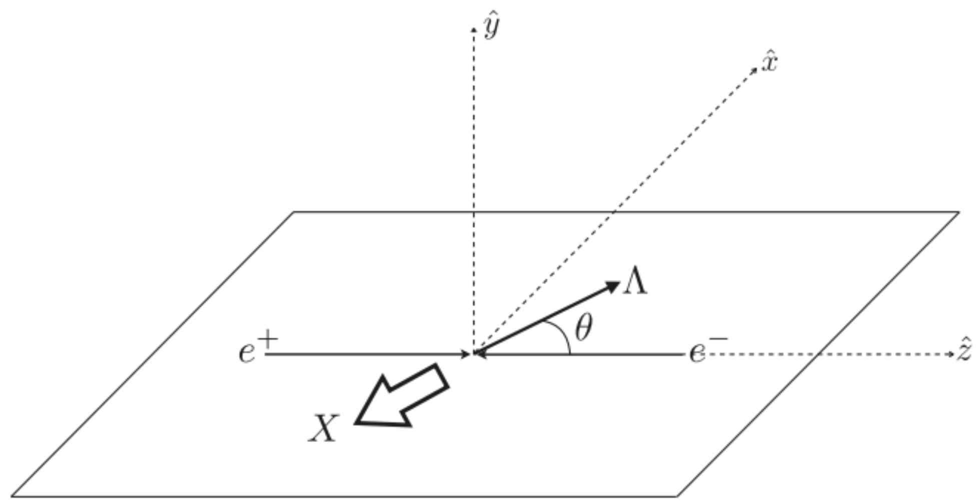
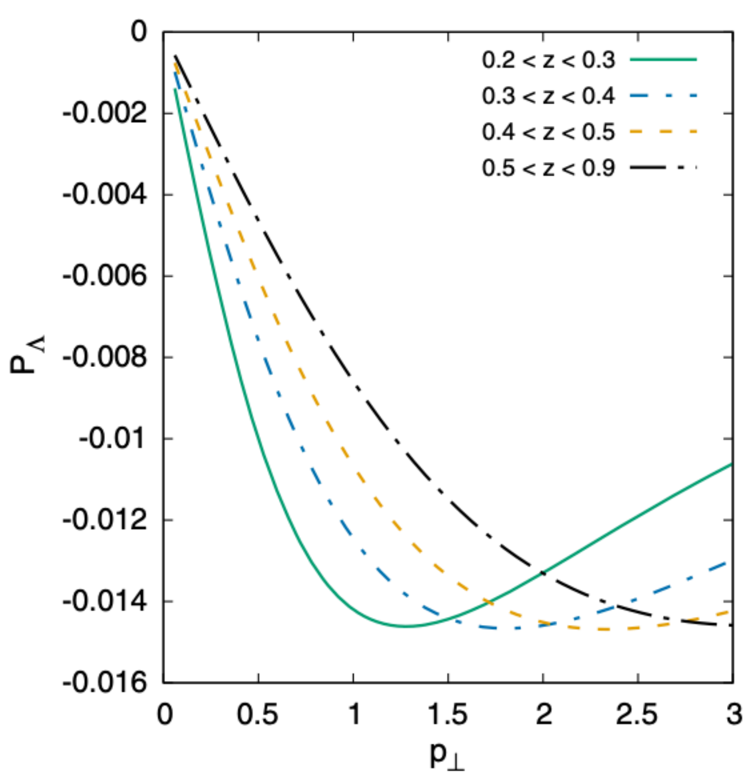
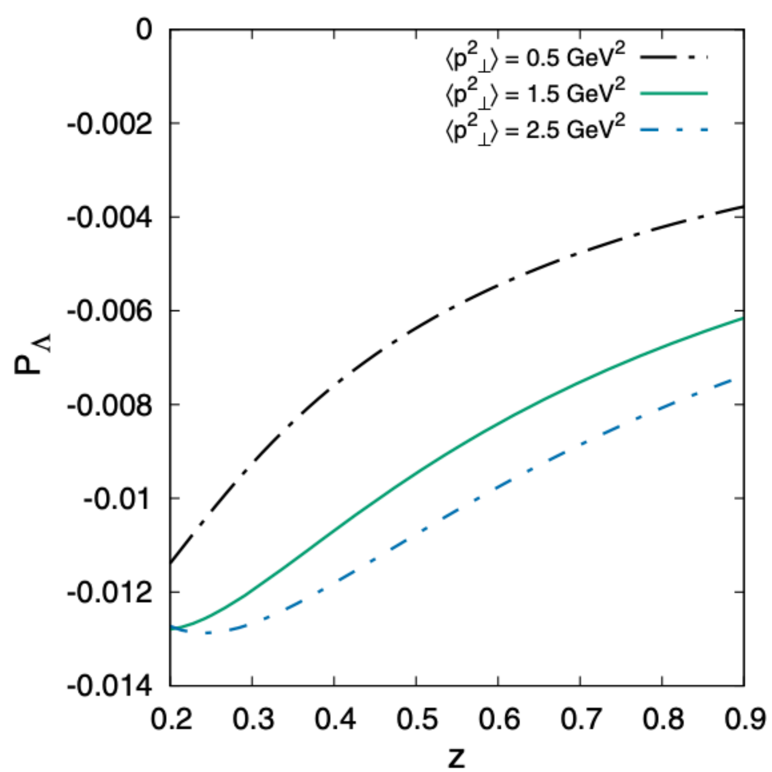


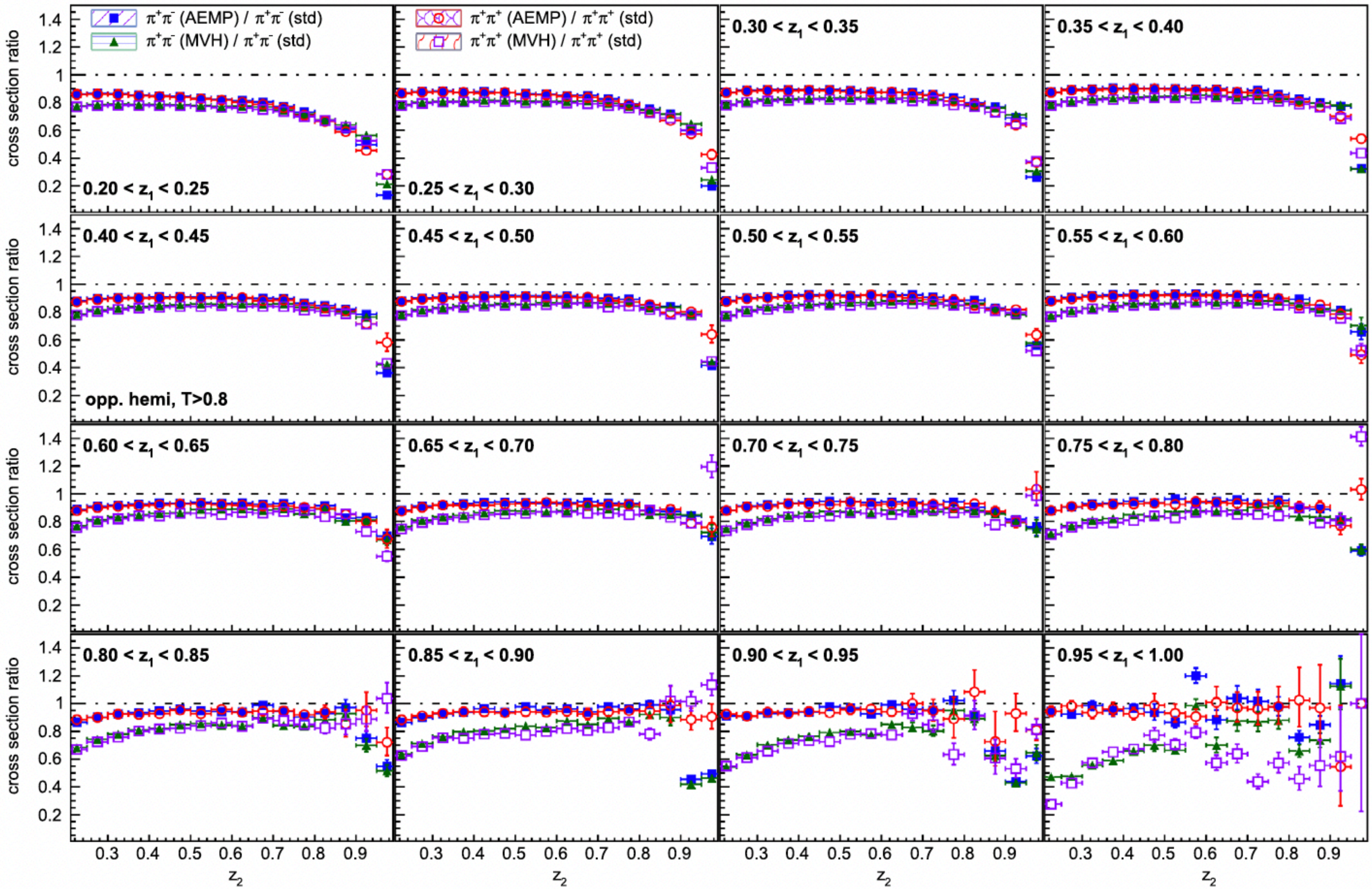
FIG. 1. Definition of the c.m. reference frame for the process $e^+e^- \to \Lambda X$.

$$P_\Lambda(z, \cos\theta) = \frac{\sum_q e_q^2 \int p_\perp dp_\perp \frac{4zp_\perp\sqrt{s}}{z^2s+4p_\perp^2} \Delta^N D_{\Lambda^\uparrow/q}(z, p_\perp)}{\sum_q e_q^2 D_{\Lambda/q}(z)} \times \frac{\sin(2\theta)}{1+\cos^2\theta} \tag{21}$$

$$P_\Lambda(z, p_\perp) = \frac{\sum_q e_q^2 \frac{4zp_\perp\sqrt{s}}{z^2s+4p_\perp^2} \Delta^N D_{\Lambda^\uparrow/q}(z, p_\perp)}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_\perp)} \times \frac{\int d(\cos\theta)\sin(2\theta)}{\int d(\cos\theta)(1+\cos^2\theta)} \tag{22}$$






π⁺π⁻ (AEMP) / π⁺π⁻ (std)
π⁺π⁻ (MVH) / π⁺π⁻ (std)
π⁺π⁺ (AEMP) / π⁺π⁺ (std)
π⁺π⁺ (MVH) / π⁺π⁺ (std)
cross section ratio
0.20 < z1 < 0.25
0.25 < z1 < 0.30
0.30 < z1 < 0.35
0.35 < z1 < 0.40
0.40 < z1 < 0.45
0.45 < z1 < 0.50
0.50 < z1 < 0.55
0.55 < z1 < 0.60
opp. hemi, T>0.8
0.60 < z1 < 0.65
0.65 < z1 < 0.70
0.70 < z1 < 0.75
0.75 < z1 < 0.80
0.80 < z1 < 0.85
0.85 < z1 < 0.90
0.90 < z1 < 0.95
0.95 < z1 < 1.00
z2